Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.2-d-x-^m-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [858]. This is test number [26].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (858)	0.00 (0)
Mathematica	100.00 (858)	0.00 (0)
Maple	99.88 (857)	0.12 (1)
Fricas	98.60 (846)	1.40 (12)
Giac	93.12 (799)	6.88 (59)
Maxima	80.19 (688)	19.81 (170)
Mupad	79.84 (685)	20.16 (173)
IntegrateAlgebraic	59.56 (511)	40.44 (347)
Sympy	53.96 (463)	% 46.04 (395)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

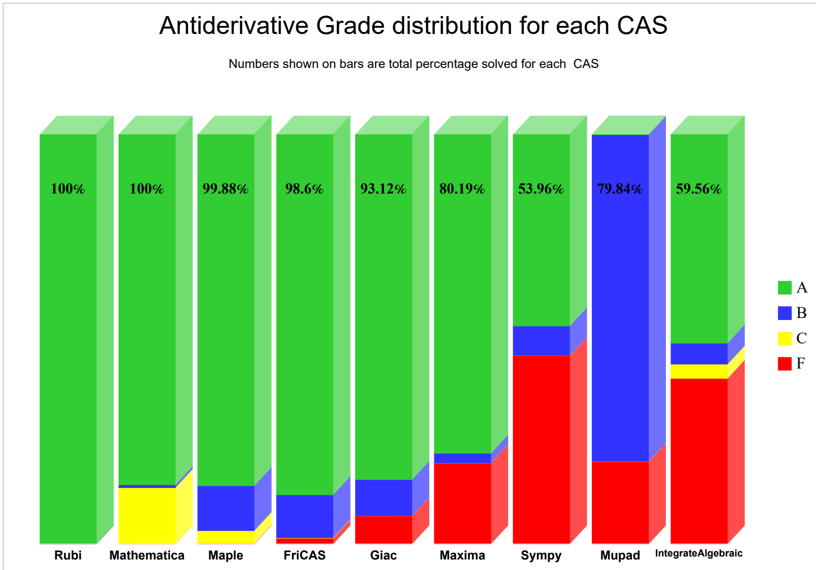
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

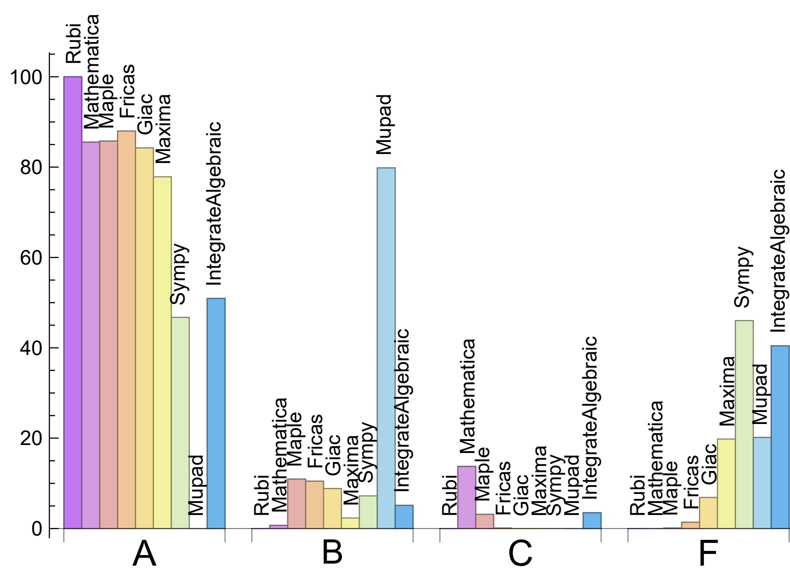
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Fricas	88.00	10.49	0.12	1.40
Maple	85.78	10.96	3.15	0.12
Mathematica	85.55	0.70	13.75	0.00
Giac	84.27	8.86	0.00	6.88
Maxima	77.86	2.33	0.00	19.81
IntegrateAlgebraic	50.93	5.13	3.50	40.44
Sympy	46.74	7.23	0.00	46.04
Mupad	N/A	79.84	0.00	20.16

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Fricas	12	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	347	100.00 %	0.00 %	0.00 %
Giac	59	44.07 %	1.69 %	54.24 %
Maxima	170	62.35 %	0.00 %	37.65 %
Sympy	395	72.66 %	27.34 %	0.00 %
Mupad	173	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

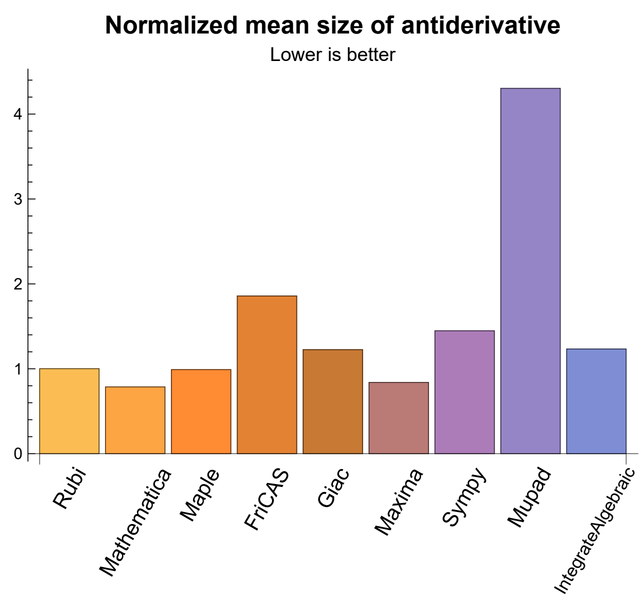
1.3 Performance

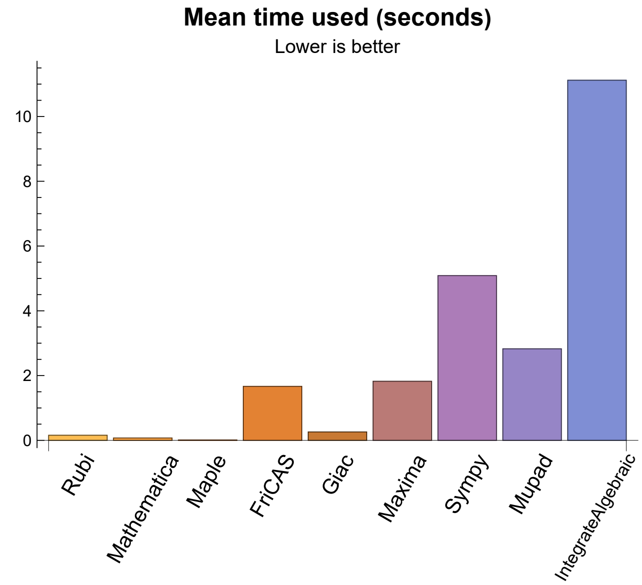
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	142.67	1.00	84.00	1.00
Mathematica	0.07	81.83	0.79	60.00	0.86
Maple	0.01	150.43	0.99	67.00	0.86
Maxima	1.82	90.16	0.84	55.00	0.88
Fricas	1.67	342.72	1.86	79.50	0.99
Sympy	5.09	110.85	1.45	53.00	0.98
Giac	0.26	182.51	1.23	69.00	0.87
Mupad	2.83	1480.55	4.30	67.00	0.88
IntegrateAlgebraic	11.12	182.00	1.23	91.00	0.82

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {14, 721}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
           leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

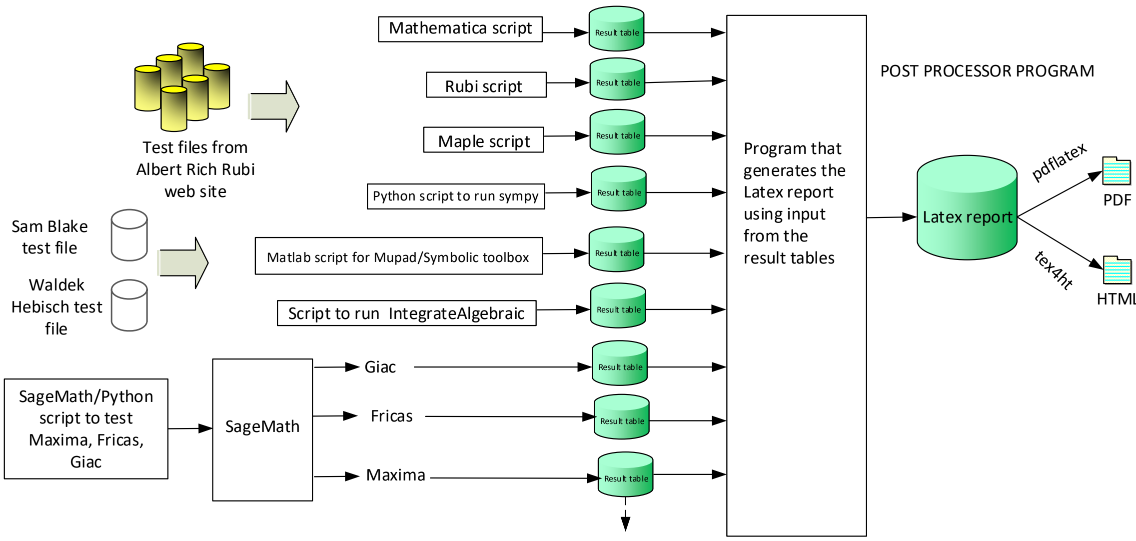
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form “{n,n,..}” which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 209, 211, 213, 215, 221, 223, 225, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 510, 512, 514, 518, 520, 522, 524, 526, 528, 532, 534, 536, 538, 540, 542, 544, 546, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 579, 581, 583, 585, 587, 591, 593, 595, 597, 599, 601, 603, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 718, 719, 720, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 856, 857, 858 }

B grade: { 11, 53, 266, 277, 295, 343 }

C grade: { 8, 10, 13, 14, 15, 118, 119, 124, 125, 137, 139, 140, 141, 154, 165, 166, 167, 204, 205, 206, 207, 208, 210, 212, 214, 216, 217, 218, 219, 220, 222, 224, 226, 228, 229, 230, 231, 232, 233, 509, 511, 513, 515, 516, 517, 519, 521, 523, 525, 527, 529, 530, 531, 533, 535, 537, 539, 541, 543, 545, 547, 548, 549, 576, 577, 578, 580, 582, 584, 586, 588, 589, 590, 592, 594, 596, 598, 600, 602, 604, 605, 606, 712, 713, 714, 715, 716, 717, 721, 722, 776, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 4, 5, 6, 7, 9, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 588, 590, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 660, 661, 662, 664, 665, 666, 667, 673, 680, 681, 691, 692, 693, 694, 695, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 709, 710, 711, 712, 713, 718, 719, 720, 721, 723, 724, 725, 726, 727, 728, 737, 738, 739, 740, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 858 }

B grade: { 8, 10, 11, 15, 43, 53, 125, 152, 169, 234, 254, 266, 277, 279, 295, 324, 343, 419, 426, 579, 580, 581, 582, 583, 584, 585, 586, 587, 589, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 658, 659, 663, 668, 669, 670, 671, 672, 674, 675, 676, 677, 678, 679, 682, 683, 684, 685, 686, 687, 688, 689, 690, 696, 705, 708, 714, 715, 716, 717, 722, 729, 730, 731, 732, 733, 734, 735, 736, 741, 742, 743, 760, 761, 772, 856, 857 }

C grade: { 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

F grade: { 3 }

2.1.4 Maxima

A grade: { 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 601, 602, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 700, 701, 702, 703, 704, 709, 710, 711, 712, 713, 718, 719, 720, 752, 753, 754, 755, 756, 757, 758, 759, 768, 769, 770, 771, 773, 775, 777, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 856, 857, 858 }

B grade: { 11, 43, 53, 92, 126, 127, 254, 266, 277, 279, 295, 324, 326, 343, 344, 346, 347, 426, 475, 778 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 116, 117, 118, 119, 136, 137, 138, 139, 140, 141, 152,

153, 154, 165, 166, 167, 579, 580, 581, 582, 587, 588, 589, 590, 591, 592, 593, 594, 603, 604, 605, 606, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 714, 715, 716, 717, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 760, 761, 762, 763, 764, 765, 766, 767, 772, 774, 776, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855
}

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 345, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 691, 692, 693, 694, 695, 700, 701, 702, 703, 704, 709, 710, 711, 712, 713, 714, 715, 718, 719, 720, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 858 }

B grade: { 8, 10, 11, 14, 43, 53, 92, 234, 254, 266, 277, 279, 295, 324, 326, 327, 328, 343, 344, 346, 347, 348, 349, 350, 426, 475, 607, 608, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 696, 697, 698, 699, 705, 706, 707, 708, 716, 717, 721, 765, 829, 830, 831, 832, 833, 834, 835, 836, 837, 839, 840, 841, 842, 843, 844, 856, 857 }

C grade: { 169 }

F grade: { 838, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

2.1.6 Sympy

A grade: { 8, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 61, 63, 65, 67, 69, 71, 72, 73, 74, 75, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202, 203, 204, 205, 206, 207, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 323, 325, 327, 328, 329, 330, 331, 332, 333, 336, 337, 338, 339, 340, 341, 342, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 513, 527, 545, 552, 574, 607, 608, 609, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 658, 659, 660, 661, 662, 663, 671, 672, 673, 696, 697, 698, 699, 702, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 722, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 856, 857, 858 }

B grade: { 9, 11, 15, 30, 43, 53, 60, 62, 64, 66, 68, 70, 78, 80, 92, 93, 254, 266, 277, 279, 295, 297, 316, 317, 324, 326, 334, 335, 343, 344, 346, 347, 651, 652, 653, 654, 655, 656, 664, 665, 666, 667, 677, 678, 679, 680, 681, 686, 691, 692, 693, 694, 695, 700, 701, 703, 704, 709, 710, 711, 712, 713 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 198, 199, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 508, 509, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 610, 611, 612, 613, 614, 615, 616, 657, 668, 669, 670, 674, 675, 676, 682, 683, 684, 685, 687, 688, 689, 690, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 793, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

2.1.7 Giac

A grade: { 1, 3, 4, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 155, 156, 158, 159, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 473, 474, 475, 476, 477, 478, 479, 480, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 612, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 664, 665, 666, 667, 668, 669, 676, 677, 678, 679, 680, 681, 682, 683, 691, 692, 693, 694, 695, 700, 701, 702, 703, 704, 709, 710, 711, 712, 713, 714, 715, 716, 718, 719, 720, 721, 722, 723, 724, 725, 726, 728, 744, 745, 746, 747, 748, 749, 752, 753, 754, 755, 756, 757, 760, 761, 762, 763, 764, 765, 766, 767, 769, 770, 771, 772, 773, 775, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828 }

B grade: { 11, 43, 53, 108, 109, 110, 126, 127, 128, 129, 130, 234, 235, 254, 266, 277, 279, 295, 343, 395, 419, 426, 607, 608, 609, 610, 611, 613, 658, 659, 660, 661, 662, 663, 670, 671, 672, 673, 674, 675, 684, 685, 686, 687, 688, 689, 690, 696, 697, 698, 699, 705, 706, 707, 708, 717, 729, 730, 731, 732, 733, 734, 735, 736, 739, 740, 741, 742, 743, 750, 751, 758, 759, 856, 857, 858 }

C grade: { }

F grade: { 2, 5, 6, 7, 10, 150, 153, 154, 157, 160, 161, 162, 165, 166, 167, 468, 469, 470, 471, 472, 481, 482, 483, 484, 485, 486, 727, 737, 738, 768, 774, 776, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 120, 121, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 144, 145, 146, 147, 148, 149, 150, 151, 153, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 382, 383, 384, 385, 389, 390, 395, 396, 397, 398, 399, 408, 409, 410, 411, 412, 419, 426, 427, 428, 429, 430, 431, 432, 445, 446, 447, 448, 449, 450, 451, 453, 454, 455, 456, 464, 465, 475, 476, 477, 478, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 553, 554, 555, 556, 560, 561, 562, 563, 567, 568, 569, 570, 607, 608, 609, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 735, 736, 746, 747, 748, 749, 754, 755, 756, 757, 762, 763, 764, 768, 769, 770, 771, 773, 774, 775, 777, 779, 780, 782, 783, 784, 785, 786, 787, 788, 789, 791, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858 }

C grade: { }

F grade: { 1, 2, 3, 107, 117, 118, 119, 122, 123, 124, 125, 136, 137, 138, 139, 140, 141, 142, 143, 152, 154, 155, 156, 165, 378, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 400, 401, 402, 403, 404, 405, 406, 407, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 425, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 452, 457, 458, 459, 460, 461, 462, 463, 466, 467, 468, 469, 470, 471, 472, 473, 474, 479, 480, 481, 482, 483, 484, 485, 486, 550, 551, 552, 557, 558, 559, 564, 565, 566, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 610, 611, 612, 729, 730, 731, 732, 733, 734, 737, 738, 739, 740, 741, 742, 743, 744, 745, 750, 751, 752, 753, 758, 759, 760, 761, 765, 766, 767, 772, 776, 778, 781, 790, 792, 793 }

2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 3, 4, 5, 6, 7, 19, 20, 28, 29, 30, 31, 40, 41, 42, 44, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 369, 370, 371, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 455, 457, 458, 459, 460, 461, 468, 469, 470, 471, 472, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828 }

B grade: { 43, 173, 372, 373, 374, 375, 376, 377, 393, 394, 395, 396, 397, 398, 399, 419, 424, 425, 426, 427, 428, 429, 430, 431, 432, 453, 454, 456, 462, 463, 464, 465, 466, 467, 473, 474, 475, 476, 477, 478, 479, 480, 752, 755 }

C grade: { 168, 753, 754, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

F grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 799, 800, 801, 802, 803, 804, 805, 806, 807, 856, 857, 858 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	97	77	0	177	0	87	-1	85
N.S.	1	1.00	0.76	0.60	0.00	1.38	0.00	0.68	-0.01	0.66
time (sec)	N/A	0.031	0.075	0.031	0.000	0.510	0.000	0.440	0.000	6.808

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	59	58	0	147	0	0	-1	73
N.S.	1	1.00	0.65	0.64	0.00	1.62	0.00	0.00	-0.01	0.80
time (sec)	N/A	0.020	0.037	0.014	0.000	0.717	0.000	0.000	0.000	6.487

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	49	0	0	90	0	24	-1	52
N.S.	1	1.00	0.82	0.00	0.00	1.50	0.00	0.40	-0.02	0.87
time (sec)	N/A	0.012	0.015	0.035	0.000	0.985	0.000	0.216	0.000	6.072

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	25	33	0	34	0	19	34	25
N.S.	1	1.00	0.74	0.97	0.00	1.00	0.00	0.56	1.00	0.74
time (sec)	N/A	0.008	0.011	0.003	0.000	0.871	0.000	0.260	4.139	6.485

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	70	40	44	0	58	0	0	45	40
N.S.	1	1.03	0.59	0.65	0.00	0.85	0.00	0.00	0.66	0.59
time (sec)	N/A	0.016	0.012	0.003	0.000	0.501	0.000	0.000	4.205	7.661

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	107	51	55	0	80	0	0	56	51
N.S.	1	1.02	0.49	0.52	0.00	0.76	0.00	0.00	0.53	0.49
time (sec)	N/A	0.024	0.015	0.004	0.000	0.776	0.000	0.000	4.210	9.923

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	148	62	66	0	102	0	0	141	62
N.S.	1	1.10	0.46	0.49	0.00	0.76	0.00	0.00	1.04	0.46
time (sec)	N/A	0.043	0.021	0.005	0.000	0.846	0.000	0.000	4.129	13.168

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	299	299	81	1099	0	583	63	75	872	0
N.S.	1	1.00	0.27	3.68	0.00	1.95	0.21	0.25	2.92	0.00
time (sec)	N/A	0.312	0.043	0.133	0.000	0.668	0.802	0.162	4.375	0.001

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	43	32	0	269	257	31	85	0
N.S.	1	1.00	0.91	0.68	0.00	5.72	5.47	0.66	1.81	0.00
time (sec)	N/A	0.026	0.023	0.009	0.000	0.809	0.633	0.149	0.102	0.001

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	299	299	52	1073	0	613	48	0	469	0
N.S.	1	1.00	0.17	3.59	0.00	2.05	0.16	0.00	1.57	0.00
time (sec)	N/A	0.308	0.033	0.106	0.000	1.339	0.581	0.000	4.356	0.001

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	37	26	25	25	26	29	11	0
N.S.	1	1.00	2.18	1.53	1.47	1.47	1.53	1.71	0.65	0.00
time (sec)	N/A	0.008	0.007	0.009	1.370	0.914	0.183	0.203	0.036	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	18	17	17	20	17	17	0
N.S.	1	1.00	1.00	0.75	0.71	0.71	0.83	0.71	0.71	0.00
time (sec)	N/A	0.015	0.013	0.006	2.997	1.157	0.161	0.169	4.116	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	91	54	53	53	70	53	83	0
N.S.	1	1.00	1.36	0.81	0.79	0.79	1.04	0.79	1.24	0.00
time (sec)	N/A	0.050	0.071	0.005	3.036	1.102	0.219	0.195	4.147	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	74	77	57	0	159	63	56	47	0
N.S.	1	1.00	1.04	0.77	0.00	2.15	0.85	0.76	0.64	0.00
time (sec)	N/A	0.049	0.070	0.036	0.000	1.637	0.206	0.189	4.186	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	41	386	0	247	899	143	210	0
N.S.	1	1.00	0.23	2.19	0.00	1.40	5.11	0.81	1.19	0.00
time (sec)	N/A	0.162	0.038	0.109	0.000	0.816	1.133	0.534	4.205	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.006	0.002	0.001	1.330	0.371	0.074	0.164	0.023	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.006	0.002	0.001	1.300	0.661	0.065	0.166	0.021	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.003	0.000	0.001	1.363	0.487	0.063	0.153	0.020	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	16
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.94
time (sec)	N/A	0.005	0.001	0.001	1.306	0.714	0.067	0.154	0.020	0.009

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	12
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	1.00
time (sec)	N/A	0.004	0.000	0.000	1.263	0.817	0.065	0.147	0.017	0.013

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	14	11	10	14	11	0
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85	0.00
time (sec)	N/A	0.005	0.001	0.003	1.347	0.508	0.099	0.147	0.022	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	13	5	10	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.00	0.00
time (sec)	N/A	0.005	0.001	0.005	1.334	0.658	0.103	0.162	0.024	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	14	17	10	20	11	0
N.S.	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	0.85	0.00
time (sec)	N/A	0.006	0.002	0.007	1.351	0.499	0.117	0.149	0.039	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	14	13	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87	0.00
time (sec)	N/A	0.006	0.002	0.005	1.296	0.626	0.125	0.181	0.027	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	14	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76	0.00
time (sec)	N/A	0.006	0.003	0.004	1.315	0.477	0.131	0.187	0.027	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	15	15	15	15	15	0
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.00
time (sec)	N/A	0.006	0.002	0.005	1.246	0.544	0.154	0.168	0.028	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.013	0.002	0.001	1.378	0.615	0.072	0.158	0.037	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	24	24	28
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.93
time (sec)	N/A	0.025	0.001	0.001	1.350	2.024	0.072	0.195	0.036	0.015

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	30
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	1.00
time (sec)	N/A	0.017	0.001	0.001	1.316	0.474	0.073	0.168	0.035	0.025

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	25	24	24	24	24	24	27
N.S.	1	1.00	1.00	1.56	1.50	1.50	1.50	1.50	1.50	1.69
time (sec)	N/A	0.009	0.002	0.001	1.311	0.439	0.097	0.152	0.032	0.015

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	22	21	21	25
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84	1.00
time (sec)	N/A	0.013	0.001	0.001	1.353	0.736	0.075	0.180	0.033	0.022

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	24	21	20	24	21	0
N.S.	1	1.00	1.00	0.96	1.04	0.91	0.87	1.04	0.91	0.00
time (sec)	N/A	0.019	0.001	0.003	1.346	0.485	0.113	0.149	0.029	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	22	25	19	22	22	0
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92	0.00
time (sec)	N/A	0.017	0.001	0.004	1.314	0.594	0.113	0.151	0.035	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	24	24	27	24	32	23	0
N.S.	1	1.00	1.00	0.89	0.89	1.00	0.89	1.19	0.85	0.00
time (sec)	N/A	0.021	0.001	0.005	1.328	0.555	0.147	0.168	0.032	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	22	26	22	22	24	0
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.04	0.00
time (sec)	N/A	0.018	0.001	0.007	1.286	0.522	0.165	0.160	0.027	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	26	28	24	34	24	0
N.S.	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00	0.00
time (sec)	N/A	0.019	0.001	0.005	1.315	0.527	0.190	0.180	0.045	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	26	26	27	26	25	0
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89	0.00
time (sec)	N/A	0.016	0.001	0.004	1.304	0.611	0.198	0.165	0.036	0.001

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	30	25	24	24	26	24	26	0
N.S.	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37	0.00
time (sec)	N/A	0.010	0.001	0.006	1.227	0.557	0.212	0.174	0.036	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	26	26	27	26	26	0
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.00
time (sec)	N/A	0.016	0.001	0.005	1.367	0.680	0.231	0.155	0.037	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	43
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	1.00
time (sec)	N/A	0.024	0.002	0.001	1.304	0.479	0.082	0.148	0.044	0.027

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	43	36	35	35	37	35	35	39
N.S.	1	1.00	1.26	1.06	1.03	1.03	1.09	1.03	1.03	1.15
time (sec)	N/A	0.039	0.002	0.001	1.322	0.462	0.077	0.177	0.043	0.017

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	39	35	35	43
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	1.00
time (sec)	N/A	0.021	0.002	0.002	1.344	0.558	0.086	0.166	0.043	0.027

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	36	35	35	37	35	35	38
N.S.	1	1.00	1.00	2.25	2.19	2.19	2.31	2.19	2.19	2.38
time (sec)	N/A	0.009	0.002	0.001	1.278	0.404	0.082	0.149	0.042	0.015

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	32	31	31	32	31	31	35
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89	1.00
time (sec)	N/A	0.017	0.001	0.000	1.318	0.637	0.081	0.162	0.040	0.023

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	34	36	33	37	36	33	0
N.S.	1	1.00	1.00	0.87	0.92	0.85	0.95	0.92	0.85	0.00
time (sec)	N/A	0.026	0.004	0.003	1.327	0.552	0.125	0.153	0.036	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	33	32	36	29	32	32	0
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	0.94	0.00
time (sec)	N/A	0.019	0.004	0.004	1.378	0.538	0.123	0.151	0.042	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	35	36	38	37	46	34	0
N.S.	1	1.00	1.00	0.88	0.90	0.95	0.92	1.15	0.85	0.00
time (sec)	N/A	0.029	0.007	0.007	1.266	0.680	0.170	0.158	0.037	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	34	34	36	36	34	36	0
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97	0.00
time (sec)	N/A	0.019	0.004	0.006	1.286	0.453	0.168	0.167	0.038	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	35	37	39	37	46	37	0
N.S.	1	1.00	1.00	0.88	0.92	0.98	0.92	1.15	0.92	0.00
time (sec)	N/A	0.026	0.004	0.006	1.343	0.603	0.226	0.150	0.035	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	33	33	37	34	33	34	0
N.S.	1	1.00	1.00	0.97	0.97	1.09	1.00	0.97	1.00	0.00
time (sec)	N/A	0.020	0.005	0.006	1.303	0.738	0.232	0.166	0.032	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	34	39	39	37	47	36	0
N.S.	1	1.00	1.00	0.87	1.00	1.00	0.95	1.21	0.92	0.00
time (sec)	N/A	0.026	0.004	0.007	1.319	0.652	0.292	0.156	0.047	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	36	37	37	39	37	35	0
N.S.	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.90	0.00
time (sec)	N/A	0.021	0.004	0.006	1.347	0.686	0.282	0.163	0.029	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	43	36	35	35	37	35	37	0
N.S.	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95	0.00
time (sec)	N/A	0.010	0.006	0.005	1.338	0.766	0.311	0.152	0.030	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	37	37	39	37	37	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.021	0.004	0.006	1.325	0.609	0.304	0.173	0.032	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	43	36	37	37	39	37	37	0
N.S.	1	1.00	1.08	0.90	0.92	0.92	0.98	0.92	0.92	0.00
time (sec)	N/A	0.023	0.004	0.005	1.313	0.697	0.325	0.150	0.033	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	60	60	148	107	65	54	0
N.S.	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.79	0.00
time (sec)	N/A	0.038	0.026	0.008	3.012	0.587	0.225	0.189	0.032	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	53	46	46	45	44	47	45	0
N.S.	1	1.00	1.00	0.87	0.87	0.85	0.83	0.89	0.85	0.00
time (sec)	N/A	0.045	0.006	0.004	1.329	1.309	0.180	0.159	0.047	0.001

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	49	50	126	95	55	43	0
N.S.	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.78	0.00
time (sec)	N/A	0.033	0.027	0.004	2.881	0.829	0.209	0.184	0.052	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	35	34	33	32	35	33	0
N.S.	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82	0.00
time (sec)	N/A	0.034	0.006	0.003	1.334	0.574	0.170	0.155	0.046	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	38	37	99	80	40	32	0
N.S.	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.76	0.00
time (sec)	N/A	0.027	0.019	0.003	2.950	0.602	0.194	0.174	0.047	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	24	23	22	20	24	22	0
N.S.	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81	0.00
time (sec)	N/A	0.027	0.005	0.002	1.313	0.608	0.166	0.157	0.036	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	27	26	82	56	26	23	0
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74	0.00
time (sec)	N/A	0.018	0.008	0.004	2.968	0.625	0.172	0.169	0.036	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	10	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.00
time (sec)	N/A	0.009	0.002	0.001	1.352	0.612	0.129	0.149	0.029	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	16	15	67	53	15	16	0
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67	0.00
time (sec)	N/A	0.012	0.004	0.003	2.968	0.621	0.151	0.151	4.198	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	23	18	15	22	18	0
N.S.	1	1.00	1.00	0.95	1.05	0.82	0.68	1.00	0.82	0.00
time (sec)	N/A	0.016	0.005	0.005	1.365	0.618	0.226	0.168	0.064	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	30	29	82	65	29	26	0
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76	0.00
time (sec)	N/A	0.014	0.012	0.005	2.884	0.598	0.190	0.165	4.273	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	32	33	33	31	43	31	0
N.S.	1	1.00	1.00	0.91	0.94	0.94	0.89	1.23	0.89	0.00
time (sec)	N/A	0.028	0.007	0.006	1.365	0.656	0.280	0.150	0.059	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	39	40	106	87	40	37	0
N.S.	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86	0.00
time (sec)	N/A	0.023	0.020	0.007	2.961	0.562	0.247	0.168	4.144	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	44	47	45	42	57	46	0
N.S.	1	1.00	1.00	0.90	0.96	0.92	0.86	1.16	0.94	0.00
time (sec)	N/A	0.035	0.007	0.006	1.317	1.083	0.356	0.172	0.060	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	58	52	52	132	100	52	48	0
N.S.	1	1.00	1.00	0.90	0.90	2.28	1.72	0.90	0.83	0.00
time (sec)	N/A	0.036	0.026	0.007	2.888	0.582	0.282	0.166	0.052	0.001

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	56	58	58	56	70	58	0
N.S.	1	1.00	1.00	0.89	0.92	0.92	0.89	1.11	0.92	0.00
time (sec)	N/A	0.041	0.007	0.007	1.319	0.737	0.398	0.147	0.068	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	71	68	71	190	124	73	66	0
N.S.	1	1.00	0.90	0.86	0.90	2.41	1.57	0.92	0.84	0.00
time (sec)	N/A	0.039	0.050	0.010	2.909	0.529	0.357	0.157	0.042	0.001

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	49	52	54	70	53	67	57	0
N.S.	1	1.00	0.86	0.91	0.95	1.23	0.93	1.18	1.00	0.00
time (sec)	N/A	0.051	0.017	0.011	1.319	0.534	0.300	0.180	4.144	0.001

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	60	57	59	164	107	61	56	0
N.S.	1	1.00	0.91	0.86	0.89	2.48	1.62	0.92	0.85	0.00
time (sec)	N/A	0.035	0.041	0.009	2.935	0.540	0.331	0.169	0.058	0.001

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	38	41	43	56	39	49	45	0
N.S.	1	1.00	0.86	0.93	0.98	1.27	0.89	1.11	1.02	0.00
time (sec)	N/A	0.037	0.015	0.008	1.324	0.615	0.276	0.174	0.042	0.001

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	51	43	45	136	83	42	43	0
N.S.	1	1.00	0.93	0.78	0.82	2.47	1.51	0.76	0.78	0.00
time (sec)	N/A	0.024	0.031	0.010	2.937	0.488	0.282	0.167	4.174	0.001

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	27	30	32	35	29	32	29	0
N.S.	1	1.00	0.82	0.91	0.97	1.06	0.88	0.97	0.88	0.00
time (sec)	N/A	0.032	0.008	0.009	1.289	0.448	0.218	0.150	4.178	0.001

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	36	36	120	78	35	33	0
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73	0.00
time (sec)	N/A	0.019	0.020	0.008	3.022	0.511	0.232	0.171	4.149	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	15	15	15	14	14	0
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.94	0.88	0.88	0.00
time (sec)	N/A	0.009	0.002	0.000	1.321	0.624	0.175	0.168	0.024	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	36	35	120	78	35	33	0
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73	0.00
time (sec)	N/A	0.017	0.024	0.005	2.998	0.713	0.251	0.154	0.037	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	33	35	37	47	34	36	34	0
N.S.	1	1.00	0.87	0.92	0.97	1.24	0.89	0.95	0.89	0.00
time (sec)	N/A	0.035	0.016	0.012	1.296	0.592	0.335	0.155	4.177	0.001

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	54	46	49	136	92	47	44	0
N.S.	1	1.00	0.95	0.81	0.86	2.39	1.61	0.82	0.77	0.00
time (sec)	N/A	0.025	0.035	0.010	2.964	0.751	0.317	0.155	0.064	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	41	46	52	73	51	50	51	0
N.S.	1	1.00	0.84	0.94	1.06	1.49	1.04	1.02	1.04	0.00
time (sec)	N/A	0.041	0.037	0.012	1.342	1.092	0.399	0.175	4.211	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	67	59	64	172	114	59	58	0
N.S.	1	1.00	0.99	0.87	0.94	2.53	1.68	0.87	0.85	0.00
time (sec)	N/A	0.029	0.038	0.014	2.958	1.026	0.392	0.152	4.171	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	57	61	70	90	68	86	67	0
N.S.	1	1.00	0.86	0.92	1.06	1.36	1.03	1.30	1.02	0.00
time (sec)	N/A	0.055	0.052	0.013	1.345	0.749	0.492	0.152	4.170	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	80	70	75	198	126	70	70	0
N.S.	1	1.00	0.99	0.86	0.93	2.44	1.56	0.86	0.86	0.00
time (sec)	N/A	0.045	0.045	0.012	3.027	0.657	0.431	0.176	4.285	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	77	77	82	230	133	73	77	0
N.S.	1	1.00	0.91	0.91	0.96	2.71	1.56	0.86	0.91	0.00
time (sec)	N/A	0.042	0.047	0.011	2.938	0.387	0.498	0.157	4.211	0.001

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	48	58	66	91	68	62	68	0
N.S.	1	1.00	0.74	0.89	1.02	1.40	1.05	0.95	1.05	0.00
time (sec)	N/A	0.054	0.056	0.010	1.358	0.534	0.431	0.195	4.257	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	66	63	68	202	107	54	64	0
N.S.	1	1.00	0.89	0.85	0.92	2.73	1.45	0.73	0.86	0.00
time (sec)	N/A	0.032	0.045	0.011	2.817	0.712	0.458	0.172	4.249	0.001

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	39	46	55	69	53	42	52	0
N.S.	1	1.00	0.80	0.94	1.12	1.41	1.08	0.86	1.06	0.00
time (sec)	N/A	0.045	0.014	0.009	1.351	0.751	0.367	0.179	4.181	0.001

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	55	47	59	188	110	45	56	0
N.S.	1	1.00	0.86	0.73	0.92	2.94	1.72	0.70	0.88	0.00
time (sec)	N/A	0.026	0.042	0.010	2.912	0.782	0.374	0.161	4.226	0.001

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	24	31	36	36	36	22	37	0
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95	0.00
time (sec)	N/A	0.011	0.008	0.008	1.331	0.811	0.307	0.159	4.179	0.001

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	58	49	62	190	110	50	55	0
N.S.	1	1.00	0.89	0.75	0.95	2.92	1.69	0.77	0.85	0.00
time (sec)	N/A	0.025	0.028	0.008	2.859	1.919	0.353	0.159	4.226	0.001

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	26	26	27	14	28	0
N.S.	1	1.00	1.00	0.94	1.62	1.62	1.69	0.88	1.75	0.00
time (sec)	N/A	0.009	0.002	0.002	1.272	0.800	0.271	0.154	0.028	0.001

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	55	51	58	188	105	45	55	0
N.S.	1	1.00	0.89	0.82	0.94	3.03	1.69	0.73	0.89	0.00
time (sec)	N/A	0.023	0.033	0.006	2.986	1.186	0.365	0.203	4.207	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	43	49	60	90	56	59	56	0
N.S.	1	1.00	0.80	0.91	1.11	1.67	1.04	1.09	1.04	0.00
time (sec)	N/A	0.044	0.031	0.012	1.378	2.310	0.455	0.155	0.055	0.001

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	68	66	71	202	116	57	66	0
N.S.	1	1.00	0.89	0.87	0.93	2.66	1.53	0.75	0.87	0.00
time (sec)	N/A	0.036	0.040	0.013	3.002	2.471	0.452	0.172	4.256	0.001

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	59	62	77	119	80	66	75	0
N.S.	1	1.00	0.88	0.93	1.15	1.78	1.19	0.99	1.12	0.00
time (sec)	N/A	0.061	0.056	0.015	1.384	0.790	0.632	0.165	0.064	0.001

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	79	79	86	238	138	71	80	0
N.S.	1	1.00	0.91	0.91	0.99	2.74	1.59	0.82	0.92	0.00
time (sec)	N/A	0.042	0.042	0.013	2.934	1.173	0.499	0.176	4.261	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	74	79	92	134	90	79	88	0
N.S.	1	1.00	0.86	0.92	1.07	1.56	1.05	0.92	1.02	0.00
time (sec)	N/A	0.072	0.047	0.014	1.351	0.733	0.569	0.156	4.248	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	90	89	97	264	150	80	92	0
N.S.	1	1.00	0.90	0.89	0.97	2.64	1.50	0.80	0.92	0.00
time (sec)	N/A	0.049	0.053	0.015	2.975	2.159	0.577	0.178	4.235	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	85	90	103	145	104	110	101	0
N.S.	1	1.00	0.89	0.95	1.08	1.53	1.09	1.16	1.06	0.00
time (sec)	N/A	0.085	0.071	0.015	1.363	1.920	0.645	0.154	0.101	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	114	124	121	188	0	101	105	98
N.S.	1	1.00	0.96	1.04	1.02	1.58	0.00	0.85	0.88	0.82
time (sec)	N/A	0.131	0.076	0.016	1.455	1.039	0.000	0.193	4.685	0.240

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	103	104	97	167	0	85	77	93
N.S.	1	1.00	1.13	1.14	1.07	1.84	0.00	0.93	0.85	1.02
time (sec)	N/A	0.100	0.061	0.008	1.447	0.820	0.000	0.185	4.364	0.215

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	90	84	73	140	0	69	64	78
N.S.	1	1.00	1.32	1.24	1.07	2.06	0.00	1.01	0.94	1.15
time (sec)	N/A	0.064	0.048	0.007	1.428	0.868	0.000	0.189	4.366	0.191

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	64	64	49	115	0	52	50	61
N.S.	1	1.00	1.16	1.16	0.89	2.09	0.00	0.95	0.91	1.11
time (sec)	N/A	0.071	0.025	0.005	1.427	1.104	0.000	0.174	4.210	0.162

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	60	84	51	115	0	61	-1	61
N.S.	1	1.00	1.15	1.62	0.98	2.21	0.00	1.17	-0.02	1.17
time (sec)	N/A	0.077	0.093	0.007	1.390	0.850	0.000	0.272	0.000	0.126

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	41	28	0	63	28	35
N.S.	1	1.00	1.00	1.16	1.64	1.12	0.00	2.52	1.12	1.40
time (sec)	N/A	0.039	0.010	0.004	1.438	0.840	0.000	0.225	4.148	0.112

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	35	39	65	42	0	120	41	46
N.S.	1	1.00	0.67	0.75	1.25	0.81	0.00	2.31	0.79	0.88
time (sec)	N/A	0.083	0.011	0.004	1.422	0.582	0.000	0.221	4.263	0.128

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	46	50	89	53	0	148	89	57
N.S.	1	1.00	0.58	0.62	1.11	0.66	0.00	1.85	1.11	0.71
time (sec)	N/A	0.120	0.013	0.005	1.426	0.586	0.000	0.238	4.338	0.137

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	57	61	113	64	0	178	113	68
N.S.	1	1.00	0.53	0.56	1.05	0.59	0.00	1.65	1.05	0.63
time (sec)	N/A	0.164	0.014	0.007	1.420	0.964	0.000	0.294	4.505	0.155

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	68	72	137	75	0	206	137	79
N.S.	1	1.00	0.50	0.53	1.01	0.55	0.00	1.51	1.01	0.58
time (sec)	N/A	0.214	0.015	0.006	1.531	0.943	0.000	0.253	4.619	0.159

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	46	50	46	53	0	60	53	57
N.S.	1	1.00	0.59	0.64	0.59	0.68	0.00	0.77	0.68	0.73
time (sec)	N/A	0.094	0.023	0.005	1.406	1.312	0.000	0.159	4.233	0.059

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	35	39	34	41	0	44	41	45
N.S.	1	1.00	0.67	0.75	0.65	0.79	0.00	0.85	0.79	0.87
time (sec)	N/A	0.049	0.018	0.005	1.394	0.930	0.000	0.159	4.135	0.055

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	14	28	0	27	29	25
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	1.16	1.00
time (sec)	N/A	0.006	0.005	0.003	1.429	0.987	0.000	0.154	4.136	0.030

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	60	65	0	117	0	69	68	50
N.S.	1	1.00	1.20	1.30	0.00	2.34	0.00	1.38	1.36	1.00
time (sec)	N/A	0.049	0.030	0.006	0.000	0.979	0.000	0.180	4.308	0.084

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	63	85	0	134	0	50	-1	56
N.S.	1	1.00	1.12	1.52	0.00	2.39	0.00	0.89	-0.02	1.00
time (sec)	N/A	0.052	0.042	0.006	0.000	3.898	0.000	0.200	0.000	0.118

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	46	106	0	159	0	78	-1	71
N.S.	1	1.00	0.55	1.26	0.00	1.89	0.00	0.93	-0.01	0.85
time (sec)	N/A	0.098	0.014	0.007	0.000	1.362	0.000	0.212	0.000	0.126

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	46	128	0	185	0	100	-1	82
N.S.	1	1.00	0.41	1.14	0.00	1.65	0.00	0.89	-0.01	0.73
time (sec)	N/A	0.145	0.014	0.010	0.000	1.133	0.000	0.267	0.000	0.175

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	126	142	142	210	0	115	134	109
N.S.	1	1.00	1.02	1.15	1.15	1.69	0.00	0.93	1.08	0.88
time (sec)	N/A	0.132	0.099	0.010	1.390	2.609	0.000	0.278	4.351	0.343

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	115	122	118	189	0	99	99	98
N.S.	1	1.00	1.14	1.21	1.17	1.87	0.00	0.98	0.98	0.97
time (sec)	N/A	0.092	0.096	0.010	1.471	1.140	0.000	0.194	4.445	0.286

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	104	102	91	166	0	84	-1	91
N.S.	1	1.00	1.18	1.16	1.03	1.89	0.00	0.95	-0.01	1.03
time (sec)	N/A	0.099	0.075	0.008	1.416	1.024	0.000	0.206	0.000	0.293

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	71	84	70	145	0	68	-1	73
N.S.	1	1.00	0.89	1.05	0.88	1.81	0.00	0.85	-0.01	0.91
time (sec)	N/A	0.100	0.103	0.005	1.445	1.582	0.000	0.248	0.000	0.287

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	54	107	71	139	0	79	-1	73
N.S.	1	1.00	0.71	1.41	0.93	1.83	0.00	1.04	-0.01	0.96
time (sec)	N/A	0.111	0.014	0.007	1.444	1.457	0.000	0.272	0.000	0.291

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	56	129	89	135	0	122	-1	73
N.S.	1	1.00	0.75	1.72	1.19	1.80	0.00	1.63	-0.01	0.97
time (sec)	N/A	0.104	0.016	0.007	1.505	1.347	0.000	0.437	0.000	0.241

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	81	39	0	92	30	46
N.S.	1	1.00	1.00	1.16	3.24	1.56	0.00	3.68	1.20	1.84
time (sec)	N/A	0.046	0.014	0.004	1.409	2.063	0.000	0.303	4.380	0.217

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	35	39	105	53	0	178	87	57
N.S.	1	1.00	0.67	0.75	2.02	1.02	0.00	3.42	1.67	1.10
time (sec)	N/A	0.093	0.014	0.005	1.463	0.780	0.000	0.255	4.579	0.234

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	46	50	129	64	0	206	111	68
N.S.	1	1.00	0.58	0.62	1.61	0.80	0.00	2.58	1.39	0.85
time (sec)	N/A	0.141	0.015	0.005	1.520	1.122	0.000	0.267	4.722	0.247

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	57	61	153	75	0	236	135	79
N.S.	1	1.00	0.53	0.56	1.42	0.69	0.00	2.19	1.25	0.73
time (sec)	N/A	0.184	0.017	0.005	1.492	1.655	0.000	0.276	4.990	0.262

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	68	72	177	86	0	264	159	90
N.S.	1	1.00	0.50	0.53	1.30	0.63	0.00	1.94	1.17	0.66
time (sec)	N/A	0.262	0.018	0.007	1.468	1.142	0.000	0.286	5.174	0.280

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	75	72	79	86	0	92	73	68
N.S.	1	1.00	0.56	0.54	0.59	0.64	0.00	0.69	0.54	0.51
time (sec)	N/A	0.253	0.036	0.007	1.534	1.193	0.000	0.187	4.478	0.350

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	64	61	68	75	0	76	62	79
N.S.	1	1.00	0.60	0.58	0.64	0.71	0.00	0.72	0.58	0.75
time (sec)	N/A	0.196	0.031	0.008	1.503	0.758	0.000	0.165	4.304	0.338

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	53	50	57	64	0	60	51	68
N.S.	1	1.00	0.66	0.62	0.71	0.80	0.00	0.75	0.64	0.85
time (sec)	N/A	0.111	0.024	0.006	1.445	1.023	0.000	0.161	4.187	0.314

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	42	39	45	52	0	44	40	56
N.S.	1	1.00	0.81	0.75	0.87	1.00	0.00	0.85	0.77	1.08
time (sec)	N/A	0.054	0.020	0.006	1.504	1.043	0.000	0.164	4.160	0.296

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	32	39	0	27	30	25
N.S.	1	1.00	1.00	1.16	1.28	1.56	0.00	1.08	1.20	1.00
time (sec)	N/A	0.048	0.008	0.003	1.483	0.963	0.000	0.156	4.149	0.269

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	76	78	0	140	0	89	-1	62
N.S.	1	1.00	1.04	1.07	0.00	1.92	0.00	1.22	-0.01	0.85
time (sec)	N/A	0.110	0.052	0.006	0.000	0.908	0.000	0.166	0.000	0.376

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	44	102	0	147	0	69	-1	66
N.S.	1	1.00	0.56	1.29	0.00	1.86	0.00	0.87	-0.01	0.84
time (sec)	N/A	0.118	0.016	0.006	0.000	2.615	0.000	0.201	0.000	0.483

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	80	125	0	164	0	76	-1	68
N.S.	1	1.00	0.99	1.54	0.00	2.02	0.00	0.94	-0.01	0.84
time (sec)	N/A	0.115	0.051	0.008	0.000	0.708	0.000	0.240	0.000	0.545

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	46	145	0	185	0	100	-1	82
N.S.	1	1.00	0.42	1.33	0.00	1.70	0.00	0.92	-0.01	0.75
time (sec)	N/A	0.164	0.018	0.011	0.000	1.285	0.000	0.235	0.000	0.638

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	46	165	0	207	0	119	-1	93
N.S.	1	1.00	0.34	1.20	0.00	1.51	0.00	0.87	-0.01	0.68
time (sec)	N/A	0.203	0.017	0.016	0.000	0.755	0.000	0.233	0.000	0.718

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	46	186	0	229	0	138	-1	104
N.S.	1	1.00	0.28	1.13	0.00	1.39	0.00	0.84	-0.01	0.63
time (sec)	N/A	0.255	0.017	0.031	0.000	1.169	0.000	0.379	0.000	0.805

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	100	105	100	166	0	87	-1	93
N.S.	1	1.00	0.88	0.92	0.88	1.46	0.00	0.76	-0.01	0.82
time (sec)	N/A	0.129	0.047	0.009	1.474	0.813	0.000	0.223	0.000	0.244

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	89	85	76	145	0	73	-1	82
N.S.	1	1.00	1.03	0.99	0.88	1.69	0.00	0.85	-0.01	0.95
time (sec)	N/A	0.103	0.036	0.009	1.453	0.781	0.000	0.215	0.000	0.221

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	73	64	52	114	0	59	53	68
N.S.	1	1.00	1.26	1.10	0.90	1.97	0.00	1.02	0.91	1.17
time (sec)	N/A	0.083	0.027	0.007	1.448	0.915	0.000	0.199	4.302	0.193

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	52	44	32	74	0	39	33	40
N.S.	1	1.00	1.68	1.42	1.03	2.39	0.00	1.26	1.06	1.29
time (sec)	N/A	0.054	0.013	0.003	1.461	0.881	0.000	0.188	4.360	0.129

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	26	21	21	0	25	21	23
N.S.	1	1.00	1.00	1.13	0.91	0.91	0.00	1.09	0.91	1.00
time (sec)	N/A	0.041	0.008	0.005	1.472	1.875	0.000	0.181	4.213	0.133

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	35	37	44	31	0	57	29	35
N.S.	1	1.00	0.67	0.71	0.85	0.60	0.00	1.10	0.56	0.67
time (sec)	N/A	0.083	0.015	0.004	1.429	0.715	0.000	0.186	4.259	0.151

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	46	50	68	42	0	90	42	46
N.S.	1	1.00	0.58	0.62	0.85	0.52	0.00	1.12	0.52	0.58
time (sec)	N/A	0.125	0.014	0.005	1.435	1.090	0.000	0.209	4.326	0.159

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	57	61	92	53	0	123	92	57
N.S.	1	1.00	0.53	0.56	0.85	0.49	0.00	1.14	0.85	0.53
time (sec)	N/A	0.169	0.015	0.007	1.408	1.259	0.000	0.200	4.273	0.164

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	34	37	34	30	0	0	33	34
N.S.	1	1.00	0.68	0.74	0.68	0.60	0.00	0.00	0.66	0.68
time (sec)	N/A	0.079	0.016	0.004	1.505	1.591	0.000	0.000	4.248	0.057

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	26	13	20	0	31	20	22
N.S.	1	1.00	1.00	1.18	0.59	0.91	0.00	1.41	0.91	1.00
time (sec)	N/A	0.017	0.005	0.003	1.419	1.064	0.000	0.185	4.258	0.041

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	52	50	0	80	0	46	-1	30
N.S.	1	1.00	1.73	1.67	0.00	2.67	0.00	1.53	-0.03	1.00
time (sec)	N/A	0.009	0.009	0.005	0.000	0.981	0.000	0.170	0.000	0.063

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	68	73	0	133	0	0	76	59
N.S.	1	1.00	1.15	1.24	0.00	2.25	0.00	0.00	1.29	1.00
time (sec)	N/A	0.055	0.060	0.006	0.000	4.015	0.000	0.000	4.472	0.123

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	44	94	0	163	0	0	-1	71
N.S.	1	1.00	0.51	1.08	0.00	1.87	0.00	0.00	-0.01	0.82
time (sec)	N/A	0.097	0.012	0.009	0.000	1.788	0.000	0.000	0.000	0.138

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	88	87	103	209	0	114	-1	102
N.S.	1	1.00	0.81	0.80	0.94	1.92	0.00	1.05	-0.01	0.94
time (sec)	N/A	0.129	0.050	0.010	1.477	0.726	0.000	0.269	0.000	0.450

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	76	73	77	180	0	99	-1	88
N.S.	1	1.00	0.94	0.90	0.95	2.22	0.00	1.22	-0.01	1.09
time (sec)	N/A	0.109	0.041	0.007	1.506	1.094	0.000	0.240	0.000	0.390

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	66	63	54	150	0	0	55	74
N.S.	1	1.00	1.20	1.15	0.98	2.73	0.00	0.00	1.00	1.35
time (sec)	N/A	0.092	0.075	0.007	1.469	1.650	0.000	0.000	4.328	0.311

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	28	20	26	0	35	26	28
N.S.	1	1.00	1.00	1.27	0.91	1.18	0.00	1.59	1.18	1.27
time (sec)	N/A	0.055	0.008	0.003	1.453	1.910	0.000	0.179	4.131	0.271

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	29	37	41	41	0	28	26	41
N.S.	1	1.00	1.04	1.32	1.46	1.46	0.00	1.00	0.93	1.46
time (sec)	N/A	0.046	0.010	0.005	1.467	2.478	0.000	0.194	4.125	0.256

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	48	45	65	54	0	0	51	55
N.S.	1	1.00	0.65	0.61	0.88	0.73	0.00	0.00	0.69	0.74
time (sec)	N/A	0.129	0.010	0.006	1.485	3.968	0.000	0.000	4.236	0.281

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	57	59	89	63	0	0	60	66
N.S.	1	1.00	0.56	0.58	0.87	0.62	0.00	0.00	0.59	0.65
time (sec)	N/A	0.185	0.011	0.005	1.511	2.280	0.000	0.000	4.305	0.298

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	68	72	113	76	0	0	114	77
N.S.	1	1.00	0.52	0.55	0.87	0.58	0.00	0.00	0.88	0.59
time (sec)	N/A	0.233	0.013	0.006	1.469	2.052	0.000	0.000	4.407	0.300

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	29	37	22	39	0	52	38	40
N.S.	1	1.00	0.62	0.79	0.47	0.83	0.00	1.11	0.81	0.85
time (sec)	N/A	0.068	0.014	0.004	1.483	0.693	0.000	0.209	4.225	0.391

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	29	14	29	0	17	30	32
N.S.	1	1.00	1.00	1.38	0.67	1.38	0.00	0.81	1.43	1.52
time (sec)	N/A	0.019	0.005	0.003	1.443	1.084	0.000	0.206	4.152	0.347

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	38	65	0	162	0	0	-1	62
N.S.	1	1.00	0.75	1.27	0.00	3.18	0.00	0.00	-0.02	1.22
time (sec)	N/A	0.062	0.009	0.007	0.000	1.676	0.000	0.000	0.000	0.420

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	40	77	0	199	0	0	42	78
N.S.	1	1.00	0.49	0.95	0.00	2.46	0.00	0.00	0.52	0.96
time (sec)	N/A	0.065	0.008	0.006	0.000	0.844	0.000	0.000	4.341	0.491

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	41	94	0	229	0	0	44	91
N.S.	1	1.00	0.38	0.86	0.00	2.10	0.00	0.00	0.40	0.83
time (sec)	N/A	0.153	0.011	0.008	0.000	0.613	0.000	0.000	4.637	0.594

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	57	48	26	37	0	26	42	55
N.S.	1	1.00	1.68	1.41	0.76	1.09	0.00	0.76	1.24	1.62
time (sec)	N/A	0.058	0.018	0.008	3.032	0.780	0.000	0.176	4.332	0.125

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	C	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	52	54	26	59	0	27	41	51
N.S.	1	1.00	1.53	1.59	0.76	1.74	0.00	0.79	1.21	1.50
time (sec)	N/A	0.057	0.018	0.006	2.980	0.913	0.000	0.207	4.363	0.118

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	51	48	41	45	0	41	40	49
N.S.	1	1.00	1.13	1.07	0.91	1.00	0.00	0.91	0.89	1.09
time (sec)	N/A	0.056	0.009	0.007	3.047	0.700	0.000	0.171	4.397	0.117

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	57	60	41	45	0	42	40	51
N.S.	1	1.00	1.27	1.33	0.91	1.00	0.00	0.93	0.89	1.13
time (sec)	N/A	0.057	0.012	0.007	3.043	0.857	0.000	0.173	4.456	0.128

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	73	64	52	114	0	59	53	68
N.S.	1	1.00	1.26	1.10	0.90	1.97	0.00	1.02	0.91	1.17
time (sec)	N/A	0.083	0.033	0.007	1.419	0.880	0.000	0.199	4.707	0.196

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	77	67	42	120	0	68	60	146
N.S.	1	1.00	1.28	1.12	0.70	2.00	0.00	1.13	1.00	2.43
time (sec)	N/A	0.082	0.042	0.009	3.026	0.920	0.000	0.201	4.621	0.296

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	18	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71	1.00
time (sec)	N/A	0.005	0.006	0.003	1.302	1.557	11.339	0.147	0.035	0.019

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	18	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71	1.00
time (sec)	N/A	0.005	0.005	0.004	1.358	1.016	5.565	0.154	0.029	0.019

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	18	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71	1.00
time (sec)	N/A	0.005	0.005	0.004	1.344	1.835	2.525	0.161	0.027	0.019

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	18	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71	1.00
time (sec)	N/A	0.005	0.005	0.003	1.327	0.577	1.726	0.212	0.028	0.016

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	18	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71	1.00
time (sec)	N/A	0.005	0.005	0.003	1.333	0.969	0.777	0.150	0.026	0.016

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	16	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.76	0.90	0.62	0.71	1.00
time (sec)	N/A	0.005	0.005	0.004	1.304	1.104	0.787	0.200	0.028	0.016

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	15	13	14	17	13	14	20
N.S.	1	1.00	1.00	0.79	0.68	0.74	0.89	0.68	0.74	1.05
time (sec)	N/A	0.005	0.004	0.003	1.310	1.334	1.001	0.153	0.030	0.016

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	13	14	17	13	15	18
N.S.	1	1.00	1.00	0.84	0.68	0.74	0.89	0.68	0.79	0.95
time (sec)	N/A	0.005	0.005	0.004	1.343	0.905	1.803	0.163	0.031	0.021

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	25	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69	0.94
time (sec)	N/A	0.016	0.009	0.006	1.278	2.049	35.136	0.147	0.049	0.025

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	25	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69	0.94
time (sec)	N/A	0.018	0.008	0.006	1.353	0.823	20.810	0.151	0.042	0.025

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	25	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69	0.94
time (sec)	N/A	0.015	0.008	0.006	1.231	2.347	11.378	0.159	4.271	0.026

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	25	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69	0.94
time (sec)	N/A	0.014	0.009	0.005	1.358	1.514	2.750	0.145	0.041	0.023

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	25	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69	0.94
time (sec)	N/A	0.014	0.008	0.006	1.251	1.306	4.975	0.151	0.040	0.025

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	26	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.72	0.94
time (sec)	N/A	0.016	0.008	0.006	1.330	1.592	5.218	0.152	4.438	0.024

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	26	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.72	0.94
time (sec)	N/A	0.015	0.008	0.004	1.349	1.462	5.988	0.149	0.043	0.024

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	27	34	24	26	34
N.S.	1	1.00	0.83	0.75	0.67	0.75	0.94	0.67	0.72	0.94
time (sec)	N/A	0.016	0.008	0.007	1.325	0.722	8.695	0.147	0.047	0.023

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	38	35	40	49	35	35	47
N.S.	1	1.00	1.00	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.021	0.013	0.006	1.345	1.324	83.180	0.182	0.050	0.029

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	38	35	40	49	35	35	47
N.S.	1	1.00	1.00	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.020	0.012	0.005	1.352	0.958	53.376	0.167	0.051	0.027

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	38	35	40	49	35	35	47
N.S.	1	1.00	1.00	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.020	0.011	0.005	1.309	0.937	33.338	0.154	0.050	0.028

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	41	38	35	40	49	35	35	47
N.S.	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.019	0.010	0.007	1.327	1.860	4.318	0.151	0.051	0.025

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	41	38	35	40	49	35	35	47
N.S.	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.020	0.011	0.005	1.311	0.690	17.612	0.151	0.047	0.025

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	41	38	35	40	49	35	35	47
N.S.	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.020	0.011	0.006	1.306	0.689	19.942	0.152	0.049	0.026

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	41	38	35	40	49	35	35	47
N.S.	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.019	0.010	0.006	1.352	0.856	22.204	0.151	0.052	0.026

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	41	38	35	40	49	35	35	47
N.S.	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.022	0.010	0.004	1.370	1.723	26.498	0.178	0.050	0.027

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	217	89	158	198	182	0	197	66	138
N.S.	1	1.00	0.41	0.73	0.91	0.84	0.00	0.91	0.30	0.64
time (sec)	N/A	0.232	0.044	0.014	3.006	0.873	0.000	0.174	0.114	0.196

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	203	152	194	170	0	196	67	134
N.S.	1	1.00	0.94	0.71	0.90	0.79	0.00	0.91	0.31	0.62
time (sec)	N/A	0.194	0.064	0.008	2.974	1.688	0.000	0.168	4.494	0.195

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	78	143	186	165	180	178	54	124
N.S.	1	1.00	0.38	0.70	0.91	0.81	0.88	0.87	0.26	0.61
time (sec)	N/A	0.186	0.018	0.006	3.124	0.861	166.485	0.177	4.359	0.176

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	189	140	185	124	172	178	55	123
N.S.	1	1.00	0.94	0.69	0.92	0.61	0.85	0.88	0.27	0.61
time (sec)	N/A	0.185	0.036	0.007	3.047	0.905	77.501	0.165	4.360	0.180

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	54	132	172	126	165	182	38	114
N.S.	1	1.00	0.28	0.69	0.90	0.66	0.86	0.95	0.20	0.59
time (sec)	N/A	0.142	0.024	0.005	3.002	1.015	48.274	0.194	0.079	0.149

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	146	132	172	126	160	182	37	113
N.S.	1	1.00	0.76	0.69	0.90	0.66	0.83	0.95	0.19	0.59
time (sec)	N/A	0.139	0.035	0.006	3.024	1.022	27.221	0.161	4.434	0.151

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	27	140	186	142	170	190	54	122
N.S.	1	1.00	0.13	0.69	0.92	0.70	0.84	0.94	0.27	0.60
time (sec)	N/A	0.164	0.005	0.008	2.917	1.040	18.320	0.167	4.530	0.181

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	29	143	187	167	178	178	53	125
N.S.	1	1.00	0.14	0.70	0.92	0.82	0.87	0.87	0.26	0.61
time (sec)	N/A	0.167	0.006	0.011	2.988	0.743	27.744	0.164	0.099	0.186

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	29	152	198	193	190	200	66	134
N.S.	1	1.00	0.13	0.71	0.92	0.90	0.88	0.93	0.31	0.62
time (sec)	N/A	0.188	0.006	0.011	3.131	1.050	48.980	0.167	0.093	0.188

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	217	29	158	201	189	197	192	65	135
N.S.	1	1.00	0.13	0.73	0.93	0.87	0.91	0.88	0.30	0.62
time (sec)	N/A	0.182	0.006	0.011	3.119	1.550	106.893	0.198	4.386	0.187

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	29	169	209	204	0	199	77	145
N.S.	1	1.00	0.13	0.73	0.91	0.89	0.00	0.87	0.33	0.63
time (sec)	N/A	0.220	0.007	0.013	3.088	0.776	0.000	0.178	4.466	0.216

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	220	172	217	227	0	216	92	245
N.S.	1	1.00	0.91	0.71	0.89	0.93	0.00	0.89	0.38	1.01
time (sec)	N/A	0.210	0.206	0.014	3.016	0.957	0.000	0.195	0.097	0.376

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	57	161	207	229	0	196	80	230
N.S.	1	1.00	0.25	0.70	0.90	1.00	0.00	0.85	0.35	1.00
time (sec)	N/A	0.182	0.015	0.011	3.046	0.863	0.000	0.180	0.107	0.335

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	221	158	206	192	0	196	80	151
N.S.	1	1.00	0.96	0.69	0.90	0.83	0.00	0.85	0.35	0.66
time (sec)	N/A	0.182	0.104	0.013	3.064	0.814	0.000	0.198	4.320	0.338

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	43	149	195	185	0	199	64	139
N.S.	1	1.00	0.20	0.68	0.89	0.85	0.00	0.91	0.29	0.64
time (sec)	N/A	0.166	0.014	0.013	3.005	1.047	0.000	0.182	0.089	0.332

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	198	158	195	187	0	199	64	139
N.S.	1	1.00	0.91	0.72	0.89	0.86	0.00	0.91	0.29	0.64
time (sec)	N/A	0.160	0.097	0.012	3.102	1.001	0.000	0.170	4.311	0.324

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	29	158	194	182	0	199	64	139
N.S.	1	1.00	0.13	0.72	0.89	0.83	0.00	0.91	0.29	0.64
time (sec)	N/A	0.163	0.005	0.010	3.041	1.926	0.000	0.181	4.340	0.312

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	199	149	194	179	0	199	64	139
N.S.	1	1.00	0.91	0.68	0.89	0.82	0.00	0.91	0.29	0.64
time (sec)	N/A	0.169	0.093	0.006	3.047	1.085	0.000	0.187	0.095	0.300

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	27	158	208	208	0	210	77	149
N.S.	1	1.00	0.12	0.69	0.90	0.90	0.00	0.91	0.33	0.65
time (sec)	N/A	0.190	0.006	0.017	2.931	1.193	0.000	0.171	0.085	0.343

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	29	161	209	228	0	196	77	149
N.S.	1	1.00	0.13	0.70	0.91	0.99	0.00	0.85	0.33	0.65
time (sec)	N/A	0.185	0.007	0.015	3.030	1.331	0.000	0.184	0.107	0.340

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	29	172	221	251	0	220	87	160
N.S.	1	1.00	0.12	0.71	0.91	1.03	0.00	0.91	0.36	0.66
time (sec)	N/A	0.217	0.006	0.020	2.969	1.038	0.000	0.194	4.369	0.345

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	29	178	224	245	0	212	87	160
N.S.	1	1.00	0.12	0.73	0.92	1.01	0.00	0.87	0.36	0.66
time (sec)	N/A	0.207	0.006	0.018	3.156	0.788	0.000	0.169	0.106	0.354

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	258	29	189	232	262	0	219	99	171
N.S.	1	1.00	0.11	0.73	0.90	1.02	0.00	0.85	0.38	0.66
time (sec)	N/A	0.235	0.007	0.017	3.006	1.157	0.000	0.174	4.369	0.395

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	220	178	229	247	0	208	101	324
N.S.	1	1.00	0.88	0.71	0.91	0.98	0.00	0.83	0.40	1.29
time (sec)	N/A	0.211	0.250	0.017	2.950	0.996	0.000	0.184	4.389	0.547

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	66	161	218	248	0	209	87	311
N.S.	1	1.00	0.28	0.67	0.91	1.04	0.00	0.87	0.36	1.30
time (sec)	N/A	0.203	0.019	0.017	2.955	1.281	0.000	0.220	4.281	0.492

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	242	170	218	254	0	209	87	311
N.S.	1	1.00	1.01	0.71	0.91	1.06	0.00	0.87	0.36	1.30
time (sec)	N/A	0.189	0.104	0.016	2.980	1.338	0.000	0.180	0.097	0.365

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	45	169	222	260	0	212	85	310
N.S.	1	1.00	0.19	0.70	0.92	1.07	0.00	0.88	0.35	1.28
time (sec)	N/A	0.188	0.016	0.017	3.086	0.959	0.000	0.185	0.087	0.290

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	223	169	221	257	0	211	85	153
N.S.	1	1.00	0.92	0.70	0.91	1.06	0.00	0.87	0.35	0.63
time (sec)	N/A	0.183	0.110	0.015	2.963	1.264	0.000	0.177	0.102	0.438

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	29	175	217	250	0	209	86	149
N.S.	1	1.00	0.12	0.73	0.91	1.05	0.00	0.87	0.36	0.62
time (sec)	N/A	0.185	0.005	0.010	3.085	0.848	0.000	0.197	0.089	0.275

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	220	166	217	241	0	209	86	149
N.S.	1	1.00	0.92	0.69	0.91	1.01	0.00	0.87	0.36	0.62
time (sec)	N/A	0.186	0.082	0.008	2.966	1.100	0.000	0.207	4.292	0.274

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	27	178	230	263	0	220	99	160
N.S.	1	1.00	0.11	0.71	0.92	1.05	0.00	0.88	0.39	0.64
time (sec)	N/A	0.217	0.006	0.019	3.136	3.192	0.000	0.187	4.366	0.489

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	29	181	231	283	0	208	99	160
N.S.	1	1.00	0.12	0.72	0.92	1.13	0.00	0.83	0.39	0.64
time (sec)	N/A	0.214	0.007	0.018	3.049	0.555	0.000	0.182	0.128	0.471

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	264	264	29	192	243	306	0	232	109	171
N.S.	1	1.00	0.11	0.73	0.92	1.16	0.00	0.88	0.41	0.65
time (sec)	N/A	0.233	0.007	0.022	3.144	0.962	0.000	0.269	0.118	0.473

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	264	264	29	198	246	300	0	224	109	171
N.S.	1	1.00	0.11	0.75	0.93	1.14	0.00	0.85	0.41	0.65
time (sec)	N/A	0.231	0.006	0.022	2.929	0.603	0.000	0.207	4.348	0.474

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	29	209	254	317	0	231	121	182
N.S.	1	1.00	0.10	0.75	0.91	1.14	0.00	0.83	0.43	0.65
time (sec)	N/A	0.268	0.007	0.022	3.042	0.874	0.000	0.186	0.139	0.483

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	29	209	257	311	0	243	121	182
N.S.	1	1.00	0.10	0.75	0.92	1.11	0.00	0.87	0.43	0.65
time (sec)	N/A	0.260	0.007	0.021	3.063	2.079	0.000	0.171	4.389	0.488

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	59	181	76	161	758	264	171	0
N.S.	1	1.00	0.81	2.48	1.04	2.21	10.38	3.62	2.34	0.00
time (sec)	N/A	0.050	0.039	0.007	1.508	0.660	5.284	0.181	4.290	0.325

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	43	96	55	89	352	141	97	0
N.S.	1	1.00	0.83	1.85	1.06	1.71	6.77	2.71	1.87	0.00
time (sec)	N/A	0.039	0.039	0.006	1.448	0.646	2.295	0.175	4.195	0.103

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	27	39	34	39	119	56	38	0
N.S.	1	1.00	0.79	1.15	1.00	1.15	3.50	1.65	1.12	0.00
time (sec)	N/A	0.014	0.016	0.003	1.426	0.804	0.757	0.197	4.148	0.051

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.010	0.002	0.001	1.361	0.682	0.067	0.149	0.038	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.009	0.001	0.000	1.328	0.474	0.068	0.148	0.033	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	16	25	24	24	24	24	24	0
N.S.	1	1.00	0.53	0.83	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.009	0.002	0.001	1.285	0.796	0.067	0.170	0.031	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	22	21	21	0
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84	0.00
time (sec)	N/A	0.005	0.000	0.001	1.350	0.410	0.070	0.150	0.028	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	24	21	20	24	21	0
N.S.	1	1.00	1.00	0.96	1.04	0.91	0.87	1.04	0.91	0.00
time (sec)	N/A	0.007	0.001	0.004	1.283	0.753	0.104	0.171	4.097	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	22	25	19	22	22	0
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92	0.00
time (sec)	N/A	0.008	0.001	0.004	1.314	0.837	0.097	0.146	0.034	0.001

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	24	24	27	24	32	23	0
N.S.	1	1.00	1.00	0.89	0.89	1.00	0.89	1.19	0.85	0.00
time (sec)	N/A	0.009	0.001	0.007	1.316	0.680	0.138	0.187	0.032	0.001

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	22	26	22	22	24	0
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.04	0.00
time (sec)	N/A	0.009	0.001	0.005	1.326	0.808	0.138	0.200	4.106	0.001

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	26	28	24	34	24	0
N.S.	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00	0.00
time (sec)	N/A	0.009	0.001	0.006	1.372	0.853	0.174	0.151	0.044	0.001

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	26	26	27	26	25	0
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89	0.00
time (sec)	N/A	0.010	0.001	0.005	1.337	1.090	0.185	0.171	0.036	0.001

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	26	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.00
time (sec)	N/A	0.009	0.001	0.005	1.328	0.769	0.196	0.177	0.035	0.001

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	26	26	27	26	26	0
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.00
time (sec)	N/A	0.009	0.001	0.005	1.348	0.699	0.208	0.168	0.034	0.001

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	47	46	46	53	46	46	0
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.82	0.00
time (sec)	N/A	0.028	0.003	0.001	1.328	0.676	0.081	0.147	0.025	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	56	47	46	46	49	46	46	0
N.S.	1	1.00	1.06	0.89	0.87	0.87	0.92	0.87	0.87	0.00
time (sec)	N/A	0.070	0.002	0.000	1.339	0.866	0.081	0.149	0.022	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	47	46	46	53	46	46	0
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.82	0.00
time (sec)	N/A	0.027	0.002	0.001	1.329	0.827	0.082	0.158	0.023	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	56	47	46	46	53	46	46	0
N.S.	1	1.00	1.65	1.38	1.35	1.35	1.56	1.35	1.35	0.00
time (sec)	N/A	0.040	0.002	0.002	1.279	0.719	0.080	0.150	0.022	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	47	46	46	53	46	46	0
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.82	0.00
time (sec)	N/A	0.029	0.002	0.001	1.323	0.430	0.080	0.148	0.022	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	45	44	44	44	44	44	0
N.S.	1	1.00	1.00	2.81	2.75	2.75	2.75	2.75	2.75	0.00
time (sec)	N/A	0.005	0.002	0.001	1.333	0.718	0.079	0.147	0.022	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	44	55	43	49	43	43	0
N.S.	1	1.00	1.00	0.86	1.08	0.84	0.96	0.84	0.84	0.00
time (sec)	N/A	0.024	0.001	0.001	1.282	0.651	0.085	0.143	0.021	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	45	47	44	49	47	44	0
N.S.	1	1.00	1.00	0.90	0.94	0.88	0.98	0.94	0.88	0.00
time (sec)	N/A	0.034	0.004	0.003	1.341	0.772	0.134	0.148	0.027	0.001

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	45	44	48	44	44	44	0
N.S.	1	1.00	1.00	0.94	0.92	1.00	0.92	0.92	0.92	0.00
time (sec)	N/A	0.026	0.007	0.003	1.338	1.021	0.130	0.148	0.025	0.001

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	45	46	49	46	56	44	0
N.S.	1	1.00	1.00	0.94	0.96	1.02	0.96	1.17	0.92	0.00
time (sec)	N/A	0.037	0.005	0.007	1.340	0.753	0.172	0.156	0.027	0.001

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	45	45	48	49	45	47	0
N.S.	1	1.00	1.00	0.90	0.90	0.96	0.98	0.90	0.94	0.00
time (sec)	N/A	0.026	0.006	0.005	1.445	0.844	0.170	0.172	0.045	0.001

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	46	48	49	49	59	48	0
N.S.	1	1.00	1.00	0.94	0.98	1.00	1.00	1.20	0.98	0.00
time (sec)	N/A	0.036	0.005	0.007	1.362	0.815	0.220	0.150	0.038	0.001

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	45	47	48	49	47	47	0
N.S.	1	1.00	1.00	0.90	0.94	0.96	0.98	0.94	0.94	0.00
time (sec)	N/A	0.026	0.009	0.006	1.371	0.853	0.227	0.152	0.045	0.001

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	46	48	50	49	57	47	0
N.S.	1	1.00	1.00	0.94	0.98	1.02	1.00	1.16	0.96	0.00
time (sec)	N/A	0.033	0.005	0.007	1.309	0.889	0.304	0.167	0.038	0.001

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	44	46	48	48	46	46	0
N.S.	1	1.00	1.00	0.94	0.98	1.02	1.02	0.98	0.98	0.00
time (sec)	N/A	0.025	0.006	0.006	1.449	0.710	0.297	0.156	4.190	0.001

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	45	50	50	49	58	47	0
N.S.	1	1.00	1.00	0.90	1.00	1.00	0.98	1.16	0.94	0.00
time (sec)	N/A	0.033	0.005	0.005	1.376	0.818	0.368	0.152	0.051	0.001

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	47	48	48	51	48	47	0
N.S.	1	1.00	1.00	0.87	0.89	0.89	0.94	0.89	0.87	0.00
time (sec)	N/A	0.026	0.008	0.007	1.426	0.705	0.369	0.172	0.034	0.001

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	52	47	46	46	49	46	46	0
N.S.	1	1.00	2.74	2.47	2.42	2.42	2.58	2.42	2.42	0.00
time (sec)	N/A	0.006	0.005	0.005	1.346	0.756	0.393	0.184	0.034	0.001

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	47	48	48	51	48	48	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.026	0.007	0.006	1.300	0.660	0.401	0.157	4.840	0.001

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	56	47	48	48	51	48	48	0
N.S.	1	1.00	1.40	1.18	1.20	1.20	1.28	1.20	1.20	0.00
time (sec)	N/A	0.025	0.004	0.005	1.351	0.821	0.428	0.148	4.218	0.001

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	47	48	48	51	48	48	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.027	0.009	0.005	1.463	0.662	0.439	0.214	0.037	0.001

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	47	48	48	51	48	48	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.036	0.004	0.007	1.434	0.594	0.445	0.154	4.329	0.001

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	47	48	48	51	48	48	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.025	0.007	0.005	1.380	0.872	0.458	0.152	4.351	0.001

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	69	68	68	80	68	68	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83	0.00
time (sec)	N/A	0.046	0.003	0.001	1.349	0.713	0.089	0.160	0.033	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	82	69	68	68	78	68	68	0
N.S.	1	1.00	1.14	0.96	0.94	0.94	1.08	0.94	0.94	0.00
time (sec)	N/A	0.117	0.003	0.001	1.431	0.655	0.087	0.152	0.031	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	79	68	67	67	76	67	67	0
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.96	0.85	0.85	0.00
time (sec)	N/A	0.038	0.003	0.001	1.390	0.906	0.088	0.150	0.031	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	82	69	68	68	80	68	68	0
N.S.	1	1.00	1.55	1.30	1.28	1.28	1.51	1.28	1.28	0.00
time (sec)	N/A	0.085	0.003	0.001	1.362	0.586	0.086	0.153	0.033	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	69	68	68	80	68	68	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83	0.00
time (sec)	N/A	0.040	0.003	0.000	1.294	0.774	0.089	0.165	0.033	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	77	68	67	67	75	67	67	0
N.S.	1	1.00	2.26	2.00	1.97	1.97	2.21	1.97	1.97	0.00
time (sec)	N/A	0.046	0.003	0.000	1.361	0.664	0.087	0.170	0.032	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	69	68	68	80	68	68	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83	0.00
time (sec)	N/A	0.038	0.003	0.002	1.357	0.758	0.086	0.151	0.032	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	69	68	68	78	68	68	0
N.S.	1	1.00	1.00	4.31	4.25	4.25	4.88	4.25	4.25	0.00
time (sec)	N/A	0.005	0.002	0.002	1.379	0.692	0.088	0.155	0.031	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	66	100	65	73	65	65	0
N.S.	1	1.00	1.00	0.90	1.37	0.89	1.00	0.89	0.89	0.00
time (sec)	N/A	0.034	0.001	0.001	1.337	0.655	0.083	0.156	0.030	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	76	67	69	66	76	69	66	0
N.S.	1	1.00	1.00	0.88	0.91	0.87	1.00	0.91	0.87	0.00
time (sec)	N/A	0.055	0.004	0.003	1.403	0.786	0.166	0.151	0.036	0.001

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	72	67	66	70	70	66	66	0
N.S.	1	1.00	1.00	0.93	0.92	0.97	0.97	0.92	0.92	0.00
time (sec)	N/A	0.039	0.008	0.005	1.349	0.914	0.158	0.156	0.034	0.001

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	68	69	72	76	79	67	0
N.S.	1	1.00	1.00	0.88	0.90	0.94	0.99	1.03	0.87	0.00
time (sec)	N/A	0.056	0.008	0.007	1.434	0.693	0.196	0.153	0.039	0.001

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	67	67	70	75	67	69	0
N.S.	1	1.00	1.00	0.91	0.91	0.95	1.01	0.91	0.93	0.00
time (sec)	N/A	0.036	0.009	0.005	1.290	0.697	0.207	0.184	0.032	0.001

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	72	67	69	71	73	80	69	0
N.S.	1	1.00	1.00	0.93	0.96	0.99	1.01	1.11	0.96	0.00
time (sec)	N/A	0.052	0.005	0.007	1.366	0.764	0.253	0.159	0.036	0.001

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	72	67	67	70	73	67	69	0
N.S.	1	1.00	1.00	0.93	0.93	0.97	1.01	0.93	0.96	0.00
time (sec)	N/A	0.041	0.007	0.006	1.315	0.778	0.263	0.156	0.033	0.001

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	79	68	70	71	76	81	70	0
N.S.	1	1.00	1.00	0.86	0.89	0.90	0.96	1.03	0.89	0.00
time (sec)	N/A	0.051	0.005	0.006	1.386	0.752	0.325	0.153	4.344	0.001

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	72	67	69	70	73	69	69	0
N.S.	1	1.00	1.00	0.93	0.96	0.97	1.01	0.96	0.96	0.00
time (sec)	N/A	0.037	0.009	0.006	1.381	0.747	0.335	0.151	0.055	0.001

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	68	70	72	73	81	69	0
N.S.	1	1.00	1.00	0.93	0.96	0.99	1.00	1.11	0.95	0.00
time (sec)	N/A	0.052	0.008	0.008	1.314	0.777	0.420	0.156	0.047	0.001

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	67	69	70	73	69	70	0
N.S.	1	1.00	1.00	0.91	0.93	0.95	0.99	0.93	0.95	0.00
time (sec)	N/A	0.039	0.009	0.007	1.413	0.819	0.435	0.240	0.053	0.001

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	68	72	72	75	81	70	0
N.S.	1	1.00	1.00	0.88	0.94	0.94	0.97	1.05	0.91	0.00
time (sec)	N/A	0.051	0.005	0.009	1.381	0.531	0.626	0.151	4.403	0.001

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	71	66	68	70	71	68	68	0
N.S.	1	1.00	1.00	0.93	0.96	0.99	1.00	0.96	0.96	0.00
time (sec)	N/A	0.037	0.006	0.007	1.314	0.629	0.521	0.153	4.300	0.001

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	76	67	72	72	73	80	69	0
N.S.	1	1.00	1.00	0.88	0.95	0.95	0.96	1.05	0.91	0.00
time (sec)	N/A	0.049	0.005	0.007	1.400	0.772	0.618	0.152	0.065	0.001

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	76	69	70	70	75	70	69	0
N.S.	1	1.00	1.00	0.91	0.92	0.92	0.99	0.92	0.91	0.00
time (sec)	N/A	0.040	0.009	0.005	1.340	0.799	0.598	0.177	0.053	0.001

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	82	69	68	68	73	68	70	0
N.S.	1	1.00	4.32	3.63	3.58	3.58	3.84	3.58	3.68	0.00
time (sec)	N/A	0.007	0.008	0.006	1.380	0.806	0.628	0.159	4.357	0.001

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	69	70	70	75	70	70	0
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85	0.00
time (sec)	N/A	0.038	0.009	0.005	1.365	1.075	0.617	0.153	0.051	0.001

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	78	69	70	70	75	70	69	0
N.S.	1	1.00	1.95	1.72	1.75	1.75	1.88	1.75	1.72	0.00
time (sec)	N/A	0.025	0.005	0.006	1.391	0.848	0.680	0.169	4.366	0.001

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	69	70	70	75	70	70	0
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85	0.00
time (sec)	N/A	0.038	0.007	0.006	1.373	0.509	0.661	0.168	0.047	0.001

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	82	69	70	70	75	70	70	0
N.S.	1	1.00	1.32	1.11	1.13	1.13	1.21	1.13	1.13	0.00
time (sec)	N/A	0.039	0.005	0.006	1.336	0.644	0.741	0.166	4.313	0.001

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	69	70	70	75	70	69	0
N.S.	1	1.00	1.00	0.86	0.88	0.88	0.94	0.88	0.86	0.00
time (sec)	N/A	0.038	0.009	0.006	1.354	0.742	0.714	0.177	0.053	0.001

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	82	69	70	70	75	70	70	0
N.S.	1	1.00	0.98	0.82	0.83	0.83	0.89	0.83	0.83	0.00
time (sec)	N/A	0.056	0.008	0.006	1.416	0.829	0.857	0.154	0.053	0.001

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	69	70	70	75	70	70	0
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85	0.00
time (sec)	N/A	0.040	0.010	0.006	1.338	0.850	0.758	0.172	0.053	0.001

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	72	74	77	93	80	92	79	0
N.S.	1	1.00	0.87	0.89	0.93	1.12	0.96	1.11	0.95	0.00
time (sec)	N/A	0.082	0.021	0.009	1.373	0.483	0.323	0.161	4.361	0.001

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	60	63	65	81	66	80	68	0
N.S.	1	1.00	0.86	0.90	0.93	1.16	0.94	1.14	0.97	0.00
time (sec)	N/A	0.063	0.022	0.010	1.353	0.594	0.288	0.173	0.044	0.001

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	49	52	54	70	53	67	57	0
N.S.	1	1.00	0.86	0.91	0.95	1.23	0.93	1.18	1.00	0.00
time (sec)	N/A	0.052	0.015	0.008	1.346	0.751	0.273	0.160	0.055	0.001

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	38	41	43	56	39	49	45	0
N.S.	1	1.00	0.86	0.93	0.98	1.27	0.89	1.11	1.02	0.00
time (sec)	N/A	0.037	0.015	0.006	1.296	0.749	0.253	0.169	0.052	0.001

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	27	30	32	35	29	30	29	0
N.S.	1	1.00	0.82	0.91	0.97	1.06	0.88	0.91	0.88	0.00
time (sec)	N/A	0.028	0.008	0.006	1.414	0.847	0.209	0.162	0.051	0.001

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	15	15	15	14	14	0
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.94	0.88	0.88	0.00
time (sec)	N/A	0.005	0.003	0.003	1.380	0.870	0.165	0.170	4.325	0.001

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	33	35	37	47	34	47	34	0
N.S.	1	1.00	0.87	0.92	0.97	1.24	0.89	1.24	0.89	0.00
time (sec)	N/A	0.038	0.015	0.012	1.359	0.910	0.322	0.152	4.388	0.001

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	41	46	52	73	51	51	51	0
N.S.	1	1.00	0.84	0.94	1.06	1.49	1.04	1.04	1.04	0.00
time (sec)	N/A	0.049	0.036	0.014	1.357	1.008	0.394	0.163	0.077	0.001

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	57	61	70	90	68	86	67	0
N.S.	1	1.00	0.86	0.92	1.06	1.36	1.03	1.30	1.02	0.00
time (sec)	N/A	0.054	0.051	0.014	1.415	1.203	0.467	0.173	0.072	0.001

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	82	78	82	212	134	84	77	0
N.S.	1	1.00	0.89	0.85	0.89	2.30	1.46	0.91	0.84	0.00
time (sec)	N/A	0.056	0.053	0.013	2.992	0.871	0.348	0.153	0.044	0.001

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	71	68	71	190	124	73	66	0
N.S.	1	1.00	0.90	0.86	0.90	2.41	1.57	0.92	0.84	0.00
time (sec)	N/A	0.045	0.046	0.010	2.946	0.889	0.323	0.151	4.274	0.001

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	60	57	59	164	107	61	56	0
N.S.	1	1.00	0.91	0.86	0.89	2.48	1.62	0.92	0.85	0.00
time (sec)	N/A	0.038	0.041	0.009	3.127	0.656	0.303	0.156	0.064	0.001

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	51	43	45	136	83	42	43	0
N.S.	1	1.00	0.93	0.78	0.82	2.47	1.51	0.76	0.78	0.00
time (sec)	N/A	0.027	0.032	0.008	3.083	0.850	0.273	0.153	4.286	0.001

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	36	36	120	78	35	33	0
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73	0.00
time (sec)	N/A	0.017	0.021	0.007	2.995	0.816	0.222	0.197	0.046	0.001

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	36	35	120	78	35	33	0
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73	0.00
time (sec)	N/A	0.017	0.024	0.004	2.846	0.808	0.220	0.153	0.043	0.001

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	54	46	49	136	92	47	44	0
N.S.	1	1.00	0.95	0.81	0.86	2.39	1.61	0.82	0.77	0.00
time (sec)	N/A	0.027	0.036	0.011	2.946	1.873	0.320	0.179	4.489	0.001

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	67	59	64	172	114	59	58	0
N.S.	1	1.00	0.99	0.87	0.94	2.53	1.68	0.87	0.85	0.00
time (sec)	N/A	0.036	0.037	0.011	3.017	0.839	0.358	0.152	4.431	0.001

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	80	70	75	198	126	70	70	0
N.S.	1	1.00	0.99	0.86	0.93	2.44	1.56	0.86	0.86	0.00
time (sec)	N/A	0.046	0.044	0.013	3.083	0.817	0.428	0.154	4.713	0.001

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	78	86	99	137	100	91	98	0
N.S.	1	1.00	0.86	0.95	1.09	1.51	1.10	1.00	1.08	0.00
time (sec)	N/A	0.093	0.031	0.013	1.386	0.836	0.627	0.175	4.483	0.001

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	59	74	88	124	90	73	88	0
N.S.	1	1.00	0.77	0.96	1.14	1.61	1.17	0.95	1.14	0.00
time (sec)	N/A	0.073	0.050	0.013	1.405	0.782	0.586	0.156	4.506	0.001

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	50	64	77	102	76	53	75	0
N.S.	1	1.00	0.70	0.90	1.08	1.44	1.07	0.75	1.06	0.00
time (sec)	N/A	0.064	0.018	0.010	1.310	0.734	0.472	0.168	4.327	0.001

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	35	48	58	58	60	33	60	0
N.S.	1	1.00	1.84	2.53	3.05	3.05	3.16	1.74	3.16	0.00
time (sec)	N/A	0.007	0.013	0.008	1.384	0.486	0.398	0.191	4.287	0.001

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	31	47	47	48	22	48	0
N.S.	1	1.00	0.71	0.91	1.38	1.38	1.41	0.65	1.41	0.00
time (sec)	N/A	0.029	0.009	0.006	1.349	0.537	0.366	0.155	4.229	0.001

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	37	37	39	14	39	0
N.S.	1	1.00	1.00	0.94	2.31	2.31	2.44	0.88	2.44	0.00
time (sec)	N/A	0.005	0.003	0.004	1.339	0.801	0.333	0.153	4.284	0.001

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	54	63	82	134	80	70	78	0
N.S.	1	1.00	0.77	0.90	1.17	1.91	1.14	1.00	1.11	0.00
time (sec)	N/A	0.076	0.041	0.014	1.418	2.417	0.556	0.153	4.467	0.001

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	70	77	99	163	102	93	97	0
N.S.	1	1.00	0.83	0.92	1.18	1.94	1.21	1.11	1.15	0.00
time (sec)	N/A	0.086	0.067	0.015	1.396	0.923	0.673	0.160	0.152	0.001

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	85	96	114	178	116	108	111	0
N.S.	1	1.00	0.84	0.95	1.13	1.76	1.15	1.07	1.10	0.00
time (sec)	N/A	0.098	0.058	0.015	1.421	4.941	0.718	0.156	4.664	0.001

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	99	108	116	322	172	96	109	0
N.S.	1	1.00	0.85	0.92	0.99	2.75	1.47	0.82	0.93	0.00
time (sec)	N/A	0.072	0.056	0.015	2.977	0.577	0.661	0.153	0.062	0.001

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	89	97	104	296	156	84	99	0
N.S.	1	1.00	0.86	0.93	1.00	2.85	1.50	0.81	0.95	0.00
time (sec)	N/A	0.059	0.046	0.013	3.010	1.177	0.628	0.163	4.359	0.001

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	77	83	90	268	131	65	86	0
N.S.	1	1.00	0.83	0.89	0.97	2.88	1.41	0.70	0.92	0.00
time (sec)	N/A	0.047	0.045	0.012	2.846	0.517	0.572	0.159	0.103	0.001

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	66	58	81	254	134	56	78	0
N.S.	1	1.00	0.80	0.70	0.98	3.06	1.61	0.67	0.94	0.00
time (sec)	N/A	0.040	0.038	0.008	3.068	0.847	0.487	0.168	4.388	0.001

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	69	58	87	258	143	62	75	0
N.S.	1	1.00	0.82	0.69	1.04	3.07	1.70	0.74	0.89	0.00
time (sec)	N/A	0.041	0.044	0.010	2.928	0.846	0.453	0.156	4.349	0.001

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	69	58	87	258	139	62	74	0
N.S.	1	1.00	0.81	0.68	1.02	3.04	1.64	0.73	0.87	0.00
time (sec)	N/A	0.043	0.038	0.011	2.985	1.136	0.437	0.185	4.313	0.001

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	66	66	80	254	129	56	77	0
N.S.	1	1.00	0.84	0.84	1.01	3.22	1.63	0.71	0.97	0.00
time (sec)	N/A	0.037	0.035	0.004	3.009	0.801	0.445	0.154	4.356	0.001

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	79	86	93	268	139	68	88	0
N.S.	1	1.00	0.83	0.91	0.98	2.82	1.46	0.72	0.93	0.00
time (sec)	N/A	0.056	0.043	0.014	3.028	0.816	0.584	0.160	4.438	0.001

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	91	99	108	304	162	82	102	0
N.S.	1	1.00	0.86	0.93	1.02	2.87	1.53	0.77	0.96	0.00
time (sec)	N/A	0.066	0.049	0.016	3.051	0.904	0.633	0.168	4.449	0.001

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	101	110	119	330	173	93	114	0
N.S.	1	1.00	0.85	0.92	1.00	2.77	1.45	0.78	0.96	0.00
time (sec)	N/A	0.079	0.052	0.015	3.008	0.874	0.710	0.159	4.465	0.001

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	114	120	143	203	150	113	142	0
N.S.	1	1.00	0.86	0.90	1.08	1.53	1.13	0.85	1.07	0.00
time (sec)	N/A	0.143	0.025	0.015	1.415	0.875	0.995	0.157	0.130	0.001

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	101	109	132	190	138	95	132	0
N.S.	1	1.00	0.86	0.92	1.12	1.61	1.17	0.81	1.12	0.00
time (sec)	N/A	0.114	0.028	0.014	1.469	0.857	0.974	0.232	4.600	0.001

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	72	98	121	168	124	75	119	0
N.S.	1	1.00	0.66	0.90	1.11	1.54	1.14	0.69	1.09	0.00
time (sec)	N/A	0.102	0.024	0.011	1.422	0.770	0.812	0.157	4.370	0.001

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	57	81	102	102	107	55	104	0
N.S.	1	1.00	3.00	4.26	5.37	5.37	5.63	2.89	5.47	0.00
time (sec)	N/A	0.007	0.016	0.008	1.369	0.851	0.718	0.197	4.446	0.001

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	46	65	91	91	95	44	93	0
N.S.	1	1.00	1.18	1.67	2.33	2.33	2.44	1.13	2.38	0.00
time (sec)	N/A	0.026	0.014	0.008	1.393	0.693	0.668	0.159	0.054	0.001

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	48	80	80	83	33	81	0
N.S.	1	1.00	0.66	0.91	1.51	1.51	1.57	0.62	1.53	0.00
time (sec)	N/A	0.045	0.013	0.009	1.347	0.810	0.618	0.192	4.618	0.001

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	31	69	69	71	22	70	0
N.S.	1	1.00	0.71	0.91	2.03	2.03	2.09	0.65	2.06	0.00
time (sec)	N/A	0.031	0.008	0.009	1.355	0.763	0.582	0.159	4.478	0.001

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	59	59	63	14	61	0
N.S.	1	1.00	1.00	0.94	3.69	3.69	3.94	0.88	3.81	0.00
time (sec)	N/A	0.005	0.003	0.005	1.314	0.770	0.500	0.194	0.058	0.001

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	76	91	126	222	128	92	122	0
N.S.	1	1.00	0.75	0.89	1.24	2.18	1.25	0.90	1.20	0.00
time (sec)	N/A	0.105	0.059	0.015	1.450	0.793	0.814	0.152	0.238	0.001

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	92	107	143	251	150	115	141	0
N.S.	1	1.00	0.79	0.92	1.23	2.16	1.29	0.99	1.22	0.00
time (sec)	N/A	0.126	0.084	0.018	1.581	0.808	0.901	0.159	4.677	0.001

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	107	129	158	266	165	130	155	0
N.S.	1	1.00	0.76	0.92	1.13	1.90	1.18	0.93	1.11	0.00
time (sec)	N/A	0.145	0.061	0.018	1.448	0.886	0.956	0.159	4.912	0.001

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	122	148	159	454	218	117	153	0
N.S.	1	1.00	0.79	0.95	1.03	2.93	1.41	0.75	0.99	0.00
time (sec)	N/A	0.106	0.064	0.016	2.885	0.816	1.086	0.169	0.106	0.001

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	111	137	148	428	204	106	143	0
N.S.	1	1.00	0.78	0.96	1.04	3.01	1.44	0.75	1.01	0.00
time (sec)	N/A	0.093	0.058	0.017	2.909	0.899	1.032	0.160	4.520	0.001

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	100	123	134	400	178	87	130	0
N.S.	1	1.00	0.76	0.94	1.02	3.05	1.36	0.66	0.99	0.00
time (sec)	N/A	0.078	0.052	0.016	2.989	0.790	0.947	0.171	0.158	0.001

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	88	80	125	386	182	78	122	0
N.S.	1	1.00	0.73	0.66	1.03	3.19	1.50	0.64	1.01	0.00
time (sec)	N/A	0.069	0.050	0.013	3.070	0.860	0.789	0.185	4.522	0.001

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	91	80	131	390	194	84	119	0
N.S.	1	1.00	0.75	0.66	1.07	3.20	1.59	0.69	0.98	0.00
time (sec)	N/A	0.072	0.058	0.012	3.027	0.851	0.757	0.160	4.421	0.001

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	91	78	133	390	196	84	117	0
N.S.	1	1.00	0.74	0.63	1.08	3.17	1.59	0.68	0.95	0.00
time (sec)	N/A	0.072	0.060	0.012	2.957	0.861	0.702	0.157	4.504	0.001

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	91	78	133	390	196	84	116	0
N.S.	1	1.00	0.73	0.63	1.07	3.15	1.58	0.68	0.94	0.00
time (sec)	N/A	0.073	0.050	0.012	3.039	1.481	0.636	0.164	4.468	0.001

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	91	80	131	390	190	84	118	0
N.S.	1	1.00	0.73	0.64	1.05	3.12	1.52	0.67	0.94	0.00
time (sec)	N/A	0.075	0.051	0.012	2.966	0.874	0.624	0.170	4.482	0.001

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	89	96	124	386	177	78	121	0
N.S.	1	1.00	0.79	0.85	1.10	3.42	1.57	0.69	1.07	0.00
time (sec)	N/A	0.066	0.045	0.006	3.042	0.732	0.676	0.168	4.707	0.001

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	101	126	137	400	187	90	132	0
N.S.	1	1.00	0.76	0.95	1.03	3.01	1.41	0.68	0.99	0.00
time (sec)	N/A	0.089	0.055	0.017	3.121	3.126	0.828	0.158	4.581	0.001

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	113	139	152	436	209	104	146	0
N.S.	1	1.00	0.78	0.97	1.06	3.03	1.45	0.72	1.01	0.00
time (sec)	N/A	0.103	0.060	0.019	3.096	0.885	0.888	0.159	4.623	0.001

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	123	150	163	462	221	115	158	0
N.S.	1	1.00	0.78	0.96	1.04	2.94	1.41	0.73	1.01	0.00
time (sec)	N/A	0.122	0.063	0.019	3.107	0.833	0.949	0.164	4.648	0.001

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	16	16	15	19	12	15	16	0
N.S.	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	0.84	0.00
time (sec)	N/A	0.003	0.006	0.005	3.056	1.979	0.110	0.149	0.034	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	8	9	11	0
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.00	0.00
time (sec)	N/A	0.002	0.001	0.005	1.347	0.847	0.086	0.149	0.018	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	21	12	15	17	0
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89	0.00
time (sec)	N/A	0.005	0.008	0.006	2.969	0.826	0.105	0.157	0.027	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	18	19	18	23	15	18	18	0
N.S.	1	1.00	0.82	0.86	0.82	1.05	0.68	0.82	0.82	0.00
time (sec)	N/A	0.011	0.005	0.004	1.276	0.857	0.094	0.196	0.035	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	11	10	9	9	8	9	11	0
N.S.	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	0.85	0.00
time (sec)	N/A	0.002	0.002	0.004	1.330	0.784	0.088	0.162	0.046	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	20	19	18	23	14	19	18	0
N.S.	1	1.00	0.83	0.79	0.75	0.96	0.58	0.79	0.75	0.00
time (sec)	N/A	0.015	0.006	0.005	1.319	1.150	0.099	0.151	4.228	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	13	13	12	29	71	39
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	0.90	0.49
time (sec)	N/A	0.064	0.015	0.008	1.353	0.761	0.105	0.157	4.455	6.678

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	39	36	13	13	12	23	59	39
N.S.	1	1.00	0.58	0.54	0.19	0.19	0.18	0.34	0.88	0.58
time (sec)	N/A	0.052	0.007	0.004	1.281	1.071	0.103	0.184	4.310	6.188

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	38	35	14	13	12	22	33	38
N.S.	1	1.00	1.06	0.97	0.39	0.36	0.33	0.61	0.92	1.06
time (sec)	N/A	0.026	0.007	0.010	1.321	0.669	0.098	0.157	4.352	5.684

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	37	34	14	11	10	30	109	197
N.S.	1	1.00	0.49	0.45	0.19	0.15	0.13	0.40	1.45	2.63
time (sec)	N/A	0.022	0.012	0.014	1.385	0.633	0.123	0.158	4.394	0.205

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	39	38	14	17	10	45	112	734
N.S.	1	1.00	0.52	0.51	0.19	0.23	0.13	0.60	1.49	9.79
time (sec)	N/A	0.022	0.010	0.009	1.402	1.966	0.150	0.235	4.450	0.515

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	37	34	13	13	14	30	33	118
N.S.	1	1.00	0.95	0.87	0.33	0.33	0.36	0.77	0.85	3.03
time (sec)	N/A	0.038	0.008	0.003	1.333	0.906	0.168	0.157	4.212	0.380

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	39	36	15	15	15	31	35	751
N.S.	1	1.00	0.54	0.50	0.21	0.21	0.21	0.43	0.49	10.43
time (sec)	N/A	0.016	0.008	0.003	1.303	0.872	0.190	0.155	4.238	3.254

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	266
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	3.37
time (sec)	N/A	0.059	0.008	0.005	1.351	0.820	0.203	0.158	4.242	0.581

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	312
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	3.95
time (sec)	N/A	0.058	0.008	0.004	1.310	0.815	0.216	0.164	4.215	0.640

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	13	13	12	29	-1	39
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01	0.49
time (sec)	N/A	0.023	0.007	0.004	1.348	0.772	0.100	0.166	0.000	4.535

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	13	13	12	29	-1	39
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01	0.49
time (sec)	N/A	0.023	0.007	0.003	1.307	1.372	0.148	0.154	0.000	4.350

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	36	33	10	10	8	20	-1	36
N.S.	1	1.00	0.49	0.45	0.14	0.14	0.11	0.27	-0.01	0.49
time (sec)	N/A	0.014	0.007	0.003	1.319	0.763	0.099	0.185	0.000	4.221

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	35	34	10	13	5	26	-1	35
N.S.	1	1.00	0.49	0.47	0.14	0.18	0.07	0.36	-0.01	0.49
time (sec)	N/A	0.020	0.008	0.003	1.345	1.101	0.125	0.156	0.000	8.057

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	37	34	13	13	14	30	33	39
N.S.	1	1.00	0.48	0.44	0.17	0.17	0.18	0.39	0.43	0.51
time (sec)	N/A	0.023	0.007	0.003	1.383	0.590	0.162	0.173	4.244	13.393

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.021	0.008	0.004	1.306	0.869	0.181	0.152	4.211	16.713

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.022	0.008	0.002	1.271	1.396	0.197	0.156	4.184	19.608

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.022	0.008	0.006	1.294	0.847	0.215	0.161	4.195	21.360

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.114	0.019	0.007	1.315	0.796	0.000	0.156	0.000	12.315

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.114	0.016	0.007	1.317	0.795	0.000	0.158	0.000	10.515

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	119	61	58	35	35	0	67	-1	61
N.S.	1	1.12	0.58	0.55	0.33	0.33	0.00	0.63	-0.01	0.58
time (sec)	N/A	0.083	0.016	0.007	1.337	0.806	0.000	0.219	0.000	9.450

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	61	58	35	35	0	45	46	61
N.S.	1	1.00	0.91	0.87	0.52	0.52	0.00	0.67	0.69	0.91
time (sec)	N/A	0.051	0.016	0.006	1.326	0.902	0.000	0.154	4.286	8.593

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	27	57	35	35	0	44	36	60
N.S.	1	1.00	0.75	1.58	0.97	0.97	0.00	1.22	1.00	1.67
time (sec)	N/A	0.026	0.012	0.005	1.356	0.793	0.000	0.151	4.253	8.061

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	60	57	33	33	0	68	-1	256
N.S.	1	1.00	0.37	0.35	0.20	0.20	0.00	0.42	-0.01	1.57
time (sec)	N/A	0.049	0.021	0.008	1.328	0.644	0.000	0.191	0.000	0.383

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	62	59	34	38	0	87	-1	320
N.S.	1	1.00	0.38	0.36	0.21	0.23	0.00	0.53	-0.01	1.95
time (sec)	N/A	0.047	0.021	0.012	1.292	0.812	0.000	0.161	0.000	0.660

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	61	60	34	39	0	87	-1	1170
N.S.	1	1.00	0.37	0.37	0.21	0.24	0.00	0.53	-0.01	7.13
time (sec)	N/A	0.049	0.016	0.014	1.312	1.477	0.000	0.173	0.000	1.546

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	63	60	33	39	0	87	-1	944
N.S.	1	1.00	0.39	0.37	0.20	0.24	0.00	0.53	-0.01	5.79
time (sec)	N/A	0.046	0.021	0.014	1.261	0.631	0.000	0.188	0.000	4.937

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	59	56	35	35	0	68	151	306
N.S.	1	1.00	1.44	1.37	0.85	0.85	0.00	1.66	3.68	7.46
time (sec)	N/A	0.039	0.016	0.005	1.358	0.763	0.000	0.174	4.243	1.189

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	61	58	35	37	0	69	151	356
N.S.	1	1.00	0.85	0.81	0.49	0.51	0.00	0.96	2.10	4.94
time (sec)	N/A	0.017	0.014	0.008	1.426	0.850	0.000	0.207	4.203	0.947

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	37	0	69	151	400
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90	2.40
time (sec)	N/A	0.105	0.014	0.007	1.310	0.968	0.000	0.162	4.207	1.053

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	37	0	69	151	444
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90	2.66
time (sec)	N/A	0.106	0.017	0.009	1.133	1.085	0.000	0.156	4.212	1.167

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	37	0	69	151	488
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90	2.92
time (sec)	N/A	0.105	0.014	0.008	1.323	0.829	0.000	0.183	4.234	1.276

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.042	0.015	0.009	1.298	0.800	0.000	0.159	0.000	7.301

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.041	0.015	0.008	1.268	0.809	0.000	0.158	0.000	6.456

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.043	0.015	0.007	1.393	0.831	0.000	0.181	0.000	5.880

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.041	0.012	0.007	1.347	0.528	0.000	0.155	0.000	5.528

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	59	56	31	31	0	63	-1	59
N.S.	1	1.00	0.37	0.35	0.19	0.19	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.033	0.012	0.004	1.364	0.791	0.000	0.184	0.000	5.302

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	60	58	32	36	0	64	-1	60
N.S.	1	1.00	0.38	0.37	0.20	0.23	0.00	0.41	-0.01	0.38
time (sec)	N/A	0.040	0.015	0.007	1.342	0.611	0.000	0.163	0.000	9.705

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	59	56	33	36	0	67	-1	60
N.S.	1	1.00	0.37	0.35	0.20	0.22	0.00	0.42	-0.01	0.37
time (sec)	N/A	0.040	0.014	0.007	1.290	0.736	0.000	0.158	0.000	14.485

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	59	56	32	37	0	66	-1	61
N.S.	1	1.00	0.37	0.35	0.20	0.23	0.00	0.42	-0.01	0.39
time (sec)	N/A	0.040	0.014	0.006	1.323	0.814	0.000	0.190	0.000	17.502

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	61	58	35	37	0	69	151	61
N.S.	1	1.00	0.37	0.36	0.21	0.23	0.00	0.42	0.93	0.37
time (sec)	N/A	0.043	0.014	0.006	1.273	0.768	0.000	0.219	4.255	17.904

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.039	0.014	0.007	1.338	1.014	0.000	0.190	4.264	18.942

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.044	0.016	0.007	1.303	1.283	0.000	0.157	4.549	19.618

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.040	0.014	0.007	1.390	0.772	0.000	0.157	4.630	20.499

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.041	0.014	0.009	1.294	1.198	0.000	0.206	4.297	21.967

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.162	0.027	0.010	1.319	0.775	0.000	0.157	0.000	30.123

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.159	0.023	0.010	1.374	0.737	0.000	0.154	0.000	22.749

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.41	0.40	0.28	0.28	0.00	0.52	-0.00	0.41
time (sec)	N/A	0.132	0.020	0.009	1.433	1.024	0.000	0.160	0.000	17.018

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.52	0.50	0.36	0.36	0.00	0.66	-0.01	0.52
time (sec)	N/A	0.118	0.022	0.008	1.400	1.109	0.000	0.156	0.000	14.638

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	83	80	56	56	0	104	-1	83
N.S.	1	1.00	0.70	0.67	0.47	0.47	0.00	0.87	-0.01	0.70
time (sec)	N/A	0.097	0.022	0.008	1.336	0.516	0.000	0.193	0.000	13.032

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	83	80	56	56	0	67	-1	83
N.S.	1	1.00	1.24	1.19	0.84	0.84	0.00	1.00	-0.01	1.24
time (sec)	N/A	0.052	0.021	0.008	1.249	1.120	0.000	0.202	0.000	12.214

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	27	79	57	57	0	66	36	82
N.S.	1	1.00	0.75	2.19	1.58	1.58	0.00	1.83	1.00	2.28
time (sec)	N/A	0.026	0.014	0.006	1.311	0.805	0.000	0.158	4.404	10.476

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	82	79	55	55	0	106	-1	314
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.42	-0.00	1.25
time (sec)	N/A	0.071	0.025	0.011	1.361	0.863	0.000	0.193	0.000	0.565

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	85	82	56	61	0	125	-1	364
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.50	-0.00	1.46
time (sec)	N/A	0.071	0.026	0.012	1.339	0.846	0.000	0.162	0.000	0.955

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	85	82	56	61	0	127	-1	366
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.51	-0.00	1.46
time (sec)	N/A	0.070	0.026	0.014	1.341	0.829	0.000	0.159	0.000	1.038

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	85	82	56	61	0	128	-1	364
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.51	-0.00	1.46
time (sec)	N/A	0.072	0.024	0.013	1.384	0.965	0.000	0.158	0.000	1.795

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	85	82	56	61	0	126	-1	2027
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.50	-0.00	8.11
time (sec)	N/A	0.071	0.020	0.013	1.359	0.832	0.000	0.192	0.000	2.753

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	85	82	55	61	0	125	-1	2386
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.50	-0.00	9.51
time (sec)	N/A	0.068	0.027	0.014	1.325	0.605	0.000	0.164	0.000	3.603

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	81	78	57	57	0	106	231	442
N.S.	1	1.00	1.98	1.90	1.39	1.39	0.00	2.59	5.63	10.78
time (sec)	N/A	0.039	0.018	0.007	1.396	0.870	0.000	0.158	4.178	2.632

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	83	80	57	59	0	107	231	488
N.S.	1	1.00	1.15	1.11	0.79	0.82	0.00	1.49	3.21	6.78
time (sec)	N/A	0.017	0.021	0.009	1.347	1.263	0.000	0.161	4.219	1.337

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	83	80	57	59	0	107	231	532
N.S.	1	1.00	0.65	0.62	0.45	0.46	0.00	0.84	1.80	4.16
time (sec)	N/A	0.091	0.018	0.008	1.351	0.808	0.000	0.213	4.235	1.444

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	59	0	107	231	576
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91	2.26
time (sec)	N/A	0.155	0.017	0.009	1.330	0.822	0.000	0.170	4.266	1.594

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	59	0	107	231	620
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91	2.43
time (sec)	N/A	0.151	0.018	0.009	1.340	0.731	0.000	0.163	4.225	1.669

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	59	0	107	231	664
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91	2.60
time (sec)	N/A	0.153	0.018	0.010	1.340	0.956	0.000	0.186	4.233	1.779

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	59	0	107	231	708
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91	2.78
time (sec)	N/A	0.151	0.022	0.011	1.376	0.756	0.000	0.166	4.223	1.905

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.062	0.022	0.008	1.283	1.066	0.000	0.157	0.000	19.388

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.058	0.020	0.009	1.355	0.975	0.000	0.160	0.000	13.147

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.061	0.020	0.007	1.327	0.466	0.000	0.164	0.000	10.428

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.058	0.020	0.008	1.288	0.942	0.000	0.172	0.000	8.623

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.059	0.020	0.010	1.334	0.968	0.000	0.158	0.000	7.545

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	83	80	56	56	0	104	-1	83
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.060	0.020	0.008	1.364	0.987	0.000	0.180	0.000	6.754

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	81	78	54	54	0	102	-1	81
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.051	0.016	0.004	1.319	0.805	0.000	0.162	0.000	6.349

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	83	80	55	59	0	103	-1	83
N.S.	1	1.00	0.34	0.32	0.22	0.24	0.00	0.42	-0.00	0.34
time (sec)	N/A	0.058	0.020	0.007	1.324	0.853	0.000	0.162	0.000	10.720

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	83	80	54	59	0	104	-1	83
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.42	-0.00	0.34
time (sec)	N/A	0.060	0.023	0.007	1.329	0.649	0.000	0.160	0.000	15.892

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	249	83	80	55	59	0	106	-1	83
N.S.	1	1.00	0.33	0.32	0.22	0.24	0.00	0.43	-0.00	0.33
time (sec)	N/A	0.058	0.020	0.007	1.347	0.870	0.000	0.174	0.000	18.615

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	83	80	55	59	0	106	-1	83
N.S.	1	1.00	0.34	0.32	0.22	0.24	0.00	0.43	-0.00	0.34
time (sec)	N/A	0.058	0.018	0.007	1.287	0.851	0.000	0.159	0.000	18.689

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	83	80	54	59	0	105	-1	83
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.43	-0.00	0.34
time (sec)	N/A	0.058	0.018	0.006	1.350	0.822	0.000	0.168	0.000	18.685

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	83	80	57	59	0	107	231	83
N.S.	1	1.00	0.33	0.32	0.23	0.24	0.00	0.43	0.92	0.33
time (sec)	N/A	0.058	0.017	0.008	1.335	0.942	0.000	0.159	4.221	18.963

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	83	80	57	59	0	107	231	83
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.060	0.017	0.007	1.335	1.065	0.000	0.178	4.350	19.154

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.058	0.017	0.009	1.352	0.716	0.000	0.160	4.208	19.784

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.059	0.017	0.009	1.355	0.771	0.000	0.168	4.315	21.835

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.058	0.019	0.008	1.345	0.737	0.000	0.199	4.267	22.459

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.058	0.018	0.009	1.337	1.513	0.000	0.161	4.337	24.640

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.058	0.018	0.010	1.376	1.340	0.000	0.160	4.311	28.041

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	55	52	34	33	32	59	-1	182
N.S.	1	1.00	0.43	0.41	0.27	0.26	0.25	0.46	-0.01	1.43
time (sec)	N/A	0.102	0.024	0.009	1.389	0.707	0.196	0.181	0.000	0.292

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	44	41	23	22	20	33	64	156
N.S.	1	1.00	0.59	0.55	0.31	0.29	0.27	0.44	0.85	2.08
time (sec)	N/A	0.056	0.012	0.007	1.311	0.673	0.183	0.171	4.519	0.223

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	35	32	13	13	10	22	33	149
N.S.	1	1.00	0.80	0.73	0.30	0.30	0.23	0.50	0.75	3.39
time (sec)	N/A	0.032	0.008	0.004	1.351	1.625	0.154	0.153	4.415	0.255

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	42	39	23	18	15	33	40	94
N.S.	1	1.00	0.52	0.49	0.29	0.22	0.19	0.41	0.50	1.18
time (sec)	N/A	0.034	0.011	0.008	1.281	0.889	0.262	0.152	4.451	0.186

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	122	54	51	33	33	31	52	75	380
N.S.	1	0.98	0.43	0.41	0.26	0.26	0.25	0.42	0.60	3.04
time (sec)	N/A	0.050	0.015	0.012	1.285	0.578	0.318	0.156	4.454	0.603

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	66	63	37	99	80	64	-1	63
N.S.	1	1.00	0.51	0.49	0.29	0.77	0.62	0.50	-0.01	0.49
time (sec)	N/A	0.046	0.026	0.009	2.931	3.012	0.214	0.160	0.000	4.970

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	54	48	26	82	56	42	-1	52
N.S.	1	1.00	0.61	0.54	0.29	0.92	0.63	0.47	-0.01	0.58
time (sec)	N/A	0.032	0.014	0.007	2.914	0.788	0.193	0.231	0.000	4.356

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	44	34	15	67	53	23	-1	44
N.S.	1	1.00	0.83	0.64	0.28	1.26	1.00	0.43	-0.02	0.83
time (sec)	N/A	0.015	0.012	0.005	3.030	1.274	0.177	0.177	0.000	4.169

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	56	50	29	82	65	37	-1	55
N.S.	1	1.00	0.61	0.54	0.32	0.89	0.71	0.40	-0.01	0.60
time (sec)	N/A	0.033	0.014	0.009	2.941	1.564	0.232	0.153	0.000	8.784

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	130	70	69	40	106	87	50	-1	66
N.S.	1	0.98	0.53	0.52	0.30	0.80	0.65	0.38	-0.01	0.50
time (sec)	N/A	0.043	0.025	0.011	3.010	1.195	0.279	0.159	0.000	17.683

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	81	103	66	91	0	83	-1	1386
N.S.	1	1.00	0.51	0.65	0.42	0.58	0.00	0.53	-0.01	8.77
time (sec)	N/A	0.131	0.031	0.018	1.359	1.493	0.000	0.248	0.000	1.264

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	61	81	55	69	0	64	-1	1590
N.S.	1	1.00	0.54	0.72	0.49	0.61	0.00	0.57	-0.01	14.07
time (sec)	N/A	0.098	0.022	0.015	1.365	1.029	0.000	0.227	0.000	1.173

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	69	39	32	36	36	0	32	42	157
N.S.	1	1.68	0.95	0.78	0.88	0.88	0.00	0.78	1.02	3.83
time (sec)	N/A	0.050	0.012	0.006	1.385	0.716	0.000	0.227	4.244	0.570

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	24	26	26	0	24	34	137
N.S.	1	1.00	0.71	0.63	0.68	0.68	0.00	0.63	0.89	3.61
time (sec)	N/A	0.026	0.008	0.004	1.322	0.875	0.000	0.204	4.343	0.521

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	74	107	57	90	0	79	-1	755
N.S.	1	1.00	0.50	0.73	0.39	0.61	0.00	0.54	-0.01	5.14
time (sec)	N/A	0.083	0.027	0.018	1.441	2.924	0.000	0.273	0.000	2.251

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	97	133	75	119	0	96	-1	796
N.S.	1	1.00	0.51	0.70	0.40	0.63	0.00	0.51	-0.01	4.21
time (sec)	N/A	0.096	0.037	0.021	1.369	1.055	0.000	0.236	0.000	3.301

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	84	97	59	188	0	0	-1	76
N.S.	1	1.00	0.66	0.76	0.46	1.47	0.00	0.00	-0.01	0.59
time (sec)	N/A	0.053	0.032	0.016	3.031	1.333	0.000	0.000	0.000	7.061

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	81	97	62	190	0	0	-1	77
N.S.	1	1.00	0.63	0.75	0.48	1.47	0.00	0.00	-0.01	0.60
time (sec)	N/A	0.049	0.028	0.018	2.935	1.213	0.000	0.000	0.000	5.915

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	83	97	58	188	0	0	-1	76
N.S.	1	1.00	0.61	0.72	0.43	1.39	0.00	0.00	-0.01	0.56
time (sec)	N/A	0.038	0.024	0.008	2.987	1.162	0.000	0.000	0.000	6.006

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	93	119	71	202	0	0	-1	89
N.S.	1	1.00	0.55	0.70	0.42	1.20	0.00	0.00	-0.01	0.53
time (sec)	N/A	0.065	0.032	0.018	2.979	1.298	0.000	0.000	0.000	11.669

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	105	139	86	238	0	0	-1	100
N.S.	1	1.00	0.50	0.67	0.41	1.14	0.00	0.00	-0.00	0.48
time (sec)	N/A	0.079	0.038	0.021	3.035	2.225	0.000	0.000	0.000	19.089

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	103	163	110	157	0	105	-1	2541
N.S.	1	1.00	0.43	0.68	0.46	0.66	0.00	0.44	-0.00	10.68
time (sec)	N/A	0.189	0.039	0.021	1.383	0.628	0.000	0.250	0.000	2.504

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	83	141	99	135	0	84	-1	3027
N.S.	1	1.00	0.42	0.72	0.51	0.69	0.00	0.43	-0.01	15.44
time (sec)	N/A	0.162	0.029	0.017	1.458	2.771	0.000	0.211	0.000	2.282

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	61	54	80	80	0	54	144	274
N.S.	1	1.00	1.49	1.32	1.95	1.95	0.00	1.32	3.51	6.68
time (sec)	N/A	0.040	0.017	0.007	1.404	1.144	0.000	0.217	4.291	0.733

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	50	43	69	69	0	43	53	256
N.S.	1	1.00	0.68	0.58	0.93	0.93	0.00	0.58	0.72	3.46
time (sec)	N/A	0.016	0.016	0.008	1.788	2.673	0.000	0.213	4.235	0.687

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	39	32	58	58	0	32	42	224
N.S.	1	1.00	0.57	0.46	0.84	0.84	0.00	0.46	0.61	3.25
time (sec)	N/A	0.054	0.013	0.008	1.364	1.212	0.000	0.214	4.260	0.688

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	24	48	48	0	24	34	200
N.S.	1	1.00	0.71	0.63	1.26	1.26	0.00	0.63	0.89	5.26
time (sec)	N/A	0.025	0.010	0.007	1.316	4.600	0.000	0.208	4.273	0.749

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	96	193	101	178	0	101	-1	3893
N.S.	1	1.00	0.43	0.87	0.45	0.80	0.00	0.45	-0.00	17.46
time (sec)	N/A	0.121	0.043	0.019	1.439	1.101	0.000	0.266	0.000	112.392

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	119	219	119	207	0	118	-1	2844
N.S.	1	1.00	0.45	0.82	0.45	0.78	0.00	0.44	-0.00	10.65
time (sec)	N/A	0.142	0.047	0.023	1.421	0.672	0.000	0.222	0.000	81.204

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	105	172	109	324	0	0	-1	101
N.S.	1	1.00	0.50	0.82	0.52	1.54	0.00	0.00	-0.00	0.48
time (sec)	N/A	0.085	0.043	0.018	3.033	3.007	0.000	0.000	0.000	10.581

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	105	172	111	324	0	0	-1	101
N.S.	1	1.00	0.50	0.81	0.52	1.53	0.00	0.00	-0.00	0.48
time (sec)	N/A	0.088	0.040	0.018	2.994	1.665	0.000	0.000	0.000	14.149

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	105	172	109	324	0	0	-1	101
N.S.	1	1.00	0.49	0.81	0.51	1.52	0.00	0.00	-0.00	0.47
time (sec)	N/A	0.084	0.034	0.017	2.902	1.270	0.000	0.000	0.000	17.354

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	105	169	102	320	0	0	-1	98
N.S.	1	1.00	0.49	0.79	0.48	1.50	0.00	0.00	-0.00	0.46
time (sec)	N/A	0.072	0.039	0.008	3.051	2.216	0.000	0.000	0.000	14.074

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	115	191	115	334	0	0	-1	111
N.S.	1	1.00	0.46	0.76	0.46	1.33	0.00	0.00	-0.00	0.44
time (sec)	N/A	0.109	0.044	0.022	3.039	3.021	0.000	0.000	0.000	16.442

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	127	211	130	370	0	0	-1	122
N.S.	1	1.00	0.44	0.73	0.45	1.27	0.00	0.00	-0.00	0.42
time (sec)	N/A	0.125	0.048	0.024	3.135	0.957	0.000	0.000	0.000	24.283

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	33	30	41	40	49	48	40	44
N.S.	1	1.00	0.65	0.59	0.80	0.78	0.96	0.94	0.78	0.86
time (sec)	N/A	0.014	0.015	0.008	1.305	1.013	2.671	0.167	0.074	0.034

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	33	30	41	34	49	42	41	49
N.S.	1	1.00	0.65	0.59	0.80	0.67	0.96	0.82	0.80	0.96
time (sec)	N/A	0.014	0.012	0.008	1.341	1.686	1.244	0.149	4.224	0.033

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	33	30	41	29	49	37	41	44
N.S.	1	1.00	0.65	0.59	0.80	0.57	0.96	0.73	0.80	0.86
time (sec)	N/A	0.013	0.009	0.008	1.364	1.251	0.482	0.150	0.050	0.030

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	33	30	41	31	48	41	41	49
N.S.	1	1.00	0.67	0.61	0.84	0.63	0.98	0.84	0.84	1.00
time (sec)	N/A	0.013	0.010	0.009	1.274	1.359	0.628	0.151	0.045	0.036

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	33	30	44	34	48	51	31	44
N.S.	1	1.00	0.67	0.61	0.90	0.69	0.98	1.04	0.63	0.90
time (sec)	N/A	0.015	0.012	0.009	1.357	1.922	0.665	0.221	0.052	0.038

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	33	30	43	34	48	53	34	44
N.S.	1	1.00	0.67	0.61	0.88	0.69	0.98	1.08	0.69	0.90
time (sec)	N/A	0.014	0.012	0.008	1.403	0.976	0.913	0.154	4.231	0.039

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	38	30	47	34	48	48	34	44
N.S.	1	1.00	0.78	0.61	0.96	0.69	0.98	0.98	0.69	0.90
time (sec)	N/A	0.013	0.013	0.006	1.301	1.025	1.969	0.155	0.050	0.047

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	55	52	73	68	90	86	71	85
N.S.	1	1.00	0.60	0.57	0.80	0.75	0.99	0.95	0.78	0.93
time (sec)	N/A	0.045	0.022	0.008	1.350	1.494	5.681	0.155	4.199	0.052

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	55	52	73	58	90	74	71	85
N.S.	1	1.00	0.60	0.57	0.80	0.64	0.99	0.81	0.78	0.93
time (sec)	N/A	0.044	0.019	0.009	1.350	2.234	2.691	0.171	0.030	0.054

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	55	52	73	51	90	69	71	85
N.S.	1	1.00	0.60	0.57	0.80	0.56	0.99	0.76	0.78	0.93
time (sec)	N/A	0.041	0.014	0.008	1.333	1.800	1.275	0.158	0.029	0.050

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	55	52	90	53	88	73	71	85
N.S.	1	1.00	0.62	0.58	1.01	0.60	0.99	0.82	0.80	0.96
time (sec)	N/A	0.041	0.016	0.009	1.357	0.875	1.346	0.260	0.031	0.050

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	55	52	76	56	88	89	71	72
N.S.	1	1.00	0.62	0.58	0.85	0.63	0.99	1.00	0.80	0.81
time (sec)	N/A	0.042	0.017	0.009	1.211	1.192	1.377	0.177	0.033	0.060

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	55	52	76	56	88	92	71	72
N.S.	1	1.00	0.62	0.58	0.85	0.63	0.99	1.03	0.80	0.81
time (sec)	N/A	0.042	0.017	0.008	1.355	0.846	1.737	0.158	0.030	0.048

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	60	52	82	56	87	95	75	72
N.S.	1	1.00	0.69	0.60	0.94	0.64	1.00	1.09	0.86	0.83
time (sec)	N/A	0.042	0.020	0.008	1.438	1.815	2.474	0.175	0.058	0.055

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	77	74	105	96	129	124	103	121
N.S.	1	1.00	0.60	0.57	0.81	0.74	1.00	0.96	0.80	0.94
time (sec)	N/A	0.066	0.030	0.009	1.401	1.709	10.798	0.161	0.039	0.065

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	77	74	105	82	131	106	103	121
N.S.	1	1.00	0.59	0.56	0.80	0.63	1.00	0.81	0.79	0.92
time (sec)	N/A	0.061	0.025	0.010	1.333	0.844	5.338	0.191	0.037	0.061

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	77	74	105	73	131	101	103	121
N.S.	1	1.00	0.59	0.56	0.80	0.56	1.00	0.77	0.79	0.92
time (sec)	N/A	0.061	0.021	0.008	1.392	2.070	3.003	0.175	0.038	0.059

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	77	74	155	75	129	105	103	121
N.S.	1	1.00	0.60	0.57	1.20	0.58	1.00	0.81	0.80	0.94
time (sec)	N/A	0.060	0.023	0.009	1.361	0.953	2.938	0.180	0.037	0.057

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	77	74	108	78	126	127	103	100
N.S.	1	1.00	0.62	0.59	0.86	0.62	1.01	1.02	0.82	0.80
time (sec)	N/A	0.060	0.022	0.010	1.389	0.994	3.006	0.199	0.039	0.061

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	77	74	108	78	128	130	103	100
N.S.	1	1.00	0.61	0.58	0.85	0.61	1.01	1.02	0.81	0.79
time (sec)	N/A	0.062	0.022	0.009	1.388	1.313	3.571	0.169	0.038	0.061

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	82	74	114	78	128	133	107	100
N.S.	1	1.00	0.65	0.58	0.90	0.61	1.01	1.05	0.84	0.79
time (sec)	N/A	0.064	0.027	0.010	1.346	0.892	4.556	0.174	0.039	0.060

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	235	242	300	283	0	297	129	218
N.S.	1	1.00	0.74	0.77	0.95	0.90	0.00	0.94	0.41	0.69
time (sec)	N/A	0.383	0.340	0.023	3.037	2.073	0.000	0.213	4.273	0.497

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	63	226	273	283	0	277	112	200
N.S.	1	1.00	0.21	0.76	0.92	0.95	0.00	0.93	0.38	0.67
time (sec)	N/A	0.299	0.019	0.018	3.123	1.637	0.000	0.192	0.124	0.508

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	244	223	282	247	0	263	112	200
N.S.	1	1.00	0.82	0.75	0.95	0.83	0.00	0.88	0.38	0.67
time (sec)	N/A	0.295	0.163	0.018	3.029	1.127	0.000	0.189	0.120	0.479

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	54	209	256	247	0	277	92	183
N.S.	1	1.00	0.19	0.74	0.91	0.88	0.00	0.99	0.33	0.65
time (sec)	N/A	0.280	0.016	0.016	3.103	1.569	0.000	0.232	4.252	0.476

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	210	212	265	234	0	261	92	183
N.S.	1	1.00	0.75	0.75	0.94	0.83	0.00	0.93	0.33	0.65
time (sec)	N/A	0.260	0.170	0.014	3.059	1.194	0.000	0.234	4.355	0.475

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	283	283	32	210	255	232	78	264	90	181
N.S.	1	1.00	0.11	0.74	0.90	0.82	0.28	0.93	0.32	0.64
time (sec)	N/A	0.271	0.006	0.012	3.055	1.598	6.667	0.190	0.112	0.462

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	283	283	211	207	261	232	0	269	90	181
N.S.	1	1.00	0.75	0.73	0.92	0.82	0.00	0.95	0.32	0.64
time (sec)	N/A	0.265	0.171	0.011	3.001	2.307	0.000	0.192	0.105	0.397

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	300	300	30	223	268	276	0	294	102	199
N.S.	1	1.00	0.10	0.74	0.89	0.92	0.00	0.98	0.34	0.66
time (sec)	N/A	0.323	0.010	0.019	3.122	0.763	0.000	0.181	0.122	0.485

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	300	300	32	226	275	300	0	276	102	199
N.S.	1	1.00	0.11	0.75	0.92	1.00	0.00	0.92	0.34	0.66
time (sec)	N/A	0.299	0.011	0.019	3.077	0.996	0.000	0.195	4.396	0.470

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	318	318	37	242	290	323	0	307	113	213
N.S.	1	1.00	0.12	0.76	0.91	1.02	0.00	0.97	0.36	0.67
time (sec)	N/A	0.343	0.010	0.021	3.008	0.791	0.000	0.185	4.350	0.482

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	368	368	347	306	361	399	0	336	188	222
N.S.	1	1.00	0.94	0.83	0.98	1.08	0.00	0.91	0.51	0.60
time (sec)	N/A	0.418	0.227	0.023	3.103	1.001	0.000	0.216	0.129	0.940

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	350	350	87	290	334	399	0	316	171	212
N.S.	1	1.00	0.25	0.83	0.95	1.14	0.00	0.90	0.49	0.61
time (sec)	N/A	0.393	0.028	0.023	3.083	2.463	0.000	0.235	4.333	0.889

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	350	350	324	287	343	363	0	302	171	227
N.S.	1	1.00	0.93	0.82	0.98	1.04	0.00	0.86	0.49	0.65
time (sec)	N/A	0.382	0.140	0.024	3.141	0.833	0.000	0.199	4.300	0.875

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	333	333	83	271	317	370	0	314	153	213
N.S.	1	1.00	0.25	0.81	0.95	1.11	0.00	0.94	0.46	0.64
time (sec)	N/A	0.345	0.027	0.019	3.444	1.895	0.000	0.202	0.110	0.855

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	333	333	299	280	326	373	0	301	153	213
N.S.	1	1.00	0.90	0.84	0.98	1.12	0.00	0.90	0.46	0.64
time (sec)	N/A	0.345	0.130	0.021	3.131	1.773	0.000	0.213	4.289	0.809

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	74	277	323	390	0	317	150	221
N.S.	1	1.00	0.22	0.82	0.96	1.16	0.00	0.94	0.45	0.66
time (sec)	N/A	0.351	0.022	0.021	3.017	1.147	0.000	0.215	4.260	0.875

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	279	277	332	389	0	304	150	221
N.S.	1	1.00	0.83	0.82	0.99	1.16	0.00	0.90	0.45	0.66
time (sec)	N/A	0.341	0.166	0.019	2.962	0.910	0.000	0.236	4.264	0.863

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	335	335	60	277	323	396	0	317	149	213
N.S.	1	1.00	0.18	0.83	0.96	1.18	0.00	0.95	0.44	0.64
time (sec)	N/A	0.348	0.020	0.019	3.019	0.882	0.000	0.218	4.232	0.785

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	335	335	260	271	332	373	0	302	149	213
N.S.	1	1.00	0.78	0.81	0.99	1.11	0.00	0.90	0.44	0.64
time (sec)	N/A	0.347	0.131	0.021	3.019	1.678	0.000	0.197	4.272	0.780

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	335	335	32	272	317	359	252	302	150	210
N.S.	1	1.00	0.10	0.81	0.95	1.07	0.75	0.90	0.45	0.63
time (sec)	N/A	0.352	0.006	0.019	2.983	1.990	28.986	0.221	0.099	0.465

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	335	335	253	269	322	357	0	308	150	210
N.S.	1	1.00	0.76	0.80	0.96	1.07	0.00	0.92	0.45	0.63
time (sec)	N/A	0.348	0.112	0.018	3.091	0.649	0.000	0.268	4.284	0.439

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	352	352	30	285	328	410	0	327	166	227
N.S.	1	1.00	0.09	0.81	0.93	1.16	0.00	0.93	0.47	0.64
time (sec)	N/A	0.402	0.011	0.025	3.071	1.673	0.000	0.195	0.138	0.829

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	352	352	32	288	335	434	0	308	166	227
N.S.	1	1.00	0.09	0.82	0.95	1.23	0.00	0.88	0.47	0.64
time (sec)	N/A	0.388	0.013	0.024	3.071	0.887	0.000	0.216	4.253	0.823

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	370	370	37	304	350	457	0	349	179	241
N.S.	1	1.00	0.10	0.82	0.95	1.24	0.00	0.94	0.48	0.65
time (sec)	N/A	0.447	0.011	0.027	3.173	2.023	0.000	0.218	4.327	0.856

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	420	420	432	370	421	515	0	374	248	244
N.S.	1	1.00	1.03	0.88	1.00	1.23	0.00	0.89	0.59	0.58
time (sec)	N/A	0.529	0.180	0.029	3.181	0.951	0.000	0.242	4.402	1.468

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	402	402	109	354	394	515	0	354	231	234
N.S.	1	1.00	0.27	0.88	0.98	1.28	0.00	0.88	0.57	0.58
time (sec)	N/A	0.469	0.034	0.031	3.204	2.337	0.000	0.224	0.236	1.460

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	402	402	408	351	403	479	0	340	231	233
N.S.	1	1.00	1.01	0.87	1.00	1.19	0.00	0.85	0.57	0.58
time (sec)	N/A	0.486	0.297	0.031	3.226	1.755	0.000	0.248	4.358	1.272

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	385	385	104	335	377	486	0	352	213	223
N.S.	1	1.00	0.27	0.87	0.98	1.26	0.00	0.91	0.55	0.58
time (sec)	N/A	0.448	0.036	0.025	3.161	1.037	0.000	0.209	0.214	1.176

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	385	385	381	344	386	489	0	339	213	222
N.S.	1	1.00	0.99	0.89	1.00	1.27	0.00	0.88	0.55	0.58
time (sec)	N/A	0.446	0.195	0.026	3.114	1.690	0.000	0.244	4.272	1.217

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	388	388	96	341	383	506	0	355	210	226
N.S.	1	1.00	0.25	0.88	0.99	1.30	0.00	0.91	0.54	0.58
time (sec)	N/A	0.449	0.031	0.026	3.043	1.046	0.000	0.210	4.289	1.035

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	388	388	359	341	392	505	0	342	210	244
N.S.	1	1.00	0.93	0.88	1.01	1.30	0.00	0.88	0.54	0.63
time (sec)	N/A	0.446	0.259	0.025	3.221	1.010	0.000	0.223	0.127	1.292

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	391	391	85	339	385	518	0	355	208	244
N.S.	1	1.00	0.22	0.87	0.98	1.32	0.00	0.91	0.53	0.62
time (sec)	N/A	0.479	0.031	0.025	3.003	1.756	0.000	0.226	4.322	1.296

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	391	391	337	339	394	513	0	342	208	244
N.S.	1	1.00	0.86	0.87	1.01	1.31	0.00	0.87	0.53	0.62
time (sec)	N/A	0.473	0.182	0.025	3.162	3.179	0.000	0.213	4.230	1.268

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	394	394	61	339	385	520	0	355	207	244
N.S.	1	1.00	0.15	0.86	0.98	1.32	0.00	0.90	0.53	0.62
time (sec)	N/A	0.461	0.026	0.028	3.173	3.275	0.000	0.259	0.116	1.171

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	394	394	317	339	394	513	0	342	207	244
N.S.	1	1.00	0.80	0.86	1.00	1.30	0.00	0.87	0.53	0.62
time (sec)	N/A	0.449	0.170	0.025	3.149	1.167	0.000	0.221	4.265	1.146

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	389	389	48	341	383	512	0	355	209	241
N.S.	1	1.00	0.12	0.88	0.98	1.32	0.00	0.91	0.54	0.62
time (sec)	N/A	0.455	0.020	0.025	3.156	0.699	0.000	0.217	4.280	1.107

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	389	389	298	335	392	485	0	340	209	241
N.S.	1	1.00	0.77	0.86	1.01	1.25	0.00	0.87	0.54	0.62
time (sec)	N/A	0.499	0.173	0.025	3.232	1.262	0.000	0.209	0.131	0.795

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	387	387	32	336	377	469	547	340	210	238
N.S.	1	1.00	0.08	0.87	0.97	1.21	1.41	0.88	0.54	0.61
time (sec)	N/A	0.490	0.006	0.026	3.192	0.997	89.238	0.221	4.247	0.566

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	387	387	295	333	382	475	0	346	210	238
N.S.	1	1.00	0.76	0.86	0.99	1.23	0.00	0.89	0.54	0.61
time (sec)	N/A	0.496	0.160	0.026	3.106	1.228	0.000	0.192	4.289	0.559

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	404	404	30	349	388	544	0	365	226	255
N.S.	1	1.00	0.07	0.86	0.96	1.35	0.00	0.90	0.56	0.63
time (sec)	N/A	0.529	0.012	0.031	3.229	1.127	0.000	0.203	0.208	1.324

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	404	404	32	352	395	568	0	356	226	255
N.S.	1	1.00	0.08	0.87	0.98	1.41	0.00	0.88	0.56	0.63
time (sec)	N/A	0.509	0.014	0.031	3.234	1.462	0.000	0.201	4.463	1.331

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	422	422	37	368	410	591	0	362	239	269
N.S.	1	1.00	0.09	0.87	0.97	1.40	0.00	0.86	0.57	0.64
time (sec)	N/A	0.554	0.013	0.041	3.317	2.517	0.000	0.203	0.273	1.373

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	44	39	25	26	0	45	-1	69
N.S.	1	1.00	0.47	0.42	0.27	0.28	0.00	0.48	-0.01	0.74
time (sec)	N/A	0.030	0.020	0.004	1.325	0.964	0.000	0.177	0.000	59.989

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	44	39	25	22	0	42	-1	69
N.S.	1	1.00	0.47	0.42	0.27	0.24	0.00	0.45	-0.01	0.74
time (sec)	N/A	0.029	0.014	0.002	1.252	1.556	0.000	0.157	0.000	31.595

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	44	39	25	18	27	37	-1	69
N.S.	1	1.00	0.47	0.42	0.27	0.19	0.29	0.40	-0.01	0.74
time (sec)	N/A	0.029	0.013	0.004	1.397	1.115	133.053	0.169	0.000	22.907

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	43	38	24	19	0	40	47	68
N.S.	1	1.00	0.47	0.42	0.26	0.21	0.00	0.44	0.52	0.75
time (sec)	N/A	0.028	0.012	0.003	1.370	1.708	0.000	0.171	4.358	23.992

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	43	39	25	22	0	41	52	67
N.S.	1	1.00	0.47	0.43	0.27	0.24	0.00	0.45	0.57	0.74
time (sec)	N/A	0.028	0.014	0.003	1.355	0.890	0.000	0.158	4.345	26.007

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	42	37	24	23	0	42	53	68
N.S.	1	1.00	0.46	0.41	0.26	0.25	0.00	0.46	0.58	0.75
time (sec)	N/A	0.028	0.016	0.003	1.403	1.919	0.000	0.162	4.377	25.425

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	42	37	25	21	0	44	56	67
N.S.	1	1.00	0.46	0.41	0.27	0.23	0.00	0.48	0.62	0.74
time (sec)	N/A	0.029	0.016	0.003	1.270	1.195	0.000	0.181	4.319	29.293

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	66	61	83	54	0	99	-1	105
N.S.	1	1.00	0.34	0.31	0.43	0.28	0.00	0.51	-0.01	0.54
time (sec)	N/A	0.059	0.030	0.006	1.458	1.652	0.000	0.163	0.000	117.046

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	66	61	83	46	0	90	-1	105
N.S.	1	1.00	0.34	0.31	0.43	0.24	0.00	0.46	-0.01	0.54
time (sec)	N/A	0.058	0.024	0.007	1.409	1.042	0.000	0.214	0.000	114.648

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	66	61	83	40	0	85	-1	96
N.S.	1	1.00	0.34	0.31	0.43	0.21	0.00	0.44	-0.01	0.49
time (sec)	N/A	0.055	0.021	0.007	1.436	1.765	0.000	0.161	0.000	83.779

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	66	61	87	42	0	89	76	105
N.S.	1	1.00	0.34	0.32	0.45	0.22	0.00	0.46	0.39	0.54
time (sec)	N/A	0.054	0.021	0.006	1.459	2.007	0.000	0.167	4.496	52.720

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	191	191	66	61	87	45	0	102	87	96
N.S.	1	1.00	0.35	0.32	0.46	0.24	0.00	0.53	0.46	0.50
time (sec)	N/A	0.058	0.024	0.006	1.424	1.024	0.000	0.168	4.535	41.075

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	66	61	86	45	0	105	88	96
N.S.	1	1.00	0.34	0.32	0.45	0.23	0.00	0.54	0.46	0.50
time (sec)	N/A	0.055	0.026	0.007	1.475	0.699	0.000	0.174	4.490	34.467

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	191	191	66	61	86	45	0	107	91	96
N.S.	1	1.00	0.35	0.32	0.45	0.24	0.00	0.56	0.48	0.50
time (sec)	N/A	0.055	0.027	0.007	1.452	2.068	0.000	0.220	4.533	28.585

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	88	83	147	82	0	153	-1	141
N.S.	1	1.00	0.30	0.28	0.49	0.28	0.00	0.52	-0.00	0.47
time (sec)	N/A	0.082	0.043	0.006	1.471	1.578	0.000	0.171	0.000	126.122

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	88	83	147	70	0	138	-1	141
N.S.	1	1.00	0.30	0.28	0.49	0.24	0.00	0.46	-0.00	0.47
time (sec)	N/A	0.077	0.035	0.007	1.416	1.101	0.000	0.202	0.000	119.004

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	88	83	147	62	0	133	-1	141
N.S.	1	1.00	0.30	0.28	0.49	0.21	0.00	0.45	-0.00	0.47
time (sec)	N/A	0.081	0.030	0.007	1.526	2.782	0.000	0.202	0.000	120.776

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	88	83	151	64	0	137	112	141
N.S.	1	1.00	0.30	0.28	0.52	0.22	0.00	0.47	0.38	0.48
time (sec)	N/A	0.079	0.031	0.006	1.450	2.239	0.000	0.174	4.565	125.047

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	295	295	88	83	151	67	0	156	116	124
N.S.	1	1.00	0.30	0.28	0.51	0.23	0.00	0.53	0.39	0.42
time (sec)	N/A	0.079	0.034	0.005	1.504	1.859	0.000	0.188	4.536	94.083

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	88	83	151	67	0	159	116	124
N.S.	1	1.00	0.30	0.28	0.52	0.23	0.00	0.54	0.40	0.42
time (sec)	N/A	0.082	0.035	0.007	1.537	0.939	0.000	0.187	4.564	74.815

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	295	295	88	83	150	67	0	162	118	124
N.S.	1	1.00	0.30	0.28	0.51	0.23	0.00	0.55	0.40	0.42
time (sec)	N/A	0.079	0.037	0.007	1.554	0.815	0.000	0.204	4.716	61.306

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	457	457	238	239	266	223	0	273	-1	217
N.S.	1	1.00	0.52	0.52	0.58	0.49	0.00	0.60	-0.00	0.47
time (sec)	N/A	0.325	0.091	0.011	2.943	0.487	0.000	0.196	0.000	54.463

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	412	412	110	221	241	219	0	254	-1	201
N.S.	1	1.00	0.27	0.54	0.58	0.53	0.00	0.62	-0.00	0.49
time (sec)	N/A	0.286	0.054	0.009	2.971	0.841	0.000	0.197	0.000	42.657

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	410	410	221	214	250	170	0	238	-1	200
N.S.	1	1.00	0.54	0.52	0.61	0.41	0.00	0.58	-0.00	0.49
time (sec)	N/A	0.275	0.076	0.008	3.103	1.913	0.000	0.243	0.000	36.806

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	368	368	85	183	216	173	41	242	-1	188
N.S.	1	1.00	0.23	0.50	0.59	0.47	0.11	0.66	-0.00	0.51
time (sec)	N/A	0.246	0.044	0.008	3.005	1.695	57.265	0.193	0.000	34.529

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	368	368	178	182	226	165	0	251	-1	187
N.S.	1	1.00	0.48	0.49	0.61	0.45	0.00	0.68	-0.00	0.51
time (sec)	N/A	0.238	0.049	0.007	3.056	1.327	0.000	0.256	0.000	30.851

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	412	412	50	224	234	198	0	264	-1	201
N.S.	1	1.00	0.12	0.54	0.57	0.48	0.00	0.64	-0.00	0.49
time (sec)	N/A	0.279	0.012	0.010	3.255	1.496	0.000	0.234	0.000	34.070

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	414	414	52	239	242	227	0	256	-1	204
N.S.	1	1.00	0.13	0.58	0.58	0.55	0.00	0.62	-0.00	0.49
time (sec)	N/A	0.276	0.013	0.010	3.076	0.963	0.000	0.245	0.000	44.929

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	459	459	52	251	259	253	0	284	-1	220
N.S.	1	1.00	0.11	0.55	0.56	0.55	0.00	0.62	-0.00	0.48
time (sec)	N/A	0.327	0.013	0.013	3.034	1.505	0.000	0.316	0.000	59.447

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	551	551	498	737	0	341	0	419	-1	269
N.S.	1	1.00	0.90	1.34	0.00	0.62	0.00	0.76	-0.00	0.49
time (sec)	N/A	0.399	0.165	0.023	0.000	1.016	0.000	0.349	0.000	111.367

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	504	504	88	679	0	341	0	399	-1	255
N.S.	1	1.00	0.17	1.35	0.00	0.68	0.00	0.79	-0.00	0.51
time (sec)	N/A	0.369	0.034	0.023	0.000	0.954	0.000	0.410	0.000	97.891

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	504	504	484	696	0	305	0	385	-1	260
N.S.	1	1.00	0.96	1.38	0.00	0.61	0.00	0.76	-0.00	0.52
time (sec)	N/A	0.371	0.166	0.022	0.000	1.012	0.000	0.314	0.000	93.380

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	458	458	84	612	0	312	0	380	-1	242
N.S.	1	1.00	0.18	1.34	0.00	0.68	0.00	0.83	-0.00	0.53
time (sec)	N/A	0.333	0.032	0.020	0.000	1.731	0.000	0.318	0.000	89.463

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	458	458	447	666	279	315	0	367	-1	242
N.S.	1	1.00	0.98	1.45	0.61	0.69	0.00	0.80	-0.00	0.53
time (sec)	N/A	0.322	0.144	0.019	3.230	1.427	0.000	0.307	0.000	79.879

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	459	459	73	617	272	326	0	383	-1	245
N.S.	1	1.00	0.16	1.34	0.59	0.71	0.00	0.83	-0.00	0.53
time (sec)	N/A	0.333	0.026	0.018	3.186	0.597	0.000	0.319	0.000	70.173

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	459	459	272	668	281	308	0	367	-1	244
N.S.	1	1.00	0.59	1.46	0.61	0.67	0.00	0.80	-0.00	0.53
time (sec)	N/A	0.327	0.205	0.019	3.220	1.068	0.000	0.337	0.000	73.264

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	460	54	617	265	304	0	368	-1	238
N.S.	1	1.00	0.12	1.34	0.58	0.66	0.00	0.80	-0.00	0.52
time (sec)	N/A	0.333	0.014	0.012	3.205	0.639	0.000	0.336	0.000	77.406

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	460	272	638	0	298	0	374	-1	238
N.S.	1	1.00	0.59	1.39	0.00	0.65	0.00	0.81	-0.00	0.52
time (sec)	N/A	0.336	0.188	0.010	0.000	1.661	0.000	0.291	0.000	83.687

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	506	506	52	645	0	343	0	410	-1	255
N.S.	1	1.00	0.10	1.27	0.00	0.68	0.00	0.81	-0.00	0.50
time (sec)	N/A	0.384	0.014	0.021	0.000	2.183	0.000	0.314	0.000	91.787

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	506	506	54	707	0	367	0	401	-1	255
N.S.	1	1.00	0.11	1.40	0.00	0.73	0.00	0.79	-0.00	0.50
time (sec)	N/A	0.376	0.014	0.022	0.000	0.805	0.000	0.351	0.000	95.786

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	553	553	54	687	0	390	0	432	-1	269
N.S.	1	1.00	0.10	1.24	0.00	0.71	0.00	0.78	-0.00	0.49
time (sec)	N/A	0.432	0.015	0.026	0.000	1.810	0.000	0.339	0.000	90.940

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	647	647	401	1287	0	457	0	457	-1	643
N.S.	1	1.00	0.62	1.99	0.00	0.71	0.00	0.71	-0.00	0.99
time (sec)	N/A	0.509	0.304	0.027	0.000	0.952	0.000	0.445	0.000	1.366

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	600	600	110	1171	0	457	0	437	-1	623
N.S.	1	1.00	0.18	1.95	0.00	0.76	0.00	0.73	-0.00	1.04
time (sec)	N/A	0.468	0.048	0.027	0.000	1.827	0.000	0.419	0.000	1.313

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	600	600	384	1202	0	421	0	423	-1	621
N.S.	1	1.00	0.64	2.00	0.00	0.70	0.00	0.70	-0.00	1.04
time (sec)	N/A	0.463	0.283	0.028	0.000	0.911	0.000	0.420	0.000	1.183

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	554	554	106	1046	0	428	0	418	-1	603
N.S.	1	1.00	0.19	1.89	0.00	0.77	0.00	0.75	-0.00	1.09
time (sec)	N/A	0.419	0.045	0.025	0.000	1.766	0.000	0.366	0.000	0.982

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	554	554	366	1134	583	431	0	405	-1	269
N.S.	1	1.00	0.66	2.05	1.05	0.78	0.00	0.73	-0.00	0.49
time (sec)	N/A	0.422	0.264	0.025	3.762	1.198	0.000	0.348	0.000	118.010

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	557	557	97	1051	577	448	0	421	-1	272
N.S.	1	1.00	0.17	1.89	1.04	0.80	0.00	0.76	-0.00	0.49
time (sec)	N/A	0.425	0.038	0.024	3.752	1.664	0.000	0.363	0.000	116.597

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	557	557	352	1136	595	447	0	408	-1	272
N.S.	1	1.00	0.63	2.04	1.07	0.80	0.00	0.73	-0.00	0.49
time (sec)	N/A	0.430	0.288	0.023	3.710	1.197	0.000	0.345	0.000	115.497

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	560	560	86	1051	584	462	0	421	-1	281
N.S.	1	1.00	0.15	1.88	1.04	0.82	0.00	0.75	-0.00	0.50
time (sec)	N/A	0.424	0.038	0.026	3.688	1.676	0.000	0.365	0.000	106.987

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	560	560	341	1136	597	455	0	408	-1	281
N.S.	1	1.00	0.61	2.03	1.07	0.81	0.00	0.73	-0.00	0.50
time (sec)	N/A	0.438	0.305	0.024	3.811	0.868	0.000	0.352	0.000	102.808

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	557	557	73	1051	582	454	0	421	-1	269
N.S.	1	1.00	0.13	1.89	1.04	0.82	0.00	0.76	-0.00	0.48
time (sec)	N/A	0.454	0.032	0.024	3.708	1.557	0.000	0.370	0.000	104.778

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	557	557	324	1136	586	429	0	406	-1	269
N.S.	1	1.00	0.58	2.04	1.05	0.77	0.00	0.73	-0.00	0.48
time (sec)	N/A	0.426	0.296	0.024	3.638	1.561	0.000	0.348	0.000	103.786

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	556	556	54	1051	569	414	0	406	-1	266
N.S.	1	1.00	0.10	1.89	1.02	0.74	0.00	0.73	-0.00	0.48
time (sec)	N/A	0.434	0.013	0.023	3.789	1.902	0.000	0.378	0.000	114.250

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	556	556	319	1133	0	416	0	412	-1	266
N.S.	1	1.00	0.57	2.04	0.00	0.75	0.00	0.74	-0.00	0.48
time (sec)	N/A	0.429	0.134	0.024	0.000	0.844	0.000	0.334	0.000	129.584

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	602	602	52	1081	0	477	0	448	-1	283
N.S.	1	1.00	0.09	1.80	0.00	0.79	0.00	0.74	-0.00	0.47
time (sec)	N/A	0.485	0.014	0.030	0.000	0.966	0.000	0.372	0.000	140.886

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	602	602	54	1183	0	501	0	439	-1	283
N.S.	1	1.00	0.09	1.97	0.00	0.83	0.00	0.73	-0.00	0.47
time (sec)	N/A	0.484	0.018	0.029	0.000	1.105	0.000	0.739	0.000	145.087

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	649	649	54	1129	0	524	0	470	-1	297
N.S.	1	1.00	0.08	1.74	0.00	0.81	0.00	0.72	-0.00	0.46
time (sec)	N/A	0.534	0.017	0.032	0.000	3.459	0.000	0.365	0.000	144.050

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	105	602	144	507	3188	847	540	0
N.S.	1	1.00	0.70	4.01	0.96	3.38	21.25	5.65	3.60	0.00
time (sec)	N/A	0.121	0.069	0.011	1.542	1.988	7.605	0.251	4.579	1.075

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	73	292	100	253	1321	415	263	0
N.S.	1	1.00	0.70	2.81	0.96	2.43	12.70	3.99	2.53	0.00
time (sec)	N/A	0.075	0.035	0.009	1.487	0.985	3.200	0.181	4.513	0.260

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	41	94	56	87	345	135	95	0
N.S.	1	1.00	0.71	1.62	0.97	1.50	5.95	2.33	1.64	0.00
time (sec)	N/A	0.023	0.033	0.006	1.397	1.305	1.007	0.160	4.270	0.088

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	111	453	243	369	0	900	-1	0
N.S.	1	1.00	0.35	1.45	0.78	1.18	0.00	2.88	-0.00	0.00
time (sec)	N/A	0.120	0.092	0.006	1.414	1.715	0.000	0.278	0.000	1.614

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	131	199	119	159	0	384	-1	0
N.S.	1	1.00	0.64	0.97	0.58	0.78	0.00	1.87	-0.00	0.00
time (sec)	N/A	0.076	0.070	0.006	1.443	0.995	0.000	0.210	0.000	1.166

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	53	56	35	35	0	83	-1	0
N.S.	1	1.00	0.55	0.58	0.36	0.36	0.00	0.86	-0.01	0.00
time (sec)	N/A	0.034	0.024	0.004	1.398	2.026	0.000	0.163	0.000	0.746

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	110	150	115	163	0	375	206	0
N.S.	1	1.00	0.63	0.86	0.66	0.94	0.00	2.16	1.18	0.00
time (sec)	N/A	0.109	0.059	0.010	1.459	1.035	0.000	0.193	4.403	0.484

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	77	96	79	108	0	235	137	0
N.S.	1	1.00	0.59	0.74	0.61	0.83	0.00	1.81	1.05	0.00
time (sec)	N/A	0.082	0.034	0.008	1.437	1.742	0.000	0.186	4.267	0.416

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	51	60	54	70	0	132	85	0
N.S.	1	1.00	0.61	0.71	0.64	0.83	0.00	1.57	1.01	0.00
time (sec)	N/A	0.057	0.021	0.007	1.431	4.013	0.000	0.217	4.264	0.141

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	29	40	30	37	0	58	46	0
N.S.	1	1.00	0.71	0.98	0.73	0.90	0.00	1.41	1.12	0.00
time (sec)	N/A	0.025	0.004	0.004	1.324	0.882	0.000	0.262	4.669	0.115

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.00
time (sec)	N/A	0.007	0.002	0.002	1.332	1.048	0.068	0.148	0.030	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.00
time (sec)	N/A	0.007	0.001	0.000	1.385	1.050	0.067	0.151	0.028	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	16	15	16	16	0
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80	0.00
time (sec)	N/A	0.003	0.000	0.001	1.352	2.470	0.064	0.148	0.024	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	20	17	17	20	17	0
N.S.	1	1.00	1.00	0.86	0.95	0.81	0.81	0.95	0.81	0.00
time (sec)	N/A	0.005	0.002	0.003	1.360	2.107	0.098	0.149	0.025	0.001

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	16	20	12	16	16	0
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.67	0.89	0.89	0.00
time (sec)	N/A	0.007	0.002	0.004	1.385	2.461	0.099	0.147	0.029	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	20	22	17	26	17	0
N.S.	1	1.00	1.00	0.86	0.95	1.05	0.81	1.24	0.81	0.00
time (sec)	N/A	0.007	0.002	0.005	1.337	1.021	0.126	0.152	0.029	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	17	21	17	17	18	0
N.S.	1	1.00	1.00	0.94	0.94	1.17	0.94	0.94	1.00	0.00
time (sec)	N/A	0.007	0.004	0.006	1.369	1.065	0.133	0.153	0.023	0.001

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	21	23	19	27	20	0
N.S.	1	1.00	1.00	0.86	1.00	1.10	0.90	1.29	0.95	0.00
time (sec)	N/A	0.007	0.003	0.005	1.305	0.810	0.237	0.163	0.043	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	21	21	22	21	20	0
N.S.	1	1.00	1.00	0.87	0.91	0.91	0.96	0.91	0.87	0.00
time (sec)	N/A	0.007	0.002	0.005	1.362	1.923	0.256	0.152	0.031	0.001

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	21	21	22	21	21	0
N.S.	1	1.00	1.00	0.80	0.84	0.84	0.88	0.84	0.84	0.00
time (sec)	N/A	0.007	0.002	0.005	1.295	1.769	0.340	0.148	0.031	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	21	21	22	21	21	0
N.S.	1	1.00	1.00	0.80	0.84	0.84	0.88	0.84	0.84	0.00
time (sec)	N/A	0.007	0.002	0.005	1.344	0.674	0.322	0.167	0.033	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	46	51	46	45	0
N.S.	1	1.00	1.00	0.83	0.81	0.85	0.94	0.85	0.83	0.00
time (sec)	N/A	0.030	0.007	0.000	1.313	1.025	0.078	0.148	0.027	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	48	45	44	46	46	46	45	0
N.S.	1	1.00	0.89	0.83	0.81	0.85	0.85	0.85	0.83	0.00
time (sec)	N/A	0.038	0.008	0.001	1.378	2.304	0.080	0.163	0.021	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	42	45	43	48	43	42	0
N.S.	1	1.00	1.00	0.86	0.92	0.88	0.98	0.88	0.86	0.00
time (sec)	N/A	0.021	0.005	0.001	1.329	1.144	0.076	0.149	0.020	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	44	44	41	42	46	42	0
N.S.	1	1.00	1.00	0.94	0.94	0.87	0.89	0.98	0.89	0.00
time (sec)	N/A	0.041	0.012	0.001	1.339	0.573	0.143	0.152	0.024	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	45	42	46	44	44	43	0
N.S.	1	1.00	1.00	0.94	0.88	0.96	0.92	0.92	0.90	0.00
time (sec)	N/A	0.021	0.018	0.004	1.220	0.717	0.136	0.184	0.023	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	46	45	44	47	44	53	43	0
N.S.	1	1.00	0.90	0.88	0.86	0.92	0.86	1.04	0.84	0.00
time (sec)	N/A	0.041	0.016	0.006	1.374	1.740	0.169	0.152	0.026	0.001

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	42	42	46	46	42	44	0
N.S.	1	1.00	1.00	0.89	0.89	0.98	0.98	0.89	0.94	0.00
time (sec)	N/A	0.024	0.019	0.006	1.371	1.941	0.180	0.150	0.041	0.001

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	41	43	45	47	44	60	43	0
N.S.	1	1.00	0.91	0.96	1.00	1.04	0.98	1.33	0.96	0.00
time (sec)	N/A	0.038	0.019	0.008	1.336	1.010	0.374	0.177	0.037	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	49	43	45	46	48	47	44	0
N.S.	1	1.00	1.02	0.90	0.94	0.96	1.00	0.98	0.92	0.00
time (sec)	N/A	0.023	0.021	0.006	1.323	1.676	0.428	0.148	0.041	0.001

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	50	46	45	48	48	54	46	0
N.S.	1	1.00	0.98	0.90	0.88	0.94	0.94	1.06	0.90	0.00
time (sec)	N/A	0.036	0.018	0.008	1.338	0.557	0.776	0.156	4.137	0.001

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	49	42	44	46	46	46	45	0
N.S.	1	1.00	1.04	0.89	0.94	0.98	0.98	0.98	0.96	0.00
time (sec)	N/A	0.024	0.023	0.006	1.342	1.526	0.759	0.167	4.173	0.001

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	50	45	48	48	48	58	45	0
N.S.	1	1.00	1.04	0.94	1.00	1.00	1.00	1.21	0.94	0.00
time (sec)	N/A	0.035	0.026	0.006	1.364	1.228	1.307	0.151	4.182	0.001

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	50	45	46	46	49	48	46	0
N.S.	1	1.00	0.96	0.87	0.88	0.88	0.94	0.92	0.88	0.00
time (sec)	N/A	0.025	0.020	0.005	1.330	2.104	1.474	0.149	0.035	0.001

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	53	45	46	46	49	48	47	0
N.S.	1	1.00	0.98	0.83	0.85	0.85	0.91	0.89	0.87	0.00
time (sec)	N/A	0.037	0.015	0.005	1.279	1.767	2.078	0.216	4.119	0.001

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	56	45	46	46	49	48	47	0
N.S.	1	1.00	1.04	0.83	0.85	0.85	0.91	0.89	0.87	0.00
time (sec)	N/A	0.025	0.025	0.006	1.380	0.941	1.958	0.195	4.160	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	50	45	46	46	49	48	47	0
N.S.	1	1.00	0.93	0.83	0.85	0.85	0.91	0.89	0.87	0.00
time (sec)	N/A	0.035	0.016	0.005	1.341	0.812	2.698	0.146	4.158	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	89	111	81	87	97	87	76	0
N.S.	1	1.00	1.00	1.25	0.91	0.98	1.09	0.98	0.85	0.00
time (sec)	N/A	0.059	0.013	0.000	1.385	1.262	0.094	0.193	0.034	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	79	111	81	87	92	87	76	0
N.S.	1	1.00	0.89	1.25	0.91	0.98	1.03	0.98	0.85	0.00
time (sec)	N/A	0.083	0.016	0.002	1.365	0.961	0.098	0.169	0.030	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	81	107	85	83	87	83	72	0
N.S.	1	1.00	1.00	1.32	1.05	1.02	1.07	1.02	0.89	0.00
time (sec)	N/A	0.045	0.010	0.001	1.355	0.753	0.094	0.150	0.030	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	85	82	79	92	87	73	0
N.S.	1	1.00	1.00	1.00	0.96	0.93	1.08	1.02	0.86	0.00
time (sec)	N/A	0.074	0.021	0.002	1.387	1.929	0.222	0.160	0.034	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	84	78	83	82	83	73	0
N.S.	1	1.00	1.00	1.05	0.98	1.04	1.02	1.04	0.91	0.00
time (sec)	N/A	0.039	0.024	0.004	1.362	0.843	0.216	0.149	0.033	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	78	87	82	85	92	98	75	0
N.S.	1	1.00	0.91	1.01	0.95	0.99	1.07	1.14	0.87	0.00
time (sec)	N/A	0.078	0.034	0.007	1.366	0.563	0.267	0.164	0.036	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	83	84	80	83	90	84	77	0
N.S.	1	1.00	1.00	1.01	0.96	1.00	1.08	1.01	0.93	0.00
time (sec)	N/A	0.041	0.025	0.006	1.345	1.273	0.239	0.177	0.031	0.001

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	93	142	0	313	391	92	842	0
N.S.	1	1.00	0.93	1.42	0.00	3.13	3.91	0.92	8.42	0.00
time (sec)	N/A	0.117	0.089	0.008	0.000	2.861	2.912	0.564	4.396	0.001

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	78	111	0	254	316	75	655	0
N.S.	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	8.09	0.00
time (sec)	N/A	0.080	0.044	0.004	0.000	1.015	2.135	0.620	4.750	0.001

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	62	60	0	197	223	59	118	0
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	1.87	0.00
time (sec)	N/A	0.055	0.024	0.002	0.000	0.976	1.032	0.573	4.263	0.001

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	39	36	0	129	131	35	41	0
N.S.	1	1.00	1.08	1.00	0.00	3.58	3.64	0.97	1.14	0.00
time (sec)	N/A	0.034	0.009	0.002	0.000	2.332	0.586	0.570	4.270	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	113	66	0	223	253	68	1014	0
N.S.	1	1.00	1.64	0.96	0.00	3.23	3.67	0.99	14.70	0.00
time (sec)	N/A	0.070	0.069	0.006	0.000	1.306	4.669	0.575	4.936	0.001

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	135	119	0	293	345	94	2033	0
N.S.	1	1.00	1.52	1.34	0.00	3.29	3.88	1.06	22.84	0.00
time (sec)	N/A	0.131	0.125	0.009	0.000	0.683	137.798	0.580	5.892	0.001

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	188	159	0	374	0	126	2451	0
N.S.	1	1.00	1.65	1.39	0.00	3.28	0.00	1.11	21.50	0.00
time (sec)	N/A	0.196	0.235	0.011	0.000	2.111	0.000	0.553	6.367	0.001

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	250	467	0	1564	194	2457	4127	0
N.S.	1	1.00	1.23	2.30	0.00	7.70	0.96	12.10	20.33	0.00
time (sec)	N/A	0.670	0.150	0.046	0.000	1.309	5.160	1.013	5.014	0.001

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	202	343	0	1059	129	2109	3026	0
N.S.	1	1.00	1.13	1.92	0.00	5.92	0.72	11.78	16.91	0.00
time (sec)	N/A	0.269	0.107	0.026	0.000	1.801	5.506	0.974	0.653	0.001

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	165	208	0	559	75	503	416	0
N.S.	1	1.00	1.10	1.39	0.00	3.73	0.50	3.35	2.77	0.00
time (sec)	N/A	0.109	0.082	0.018	0.000	1.551	2.620	1.050	4.457	0.001

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	129	116	0	613	87	1024	763	0
N.S.	1	1.00	0.86	0.77	0.00	4.09	0.58	6.83	5.09	0.00
time (sec)	N/A	0.086	0.075	0.016	0.000	0.746	2.840	0.575	4.612	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	191	232	0	1116	148	1839	2997	0
N.S.	1	1.00	1.10	1.33	0.00	6.41	0.85	10.57	17.22	0.00
time (sec)	N/A	0.222	0.383	0.022	0.000	0.811	4.919	0.967	4.854	0.001

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	216	368	0	1622	211	1640	4160	0
N.S.	1	1.00	1.10	1.88	0.00	8.28	1.08	8.37	21.22	0.00
time (sec)	N/A	0.423	0.138	0.023	0.000	0.853	16.671	1.158	0.788	0.001

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	121	222	0	663	745	152	1336	0
N.S.	1	1.00	0.92	1.68	0.00	5.02	5.64	1.15	10.12	0.00
time (sec)	N/A	0.168	0.168	0.016	0.000	0.974	40.654	0.600	5.098	0.001

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	93	104	0	407	282	96	187	0
N.S.	1	1.00	1.19	1.33	0.00	5.22	3.62	1.23	2.40	0.00
time (sec)	N/A	0.066	0.085	0.010	0.000	0.985	3.905	0.913	0.177	0.001

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	79	77	0	360	269	82	178	0
N.S.	1	1.00	1.05	1.03	0.00	4.80	3.59	1.09	2.37	0.00
time (sec)	N/A	0.061	0.063	0.006	0.000	0.863	1.885	0.619	4.566	0.001

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	79	75	0	361	267	82	172	0
N.S.	1	1.00	1.07	1.01	0.00	4.88	3.61	1.11	2.32	0.00
time (sec)	N/A	0.058	0.078	0.006	0.000	1.733	2.777	0.577	4.311	0.001

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	207	253	0	813	0	166	5048	0
N.S.	1	1.00	1.70	2.07	0.00	6.66	0.00	1.36	41.38	0.00
time (sec)	N/A	0.198	0.326	0.018	0.000	0.945	0.000	0.556	8.292	0.001

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	248	352	0	1007	0	182	5491	0
N.S.	1	1.00	1.53	2.17	0.00	6.22	0.00	1.12	33.90	0.00
time (sec)	N/A	0.251	0.268	0.020	0.000	1.813	0.000	0.589	8.812	0.001

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	327	844	0	2856	0	3339	7599	0
N.S.	1	1.00	0.99	2.55	0.00	8.63	0.00	10.09	22.96	0.00
time (sec)	N/A	0.844	0.657	0.038	0.000	2.055	0.000	1.168	1.566	0.001

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	282	602	0	2257	379	2736	6293	0
N.S.	1	1.00	1.04	2.22	0.00	8.33	1.40	10.10	23.22	0.00
time (sec)	N/A	0.572	0.504	0.033	0.000	1.440	51.730	1.060	5.999	0.001

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	235	452	0	1668	296	2132	4973	0
N.S.	1	1.00	0.99	1.91	0.00	7.04	1.25	9.00	20.98	0.00
time (sec)	N/A	0.411	0.402	0.027	0.000	1.113	9.011	1.055	5.910	0.001

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	222	342	0	1680	298	1970	4854	0
N.S.	1	1.00	1.00	1.55	0.00	7.60	1.35	8.91	21.96	0.00
time (sec)	N/A	0.260	0.432	0.083	0.000	1.573	20.750	0.984	1.348	0.001

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	243	733	0	2309	0	2682	6404	0
N.S.	1	1.00	0.96	2.91	0.00	9.16	0.00	10.64	25.41	0.00
time (sec)	N/A	0.513	0.419	0.065	0.000	1.004	0.000	0.864	5.996	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	308	308	302	712	0	2912	0	3087	7555	0
N.S.	1	1.00	0.98	2.31	0.00	9.45	0.00	10.02	24.53	0.00
time (sec)	N/A	1.443	0.601	0.038	0.000	1.187	0.000	1.339	6.716	0.001

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	244	547	0	1631	0	306	2588	0
N.S.	1	1.00	1.17	2.62	0.00	7.80	0.00	1.46	12.38	0.00
time (sec)	N/A	0.401	0.325	0.024	0.000	2.360	0.000	1.841	7.296	0.001

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	194	267	0	973	554	212	444	0
N.S.	1	1.00	1.60	2.21	0.00	8.04	4.58	1.75	3.67	0.00
time (sec)	N/A	0.112	0.172	0.017	0.000	0.725	4.641	1.872	4.532	0.001

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	137	230	0	892	524	171	423	0
N.S.	1	1.00	1.15	1.93	0.00	7.50	4.40	1.44	3.55	0.00
time (sec)	N/A	0.104	0.193	0.016	0.000	0.957	3.811	1.772	4.444	0.001

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	145	270	0	907	580	161	460	0
N.S.	1	1.00	1.12	2.08	0.00	6.98	4.46	1.24	3.54	0.00
time (sec)	N/A	0.129	0.134	0.016	0.000	0.602	5.406	1.813	4.457	0.001

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	114	142	0	808	491	143	400	0
N.S.	1	1.00	1.01	1.26	0.00	7.15	4.35	1.27	3.54	0.00
time (sec)	N/A	0.090	0.099	0.009	0.000	2.674	3.041	1.786	4.392	0.001

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	106	141	0	809	481	144	386	0
N.S.	1	1.00	0.94	1.25	0.00	7.16	4.26	1.27	3.42	0.00
time (sec)	N/A	0.088	0.099	0.008	0.000	1.061	2.908	1.774	4.337	0.001

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	342	822	0	2017	0	323	9339	0
N.S.	1	1.00	1.71	4.11	0.00	10.08	0.00	1.62	46.70	0.00
time (sec)	N/A	0.298	0.522	0.028	0.000	1.894	0.000	1.879	10.945	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	402	1002	0	2312	0	382	10074	0
N.S.	1	1.00	1.58	3.93	0.00	9.07	0.00	1.50	39.51	0.00
time (sec)	N/A	0.391	0.621	0.033	0.000	2.666	0.000	1.801	11.756	0.001

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	400	400	455	1141	0	4279	0	2430	10912	0
N.S.	1	1.00	1.14	2.85	0.00	10.70	0.00	6.08	27.28	0.00
time (sec)	N/A	1.727	1.166	0.049	0.000	2.006	0.000	3.626	9.036	0.001

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	348	348	381	953	0	3725	0	4558	9575	0
N.S.	1	1.00	1.09	2.74	0.00	10.70	0.00	13.10	27.51	0.00
time (sec)	N/A	0.886	0.961	0.045	0.000	2.289	0.000	2.457	8.537	0.001

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	343	753	0	3128	627	1750	8521	0
N.S.	1	1.00	1.15	2.53	0.00	10.50	2.10	5.87	28.59	0.00
time (sec)	N/A	0.683	0.845	0.038	0.000	1.660	23.392	2.817	8.179	0.001

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	285	617	0	3128	0	1861	8397	0
N.S.	1	1.00	0.99	2.13	0.00	10.82	0.00	6.44	29.06	0.00
time (sec)	N/A	0.705	0.710	0.038	0.000	1.101	0.000	2.642	7.590	0.001

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	311	311	334	2958	0	3777	0	4270	9731	0
N.S.	1	1.00	1.07	9.51	0.00	12.14	0.00	13.73	31.29	0.00
time (sec)	N/A	0.701	0.851	0.161	0.000	1.685	0.000	2.454	8.367	0.001

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	355	355	372	3360	0	4323	0	2705	10979	0
N.S.	1	1.00	1.05	9.46	0.00	12.18	0.00	7.62	30.93	0.00
time (sec)	N/A	1.838	1.023	0.131	0.000	1.442	0.000	1.433	8.997	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	425	425	454	1567	0	4924	0	5273	12130	0
N.S.	1	1.00	1.07	3.69	0.00	11.59	0.00	12.41	28.54	0.00
time (sec)	N/A	0.963	1.758	0.055	0.000	2.590	0.000	2.616	9.370	0.001

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	80	116	0	259	311	78	656	0
N.S.	1	1.00	0.98	1.41	0.00	3.16	3.79	0.95	8.00	0.00
time (sec)	N/A	0.092	0.052	0.005	0.000	1.056	2.761	0.533	4.736	0.001

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	65	63	0	206	223	62	120	0
N.S.	1	1.00	1.02	0.98	0.00	3.22	3.48	0.97	1.88	0.00
time (sec)	N/A	0.062	0.023	0.004	0.000	0.693	1.448	0.566	4.398	0.001

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	41	38	0	134	131	37	42	0
N.S.	1	1.00	1.17	1.09	0.00	3.83	3.74	1.06	1.20	0.00
time (sec)	N/A	0.043	0.008	0.002	0.000	0.870	0.721	0.573	4.297	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	117	69	0	230	253	71	1015	0
N.S.	1	1.00	1.67	0.99	0.00	3.29	3.61	1.01	14.50	0.00
time (sec)	N/A	0.084	0.074	0.008	0.000	0.876	5.742	0.569	4.892	0.001

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	139	123	0	298	350	95	2032	0
N.S.	1	1.00	1.56	1.38	0.00	3.35	3.93	1.07	22.83	0.00
time (sec)	N/A	0.140	0.141	0.008	0.000	1.625	142.971	0.588	5.844	0.001

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	208	343	0	1051	129	2153	3000	0
N.S.	1	1.00	1.16	1.92	0.00	5.87	0.72	12.03	16.76	0.00
time (sec)	N/A	0.365	0.123	0.028	0.000	1.156	2.775	0.984	0.673	0.001

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	137	208	0	551	75	513	416	0
N.S.	1	1.00	0.91	1.39	0.00	3.67	0.50	3.42	2.77	0.00
time (sec)	N/A	0.109	0.107	0.013	0.000	0.818	1.245	1.063	4.537	0.001

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	137	116	0	605	87	1050	763	0
N.S.	1	1.00	0.91	0.77	0.00	4.03	0.58	7.00	5.09	0.00
time (sec)	N/A	0.072	0.079	0.013	0.000	0.824	1.245	0.567	0.486	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	199	232	0	1108	148	1877	2979	0
N.S.	1	1.00	1.16	1.35	0.00	6.44	0.86	10.91	17.32	0.00
time (sec)	N/A	0.203	0.399	0.016	0.000	0.838	3.834	1.014	4.932	0.001

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	62	86	74	156	138	60	166	0
N.S.	1	1.00	0.90	1.25	1.07	2.26	2.00	0.87	2.41	0.00
time (sec)	N/A	0.084	0.038	0.005	2.998	1.048	1.785	0.260	0.391	0.001

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	51	49	60	134	110	46	153	0
N.S.	1	1.00	0.91	0.88	1.07	2.39	1.96	0.82	2.73	0.00
time (sec)	N/A	0.049	0.019	0.002	2.971	0.956	0.845	0.260	0.170	0.001

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	26	37	91	53	23	31	0
N.S.	1	1.00	1.00	0.84	1.19	2.94	1.71	0.74	1.00	0.00
time (sec)	N/A	0.028	0.008	0.002	3.015	1.112	0.339	0.262	4.341	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	90	71	85	151	184	71	183	0
N.S.	1	1.00	1.17	0.92	1.10	1.96	2.39	0.92	2.38	0.00
time (sec)	N/A	0.072	0.049	0.006	3.125	0.970	5.311	0.328	4.563	0.001

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	146	122	123	209	372	126	389	0
N.S.	1	1.00	1.51	1.26	1.27	2.15	3.84	1.30	4.01	0.00
time (sec)	N/A	0.141	0.094	0.010	2.970	0.783	32.928	0.282	4.870	0.001

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	144	210	0	603	105	511	1097	0
N.S.	1	1.00	1.26	1.84	0.00	5.29	0.92	4.48	9.62	0.00
time (sec)	N/A	0.165	0.086	0.030	0.000	0.847	2.019	0.357	4.791	0.001

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	128	134	0	267	44	199	216	0
N.S.	1	1.00	1.17	1.23	0.00	2.45	0.40	1.83	1.98	0.00
time (sec)	N/A	0.054	0.104	0.011	0.000	1.756	0.596	0.360	0.297	0.001

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	105	74	0	553	63	299	322	0
N.S.	1	1.00	0.96	0.68	0.00	5.07	0.58	2.74	2.95	0.00
time (sec)	N/A	0.047	0.063	0.012	0.000	0.780	0.947	0.247	5.779	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	143	180	0	1612	134	698	2774	0
N.S.	1	1.00	1.18	1.49	0.00	13.32	1.11	5.77	22.93	0.00
time (sec)	N/A	0.113	0.146	0.013	0.000	1.508	6.144	0.384	5.116	0.001

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	62	84	58	157	144	58	302	0
N.S.	1	1.00	0.90	1.22	0.84	2.28	2.09	0.84	4.38	0.00
time (sec)	N/A	0.076	0.036	0.006	3.028	0.825	1.604	0.248	0.180	0.001

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	49	47	42	131	117	42	85	0
N.S.	1	1.00	0.91	0.87	0.78	2.43	2.17	0.78	1.57	0.00
time (sec)	N/A	0.045	0.018	0.003	2.875	0.726	0.600	0.230	0.086	0.001

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	26	21	91	60	21	24	0
N.S.	1	1.00	1.00	0.84	0.68	2.94	1.94	0.68	0.77	0.00
time (sec)	N/A	0.026	0.007	0.003	2.940	0.878	0.459	0.241	0.051	0.001

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	105	63	61	147	194	61	71	0
N.S.	1	1.00	1.52	0.91	0.88	2.13	2.81	0.88	1.03	0.00
time (sec)	N/A	0.067	0.057	0.009	3.013	0.575	5.964	0.234	4.641	0.001

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	163	110	104	208	386	125	3313	0
N.S.	1	1.00	1.83	1.24	1.17	2.34	4.34	1.40	37.22	0.00
time (sec)	N/A	0.132	0.098	0.010	2.904	1.072	41.752	0.277	7.390	0.001

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	432	432	164	1658	0	615	105	533	1147	0
N.S.	1	1.00	0.38	3.84	0.00	1.42	0.24	1.23	2.66	0.00
time (sec)	N/A	0.891	0.097	0.138	0.000	2.046	2.197	0.346	4.650	0.001

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	143	724	0	279	44	203	222	0
N.S.	1	1.00	0.43	2.19	0.00	0.84	0.13	0.61	0.67	0.00
time (sec)	N/A	0.256	0.116	0.059	0.000	1.650	0.827	0.343	0.283	0.001

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	359	359	119	913	0	567	63	307	986	0
N.S.	1	1.00	0.33	2.54	0.00	1.58	0.18	0.86	2.75	0.00
time (sec)	N/A	0.261	0.070	0.079	0.000	1.122	1.235	0.249	5.157	0.001

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	433	433	174	3318	0	1582	134	742	2848	0
N.S.	1	1.00	0.40	7.66	0.00	3.65	0.31	1.71	6.58	0.00
time (sec)	N/A	0.519	0.150	0.070	0.000	0.757	4.537	0.375	5.274	0.001

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	19	18	18	26	18	20	0
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.30	0.90	1.00	0.00
time (sec)	N/A	0.020	0.006	0.002	2.837	0.934	0.168	0.151	0.057	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	11	10	10	10	10	10	0
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71	0.00
time (sec)	N/A	0.016	0.005	0.003	2.920	0.820	0.124	0.586	0.059	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	20	18	18	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.87	0.78	0.78	0.00
time (sec)	N/A	0.012	0.014	0.009	2.982	1.648	0.209	0.192	4.368	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	74	94	57	0	159	63	56	44	0
N.S.	1	1.00	1.27	0.77	0.00	2.15	0.85	0.76	0.59	0.00
time (sec)	N/A	0.050	0.144	0.022	0.000	0.823	0.308	0.172	0.082	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	39	308	0	247	24	147	101	0
N.S.	1	1.00	0.21	1.64	0.00	1.31	0.13	0.78	0.54	0.00
time (sec)	N/A	0.177	0.031	0.105	0.000	0.812	0.830	0.849	4.371	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	164	296	0	367	0	172	315	170
N.S.	1	1.00	0.96	1.73	0.00	2.15	0.00	1.01	1.84	0.99
time (sec)	N/A	0.155	0.147	0.024	0.000	0.936	0.000	0.250	5.313	0.455

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	136	247	0	303	0	134	193	132
N.S.	1	1.00	0.89	1.61	0.00	1.98	0.00	0.88	1.26	0.86
time (sec)	N/A	0.129	0.067	0.017	0.000	1.740	0.000	0.232	4.639	0.349

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	101	139	0	237	0	98	87	107
N.S.	1	1.00	0.94	1.29	0.00	2.19	0.00	0.91	0.81	0.99
time (sec)	N/A	0.081	0.049	0.014	0.000	1.062	0.000	0.215	4.520	0.274

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	83	101	0	197	0	76	72	85
N.S.	1	1.00	1.00	1.22	0.00	2.37	0.00	0.92	0.87	1.02
time (sec)	N/A	0.055	0.022	0.010	0.000	0.928	0.000	0.204	4.622	0.213

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	106	91	0	566	0	0	88	108
N.S.	1	1.00	0.97	0.83	0.00	5.19	0.00	0.00	0.81	0.99
time (sec)	N/A	0.109	0.043	0.012	0.000	0.686	0.000	0.000	4.423	0.247

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	112	140	0	601	0	148	91	114
N.S.	1	1.00	1.00	1.25	0.00	5.37	0.00	1.32	0.81	1.02
time (sec)	N/A	0.104	0.049	0.011	0.000	1.081	0.000	0.292	4.553	0.223

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	88	193	0	215	0	241	-1	91
N.S.	1	1.00	1.00	2.19	0.00	2.44	0.00	2.74	-0.01	1.03
time (sec)	N/A	0.071	0.040	0.012	0.000	1.171	0.000	0.224	0.000	0.329

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	108	222	0	261	0	359	-1	108
N.S.	1	1.00	0.93	1.91	0.00	2.25	0.00	3.09	-0.01	0.93
time (sec)	N/A	0.096	0.075	0.013	0.000	0.990	0.000	0.301	0.000	0.567

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	141	387	0	325	0	617	-1	141
N.S.	1	1.00	0.88	2.40	0.00	2.02	0.00	3.83	-0.01	0.88
time (sec)	N/A	0.150	0.095	0.017	0.000	0.765	0.000	0.281	0.000	0.743

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	173	442	0	389	0	842	-1	176
N.S.	1	1.00	0.87	2.22	0.00	1.95	0.00	4.23	-0.01	0.88
time (sec)	N/A	0.229	0.120	0.020	0.000	2.671	0.000	0.357	0.000	1.005

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	192	534	0	535	0	669	-1	255
N.S.	1	1.00	0.86	2.39	0.00	2.40	0.00	3.00	-0.00	1.14
time (sec)	N/A	0.212	0.253	0.035	0.000	1.295	0.000	0.405	0.000	0.979

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	175	432	0	451	0	535	-1	209
N.S.	1	1.00	0.86	2.12	0.00	2.21	0.00	2.62	-0.00	1.02
time (sec)	N/A	0.182	0.165	0.024	0.000	2.368	0.000	0.397	0.000	0.771

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	149	316	0	361	0	414	223	162
N.S.	1	1.00	0.99	2.11	0.00	2.41	0.00	2.76	1.49	1.08
time (sec)	N/A	0.117	0.142	0.020	0.000	0.739	0.000	0.394	4.880	0.600

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	126	242	0	297	0	317	115	130
N.S.	1	1.00	1.02	1.95	0.00	2.40	0.00	2.56	0.93	1.05
time (sec)	N/A	0.086	0.086	0.015	0.000	0.855	0.000	0.389	4.965	0.473

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	143	192	0	727	0	0	-1	148
N.S.	1	1.00	0.92	1.24	0.00	4.69	0.00	0.00	-0.01	0.95
time (sec)	N/A	0.184	0.145	0.019	0.000	1.457	0.000	0.000	0.000	0.618

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	134	170	0	713	0	0	-1	138
N.S.	1	1.00	0.89	1.13	0.00	4.75	0.00	0.00	-0.01	0.92
time (sec)	N/A	0.169	0.118	0.019	0.000	2.126	0.000	0.000	0.000	0.672

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	134	174	0	713	0	302	-1	134
N.S.	1	1.00	0.89	1.15	0.00	4.72	0.00	2.00	-0.01	0.89
time (sec)	N/A	0.163	0.170	0.018	0.000	2.075	0.000	0.446	0.000	0.714

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	149	202	0	771	0	412	-1	148
N.S.	1	1.00	0.91	1.24	0.00	4.73	0.00	2.53	-0.01	0.91
time (sec)	N/A	0.184	0.216	0.019	0.000	1.557	0.000	0.677	0.000	0.877

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	138	260	0	319	0	606	-1	139
N.S.	1	1.00	1.04	1.95	0.00	2.40	0.00	4.56	-0.01	1.05
time (sec)	N/A	0.117	0.167	0.020	0.000	1.170	0.000	0.495	0.000	0.996

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	167	337	0	383	0	832	-1	174
N.S.	1	1.00	1.03	2.08	0.00	2.36	0.00	5.14	-0.01	1.07
time (sec)	N/A	0.140	0.140	0.024	0.000	1.557	0.000	0.530	0.000	1.365

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	216	206	457	0	473	0	1235	-1	221
N.S.	1	1.00	0.95	2.12	0.00	2.19	0.00	5.72	-0.00	1.02
time (sec)	N/A	0.217	0.212	0.027	0.000	1.512	0.000	0.701	0.000	1.938

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	104	162	0	241	0	103	-1	101
N.S.	1	1.00	0.86	1.34	0.00	1.99	0.00	0.85	-0.01	0.83
time (sec)	N/A	0.114	0.063	0.017	0.000	2.960	0.000	0.211	0.000	0.286

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	88	116	0	203	0	82	-1	91
N.S.	1	1.00	0.85	1.12	0.00	1.95	0.00	0.79	-0.01	0.88
time (sec)	N/A	0.099	0.034	0.014	0.000	1.123	0.000	0.240	0.000	0.251

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	56	0	161	0	61	55	70
N.S.	1	1.00	1.00	0.82	0.00	2.37	0.00	0.90	0.81	1.03
time (sec)	N/A	0.054	0.014	0.011	0.000	0.862	0.000	0.213	4.428	0.170

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	35	0	118	0	40	34	41
N.S.	1	1.00	1.00	0.81	0.00	2.74	0.00	0.93	0.79	0.95
time (sec)	N/A	0.031	0.006	0.009	0.000	0.912	0.000	0.211	4.690	0.115

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	39	0	124	0	38	44	45
N.S.	1	1.00	1.00	0.89	0.00	2.82	0.00	0.86	1.00	1.02
time (sec)	N/A	0.041	0.011	0.011	0.000	1.018	0.000	0.250	4.441	0.108

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	72	63	0	179	0	114	56	76
N.S.	1	1.00	1.00	0.88	0.00	2.49	0.00	1.58	0.78	1.06
time (sec)	N/A	0.061	0.021	0.013	0.000	0.581	0.000	0.434	4.484	0.198

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	91	127	0	221	0	221	-1	91
N.S.	1	1.00	0.84	1.18	0.00	2.05	0.00	2.05	-0.01	0.84
time (sec)	N/A	0.105	0.054	0.014	0.000	1.196	0.000	0.298	0.000	0.306

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	112	176	0	265	0	335	-1	110
N.S.	1	1.00	0.77	1.21	0.00	1.83	0.00	2.31	-0.01	0.76
time (sec)	N/A	0.166	0.079	0.016	0.000	1.446	0.000	0.265	0.000	0.466

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	107	168	153	249	0	112	-1	3247
N.S.	1	1.00	0.86	1.35	1.23	2.01	0.00	0.90	-0.01	26.19
time (sec)	N/A	0.113	0.077	0.022	2.430	1.213	0.000	0.259	0.000	23.348

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	89	120	105	211	0	91	-1	91
N.S.	1	1.00	0.83	1.12	0.98	1.97	0.00	0.85	-0.01	0.85
time (sec)	N/A	0.093	0.044	0.017	2.420	0.651	0.000	0.212	0.000	30.868

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	58	50	169	0	70	62	77
N.S.	1	1.00	1.00	0.83	0.71	2.41	0.00	1.00	0.89	1.10
time (sec)	N/A	0.058	0.016	0.014	2.466	0.732	0.000	0.265	4.593	13.917

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	36	28	124	0	45	40	127
N.S.	1	1.00	1.00	0.82	0.64	2.82	0.00	1.02	0.91	2.89
time (sec)	N/A	0.038	0.006	0.010	2.394	0.847	0.000	0.205	4.789	0.196

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	46	45	36	129	0	36	52	48
N.S.	1	1.00	0.98	0.96	0.77	2.74	0.00	0.77	1.11	1.02
time (sec)	N/A	0.042	0.014	0.016	2.330	1.055	0.000	0.207	4.520	0.110

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	76	74	62	188	0	111	64	80
N.S.	1	1.00	0.99	0.96	0.81	2.44	0.00	1.44	0.83	1.04
time (sec)	N/A	0.062	0.022	0.012	2.410	1.107	0.000	0.217	4.546	0.182

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	95	149	126	230	0	224	-1	95
N.S.	1	1.00	0.83	1.30	1.10	2.00	0.00	1.95	-0.01	0.83
time (sec)	N/A	0.123	0.046	0.014	2.432	1.239	0.000	0.225	0.000	0.274

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	116	202	179	272	0	344	-1	114
N.S.	1	1.00	0.75	1.31	1.16	1.77	0.00	2.23	-0.01	0.74
time (sec)	N/A	0.172	0.076	0.018	2.258	4.006	0.000	0.267	0.000	0.408

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	181	354	0	591	0	215	-1	169
N.S.	1	1.00	0.95	1.86	0.00	3.11	0.00	1.13	-0.01	0.89
time (sec)	N/A	0.237	0.194	0.020	0.000	1.834	0.000	0.274	0.000	0.756

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	137	264	0	459	0	154	-1	131
N.S.	1	1.00	1.02	1.97	0.00	3.43	0.00	1.15	-0.01	0.98
time (sec)	N/A	0.111	0.117	0.015	0.000	1.578	0.000	0.273	0.000	0.583

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	107	149	0	387	0	101	84	96
N.S.	1	1.00	0.93	1.30	0.00	3.37	0.00	0.88	0.73	0.83
time (sec)	N/A	0.092	0.098	0.016	0.000	1.623	0.000	0.259	4.765	0.459

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	38	0	67	0	44	37	38
N.S.	1	1.00	1.00	1.06	0.00	1.86	0.00	1.22	1.03	1.06
time (sec)	N/A	0.029	0.096	0.006	0.000	1.754	0.000	0.284	4.474	0.366

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	37	36	0	67	0	45	35	37
N.S.	1	1.00	1.03	1.00	0.00	1.86	0.00	1.25	0.97	1.03
time (sec)	N/A	0.023	0.020	0.004	0.000	1.075	0.000	0.195	4.362	0.311

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	89	99	0	389	0	110	-1	95
N.S.	1	1.00	1.00	1.11	0.00	4.37	0.00	1.24	-0.01	1.07
time (sec)	N/A	0.081	0.103	0.014	0.000	3.184	0.000	0.203	0.000	0.474

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	137	195	0	485	0	200	-1	134
N.S.	1	1.00	0.99	1.40	0.00	3.49	0.00	1.44	-0.01	0.96
time (sec)	N/A	0.126	0.084	0.015	0.000	1.449	0.000	0.276	0.000	0.583

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	179	314	0	615	0	350	-1	175
N.S.	1	1.00	0.92	1.61	0.00	3.15	0.00	1.79	-0.01	0.90
time (sec)	N/A	0.211	0.126	0.018	0.000	2.741	0.000	0.377	0.000	0.867

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	34	37	34	30	0	0	33	34
N.S.	1	1.00	0.68	0.74	0.68	0.60	0.00	0.00	0.66	0.68
time (sec)	N/A	0.068	0.021	0.005	1.264	0.968	0.000	0.000	4.604	0.039

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	73	64	52	114	0	59	53	68
N.S.	1	1.00	1.26	1.10	0.90	1.97	0.00	1.02	0.91	1.17
time (sec)	N/A	0.084	0.036	0.007	1.157	1.022	0.000	0.198	4.613	0.202

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	26	13	20	0	31	20	22
N.S.	1	1.00	1.00	1.18	0.59	0.91	0.00	1.41	0.91	1.00
time (sec)	N/A	0.017	0.005	0.005	1.219	0.954	0.000	0.183	4.369	0.031

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	52	44	32	74	0	39	33	40
N.S.	1	1.00	1.68	1.42	1.03	2.39	0.00	1.26	1.06	1.29
time (sec)	N/A	0.049	0.012	0.003	1.055	0.920	0.000	0.185	4.563	0.141

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	52	50	0	80	0	46	-1	30
N.S.	1	1.00	1.73	1.67	0.00	2.67	0.00	1.53	-0.03	1.00
time (sec)	N/A	0.010	0.011	0.005	0.000	0.994	0.000	0.189	0.000	0.044

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	26	21	21	0	25	21	23
N.S.	1	1.00	1.00	1.13	0.91	0.91	0.00	1.09	0.91	1.00
time (sec)	N/A	0.040	0.007	0.004	1.022	0.842	0.000	0.180	4.315	0.135

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	68	73	0	133	0	0	76	59
N.S.	1	1.00	1.15	1.24	0.00	2.25	0.00	0.00	1.29	1.00
time (sec)	N/A	0.056	0.064	0.008	0.000	1.071	0.000	0.000	4.637	0.063

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	35	37	44	31	0	57	29	35
N.S.	1	1.00	0.67	0.71	0.85	0.60	0.00	1.10	0.56	0.67
time (sec)	N/A	0.084	0.015	0.005	1.086	0.823	0.000	0.185	4.471	0.155

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	44	94	0	163	0	0	-1	71
N.S.	1	1.00	0.51	1.08	0.00	1.87	0.00	0.00	-0.01	0.82
time (sec)	N/A	0.100	0.012	0.007	0.000	1.001	0.000	0.000	0.000	0.068

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	22	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	0.78	1.00
time (sec)	N/A	0.004	0.004	0.006	0.969	1.203	0.886	0.187	4.663	0.021

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	24	45	63	20	25	-1	32
N.S.	1	1.00	1.00	0.80	1.50	2.10	0.67	0.83	-0.03	1.07
time (sec)	N/A	0.016	0.006	0.008	2.426	2.244	1.099	0.164	0.000	0.038

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	29	37	63	22	23	19	27
N.S.	1	1.00	1.00	1.07	1.37	2.33	0.81	0.85	0.70	1.00
time (sec)	N/A	0.020	0.006	0.011	2.294	1.144	1.251	0.152	4.549	0.039

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	20	31	17	21
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	1.48	0.81	1.00
time (sec)	N/A	0.005	0.005	0.005	1.081	2.549	0.843	0.169	4.514	0.057

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	62	59	51	124	95	54	-1	63
N.S.	1	1.00	0.85	0.81	0.70	1.70	1.30	0.74	-0.01	0.86
time (sec)	N/A	0.022	0.028	0.007	0.971	0.619	4.612	0.293	0.000	0.080

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	27	25	33	23	44	30	24	27
N.S.	1	1.00	0.75	0.69	0.92	0.64	1.22	0.83	0.67	0.75
time (sec)	N/A	0.023	0.014	0.005	1.022	1.097	0.545	0.150	4.599	0.028

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	39	31	93	42	40	56	51
N.S.	1	1.00	1.00	0.80	0.63	1.90	0.86	0.82	1.14	1.04
time (sec)	N/A	0.013	0.019	0.006	1.024	0.778	2.911	0.185	4.640	0.061

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	20	13	13	15
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87	1.00
time (sec)	N/A	0.003	0.002	0.003	1.048	0.901	0.400	0.155	4.331	0.018

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	21	13	59	17	23	20	28
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	0.92	0.80	1.12
time (sec)	N/A	0.006	0.005	0.003	1.018	1.582	1.197	0.183	0.124	0.033

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	17	60	19	22	19	25
N.S.	1	1.00	1.00	1.16	0.68	2.40	0.76	0.88	0.76	1.00
time (sec)	N/A	0.017	0.006	0.005	1.079	0.717	1.203	0.158	4.573	0.026

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	18	17	17	19	30	17	19
N.S.	1	1.00	1.00	0.95	0.89	0.89	1.00	1.58	0.89	1.00
time (sec)	N/A	0.004	0.004	0.005	1.005	0.971	0.749	0.181	0.040	0.052

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	61	48	36	105	42	51	38	50
N.S.	1	1.00	1.22	0.96	0.72	2.10	0.84	1.02	0.76	1.00
time (sec)	N/A	0.027	0.054	0.006	1.029	2.013	3.473	0.168	4.535	0.072

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	29	26	36	27	46	55	25	31
N.S.	1	1.00	0.66	0.59	0.82	0.61	1.05	1.25	0.57	0.70
time (sec)	N/A	0.010	0.006	0.005	1.000	0.896	1.049	0.283	4.552	0.068

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	13	15	8	-1	17
N.S.	1	1.00	1.00	0.81	0.75	0.81	0.94	0.50	-0.06	1.06
time (sec)	N/A	0.002	0.002	0.004	0.965	1.788	0.627	0.151	0.000	0.016

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	15	8	10	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.50	0.62	1.00
time (sec)	N/A	0.002	0.002	0.003	1.038	0.835	0.527	0.148	4.505	0.013

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	14	14	5	-1	16
N.S.	1	1.00	1.00	0.92	0.85	1.08	1.08	0.38	-0.08	1.23
time (sec)	N/A	0.001	0.001	0.003	0.984	0.966	0.482	0.170	0.000	0.016

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	16	0	7	-1	18
N.S.	1	1.00	1.00	0.93	0.87	1.07	0.00	0.47	-0.07	1.20
time (sec)	N/A	0.001	0.002	0.003	0.985	1.058	0.000	0.150	0.000	0.017

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	15	14	8	13	17
N.S.	1	1.00	1.00	0.92	0.83	1.25	1.17	0.67	1.08	1.42
time (sec)	N/A	0.001	0.001	0.002	0.977	1.917	0.472	0.150	4.300	0.017

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	15	15	8	10	19
N.S.	1	1.00	1.00	0.77	0.69	1.15	1.15	0.62	0.77	1.46
time (sec)	N/A	0.001	0.001	0.002	1.026	0.951	0.514	0.159	4.338	0.014

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	15	17	8	13	19
N.S.	1	1.00	1.00	0.81	0.75	0.94	1.06	0.50	0.81	1.19
time (sec)	N/A	0.001	0.002	0.002	1.075	1.818	0.556	0.155	4.305	0.017

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	17	13	12	15	19	8	13	19
N.S.	1	1.00	1.06	0.81	0.75	0.94	1.19	0.50	0.81	1.19
time (sec)	N/A	0.002	0.004	0.003	1.048	0.796	0.658	0.162	4.272	0.018

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	15	13	12	15	19	8	13	19
N.S.	1	1.00	0.94	0.81	0.75	0.94	1.19	0.50	0.81	1.19
time (sec)	N/A	0.002	0.002	0.003	1.064	1.886	0.708	0.153	4.331	0.018

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	9	8	8	8	8	8	0
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67	0.00
time (sec)	N/A	0.001	0.001	0.001	1.009	1.837	0.066	0.149	0.016	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	9	8	8	8	8	8	0
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67	0.00
time (sec)	N/A	0.001	0.001	0.000	1.040	0.882	0.063	0.175	0.027	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	9	8	8	8	8	8	0
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67	0.00
time (sec)	N/A	0.001	0.001	0.001	0.949	1.056	0.066	0.150	0.013	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	9	8	8	8	8	8	0
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67	0.00
time (sec)	N/A	0.001	0.001	0.000	1.074	1.867	0.081	0.150	0.016	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	5	5	5	0
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.71	0.71	0.71	0.00
time (sec)	N/A	0.001	0.000	0.000	0.981	2.584	0.135	0.150	0.002	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	7	7	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.88	0.75	0.00
time (sec)	N/A	0.001	0.000	0.001	1.032	1.142	0.081	0.148	4.241	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	9	8	8	8	8	8	0
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.001	0.001	0.000	0.882	2.656	0.077	0.182	0.028	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	9	8	8	12	8	8	0
N.S.	1	1.00	1.00	0.75	0.67	0.67	1.00	0.67	0.67	0.00
time (sec)	N/A	0.002	0.001	0.001	1.038	1.845	0.075	0.148	4.400	0.001

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	9	8	8	12	8	8	0
N.S.	1	1.00	1.00	0.75	0.67	0.67	1.00	0.67	0.67	0.00
time (sec)	N/A	0.001	0.001	0.000	1.003	1.709	0.074	0.178	4.329	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	25	22	19	24	29	19	21	29
N.S.	1	1.00	0.81	0.71	0.61	0.77	0.94	0.61	0.68	0.94
time (sec)	N/A	0.006	0.007	0.003	0.979	1.379	6.725	0.164	4.291	0.017

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	25	22	19	24	29	19	21	29
N.S.	1	1.00	0.81	0.71	0.61	0.77	0.94	0.61	0.68	0.94
time (sec)	N/A	0.007	0.009	0.004	1.042	1.644	2.672	0.209	0.036	0.017

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	25	22	19	22	29	19	21	29
N.S.	1	1.00	0.81	0.71	0.61	0.71	0.94	0.61	0.68	0.94
time (sec)	N/A	0.007	0.007	0.004	1.013	2.211	2.102	0.169	0.034	0.016

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	25	22	19	21	27	19	21	29
N.S.	1	1.00	0.86	0.76	0.66	0.72	0.93	0.66	0.72	1.00
time (sec)	N/A	0.006	0.008	0.005	1.028	0.868	0.818	0.152	0.031	0.016

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	25	22	19	21	27	19	21	25
N.S.	1	1.00	0.86	0.76	0.66	0.72	0.93	0.66	0.72	0.86
time (sec)	N/A	0.006	0.009	0.004	1.095	0.996	1.041	0.149	0.039	0.020

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	25	22	19	21	27	19	21	25
N.S.	1	1.00	0.86	0.76	0.66	0.72	0.93	0.66	0.72	0.86
time (sec)	N/A	0.006	0.009	0.004	1.023	1.063	1.282	0.326	0.034	0.020

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	25	22	20	21	27	20	21	25
N.S.	1	1.00	0.86	0.76	0.69	0.72	0.93	0.69	0.72	0.86
time (sec)	N/A	0.006	0.009	0.004	1.081	1.753	1.871	0.209	4.326	0.022

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	64	49	44	49	70	46	45	62
N.S.	1	1.00	1.00	0.77	0.69	0.77	1.09	0.72	0.70	0.97
time (sec)	N/A	0.023	3.696	0.006	1.026	0.933	22.351	0.206	4.415	0.030

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	66	49	44	49	70	46	45	62
N.S.	1	1.00	1.03	0.77	0.69	0.77	1.09	0.72	0.70	0.97
time (sec)	N/A	0.022	0.057	0.007	1.160	0.809	12.365	0.172	0.026	0.030

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	50	49	44	47	63	46	45	62
N.S.	1	1.00	0.78	0.77	0.69	0.73	0.98	0.72	0.70	0.97
time (sec)	N/A	0.023	3.384	0.006	1.112	0.582	3.454	0.150	0.028	0.030

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	63	49	48	46	68	46	45	62
N.S.	1	1.00	1.02	0.79	0.77	0.74	1.10	0.74	0.73	1.00
time (sec)	N/A	0.022	0.042	0.006	1.139	0.974	4.996	0.151	0.025	0.028

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	54	49	44	46	68	46	45	52
N.S.	1	1.00	0.87	0.79	0.71	0.74	1.10	0.74	0.73	0.84
time (sec)	N/A	0.022	0.044	0.007	1.113	0.825	5.654	0.154	0.026	0.034

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	53	49	44	46	68	46	45	52
N.S.	1	1.00	0.85	0.79	0.71	0.74	1.10	0.74	0.73	0.84
time (sec)	N/A	0.022	0.065	0.007	1.130	0.957	6.892	0.151	0.027	0.034

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	50	49	45	46	68	47	48	52
N.S.	1	1.00	0.81	0.79	0.73	0.74	1.10	0.76	0.77	0.84
time (sec)	N/A	0.023	0.049	0.007	1.118	1.496	9.109	0.160	0.046	0.039

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	103	90	81	86	129	87	76	111
N.S.	1	1.00	1.00	0.87	0.79	0.83	1.25	0.84	0.74	1.08
time (sec)	N/A	0.048	3.724	0.008	0.999	0.930	60.632	0.157	0.041	0.052

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	105	90	81	86	129	87	76	111
N.S.	1	1.00	1.02	0.87	0.79	0.83	1.25	0.84	0.74	1.08
time (sec)	N/A	0.043	0.098	0.007	1.061	0.889	39.321	0.166	0.037	0.047

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	103	90	81	84	112	87	76	111
N.S.	1	1.00	1.00	0.87	0.79	0.82	1.09	0.84	0.74	1.08
time (sec)	N/A	0.042	3.379	0.008	1.043	2.685	6.047	0.160	0.035	0.048

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	102	90	88	83	128	87	76	111
N.S.	1	1.00	1.01	0.89	0.87	0.82	1.27	0.86	0.75	1.10
time (sec)	N/A	0.044	0.074	0.007	1.043	1.610	23.501	0.171	0.035	0.045

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	100	90	81	83	126	87	76	93
N.S.	1	1.00	1.01	0.91	0.82	0.84	1.27	0.88	0.77	0.94
time (sec)	N/A	0.043	0.084	0.006	0.982	0.933	19.835	0.156	0.038	0.060

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	103	90	81	83	128	87	76	93
N.S.	1	1.00	1.02	0.89	0.80	0.82	1.27	0.86	0.75	0.92
time (sec)	N/A	0.044	0.075	0.008	1.063	1.139	25.437	0.198	0.037	0.057

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	100	90	82	83	124	88	79	93
N.S.	1	1.00	1.01	0.91	0.83	0.84	1.25	0.89	0.80	0.94
time (sec)	N/A	0.042	0.077	0.007	1.055	1.918	31.860	0.174	0.036	0.062

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	389	389	80	65	0	6649	0	0	12789	83
N.S.	1	1.00	0.21	0.17	0.00	17.09	0.00	0.00	32.88	0.21
time (sec)	N/A	0.860	0.053	0.059	0.000	11.734	0.000	0.000	5.820	0.132

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	385	385	80	64	0	5319	0	0	10449	83
N.S.	1	1.00	0.21	0.17	0.00	13.82	0.00	0.00	27.14	0.22
time (sec)	N/A	0.797	0.045	0.010	0.000	4.478	0.000	0.000	6.858	0.077

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	48	45	0	4058	0	0	8093	48
N.S.	1	1.00	0.15	0.14	0.00	12.26	0.00	0.00	24.45	0.15
time (sec)	N/A	0.442	0.029	0.008	0.000	1.962	0.000	0.000	6.512	0.085

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	46	45	0	2482	0	0	8229	46
N.S.	1	1.00	0.14	0.14	0.00	7.50	0.00	0.00	24.86	0.14
time (sec)	N/A	0.403	0.028	0.008	0.000	1.113	0.000	0.000	6.024	0.076

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	47	45	0	2769	0	0	6133	47
N.S.	1	1.00	0.14	0.14	0.00	8.37	0.00	0.00	18.53	0.14
time (sec)	N/A	0.366	0.029	0.007	0.000	0.901	0.000	0.000	5.308	0.081

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	49	42	0	4045	0	0	10401	49
N.S.	1	1.00	0.15	0.13	0.00	12.22	0.00	0.00	31.42	0.15
time (sec)	N/A	0.415	0.032	0.006	0.000	3.461	0.000	0.000	6.258	0.054

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	371	371	78	65	0	5384	0	0	10573	81
N.S.	1	1.00	0.21	0.18	0.00	14.51	0.00	0.00	28.50	0.22
time (sec)	N/A	0.567	0.049	0.010	0.000	4.524	0.000	0.000	5.737	0.119

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	371	371	82	64	0	6671	0	0	16557	85
N.S.	1	1.00	0.22	0.17	0.00	17.98	0.00	0.00	44.63	0.23
time (sec)	N/A	0.512	0.054	0.011	0.000	9.253	0.000	0.000	8.637	0.075

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	412	412	107	82	0	7995	0	0	15149	109
N.S.	1	1.00	0.26	0.20	0.00	19.41	0.00	0.00	36.77	0.26
time (sec)	N/A	0.981	0.073	0.013	0.000	32.952	0.000	0.000	6.479	0.154

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	544	544	144	149	0	0	0	0	28774	257
N.S.	1	1.00	0.26	0.27	0.00	0.00	0.00	0.00	52.89	0.47
time (sec)	N/A	2.577	0.281	0.021	0.000	0.000	0.000	0.000	7.012	0.653

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	520	520	144	146	0	11906	0	0	31964	262
N.S.	1	1.00	0.28	0.28	0.00	22.90	0.00	0.00	61.47	0.50
time (sec)	N/A	1.371	0.266	0.020	0.000	75.662	0.000	0.000	11.849	0.503

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	471	471	124	120	0	11032	0	0	23808	196
N.S.	1	1.00	0.26	0.25	0.00	23.42	0.00	0.00	50.55	0.42
time (sec)	N/A	0.917	0.213	0.019	0.000	79.680	0.000	0.000	6.445	0.465

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	483	483	127	118	0	9245	0	0	26432	201
N.S.	1	1.00	0.26	0.24	0.00	19.14	0.00	0.00	54.72	0.42
time (sec)	N/A	1.032	0.213	0.020	0.000	12.624	0.000	0.000	10.885	0.378

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	450	450	109	121	0	9757	0	0	21913	121
N.S.	1	1.00	0.24	0.27	0.00	21.68	0.00	0.00	48.70	0.27
time (sec)	N/A	0.710	0.214	0.019	0.000	31.669	0.000	0.000	6.058	0.320

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	442	442	111	118	0	10570	0	0	28713	122
N.S.	1	1.00	0.25	0.27	0.00	23.91	0.00	0.00	64.96	0.28
time (sec)	N/A	0.705	0.230	0.020	0.000	29.109	0.000	0.000	10.634	0.274

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	489	489	149	146	0	12411	0	0	26373	156
N.S.	1	1.00	0.30	0.30	0.00	25.38	0.00	0.00	53.93	0.32
time (sec)	N/A	0.998	0.243	0.020	0.000	152.988	0.000	0.000	6.560	0.354

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	503	503	153	144	0	0	0	0	35171	160
N.S.	1	1.00	0.30	0.29	0.00	0.00	0.00	0.00	69.92	0.32
time (sec)	N/A	1.276	0.249	0.019	0.000	0.000	0.000	0.000	7.286	0.302

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	573	573	190	245	0	0	0	0	31145	280
N.S.	1	1.00	0.33	0.43	0.00	0.00	0.00	0.00	54.35	0.49
time (sec)	N/A	2.446	0.339	0.027	0.000	0.000	0.000	0.000	11.419	0.539

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	621	621	254	275	0	0	0	0	50970	527
N.S.	1	1.00	0.41	0.44	0.00	0.00	0.00	0.00	82.08	0.85
time (sec)	N/A	1.772	0.442	0.039	0.000	0.000	0.000	0.000	9.350	1.190

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	569	569	216	242	0	0	0	0	39697	397
N.S.	1	1.00	0.38	0.43	0.00	0.00	0.00	0.00	69.77	0.70
time (sec)	N/A	1.908	0.408	0.038	0.000	0.000	0.000	0.000	8.015	1.116

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	569	569	219	241	0	0	0	0	45495	413
N.S.	1	1.00	0.38	0.42	0.00	0.00	0.00	0.00	79.96	0.73
time (sec)	N/A	1.965	0.387	0.036	0.000	0.000	0.000	0.000	8.524	0.933

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	533	533	176	244	0	0	0	0	37678	344
N.S.	1	1.00	0.33	0.46	0.00	0.00	0.00	0.00	70.69	0.65
time (sec)	N/A	1.452	0.380	0.036	0.000	0.000	0.000	0.000	7.659	0.970

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	533	533	177	237	0	0	0	0	47803	350
N.S.	1	1.00	0.33	0.44	0.00	0.00	0.00	0.00	89.69	0.66
time (sec)	N/A	1.363	0.441	0.036	0.000	0.000	0.000	0.000	8.384	0.801

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	594	594	222	277	0	0	0	0	42197	245
N.S.	1	1.00	0.37	0.47	0.00	0.00	0.00	0.00	71.04	0.41
time (sec)	N/A	2.310	0.406	0.037	0.000	0.000	0.000	0.000	8.019	0.557

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	594	594	224	270	0	0	0	0	54027	245
N.S.	1	1.00	0.38	0.45	0.00	0.00	0.00	0.00	90.95	0.41
time (sec)	N/A	2.366	0.412	0.037	0.000	0.000	0.000	0.000	9.347	0.485

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	658	658	254	321	0	0	0	0	46948	294
N.S.	1	1.00	0.39	0.49	0.00	0.00	0.00	0.00	71.35	0.45
time (sec)	N/A	5.485	0.495	0.053	0.000	0.000	0.000	0.000	8.746	0.677

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	658	658	258	316	0	0	0	0	60099	296
N.S.	1	1.00	0.39	0.48	0.00	0.00	0.00	0.00	91.34	0.45
time (sec)	N/A	5.792	0.464	0.038	0.000	0.000	0.000	0.000	9.847	0.593

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	111	782	195	594	4451	1132	546	0
N.S.	1	1.00	0.71	5.01	1.25	3.81	28.53	7.26	3.50	0.00
time (sec)	N/A	0.100	0.129	0.009	1.308	0.761	7.268	0.226	4.834	0.683

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	70	301	110	241	1486	449	260	0
N.S.	1	1.00	0.69	2.98	1.09	2.39	14.71	4.45	2.57	0.00
time (sec)	N/A	0.053	0.054	0.008	1.107	0.750	2.871	0.181	4.584	0.222

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	35	78	50	71	314	119	89	0
N.S.	1	1.00	0.67	1.50	0.96	1.37	6.04	2.29	1.71	0.00
time (sec)	N/A	0.020	0.029	0.004	1.087	0.787	0.989	0.160	4.395	0.064

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [209] had the largest ratio of [.5263]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	22	0.136
2	A	3	3	1.00	22	0.136
3	A	2	2	1.00	22	0.091
4	A	2	2	1.00	22	0.091
5	A	3	3	1.03	22	0.136
6	A	4	3	1.02	22	0.136
7	A	5	3	1.10	22	0.136
8	A	9	5	1.00	16	0.312
9	A	3	3	1.00	16	0.188
10	A	9	5	1.00	16	0.312
11	A	3	2	1.00	12	0.167
12	A	3	2	1.00	12	0.167
13	A	9	5	1.00	12	0.417
14	A	9	5	1.00	12	0.417
15	A	9	5	1.00	12	0.417
16	A	2	1	1.00	15	0.067
17	A	2	1	1.00	13	0.077
18	A	1	0	1.00	11	0.000
19	A	2	1	1.00	15	0.067
20	A	2	1	1.00	15	0.067
21	A	2	1	1.00	15	0.067
22	A	2	1	1.00	15	0.067
23	A	2	1	1.00	15	0.067
24	A	2	1	1.00	15	0.067
25	A	2	1	1.00	15	0.067
26	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	3	2	1.00	13	0.154
28	A	4	3	1.00	17	0.176
29	A	3	2	1.00	17	0.118
30	A	2	2	1.00	17	0.118
31	A	3	2	1.00	17	0.118
32	A	4	3	1.00	17	0.176
33	A	3	2	1.00	17	0.118
34	A	4	3	1.00	17	0.176
35	A	3	2	1.00	17	0.118
36	A	4	3	1.00	17	0.176
37	A	3	2	1.00	17	0.118
38	A	2	2	1.00	17	0.118
39	A	3	2	1.00	17	0.118
40	A	3	2	1.00	17	0.118
41	A	4	3	1.00	17	0.176
42	A	3	2	1.00	17	0.118
43	A	2	2	1.00	17	0.118
44	A	3	2	1.00	17	0.118
45	A	4	3	1.00	17	0.176
46	A	3	2	1.00	17	0.118
47	A	4	3	1.00	17	0.176
48	A	3	2	1.00	17	0.118
49	A	4	3	1.00	17	0.176
50	A	3	2	1.00	17	0.118
51	A	4	3	1.00	17	0.176
52	A	3	2	1.00	17	0.118
53	A	2	2	1.00	17	0.118
54	A	3	2	1.00	17	0.118
55	A	4	4	1.00	17	0.235
56	A	4	3	1.00	17	0.176
57	A	4	3	1.00	17	0.176
58	A	4	3	1.00	17	0.176
59	A	4	3	1.00	17	0.176
60	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
61	A	4	3	1.00	17	0.176
62	A	3	3	1.00	17	0.176
63	A	2	2	1.00	17	0.118
64	A	2	2	1.00	17	0.118
65	A	5	5	1.00	15	0.333
66	A	3	3	1.00	13	0.231
67	A	4	3	1.00	17	0.176
68	A	4	3	1.00	17	0.176
69	A	4	3	1.00	17	0.176
70	A	5	3	1.00	17	0.176
71	A	4	3	1.00	17	0.176
72	A	5	4	1.00	17	0.235
73	A	4	3	1.00	17	0.176
74	A	5	4	1.00	17	0.235
75	A	4	3	1.00	17	0.176
76	A	4	4	1.00	17	0.235
77	A	4	3	1.00	17	0.176
78	A	3	3	1.00	17	0.176
79	A	2	2	1.00	17	0.118
80	A	3	3	1.00	17	0.176
81	A	4	3	1.00	17	0.176
82	A	4	4	1.00	17	0.235
83	A	4	3	1.00	15	0.200
84	A	5	4	1.00	13	0.308
85	A	4	3	1.00	17	0.176
86	A	6	4	1.00	17	0.235
87	A	6	4	1.00	17	0.235
88	A	4	3	1.00	17	0.176
89	A	5	4	1.00	17	0.235
90	A	4	3	1.00	17	0.176
91	A	4	3	1.00	17	0.176
92	A	2	2	1.00	17	0.118
93	A	4	4	1.00	17	0.235
94	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	4	3	1.00	17	0.176
96	A	4	3	1.00	17	0.176
97	A	5	4	1.00	17	0.235
98	A	4	3	1.00	17	0.176
99	A	6	4	1.00	17	0.235
100	A	4	3	1.00	15	0.200
101	A	7	4	1.00	13	0.308
102	A	4	3	1.00	17	0.176
103	A	6	6	1.00	19	0.316
104	A	5	5	1.00	19	0.263
105	A	4	4	1.00	17	0.235
106	A	4	4	1.00	19	0.210
107	A	4	4	1.00	19	0.210
108	A	1	1	1.00	19	0.053
109	A	2	2	1.00	19	0.105
110	A	3	2	1.00	19	0.105
111	A	4	2	1.00	19	0.105
112	A	5	2	1.00	19	0.105
113	A	3	2	1.00	19	0.105
114	A	2	2	1.00	19	0.105
115	A	1	1	1.00	15	0.067
116	A	3	3	1.00	19	0.158
117	A	3	3	1.00	19	0.158
118	A	4	4	1.00	19	0.210
119	A	5	4	1.00	19	0.210
120	A	6	5	1.00	19	0.263
121	A	5	4	1.00	17	0.235
122	A	5	5	1.00	19	0.263
123	A	5	4	1.00	19	0.210
124	A	5	5	1.00	19	0.263
125	A	5	4	1.00	19	0.210
126	A	1	1	1.00	19	0.053
127	A	2	2	1.00	19	0.105
128	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	4	2	1.00	19	0.105
130	A	5	2	1.00	19	0.105
131	A	5	3	1.00	19	0.158
132	A	4	3	1.00	19	0.158
133	A	3	3	1.00	19	0.158
134	A	2	2	1.00	15	0.133
135	A	1	1	1.00	19	0.053
136	A	4	3	1.00	19	0.158
137	A	4	4	1.00	19	0.210
138	A	4	3	1.00	19	0.158
139	A	5	4	1.00	19	0.210
140	A	6	4	1.00	19	0.210
141	A	7	4	1.00	19	0.210
142	A	6	5	1.00	19	0.263
143	A	5	5	1.00	19	0.263
144	A	4	4	1.00	19	0.210
145	A	3	3	1.00	17	0.176
146	A	1	1	1.00	19	0.053
147	A	2	2	1.00	19	0.105
148	A	3	2	1.00	19	0.105
149	A	4	2	1.00	19	0.105
150	A	2	2	1.00	19	0.105
151	A	1	1	1.00	19	0.053
152	A	2	2	1.00	15	0.133
153	A	3	3	1.00	19	0.158
154	A	4	3	1.00	19	0.158
155	A	6	6	1.00	19	0.316
156	A	5	5	1.00	19	0.263
157	A	4	4	1.00	19	0.210
158	A	1	1	1.00	19	0.053
159	A	2	2	1.00	17	0.118
160	A	3	3	1.00	19	0.158
161	A	4	3	1.00	19	0.158
162	A	5	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
163	A	2	2	1.00	19	0.105
164	A	1	1	1.00	19	0.053
165	A	3	3	1.00	19	0.158
166	A	4	4	1.00	15	0.267
167	A	5	4	1.00	19	0.210
168	A	4	4	1.00	19	0.210
169	A	4	4	1.00	19	0.210
170	A	4	4	1.00	19	0.210
171	A	4	4	1.00	19	0.210
172	A	4	4	1.00	19	0.210
173	A	4	4	1.00	20	0.200
174	A	2	1	1.00	17	0.059
175	A	2	1	1.00	17	0.059
176	A	2	1	1.00	17	0.059
177	A	2	1	1.00	17	0.059
178	A	2	1	1.00	17	0.059
179	A	2	1	1.00	17	0.059
180	A	2	1	1.00	17	0.059
181	A	2	1	1.00	17	0.059
182	A	3	2	1.00	19	0.105
183	A	3	2	1.00	19	0.105
184	A	3	2	1.00	19	0.105
185	A	3	2	1.00	19	0.105
186	A	3	2	1.00	19	0.105
187	A	3	2	1.00	19	0.105
188	A	3	2	1.00	19	0.105
189	A	3	2	1.00	19	0.105
190	A	3	2	1.00	19	0.105
191	A	3	2	1.00	19	0.105
192	A	3	2	1.00	19	0.105
193	A	3	2	1.00	19	0.105
194	A	3	2	1.00	19	0.105
195	A	3	2	1.00	19	0.105
196	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	3	2	1.00	19	0.105
198	A	13	9	1.00	19	0.474
199	A	13	9	1.00	19	0.474
200	A	12	9	1.00	19	0.474
201	A	12	9	1.00	19	0.474
202	A	11	8	1.00	19	0.421
203	A	11	8	1.00	19	0.421
204	A	12	9	1.00	19	0.474
205	A	12	9	1.00	19	0.474
206	A	13	9	1.00	19	0.474
207	A	13	9	1.00	19	0.474
208	A	14	9	1.00	19	0.474
209	A	14	10	1.00	19	0.526
210	A	13	10	1.00	19	0.526
211	A	13	10	1.00	19	0.526
212	A	12	9	1.00	19	0.474
213	A	12	9	1.00	19	0.474
214	A	12	9	1.00	19	0.474
215	A	12	9	1.00	19	0.474
216	A	13	10	1.00	19	0.526
217	A	13	10	1.00	19	0.526
218	A	14	10	1.00	19	0.526
219	A	14	10	1.00	19	0.526
220	A	15	10	1.00	19	0.526
221	A	14	10	1.00	19	0.526
222	A	13	9	1.00	19	0.474
223	A	13	9	1.00	19	0.474
224	A	13	10	1.00	19	0.526
225	A	13	10	1.00	19	0.526
226	A	13	9	1.00	19	0.474
227	A	13	9	1.00	19	0.474
228	A	14	10	1.00	19	0.526
229	A	14	10	1.00	19	0.526
230	A	15	10	1.00	19	0.526

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	15	10	1.00	19	0.526
232	A	16	10	1.00	19	0.526
233	A	16	10	1.00	19	0.526
234	A	4	3	1.00	19	0.158
235	A	4	3	1.00	19	0.158
236	A	2	1	1.00	17	0.059
237	A	2	1	1.00	22	0.045
238	A	2	1	1.00	22	0.045
239	A	2	1	1.00	20	0.050
240	A	1	0	1.00	18	0.000
241	A	2	1	1.00	22	0.045
242	A	2	1	1.00	22	0.045
243	A	2	1	1.00	22	0.045
244	A	2	1	1.00	22	0.045
245	A	2	1	1.00	22	0.045
246	A	2	1	1.00	22	0.045
247	A	2	1	1.00	22	0.045
248	A	2	1	1.00	22	0.045
249	A	3	2	1.00	24	0.083
250	A	4	3	1.00	24	0.125
251	A	3	2	1.00	24	0.083
252	A	4	3	1.00	24	0.125
253	A	3	2	1.00	24	0.083
254	A	2	2	1.00	22	0.091
255	A	3	2	1.00	20	0.100
256	A	4	3	1.00	24	0.125
257	A	3	2	1.00	24	0.083
258	A	4	3	1.00	24	0.125
259	A	3	2	1.00	24	0.083
260	A	4	3	1.00	24	0.125
261	A	3	2	1.00	24	0.083
262	A	4	3	1.00	24	0.125
263	A	3	2	1.00	24	0.083
264	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	3	2	1.00	24	0.083
266	A	2	2	1.00	24	0.083
267	A	3	2	1.00	24	0.083
268	A	4	4	1.00	24	0.167
269	A	3	2	1.00	24	0.083
270	A	4	3	1.00	24	0.125
271	A	3	2	1.00	24	0.083
272	A	3	2	1.00	24	0.083
273	A	4	3	1.00	24	0.125
274	A	3	2	1.00	24	0.083
275	A	4	3	1.00	24	0.125
276	A	3	2	1.00	24	0.083
277	A	4	3	1.00	24	0.125
278	A	3	2	1.00	24	0.083
279	A	2	2	1.00	22	0.091
280	A	3	2	1.00	20	0.100
281	A	4	3	1.00	24	0.125
282	A	3	2	1.00	24	0.083
283	A	4	3	1.00	24	0.125
284	A	3	2	1.00	24	0.083
285	A	4	3	1.00	24	0.125
286	A	3	2	1.00	24	0.083
287	A	4	3	1.00	24	0.125
288	A	3	2	1.00	24	0.083
289	A	4	3	1.00	24	0.125
290	A	3	2	1.00	24	0.083
291	A	4	3	1.00	24	0.125
292	A	3	2	1.00	24	0.083
293	A	4	3	1.00	24	0.125
294	A	3	2	1.00	24	0.083
295	A	2	2	1.00	24	0.083
296	A	3	2	1.00	24	0.083
297	A	4	4	1.00	24	0.167
298	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
299	A	5	4	1.00	24	0.167
300	A	3	2	1.00	24	0.083
301	A	6	4	1.00	24	0.167
302	A	3	2	1.00	24	0.083
303	A	4	3	1.00	24	0.125
304	A	4	3	1.00	24	0.125
305	A	4	3	1.00	24	0.125
306	A	4	3	1.00	24	0.125
307	A	4	3	1.00	24	0.125
308	A	2	2	1.00	22	0.091
309	A	4	3	1.00	24	0.125
310	A	4	3	1.00	24	0.125
311	A	4	3	1.00	24	0.125
312	A	5	4	1.00	24	0.167
313	A	5	4	1.00	24	0.167
314	A	5	4	1.00	24	0.167
315	A	4	4	1.00	24	0.167
316	A	3	3	1.00	24	0.125
317	A	3	3	1.00	20	0.150
318	A	4	4	1.00	24	0.167
319	A	5	4	1.00	24	0.167
320	A	6	4	1.00	24	0.167
321	A	4	3	1.00	24	0.125
322	A	4	3	1.00	24	0.125
323	A	4	3	1.00	24	0.125
324	A	2	2	1.00	24	0.083
325	A	4	3	1.00	24	0.125
326	A	2	2	1.00	22	0.091
327	A	4	3	1.00	24	0.125
328	A	4	3	1.00	24	0.125
329	A	4	3	1.00	24	0.125
330	A	7	4	1.00	24	0.167
331	A	7	4	1.00	24	0.167
332	A	6	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
333	A	5	3	1.00	24	0.125
334	A	5	4	1.00	24	0.167
335	A	5	4	1.00	24	0.167
336	A	5	3	1.00	20	0.150
337	A	6	4	1.00	24	0.167
338	A	7	4	1.00	24	0.167
339	A	8	4	1.00	24	0.167
340	A	4	3	1.00	24	0.125
341	A	4	3	1.00	24	0.125
342	A	4	3	1.00	24	0.125
343	A	2	2	1.00	24	0.083
344	A	4	4	1.00	24	0.167
345	A	4	3	1.00	24	0.125
346	A	4	3	1.00	24	0.125
347	A	2	2	1.00	22	0.091
348	A	4	3	1.00	24	0.125
349	A	4	3	1.00	24	0.125
350	A	4	3	1.00	24	0.125
351	A	9	4	1.00	24	0.167
352	A	9	4	1.00	24	0.167
353	A	8	4	1.00	24	0.167
354	A	7	3	1.00	24	0.125
355	A	7	4	1.00	24	0.167
356	A	7	4	1.00	24	0.167
357	A	7	4	1.00	24	0.167
358	A	7	4	1.00	24	0.167
359	A	7	3	1.00	20	0.150
360	A	8	4	1.00	24	0.167
361	A	9	4	1.00	24	0.167
362	A	10	4	1.00	24	0.167
363	A	3	3	1.00	12	0.250
364	A	2	2	1.00	14	0.143
365	A	3	3	1.00	16	0.188
366	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
367	A	2	2	1.00	14	0.143
368	A	4	3	1.00	16	0.188
369	A	4	3	1.00	26	0.115
370	A	3	3	1.00	26	0.115
371	A	2	2	1.00	24	0.083
372	A	3	2	1.00	26	0.077
373	A	3	2	1.00	26	0.077
374	A	3	3	1.00	26	0.115
375	A	1	1	1.00	26	0.038
376	A	4	3	1.00	26	0.115
377	A	4	3	1.00	26	0.115
378	A	3	2	1.00	26	0.077
379	A	3	2	1.00	26	0.077
380	A	2	1	1.00	22	0.045
381	A	3	2	1.00	26	0.077
382	A	3	2	1.00	26	0.077
383	A	3	2	1.00	26	0.077
384	A	3	2	1.00	26	0.077
385	A	3	2	1.00	26	0.077
386	A	4	3	1.00	26	0.115
387	A	4	3	1.00	26	0.115
388	A	3	2	1.12	26	0.077
389	A	3	3	1.00	26	0.115
390	A	2	2	1.00	24	0.083
391	A	4	3	1.00	26	0.115
392	A	4	3	1.00	26	0.115
393	A	4	3	1.00	26	0.115
394	A	4	3	1.00	26	0.115
395	A	3	3	1.00	26	0.115
396	A	1	1	1.00	26	0.038
397	A	4	3	1.00	26	0.115
398	A	4	3	1.00	26	0.115
399	A	4	3	1.00	26	0.115
400	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	3	2	1.00	26	0.077
402	A	3	2	1.00	26	0.077
403	A	3	2	1.00	26	0.077
404	A	3	2	1.00	22	0.091
405	A	3	2	1.00	26	0.077
406	A	3	2	1.00	26	0.077
407	A	3	2	1.00	26	0.077
408	A	3	2	1.00	26	0.077
409	A	3	2	1.00	26	0.077
410	A	3	2	1.00	26	0.077
411	A	3	2	1.00	26	0.077
412	A	3	2	1.00	26	0.077
413	A	4	3	1.00	26	0.115
414	A	4	3	1.00	26	0.115
415	A	3	2	1.00	26	0.077
416	A	4	3	1.00	26	0.115
417	A	4	3	1.00	26	0.115
418	A	3	3	1.00	26	0.115
419	A	2	2	1.00	24	0.083
420	A	4	3	1.00	26	0.115
421	A	4	3	1.00	26	0.115
422	A	4	3	1.00	26	0.115
423	A	4	3	1.00	26	0.115
424	A	4	3	1.00	26	0.115
425	A	4	3	1.00	26	0.115
426	A	3	3	1.00	26	0.115
427	A	1	1	1.00	26	0.038
428	A	5	4	1.00	26	0.154
429	A	4	3	1.00	26	0.115
430	A	4	3	1.00	26	0.115
431	A	4	3	1.00	26	0.115
432	A	4	3	1.00	26	0.115
433	A	3	2	1.00	26	0.077
434	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
435	A	3	2	1.00	26	0.077
436	A	3	2	1.00	26	0.077
437	A	3	2	1.00	26	0.077
438	A	3	2	1.00	26	0.077
439	A	3	2	1.00	22	0.091
440	A	3	2	1.00	26	0.077
441	A	3	2	1.00	26	0.077
442	A	3	2	1.00	26	0.077
443	A	3	2	1.00	26	0.077
444	A	3	2	1.00	26	0.077
445	A	3	2	1.00	26	0.077
446	A	3	2	1.00	26	0.077
447	A	3	2	1.00	26	0.077
448	A	3	2	1.00	26	0.077
449	A	3	2	1.00	26	0.077
450	A	3	2	1.00	26	0.077
451	A	3	2	1.00	26	0.077
452	A	4	3	1.00	26	0.115
453	A	4	4	1.00	26	0.154
454	A	3	3	1.00	24	0.125
455	A	5	5	1.00	26	0.192
456	A	4	3	0.98	26	0.115
457	A	4	3	1.00	26	0.115
458	A	3	3	1.00	26	0.115
459	A	2	2	1.00	22	0.091
460	A	3	3	1.00	26	0.115
461	A	4	3	0.98	26	0.115
462	A	4	3	1.00	26	0.115
463	A	4	3	1.00	26	0.115
464	A	3	3	1.68	26	0.115
465	A	2	2	1.00	24	0.083
466	A	4	3	1.00	26	0.115
467	A	4	3	1.00	26	0.115
468	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	4	4	1.00	26	0.154
470	A	4	3	1.00	22	0.136
471	A	5	4	1.00	26	0.154
472	A	6	4	1.00	26	0.154
473	A	4	3	1.00	26	0.115
474	A	4	3	1.00	26	0.115
475	A	3	3	1.00	26	0.115
476	A	1	1	1.00	26	0.038
477	A	3	3	1.00	26	0.115
478	A	2	2	1.00	24	0.083
479	A	4	3	1.00	26	0.115
480	A	4	3	1.00	26	0.115
481	A	6	4	1.00	26	0.154
482	A	6	4	1.00	26	0.154
483	A	6	4	1.00	26	0.154
484	A	6	3	1.00	22	0.136
485	A	7	4	1.00	26	0.154
486	A	8	4	1.00	26	0.154
487	A	2	1	1.00	26	0.038
488	A	2	1	1.00	26	0.038
489	A	2	1	1.00	26	0.038
490	A	2	1	1.00	26	0.038
491	A	2	1	1.00	26	0.038
492	A	2	1	1.00	26	0.038
493	A	2	1	1.00	26	0.038
494	A	3	2	1.00	28	0.071
495	A	3	2	1.00	28	0.071
496	A	3	2	1.00	28	0.071
497	A	3	2	1.00	28	0.071
498	A	3	2	1.00	28	0.071
499	A	3	2	1.00	28	0.071
500	A	3	2	1.00	28	0.071
501	A	3	2	1.00	28	0.071
502	A	3	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
503	A	3	2	1.00	28	0.071
504	A	3	2	1.00	28	0.071
505	A	3	2	1.00	28	0.071
506	A	3	2	1.00	28	0.071
507	A	3	2	1.00	28	0.071
508	A	14	10	1.00	28	0.357
509	A	13	10	1.00	28	0.357
510	A	13	10	1.00	28	0.357
511	A	12	9	1.00	28	0.321
512	A	12	9	1.00	28	0.321
513	A	12	9	1.00	28	0.321
514	A	12	9	1.00	28	0.321
515	A	13	10	1.00	28	0.357
516	A	13	10	1.00	28	0.357
517	A	14	10	1.00	28	0.357
518	A	16	10	1.00	28	0.357
519	A	15	10	1.00	28	0.357
520	A	15	10	1.00	28	0.357
521	A	14	9	1.00	28	0.321
522	A	14	9	1.00	28	0.321
523	A	14	10	1.00	28	0.357
524	A	14	10	1.00	28	0.357
525	A	14	10	1.00	28	0.357
526	A	14	10	1.00	28	0.357
527	A	14	9	1.00	28	0.321
528	A	14	9	1.00	28	0.321
529	A	15	10	1.00	28	0.357
530	A	15	10	1.00	28	0.357
531	A	16	10	1.00	28	0.357
532	A	18	10	1.00	28	0.357
533	A	17	10	1.00	28	0.357
534	A	17	10	1.00	28	0.357
535	A	16	9	1.00	28	0.321
536	A	16	9	1.00	28	0.321

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
537	A	16	10	1.00	28	0.357
538	A	16	10	1.00	28	0.357
539	A	16	10	1.00	28	0.357
540	A	16	10	1.00	28	0.357
541	A	16	10	1.00	28	0.357
542	A	16	10	1.00	28	0.357
543	A	16	10	1.00	28	0.357
544	A	16	10	1.00	28	0.357
545	A	16	9	1.00	28	0.321
546	A	16	9	1.00	28	0.321
547	A	17	10	1.00	28	0.357
548	A	17	10	1.00	28	0.357
549	A	18	10	1.00	28	0.357
550	A	3	2	1.00	30	0.067
551	A	3	2	1.00	30	0.067
552	A	3	2	1.00	30	0.067
553	A	3	2	1.00	30	0.067
554	A	3	2	1.00	30	0.067
555	A	3	2	1.00	30	0.067
556	A	3	2	1.00	30	0.067
557	A	3	2	1.00	30	0.067
558	A	3	2	1.00	30	0.067
559	A	3	2	1.00	30	0.067
560	A	3	2	1.00	30	0.067
561	A	3	2	1.00	30	0.067
562	A	3	2	1.00	30	0.067
563	A	3	2	1.00	30	0.067
564	A	3	2	1.00	30	0.067
565	A	3	2	1.00	30	0.067
566	A	3	2	1.00	30	0.067
567	A	3	2	1.00	30	0.067
568	A	3	2	1.00	30	0.067
569	A	3	2	1.00	30	0.067
570	A	3	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
571	A	13	9	1.00	30	0.300
572	A	12	9	1.00	30	0.300
573	A	12	9	1.00	30	0.300
574	A	11	8	1.00	30	0.267
575	A	11	8	1.00	30	0.267
576	A	12	9	1.00	30	0.300
577	A	12	9	1.00	30	0.300
578	A	13	9	1.00	30	0.300
579	A	15	10	1.00	30	0.333
580	A	14	10	1.00	30	0.333
581	A	14	10	1.00	30	0.333
582	A	13	9	1.00	30	0.300
583	A	13	9	1.00	30	0.300
584	A	13	10	1.00	30	0.333
585	A	13	10	1.00	30	0.333
586	A	13	9	1.00	30	0.300
587	A	13	9	1.00	30	0.300
588	A	14	10	1.00	30	0.333
589	A	14	10	1.00	30	0.333
590	A	15	10	1.00	30	0.333
591	A	17	10	1.00	30	0.333
592	A	16	10	1.00	30	0.333
593	A	16	10	1.00	30	0.333
594	A	15	9	1.00	30	0.300
595	A	15	9	1.00	30	0.300
596	A	15	10	1.00	30	0.333
597	A	15	10	1.00	30	0.333
598	A	15	10	1.00	30	0.333
599	A	15	10	1.00	30	0.333
600	A	15	10	1.00	30	0.333
601	A	15	10	1.00	30	0.333
602	A	15	9	1.00	30	0.300
603	A	15	9	1.00	30	0.300
604	A	16	10	1.00	30	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
605	A	16	10	1.00	30	0.333
606	A	17	10	1.00	30	0.333
607	A	3	2	1.00	26	0.077
608	A	3	2	1.00	26	0.077
609	A	2	1	1.00	24	0.042
610	A	3	2	1.00	28	0.071
611	A	3	2	1.00	28	0.071
612	A	3	2	1.00	28	0.071
613	A	4	3	1.00	24	0.125
614	A	4	3	1.00	24	0.125
615	A	4	3	1.00	24	0.125
616	A	2	2	1.00	22	0.091
617	A	2	1	1.00	16	0.062
618	A	2	1	1.00	14	0.071
619	A	1	0	1.00	12	0.000
620	A	2	1	1.00	16	0.062
621	A	2	1	1.00	16	0.062
622	A	2	1	1.00	16	0.062
623	A	2	1	1.00	16	0.062
624	A	2	1	1.00	16	0.062
625	A	2	1	1.00	16	0.062
626	A	2	1	1.00	16	0.062
627	A	2	1	1.00	16	0.062
628	A	2	1	1.00	18	0.056
629	A	3	2	1.00	16	0.125
630	A	2	1	1.00	14	0.071
631	A	3	2	1.00	18	0.111
632	A	2	1	1.00	18	0.056
633	A	3	2	1.00	18	0.111
634	A	2	1	1.00	18	0.056
635	A	3	2	1.00	18	0.111
636	A	2	1	1.00	18	0.056
637	A	3	2	1.00	18	0.111
638	A	2	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
639	A	3	2	1.00	18	0.111
640	A	2	1	1.00	18	0.056
641	A	3	2	1.00	18	0.111
642	A	2	1	1.00	18	0.056
643	A	3	2	1.00	18	0.111
644	A	2	1	1.00	18	0.056
645	A	3	2	1.00	16	0.125
646	A	2	1	1.00	14	0.071
647	A	3	2	1.00	18	0.111
648	A	2	1	1.00	18	0.056
649	A	3	2	1.00	18	0.111
650	A	2	1	1.00	18	0.056
651	A	7	6	1.00	18	0.333
652	A	6	6	1.00	18	0.333
653	A	5	5	1.00	18	0.278
654	A	3	3	1.00	16	0.188
655	A	7	7	1.00	18	0.389
656	A	8	7	1.00	18	0.389
657	A	8	7	1.00	18	0.389
658	A	5	4	1.00	18	0.222
659	A	4	3	1.00	18	0.167
660	A	3	2	1.00	18	0.111
661	A	3	2	1.00	14	0.143
662	A	4	3	1.00	18	0.167
663	A	5	4	1.00	18	0.222
664	A	7	7	1.00	18	0.389
665	A	4	4	1.00	18	0.222
666	A	4	4	1.00	18	0.222
667	A	4	4	1.00	16	0.250
668	A	8	7	1.00	18	0.389
669	A	8	7	1.00	18	0.389
670	A	6	4	1.00	18	0.222
671	A	5	4	1.00	18	0.222
672	A	4	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
673	A	4	3	1.00	18	0.167
674	A	4	3	1.00	14	0.214
675	A	5	4	1.00	18	0.222
676	A	8	8	1.00	18	0.444
677	A	5	4	1.00	18	0.222
678	A	5	5	1.00	18	0.278
679	A	5	5	1.00	18	0.278
680	A	5	5	1.00	18	0.278
681	A	5	4	1.00	16	0.250
682	A	9	8	1.00	18	0.444
683	A	9	8	1.00	18	0.444
684	A	7	5	1.00	18	0.278
685	A	6	5	1.00	18	0.278
686	A	5	4	1.00	18	0.222
687	A	5	4	1.00	18	0.222
688	A	5	4	1.00	18	0.222
689	A	5	4	1.00	14	0.286
690	A	6	5	1.00	18	0.278
691	A	6	6	1.00	19	0.316
692	A	5	5	1.00	19	0.263
693	A	3	3	1.00	17	0.176
694	A	7	7	1.00	19	0.368
695	A	8	7	1.00	19	0.368
696	A	4	3	1.00	19	0.158
697	A	3	2	1.00	19	0.105
698	A	3	2	1.00	15	0.133
699	A	4	3	1.00	19	0.158
700	A	6	6	1.00	22	0.273
701	A	5	5	1.00	22	0.227
702	A	3	3	1.00	20	0.150
703	A	7	7	1.00	22	0.318
704	A	8	7	1.00	22	0.318
705	A	4	3	1.00	22	0.136
706	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
707	A	3	2	1.00	18	0.111
708	A	4	3	1.00	22	0.136
709	A	6	6	1.00	20	0.300
710	A	5	5	1.00	20	0.250
711	A	3	3	1.00	18	0.167
712	A	7	7	1.00	20	0.350
713	A	8	7	1.00	20	0.350
714	A	10	6	1.00	20	0.300
715	A	9	5	1.00	20	0.250
716	A	9	5	1.00	16	0.312
717	A	10	6	1.00	20	0.300
718	A	3	3	1.00	12	0.250
719	A	3	3	1.00	14	0.214
720	A	3	2	1.00	16	0.125
721	A	9	6	1.00	16	0.375
722	A	9	6	1.00	16	0.375
723	A	6	6	1.00	20	0.300
724	A	6	6	1.00	20	0.300
725	A	5	5	1.00	20	0.250
726	A	4	4	1.00	18	0.222
727	A	7	6	1.00	20	0.300
728	A	7	6	1.00	20	0.300
729	A	4	4	1.00	20	0.200
730	A	5	5	1.00	20	0.250
731	A	6	6	1.00	20	0.300
732	A	7	7	1.00	20	0.350
733	A	7	6	1.00	20	0.300
734	A	7	6	1.00	20	0.300
735	A	6	5	1.00	20	0.250
736	A	5	4	1.00	18	0.222
737	A	8	7	1.00	20	0.350
738	A	8	7	1.00	20	0.350
739	A	8	7	1.00	20	0.350
740	A	8	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
741	A	5	4	1.00	20	0.200
742	A	6	5	1.00	20	0.250
743	A	7	6	1.00	20	0.300
744	A	5	5	1.00	20	0.250
745	A	5	5	1.00	20	0.250
746	A	4	4	1.00	20	0.200
747	A	3	3	1.00	18	0.167
748	A	3	3	1.00	20	0.150
749	A	4	4	1.00	20	0.200
750	A	5	5	1.00	20	0.250
751	A	6	6	1.00	20	0.300
752	A	5	5	1.00	21	0.238
753	A	5	5	1.00	21	0.238
754	A	4	4	1.00	21	0.190
755	A	3	3	1.00	19	0.158
756	A	3	3	1.00	22	0.136
757	A	4	4	1.00	22	0.182
758	A	5	5	1.00	22	0.227
759	A	6	6	1.00	22	0.273
760	A	6	6	1.00	20	0.300
761	A	5	5	1.00	20	0.250
762	A	5	5	1.00	20	0.250
763	A	2	2	1.00	20	0.100
764	A	2	2	1.00	18	0.111
765	A	5	5	1.00	20	0.250
766	A	5	5	1.00	20	0.250
767	A	6	6	1.00	20	0.300
768	A	3	3	1.00	28	0.107
769	A	5	5	1.00	28	0.179
770	A	2	2	1.00	28	0.071
771	A	4	4	1.00	26	0.154
772	A	3	3	1.00	24	0.125
773	A	2	2	1.00	28	0.071
774	A	4	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
775	A	3	3	1.00	28	0.107
776	A	5	4	1.00	28	0.143
777	A	2	2	1.00	29	0.069
778	A	4	4	1.00	27	0.148
779	A	4	4	1.00	29	0.138
780	A	2	2	1.00	29	0.069
781	A	5	4	1.00	29	0.138
782	A	4	3	1.00	29	0.103
783	A	4	4	1.00	29	0.138
784	A	2	2	1.00	27	0.074
785	A	3	3	1.00	25	0.120
786	A	4	4	1.00	29	0.138
787	A	2	2	1.00	29	0.069
788	A	5	5	1.00	29	0.172
789	A	3	3	1.00	29	0.103
790	A	3	3	1.00	23	0.130
791	A	3	3	1.00	23	0.130
792	A	3	3	1.00	23	0.130
793	A	3	3	1.00	21	0.143
794	A	3	3	1.00	19	0.158
795	A	3	3	1.00	23	0.130
796	A	3	3	1.00	23	0.130
797	A	3	3	1.00	23	0.130
798	A	3	3	1.00	23	0.130
799	A	3	3	1.00	24	0.125
800	A	3	3	1.00	24	0.125
801	A	3	3	1.00	24	0.125
802	A	3	3	1.00	22	0.136
803	A	2	2	1.00	20	0.100
804	A	3	3	1.00	24	0.125
805	A	3	3	1.00	24	0.125
806	A	3	3	1.00	24	0.125
807	A	3	3	1.00	24	0.125
808	A	2	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
809	A	2	1	1.00	18	0.056
810	A	2	1	1.00	18	0.056
811	A	2	1	1.00	18	0.056
812	A	2	1	1.00	18	0.056
813	A	2	1	1.00	18	0.056
814	A	2	1	1.00	18	0.056
815	A	2	1	1.00	20	0.050
816	A	2	1	1.00	20	0.050
817	A	2	1	1.00	20	0.050
818	A	2	1	1.00	20	0.050
819	A	2	1	1.00	20	0.050
820	A	2	1	1.00	20	0.050
821	A	2	1	1.00	20	0.050
822	A	2	1	1.00	20	0.050
823	A	2	1	1.00	20	0.050
824	A	2	1	1.00	20	0.050
825	A	2	1	1.00	20	0.050
826	A	2	1	1.00	20	0.050
827	A	2	1	1.00	20	0.050
828	A	2	1	1.00	20	0.050
829	A	9	6	1.00	20	0.300
830	A	9	6	1.00	20	0.300
831	A	8	5	1.00	20	0.250
832	A	8	5	1.00	20	0.250
833	A	8	5	1.00	20	0.250
834	A	8	5	1.00	20	0.250
835	A	9	6	1.00	20	0.300
836	A	9	6	1.00	20	0.300
837	A	10	7	1.00	20	0.350
838	A	10	7	1.00	20	0.350
839	A	10	7	1.00	20	0.350
840	A	9	6	1.00	20	0.300
841	A	9	6	1.00	20	0.300
842	A	9	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
843	A	9	6	1.00	20	0.300
844	A	9	6	1.00	20	0.300
845	A	9	6	1.00	20	0.300
846	A	10	7	1.00	20	0.350
847	A	11	8	1.00	20	0.400
848	A	10	7	1.00	20	0.350
849	A	10	7	1.00	20	0.350
850	A	10	7	1.00	20	0.350
851	A	10	7	1.00	20	0.350
852	A	10	7	1.00	20	0.350
853	A	10	7	1.00	20	0.350
854	A	10	7	1.00	20	0.350
855	A	10	7	1.00	20	0.350
856	A	2	1	1.00	20	0.050
857	A	2	1	1.00	20	0.050
858	A	2	1	1.00	18	0.056

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(a^2+2abx^2+b^2x^4\right)^{3/4} dx$

Optimal. Leaf size=128

$$\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3\sqrt{a}\left(a^2+2abx^2+b^2x^4\right)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 195, 215}

$$\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3\sqrt{a}\left(a^2+2abx^2+b^2x^4\right)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))/4 + (3*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4
)^(3/4))/(8*(a + b*x^2)) + (3*Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)*Arc
Sinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[b]*(1 + (b*x^2)/a)^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4} dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/4} \int \left(1+\frac{bx^2}{a}\right)^{3/2} dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4} + \frac{\left(3\left(a^2+2abx^2+b^2x^4\right)^{3/4}\right)\int \sqrt{1+\frac{bx^2}{a}}\, dx}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4} + \frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)} + \frac{\left(3\left(a^2+2abx^2+b^2x^4\right)\right.}{8\left(1+\frac{bx^2}{a}\right.}$$

$$= \frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4} + \frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)} + \frac{3\sqrt{a}\left(a^2+2abx^2+b^2\right.}{8\sqrt{b}\left(1+\right.}$$

Mathematica [A] time = 0.08, size = 97, normalized size = 0.76

$$\frac{\left(\left(a+bx^2\right)^2\right)^{3/4}\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x\left(5a+2bx^2\right)\sqrt{\frac{bx^2}{a}+1}\right)}{8\sqrt{b}\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (((a + b*x^2)^2)^(3/4)*(Sqrt[b]*x*(5*a + 2*b*x^2)*Sqrt[1 + (b*x^2)/a] + 3*a
^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/(8*Sqrt[b]*(a + b*x^2)*Sqrt[1 + (b*x^
2)/a])
```

IntegrateAlgebraic [A] time = 6.81, size = 85, normalized size = 0.66

$$\frac{\left(\left(a+bx^2\right)^2\right)^{3/4}\left(\frac{1}{8}\sqrt{a+bx^2}\left(5ax+2bx^3\right)-\frac{3a^2\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8\sqrt{b}}\right)}{\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (((a + b*x^2)^2)^(3/4)*((Sqrt[a + b*x^2]*(5*a*x + 2*b*x^3))/8 - (3*a^2*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*Sqrt[b])))/(a + b*x^2)^(3/2)
```

fricas [A] time = 0.51, size = 177, normalized size = 1.38

$$\left[\frac{3\,a^{2}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{1}{4}}\sqrt{b}\,x-a\right)+2\,\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{1}{4}}\left(2\,b^{2}x^{3}+5\,abx\right)}{16\,b}, -\frac{3\,a^{2}\sqrt{-b}\,\arctan\left(\frac{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{1}{4}}\sqrt{-b}\,x}{bx^{2}+a}\right)-\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{1}{4}}\left(2\,b^{2}x^{3}+5\,abx\right)}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqr
t(b)*x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/b,
 -1/8*(3*a^2*sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(
b*x^2 + a)) - (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/b]
```

giac [A] time = 0.44, size = 87, normalized size = 0.68

$$-\frac{\frac{3\,a^{3}\arctan\left(\frac{\sqrt{-\frac{bx^{2}+a}{x^{2}}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{\left(5\,a^{3}\left(b+\frac{a}{x^{2}}\right)\sqrt{-\frac{bx^{2}+a}{x^{2}}}-3\,a^{3}b\sqrt{-\frac{bx^{2}+a}{x^{2}}}\right)x^{4}}{a^{2}}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
```

```
[Out] -1/8*(3*a^3*arctan(sqrt(-(b*x^2 + a)/x^2)/sqrt(b))/sqrt(b) + (5*a^3*(b + a/
x^2)*sqrt(-(b*x^2 + a)/x^2) - 3*a^3*b*sqrt(-(b*x^2 + a)/x^2))*x^4/a^2)/a
```

maple [A] time = 0.03, size = 77, normalized size = 0.60

$$\frac{3\sqrt{b\,x^{2}+a}\;a^{2}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{8\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{1}{4}}\sqrt{b}}+\frac{\left(2b\,x^{2}+5a\right)\left(b\,x^{2}+a\right)x}{8\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
```

```
[Out] 1/8*x*(2*b*x^2+5*a)*(b*x^2+a)/((b*x^2+a)^2)^(1/4)+3/8*a^2*ln(x*b^(1/2)+(b*x
^2+a)^(1/2))/b^(1/2)/((b*x^2+a)^2)^(1/4)*(b*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4),x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4), x)
```

3.2 $\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\,\sqrt[4]{a^2+2abx^2+b^2x^4}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 195, 215}

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\,\sqrt[4]{a^2+2abx^2+b^2x^4}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))/2 + (Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(1/4)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b]*Sqrt[1 + (b*x^2)/a])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\int \sqrt{1+\frac{bx^2}{a}}\,dx}{\sqrt{1+\frac{bx^2}{a}}}$$

$$= \frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4} + \frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{2\sqrt{1+\frac{bx^2}{a}}}$$

$$= \frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4} + \frac{\sqrt{a}\,\sqrt[4]{a^2+2abx^2+b^2x^4}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\,\sqrt{1+\frac{bx^2}{a}}}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.65

$$\frac{1}{2}\sqrt[4]{\left(a+bx^2\right)^2}\left(\frac{a\log\left(\sqrt{b}\,\sqrt{a+bx^2}+bx\right)}{\sqrt{b}\,\sqrt{a+bx^2}}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

```
[Out] (((a + b*x^2)^2)^(1/4)*(x + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(Sqrt[b]
*Sqrt[a + b*x^2])))/2
```

IntegrateAlgebraic [A] time = 6.49, size = 73, normalized size = 0.80

$$\frac{\sqrt[4]{\left(a+bx^2\right)^2}\left(\frac{1}{2}x\sqrt{a+bx^2}-\frac{a\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2\sqrt{b}}\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

```
[Out] (((a + b*x^2)^2)^(1/4)*((x*Sqrt[a + b*x^2])/2 - (a*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/(2*Sqrt[b])))/Sqrt[a + b*x^2]
```

fricas [A] time = 0.72, size = 147, normalized size = 1.62

$$\left[\frac{a\sqrt{b}\,\log\left(-2\,bx^2-2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{b}\,x-a\right)+2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}bx}{4\,b}, -\frac{a\sqrt{-b}\,\arctan\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{-b}\,x}{bx^2+a}\right)-\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}bx}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)*
x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x)/b, -1/2*(a*sqrt(-b)*arcta
n((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2 + a)) - (b^2*x^4 + 2*
a*b*x^2 + a^2)^(1/4)*b*x)/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

maple [A] time = 0.01, size = 58, normalized size = 0.64

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{1}{4}} a \ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b\,x^2+a}\;\sqrt{b}}+\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{1}{4}} x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

```
[Out] 1/2*x*((b*x^2+a)^2)^(1/4)+1/2*a*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)*((b*x
^2+a)^2)^(1/4)/(b*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{a^2 + 2abx^2 + b^2x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4), x)
```

3.3 $\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1089, 215}

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(a^2 +
2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\\ &= \frac{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt[4]{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] (Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*((a + b*x^2
)^2)^(1/4))
```

IntegrateAlgebraic [A] time = 6.07, size = 52, normalized size = 0.87

$$-\frac{\left(\left(a+bx^2\right)^2\right)^{3/4}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{\sqrt{b}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] -((((a + b*x^2)^2)^(3/4)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(Sqrt[b]*(a +
 b*x^2)^(3/2)))
```

fricas [A] time = 0.99, size = 90, normalized size = 1.50

$$\left[\frac{\log\left(-2\,bx^2-2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{b}\,x-a\right)}{2\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{-b}\,x}{bx^2+a}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)*x - a)/sqrt
(b), -sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2 +
 a))/b]
```

giac [A] time = 0.22, size = 24, normalized size = 0.40

$$-\frac{\arctan\left(\frac{\sqrt{-\frac{bx^2+a}{x^2}}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
[Out] -arctan(sqrt(-(b*x^2 + a)/x^2)/sqrt(b))/sqrt(b)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2ab\,x^2 + a^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4),x)
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a^2 + 2abx^2 + b^2x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/4), x)
```

3.4 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\,dx$

Optimal. Leaf size=34

$$\frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1089, 191}

$$\frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
```

```
[Out] (x*(a + b*x^2))/(a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/2}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\\ &= \frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\end{aligned}$$

Mathematica [A]   time = 0.01, size = 25, normalized size = 0.74

$$\frac{x\left(a+bx^2\right)}{a\left(\left(a+bx^2\right)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
[Out] (x*(a + b*x^2))/(a*((a + b*x^2)^2)^(3/4))
```

IntegrateAlgebraic [A]   time = 6.49, size = 25, normalized size = 0.74

$$\frac{x\left(a+bx^2\right)}{a\left(\left(a+bx^2\right)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
[Out] (x*(a + b*x^2))/(a*((a + b*x^2)^2)^(3/4))
```

fricas [A]   time = 0.87, size = 34, normalized size = 1.00

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
[Out] (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*x/(a*b*x^2 + a^2)
```

giac [A]   time = 0.26, size = 19, normalized size = 0.56

$$-\frac{1}{a\sqrt{-\frac{bx^2+a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
[Out] -1/(a*sqrt(-(b*x^2 + a)/x^2))
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$\frac{\left(b\,x^{2}+a\right)x}{\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{4}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
[Out] x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4), x)
```

mupad [B] time = 4.14, size = 34, normalized size = 1.00

$$\frac{x\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{1/4}}{a\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4),x)
[Out] (x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4))/(a*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/4), x)
```

3.5 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/4}}\,dx$

Optimal. Leaf size=68

$$\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x\left(a+bx^2\right)}{3a\left(a^2+2abx^2+b^2x^4\right)^{5/4}}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 192, 191}

$$\frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

```
[Out] (2*x)/(3*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(3*a*(a + b*x^2)*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{\left(2\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.59

$$\frac{x\left(3a+2bx^2\right)}{3a^2\left(a+bx^2\right)\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)*((a + b*x^2)^2)^(1/4))
```

IntegrateAlgebraic [A] time = 7.66, size = 40, normalized size = 0.59

$$\frac{x\left(\left(a+bx^2\right)^2\right)^{3/4}\left(3a+2bx^2\right)}{3a^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
[Out] (x*((a + b*x^2)^2)^(3/4)*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^3)
```

fricas [A] time = 0.50, size = 58, normalized size = 0.85

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\left(2\,bx^3+3\,ax\right)}{3\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b*x^3 + 3*a*x)/(a^2*b^2*x^4 + 2*a^
3*b*x^2 + a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

maple [A] time = 0.00, size = 44, normalized size = 0.65

$$\frac{\left(b\,x^2 + a\right)\left(2b\,x^2 + 3a\right)x}{3\left(b^2x^4 + 2ab\,x^2 + a^2\right)^{\frac{5}{4}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x)
```

```
[Out] 1/3*(b*x^2+a)*x*(2*b*x^2+3*a)/a^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

mupad [B] time = 4.20, size = 45, normalized size = 0.66

$$\frac{x\,\left(2\,b\,x^2 + 3\,a\right)\,\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/4}}{3\,a^2\,\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/4),x)
```

```
[Out] (x*(3*a + 2*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4))/(3*a^2*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/4), x)
```

3.6 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx$

Optimal. Leaf size=105

$$\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{x\left(a+bx^2\right)}{5a\left(a^2+2abx^2+b^2x^4\right)^{7/4}}+\frac{8x\left(a+bx^2\right)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Rubi [A] time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 192, 191}

$$\frac{8x\left(a+bx^2\right)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (4*x)/(15*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)) + x/(5*a*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/4)) + (8*x*(a + b*x^2))/(15*a^3*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/2}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{7/2}}\,dx}{a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

$$= \frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{\left(4\left(1+\frac{bx^2}{a}\right)^{3/2}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{5a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

$$= \frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{\left(8\left(1+\frac{bx^2}{a}\right.\right.}{15a^2\left(a^2\right.}$$

$$= \frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{8x}{15a^3\left(a^2+\right.}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.49

$$\frac{x\left(15a^2+20abx^2+8b^2x^4\right)}{15a^3\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (x*(15*a^2 + 20*a*b*x^2 + 8*b^2*x^4))/(15*a^3*(a + b*x^2)*((a + b*x^2)^2)^(
3/4))
```

IntegrateAlgebraic [A] time = 9.92, size = 51, normalized size = 0.49

$$\frac{x\sqrt[4]{\left(a+bx^2\right)^2}\left(15a^2+20abx^2+8b^2x^4\right)}{15a^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (x*((a + b*x^2)^2)^(1/4)*(15*a^2 + 20*a*b*x^2 + 8*b^2*x^4))/(15*a^3*(a + b*
x^2)^3)
```

fricas [A] time = 0.78, size = 80, normalized size = 0.76

$$\frac{\left(8\,b^2x^5+20\,abx^3+15\,a^2x\right)\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}{15\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/15*(8*b^2*x^5 + 20*a*b*x^3 + 15*a^2*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)/
(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

maple [A] time = 0.00, size = 55, normalized size = 0.52

$$\frac{\left(b\,x^2+a\right)\left(8b^2x^4+20ab\,x^2+15a^2\right)x}{15\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{7}{4}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x)
```

```
[Out] 1/15*(b*x^2+a)*x*(8*b^2*x^4+20*a*b*x^2+15*a^2)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^
(7/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

mupad [B] time = 4.21, size = 56, normalized size = 0.53

$$\frac{x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{1/4}\,\left(15\,a^2+20\,a\,b\,x^2+8\,b^2\,x^4\right)}{15\,a^3\,\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(7/4),x)
```

```
[Out] (x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4)*(15*a^2 + 8*b^2*x^4 + 20*a*b*x^2))/(15
*a^3*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(7/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-7/4), x)
```

3.7 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{9/4}}\,dx$

Optimal. Leaf size=135

$$\frac{6x}{35a^2\left(a^2+2abx^2+b^2x^4\right)^{5/4}}+\frac{x\left(a+bx^2\right)}{7a\left(a^2+2abx^2+b^2x^4\right)^{9/4}}+\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{8x\left(a+bx^2\right)}{35a^3\left(a^2+2abx^2+b^2x^4\right)^{5/4}}$$

Rubi [A] time = 0.04, antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 192, 191}

$$\frac{8x}{35a^3\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (16*x)/(35*a^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(7*a*(a + b*x^2)^3*(a
^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + (6*x)/(35*a^2*(a + b*x^2)^2*(a^2 + 2*a*b
*x^2 + b^2*x^4)^(1/4)) + (8*x)/(35*a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{9/4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{9/2}}\,dx}{a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{\left(6\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{7/2}}\,dx}{7a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{\left(2\right.}{3}$$

$$= \frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{}{35}$$

$$= \frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{}{35a^2\left(a+bx^2\right.}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.46

$$\frac{x\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^3\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a + b*x^2)
^3*((a + b*x^2)^2)^(1/4))
```

IntegrateAlgebraic [A] time = 13.17, size = 62, normalized size = 0.46

$$\frac{x\left(\left(a+bx^2\right)^2\right)^{3/4}\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (x*((a + b*x^2)^2)^(3/4)*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6
))/(35*a^4*(a + b*x^2)^5)
```

fricas [A] time = 0.85, size = 102, normalized size = 0.76

$$\frac{\left(16 b^3 x^7 + 56 a b^2 x^5 + 70 a^2 b x^3 + 35 a^3 x\right)\left(b^2 x^4 + 2 a b x^2 + a^2\right)^{\frac{1}{4}}}{35 \left(a^4 b^4 x^8 + 4 a^5 b^3 x^6 + 6 a^6 b^2 x^4 + 4 a^7 b x^2 + a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*(b^2*x^4 + 2*a*b
*x^2 + a^2)^(1/4)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^
2 + a^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2 x^4 + 2 a b x^2 + a^2\right)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

maple [A] time = 0.00, size = 66, normalized size = 0.49

$$\frac{\left(b x^2 + a\right)\left(16 b^3 x^6 + 56 b^2 x^4 a + 70 a^2 b x^2 + 35 a^3\right) x}{35 \left(b^2 x^4 + 2 a b x^2 + a^2\right)^{\frac{9}{4}} a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x)
```

```
[Out] 1/35*(b*x^2+a)*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(b^2*x^4
+2*a*b*x^2+a^2)^(9/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

mupad [B] time = 4.13, size = 141, normalized size = 1.04

$$\frac{x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}}{7\,a\,\left(b\,x^2+a\right)^5}+\frac{6\,x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}}{35\,a^2\,\left(b\,x^2+a\right)^4}+\frac{8\,x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}}{35\,a^3\,\left(b\,x^2+a\right)^3}+\frac{16\,x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}}{35\,a^4\,\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(9/4),x)
```

```
[Out] (x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4))/(7*a*(a + b*x^2)^5) + (6*x*(a^2 + b^2
*x^4 + 2*a*b*x^2)^(3/4))/(35*a^2*(a + b*x^2)^4) + (8*x*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(3/4))/(35*a^3*(a + b*x^2)^3) + (16*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
3/4))/(35*a^4*(a + b*x^2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(9/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-9/4), x)
```

3.8 $\int \frac{1}{a^2+b+2ax^2+x^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\log\left(-\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\log\left(\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a-\cdots}}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+\cdots}}}$$

Rubi [A] time = 0.31, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\log\left(\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}+a}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}+\sqrt{2}x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[a^2 + b]] - Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/(2*
Sqrt[2]*Sqrt[a^2 + b]*Sqrt[a + Sqrt[a^2 + b]]) + ArcTan[(Sqrt[-a + Sqrt[a^2
 + b]] + Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/(2*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[
a + Sqrt[a^2 + b]]) - Log[Sqrt[a^2 + b] - Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*
x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[-a + Sqrt[a^2 + b]]) + Log[Sqrt[a^2
+ b] + Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*S
qrt[-a + Sqrt[a^2 + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}-x}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}+x}{\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}}$$

$$= \frac{\int \frac{1}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{4\sqrt{a^2+b}} + \frac{\int \frac{1}{\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{4\sqrt{a^2+b}} - \frac{\int \frac{-\sqrt{2}\sqrt{-a+\sqrt{a^2}}}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+}}}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-}}$$

$$= -\frac{\log\left(\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \frac{\log\left(\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+}}\right.}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}-\sqrt{2}x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}+\sqrt{2}x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} - \frac{\log\left(\sqrt{a^2+b}-\sqrt{2}\right.}{4\sqrt{2}\sqrt{a^2+}}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 0.27

$$-\frac{i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-i\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+i\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{b}}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ((-1/2*I)*(ArcTan[x/Sqrt[a - I*Sqrt[b]]]/Sqrt[a - I*Sqrt[b]] - ArcTan[x/Sqr
t[a + I*Sqrt[b]]]/Sqrt[a + I*Sqrt[b]]))/Sqrt[b]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a^2 + b + 2*a*x^2 + x^4)^(-1), x]
```

fricas [B] time = 0.67, size = 583, normalized size = 1.95

$$\frac{1}{4}\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+a}{a^2b+b^2}}\log\left(\left((a^3b+ab^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+b\right)\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+a}{a^2b+b^2}}+x\right)-\frac{1}{4}\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+a}{a^2b+b^2}}\log\left(-\left((a^3b+ab^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+b\right)\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+a}{a^2b+b^2}}+x\right)-\frac{1}{4}\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-a}{a^2b+b^2}}\log\left(\left((a^3b+ab^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-b\right)\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-a}{a^2b+b^2}}+x\right)+\frac{1}{4}\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-a}{a^2b+b^2}}\log\left(-\left((a^3b+ab^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-b\right)\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-a}{a^2b+b^2}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^
2))*log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a^2
*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) - 1/4
*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2))
*log(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a^2*b
 + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) - 1/4*s
qrt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2))*
log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-((a^2*b
+ b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x) + 1/4*sq
rt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2))*l
og(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-((a^2*b
+ b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x)
```

giac [A] time = 0.16, size = 75, normalized size = 0.25

$$-\frac{\sqrt{a+\sqrt{-b}}\,\arctan\left(\frac{x}{\sqrt{a+\sqrt{-b}}}\right)}{2\left(a\sqrt{-b}-b\right)}+\frac{\sqrt{a-\sqrt{-b}}\,\arctan\left(\frac{x}{\sqrt{a-\sqrt{-b}}}\right)}{2\left(a\sqrt{-b}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + sqrt(-b))*arctan(x/sqrt(a + sqrt(-b)))/(a*sqrt(-b) - b) + 1/2
*sqrt(a - sqrt(-b))*arctan(x/sqrt(a - sqrt(-b)))/(a*sqrt(-b) + b)
```

maple [B] time = 0.13, size = 1099, normalized size = 3.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2+b),x)
```

```
[Out] 1/8/b/(a^2+b)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)^(1/2))*(2*(a^2+b
)^(1/2)-2*a)^(1/2)*a^2+1/8/b/(a^2+b)^(3/2)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(
1/2)+(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^3+1/8/(a^2+b)*ln(x^2+x*(2
*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)+1/8/(a
^2+b)^(3/2)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)^(1/2))*(2*(a^2+b)^
(1/2)-2*a)^(1/2)*a-1/2/b/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((
2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2+1/2/b/(a^
2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1
/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^4-1/2/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*
a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/
2))+3/2/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1
/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2+b/(a^2+b)^(3/2)/(2*(a^2+b)
^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)
+2*a)^(1/2))-1/8/b/(a^2+b)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/2)-x^2-(a^2+b)^(1/
2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^2-1/8/b/(a^2+b)^(3/2)*ln(x*(2*(a^2+b)^(1/
2)-2*a)^(1/2)-x^2-(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^3-1/8/(a^2+b
)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/2)-x^2-(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)
^(1/2)-1/8/(a^2+b)^(3/2)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/2)-x^2-(a^2+b)^(1/2)
)*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a+1/2/b/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*a)^(
1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*
a^2-1/2/b/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2
)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^4+1/2/(a^2+b)^(1/2)/(2*(a^
2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(
1/2)+2*a)^(1/2))-3/2/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(
```

```
a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2-b/(a^2+b)^(3/
2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*
(a^2+b)^(1/2)+2*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 + 2\,ax^2 + a^2 + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + b), x)
```

mupad [B] time = 4.38, size = 872, normalized size = 2.92

$$-2\,\mathrm{atanh}\left(\frac{8\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}-\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a\,b^2}{a^2\,b^2+b^3}}-\frac{8\,a^2\,b^2\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}-\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}-\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}+\frac{8\,a\,b\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}-\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}\,\sqrt{-b^3}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}-\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}\right)\,\sqrt{\frac{a\,b-\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}-2\,\mathrm{atanh}\left(\frac{8\,a^2\,b^2\,x\,\sqrt{\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}+\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}+\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}+\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}-\frac{8\,x\,\sqrt{\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}+\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b\,\sqrt{-b^3}}{a^2\,b^2+b^3}+\frac{2\,a\,b^2}{a^2\,b^2+b^3}}+\frac{8\,a\,b\,x\,\sqrt{\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}+\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}}\,\sqrt{-b^3}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}+\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}+\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}\right)\,\sqrt{\frac{a\,b+\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x^2 + a^2 + x^4),x)
```

```
[Out] - 2*atanh((8*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*b^
2)))^(1/2))/((2*b*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a*b^2)/(b^3 + a^2*b^2)
) - (8*a^2*b^2*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*
b^2)))^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a^3*b^4)/(b^3 + a^
2*b^2) - (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a^2*b^
2)) + (8*a*b*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*b^
2)))^(1/2)*(-b^3)^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a^3*b^4
)/(b^3 + a^2*b^2) - (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b
^3 + a^2*b^2)))*((a*b - (-b^3)^(1/2))/(16*(b^3 + a^2*b^2)))^(1/2) - 2*atanh
((8*a^2*b^2*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 + a^2*b^2
)))^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a^3*b^4)/(b^3 + a^2*b
^2) + (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a^2*b^2))
 - (8*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 + a^2*b^2)))^(1
/2))/((2*b*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a*b^2)/(b^3 + a^2*b^2)) + (8*
a*b*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 + a^2*b^2)))^(1/2
)*(-b^3)^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a^3*b^4)/(b^3 +
a^2*b^2) + (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a^2*
b^2)))*((a*b + (-b^3)^(1/2))/(16*(b^3 + a^2*b^2)))^(1/2)
```

sympy [A] time = 0.80, size = 63, normalized size = 0.21

$$\mathrm{RootSum}\left(t^4\left(256a^2b^2+256b^3\right)-32t^2ab+1,\left(t\mapsto t\log\left(64t^3a^3b+64t^3ab^2-4ta^2+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2+b),x)

[Out] RootSum(_t**4*(256*a**2*b**2 + 256*b**3) - 32*_t**2*a*b + 1, Lambda(_t, _t*
log(64*_t**3*a**3*b + 64*_t**3*a*b**2 - 4*_t*a**2 + 4*_t*b + x)))
```

3.9 $\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1093, 207, 203}

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 + a]) - ArcTanh[x/Sqrt[1 - a]]/(2*Sqrt[1 -
 a])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{-1+a+x^2}\,dx - \frac{1}{2}\int \frac{1}{1+a+x^2}\,dx$$
$$= -\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.91

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + a]]/(2*Sqrt[-1 + a]) - ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 +
 a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(-1 + a^2 + 2*a*x^2 + x^4)^(-1), x]
```

fricas [A] time = 0.81, size = 269, normalized size = 5.72

$$\left[-\frac{(a-1)\sqrt{-a-1}\log\left(\frac{x^2+2\sqrt{-a-1}\,x-a-1}{x^2+a+1}\right)+(a+1)\sqrt{-a+1}\log\left(\frac{x^2-2\sqrt{-a+1}\,x-a+1}{x^2+a-1}\right)}{4\left(a^2-1\right)}, \frac{2(a+1)\sqrt{a-1}\arctan\left(\frac{x}{\sqrt{a-1}}\right)-(a-1)\sqrt{-a-1}\log\left(\frac{x^2+2\sqrt{-a-1}\,x-a-1}{x^2+a+1}\right)}{4\left(a^2-1\right)}, -\frac{2\sqrt{a+1}(a-1)\arctan\left(\frac{x}{\sqrt{a+1}}\right)+(a+1)\sqrt{-a+1}\log\left(\frac{x^2-2\sqrt{-a+1}\,x-a+1}{x^2+a-1}\right)}{4\left(a^2-1\right)}, -\frac{\sqrt{a+1}(a-1)\arctan\left(\frac{x}{\sqrt{a+1}}\right)-(a+1)\sqrt{a-1}\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\left(a^2-1\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - 1)*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a +
 1)) + (a + 1)*sqrt(-a + 1)*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a -
 1)))/(a^2 - 1), 1/4*(2*(a + 1)*sqrt(a - 1)*arctan(x/sqrt(a - 1)) - (a - 1)
*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a + 1)))/(a^2 - 1
), -1/4*(2*sqrt(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) + (a + 1)*sqrt(-a + 1)
*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a - 1)))/(a^2 - 1), -1/2*(sqrt
```

```
(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) - (a + 1)*sqrt(a - 1)*arctan(x/sqrt(a
 - 1)))/(a^2 - 1)]
```

giac [A] time = 0.15, size = 31, normalized size = 0.66

$$-\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}+\frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x/sqrt(a + 1))/sqrt(a + 1) + 1/2*arctan(x/sqrt(a - 1))/sqrt(a -
 1)
```

maple [A] time = 0.01, size = 32, normalized size = 0.68

$$-\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}+\frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2-1),x)
```

```
[Out] -1/2*arctan(x/(1+a)^(1/2))/(1+a)^(1/2)+1/2/(a-1)^(1/2)*arctan(x/(a-1)^(1/2)
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1.0>0)', see `assume?` for more d
etails)Is a-1.0 positive or negative?
```

mupad [B] time = 0.10, size = 85, normalized size = 1.81

$$\frac{\mathrm{atanh}\left(\frac{2\,x\,\left(\frac{a}{2}-\frac{1}{2}\right)}{\sqrt{1-a}}+\frac{2\,a\,x\,\left(\frac{a}{2}-\frac{1}{2}\right)}{{\left(1-a\right)}^{3/2}}\right)}{2\,\sqrt{1-a}}+\frac{\mathrm{atanh}\left(\frac{2\,x\,\left(\frac{a}{2}+\frac{1}{2}\right)}{\sqrt{-a-1}}+\frac{2\,a\,x\,\left(\frac{a}{2}+\frac{1}{2}\right)}{{\left(-a-1\right)}^{3/2}}\right)}{2\,\sqrt{-a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a*x^2 + a^2 + x^4 - 1),x)
```

```
[Out] atanh((2*x*(a/2 - 1/2))/(1 - a)^(1/2) + (2*a*x*(a/2 - 1/2))/(1 - a)^(3/2))/
(2*(1 - a)^(1/2)) + atanh((2*x*(a/2 + 1/2))/(- a - 1)^(1/2) + (2*a*x*(a/2 +
 1/2))/(- a - 1)^(3/2))/(2*(- a - 1)^(1/2))
```

sympy [B] time = 0.63, size = 257, normalized size = 5.47

$$\frac{\sqrt{-\frac{1}{a-1}}\log\left(-a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}-a^2\sqrt{-\frac{1}{a-1}}+a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x-\sqrt{-\frac{1}{a-1}}\right)}{4}-\frac{\sqrt{-\frac{1}{a-1}}\log\left(a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+a^2\sqrt{-\frac{1}{a-1}}-a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x+\sqrt{-\frac{1}{a-1}}\right)}{4}+\frac{\sqrt{-\frac{1}{a+1}}\log\left(-a^3\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}-a^2\sqrt{-\frac{1}{a+1}}+a\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+x-\sqrt{-\frac{1}{a+1}}\right)}{4}-\frac{\sqrt{-\frac{1}{a+1}}\log\left(a^3\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+a^2\sqrt{-\frac{1}{a+1}}-a\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+x+\sqrt{-\frac{1}{a+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2-1),x)
```

```
[Out] sqrt(-1/(a - 1))*log(-a**3*(-1/(a - 1))**(3/2) - a**2*sqrt(-1/(a - 1)) + a*
(-1/(a - 1))**(3/2) + x - sqrt(-1/(a - 1)))/4 - sqrt(-1/(a - 1))*log(a**3*(
-1/(a - 1))**(3/2) + a**2*sqrt(-1/(a - 1)) - a*(-1/(a - 1))**(3/2) + x + sq
rt(-1/(a - 1)))/4 + sqrt(-1/(a + 1))*log(-a**3*(-1/(a + 1))**(3/2) - a**2*s
qrt(-1/(a + 1)) + a*(-1/(a + 1))**(3/2) + x - sqrt(-1/(a + 1)))/4 - sqrt(-1
/(a + 1))*log(a**3*(-1/(a + 1))**(3/2) + a**2*sqrt(-1/(a + 1)) - a*(-1/(a +
 1))**(3/2) + x + sqrt(-1/(a + 1)))/4
```

3.10 $\int \frac{1}{1+a^2+2ax^2+x^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a-}}{\sqrt{\sqrt{a^2+1}+a}}\right.}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+}}}$$

Rubi [A] time = 0.31, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a}+\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[1 + a^2]] - Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/(2*
Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a + Sqrt[1 + a^2]]) + ArcTan[(Sqrt[-a + Sqrt[1 +
 a^2]] + Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/(2*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[
a + Sqrt[1 + a^2]]) - Log[Sqrt[1 + a^2] - Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*
x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[-a + Sqrt[1 + a^2]]) + Log[Sqrt[1 +
a^2] + Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*S
qrt[-a + Sqrt[1 + a^2]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}-x}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}+x}{\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}}$$

$$= \frac{\int \frac{1}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{4\sqrt{1+a^2}} + \frac{\int \frac{1}{\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{4\sqrt{1+a^2}} - \frac{\int \frac{-\sqrt{2}\sqrt{-a+\sqrt{1-}}}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+}}}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-}}$$

$$= -\frac{\log\left(\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2\right)}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \frac{\log\left(\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\right.}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}-\sqrt{2}\,x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}+\sqrt{2}\,x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} - \frac{\log\left(\sqrt{1+a^2}-\sqrt{2}\right.}{4\sqrt{2}\sqrt{1+a}}$$

Mathematica [C]   time = 0.03, size = 52, normalized size = 0.17

$$-\frac{1}{2}i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-i}}\right)}{\sqrt{a-i}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+i}}\right)}{\sqrt{a+i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] (-1/2*I)*(ArcTan[x/Sqrt[-I + a]]/Sqrt[-I + a] - ArcTan[x/Sqrt[I + a]]/Sqrt[
I + a])
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + a^2 + 2*a*x^2 + x^4)^(-1), x]
```

fricas [B]   time = 1.34, size = 613, normalized size = 2.05

$$\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\left(\frac{a}{\sqrt{a^2+1}}+1\right)\log\left(x^2+\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{1}{4}}}+\sqrt{a^2+1}\right)}{8(a^2+1)^{\frac{1}{4}}}-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\left(\frac{a}{\sqrt{a^2+1}}+1\right)\log\left(x^2-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{1}{4}}}+\sqrt{a^2+1}\right)}{8(a^2+1)^{\frac{1}{4}}}-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,(a^2+1)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{a^4+2a^2+1}\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{5}{4}}}+\frac{a^3+a}{\sqrt{a^4+2a^2+1}}+\frac{\sqrt{a^4+2a^2+1}\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\sqrt{x^2+\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{1}{4}}}+\sqrt{a^2+1}}}{(a^2+1)^{\frac{5}{4}}}-\frac{\sqrt{a^4+2a^2+1}}{\sqrt{a^2+1}}\right)}{2\sqrt{a^4+2a^2+1}}-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,(a^2+1)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{a^4+2a^2+1}\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{5}{4}}}-\frac{a^3+a}{\sqrt{a^4+2a^2+1}}+\frac{\sqrt{a^4+2a^2+1}\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\sqrt{x^2-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{1}{4}}}+\sqrt{a^2+1}}}{(a^2+1)^{\frac{5}{4}}}+\frac{\sqrt{a^4+2a^2+1}}{\sqrt{a^2+1}}\right)}{2\sqrt{a^4+2a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*(a/sqrt(a^2 + 1) + 1)*log(x
^2 + sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(1/4) + sqrt(a
^2 + 1))/(a^2 + 1)^(1/4) - 1/8*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*
(a/sqrt(a^2 + 1) + 1)*log(x^2 - sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)
*x/(a^2 + 1)^(1/4) + sqrt(a^2 + 1))/(a^2 + 1)^(1/4) - 1/2*sqrt(2*a^2 - 2*(a
^3 + a)/sqrt(a^2 + 1) + 2)*(a^2 + 1)^(1/4)*arctan(-sqrt(a^4 + 2*a^2 + 1)*sq
rt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(5/4) + (a^3 + a)/sqr
t(a^4 + 2*a^2 + 1) + sqrt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^
2 + 1) + 2)*sqrt(x^2 + sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 +
 1)^(1/4) + sqrt(a^2 + 1))/(a^2 + 1)^(5/4) - sqrt(a^4 + 2*a^2 + 1)/sqrt(a^2
 + 1))/sqrt(a^4 + 2*a^2 + 1) - 1/2*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) +
 2)*(a^2 + 1)^(1/4)*arctan(-sqrt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/
sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(5/4) - (a^3 + a)/sqrt(a^4 + 2*a^2 + 1) + sq
rt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*sqrt(x^2 -
```

```
sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(1/4) + sqrt(a^2 +
1))/(a^2 + 1)^(5/4) + sqrt(a^4 + 2*a^2 + 1)/sqrt(a^2 + 1))/sqrt(a^4 + 2*a^2
 + 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.11, size = 1073, normalized size = 3.59

$$\frac{a^{4}\arctan\left(\frac{2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\left(a^{2}+1\right)^{\frac{3}{2}}\sqrt{2a+2\sqrt{a^{2}+1}}}-\frac{a^{4}\arctan\left(\frac{-2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\left(a^{2}+1\right)^{\frac{3}{2}}\sqrt{2a+2\sqrt{a^{2}+1}}}+\frac{\sqrt{-2a+2\sqrt{a^{2}+1}}\,a^{3}\ln\left(x^{2}+\sqrt{-2a+2\sqrt{a^{2}+1}}\,x+\sqrt{a^{2}+1}\right)}{8\left(a^{2}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{-2a+2\sqrt{a^{2}+1}}\,a^{3}\ln\left(-x^{2}+\sqrt{-2a+2\sqrt{a^{2}+1}}\,x-\sqrt{a^{2}+1}\right)}{8\left(a^{2}+1\right)^{\frac{3}{2}}}-\frac{a^{2}\arctan\left(\frac{2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\sqrt{a^{2}+1}\,\sqrt{2a+2\sqrt{a^{2}+1}}}+\frac{3a^{2}\arctan\left(\frac{2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\left(a^{2}+1\right)^{\frac{3}{2}}\sqrt{2a+2\sqrt{a^{2}+1}}}+\frac{a^{2}\arctan\left(\frac{-2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\sqrt{a^{2}+1}\,\sqrt{2a+2\sqrt{a^{2}+1}}}-\frac{3a^{2}\arctan\left(\frac{-2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\left(a^{2}+1\right)^{\frac{3}{2}}\sqrt{2a+2\sqrt{a^{2}+1}}}+\frac{\sqrt{-2a+2\sqrt{a^{2}+1}}\,a^{2}\ln\left(x^{2}+\sqrt{-2a+2\sqrt{a^{2}+1}}\,x+\sqrt{a^{2}+1}\right)}{8a^{2}+8}-\frac{\sqrt{-2a+2\sqrt{a^{2}+1}}\,a^{2}\ln\left(-x^{2}+\sqrt{-2a+2\sqrt{a^{2}+1}}\,x-\sqrt{a^{2}+1}\right)}{8\left(a^{2}+1\right)}+\frac{\sqrt{-2a+2\sqrt{a^{2}+1}}\,a\ln\left(x^{2}+\sqrt{-2a+2\sqrt{a^{2}+1}}\,x+\sqrt{a^{2}+1}\right)}{8\left(a^{2}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{-2a+2\sqrt{a^{2}+1}}\,a\ln\left(-x^{2}+\sqrt{-2a+2\sqrt{a^{2}+1}}\,x-\sqrt{a^{2}+1}\right)}{8\left(a^{2}+1\right)^{\frac{3}{2}}}-\frac{\arctan\left(\frac{2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\sqrt{a^{2}+1}\,\sqrt{2a+2\sqrt{a^{2}+1}}}+\frac{\arctan\left(\frac{2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{\left(a^{2}+1\right)^{\frac{3}{2}}\sqrt{2a+2\sqrt{a^{2}+1}}}+\frac{\arctan\left(\frac{-2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\sqrt{a^{2}+1}\,\sqrt{2a+2\sqrt{a^{2}+1}}}-\frac{\arctan\left(\frac{-2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{\left(a^{2}+1\right)^{\frac{3}{2}}\sqrt{2a+2\sqrt{a^{2}+1}}}+\frac{\sqrt{-2a+2\sqrt{a^{2}+1}}\,\ln\left(x^{2}+\sqrt{-2a+2\sqrt{a^{2}+1}}\,x+\sqrt{a^{2}+1}\right)}{8a^{2}+8}-\frac{\sqrt{-2a+2\sqrt{a^{2}+1}}\,\ln\left(-x^{2}+\sqrt{-2a+2\sqrt{a^{2}+1}}\,x-\sqrt{a^{2}+1}\right)}{8\left(a^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2+1),x)
```

```
[Out] -1/8/(a^2+1)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))*(2*(a^2+1)
^(1/2)-2*a)^(1/2)*a^2-1/8/(a^2+1)^(3/2)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^
2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)*a^3-1/8/(a^2+1)*ln(x*(2*(a^2+1
)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)-1/8/(a^2+
1)^(3/2)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/
2)-2*a)^(1/2)*a+1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a
^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2-1/2/(a^2+1)^(3
/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2
*(a^2+1)^(1/2)+2*a)^(1/2))*a^4+1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2
)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))-3/2
/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1
/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2-1/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+
2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(
1/2))+1/8/(a^2+1)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2))*(2*(a
^2+1)^(1/2)-2*a)^(1/2)*a^2+1/8/(a^2+1)^(3/2)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)
^(1/2)+(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)*a^3+1/8/(a^2+1)*ln(x^2+x*
(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)+1/8/
(a^2+1)^(3/2)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2))*(2*(a^2+1
)^(1/2)-2*a)^(1/2)*a-1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((
2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2+1/2/(a^2+
1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2
))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^4-1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)
^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2)
)+3/2/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2
```

```
)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2+1/(a^2+1)^(3/2)/(2*(a^2+1)^(
1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2
*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 + 2\,ax^2 + a^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + 1), x)
```

mupad [B] time = 4.36, size = 469, normalized size = 1.57

$$-\frac{\operatorname{atanh}\left(-\frac{2\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1\mathrm{i}}{a^2+1}}}{\frac{2\,a}{a^2+1}+\frac{2\mathrm{i}}{a^2+1}}+\frac{a\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1\mathrm{i}}{a^2+1}}\,2\mathrm{i}}{\frac{2\,a}{a^2+1}+\frac{2\,a^3}{a^2+1}+\frac{2\mathrm{i}}{a^2+1}+\frac{a^2\,2\mathrm{i}}{a^2+1}}+\frac{2\,a^2\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1\mathrm{i}}{a^2+1}}}{\frac{2\,a}{a^2+1}+\frac{2\,a^3}{a^2+1}+\frac{2\mathrm{i}}{a^2+1}+\frac{a^2\,2\mathrm{i}}{a^2+1}}\right)\sqrt{\frac{a+1\mathrm{i}}{a^2+1}}}{2}+2\,\operatorname{atanh}\left(\frac{8\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1\mathrm{i}}{16\,a^2+16}}}{\frac{32\,a}{16\,a^2+16}-\frac{32\mathrm{i}}{16\,a^2+16}}+\frac{a\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1\mathrm{i}}{16\,a^2+16}}\,128\mathrm{i}}{\frac{512\,a}{16\,a^2+16}+\frac{512\,a^3}{16\,a^2+16}-\frac{512\mathrm{i}}{16\,a^2+16}-\frac{a^2\,512\mathrm{i}}{16\,a^2+16}}-\frac{128\,a^2\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1\mathrm{i}}{16\,a^2+16}}}{\frac{512\,a}{16\,a^2+16}+\frac{512\,a^3}{16\,a^2+16}-\frac{512\mathrm{i}}{16\,a^2+16}-\frac{a^2\,512\mathrm{i}}{16\,a^2+16}}\right)\sqrt{\frac{a-\mathrm{i}}{16\,a^2+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a*x^2 + a^2 + x^4 + 1),x)
```

```
[Out] 2*atanh((8*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 16))^(1/2))/((32*a)/(16*a^2 +
16) - 32i/(16*a^2 + 16)) + (a*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 16))^(1/2)*
128i)/((512*a)/(16*a^2 + 16) - 512i/(16*a^2 + 16) - (a^2*512i)/(16*a^2 + 16
) + (512*a^3)/(16*a^2 + 16)) - (128*a^2*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 1
6))^(1/2))/((512*a)/(16*a^2 + 16) - 512i/(16*a^2 + 16) - (a^2*512i)/(16*a^2
 + 16) + (512*a^3)/(16*a^2 + 16)))*((a - 1i)/(16*a^2 + 16))^(1/2) - (atanh(
(a*x*(a/(a^2 + 1) + 1i/(a^2 + 1))^(1/2)*2i)/((2*a)/(a^2 + 1) + 2i/(a^2 + 1)
 + (a^2*2i)/(a^2 + 1) + (2*a^3)/(a^2 + 1)) - (2*x*(a/(a^2 + 1) + 1i/(a^2 +
1))^(1/2))/((2*a)/(a^2 + 1) + 2i/(a^2 + 1)) + (2*a^2*x*(a/(a^2 + 1) + 1i/(a
^2 + 1))^(1/2))/((2*a)/(a^2 + 1) + 2i/(a^2 + 1) + (a^2*2i)/(a^2 + 1) + (2*a
^3)/(a^2 + 1)))*((a + 1i)/(a^2 + 1))^(1/2))/2
```

sympy [A] time = 0.58, size = 48, normalized size = 0.16

$$\operatorname{RootSum}\left(t^4\left(256a^2+256\right)-32t^2a+1,\left(t\mapsto t\log\left(64t^3a^3+64t^3a-4ta^2+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2+1),x)
```

```
[Out] RootSum(_t**4*(256*a**2 + 256) - 32*_t**2*a + 1, Lambda(_t, _t*log(64*_t**3
*a**3 + 64*_t**3*a - 4*_t*a**2 + 4*_t + x)))
```

3.11 $\int \frac{1}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}\tanh^{-1}(x) - \frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 207}

$$\frac{1}{3}\tanh^{-1}(x) - \frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTanh[x/2]/6 + ArcTanh[x]/3
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-5x^2+x^4} \, dx &= \frac{1}{3}\int \frac{1}{-4+x^2} \, dx - \frac{1}{3}\int \frac{1}{-1+x^2} \, dx \\ &= -\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 2.18

$$-\frac{1}{6}\log(1-x) + \frac{1}{12}\log(2-x) + \frac{1}{6}\log(x+1) - \frac{1}{12}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 5*x^2 + x^4)^(-1),x]
```

```
[Out] -1/6*Log[1 - x] + Log[2 - x]/12 + Log[1 + x]/6 - Log[2 + x]/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4-5x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - 5*x^2 + x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(4 - 5*x^2 + x^4)^(-1), x]
```

fricas [B] time = 0.91, size = 25, normalized size = 1.47

$$-\frac{1}{12}\log\left(x+2\right)+\frac{1}{6}\log\left(x+1\right)-\frac{1}{6}\log\left(x-1\right)+\frac{1}{12}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

giac [B] time = 0.20, size = 29, normalized size = 1.71

$$-\frac{1}{12}\log\left(|x+2|\right)+\frac{1}{6}\log\left(|x+1|\right)-\frac{1}{6}\log\left(|x-1|\right)+\frac{1}{12}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*log(abs(x + 2)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1)) + 1/12*lo
g(abs(x - 2))
```

maple [B] time = 0.01, size = 26, normalized size = 1.53

$$\frac{\ln\left(x+1\right)}{6}-\frac{\ln\left(x+2\right)}{12}+\frac{\ln\left(x-2\right)}{12}-\frac{\ln\left(x-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(x+2)+1/6*ln(1+x)+1/12*ln(x-2)-1/6*ln(x-1)
```

maxima [B] time = 1.37, size = 25, normalized size = 1.47

$$-\frac{1}{12}\log(x+2)+\frac{1}{6}\log(x+1)-\frac{1}{6}\log(x-1)+\frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.65

$$\frac{\operatorname{atanh}(x)}{3}-\frac{\operatorname{atanh}\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 5*x^2 + 4),x)
[Out] atanh(x)/3 - atanh(x/2)/6
```

sympy [B] time = 0.18, size = 26, normalized size = 1.53

$$\frac{\log(x-2)}{12}-\frac{\log(x-1)}{6}+\frac{\log(x+1)}{6}-\frac{\log(x+2)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-5*x**2+4),x)
[Out] log(x - 2)/12 - log(x - 1)/6 + log(x + 1)/6 - log(x + 2)/12
```

3.12 $\int \frac{1}{3+4x^2+x^4}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 203}

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+4x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{1}{3+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^2 + x^4)^(-1),x]
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3+4x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 4*x^2 + x^4)^(-1),x]
[Out] IntegrateAlgebraic[(3 + 4*x^2 + x^4)^(-1), x]
```

fricas [A] time = 1.16, size = 17, normalized size = 0.71

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="fricas")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

giac [A] time = 0.17, size = 17, normalized size = 0.71

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="giac")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.75

$$\frac{\arctan(x)}{2}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^2+3),x)
```

```
[Out] 1/2*arctan(x)-1/6*arctan(1/3*x*3^(1/2))*3^(1/2)
```

maxima [A] time = 3.00, size = 17, normalized size = 0.71

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

mupad [B] time = 4.12, size = 17, normalized size = 0.71

$$\frac{\operatorname{atan}(x)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + x^4 + 3),x)
```

```
[Out] atan(x)/2 - (3^(1/2)*atan((3^(1/2)*x)/3))/6
```

sympy [A] time = 0.16, size = 20, normalized size = 0.83

$$\frac{\operatorname{atan}(x)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**2+3),x)
```

```
[Out] atan(x)/2 - sqrt(3)*atan(sqrt(3)*x/3)/6
```

3.13 $\int \frac{1}{9+5x^2+x^4} \, dx$

Optimal. Leaf size=67

$$-\frac{1}{12}\log\left(x^2-x+3\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)}{6\sqrt{11}}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{1}{12}\log\left(x^2-x+3\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)}{6\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[11]]/(6*Sqrt[11]) + ArcTan[(1 + 2*x)/Sqrt[11]]/(6*Sq
rt[11]) - Log[3 - x + x^2]/12 + Log[3 + x + x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{9+5x^2+x^4}\,dx &= \frac{1}{6}\int \frac{1-x}{3-x+x^2}\,dx+\frac{1}{6}\int \frac{1+x}{3+x+x^2}\,dx\\ &= \frac{1}{12}\int \frac{1}{3-x+x^2}\,dx-\frac{1}{12}\int \frac{-1+2x}{3-x+x^2}\,dx+\frac{1}{12}\int \frac{1}{3+x+x^2}\,dx+\frac{1}{12}\int \frac{1+2x}{3+x+x^2}\,dx\\ &= -\frac{1}{12}\log\left(3-x+x^2\right)+\frac{1}{12}\log\left(3+x+x^2\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx,x,-1+2x\right)-\frac{1}{6}\text{Sub}\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{11}}\right)}{6\sqrt{11}}-\frac{1}{12}\log\left(3-x+x^2\right)+\frac{1}{12}\log\left(3+x+x^2\right)\end{aligned}$$

Mathematica [C] time = 0.07, size = 91, normalized size = 1.36

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5+i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5+i\sqrt{11}\right)}}-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5-i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5-i\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(5 - I*Sqrt[11])/2]])/Sqrt[(11*(5 - I*Sqrt[11]))/2] + (
I*ArcTan[x/Sqrt[(5 + I*Sqrt[11])/2]])/Sqrt[(11*(5 + I*Sqrt[11]))/2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{9+5x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(9 + 5*x^2 + x^4)^(-1), x]
```

fricas [A] time = 1.10, size = 53, normalized size = 0.79

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="fricas")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

giac [A] time = 0.20, size = 53, normalized size = 0.79

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="giac")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

maple [A] time = 0.00, size = 54, normalized size = 0.81

$$\frac{\sqrt{11}\,\arctan\left(\frac{(2x+1)\sqrt{11}}{11}\right)}{66}+\frac{\sqrt{11}\,\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)}{66}-\frac{\ln\left(x^2-x+3\right)}{12}+\frac{\ln\left(x^2+x+3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+5*x^2+9),x)
```

```
[Out] -1/12*ln(x^2-x+3)+1/66*11^(1/2)*arctan(1/11*(2*x-1)*11^(1/2))+1/12*ln(x^2+x
+3)+1/66*arctan(1/11*(2*x+1)*11^(1/2))*11^(1/2)
```

maxima [A] time = 3.04, size = 53, normalized size = 0.79

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="maxima")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

mupad [B] time = 4.15, size = 83, normalized size = 1.24

$$\mathrm{atan}\left(\frac{x\,8\mathrm{i}}{27\left(-\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}-\frac{2\sqrt{11}\,x}{27\left(-\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{66}+\frac{1}{6}\mathrm{i}\right)+\mathrm{atan}\left(\frac{x\,8\mathrm{i}}{27\left(\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}+\frac{2\sqrt{11}\,x}{27\left(\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{66}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + x^4 + 9),x)
```

```
[Out] atan((x*8i)/(27*((11^(1/2)*1i)/9 - 5/9)) - (2*11^(1/2)*x)/(27*((11^(1/2)*1i
)/9 - 5/9)))*(11^(1/2)/66 + 1i/6) + atan((x*8i)/(27*((11^(1/2)*1i)/9 + 5/9)
) + (2*11^(1/2)*x)/(27*((11^(1/2)*1i)/9 + 5/9)))*(11^(1/2)/66 - 1i/6)
```

sympy [A] time = 0.22, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+3\right)}{12}+\frac{\log\left(x^2+x+3\right)}{12}+\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}\,x}{11}-\frac{\sqrt{11}}{11}\right)}{66}+\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}\,x}{11}+\frac{\sqrt{11}}{11}\right)}{66}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+5*x**2+9),x)
```

```
[Out] -log(x**2 - x + 3)/12 + log(x**2 + x + 3)/12 + sqrt(11)*atan(2*sqrt(11)*x/1
1 - sqrt(11)/11)/66 + sqrt(11)*atan(2*sqrt(11)*x/11 + sqrt(11)/11)/66
```

3.14 $\int \frac{1}{1-x^2+x^4} \, dx$

Optimal. Leaf size=74

$$-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 - Log[1 - Sqrt[3]*x + x^
2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-x^2+x^4}\,dx &= \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{2\sqrt{3}}\\ &= \frac{1}{4}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx-\frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)-\frac{1}{2}\,\text{Su}\\ &= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.07, size = 77, normalized size = 1.04

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-\sqrt{-1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - Sqrt[-1 + I*Sqrt[3
]]*ArcTan[((1 + I*Sqrt[3])*x)/2]))/Sqrt[6]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^2 + x^4)^(-1), x]
```

fricas [B] time = 1.64, size = 159, normalized size = 2.15

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^2+2}+\sqrt{3}\right)+\frac{1}{24}\sqrt{6}\sqrt{2}\log\left(\sqrt{6}\sqrt{2}x+2x^2+2\right)-\frac{1}{24}\sqrt{6}\sqrt{2}\log\left(-\sqrt{6}\sqrt{2}x+2x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/6*sqrt(6)*
sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)
*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 1/24*sqrt(6)*sqrt(2)*log
(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/24*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2)
*x + 2*x^2 + 2)
```

giac [A] time = 0.19, size = 56, normalized size = 0.76

$$\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{2}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\arctan\left(2x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
1) + 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(2*x - sqrt(3))
```

maple [A] time = 0.04, size = 57, normalized size = 0.77

$$\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}-\frac{\sqrt{3}\ \ln\left(x^2-\sqrt{3}\ x+1\right)}{12}+\frac{\sqrt{3}\ \ln\left(x^2+\sqrt{3}\ x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2+1),x)
```

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))-1/12*ln(1+x^2-3^(1/2)*x)*3^
(1/2)+1/12*ln(1+x^2+3^(1/2)*x)*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - x^2 + 1), x)
```

mupad [B] time = 4.19, size = 47, normalized size = 0.64

$$\operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - x^2 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atan((2*x)/(3^(1/2)*1
i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

sympy [A] time = 0.21, size = 63, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

3.15 $\int \frac{1}{2+2x^2+x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{\log\left(x^2-\sqrt{2\left(\sqrt{2}-1\right)}\,x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\log\left(x^2+\sqrt{2\left(\sqrt{2}-1\right)}\,x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}-\frac{1}{4}\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+$$

Rubi [A] time = 0.16, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2\left(\sqrt{2}-1\right)}\,x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\log\left(x^2+\sqrt{2\left(\sqrt{2}-1\right)}\,x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}-\frac{1}{4}\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{4}\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] - 2*x)/Sqrt[2*(1 + Sqrt
[2])]])/4 + (Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] + 2*x)/Sqrt[
2*(1 + Sqrt[2])]])/4 - Log[Sqrt[2] - Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqr
t[-1 + Sqrt[2]]) + Log[Sqrt[2] + Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqrt[-1
 + Sqrt[2]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2+2x^2+x^4}\,dx = \frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{-1+\sqrt{2}}} + \frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}+x}{\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{-1+\sqrt{2}}}$$

$$= \frac{\int \frac{1}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{1}{\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{-\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{8\sqrt{-1+\sqrt{2}}} + \frac{\int \frac{\sqrt{2\left(-1\right.}}{\sqrt{2}+\sqrt{2(}}}{8\sqrt{-}}$$

$$= -\frac{\log\left(\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2\right)}{8\sqrt{-1+\sqrt{2}}} + \frac{\log\left(\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2\right)}{8\sqrt{-1+\sqrt{2}}} - \frac{\text{Subst}\left(\int \frac{}{-2(1+}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{1+\sqrt{2}}} - \frac{\log\left(\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2\right)}{8\sqrt{-1+\sqrt{2}}} + \frac{\text{lo}}{}$$

Mathematica [C] time = 0.04, size = 41, normalized size = 0.23

$$\frac{1}{4}\left((1-i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1+i}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] ((1 - I)^(3/2)*ArcTan[x/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTan[x/Sqrt[1 + I]])
/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+2x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(2 + 2*x^2 + x^4)^(-1), x]
```

fricas [A] time = 0.82, size = 247, normalized size = 1.40

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\log\left(2^{\frac{3}{4}}x\sqrt{-2\sqrt{2}+4}+2x^2+2\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\log\left(-2^{\frac{3}{4}}x\sqrt{-2\sqrt{2}+4}+2x^2+2\sqrt{2}\right)-\frac{1}{4}\cdot 2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{-2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}x\sqrt{-2\sqrt{2}+4}+2x^2+2\sqrt{2}}\sqrt{-2\sqrt{2}+4}-\sqrt{2}+1\right)-\frac{1}{4}\cdot 2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{-2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}x\sqrt{-2\sqrt{2}+4}+2x^2+2\sqrt{2}}\sqrt{-2\sqrt{2}+4}+\sqrt{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(2^(3/4)*x*sqrt(-2*sqrt(
2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) +
 4)*log(-2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(1/4)*
sqrt(-2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 1/2*2^(1/
4)*sqrt(2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(-2*sqrt(2)
 + 4) - sqrt(2) + 1) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)
*x*sqrt(-2*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(-2*sqrt(2) + 4)
+ 2*x^2 + 2*sqrt(2))*sqrt(-2*sqrt(2) + 4) + sqrt(2) - 1)
```

giac [A] time = 0.53, size = 143, normalized size = 0.81

$$\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{2\sqrt{\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{2\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{-\sqrt{2}+2}+\sqrt{2}\right)-\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{-\sqrt{2}+2}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(-sqrt(2) + 2))
/sqrt(sqrt(2) + 2)) + 1/4*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2*x - 2^(1/
4)*sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(sqrt(2) + 1)*log(x^2 +
 2^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(2)) - 1/8*sqrt(sqrt(2) + 1)*log(x^2 -
2^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(2))
```

maple [B] time = 0.11, size = 386, normalized size = 2.19

$$-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)}{8\sqrt{2+2\sqrt{2}}}-\frac{\left(-2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)}{4\sqrt{2+2\sqrt{2}}}+\frac{\sqrt{2}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)}{2\sqrt{2+2\sqrt{2}}}-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)}{8\sqrt{2+2\sqrt{2}}}-\frac{\left(-2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)}{4\sqrt{2+2\sqrt{2}}}+\frac{\sqrt{2}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)}{2\sqrt{2+2\sqrt{2}}}-\frac{\sqrt{-2+2\sqrt{2}}\sqrt{2}\ln\left(x^2-\sqrt{-2+2\sqrt{2}}\,x+\sqrt{2}\right)}{16}-\frac{\sqrt{-2+2\sqrt{2}}\ln\left(x^2-\sqrt{-2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}+\frac{\sqrt{-2+2\sqrt{2}}\sqrt{2}\ln\left(x^2+\sqrt{-2+2\sqrt{2}}\,x+\sqrt{2}\right)}{16}+\frac{\sqrt{-2+2\sqrt{2}}\ln\left(x^2+\sqrt{-2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*x^2+2),x)
```

```
[Out] -1/16*ln(x^2+2^(1/2)-x*(-2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)*2^(1/2)-1
/8*ln(x^2+2^(1/2)-x*(-2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)-1/8/(2+2*2^(
1/2))^(1/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(-2+2*2^
(1/2))*2^(1/2)-1/4/(2+2*2^(1/2))^(1/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2))/(2
+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))+1/2/(2+2*2^(1/2))^(1/2)*arctan((2*x-(-2+2
*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*2^(1/2)+1/16*ln(x^2+2^(1/2)+x*(-2+2*2
^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)*2^(1/2)+1/8*ln(x^2+2^(1/2)+x*(-2+2*2^(1
/2))^(1/2))*(-2+2*2^(1/2))^(1/2)-1/8/(2+2*2^(1/2))^(1/2)*arctan((2*x+(-2+2*
2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))*2^(1/2)-1/4/(2+2*2^(1/2
))^(1/2)*arctan((2*x+(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(-2+2*2^(1/
2))+1/2/(2+2*2^(1/2))^(1/2)*arctan((2*x+(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))
^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4+2\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*x^2 + 2), x)
```

mupad [B] time = 4.21, size = 210, normalized size = 1.19

$$\operatorname{atanh}\left(\frac{4\sqrt{2}\,x\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}+\frac{4\sqrt{2}\,x\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}-2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)-\operatorname{atanh}\left(\frac{4\sqrt{2}\,x\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1}-\frac{4\sqrt{2}\,x\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}+2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + x^4 + 2),x)
```

```
[Out] atanh((4*2^(1/2)*x*(1/64 - 2^(1/2)/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)
*(2^(1/2)/64 + 1/64)^(1/2) - 1) + (4*2^(1/2)*x*(2^(1/2)/64 + 1/64)^(1/2))/(
64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) - 1))*(2*(1/64 - 2^(
1/2)/64)^(1/2) - 2*(2^(1/2)/64 + 1/64)^(1/2)) - atanh((4*2^(1/2)*x*(1/64 -
2^(1/2)/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2)
+ 1) - (4*2^(1/2)*x*(2^(1/2)/64 + 1/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2
)*(2^(1/2)/64 + 1/64)^(1/2) + 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) + 2*(2^(1/2)
/64 + 1/64)^(1/2))
```

sympy [B] time = 1.13, size = 899, normalized size = 5.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*x**2+2),x)
```

```
[Out] sqrt(1/64 + sqrt(2)/64)*log(x**2 + x*(-4*sqrt(2)*sqrt(1 + sqrt(2)) - sqrt(1
 + sqrt(2)) + 3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3)) - 15*sqrt(2*sqrt(2)
+ 3) - 7*sqrt(2)*sqrt(2*sqrt(2) + 3) + 29 + 23*sqrt(2)) - sqrt(1/64 + sqrt(
2)/64)*log(x**2 + x*(-3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3) + sqrt(1 + sq
rt(2)) + 4*sqrt(2)*sqrt(1 + sqrt(2))) - 15*sqrt(2*sqrt(2) + 3) - 7*sqrt(2)*
sqrt(2*sqrt(2) + 3) + 29 + 23*sqrt(2)) + 2*sqrt(-sqrt(2*sqrt(2) + 3)/32 + 1
/64 + 3*sqrt(2)/64)*atan(2*x/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))
+ sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) - 4*sqr
t(2)*sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt
(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) - sqrt(1 + sq
rt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*
sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) + 3*sqrt(1 + sqrt(2))*sqrt(2*
sqrt(2) + 3)/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2)
 + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)))) + 2*sqrt(-sqrt(2*sqrt(
2) + 3)/32 + 1/64 + 3*sqrt(2)/64)*atan(2*x/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1
 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqr
t(2))) - 3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3)/(sqrt(-2*sqrt(2*sqrt(2) +
3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 +
 3*sqrt(2))) + sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(
2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) + 4
*sqrt(2)*sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) +
sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))))
```

3.16 $\int x^2 \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=17

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(bx^2+cx^4\right) dx &= \int \left(bx^4+cx^6\right) dx \\ &= \frac{bx^5}{5}+\frac{cx^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(bx^2 + cx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x^2*(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.37, size = 13, normalized size = 0.76

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/7*x^7*c + 1/5*x^5*b
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{7}c\,x^7 + \frac{1}{5}b\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2),x)
[Out] 1/5*b*x^5+1/7*c*x^7
```

maxima [A] time = 1.33, size = 13, normalized size = 0.76

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^7}{7}+\frac{b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2 + c*x^4),x)
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2),x)
```

```
[Out] b*x**5/5 + c*x**7/7
```

3.17 $\int x \left(bx^2+cx^4\right) \, dx$

Optimal. Leaf size=17

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x \left(bx^2+cx^4\right) \, dx &= \int \left(bx^3+cx^5\right) \, dx\\ &= \frac{bx^4}{4}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(bx^2 + cx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x*(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.66, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/6*x^6*c + 1/4*x^4*b
```

giac [A] time = 0.17, size = 13, normalized size = 0.76

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{6}c\,x^6 + \frac{1}{4}b\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2),x)
[Out] 1/4*b*x^4+1/6*c*x^6
```

maxima [A] time = 1.30, size = 13, normalized size = 0.76

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^6}{6}+\frac{b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2 + c*x^4),x)
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2),x)
```

```
[Out] b*x**4/4 + c*x**6/6
```

3.18 $\int \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=17

$$\frac{bx^3}{3}+\frac{cx^5}{5}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[b*x^2 + c*x^4,x]
[Out] (b*x^3)/3 + (c*x^5)/5
```

Rubi steps

$$\int \left(bx^2+cx^4\right) dx=\frac{bx^3}{3}+\frac{cx^5}{5}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x^2 + c*x^4,x]
[Out] (b*x^3)/3 + (c*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+cx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[b*x^2 + c*x^4,x]
[Out] IntegrateAlgebraic[b*x^2 + c*x^4, x]
```

fricas [**A**] time = 0.49, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5c + \frac{1}{3}x^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="fricas")
[Out] 1/5*x^5*c + 1/3*x^3*b
```

giac [**A**] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="giac")
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

maple [**A**] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{5}c\,x^5 + \frac{1}{3}b\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^2,x)
[Out] 1/3*b*x^3+1/5*c*x^5
```

maxima [**A**] time = 1.36, size = 13, normalized size = 0.76

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="maxima")
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

mupad [**B**] time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^5}{5} + \frac{b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^2 + c*x^4,x)
```

```
[Out] (b*x^3)/3 + (c*x^5)/5
```

sympy **[A]** time = 0.06, size = 12, normalized size = 0.71

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**2,x)
```

```
[Out] b*x**3/3 + c*x**5/5
```

3.19 $\int \frac{bx^2+cx^4}{x} dx$

Optimal. Leaf size=17

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2 + cx^4}{x} dx &= \int \left(bx + cx^3\right) dx \\ &= \frac{bx^2}{2} + \frac{cx^4}{4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 0.94

$$\frac{1}{4}x^2\left(2b+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x,x]
[Out] (x^2*(2*b + c*x^2))/4
```

fricas [A] time = 0.71, size = 13, normalized size = 0.76

$$\frac{1}{4}cx^4+\frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="fricas")
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{4}cx^4+\frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="giac")
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4}c\,x^4+\frac{1}{2}b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x,x)
[Out] 1/2*b*x^2+1/4*c*x^4
```

maxima [A] time = 1.31, size = 13, normalized size = 0.76

$$\frac{1}{4}cx^4+\frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^4}{4}+\frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x,x)
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{bx^2}{2}+\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x,x)
```

```
[Out] b*x**2/2 + c*x**4/4
```

3.20 $\int \frac{bx^2+cx^4}{x^2}\,dx$

Optimal. Leaf size=12

$$bx + \frac{cx^3}{3}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^2,x]
```

```
[Out] b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^2}\,dx &= \int \left(b+cx^2\right)\,dx\\ &= bx+\frac{cx^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^2,x]
```

```
[Out] b*x + (c*x^3)/3
```

IntegrateAlgebraic [A] time = 0.01, size = 12, normalized size = 1.00

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^2,x]
[Out] b*x + (c*x^3)/3
```

fricas [A] time = 0.82, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="fricas")
[Out] 1/3*c*x^3 + b*x
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="giac")
[Out] 1/3*c*x^3 + b*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{3}c\,x^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^2,x)
[Out] b*x+1/3*c*x^3
```

maxima [A] time = 1.26, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + b*x
```

mupad [B]   time = 0.02, size = 10, normalized size = 0.83

$$\frac{c\,x^3}{3}+b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^2,x)
```

```
[Out] b*x + (c*x^3)/3
```

sympy [A]   time = 0.07, size = 8, normalized size = 0.67

$$bx+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**2,x)
```

```
[Out] b*x + c*x**3/3
```

3.21 $\int \frac{bx^2+cx^4}{x^3}\,dx$

Optimal. Leaf size=13

$$b\log(x)+\frac{cx^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$b\log(x)+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^3,x]
```

```
[Out] (c*x^2)/2 + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^3}\,dx &= \int \left(\frac{b}{x}+cx\right)dx\\ &= \frac{cx^2}{2}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$b\log(x)+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^3,x]
```

```
[Out] (c*x^2)/2 + b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2 + cx^4}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^3,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^3, x]
```

fricas [A] time = 0.51, size = 11, normalized size = 0.85

$$\frac{1}{2} cx^2 + b \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="fricas")
[Out] 1/2*c*x^2 + b*log(x)
```

giac [A] time = 0.15, size = 14, normalized size = 1.08

$$\frac{1}{2} cx^2 + \frac{1}{2} b \log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="giac")
[Out] 1/2*c*x^2 + 1/2*b*log(x^2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{c\,x^2}{2} + b \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^3,x)
[Out] 1/2*c*x^2+b*ln(x)
```

maxima [A] time = 1.35, size = 14, normalized size = 1.08

$$\frac{1}{2} cx^2 + \frac{1}{2} b \log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$\frac{c\,x^2}{2}+b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^3,x)
```

```
[Out] (c*x^2)/2 + b*log(x)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$b\log{(x)}+\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**3,x)
```

```
[Out] b*log(x) + c*x**2/2
```

3.22 $\int \frac{bx^2+cx^4}{x^4}\,dx$

Optimal. Leaf size=10

$$cx-\frac{b}{x}$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$cx-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -(b/x) + c*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^4}\,dx &= \int \left(c+\frac{b}{x^2}\right)dx\\ &= -\frac{b}{x}+cx\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$cx-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -(b/x) + c*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2 + cx^4}{x^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^4,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^4, x]
```

fricas [A] time = 0.66, size = 13, normalized size = 1.30

$$\frac{cx^2 - b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="fricas")
[Out] (c*x^2 - b)/x
```

giac [A] time = 0.16, size = 10, normalized size = 1.00

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="giac")
[Out] c*x - b/x
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^4,x)
[Out] -b/x+c*x
```

maxima [A] time = 1.33, size = 10, normalized size = 1.00

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] c*x - b/x
```

mupad [B] time = 0.02, size = 10, normalized size = 1.00

$$c\,x-\frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^4,x)
```

```
[Out] c*x - b/x
```

sympy [A] time = 0.10, size = 5, normalized size = 0.50

$$-\frac{b}{x}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**4,x)
```

```
[Out] -b/x + c*x
```

3.23 $\int \frac{bx^2+cx^4}{x^5} \, dx$

Optimal. Leaf size=13

$$c\log(x)-\frac{b}{2x^2}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$c\log(x)-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -b/(2*x^2) + c*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^5}\,dx &= \int \left(\frac{b}{x^3}+\frac{c}{x}\right)dx\\ &= -\frac{b}{2x^2}+c\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$c\log(x)-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -1/2*b/x^2 + c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2 + cx^4}{x^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^5,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^5, x]
```

fricas [A] time = 0.50, size = 17, normalized size = 1.31

$$\frac{2\,cx^2\log(x) - b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="fricas")
[Out] 1/2*(2*c*x^2*log(x) - b)/x^2
```

giac [A] time = 0.15, size = 20, normalized size = 1.54

$$\frac{1}{2}\,c\log\left(x^2\right) - \frac{cx^2 + b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="giac")
[Out] 1/2*c*log(x^2) - 1/2*(c*x^2 + b)/x^2
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$c\ln(x) - \frac{b}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^5,x)
[Out] -1/2*b/x^2+c*ln(x)
```

maxima [A] time = 1.35, size = 14, normalized size = 1.08

$$\frac{1}{2}\,c\log\left(x^2\right) - \frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2) - 1/2*b/x^2
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$c\,\ln(x)-\frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^5,x)
```

```
[Out] c*log(x) - b/(2*x^2)
```

sympy [A] time = 0.12, size = 10, normalized size = 0.77

$$-\frac{b}{2x^2}+c\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**5,x)
```

```
[Out] -b/(2*x**2) + c*log(x)
```

3.24 $\int \frac{bx^2+cx^4}{x^6}\,dx$

Optimal. Leaf size=15

$$-\frac{b}{3x^3}-\frac{c}{x}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -b/(3*x^3) - c/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^6}\,dx &= \int \left(\frac{b}{x^4}+\frac{c}{x^2}\right)dx\\ &= -\frac{b}{3x^3}-\frac{c}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -1/3*b/x^3 - c/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2 + cx^4}{x^6} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^6,x]
```

```
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^6, x]
```

fricas [A] time = 0.63, size = 13, normalized size = 0.87

$$-\frac{3\,cx^2 + b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

giac [A] time = 0.18, size = 13, normalized size = 0.87

$$-\frac{3\,cx^2 + b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="giac")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{c}{x} - \frac{b}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^6,x)
```

```
[Out] -1/3*b/x^3-c/x
```

maxima [A] time = 1.30, size = 13, normalized size = 0.87

$$-\frac{3\,cx^2 + b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$-\frac{3\,c\,x^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^6,x)
```

```
[Out] -(b + 3*c*x^2)/(3*x^3)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.93

$$\frac{-b-3cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**6,x)
```

```
[Out] (-b - 3*c*x**2)/(3*x**3)
```

3.25 $\int \frac{bx^2+cx^4}{x^7} dx$

Optimal. Leaf size=17

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^7,x]
[Out] -b/(4*x^4) - c/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^7}\,dx &= \int \left(\frac{b}{x^5}+\frac{c}{x^3}\right) dx\\ &= -\frac{b}{4x^4}-\frac{c}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^7,x]
[Out] -1/4*b/x^4 - c/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2 + cx^4}{x^7} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^7,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^7, x]
```

fricas [A] time = 0.48, size = 13, normalized size = 0.76

$$-\frac{2\,cx^2 + b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="fricas")
[Out] -1/4*(2*c*x^2 + b)/x^4
```

giac [A] time = 0.19, size = 13, normalized size = 0.76

$$-\frac{2\,cx^2 + b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="giac")
[Out] -1/4*(2*c*x^2 + b)/x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{c}{2x^2} - \frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^7,x)
[Out] -1/4*b/x^4-1/2*c/x^2
```

maxima [A] time = 1.31, size = 13, normalized size = 0.76

$$-\frac{2\,cx^2 + b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,c\,x^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^7,x)
```

```
[Out] -(b + 2*c*x^2)/(4*x^4)
```

sympy [A] time = 0.13, size = 14, normalized size = 0.82

$$\frac{-b-2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**7,x)
```

```
[Out] (-b - 2*c*x**2)/(4*x**4)
```

3.26 $\int \frac{bx^2+cx^4}{x^8}\,dx$

Optimal. Leaf size=17

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -b/(5*x^5) - c/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^8}\,dx &= \int \left(\frac{b}{x^6}+\frac{c}{x^4}\right)dx\\ &= -\frac{b}{5x^5}-\frac{c}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -1/5*b/x^5 - c/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2 + cx^4}{x^8}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^8,x]
```

```
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^8, x]
```

fricas [A] time = 0.54, size = 15, normalized size = 0.88

$$-\frac{5\,cx^2 + 3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

giac [A] time = 0.17, size = 15, normalized size = 0.88

$$-\frac{5\,cx^2 + 3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="giac")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{c}{3x^3} - \frac{b}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^8,x)
```

```
[Out] -1/5*b/x^5-1/3*c/x^3
```

maxima [A] time = 1.25, size = 15, normalized size = 0.88

$$-\frac{5\,cx^2 + 3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{5\,c\,x^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^8,x)
```

```
[Out] -(3*b + 5*c*x^2)/(15*x^5)
```

sympy [A] time = 0.15, size = 15, normalized size = 0.88

$$\frac{-3b-5cx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**8,x)
```

```
[Out] (-3*b - 5*c*x**2)/(15*x**5)
```

3.27 $\int \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 270}

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(bx^2+cx^4\right)^2 \, dx &= \int x^4\left(b+cx^2\right)^2 \, dx\\ &= \int \left(b^2x^4+2bcx^6+c^2x^8\right) dx\\ &= \frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + cx^4\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.62, size = 24, normalized size = 0.80

$$\frac{1}{9}x^9c^2 + \frac{2}{7}x^7cb + \frac{1}{5}x^5b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.80

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bc\,x^7 + \frac{1}{5}b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*b^2*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

maxima [A] time = 1.38, size = 24, normalized size = 0.80

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{b^2\,x^5}{5} + \frac{2\,b\,c\,x^7}{7} + \frac{c^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2,x)
```

```
[Out] (b^2*x^5)/5 + (c^2*x^9)/9 + (2*b*c*x^7)/7
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{b^2x^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2,x)
```

```
[Out] b**2*x**5/5 + 2*b*c*x**7/7 + c**2*x**9/9
```

3.28 $\int \frac{(bx^2+cx^4)^2}{x} dx$

Optimal. Leaf size=30

$$\frac{b^2x^4}{4} + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^2x^4}{4} + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x,x]
```

```
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^2}{x}\,dx &= \int x^3\left(b+cx^2\right)^2\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int x(b+cx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(b^2x+2bcx^2+c^2x^3\right)\,dx,x,x^2\right)\\ &= \frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x,x]
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 0.93

$$\frac{1}{24}x^4\left(6b^2+8bcx^2+3c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x,x]
[Out] (x^4*(6*b^2 + 8*b*c*x^2 + 3*c^2*x^4))/24
```

fricas [A] time = 2.02, size = 24, normalized size = 0.80

$$\frac{1}{8}\,c^2x^8+\frac{1}{3}\,bcx^6+\frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="fricas")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

giac [A] time = 0.19, size = 24, normalized size = 0.80

$$\frac{1}{8}\,c^2x^8+\frac{1}{3}\,bcx^6+\frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="giac")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bc\,x^6 + \frac{1}{4}b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x,x)
[Out] 1/4*b^2*x^4+1/3*b*c*x^6+1/8*c^2*x^8
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$\frac{1}{8}\,c^2x^8 + \frac{1}{3}\,bcx^6 + \frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="maxima")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{b^2\,x^4}{4} + \frac{b\,c\,x^6}{3} + \frac{c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x,x)
[Out] (b^2*x^4)/4 + (c^2*x^8)/8 + (b*c*x^6)/3
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{b^2x^4}{4} + \frac{bcx^6}{3} + \frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x,x)
[Out] b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8
```

3.29 $\int \frac{(bx^2+cx^4)^2}{x^2} dx$

Optimal. Leaf size=30

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^2} dx &= \int x^2 \left(b+cx^2\right)^2 dx\\ &= \int \left(b^2x^2+2bcx^4+c^2x^6\right) dx\\ &= \frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

IntegrateAlgebraic [A] time = 0.03, size = 30, normalized size = 1.00

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

fricas [A] time = 0.47, size = 24, normalized size = 0.80

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

giac [A] time = 0.17, size = 24, normalized size = 0.80

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{7}c^2x^7 + \frac{2}{5}bc\,x^5 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^2,x)
```

```
[Out] 1/3*b^2*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

maxima [A] time = 1.32, size = 24, normalized size = 0.80

$$\frac{1}{7}c^2x^7 + \frac{2}{5}bcx^5 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{b^2\,x^3}{3} + \frac{2\,b\,c\,x^5}{5} + \frac{c^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^2,x)
```

```
[Out] (b^2*x^3)/3 + (c^2*x^7)/7 + (2*b*c*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{b^2x^3}{3} + \frac{2bcx^5}{5} + \frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**2,x)
```

```
[Out] b**2*x**3/3 + 2*b*c*x**5/5 + c**2*x**7/7
```

3.30 $\int \frac{(bx^2+cx^4)^2}{x^3} \, dx$

Optimal. Leaf size=16

$$\frac{(b+cx^2)^3}{6c}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$\frac{(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] (b + c*x^2)^3/(6*c)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^3} \, dx &= \int x(b+cx^2)^2 \, dx\\ &= \frac{(b+cx^2)^3}{6c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{(b + cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^3,x]
[Out] (b + c*x^2)^3/(6*c)
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 1.69

$$\frac{1}{6}x^2\left(3b^2 + 3bcx^2 + c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^3,x]
[Out] (x^2*(3*b^2 + 3*b*c*x^2 + c^2*x^4))/6
```

fricas [A] time = 0.44, size = 24, normalized size = 1.50

$$\frac{1}{6}c^2x^6 + \frac{1}{2}bcx^4 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

giac [A] time = 0.15, size = 24, normalized size = 1.50

$$\frac{1}{6}c^2x^6 + \frac{1}{2}bcx^4 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$\frac{1}{6}c^2x^6 + \frac{1}{2}bc\,x^4 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^3,x)
```

```
[Out] 1/6*c^2*x^6+1/2*b*c*x^4+1/2*b^2*x^2
```

maxima [A] time = 1.31, size = 24, normalized size = 1.50

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 1.50

$$\frac{b^2\,x^2}{2}+\frac{b\,c\,x^4}{2}+\frac{c^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^3,x)
```

```
[Out] (b^2*x^2)/2 + (c^2*x^6)/6 + (b*c*x^4)/2
```

sympy [B] time = 0.10, size = 24, normalized size = 1.50

$$\frac{b^2x^2}{2}+\frac{bcx^4}{2}+\frac{c^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**3,x)
```

```
[Out] b**2*x**2/2 + b*c*x**4/2 + c**2*x**6/6
```

3.31 $\int \frac{(bx^2+cx^4)^2}{x^4} dx$

Optimal. Leaf size=25

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 194}

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{(bx^2+cx^4)^2}{x^4} dx &= \int (b+cx^2)^2 dx \\ &= \int (b^2+2bcx^2+c^2x^4) dx \\ &= b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.00

$$b^2x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

fricas [A] time = 0.74, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5+\frac{2}{3}\,bcx^3+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

giac [A] time = 0.18, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5+\frac{2}{3}\,bcx^3+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}c^2x^5+\frac{2}{3}bc\,x^3+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^4,x)
```

```
[Out] b^2*x+2/3*b*c*x^3+1/5*c^2*x^5
```

maxima [A] time = 1.35, size = 21, normalized size = 0.84

$$\frac{1}{5}c^2x^5 + \frac{2}{3}bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$b^2\,x + \frac{2\,b\,c\,x^3}{3} + \frac{c^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^4,x)
```

```
[Out] b^2*x + (c^2*x^5)/5 + (2*b*c*x^3)/3
```

sympy [A] time = 0.08, size = 22, normalized size = 0.88

$$b^2x + \frac{2bcx^3}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**4,x)
```

```
[Out] b**2*x + 2*b*c*x**3/3 + c**2*x**5/5
```

3.32 $\int \frac{(bx^2+cx^4)^2}{x^5} \, dx$

Optimal. Leaf size=23

$$b^2 \log(x) + bcx^2 + \frac{c^2x^4}{4}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$b^2 \log(x) + bcx^2 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^2}{x^5}\,dx &= \int \frac{\left(b+cx^2\right)^2}{x}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+cx)^2}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(2bc+\frac{b^2}{x}+c^2x\right)\,dx,x,x^2\right)\\ &= bcx^2+\frac{c^2x^4}{4}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$b^2\log(x)+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^5, x]
```

fricas [A] time = 0.48, size = 21, normalized size = 0.91

$$\frac{1}{4}\,c^2x^4+bcx^2+b^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + b^2*log(x)
```

giac [A] time = 0.15, size = 24, normalized size = 1.04

$$\frac{1}{4}\,c^2x^4 + bcx^2 + \frac{1}{2}\,b^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{c^2x^4}{4} + bc\,x^2 + b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^5,x)
```

```
[Out] b*c*x^2+1/4*c^2*x^4+b^2*ln(x)
```

maxima [A] time = 1.35, size = 24, normalized size = 1.04

$$\frac{1}{4}\,c^2x^4 + bcx^2 + \frac{1}{2}\,b^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.91

$$b^2\,\ln(x) + \frac{c^2\,x^4}{4} + b\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^5,x)
```

```
[Out] b^2*log(x) + (c^2*x^4)/4 + b*c*x^2
```

sympy [A] time = 0.11, size = 20, normalized size = 0.87

$$b^2\log{(x)} + bcx^2 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**5,x)
```

```
[Out] b**2*log(x) + b*c*x**2 + c**2*x**4/4
```

3.33 $\int \frac{(bx^2+cx^4)^2}{x^6} dx$

Optimal. Leaf size=24

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^6} dx &= \int \frac{(b+cx^2)^2}{x^2} dx \\
&= \int \left(2bc + \frac{b^2}{x^2} + c^2x^2\right) dx \\
&= -\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^6,x]
[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + cx^4)^2}{x^6} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^6,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^6, x]
```

fricas [A] time = 0.59, size = 25, normalized size = 1.04

$$\frac{c^2x^4 + 6\,bcx^2 - 3\,b^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
[Out] 1/3*(c^2*x^4 + 6*b*c*x^2 - 3*b^2)/x
```

giac [A] time = 0.15, size = 22, normalized size = 0.92

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{c^2x^3}{3} + 2bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^6,x)
```

```
[Out] -b^2/x+2*b*c*x+1/3*c^2*x^3
```

maxima [A] time = 1.31, size = 22, normalized size = 0.92

$$\frac{1}{3}c^2x^3 + 2\,bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

mupad [B] time = 0.04, size = 22, normalized size = 0.92

$$\frac{c^2\,x^3}{3} - \frac{b^2}{x} + 2\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^6,x)
```

```
[Out] (c^2*x^3)/3 - b^2/x + 2*b*c*x
```

sympy [A] time = 0.11, size = 19, normalized size = 0.79

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**6,x)
```

```
[Out] -b**2/x + 2*b*c*x + c**2*x**3/3
```

3.34 $\int \frac{(bx^2+cx^4)^2}{x^7} dx$

Optimal. Leaf size=27

$$-\frac{b^2}{2x^2} + 2bc\log(x) + \frac{c^2x^2}{2}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{2x^2} + 2bc\log(x) + \frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -b^2/(2*x^2) + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^2}{x^7}\,dx &= \int \frac{\left(b+cx^2\right)^2}{x^3}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^2}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(c^2+\frac{b^2}{x^2}+\frac{2bc}{x}\right)dx, x, x^2\right)\\
&= -\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)
\end{aligned}
$$

Mathematica [**A**] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -1/2*b^2/x^2 + (c^2*x^2)/2 + 2*b*c*Log[x]
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^7, x]
```

fricas [**A**] time = 0.56, size = 27, normalized size = 1.00

$$\frac{c^2x^4+4\,bcx^2\log(x)-b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^4 + 4*b*c*x^2*log(x) - b^2)/x^2
```

giac [A] time = 0.17, size = 32, normalized size = 1.19

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{2\,bcx^2 + b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/2*(2*b*c*x^2 + b^2)/x^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{c^2x^2}{2} + 2bc\ln(x) - \frac{b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^7,x)
```

```
[Out] -1/2*b^2/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

maxima [A] time = 1.33, size = 24, normalized size = 0.89

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/2*b^2/x^2
```

mupad [B] time = 0.03, size = 23, normalized size = 0.85

$$\frac{c^2\,x^2}{2} - \frac{b^2}{2\,x^2} + 2\,b\,c\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^7,x)
```

```
[Out] (c^2*x^2)/2 - b^2/(2*x^2) + 2*b*c*log(x)
```

sympy [A] time = 0.15, size = 24, normalized size = 0.89

$$-\frac{b^2}{2x^2} + 2bc\log\left(x\right) + \frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**7,x)
```

```
[Out] -b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2
```

3.35 $\int \frac{(bx^2+cx^4)^2}{x^8} dx$

Optimal. Leaf size=23

$$-\frac{b^2}{3x^3} - \frac{2bc}{x} + c^2x$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{3x^3} - \frac{2bc}{x} + c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -b^2/(3*x^3) - (2*b*c)/x + c^2*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^8} dx &= \int \frac{(b+cx^2)^2}{x^4} dx \\
&= \int \left(c^2 + \frac{b^2}{x^4} + \frac{2bc}{x^2}\right) dx \\
&= -\frac{b^2}{3x^3} - \frac{2bc}{x} + c^2x
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{b^2}{3x^3} - \frac{2bc}{x} + c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^8,x]
[Out] -1/3*b^2/x^3 - (2*b*c)/x + c^2*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + cx^4)^2}{x^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^8,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^8, x]
```

fricas [A] time = 0.52, size = 26, normalized size = 1.13

$$\frac{3\,c^2x^4 - 6\,bcx^2 - b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
[Out] 1/3*(3*c^2*x^4 - 6*b*c*x^2 - b^2)/x^3
```

giac [A] time = 0.16, size = 22, normalized size = 0.96

$$c^2x - \frac{6\,bcx^2 + b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$c^2x - \frac{2bc}{x} - \frac{b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^8,x)
```

```
[Out] -1/3*b^2/x^3-2*b*c/x+c^2*x
```

maxima [A] time = 1.29, size = 22, normalized size = 0.96

$$c^2x - \frac{6\,bcx^2 + b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 1.04

$$c^2\,x - \frac{\frac{b^2}{3} + 2\,c\,b\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^8,x)
```

```
[Out] c^2*x - (b^2/3 + 2*b*c*x^2)/x^3
```

sympy [A] time = 0.17, size = 22, normalized size = 0.96

$$c^2x + \frac{-b^2 - 6bcx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] c**2*x + (-b**2 - 6*b*c*x**2)/(3*x**3)
```

3.36 $\int \frac{(bx^2+cx^4)^2}{x^9} \, dx$

Optimal. Leaf size=24

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -b^2/(4*x^4) - (b*c)/x^2 + c^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^9}\,dx &= \int \frac{(b+cx^2)^2}{x^5}\,dx\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(b+cx)^2}{x^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2bc}{x^2}+\frac{c^2}{x}\right)dx, x, x^2\right)\\ &= -\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^9,x]
[Out] -1/4*b^2/x^4 - (b*c)/x^2 + c^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+cx^4)^2}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^9,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^9, x]
```

fricas [A] time = 0.53, size = 28, normalized size = 1.17

$$\frac{4\,c^2x^4\log(x)-4\,bcx^2-b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
[Out] 1/4*(4*c^2*x^4*log(x) - 4*b*c*x^2 - b^2)/x^4
```

giac [A] time = 0.18, size = 34, normalized size = 1.42

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{3\,c^2x^4+4\,bcx^2+b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
[Out] 1/2*c^2*log(x^2) - 1/4*(3*c^2*x^4 + 4*b*c*x^2 + b^2)/x^4
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$c^2\ln(x)-\frac{bc}{x^2}-\frac{b^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^9,x)
[Out] -1/4*b^2/x^4-b*c/x^2+c^2*ln(x)
```

maxima [A] time = 1.32, size = 26, normalized size = 1.08

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{4\,bcx^2+b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
[Out] 1/2*c^2*log(x^2) - 1/4*(4*b*c*x^2 + b^2)/x^4
```

mupad [B] time = 0.05, size = 24, normalized size = 1.00

$$c^2\,\ln(x)-\frac{\frac{b^2}{4}+c\,b\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^9,x)
[Out] c^2*log(x) - (b^2/4 + b*c*x^2)/x^4
```

sympy [A] time = 0.19, size = 24, normalized size = 1.00

$$c^2\log\left(x\right)+\frac{-b^2-4bcx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**9,x)
```

```
[Out] c**2*log(x) + (-b**2 - 4*b*c*x**2)/(4*x**4)
```

3.37 $\int \frac{(bx^2+cx^4)^2}{x^{10}} dx$

Optimal. Leaf size=28

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -b^2/(5*x^5) - (2*b*c)/(3*x^3) - c^2/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx &= \int \frac{(b+cx^2)^2}{x^6}\,dx\\ &= \int \left(\frac{b^2}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx\\ &= -\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^10,x]
[Out] -1/5*b^2/x^5 - (2*b*c)/(3*x^3) - c^2/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^10,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^10, x]
```

fricas [A] time = 0.61, size = 26, normalized size = 0.93

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

giac [A] time = 0.17, size = 26, normalized size = 0.93

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$-\frac{c^2}{x}-\frac{2bc}{3x^3}-\frac{b^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^10,x)
```

```
[Out] -1/5*b^2/x^5-2/3*b*c/x^3-c^2/x
```

maxima [A] time = 1.30, size = 26, normalized size = 0.93

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$-\frac{\frac{b^2}{5}+\frac{2\,b\,c\,x^2}{3}+c^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^10,x)
```

```
[Out] -(b^2/5 + c^2*x^4 + (2*b*c*x^2)/3)/x^5
```

sympy [A] time = 0.20, size = 27, normalized size = 0.96

$$\frac{-3b^2-10bcx^2-15c^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**10,x)
```

```
[Out] (-3*b**2 - 10*b*c*x**2 - 15*c**2*x**4)/(15*x**5)
```

3.38 $\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^3}{6bx^6}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$-\frac{(b+cx^2)^3}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -(b + c*x^2)^3/(6*b*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx = \int \frac{(b+cx^2)^2}{x^7}\,dx$$
$$= -\frac{(b+cx^2)^3}{6bx^6}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.58

$$-\frac{b^2}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^11,x]
[Out] -1/6*b^2/x^6 - (b*c)/(2*x^4) - c^2/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^11,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^11, x]
```

fricas [A] time = 0.56, size = 24, normalized size = 1.26

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

giac [A] time = 0.17, size = 24, normalized size = 1.26

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

maple [A] time = 0.01, size = 25, normalized size = 1.32

$$-\frac{c^2}{2x^2}-\frac{bc}{2x^4}-\frac{b^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^11,x)
```

```
[Out] -1/2*b*c/x^4-1/2*c^2/x^2-1/6*b^2/x^6
```

maxima [A] time = 1.23, size = 24, normalized size = 1.26

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

mupad [B] time = 0.04, size = 26, normalized size = 1.37

$$-\frac{\frac{b^2}{6}+\frac{b\,c\,x^2}{2}+\frac{c^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^11,x)
[Out] -(b^2/6 + (c^2*x^4)/2 + (b*c*x^2)/2)/x^6
```

sympy [A] time = 0.21, size = 26, normalized size = 1.37

$$\frac{-b^2-3bcx^2-3c^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**11,x)
[Out] (-b**2 - 3*b*c*x**2 - 3*c**2*x**4)/(6*x**6)
```

3.39 $\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=30

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -b^2/(7*x^7) - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx &= \int \frac{(b+cx^2)^2}{x^8}\,dx\\
&= \int \left(\frac{b^2}{x^8}+\frac{2bc}{x^6}+\frac{c^2}{x^4}\right)dx\\
&= -\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^12,x]
[Out] -1/7*b^2/x^7 - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^12,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^12, x]
```

fricas [A] time = 0.68, size = 26, normalized size = 0.87

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

giac [A] time = 0.15, size = 26, normalized size = 0.87

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{c^2}{3x^3}-\frac{2bc}{5x^5}-\frac{b^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^12,x)
```

```
[Out] -1/7*b^2/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

maxima [A] time = 1.37, size = 26, normalized size = 0.87

$$-\frac{35\,c^2x^4 + 42\,bcx^2 + 15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{b^2}{7} + \frac{2\,b\,c\,x^2}{5} + \frac{c^2\,x^4}{3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^12,x)
```

```
[Out] -(b^2/7 + (c^2*x^4)/3 + (2*b*c*x^2)/5)/x^7
```

sympy [A] time = 0.23, size = 27, normalized size = 0.90

$$\frac{-15b^2 - 42bcx^2 - 35c^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**12,x)
```

```
[Out] (-15*b**2 - 42*b*c*x**2 - 35*c**2*x**4)/(105*x**7)
```

3.40 $\int \frac{\left(bx^2+cx^4\right)^3}{x^2}\,dx$

Optimal. Leaf size=43

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$\frac{3}{7}b^2cx^7+\frac{b^3x^5}{5}+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^2}\,dx &= \int x^4\left(b+cx^2\right)^3\,dx\\
&= \int \left(b^3x^4+3b^2cx^6+3bc^2x^8+c^3x^{10}\right)dx\\
&= \frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^2,x]
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

IntegrateAlgebraic [A] time = 0.03, size = 43, normalized size = 1.00

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^2,x]
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

fricas [A] time = 0.48, size = 35, normalized size = 0.81

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{11}c^3x^{11}+\frac{1}{3}b\,c^2x^9+\frac{3}{7}b^2c\,x^7+\frac{1}{5}b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^2,x)
```

```
[Out] 1/5*b^3*x^5+3/7*b^2*c*x^7+1/3*b*c^2*x^9+1/11*c^3*x^11
```

maxima [A] time = 1.30, size = 35, normalized size = 0.81

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{b^3\,x^5}{5}+\frac{3\,b^2\,c\,x^7}{7}+\frac{b\,c^2\,x^9}{3}+\frac{c^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^2,x)
```

```
[Out] (b^3*x^5)/5 + (c^3*x^11)/11 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3
```

sympy [A] time = 0.08, size = 37, normalized size = 0.86

$$\frac{b^3x^5}{5}+\frac{3b^2cx^7}{7}+\frac{bc^2x^9}{3}+\frac{c^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**2,x)
```

```
[Out] b**3*x**5/5 + 3*b**2*c*x**7/7 + b*c**2*x**9/3 + c**3*x**11/11
```

3.41 $\int \frac{\left(bx^2+cx^4\right)^3}{x^3}\,dx$

Optimal. Leaf size=34

$$\frac{\left(b+cx^2\right)^5}{10c^2}-\frac{b\left(b+cx^2\right)^4}{8c^2}$$

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{\left(b+cx^2\right)^5}{10c^2}-\frac{b\left(b+cx^2\right)^4}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] -(b*(b + c*x^2)^4)/(8*c^2) + (b + c*x^2)^5/(10*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^3}\,dx = \int x^3\left(b+cx^2\right)^3\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int x(b+cx)^3\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{b(b+cx)^3}{c}+\frac{(b+cx)^4}{c}\right)dx, x, x^2\right)$$
$$= -\frac{b\left(b+cx^2\right)^4}{8c^2}+\frac{\left(b+cx^2\right)^5}{10c^2}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.26

$$\frac{b^3x^4}{4}+\frac{1}{2}b^2cx^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] (b^3*x^4)/4 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 1.15

$$\frac{1}{40}x^4\left(10b^3+20b^2cx^2+15bc^2x^4+4c^3x^6\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] (x^4*(10*b^3 + 20*b^2*c*x^2 + 15*b*c^2*x^4 + 4*c^3*x^6))/40
```

fricas [A] time = 0.46, size = 35, normalized size = 1.03

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

giac [A] time = 0.18, size = 35, normalized size = 1.03

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 1.06

$$\frac{1}{10}c^3x^{10}+\frac{3}{8}b\,c^2x^8+\frac{1}{2}b^2c\,x^6+\frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^3,x)
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*b^2*c*x^6+1/4*b^3*x^4
```

maxima [A] time = 1.32, size = 35, normalized size = 1.03

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

mupad [B] time = 0.04, size = 35, normalized size = 1.03

$$\frac{b^3\,x^4}{4}+\frac{b^2\,c\,x^6}{2}+\frac{3\,b\,c^2\,x^8}{8}+\frac{c^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^3,x)
[Out] (b^3*x^4)/4 + (c^3*x^10)/10 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8
```

sympy [A] time = 0.08, size = 37, normalized size = 1.09

$$\frac{b^3x^4}{4}+\frac{b^2cx^6}{2}+\frac{3bc^2x^8}{8}+\frac{c^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**3,x)
```

```
[Out] b**3*x**4/4 + b**2*c*x**6/2 + 3*b*c**2*x**8/8 + c**3*x**10/10
```

3.42 $\int \frac{(bx^2+cx^4)^3}{x^4}\,dx$

Optimal. Leaf size=43

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$\frac{3}{5}b^2cx^5+\frac{b^3x^3}{3}+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^4}\,dx &= \int x^2\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^2+3b^2cx^4+3bc^2x^6+c^3x^8\right)dx\\ &= \frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{b^3x^3}{3} + \frac{3}{5}b^2cx^5 + \frac{3}{7}bc^2x^7 + \frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^4,x]
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

IntegrateAlgebraic [A] time = 0.03, size = 43, normalized size = 1.00

$$\frac{b^3x^3}{3} + \frac{3}{5}b^2cx^5 + \frac{3}{7}bc^2x^7 + \frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^4,x]
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

fricas [A] time = 0.56, size = 35, normalized size = 0.81

$$\frac{1}{9}c^3x^9 + \frac{3}{7}bc^2x^7 + \frac{3}{5}b^2cx^5 + \frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

giac [A] time = 0.17, size = 35, normalized size = 0.81

$$\frac{1}{9}c^3x^9 + \frac{3}{7}bc^2x^7 + \frac{3}{5}b^2cx^5 + \frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{9}c^3x^9 + \frac{3}{7}b\,c^2x^7 + \frac{3}{5}b^2c\,x^5 + \frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^4,x)
```

```
[Out] 1/3*b^3*x^3+3/5*b^2*c*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

maxima [A] time = 1.34, size = 35, normalized size = 0.81

$$\frac{1}{9}\,c^3x^9 + \frac{3}{7}\,bc^2x^7 + \frac{3}{5}\,b^2cx^5 + \frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{b^3\,x^3}{3} + \frac{3\,b^2\,c\,x^5}{5} + \frac{3\,b\,c^2\,x^7}{7} + \frac{c^3\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^4,x)
```

```
[Out] (b^3*x^3)/3 + (c^3*x^9)/9 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7
```

sympy [A] time = 0.09, size = 39, normalized size = 0.91

$$\frac{b^3x^3}{3} + \frac{3b^2cx^5}{5} + \frac{3bc^2x^7}{7} + \frac{c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**4,x)
```

```
[Out] b**3*x**3/3 + 3*b**2*c*x**5/5 + 3*b*c**2*x**7/7 + c**3*x**9/9
```

3.43 $\int \frac{(bx^2+cx^4)^3}{x^5} \, dx$

Optimal. Leaf size=16

$$\frac{(b + cx^2)^4}{8c}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$\frac{(b + cx^2)^4}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^5,x]
```

```
[Out] (b + c*x^2)^4/(8*c)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{(bx^2 + cx^4)^3}{x^5} \, dx &= \int x (b + cx^2)^3 \, dx \\ &= \frac{(b + cx^2)^4}{8c} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{(b + cx^2)^4}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^5,x]
[Out] (b + c*x^2)^4/(8*c)
```

IntegrateAlgebraic [B] time = 0.01, size = 38, normalized size = 2.38

$$\frac{1}{8}x^2\left(4b^3 + 6b^2cx^2 + 4bc^2x^4 + c^3x^6\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^5,x]
[Out] (x^2*(4*b^3 + 6*b^2*c*x^2 + 4*b*c^2*x^4 + c^3*x^6))/8
```

fricas [B] time = 0.40, size = 35, normalized size = 2.19

$$\frac{1}{8}c^3x^8 + \frac{1}{2}bc^2x^6 + \frac{3}{4}b^2cx^4 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

giac [B] time = 0.15, size = 35, normalized size = 2.19

$$\frac{1}{8}c^3x^8 + \frac{1}{2}bc^2x^6 + \frac{3}{4}b^2cx^4 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

maple [B] time = 0.00, size = 36, normalized size = 2.25

$$\frac{1}{8}c^3x^8 + \frac{1}{2}b\,c^2x^6 + \frac{3}{4}b^2c\,x^4 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^5,x)
```

```
[Out] 1/8*c^3*x^8+1/2*b*c^2*x^6+3/4*b^2*c*x^4+1/2*b^3*x^2
```

maxima [B] time = 1.28, size = 35, normalized size = 2.19

$$\frac{1}{8}\,c^3x^8 + \frac{1}{2}\,bc^2x^6 + \frac{3}{4}\,b^2cx^4 + \frac{1}{2}\,b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

mupad [B] time = 0.04, size = 35, normalized size = 2.19

$$\frac{b^3\,x^2}{2} + \frac{3\,b^2\,c\,x^4}{4} + \frac{b\,c^2\,x^6}{2} + \frac{c^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^5,x)
```

```
[Out] (b^3*x^2)/2 + (c^3*x^8)/8 + (3*b^2*c*x^4)/4 + (b*c^2*x^6)/2
```

sympy [B] time = 0.08, size = 37, normalized size = 2.31

$$\frac{b^3x^2}{2} + \frac{3b^2cx^4}{4} + \frac{bc^2x^6}{2} + \frac{c^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**5,x)
```

```
[Out] b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8
```

3.44 $\int \frac{(bx^2+cx^4)^3}{x^6} \, dx$

Optimal. Leaf size=35

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 194}

$$b^2cx^3 + b^3x + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^6,x]
```

```
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^6} \, dx &= \int (b+cx^2)^3 \, dx \\
&= \int (b^3 + 3b^2cx^2 + 3bc^2x^4 + c^3x^6) \, dx \\
&= b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^6,x]
```

```
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

IntegrateAlgebraic [A] time = 0.02, size = 35, normalized size = 1.00

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^6,x]
```

```
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

fricas [A] time = 0.64, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

giac [A] time = 0.16, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{1}{7}c^3x^7 + \frac{3}{5}b\,c^2x^5 + b^2c\,x^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^6,x)
```

```
[Out] b^3*x+b^2*c*x^3+3/5*b*c^2*x^5+1/7*c^3*x^7
```

maxima [A] time = 1.32, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$b^3\,x + b^2\,c\,x^3 + \frac{3\,b\,c^2\,x^5}{5} + \frac{c^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^6,x)
```

```
[Out] b^3*x + (c^3*x^7)/7 + b^2*c*x^3 + (3*b*c^2*x^5)/5
```

sympy [A] time = 0.08, size = 32, normalized size = 0.91

$$b^3x + b^2cx^3 + \frac{3bc^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**6,x)
```

```
[Out] b**3*x + b**2*c*x**3 + 3*b*c**2*x**5/5 + c**3*x**7/7
```

3.45 $\int \frac{(bx^2+cx^4)^3}{x^7} dx$

Optimal. Leaf size=39

$$b^3\log(x) + \frac{3}{2}b^2cx^2 + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{3}{2}b^2cx^2 + b^3\log(x) + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^7,x]
```

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^3}{x^7}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(3b^2c+\frac{b^3}{x}+3bc^2x+c^3x^2\right)dx,x,x^2\right)\\ &= \frac{3}{2}b^2cx^2+\frac{3}{4}bc^2x^4+\frac{c^3x^6}{6}+b^3\log(x)\end{aligned}$$

Mathematica [**A**] time = 0.00, size = 39, normalized size = 1.00

$$b^3\log(x)+\frac{3}{2}b^2cx^2+\frac{3}{4}bc^2x^4+\frac{c^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^7,x]
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^7,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^7, x]
```

fricas [**A**] time = 0.55, size = 33, normalized size = 0.85

$$\frac{1}{6}\,c^3x^6+\frac{3}{4}\,bc^2x^4+\frac{3}{2}\,b^2cx^2+b^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + b^3*log(x)
```

giac [A] time = 0.15, size = 36, normalized size = 0.92

$$\frac{1}{6}\,c^3x^6 + \frac{3}{4}\,bc^2x^4 + \frac{3}{2}\,b^2cx^2 + \frac{1}{2}\,b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$\frac{c^3x^6}{6} + \frac{3b\,c^2x^4}{4} + \frac{3b^2c\,x^2}{2} + b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^7,x)
```

```
[Out] 3/2*b^2*c*x^2+3/4*b*c^2*x^4+1/6*c^3*x^6+b^3*ln(x)
```

maxima [A] time = 1.33, size = 36, normalized size = 0.92

$$\frac{1}{6}\,c^3x^6 + \frac{3}{4}\,bc^2x^4 + \frac{3}{2}\,b^2cx^2 + \frac{1}{2}\,b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.85

$$b^3\,\ln(x) + \frac{c^3\,x^6}{6} + \frac{3\,b^2\,c\,x^2}{2} + \frac{3\,b\,c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^7,x)
```

```
[Out] b^3*log(x) + (c^3*x^6)/6 + (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4
```

sympy [A] time = 0.12, size = 37, normalized size = 0.95

$$b^3\log{(x)} + \frac{3b^2cx^2}{2} + \frac{3bc^2x^4}{4} + \frac{c^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**7,x)
```

```
[Out] b**3*log(x) + 3*b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6
```

3.46 $\int \frac{(bx^2+cx^4)^3}{x^8} \, dx$

Optimal. Leaf size=34

$$-\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$3b^2cx - \frac{b^3}{x} + bc^2x^3 + \frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^8,x]
```

```
[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^8} \, dx &= \int \frac{(b+cx^2)^3}{x^2} \, dx \\
&= \int \left(3b^2c + \frac{b^3}{x^2} + 3bc^2x^2 + c^3x^4\right) dx \\
&= -\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^8,x]
[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^8,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^8, x]
```

fricas [A] time = 0.54, size = 36, normalized size = 1.06

$$\frac{c^3x^6+5\,bc^2x^4+15\,b^2cx^2-5\,b^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
[Out] 1/5*(c^3*x^6 + 5*b*c^2*x^4 + 15*b^2*c*x^2 - 5*b^3)/x
```

giac [A] time = 0.15, size = 32, normalized size = 0.94

$$\frac{1}{5}\,c^3x^5+bc^2x^3+3\,b^2cx-\frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$\frac{c^3x^5}{5}+b\,c^2x^3+3b^2cx-\frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^8,x)
```

```
[Out] -b^3/x+3*b^2*c*x+b*c^2*x^3+1/5*c^3*x^5
```

maxima [**A**] time = 1.38, size = 32, normalized size = 0.94

$$\frac{1}{5}c^3x^5 + bc^2x^3 + 3b^2cx - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

mupad [**B**] time = 0.04, size = 32, normalized size = 0.94

$$\frac{c^3 x^5}{5} - \frac{b^3}{x} + b c^2 x^3 + 3 b^2 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^8,x)
```

```
[Out] (c^3*x^5)/5 - b^3/x + b*c^2*x^3 + 3*b^2*c*x
```

sympy [**A**] time = 0.12, size = 29, normalized size = 0.85

$$-\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**8,x)
```

```
[Out] -b**3/x + 3*b**2*c*x + b*c**2*x**3 + c**3*x**5/5
```

3.47 $\int \frac{(bx^2+cx^4)^3}{x^9}\,dx$

Optimal. Leaf size=40

$$-\frac{b^3}{2x^2}+3b^2c\log(x)+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$3b^2c\log(x)-\frac{b^3}{2x^2}+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^9,x]
```

```
[Out] -b^3/(2*x^2) + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^9}\,dx = \int \frac{\left(b+cx^2\right)^3}{x^3}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+cx)^3}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(3bc^2+\frac{b^3}{x^2}+\frac{3b^2c}{x}+c^3x\right)dx, x, x^2\right)$$
$$= -\frac{b^3}{2x^2}+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}+3b^2c\log(x)$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{b^3}{2x^2}+3b^2c\log(x)+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^9,x]
[Out] -1/2*b^3/x^2 + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^9,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^9, x]
```

fricas [A] time = 0.68, size = 38, normalized size = 0.95

$$\frac{c^3x^6+6\,bc^2x^4+12\,b^2cx^2\log(x)-2\,b^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
[Out] 1/4*(c^3*x^6 + 6*b*c^2*x^4 + 12*b^2*c*x^2*log(x) - 2*b^3)/x^2
```

giac [A] time = 0.16, size = 46, normalized size = 1.15

$$\frac{1}{4}\,c^3x^4 + \frac{3}{2}\,bc^2x^2 + \frac{3}{2}\,b^2c\log\left(x^2\right) - \frac{3\,b^2cx^2 + b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*(3*b^2*c*x^2 + b^3)/
x^2
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{c^3x^4}{4} + \frac{3b\,c^2x^2}{2} + 3b^2c\ln(x) - \frac{b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^9,x)
```

```
[Out] -1/2*b^3/x^2+3/2*b*c^2*x^2+1/4*c^3*x^4+3*b^2*c*ln(x)
```

maxima [A] time = 1.27, size = 36, normalized size = 0.90

$$\frac{1}{4}\,c^3x^4 + \frac{3}{2}\,bc^2x^2 + \frac{3}{2}\,b^2c\log\left(x^2\right) - \frac{b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*b^3/x^2
```

mupad [B] time = 0.04, size = 34, normalized size = 0.85

$$\frac{c^3\,x^4}{4} - \frac{b^3}{2\,x^2} + \frac{3\,b\,c^2\,x^2}{2} + 3\,b^2\,c\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^9,x)
```

```
[Out] (c^3*x^4)/4 - b^3/(2*x^2) + (3*b*c^2*x^2)/2 + 3*b^2*c*log(x)
```

sympy [A] time = 0.17, size = 37, normalized size = 0.92

$$-\frac{b^3}{2x^2} + 3b^2c\log\left(x\right) + \frac{3bc^2x^2}{2} + \frac{c^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**9,x)
```

```
[Out] -b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4
```

3.48 $\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx$

Optimal. Leaf size=37

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{3b^2c}{x}-\frac{b^3}{3x^3}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^10,x]
```

```
[Out] -b^3/(3*x^3) - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx &= \int \frac{(b+cx^2)^3}{x^4}\,dx\\ &= \int \left(3bc^2+\frac{b^3}{x^4}+\frac{3b^2c}{x^2}+c^3x^2\right)dx\\ &= -\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}\end{aligned}$$

Mathematica [A]   time = 0.00, size = 37, normalized size = 1.00

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^10,x]
```

```
[Out] -1/3*b^3/x^3 - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^10,x]
```

```
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^10, x]
```

fricas [A]   time = 0.45, size = 36, normalized size = 0.97

$$\frac{c^3x^6+9\,bc^2x^4-9\,b^2cx^2-b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/3*(c^3*x^6 + 9*b*c^2*x^4 - 9*b^2*c*x^2 - b^3)/x^3
```

giac [A]   time = 0.17, size = 34, normalized size = 0.92

$$\frac{1}{3}\,c^3x^3+3\,bc^2x-\frac{9\,b^2cx^2+b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

maple [A]   time = 0.01, size = 34, normalized size = 0.92

$$\frac{c^3x^3}{3}+3b\,c^2x-\frac{3b^2c}{x}-\frac{b^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^10,x)
```

```
[Out] -1/3*b^3/x^3-3*b^2*c/x+3*b*c^2*x+1/3*c^3*x^3
```

maxima [A] time = 1.29, size = 34, normalized size = 0.92

$$\frac{1}{3}\,c^3x^3 + 3\,bc^2x - \frac{9\,b^2cx^2 + b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

mupad [B] time = 0.04, size = 36, normalized size = 0.97

$$\frac{c^3\,x^3}{3} - \frac{\frac{b^3}{3} + 3\,c\,b^2\,x^2}{x^3} + 3\,b\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^10,x)
```

```
[Out] (c^3*x^3)/3 - (b^3/3 + 3*b^2*c*x^2)/x^3 + 3*b*c^2*x
```

sympy [A] time = 0.17, size = 36, normalized size = 0.97

$$3bc^2x + \frac{c^3x^3}{3} + \frac{-b^3 - 9b^2cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**10,x)
```

```
[Out] 3*b*c**2*x + c**3*x**3/3 + (-b**3 - 9*b**2*c*x**2)/(3*x**3)
```

3.49 $\int \frac{(bx^2+cx^4)^3}{x^{11}} dx$

Optimal. Leaf size=40

$$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{3b^2c}{2x^2}-\frac{b^3}{4x^4}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^11,x]
```

```
[Out] -b^3/(4*x^4) - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{11}}\,dx = \int \frac{\left(b+cx^2\right)^3}{x^5}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+cx)^3}{x^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(c^3+\frac{b^3}{x^3}+\frac{3b^2c}{x^2}+\frac{3bc^2}{x}\right)dx, x, x^2\right)$$
$$= -\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+\frac{c^3x^2}{2}+3bc^2\log(x)$$

Mathematica [A] time = 0.00, size = 40, normalized size = 1.00

$$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^11,x]
```

```
[Out] -1/4*b^3/x^4 - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^11,x]
```

```
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^11, x]
```

fricas [A] time = 0.60, size = 39, normalized size = 0.98

$$\frac{2\,c^3x^6+12\,bc^2x^4\log(x)-6\,b^2cx^2-b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*x^6 + 12*b*c^2*x^4*log(x) - 6*b^2*c*x^2 - b^3)/x^4
```

giac [A] time = 0.15, size = 46, normalized size = 1.15

$$\frac{1}{2}c^3x^2 + \frac{3}{2}bc^2\log\left(x^2\right) - \frac{9\,bc^2x^4 + 6\,b^2cx^2 + b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(9*b*c^2*x^4 + 6*b^2*c*x^2 + b^3)/x^
4
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{c^3x^2}{2} + 3b\,c^2\ln(x) - \frac{3b^2c}{2x^2} - \frac{b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^11,x)
```

```
[Out] -1/4*b^3/x^4-3/2*b^2*c/x^2+1/2*c^3*x^2+3*b*c^2*ln(x)
```

maxima [A] time = 1.34, size = 37, normalized size = 0.92

$$\frac{1}{2}c^3x^2 + \frac{3}{2}bc^2\log\left(x^2\right) - \frac{6\,b^2cx^2 + b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(6*b^2*c*x^2 + b^3)/x^4
```

mupad [B] time = 0.03, size = 37, normalized size = 0.92

$$\frac{c^3\,x^2}{2} - \frac{\frac{b^3}{4} + \frac{3\,c\,b^2\,x^2}{2}}{x^4} + 3\,b\,c^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^11,x)
```

```
[Out] (c^3*x^2)/2 - (b^3/4 + (3*b^2*c*x^2)/2)/x^4 + 3*b*c^2*log(x)
```

sympy [A] time = 0.23, size = 37, normalized size = 0.92

$$3bc^2\log\left(x\right) + \frac{c^3x^2}{2} + \frac{-b^3 - 6b^2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**11,x)
```

```
[Out] 3*b*c**2*log(x) + c**3*x**2/2 + (-b**3 - 6*b**2*c*x**2)/(4*x**4)
```

3.50 $\int \frac{(bx^2+cx^4)^3}{x^{12}} dx$

Optimal. Leaf size=34

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2c}{x^3}-\frac{b^3}{5x^5}-\frac{3bc^2}{x}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^12,x]
```

```
[Out] -b^3/(5*x^5) - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx &= \int \frac{(b+cx^2)^3}{x^6}\,dx\\ &= \int \left(c^3+\frac{b^3}{x^6}+\frac{3b^2c}{x^4}+\frac{3bc^2}{x^2}\right)dx\\ &= -\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^12,x]
[Out] -1/5*b^3/x^5 - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^12,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^12, x]
```

fricas [A] time = 0.74, size = 37, normalized size = 1.09

$$\frac{5\,c^3x^6-15\,bc^2x^4-5\,b^2cx^2-b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
[Out] 1/5*(5*c^3*x^6 - 15*b*c^2*x^4 - 5*b^2*c*x^2 - b^3)/x^5
```

giac [A] time = 0.17, size = 33, normalized size = 0.97

$$c^3x-\frac{15\,bc^2x^4+5\,b^2cx^2+b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

maple [A] time = 0.01, size = 33, normalized size = 0.97

$$c^3x-\frac{3b\,c^2}{x}-\frac{b^2c}{x^3}-\frac{b^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^12,x)
```

```
[Out] -1/5*b^3/x^5-b^2*c/x^3-3*b*c^2/x+c^3*x
```

maxima [A] time = 1.30, size = 33, normalized size = 0.97

$$c^3x - \frac{15\,bc^2x^4 + 5\,b^2cx^2 + b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

mupad [B] time = 0.03, size = 34, normalized size = 1.00

$$c^3\,x - \frac{\frac{b^3}{5} + b^2\,c\,x^2 + 3\,b\,c^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^12,x)
```

```
[Out] c^3*x - (b^3/5 + b^2*c*x^2 + 3*b*c^2*x^4)/x^5
```

sympy [A] time = 0.23, size = 34, normalized size = 1.00

$$c^3x + \frac{-b^3 - 5b^2cx^2 - 15bc^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**12,x)
```

```
[Out] c**3*x + (-b**3 - 5*b**2*c*x**2 - 15*b*c**2*x**4)/(5*x**5)
```

3.51 $\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx$

Optimal. Leaf size=39

$$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{3b^2c}{4x^4}-\frac{b^3}{6x^6}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^13,x]
```

```
[Out] -b^3/(6*x^6) - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^{13}}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^7}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+cx)^3}{x^4}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3b^2c}{x^3}+\frac{3bc^2}{x^2}+\frac{c^3}{x}\right)dx, x, x^2\right) \\
&= -\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$
-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^13,x]
[Out] -1/6*b^3/x^6 - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(bx^2+cx^4\right)^3}{x^{13}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^13,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^13, x]
```

fricas [A] time = 0.65, size = 39, normalized size = 1.00

$$
\frac{12\,c^3x^6\log(x)-18\,bc^2x^4-9\,b^2cx^2-2\,b^3}{12\,x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
[Out] 1/12*(12*c^3*x^6*log(x) - 18*b*c^2*x^4 - 9*b^2*c*x^2 - 2*b^3)/x^6
```

giac [A] time = 0.16, size = 47, normalized size = 1.21

$$\frac{1}{2}\,c^3\log\left(x^2\right)-\frac{11\,c^3x^6+18\,bc^2x^4+9\,b^2cx^2+2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*c^3*log(x^2) - 1/12*(11*c^3*x^6 + 18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x
^6
```

maple [A] time = 0.01, size = 34, normalized size = 0.87

$$c^3\ln(x)-\frac{3b\,c^2}{2x^2}-\frac{3b^2c}{4x^4}-\frac{b^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^13,x)
```

```
[Out] -1/6*b^3/x^6-3/4*b^2*c/x^4-3/2*b*c^2/x^2+c^3*ln(x)
```

maxima [A] time = 1.32, size = 39, normalized size = 1.00

$$\frac{1}{2}\,c^3\log\left(x^2\right)-\frac{18\,bc^2x^4+9\,b^2cx^2+2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(x^2) - 1/12*(18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x^6
```

mupad [B] time = 0.05, size = 36, normalized size = 0.92

$$c^3\,\ln(x)-\frac{\frac{b^3}{6}+\frac{3\,b^2\,c\,x^2}{4}+\frac{3\,b\,c^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^13,x)
```

```
[Out] c^3*log(x) - (b^3/6 + (3*b^2*c*x^2)/4 + (3*b*c^2*x^4)/2)/x^6
```

sympy [A] time = 0.29, size = 37, normalized size = 0.95

$$c^3\log\left(x\right)+\frac{-2b^3-9b^2cx^2-18bc^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**13,x)

[Out] c**3*log(x) + (-2*b**3 - 9*b**2*c*x**2 - 18*b*c**2*x**4)/(12*x**6)
```

3.52 $\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx$

Optimal. Leaf size=39

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{3b^2c}{5x^5}-\frac{b^3}{7x^7}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^14,x]
```

```
[Out] -b^3/(7*x^7) - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx &= \int \frac{(b+cx^2)^3}{x^8}\,dx\\ &= \int \left(\frac{b^3}{x^8}+\frac{3b^2c}{x^6}+\frac{3bc^2}{x^4}+\frac{c^3}{x^2}\right)dx\\ &= -\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^14,x]
[Out] -1/7*b^3/x^7 - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^14,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^14, x]
```

fricas [A] time = 0.69, size = 37, normalized size = 0.95

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

giac [A] time = 0.16, size = 37, normalized size = 0.95

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

maple [A] time = 0.01, size = 36, normalized size = 0.92

$$-\frac{c^3}{x}-\frac{b\,c^2}{x^3}-\frac{3b^2c}{5x^5}-\frac{b^3}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^14,x)
```

```
[Out] -1/7*b^3/x^7-3/5*b^2*c/x^5-b*c^2/x^3-c^3/x
```

maxima [A] time = 1.35, size = 37, normalized size = 0.95

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

mupad [B] time = 0.03, size = 35, normalized size = 0.90

$$-\frac{\frac{b^3}{7}+\frac{3\,b^2\,c\,x^2}{5}+b\,c^2\,x^4+c^3\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^14,x)
```

```
[Out] -(b^3/7 + c^3*x^6 + (3*b^2*c*x^2)/5 + b*c^2*x^4)/x^7
```

sympy [A] time = 0.28, size = 39, normalized size = 1.00

$$\frac{-5b^3-21b^2cx^2-35bc^2x^4-35c^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**14,x)
```

```
[Out] (-5*b**3 - 21*b**2*c*x**2 - 35*b*c**2*x**4 - 35*c**3*x**6)/(35*x**7)
```

3.53 $\int \frac{(bx^2+cx^4)^3}{x^{15}} dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^4}{8bx^8}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$-\frac{(b+cx^2)^4}{8bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^15,x]
```

```
[Out] -(b + c*x^2)^4/(8*b*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{15}} dx &= \int \frac{(b+cx^2)^3}{x^9} dx\\ &= -\frac{(b+cx^2)^4}{8bx^8}\end{aligned}$$

Mathematica [B] time = 0.01, size = 43, normalized size = 2.26

$$-\frac{b^3}{8x^8}-\frac{b^2c}{2x^6}-\frac{3bc^2}{4x^4}-\frac{c^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^15,x]
[Out] -1/8*b^3/x^8 - (b^2*c)/(2*x^6) - (3*b*c^2)/(4*x^4) - c^3/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + cx^4)^3}{x^{15}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^15,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^15, x]
```

fricas [B] time = 0.77, size = 35, normalized size = 1.84

$$-\frac{4 c^3 x^6 + 6 bc^2 x^4 + 4 b^2 c x^2 + b^3}{8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

giac [B] time = 0.15, size = 35, normalized size = 1.84

$$-\frac{4 c^3 x^6 + 6 bc^2 x^4 + 4 b^2 c x^2 + b^3}{8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

maple [B] time = 0.00, size = 36, normalized size = 1.89

$$-\frac{c^3}{2x^2} - \frac{3b c^2}{4x^4} - \frac{b^2 c}{2x^6} - \frac{b^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^15,x)
```

```
[Out] -1/2*c^3/x^2-1/2*b^2*c/x^6-3/4*b*c^2/x^4-1/8*b^3/x^8
```

maxima [B] time = 1.34, size = 35, normalized size = 1.84

$$-\frac{4\,c^3x^6 + 6\,bc^2x^4 + 4\,b^2cx^2 + b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

mupad [B] time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{b^3}{8} + \frac{b^2\,c\,x^2}{2} + \frac{3\,b\,c^2\,x^4}{4} + \frac{c^3\,x^6}{2}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^15,x)
[Out] -(b^3/8 + (c^3*x^6)/2 + (b^2*c*x^2)/2 + (3*b*c^2*x^4)/4)/x^8
```

sympy [B] time = 0.31, size = 37, normalized size = 1.95

$$\frac{-b^3 - 4b^2cx^2 - 6bc^2x^4 - 4c^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**15,x)
[Out] (-b**3 - 4*b**2*c*x**2 - 6*b*c**2*x**4 - 4*c**3*x**6)/(8*x**8)
```

3.54 $\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx$

Optimal. Leaf size=43

$$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{3b^2c}{7x^7}-\frac{b^3}{9x^9}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^16,x]
```

```
[Out] -b^3/(9*x^9) - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx &= \int \frac{(b+cx^2)^3}{x^{10}}\,dx\\ &= \int \left(\frac{b^3}{x^{10}}+\frac{3b^2c}{x^8}+\frac{3bc^2}{x^6}+\frac{c^3}{x^4}\right)dx\\ &= -\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}\end{aligned}$$

Mathematica [A]   time = 0.00, size = 43, normalized size = 1.00

$$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^16,x]
[Out] -1/9*b^3/x^9 - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^16,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^16, x]
```

fricas [A]   time = 0.61, size = 37, normalized size = 0.86

$$-\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

giac [A]   time = 0.17, size = 37, normalized size = 0.86

$$-\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

maple [A]   time = 0.01, size = 36, normalized size = 0.84

$$-\frac{c^3}{3x^3}-\frac{3b\,c^2}{5x^5}-\frac{3b^2c}{7x^7}-\frac{b^3}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^16,x)
```

```
[Out] -1/9*b^3/x^9-3/7*b^2*c/x^7-3/5*b*c^2/x^5-1/3*c^3/x^3
```

maxima [A] time = 1.33, size = 37, normalized size = 0.86

$$-\frac{105\,c^3x^6 + 189\,bc^2x^4 + 135\,b^2cx^2 + 35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{b^3}{9} + \frac{3\,b^2\,c\,x^2}{7} + \frac{3\,b\,c^2\,x^4}{5} + \frac{c^3\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^16,x)
```

```
[Out] -(b^3/9 + (c^3*x^6)/3 + (3*b^2*c*x^2)/7 + (3*b*c^2*x^4)/5)/x^9
```

sympy [A] time = 0.30, size = 39, normalized size = 0.91

$$\frac{-35b^3 - 135b^2cx^2 - 189bc^2x^4 - 105c^3x^6}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**16,x)
```

```
[Out] (-35*b**3 - 135*b**2*c*x**2 - 189*b*c**2*x**4 - 105*c**3*x**6)/(315*x**9)
```

3.55 $\int \frac{(bx^2+cx^4)^3}{x^{17}}\, dx$

Optimal. Leaf size=40

$$\frac{c(b+cx^2)^4}{40b^2x^8}-\frac{(b+cx^2)^4}{10bx^{10}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 266, 45, 37}

$$\frac{c(b+cx^2)^4}{40b^2x^8}-\frac{(b+cx^2)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^17,x]
```

```
[Out] -(b + c*x^2)^4/(10*b*x^10) + (c*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^{17}}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^{11}}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x^6}\,dx, x, x^2\right) \\
&= -\frac{\left(b+cx^2\right)^4}{10bx^{10}} - \frac{c\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x^5}\,dx, x, x^2\right)}{10b} \\
&= -\frac{\left(b+cx^2\right)^4}{10bx^{10}} + \frac{c\left(b+cx^2\right)^4}{40b^2x^8}
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.08

$$-\frac{b^3}{10x^{10}} - \frac{3b^2c}{8x^8} - \frac{bc^2}{2x^6} - \frac{c^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^17,x]
[Out] -1/10*b^3/x^10 - (3*b^2*c)/(8*x^8) - (b*c^2)/(2*x^6) - c^3/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^17,x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^17, x]
```

fricas [A] time = 0.70, size = 37, normalized size = 0.92

$$-\frac{10\,c^3x^6 + 20\,bc^2x^4 + 15\,b^2cx^2 + 4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

giac [A] time = 0.15, size = 37, normalized size = 0.92

$$-\frac{10\,c^{3}x^{6}+20\,bc^{2}x^{4}+15\,b^{2}cx^{2}+4\,b^{3}}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.90

$$-\frac{c^{3}}{4x^{4}}-\frac{b\,c^{2}}{2x^{6}}-\frac{3b^{2}c}{8x^{8}}-\frac{b^{3}}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^17,x)
```

```
[Out] -3/8*b^2*c/x^8-1/2*b*c^2/x^6-1/4*c^3/x^4-1/10*b^3/x^10
```

maxima [A] time = 1.31, size = 37, normalized size = 0.92

$$-\frac{10\,c^{3}x^{6}+20\,bc^{2}x^{4}+15\,b^{2}cx^{2}+4\,b^{3}}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

mupad [B] time = 0.03, size = 37, normalized size = 0.92

$$-\frac{\frac{b^{3}}{10}+\frac{3\,b^{2}\,c\,x^{2}}{8}+\frac{b\,c^{2}\,x^{4}}{2}+\frac{c^{3}\,x^{6}}{4}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^17,x)
```

```
[Out] -(b^3/10 + (c^3*x^6)/4 + (3*b^2*c*x^2)/8 + (b*c^2*x^4)/2)/x^10
```

sympy [A] time = 0.32, size = 39, normalized size = 0.98

$$\frac{-4b^3-15b^2cx^2-20bc^2x^4-10c^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**17,x)
```

```
[Out] (-4*b**3 - 15*b**2*c*x**2 - 20*b*c**2*x**4 - 10*c**3*x**6)/(40*x**10)
```

3.56 $\int \frac{x^{10}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=68

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 302, 205}

$$\frac{b^2x^3}{3c^3}-\frac{b^3x}{c^4}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}}{bx^2+cx^4}\,dx &= \int \frac{x^8}{b+cx^2}\,dx\\
&= \int \left(-\frac{b^3}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{c^2}+\frac{x^6}{c}+\frac{b^4}{c^4\left(b+cx^2\right)}\right)dx\\
&= -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^4\int \frac{1}{b+cx^2}\,dx}{c^4}\\
&= -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$
\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^{10}}{bx^2+cx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^10/(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.59, size = 148, normalized size = 2.18

$$
\left[\frac{30\,c^3x^7-42\,bc^2x^5+70\,b^2cx^3+105\,b^3\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-210\,b^3x}{210\,c^4},\ \frac{15\,c^3x^7-21\,bc^2x^5+35\,b^2cx^3+105\,b^3\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-105\,b^3x}{105\,c^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^3*x^7 - 42*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*sqrt(-b/c)*log((
c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) - 210*b^3*x)/c^4, 1/105*(15*c^3*
x^7 - 21*b*c^2*x^5 + 35*b^2*c*x^3 + 105*b^3*sqrt(b/c)*arctan(c*x*sqrt(b/c)/
b) - 105*b^3*x)/c^4]
```

giac [A] time = 0.19, size = 65, normalized size = 0.96

$$\frac{b^4 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, c^4} + \frac{15\, c^6 x^7 - 21\, bc^5 x^5 + 35\, b^2 c^4 x^3 - 105\, b^3 c^3 x}{105\, c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^6*x^7 - 21*b*c^5*x^
5 + 35*b^2*c^4*x^3 - 105*b^3*c^3*x)/c^7
```

maple [A] time = 0.01, size = 60, normalized size = 0.88

$$\frac{x^7}{7c} - \frac{b\, x^5}{5c^2} + \frac{b^2 x^3}{3c^3} + \frac{b^4 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\; c^4} - \frac{b^3 x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2),x)
```

```
[Out] 1/7*x^7/c-1/5*b*x^5/c^2+1/3*b^2*x^3/c^3-b^3*x/c^4+b^4/c^4/(b*c)^(1/2)*arcta
n(x*c/(b*c)^(1/2))
```

maxima [A] time = 3.01, size = 60, normalized size = 0.88

$$\frac{b^4 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, c^4} + \frac{15\, c^3 x^7 - 21\, bc^2 x^5 + 35\, b^2 c x^3 - 105\, b^3 x}{105\, c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^3*x^7 - 21*b*c^2*x^
5 + 35*b^2*c*x^3 - 105*b^3*x)/c^4
```

mupad [B] time = 0.03, size = 54, normalized size = 0.79

$$\frac{x^7}{7\,c} - \frac{b\,x^5}{5\,c^2} - \frac{b^3\,x}{c^4} + \frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}} + \frac{b^2\,x^3}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2 + c*x^4),x)
```

```
[Out] x^7/(7*c) - (b*x^5)/(5*c^2) - (b^3*x)/c^4 + (b^(7/2)*atan((c^(1/2)*x)/b^(1/
2)))/c^(9/2) + (b^2*x^3)/(3*c^3)
```

sympy [A] time = 0.22, size = 107, normalized size = 1.57

$$-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}-\frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2}+\frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2}+\frac{x^7}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*x/c**4 + b**2*x**3/(3*c**3) - b*x**5/(5*c**2) - sqrt(-b**7/c**9)*log(
x - c**4*sqrt(-b**7/c**9)/b**3)/2 + sqrt(-b**7/c**9)*log(x + c**4*sqrt(-b**
7/c**9)/b**3)/2 + x**7/(7*c)
```

3.57 $\int \frac{x^9}{bx^2+cx^4} dx$

Optimal. Leaf size=53

$$-\frac{b^3 \log \left(b+cx^2\right)}{2c^4}+\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^2x^2}{2c^3}-\frac{b^3 \log \left(b+cx^2\right)}{2c^4}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*c
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{bx^2+cx^4}\,dx &= \int \frac{x^7}{b+cx^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{b+cx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^2}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}-\frac{b^3}{c^3(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}-\frac{b^3\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.00

$$-\frac{b^3\log\left(b+cx^2\right)}{2c^4}+\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4),x]
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*c
^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x^9/(b*x^2 + c*x^4), x]
```

fricas [A] time = 1.31, size = 45, normalized size = 0.85

$$\frac{2\,c^3x^6-3\,bc^2x^4+6\,b^2cx^2-6\,b^3\log\left(cx^2+b\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/12*(2*c^3*x^6 - 3*b*c^2*x^4 + 6*b^2*c*x^2 - 6*b^3*log(c*x^2 + b))/c^4
```

giac [A] time = 0.16, size = 47, normalized size = 0.89

$$-\frac{b^{3} \log \left(\left|cx^{2}+b\right|\right)}{2 \, c^{4}}+\frac{2 \, c^{2} x^{6}-3 \, b c x^{4}+6 \, b^{2} x^{2}}{12 \, c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(abs(c*x^2 + b))/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2)
/c^3
```

maple [A] time = 0.00, size = 46, normalized size = 0.87

$$\frac{x^{6}}{6c}-\frac{b\,x^{4}}{4c^{2}}+\frac{b^{2}x^{2}}{2c^{3}}-\frac{b^{3}\ln\left(c\,x^{2}+b\right)}{2c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*b^2*x^2/c^3-1/4*b*x^4/c^2+1/6*x^6/c-1/2*b^3*ln(c*x^2+b)/c^4
```

maxima [A] time = 1.33, size = 46, normalized size = 0.87

$$-\frac{b^{3} \log \left(cx^{2}+b\right)}{2 \, c^{4}}+\frac{2 \, c^{2} x^{6}-3 \, b c x^{4}+6 \, b^{2} x^{2}}{12 \, c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(c*x^2 + b)/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2)/c^3
```

mupad [B] time = 0.05, size = 45, normalized size = 0.85

$$\frac{x^{6}}{6\,c}-\frac{b\,x^{4}}{4\,c^{2}}-\frac{b^{3}\,\ln\left(c\,x^{2}+b\right)}{2\,c^{4}}+\frac{b^{2}\,x^{2}}{2\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4),x)
```

```
[Out] x^6/(6*c) - (b*x^4)/(4*c^2) - (b^3*log(b + c*x^2))/(2*c^4) + (b^2*x^2)/(2*c
^3)
```

sympy [A] time = 0.18, size = 44, normalized size = 0.83

$$-\frac{b^3 \log \left(b + cx^2\right)}{2c^4} + \frac{b^2x^2}{2c^3} - \frac{bx^4}{4c^2} + \frac{x^6}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*log(b + c*x**2)/(2*c**4) + b**2*x**2/(2*c**3) - b*x**4/(4*c**2) + x**
6/(6*c)
```

3.58 $\int \frac{x^8}{bx^2+cx^4}\,dx$

Optimal. Leaf size=55

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 302, 205}

$$\frac{b^2x}{c^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/c^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^8}{bx^2+cx^4}\,dx = \int \frac{x^6}{b+cx^2}\,dx$$
$$= \int \left(\frac{b^2}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{c}-\frac{b^3}{c^3\left(b+cx^2\right)}\right)dx$$
$$= \frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}-\frac{b^3\int\frac{1}{b+cx^2}\,dx}{c^3}$$
$$= \frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/c^(7/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^8/(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.83, size = 126, normalized size = 2.29

$$\left[\frac{6\,c^2x^5-10\,bcx^3+15\,b^2\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+30\,b^2x}{30\,c^3},\ \frac{3\,c^2x^5-5\,bcx^3-15\,b^2\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+15\,b^2x}{15\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*c^2*x^5 - 10*b*c*x^3 + 15*b^2*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-
b/c) - b)/(c*x^2 + b)) + 30*b^2*x)/c^3, 1/15*(3*c^2*x^5 - 5*b*c*x^3 - 15*b^
2*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*b^2*x)/c^3]
```

giac [A] time = 0.18, size = 55, normalized size = 1.00

$$-\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3}+\frac{3\,c^4x^5-5\,bc^3x^3+15\,b^2c^2x}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^4*x^5 - 5*b*c^3*x^3
+ 15*b^2*c^2*x)/c^5
```

maple [A] time = 0.00, size = 49, normalized size = 0.89

$$\frac{x^5}{5c}-\frac{b\,x^3}{3c^2}-\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^3}+\frac{b^2x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2),x)
```

```
[Out] 1/5*x^5/c-1/3*b*x^3/c^2+b^2*x/c^3-b^3/c^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*
c*x)
```

maxima [A] time = 2.88, size = 50, normalized size = 0.91

$$-\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3}+\frac{3\,c^2x^5-5\,bcx^3+15\,b^2x}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^2*x^5 - 5*b*c*x^3 +
15*b^2*x)/c^3
```

mupad [B] time = 0.05, size = 43, normalized size = 0.78

$$\frac{x^5}{5\,c}-\frac{b\,x^3}{3\,c^2}+\frac{b^2\,x}{c^3}-\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2 + c*x^4),x)
```

```
[Out] x^5/(5*c) - (b*x^3)/(3*c^2) + (b^2*x)/c^3 - (b^(5/2)*atan((c^(1/2)*x)/b^(1/
2)))/c^(7/2)
```

sympy [A] time = 0.21, size = 95, normalized size = 1.73

$$\frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x - \frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2} - \frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x + \frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2} + \frac{x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2),x)
```

```
[Out] b**2*x/c**3 - b*x**3/(3*c**2) + sqrt(-b**5/c**7)*log(x - c**3*sqrt(-b**5/c*
*7)/b**2)/2 - sqrt(-b**5/c**7)*log(x + c**3*sqrt(-b**5/c**7)/b**2)/2 + x**5
/(5*c)
```

3.59 $\int \frac{x^7}{bx^2+cx^4}\,dx$

Optimal. Leaf size=40

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^7}{bx^2+cx^4}\,dx = \int \frac{x^5}{b+cx^2}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{b+cx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{b^2}{c^2(b+cx)}\right)dx, x, x^2\right)$$
$$= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b^2\log\left(b+cx^2\right)}{2c^3}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4),x]
[Out] -1/2*(b*x^2)/c^2 + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x^7/(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.57, size = 33, normalized size = 0.82

$$\frac{c^2x^4-2\,bcx^2+2\,b^2\log\left(cx^2+b\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/4*(c^2*x^4 - 2*b*c*x^2 + 2*b^2*log(c*x^2 + b))/c^3
```

giac [A] time = 0.15, size = 35, normalized size = 0.88

$$\frac{b^2 \log\left(\left|cx^2 + b\right|\right)}{2\,c^3} + \frac{cx^4 - 2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/2*b^2*log(abs(c*x^2 + b))/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.88

$$\frac{x^4}{4c} - \frac{b\,x^2}{2c^2} + \frac{b^2 \ln\left(c\,x^2 + b\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2),x)
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/2*b^2*ln(c*x^2+b)/c^3
```

maxima [A] time = 1.33, size = 34, normalized size = 0.85

$$\frac{b^2 \log\left(cx^2 + b\right)}{2\,c^3} + \frac{cx^4 - 2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*b^2*log(c*x^2 + b)/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

mupad [B] time = 0.05, size = 33, normalized size = 0.82

$$\frac{2\,b^2\,\ln\left(c\,x^2 + b\right) + c^2\,x^4 - 2\,b\,c\,x^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4),x)
[Out] (2*b^2*log(b + c*x^2) + c^2*x^4 - 2*b*c*x^2)/(4*c^3)
```

sympy [A] time = 0.17, size = 32, normalized size = 0.80

$$\frac{b^2 \log\left(b + cx^2\right)}{2c^3} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2),x)
```

```
[Out] b**2*log(b + c*x**2)/(2*c**3) - b*x**2/(2*c**2) + x**4/(4*c)
```

3.60 $\int \frac{x^6}{bx^2+cx^4} dx$

Optimal. Leaf size=42

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 302, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^6}{bx^2+cx^4}\,dx = \int \frac{x^4}{b+cx^2}\,dx$$
$$= \int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right)dx$$
$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^2\int \frac{1}{b+cx^2}\,dx}{c^2}$$
$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}$$

Mathematica [**A**] time = 0.02, size = 42, normalized size = 1.00

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4),x]
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x^6/(b*x^2 + c*x^4), x]
```

fricas [**A**] time = 0.60, size = 99, normalized size = 2.36

$$\left[\frac{2\,cx^3+3\,b\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-6\,bx}{6\,c^2},\ \frac{cx^3+3\,b\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-3\,bx}{3\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*x^3 + 3*b*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
b)) - 6*b*x)/c^2, 1/3*(c*x^3 + 3*b*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*b*
x)/c^2]
```

giac [A] time = 0.17, size = 40, normalized size = 0.95

$$\frac{b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, c^2} + \frac{c^2x^3 - 3\, bcx}{3\, c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

maple [A] time = 0.00, size = 38, normalized size = 0.90

$$\frac{x^3}{3c} + \frac{b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\; c^2} - \frac{bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2),x)
```

```
[Out] 1/3*x^3/c-b*x/c^2+b^2/c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.95, size = 37, normalized size = 0.88

$$\frac{b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, c^2} + \frac{cx^3 - 3\, bx}{3\, c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c*x^3 - 3*b*x)/c^2
```

mupad [B] time = 0.05, size = 32, normalized size = 0.76

$$\frac{x^3}{3\, c} + \frac{b^{3/2}\, \mathrm{atan}\left(\frac{\sqrt{c}\, x}{\sqrt{b}}\right)}{c^{5/2}} - \frac{b\, x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4),x)
```

```
[Out] x^3/(3*c) + (b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/c^(5/2) - (b*x)/c^2
```

sympy [B] time = 0.19, size = 80, normalized size = 1.90

$$-\frac{bx}{c^2} - \frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x - \frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2} + \frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x + \frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2} + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2),x)
```

```
[Out] -b*x/c**2 - sqrt(-b**3/c**5)*log(x - c**2*sqrt(-b**3/c**5)/b)/2 + sqrt(-b**
3/c**5)*log(x + c**2*sqrt(-b**3/c**5)/b)/2 + x**3/(3*c)
```

3.61 $\int \frac{x^5}{bx^2+cx^4}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2c} - \frac{b\log\left(b+cx^2\right)}{2c^2}$$

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{x^2}{2c} - \frac{b\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{bx^2+cx^4}\,dx &= \int \frac{x^3}{b+cx^2}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{b+cx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{c}-\frac{b}{c(b+cx)}\right)dx, x, x^2\right)\\
&= \frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4),x]
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x^5/(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.61, size = 22, normalized size = 0.81

$$\frac{cx^2-b\log\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*(c*x^2 - b*log(c*x^2 + b))/c^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,c} - \frac{b\log\left(\left|cx^2 + b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/2*x^2/c - 1/2*b*log(abs(c*x^2 + b))/c^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^2}{2c} - \frac{b\ln\left(c\,x^2 + b\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2),x)
[Out] 1/2*x^2/c-1/2*b*ln(c*x^2+b)/c^2
```

maxima [A] time = 1.31, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,c} - \frac{b\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*x^2/c - 1/2*b*log(c*x^2 + b)/c^2
```

mupad [B] time = 0.04, size = 22, normalized size = 0.81

$$-\frac{b\,\ln\left(c\,x^2 + b\right) - c\,x^2}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4),x)
[Out] -(b*log(b + c*x^2) - c*x^2)/(2*c^2)
```

sympy [A] time = 0.17, size = 20, normalized size = 0.74

$$-\frac{b\log\left(b + cx^2\right)}{2c^2} + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2),x)
```

```
[Out] -b*log(b + c*x**2)/(2*c**2) + x**2/(2*c)
```

3.62 $\int \frac{x^4}{bx^2+cx^4} dx$

Optimal. Leaf size=31

$$\frac{x}{c} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 321, 205}

$$\frac{x}{c} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^4}{bx^2+cx^4}\,dx = \int \frac{x^2}{b+cx^2}\,dx$$
$$= \frac{x}{c} - \frac{b\int \frac{1}{b+cx^2}\,dx}{c}$$
$$= \frac{x}{c} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{c} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^4/(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.62, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+2\,x}{2\,c}, -\frac{\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-x}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 2*x)/c,
-(sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - x)/c]
```

giac [A] time = 0.17, size = 26, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

maple [A] time = 0.00, size = 27, normalized size = 0.87

$$-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2),x)
```

```
[Out] x/c-b/c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.97, size = 26, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

mupad [B] time = 0.04, size = 23, normalized size = 0.74

$$\frac{x}{c}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4),x)
```

[Out] x/c - (b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/c^(3/2)

sympy [B] time = 0.17, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{b}{c^3}}\log\left(-c\sqrt{-\frac{b}{c^3}}+x\right)}{2}-\frac{\sqrt{-\frac{b}{c^3}}\log\left(c\sqrt{-\frac{b}{c^3}}+x\right)}{2}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**4+b*x**2),x)

[Out] sqrt(-b/c**3)*log(-c*sqrt(-b/c**3) + x)/2 - sqrt(-b/c**3)*log(c*sqrt(-b/c**3) + x)/2 + x/c

3.63 $\int \frac{x^3}{bx^2+cx^4}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^2)}{2c}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 260}

$$\frac{\log(b+cx^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] Log[b + c*x^2]/(2*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{bx^2+cx^4}\,dx &= \int \frac{x}{b+cx^2}\,dx\\ &= \frac{\log(b+cx^2)}{2c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] Log[b + c*x^2]/(2*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{bx^2 + cx^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^3/(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.61, size = 13, normalized size = 0.87

$$\frac{\log\left(cx^2 + b\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|cx^2 + b\right|\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^2 + b))/c
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(c\,x^2 + b\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*ln(c*x^2+b)/c
```

maxima [A] time = 1.35, size = 13, normalized size = 0.87

$$\frac{\log\left(cx^2+b\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^2+b\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4),x)
```

```
[Out] log(b + c*x^2)/(2*c)
```

sympy [A] time = 0.13, size = 10, normalized size = 0.67

$$\frac{\log\left(b+cx^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2),x)
```

```
[Out] log(b + c*x**2)/(2*c)
```

3.64 $\int \frac{x^2}{bx^2+cx^4}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{bx^2+cx^4}\,dx &= \int \frac{1}{b+cx^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{bx^2 + cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.62, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{2\,bc},\ \frac{\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b))/(b*c), sqrt(
b*c)*arctan(sqrt(b*c)*x/b)/(b*c)]
```

giac [A] time = 0.15, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

maple [A]   time = 0.00, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2),x)
```

```
[Out] 1/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A]   time = 2.97, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

mupad [B]   time = 4.20, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4),x)
```

```
[Out] atan((c^(1/2)*x)/b^(1/2))/(b^(1/2)*c^(1/2))
```

sympy [B]   time = 0.15, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{bc}}\log\left(-b\sqrt{-\frac{1}{bc}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{bc}}\log\left(b\sqrt{-\frac{1}{bc}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-1/(b*c))*log(-b*sqrt(-1/(b*c)) + x)/2 + sqrt(-1/(b*c))*log(b*sqrt(-1
/(b*c)) + x)/2
```

3.65 $\int \frac{x}{bx^2+cx^4} dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b} - \frac{\log\left(b + cx^2\right)}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1584, 266, 36, 29, 31}

$$\frac{\log(x)}{b} - \frac{\log\left(b + cx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{bx^2+cx^4}\,dx &= \int \frac{1}{x\left(b+cx^2\right)}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(b+cx)}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2b} - \frac{c\,\text{Subst}\left(\int \frac{1}{b+cx}\,dx, x, x^2\right)}{2b}\\ &= \frac{\log(x)}{b} - \frac{\log\left(b+cx^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{b} - \frac{\log\left(b+cx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x/(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.62, size = 18, normalized size = 0.82

$$-\frac{\log\left(cx^2+b\right) - 2\,\log(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(log(c*x^2 + b) - 2*log(x))/b
```

giac [A] time = 0.17, size = 22, normalized size = 1.00

$$-\frac{\log\left(\left|cx^2+b\right|\right)}{2\,b}+\frac{\log\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="giac")
[Out] -1/2*log(abs(c*x^2 + b))/b + log(abs(x))/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{b}-\frac{\ln\left(c\,x^2+b\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2),x)
[Out] ln(x)/b-1/2*ln(c*x^2+b)/b
```

maxima [A] time = 1.36, size = 23, normalized size = 1.05

$$-\frac{\log\left(cx^2+b\right)}{2\,b}+\frac{\log\left(x^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] -1/2*log(c*x^2 + b)/b + 1/2*log(x^2)/b
```

mupad [B] time = 0.06, size = 18, normalized size = 0.82

$$-\frac{\ln\left(c\,x^2+b\right)-2\,\ln(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4),x)
[Out] -(log(b + c*x^2) - 2*log(x))/(2*b)
```

sympy [A] time = 0.23, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{b}{c}+x^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2),x)
```

```
[Out] log(x)/b - log(b/c + x**2)/(2*b)
```

3.66 $\int \frac{1}{bx^2+cx^4} \, dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 325, 205}

$$-\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-1),x]
```

```
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{bx^2+cx^4}\,dx = \int \frac{1}{x^2\left(b+cx^2\right)}\,dx$$
$$= -\frac{1}{bx} - \frac{c\int \frac{1}{b+cx^2}\,dx}{b}$$
$$= -\frac{1}{bx} - \frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}} - \frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-1),x]
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(-1),x]
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^(-1), x]
```

fricas [A] time = 0.60, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-2}{2\,bx}, -\frac{x\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+1}{bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 2)/(b*
x), -(x*sqrt(c/b)*arctan(x*sqrt(c/b)) + 1)/(b*x)]
```

giac [A] time = 0.16, size = 29, normalized size = 0.85

$$-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="giac")
[Out] -c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

maple [A] time = 0.00, size = 30, normalized size = 0.88

$$-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2),x)
[Out] -c/b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/b/x
```

maxima [A] time = 2.88, size = 29, normalized size = 0.85

$$-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] -c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

mupad [B] time = 4.27, size = 26, normalized size = 0.76

$$-\frac{1}{b\,x}-\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4),x)
```

```
[Out] - 1/(b*x) - (c^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/b^(3/2)
```

sympy [B] time = 0.19, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{c}{b^3}}\log\left(-\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{\sqrt{-\frac{c}{b^3}}\log\left(\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c/b**3)*log(-b**2*sqrt(-c/b**3)/c + x)/2 - sqrt(-c/b**3)*log(b**2*sqr
t(-c/b**3)/c + x)/2 - 1/(b*x)
```

3.67 $\int \frac{1}{x(bx^2+cx^4)}\,dx$

Optimal. Leaf size=35

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)),x]

[Out] -1/(2*b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^3\left(b+cx^2\right)}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(b+cx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bx^2}-\frac{c}{b^2x}+\frac{c^2}{b^2(b+cx)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2bx^2}-\frac{c\log(x)}{b^2}+\frac{c\log\left(b+cx^2\right)}{2b^2}$$

Mathematica [A]   time = 0.01, size = 35, normalized size = 1.00

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)),x]
[Out] -1/2*1/(b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(b*x^2 + c*x^4)),x]
[Out] IntegrateAlgebraic[1/(x*(b*x^2 + c*x^4)), x]
```

fricas [A]   time = 0.66, size = 33, normalized size = 0.94

$$\frac{cx^2\log\left(cx^2+b\right)-2\,cx^2\log(x)-b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*(c*x^2*log(c*x^2 + b) - 2*c*x^2*log(x) - b)/(b^2*x^2)
```

giac [A] time = 0.15, size = 43, normalized size = 1.23

$$-\frac{c\log\left(x^2\right)}{2\,b^2}+\frac{c\log\left(\left|cx^2+b\right|\right)}{2\,b^2}+\frac{cx^2-b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*c*log(x^2)/b^2 + 1/2*c*log(abs(c*x^2 + b))/b^2 + 1/2*(c*x^2 - b)/(b^2*
x^2)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{c\ln(x)}{b^2}+\frac{c\ln\left(c\,x^2+b\right)}{2b^2}-\frac{1}{2b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2),x)
```

```
[Out] -1/2/b/x^2-c*ln(x)/b^2+1/2*c*ln(c*x^2+b)/b^2
```

maxima [A] time = 1.37, size = 33, normalized size = 0.94

$$\frac{c\log\left(cx^2+b\right)}{2\,b^2}-\frac{c\log\left(x^2\right)}{2\,b^2}-\frac{1}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(c*x^2 + b)/b^2 - 1/2*c*log(x^2)/b^2 - 1/2/(b*x^2)
```

mupad [B] time = 0.06, size = 31, normalized size = 0.89

$$\frac{c\,\ln\left(c\,x^2+b\right)}{2\,b^2}-\frac{1}{2\,b\,x^2}-\frac{c\,\ln(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)),x)
```

```
[Out] (c*log(b + c*x^2))/(2*b^2) - 1/(2*b*x^2) - (c*log(x))/b^2
```

sympy [A] time = 0.28, size = 31, normalized size = 0.89

$$-\frac{1}{2bx^2}-\frac{c\log\left(x\right)}{b^2}+\frac{c\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2),x)
```

```
[Out] -1/(2*b*x**2) - c*log(x)/b**2 + c*log(b/c + x**2)/(2*b**2)
```

3.68 $\int \frac{1}{x^2\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=43

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 325, 205}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^2(bx^2+cx^4)}\,dx = \int \frac{1}{x^4(b+cx^2)}\,dx$$
$$= -\frac{1}{3bx^3} - \frac{c\int \frac{1}{x^2(b+cx^2)}\,dx}{b}$$
$$= -\frac{1}{3bx^3} + \frac{c}{b^2x} + \frac{c^2\int \frac{1}{b+cx^2}\,dx}{b^2}$$
$$= -\frac{1}{3bx^3} + \frac{c}{b^2x} + \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}} + \frac{c}{b^2x} - \frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)),x]
[Out] -1/3*1/(b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(bx^2+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(b*x^2 + c*x^4)),x]
[Out] IntegrateAlgebraic[1/(x^2*(b*x^2 + c*x^4)), x]
```

fricas [A] time = 0.56, size = 106, normalized size = 2.47

$$\left[\frac{3\,cx^3\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+6\,cx^2-2\,b}{6\,b^2x^3}, \frac{3\,cx^3\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,cx^2-b}{3\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*c*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) +
6*c*x^2 - 2*b)/(b^2*x^3), 1/3*(3*c*x^3*sqrt(c/b)*arctan(x*sqrt(c/b)) + 3*c*
x^2 - b)/(b^2*x^3)]
```

giac [A] time = 0.17, size = 40, normalized size = 0.93

$$\frac{c^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, b^2} + \frac{3\, cx^2 - b}{3\, b^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^2*x^3)
```

maple [A] time = 0.01, size = 39, normalized size = 0.91

$$\frac{c^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\; b^2} + \frac{c}{b^2 x} - \frac{1}{3b\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2),x)
```

```
[Out] c^2/b^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/3/b/x^3+c/b^2/x
```

maxima [A] time = 2.96, size = 40, normalized size = 0.93

$$\frac{c^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, b^2} + \frac{3\, cx^2 - b}{3\, b^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^2*x^3)
```

mupad [B] time = 4.14, size = 37, normalized size = 0.86

$$\frac{c^{3/2} \operatorname{atan}\left(\frac{\sqrt{c}\, x}{\sqrt{b}}\right)}{b^{5/2}} - \frac{\frac{1}{3\, b} - \frac{c\, x^2}{b^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/b^(5/2) - (1/(3*b) - (c*x^2)/b^2)/x^3
```

sympy [B] time = 0.25, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{c^3}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2}+x\right)}{2}+\frac{\sqrt{-\frac{c^3}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2}+x\right)}{2}+\frac{-b+3cx^2}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-c**3/b**5)*log(-b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + sqrt(-c**3/b**5)
*log(b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + (-b + 3*c*x**2)/(3*b**2*x**3)
```

3.69 $\int \frac{1}{x^3(bx^2+cx^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{c^2\log\left(b+cx^2\right)}{2b^3}+\frac{c^2\log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{c^2\log\left(b+cx^2\right)}{2b^3}+\frac{c^2\log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(2*b
^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^3 \left(bx^2 + cx^4\right)} \, dx = \int \frac{1}{x^5 \left(b + cx^2\right)} \, dx$$
$$= \frac{1}{2} \operatorname{Subst}\left(\int \frac{1}{x^3(b + cx)} \, dx, x, x^2\right)$$
$$= \frac{1}{2} \operatorname{Subst}\left(\int \left(\frac{1}{bx^3} - \frac{c}{b^2x^2} + \frac{c^2}{b^3x} - \frac{c^3}{b^3(b + cx)}\right) dx, x, x^2\right)$$
$$= -\frac{1}{4bx^4} + \frac{c}{2b^2x^2} + \frac{c^2 \log(x)}{b^3} - \frac{c^2 \log\left(b + cx^2\right)}{2b^3}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{c^2 \log\left(b + cx^2\right)}{2b^3} + \frac{c^2 \log(x)}{b^3} + \frac{c}{2b^2x^2} - \frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^2 + c*x^4)),x]
[Out] -1/4*1/(b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(2
*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(bx^2 + cx^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(b*x^2 + c*x^4)),x]
[Out] IntegrateAlgebraic[1/(x^3*(b*x^2 + c*x^4)), x]
```

fricas [A] time = 1.08, size = 45, normalized size = 0.92

$$-\frac{2\,c^2x^4 \log\left(cx^2 + b\right) - 4\,c^2x^4 \log(x) - 2\,bcx^2 + b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*c^2*x^4*log(c*x^2 + b) - 4*c^2*x^4*log(x) - 2*b*c*x^2 + b^2)/(b^3*x
^4)
```

giac [A] time = 0.17, size = 57, normalized size = 1.16

$$\frac{c^2 \log\left(x^2\right)}{2\,b^3} - \frac{c^2 \log\left(\left|cx^2 + b\right|\right)}{2\,b^3} - \frac{3\,c^2x^4 - 2\,bcx^2 + b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2)/b^3 - 1/2*c^2*log(abs(c*x^2 + b))/b^3 - 1/4*(3*c^2*x^4 - 2
*b*c*x^2 + b^2)/(b^3*x^4)
```

maple [A] time = 0.01, size = 44, normalized size = 0.90

$$\frac{c^2 \ln(x)}{b^3} - \frac{c^2 \ln\left(c\,x^2 + b\right)}{2b^3} + \frac{c}{2b^2x^2} - \frac{1}{4b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2),x)
```

```
[Out] -1/4/b/x^4+1/2*c/b^2/x^2+c^2*ln(x)/b^3-1/2*c^2*ln(c*x^2+b)/b^3
```

maxima [A] time = 1.32, size = 47, normalized size = 0.96

$$-\frac{c^2 \log\left(cx^2 + b\right)}{2\,b^3} + \frac{c^2 \log\left(x^2\right)}{2\,b^3} + \frac{2\,cx^2 - b}{4\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*log(c*x^2 + b)/b^3 + 1/2*c^2*log(x^2)/b^3 + 1/4*(2*c*x^2 - b)/(b^2
*x^4)
```

mupad [B] time = 0.06, size = 46, normalized size = 0.94

$$\frac{c^2\,\ln(x)}{b^3} - \frac{c^2\,\ln\left(c\,x^2 + b\right)}{2\,b^3} - \frac{\frac{1}{4\,b} - \frac{c\,x^2}{2\,b^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^2*log(x))/b^3 - (c^2*log(b + c*x^2))/(2*b^3) - (1/(4*b) - (c*x^2)/(2*b^2
))/x^4
```

sympy [**A**] time = 0.36, size = 42, normalized size = 0.86

$$\frac{-b + 2cx^2}{4b^2x^4} + \frac{c^2 \log(x)}{b^3} - \frac{c^2 \log\left(\frac{b}{c} + x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2),x)
```

```
[Out] (-b + 2*c*x**2)/(4*b**2*x**4) + c**2*log(x)/b**3 - c**2*log(b/c + x**2)/(2*
b**3)
```

3.70 $\int \frac{1}{x^4\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=58

$$-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{c^2}{b^3x}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 325, 205}

$$-\frac{c^2}{b^3x}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(5*b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/b^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(bx^2+cx^4\right)}\,dx &= \int \frac{1}{x^6\left(b+cx^2\right)}\,dx\\ &= -\frac{1}{5bx^5}-\frac{c\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{b}\\ &= -\frac{1}{5bx^5}+\frac{c}{3b^2x^3}+\frac{c^2\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{b^2}\\ &= -\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^3\int \frac{1}{b+cx^2}\,dx}{b^3}\\ &= -\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.00

$$-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{c^2}{b^3x}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^2 + c*x^4)),x]
[Out] -1/5*1/(b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)/
Sqrt[b]])/b^(7/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(b*x^2 + c*x^4)),x]
[Out] IntegrateAlgebraic[1/(x^4*(b*x^2 + c*x^4)), x]
```

fricas [A] time = 0.58, size = 132, normalized size = 2.28

$$\left[\frac{15\,c^2x^5\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-30\,c^2x^4+10\,bcx^2-6\,b^2}{30\,b^3x^5},\,-\frac{15\,c^2x^5\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+15\,c^2x^4-5\,bcx^2+3\,b^2}{15\,b^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*c^2*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)
) - 30*c^2*x^4 + 10*b*c*x^2 - 6*b^2)/(b^3*x^5), -1/15*(15*c^2*x^5*sqrt(c/b)
*arctan(x*sqrt(c/b)) + 15*c^2*x^4 - 5*b*c*x^2 + 3*b^2)/(b^3*x^5)]
```

giac [A] time = 0.17, size = 52, normalized size = 0.90

$$-\frac{c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, b^3} - \frac{15\, c^2 x^4 - 5\, bcx^2 + 3\, b^2}{15\, b^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5*b*c*x^2 +
 3*b^2)/(b^3*x^5)
```

maple [A] time = 0.01, size = 52, normalized size = 0.90

$$-\frac{c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\; b^3} - \frac{c^2}{b^3 x} + \frac{c}{3b^2 x^3} - \frac{1}{5b\, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2),x)
```

```
[Out] -c^3/b^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/5/b/x^5-c^2/b^3/x+1/3*c/b^
2/x^3
```

maxima [A] time = 2.89, size = 52, normalized size = 0.90

$$-\frac{c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, b^3} - \frac{15\, c^2 x^4 - 5\, bcx^2 + 3\, b^2}{15\, b^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5*b*c*x^2 +
 3*b^2)/(b^3*x^5)
```

mupad [B] time = 0.05, size = 48, normalized size = 0.83

$$-\frac{\frac{1}{5\,b}-\frac{c\,x^2}{3\,b^2}+\frac{c^2\,x^4}{b^3}}{x^5}-\frac{c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^2 + c*x^4)),x)
```

```
[Out] - (1/(5*b) - (c*x^2)/(3*b^2) + (c^2*x^4)/b^3)/x^5 - (c^(5/2)*atan((c^(1/2)*
x)/b^(1/2)))/b^(7/2)
```

sympy [B] time = 0.28, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{c^5}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c^5}{b^7}}}{c^3}+x\right)}{2}-\frac{\sqrt{-\frac{c^5}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c^5}{b^7}}}{c^3}+x\right)}{2}+\frac{-3b^2+5bcx^2-15c^2x^4}{15b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c**5/b**7)*log(-b**4*sqrt(-c**5/b**7)/c**3 + x)/2 - sqrt(-c**5/b**7)*
log(b**4*sqrt(-c**5/b**7)/c**3 + x)/2 + (-3*b**2 + 5*b*c*x**2 - 15*c**2*x**
4)/(15*b**3*x**5)
```

3.71 $\int \frac{1}{x^5(bx^2+cx^4)}\,dx$

Optimal. Leaf size=63

$$\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}-\frac{c^2}{2b^3x^2}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{c^2}{2b^3x^2}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(6*b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4 + (c^3*Lo
g[b + c*x^2])/(2*b^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^5 \left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^7 \left(b+cx^2\right)}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4(b+cx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bx^4}-\frac{c}{b^2x^3}+\frac{c^2}{b^3x^2}-\frac{c^3}{b^4x}+\frac{c^4}{b^4(b+cx)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{6bx^6}+\frac{c}{4b^2x^4}-\frac{c^2}{2b^3x^2}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.00

$$\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}-\frac{c^2}{2b^3x^2}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*1/(b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4 + (c^3*
Log[b + c*x^2])/(2*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(b*x^2 + c*x^4)), x]
```

fricas [A] time = 0.74, size = 58, normalized size = 0.92

$$\frac{6\,c^3x^6\log\left(cx^2+b\right)-12\,c^3x^6\log(x)-6\,bc^2x^4+3\,b^2cx^2-2\,b^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*c^3*x^6*log(c*x^2 + b) - 12*c^3*x^6*log(x) - 6*b*c^2*x^4 + 3*b^2*c*
x^2 - 2*b^3)/(b^4*x^6)
```

giac [A] time = 0.15, size = 70, normalized size = 1.11

$$-\frac{c^3 \log\left(x^2\right)}{2\, b^4} + \frac{c^3 \log\left(\left|cx^2 + b\right|\right)}{2\, b^4} + \frac{11\, c^3 x^6 - 6\, bc^2 x^4 + 3\, b^2 c x^2 - 2\, b^3}{12\, b^4 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*c^3*log(x^2)/b^4 + 1/2*c^3*log(abs(c*x^2 + b))/b^4 + 1/12*(11*c^3*x^6
- 6*b*c^2*x^4 + 3*b^2*c*x^2 - 2*b^3)/(b^4*x^6)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$-\frac{c^3 \ln(x)}{b^4} + \frac{c^3 \ln\left(c\, x^2 + b\right)}{2 b^4} - \frac{c^2}{2 b^3 x^2} + \frac{c}{4 b^2 x^4} - \frac{1}{6 b\, x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2),x)
```

```
[Out] -1/6/b/x^6+1/4*c/b^2/x^4-1/2*c^2/b^3/x^2-c^3*ln(x)/b^4+1/2*c^3*ln(c*x^2+b)/
b^4
```

maxima [A] time = 1.32, size = 58, normalized size = 0.92

$$\frac{c^3 \log\left(cx^2 + b\right)}{2\, b^4} - \frac{c^3 \log\left(x^2\right)}{2\, b^4} - \frac{6\, c^2 x^4 - 3\, bcx^2 + 2\, b^2}{12\, b^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(c*x^2 + b)/b^4 - 1/2*c^3*log(x^2)/b^4 - 1/12*(6*c^2*x^4 - 3*b*c
*x^2 + 2*b^2)/(b^3*x^6)
```

mupad [B] time = 0.07, size = 58, normalized size = 0.92

$$\frac{c^3\, \ln\left(c\, x^2 + b\right)}{2\, b^4} - \frac{\frac{1}{6\, b} - \frac{c\, x^2}{4\, b^2} + \frac{c^2\, x^4}{2\, b^3}}{x^6} - \frac{c^3\, \ln(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^3*log(b + c*x^2))/(2*b^4) - (1/(6*b) - (c*x^2)/(4*b^2) + (c^2*x^4)/(2*b^
3))/x^6 - (c^3*log(x))/b^4
```

sympy [A] time = 0.40, size = 56, normalized size = 0.89

$$\frac{-2b^2 + 3bcx^2 - 6c^2x^4}{12b^3x^6} - \frac{c^3 \log{(x)}}{b^4} + \frac{c^3 \log{\left(\frac{b}{c} + x^2 \right)}}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2),x)
```

```
[Out] (-2*b**2 + 3*b*c*x**2 - 6*c**2*x**4)/(12*b**3*x**6) - c**3*log(x)/b**4 + c*
*3*log(b/c + x**2)/(2*b**4)
```

3.72 $\int \frac{x^{12}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}+\frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}-\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7x^5}{10c^2}$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 302, 205}

$$\frac{7b^2x}{2c^4}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}-\frac{7bx^3}{6c^3}-\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7x^5}{10c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (7*b^2*x)/(2*c^4) - (7*b*x^3)/(6*c^3) + (7*x^5)/(10*c^2) - x^7/(2*c*(b + c*
x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^8}{\left(b+cx^2\right)^2}\,dx\\
&= -\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7\int \frac{x^6}{b+cx^2}\,dx}{2c}\\
&= -\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7\int \left(\frac{b^2}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{c}-\frac{b^3}{c^3\left(b+cx^2\right)}\right)dx}{2c}\\
&= \frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}+\frac{7x^5}{10c^2}-\frac{x^7}{2c\left(b+cx^2\right)}-\frac{\left(7b^3\right)\int \frac{1}{b+cx^2}\,dx}{2c^4}\\
&= \frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}+\frac{7x^5}{10c^2}-\frac{x^7}{2c\left(b+cx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.90

$$\frac{x\left(\frac{15b^3}{b+cx^2}+90b^2-20bcx^2+6c^2x^4\right)}{30c^4}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(90*b^2 - 20*b*c*x^2 + 6*c^2*x^4 + (15*b^3)/(b + c*x^2)))/(30*c^4) - (7*
b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^12/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.53, size = 190, normalized size = 2.41

$$\left[\frac{12\,c^{3}x^{7}-28\,bc^{2}x^{5}+140\,b^{2}cx^{3}+210\,b^{3}x+105\,\left(b^{2}cx^{2}+b^{3}\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^{2}-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^{2}+b}\right)}{60\,\left(c^{5}x^{2}+bc^{4}\right)},\ \frac{6\,c^{3}x^{7}-14\,bc^{2}x^{5}+70\,b^{2}cx^{3}+105\,b^{3}x-105\,\left(b^{2}cx^{2}+b^{3}\right)\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)}{30\,\left(c^{5}x^{2}+bc^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^3*x^7 - 28*b*c^2*x^5 + 140*b^2*c*x^3 + 210*b^3*x + 105*(b^2*c*x
^2 + b^3)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^5*
x^2 + b*c^4), 1/30*(6*c^3*x^7 - 14*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*x - 1
05*(b^2*c*x^2 + b^3)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^2 + b*c^4)]
```

giac [A] time = 0.16, size = 73, normalized size = 0.92

$$-\frac{7\,b^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^{4}}+\frac{b^{3}x}{2\,\left(cx^{2}+b\right)c^{4}}+\frac{3\,c^{8}x^{5}-10\,bc^{7}x^{3}+45\,b^{2}c^{6}x}{15\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*b^3*x/((c*x^2 + b)*c^4
) + 1/15*(3*c^8*x^5 - 10*b*c^7*x^3 + 45*b^2*c^6*x)/c^10
```

maple [A] time = 0.01, size = 68, normalized size = 0.86

$$\frac{x^{5}}{5c^{2}}-\frac{2b\,x^{3}}{3c^{3}}+\frac{b^{3}x}{2\left(c\,x^{2}+b\right)c^{4}}-\frac{7b^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^{4}}+\frac{3b^{2}x}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*x^5/c^2-2/3*b*x^3/c^3+3*b^2*x/c^4+1/2/c^4*b^3*x/(c*x^2+b)-7/2/c^4*b^3/(
b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.91, size = 71, normalized size = 0.90

$$\frac{b^{3}x}{2\,\left(c^{5}x^{2}+bc^{4}\right)}-\frac{7\,b^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^{4}}+\frac{3\,c^{2}x^{5}-10\,bcx^{3}+45\,b^{2}x}{15\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x/(c^5*x^2 + b*c^4) - 7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4)
 + 1/15*(3*c^2*x^5 - 10*b*c*x^3 + 45*b^2*x)/c^4
```

mupad [B] time = 0.04, size = 66, normalized size = 0.84

$$\frac{x^5}{5\,c^2}-\frac{2\,b\,x^3}{3\,c^3}+\frac{3\,b^2\,x}{c^4}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{9/2}}+\frac{b^3\,x}{2\,\left(c^5\,x^2+b\,c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5/(5*c^2) - (2*b*x^3)/(3*c^3) + (3*b^2*x)/c^4 - (7*b^(5/2)*atan((c^(1/2)*
x)/b^(1/2)))/(2*c^(9/2)) + (b^3*x)/(2*(b*c^4 + c^5*x^2))
```

sympy [A] time = 0.36, size = 124, normalized size = 1.57

$$\frac{b^3x}{2bc^4+2c^5x^2}+\frac{3b^2x}{c^4}-\frac{2bx^3}{3c^3}+\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}-\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}+\frac{x^5}{5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3*x/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*x/c**4 - 2*b*x**3/(3*c**3) + 7*sqr
t(-b**5/c**9)*log(x - c**4*sqrt(-b**5/c**9)/b**2)/4 - 7*sqrt(-b**5/c**9)*lo
g(x + c**4*sqrt(-b**5/c**9)/b**2)/4 + x**5/(5*c**2)
```

3.73 $\int \frac{x^{11}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=57

$$\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}-\frac{bx^2}{c^3}+\frac{x^4}{4c^2}$$

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}-\frac{bx^2}{c^3}+\frac{x^4}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*x^2)/c^3) + x^4/(4*c^2) + b^3/(2*c^4*(b + c*x^2)) + (3*b^2*Log[b + c*x
^2])/(2*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^7}{\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{2b}{c^3}+\frac{x}{c^2}-\frac{b^3}{c^3(b+cx)^2}+\frac{3b^2}{c^3(b+cx)}\right)dx, x, x^2\right)$$
$$= -\frac{bx^2}{c^3}+\frac{x^4}{4c^2}+\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{\frac{2b^3}{b+cx^2}+6b^2\log\left(b+cx^2\right)-4bcx^2+c^2x^4}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-4*b*c*x^2 + c^2*x^4 + (2*b^3)/(b + c*x^2) + 6*b^2*Log[b + c*x^2])/(4*c^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^11/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.53, size = 70, normalized size = 1.23

$$\frac{c^3x^6-3\,bc^2x^4-4\,b^2cx^2+2\,b^3+6\left(b^2cx^2+b^3\right)\log\left(cx^2+b\right)}{4\left(c^5x^2+bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*x^6 - 3*b*c^2*x^4 - 4*b^2*c*x^2 + 2*b^3 + 6*(b^2*c*x^2 + b^3)*log(
c*x^2 + b))/(c^5*x^2 + b*c^4)
```

giac [A] time = 0.18, size = 67, normalized size = 1.18

$$\frac{3\,b^2\log\left(\left|cx^2+b\right|\right)}{2\,c^4}+\frac{c^2x^4-4\,bcx^2}{4\,c^4}-\frac{3\,b^2cx^2+2\,b^3}{2\left(cx^2+b\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(abs(c*x^2 + b))/c^4 + 1/4*(c^2*x^4 - 4*b*c*x^2)/c^4 - 1/2*(3*b^
2*c*x^2 + 2*b^3)/((c*x^2 + b)*c^4)
```

maple [A] time = 0.01, size = 52, normalized size = 0.91

$$\frac{x^4}{4c^2}-\frac{b\,x^2}{c^3}+\frac{b^3}{2\left(c\,x^2+b\right)c^4}+\frac{3b^2\ln\left(c\,x^2+b\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2)^2,x)
```

```
[Out] -b*x^2/c^3+1/4*x^4/c^2+1/2*b^3/c^4/(c*x^2+b)+3/2*b^2*ln(c*x^2+b)/c^4
```

maxima [A] time = 1.32, size = 54, normalized size = 0.95

$$\frac{b^3}{2\left(c^5x^2+bc^4\right)}+\frac{3\,b^2\log\left(cx^2+b\right)}{2\,c^4}+\frac{cx^4-4\,bx^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3/(c^5*x^2 + b*c^4) + 3/2*b^2*log(c*x^2 + b)/c^4 + 1/4*(c*x^4 - 4*b*x
^2)/c^3
```

mupad [B] time = 4.14, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,c^2}+\frac{b^3}{2\,c\,\left(c^4\,x^2+b\,c^3\right)}-\frac{b\,x^2}{c^3}+\frac{3\,b^2\,\ln\left(c\,x^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^4/(4*c^2) + b^3/(2*c*(b*c^3 + c^4*x^2)) - (b*x^2)/c^3 + (3*b^2*log(b + c*
x^2))/(2*c^4)
```

sympy [A] time = 0.30, size = 53, normalized size = 0.93

$$\frac{b^3}{2bc^4 + 2c^5x^2} + \frac{3b^2\log\left(b + cx^2\right)}{2c^4} - \frac{bx^2}{c^3} + \frac{x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*log(b + c*x**2)/(2*c**4) - b*x**2/c*
*3 + x**4/(4*c**2)
```

3.74 $\int \frac{x^{10}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{5bx}{2c^3}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5x^3}{6c^2}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 302, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{5bx}{2c^3}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5x^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-5*b*x)/(2*c^3) + (5*x^3)/(6*c^2) - x^5/(2*c*(b + c*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^6}{\left(b+cx^2\right)^2}\,dx\\ &= -\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5\int \frac{x^4}{b+cx^2}\,dx}{2c}\\ &= -\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right)dx}{2c}\\ &= -\frac{5bx}{2c^3}+\frac{5x^3}{6c^2}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{\left(5b^2\right)\int \frac{1}{b+cx^2}\,dx}{2c^3}\\ &= -\frac{5bx}{2c^3}+\frac{5x^3}{6c^2}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.91

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}+\frac{x\left(-\frac{3b^2}{b+cx^2}-12b+2cx^2\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(-12*b + 2*c*x^2 - (3*b^2)/(b + c*x^2)))/(6*c^3) + (5*b^(3/2)*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^10/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.54, size = 164, normalized size = 2.48

$$\left[\frac{4\,c^2x^5 - 20\,bcx^3 - 30\,b^2x + 15\left(bcx^2 + b^2\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2 + 2\,cx\sqrt{-\frac{b}{c}} - b}{cx^2 + b}\right)}{12\left(c^4x^2 + bc^3\right)}, \frac{2\,c^2x^5 - 10\,bcx^3 - 15\,b^2x + 15\left(bcx^2 + b^2\right)\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)}{6\left(c^4x^2 + bc^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*c^2*x^5 - 20*b*c*x^3 - 30*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(-b/c)*lo
g((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^4*x^2 + b*c^3), 1/6*(2*c^
2*x^5 - 10*b*c*x^3 - 15*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*sqr
t(b/c)/b))/(c^4*x^2 + b*c^3)]
```

giac [A] time = 0.17, size = 61, normalized size = 0.92

$$\frac{5\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3} - \frac{b^2x}{2\left(cx^2 + b\right)c^3} + \frac{c^4x^3 - 6\,bc^3x}{3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*b^2*x/((c*x^2 + b)*c^3)
 + 1/3*(c^4*x^3 - 6*b*c^3*x)/c^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{x^3}{3c^2} - \frac{b^2x}{2\left(c\,x^2 + b\right)c^3} + \frac{5b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^3} - \frac{2bx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/3*x^3/c^2-2*b*x/c^3-1/2/c^3*b^2*x/(c*x^2+b)+5/2/c^3*b^2/(b*c)^(1/2)*arcta
n(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.93, size = 59, normalized size = 0.89

$$-\frac{b^2x}{2\left(c^4x^2+bc^3\right)}+\frac{5\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3}+\frac{cx^3-6\,bx}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*x/(c^4*x^2 + b*c^3) + 5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3
) + 1/3*(c*x^3 - 6*b*x)/c^3
```

mupad [B] time = 0.06, size = 56, normalized size = 0.85

$$\frac{x^3}{3\,c^2}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{7/2}}-\frac{b^2\,x}{2\,\left(c^4\,x^2+b\,c^3\right)}-\frac{2\,b\,x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^3/(3*c^2) + (5*b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*c^(7/2)) - (b^2*x)/(
2*(b*c^3 + c^4*x^2)) - (2*b*x)/c^3
```

sympy [A] time = 0.33, size = 107, normalized size = 1.62

$$-\frac{b^2x}{2bc^3+2c^4x^2}-\frac{2bx}{c^3}-\frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x-\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x+\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{x^3}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2)**2,x)
```

```
[Out] -b**2*x/(2*b*c**3 + 2*c**4*x**2) - 2*b*x/c**3 - 5*sqrt(-b**3/c**7)*log(x -
c**3*sqrt(-b**3/c**7)/b)/4 + 5*sqrt(-b**3/c**7)*log(x + c**3*sqrt(-b**3/c**
7)/b)/4 + x**3/(3*c**2)
```

3.75 $\int \frac{x^9}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{b^2}{2c^3\left(b+cx^2\right)}-\frac{b\log\left(b+cx^2\right)}{c^3}+\frac{x^2}{2c^2}$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{2c^3\left(b+cx^2\right)}-\frac{b\log\left(b+cx^2\right)}{c^3}+\frac{x^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^2/(2*c^2) - b^2/(2*c^3*(b + c*x^2)) - (b*Log[b + c*x^2])/c^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^5}{\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{c^2}+\frac{b^2}{c^2(b+cx)^2}-\frac{2b}{c^2(b+cx)}\right)dx, x, x^2\right)$$
$$= \frac{x^2}{2c^2}-\frac{b^2}{2c^3\left(b+cx^2\right)}-\frac{b\log\left(b+cx^2\right)}{c^3}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.86

$$\frac{-\frac{b^2}{b+cx^2}-2b\log\left(b+cx^2\right)+cx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (c*x^2 - b^2/(b + c*x^2) - 2*b*Log[b + c*x^2])/(2*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^9/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.62, size = 56, normalized size = 1.27

$$\frac{c^2x^4+bcx^2-b^2-2\left(bcx^2+b^2\right)\log\left(cx^2+b\right)}{2\left(c^4x^2+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^4 + b*c*x^2 - b^2 - 2*(b*c*x^2 + b^2)*log(c*x^2 + b))/(c^4*x^2 +
 b*c^3)
```

giac [A] time = 0.17, size = 49, normalized size = 1.11

$$\frac{x^2}{2\,c^2} - \frac{b\log\left(\left|cx^2 + b\right|\right)}{c^3} + \frac{2\,bcx^2 + b^2}{2\left(cx^2 + b\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^2 - b*log(abs(c*x^2 + b))/c^3 + 1/2*(2*b*c*x^2 + b^2)/((c*x^2 + b
)*c^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.93

$$\frac{x^2}{2c^2} - \frac{b^2}{2\left(c\,x^2 + b\right)c^3} - \frac{b\ln\left(c\,x^2 + b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^2/c^2-1/2*b^2/c^3/(c*x^2+b)-b*ln(c*x^2+b)/c^3
```

maxima [A] time = 1.32, size = 43, normalized size = 0.98

$$-\frac{b^2}{2\left(c^4x^2 + bc^3\right)} + \frac{x^2}{2\,c^2} - \frac{b\log\left(cx^2 + b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(c^4*x^2 + b*c^3) + 1/2*x^2/c^2 - b*log(c*x^2 + b)/c^3
```

mupad [B] time = 0.04, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,c^2} - \frac{b^2}{2\,\left(c^4\,x^2 + b\,c^3\right)} - \frac{b\,\ln\left(c\,x^2 + b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2/(2*c^2) - b^2/(2*(b*c^3 + c^4*x^2)) - (b*log(b + c*x^2))/c^3
```

sympy [A] time = 0.28, size = 39, normalized size = 0.89

$$-\frac{b^2}{2bc^3 + 2c^4x^2} - \frac{b\log\left(b + cx^2\right)}{c^3} + \frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**2,x)
[Out] -b**2/(2*b*c**3 + 2*c**4*x**2) - b*log(b + c*x**2)/c**3 + x**2/(2*c**2)
```

3.76 $\int \frac{x^8}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}}-\frac{x^3}{2c\left(b+cx^2\right)}+\frac{3x}{2c^2}$$

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 321, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}}-\frac{x^3}{2c\left(b+cx^2\right)}+\frac{3x}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*x)/(2*c^2) - x^3/(2*c*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[
b]])/(2*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^4}{\left(b+cx^2\right)^2}\,dx\\ &= -\frac{x^3}{2c\left(b+cx^2\right)}+\frac{3\int \frac{x^2}{b+cx^2}\,dx}{2c}\\ &= \frac{3x}{2c^2}-\frac{x^3}{2c\left(b+cx^2\right)}-\frac{(3b)\int \frac{1}{b+cx^2}\,dx}{2c^2}\\ &= \frac{3x}{2c^2}-\frac{x^3}{2c\left(b+cx^2\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}}+\frac{bx}{2c^2\left(b+cx^2\right)}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/c^2 + (b*x)/(2*c^2*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])
/(2*c^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^8/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.49, size = 136, normalized size = 2.47

$$\left[\frac{4\,cx^3 + 3\left(cx^2 + b\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2 - 2\,cx\sqrt{-\frac{b}{c}} - b}{cx^2 + b}\right) + 6\,bx}{4\left(c^3x^2 + bc^2\right)}, \frac{2\,cx^3 - 3\left(cx^2 + b\right)\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right) + 3\,bx}{2\left(c^3x^2 + bc^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*x^3 + 3*(c*x^2 + b)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)
/(c*x^2 + b)) + 6*b*x)/(c^3*x^2 + b*c^2), 1/2*(2*c*x^3 - 3*(c*x^2 + b)*sqrt
(b/c)*arctan(c*x*sqrt(b/c)/b) + 3*b*x)/(c^3*x^2 + b*c^2)]
```

giac [A] time = 0.17, size = 42, normalized size = 0.76

$$-\frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2} + \frac{bx}{2\left(cx^2 + b\right)c^2} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*b*x/((c*x^2 + b)*c^2) +
x/c^2
```

maple [A] time = 0.01, size = 43, normalized size = 0.78

$$\frac{bx}{2\left(c\,x^2 + b\right)c^2} - \frac{3b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^2} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2)^2,x)
```

```
[Out] x/c^2+1/2/c^2*b*x/(c*x^2+b)-3/2/c^2*b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.94, size = 45, normalized size = 0.82

$$\frac{bx}{2\left(c^3x^2 + bc^2\right)} - \frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x/(c^3*x^2 + b*c^2) - 3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + x
/c^2
```

mupad [B] time = 4.17, size = 43, normalized size = 0.78

$$\frac{x}{c^2}+\frac{b\,x}{2\,\left(c^3\,x^2+b\,c^2\right)}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x/c^2 + (b*x)/(2*(b*c^2 + c^3*x^2)) - (3*b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))
/(2*c^(5/2))
```

sympy [A] time = 0.28, size = 83, normalized size = 1.51

$$\frac{bx}{2bc^2+2c^3x^2}+\frac{3\sqrt{-\frac{b}{c^5}}\log\left(-c^2\sqrt{-\frac{b}{c^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{c^5}}\log\left(c^2\sqrt{-\frac{b}{c^5}}+x\right)}{4}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2)**2,x)
```

```
[Out] b*x/(2*b*c**2 + 2*c**3*x**2) + 3*sqrt(-b/c**5)*log(-c**2*sqrt(-b/c**5) + x)
/4 - 3*sqrt(-b/c**5)*log(c**2*sqrt(-b/c**5) + x)/4 + x/c**2
```

3.77 $\int \frac{x^7}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=33

$$\frac{b}{2c^2\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^2}$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b}{2c^2\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^2,x]
```

```
[Out] b/(2*c^2*(b + c*x^2)) + Log[b + c*x^2]/(2*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^3}{\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(b+cx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b}{c(b+cx)^2}+\frac{1}{c(b+cx)}\right)dx,x,x^2\right)\\ &= \frac{b}{2c^2\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{b}{b+cx^2}+\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b/(b + c*x^2) + Log[b + c*x^2])/(2*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^7/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.45, size = 35, normalized size = 1.06

$$\frac{\left(cx^2+b\right)\log\left(cx^2+b\right)+b}{2\left(c^3x^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((c*x^2 + b)*log(c*x^2 + b) + b)/(c^3*x^2 + b*c^2)
```

giac [A] time = 0.15, size = 32, normalized size = 0.97

$$-\frac{x^2}{2\left(cx^2+b\right)c}+\frac{\log\left(\left|cx^2+b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] -1/2*x^2/((c*x^2 + b)*c) + 1/2*log(abs(c*x^2 + b))/c^2
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{b}{2\left(c\,x^2+b\right)c^2}+\frac{\ln\left(c\,x^2+b\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^2,x)
[Out] 1/2*b/c^2/(c*x^2+b)+1/2*ln(c*x^2+b)/c^2
```

maxima [A] time = 1.29, size = 32, normalized size = 0.97

$$\frac{b}{2\left(c^3x^2+bc^2\right)}+\frac{\log\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 1/2*b/(c^3*x^2 + b*c^2) + 1/2*log(c*x^2 + b)/c^2
```

mupad [B] time = 4.18, size = 29, normalized size = 0.88

$$\frac{\ln\left(c\,x^2+b\right)}{2\,c^2}+\frac{b}{2\,c^2\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^2,x)
[Out] log(b + c*x^2)/(2*c^2) + b/(2*c^2*(b + c*x^2))
```

sympy [A] time = 0.22, size = 29, normalized size = 0.88

$$\frac{b}{2bc^2 + 2c^3x^2} + \frac{\log\left(b + cx^2\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**2,x)
```

```
[Out] b/(2*b*c**2 + 2*c**3*x**2) + log(b + c*x**2)/(2*c**2)
```

3.78 $\int \frac{x^6}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}-\frac{x}{2c\left(b+cx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}-\frac{x}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -x/(2*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^2}{\left(b+cx^2\right)^2}\,dx$$
$$= -\frac{x}{2c\left(b+cx^2\right)} + \frac{\int \frac{1}{b+cx^2}\,dx}{2c}$$
$$= -\frac{x}{2c\left(b+cx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}} - \frac{x}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^2,x]
[Out] -1/2*x/(c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(b*x^2 + c*x^4)^2,x]
[Out] IntegrateAlgebraic[x^6/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.51, size = 120, normalized size = 2.67

$$\left[-\frac{2\,bcx + \left(cx^2+b\right)\sqrt{-bc}\,\log\left(\frac{cx^2 - 2\,\sqrt{-bc}\,x - b}{cx^2+b}\right)}{4\left(bc^3x^2+b^2c^2\right)}, -\frac{bcx - \left(cx^2+b\right)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{2\left(bc^3x^2+b^2c^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b*c*x + (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c
*x^2 + b)))/(b*c^3*x^2 + b^2*c^2), -1/2*(b*c*x - (c*x^2 + b)*sqrt(b*c)*arct
an(sqrt(b*c)*x/b))/(b*c^3*x^2 + b^2*c^2)]
```

giac [A] time = 0.17, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,c} - \frac{x}{2\left(cx^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) - 1/2*x/((c*x^2 + b)*c)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(c\,x^2+b\right)c} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x/c/(c*x^2+b)+1/2/c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 3.02, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(c^2x^2+bc\right)} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(c^2*x^2 + b*c) + 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

mupad [B] time = 4.15, size = 33, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}} - \frac{x}{2\,c\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4)^2,x)
```

```
[Out] atan((c^(1/2)*x)/b^(1/2))/(2*b^(1/2)*c^(3/2)) - x/(2*c*(b + c*x^2))
```

sympy **[B]** time = 0.23, size = 78, normalized size = 1.73

$$-\frac{x}{2bc+2c^2x^2}-\frac{\sqrt{-\frac{1}{bc^3}}\log\left(-bc\sqrt{-\frac{1}{bc^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{bc^3}}\log\left(bc\sqrt{-\frac{1}{bc^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**2,x)
```

```
[Out] -x/(2*b*c + 2*c**2*x**2) - sqrt(-1/(b*c**3))*log(-b*c*sqrt(-1/(b*c**3)) + x
)/4 + sqrt(-1/(b*c**3))*log(b*c*sqrt(-1/(b*c**3)) + x)/4
```

3.79 $\int \frac{x^5}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=16

$$-\frac{1}{2c(b+cx^2)}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$-\frac{1}{2c(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/(2*c*(b + c*x^2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(bx^2+cx^4)^2} dx &= \int \frac{x}{(b+cx^2)^2} dx \\ &= -\frac{1}{2c(b+cx^2)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*1/(c*(b + c*x^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^5/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.62, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(c^2*x^2 + b*c)
```

giac [A] time = 0.17, size = 14, normalized size = 0.88

$$-\frac{1}{2\left(cx^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2/((c*x^2 + b)*c)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(c\,x^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^2,x)
[Out] -1/2/c/(c*x^2+b)
```

maxima [A] time = 1.32, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(c^2x^2 + bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] -1/2/(c^2*x^2 + b*c)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.88

$$-\frac{1}{2\,c\,\left(c\,x^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^2,x)
[Out] -1/(2*c*(b + c*x^2))
```

sympy [A] time = 0.18, size = 15, normalized size = 0.94

$$-\frac{1}{2bc + 2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**2,x)
[Out] -1/(2*b*c + 2*c**2*x**2)
```

3.80 $\int \frac{x^4}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{1}{\left(b+cx^2\right)^2}\,dx$$
$$=\frac{x}{2b\left(b+cx^2\right)}+\frac{\int \frac{1}{b+cx^2}\,dx}{2b}$$
$$=\frac{x}{2b\left(b+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^2,x]
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(b*x^2 + c*x^4)^2,x]
[Out] IntegrateAlgebraic[x^4/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.71, size = 120, normalized size = 2.67

$$\left[\frac{2\,bcx-\left(cx^2+b\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{4\left(b^2c^2x^2+b^3c\right)},\frac{bcx+\left(cx^2+b\right)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{2\left(b^2c^2x^2+b^3c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c*x - (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*
x^2 + b)))/(b^2*c^2*x^2 + b^3*c), 1/2*(b*c*x + (c*x^2 + b)*sqrt(b*c)*arctan
(sqrt(b*c)*x/b))/(b^2*c^2*x^2 + b^3*c)]
```

giac [A] time = 0.15, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,b} + \frac{x}{2\left(cx^2+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) + 1/2*x/((c*x^2 + b)*b)
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$\frac{x}{2\left(c\,x^2+b\right)b} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x/b/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 3.00, size = 35, normalized size = 0.78

$$\frac{x}{2\left(bcx^2+b^2\right)} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(b*c*x^2 + b^2) + 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.73

$$\frac{x}{2\,b\,\left(c\,x^2+b\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{3/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x/(2*b*(b + c*x^2)) + atan((c^(1/2)*x)/b^(1/2))/(2*b^(3/2)*c^(1/2))
```

sympy [B] time = 0.25, size = 78, normalized size = 1.73

$$\frac{x}{2b^2 + 2bcx^2} - \frac{\sqrt{-\frac{1}{b^3c}}\log\left(-b^2\sqrt{-\frac{1}{b^3c}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{b^3c}}\log\left(b^2\sqrt{-\frac{1}{b^3c}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**2,x)
```

```
[Out] x/(2*b**2 + 2*b*c*x**2) - sqrt(-1/(b**3*c))*log(-b**2*sqrt(-1/(b**3*c)) + x
)/4 + sqrt(-1/(b**3*c))*log(b**2*sqrt(-1/(b**3*c)) + x)/4
```

3.81 $\int \frac{x^3}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)}{b^2}+\frac{1}{2b\left(b+cx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)}{b^2}+\frac{1}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^2,x]
```

```
[Out] 1/(2*b*(b + c*x^2)) + Log[x]/b^2 - Log[b + c*x^2]/(2*b^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x\left(b+cx^2\right)^2}\,dx \\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x(b+cx)^2}\,dx,x,x^2\right) \\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{1}{b^2x}-\frac{c}{b(b+cx)^2}-\frac{c}{b^2(b+cx)}\right)dx,x,x^2\right) \\ &= \frac{1}{2b\left(b+cx^2\right)}+\frac{\log(x)}{b^2}-\frac{\log\left(b+cx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{b}{b+cx^2}-\log\left(b+cx^2\right)+2\log(x)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^2,x]
[Out] (b/(b + c*x^2) + 2*Log[x] - Log[b + c*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(b*x^2 + c*x^4)^2,x]
[Out] IntegrateAlgebraic[x^3/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.59, size = 47, normalized size = 1.24

$$-\frac{\left(cx^2+b\right)\log\left(cx^2+b\right)-2\left(cx^2+b\right)\log(x)-b}{2\left(b^2cx^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((c*x^2 + b)*log(c*x^2 + b) - 2*(c*x^2 + b)*log(x) - b)/(b^2*c*x^2 + b
^3)
```

giac [A] time = 0.16, size = 36, normalized size = 0.95

$$-\frac{\log\left(\left|cx^2+b\right|\right)}{2\,b^2}+\frac{\log\left(|x|\right)}{b^2}+\frac{1}{2\left(cx^2+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*log(abs(c*x^2 + b))/b^2 + log(abs(x))/b^2 + 1/2/((c*x^2 + b)*b)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{2\left(c\,x^2+b\right)b}+\frac{\ln(x)}{b^2}-\frac{\ln\left(c\,x^2+b\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b/(c*x^2+b)+ln(x)/b^2-1/2*ln(c*x^2+b)/b^2
```

maxima [A] time = 1.30, size = 37, normalized size = 0.97

$$\frac{1}{2\left(bcx^2+b^2\right)}-\frac{\log\left(cx^2+b\right)}{2\,b^2}+\frac{\log\left(x^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(b*c*x^2 + b^2) - 1/2*log(c*x^2 + b)/b^2 + 1/2*log(x^2)/b^2
```

mupad [B] time = 4.18, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{b^2}+\frac{1}{2\,b\,\left(c\,x^2+b\right)}-\frac{\ln\left(c\,x^2+b\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^2,x)
```

```
[Out] log(x)/b^2 + 1/(2*b*(b + c*x^2)) - log(b + c*x^2)/(2*b^2)
```

sympy [A] time = 0.34, size = 34, normalized size = 0.89

$$\frac{1}{2b^2 + 2bcx^2} + \frac{\log(x)}{b^2} - \frac{\log\left(\frac{b}{c} + x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b**2 + 2*b*c*x**2) + log(x)/b**2 - log(b/c + x**2)/(2*b**2)
```

3.82 $\int \frac{x^2}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -3/(2*b^2*x) + 1/(2*b*x*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b
]])/(2*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^2\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2bx\left(b+cx^2\right)}+\frac{3\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{2b}\\ &= -\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}-\frac{(3c)\int \frac{1}{b+cx^2}\,dx}{2b^2}\\ &= -\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{cx}{2b^2\left(b+cx^2\right)}-\frac{1}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(1/(b^2*x)) - (c*x)/(2*b^2*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(2*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.75, size = 136, normalized size = 2.39

$$\left[-\frac{6\,cx^2-3\left(cx^3+bx\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+4\,b}{4\left(b^2cx^3+b^3x\right)},-\frac{3\,cx^2+3\left(cx^3+bx\right)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+2\,b}{2\left(b^2cx^3+b^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*c*x^2 - 3*(c*x^3 + b*x)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) -
 b)/(c*x^2 + b)) + 4*b)/(b^2*c*x^3 + b^3*x), -1/2*(3*c*x^2 + 3*(c*x^3 + b*x
)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 2*b)/(b^2*c*x^3 + b^3*x)]
```

giac [A] time = 0.16, size = 47, normalized size = 0.82

$$-\frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^2}-\frac{3\,cx^2+2\,b}{2\left(cx^3+bx\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/2*(3*c*x^2 + 2*b)/((c*x^3
+ b*x)*b^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.81

$$-\frac{cx}{2\left(c\,x^2+b\right)b^2}-\frac{3c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b^2}-\frac{1}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*c*x/(c*x^2+b)-3/2/b^2*c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/b^
2/x
```

maxima [A] time = 2.96, size = 49, normalized size = 0.86

$$-\frac{3\,cx^2+2\,b}{2\left(b^2cx^3+b^3x\right)}-\frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*c*x^2 + 2*b)/(b^2*c*x^3 + b^3*x) - 3/2*c*arctan(c*x/sqrt(b*c))/(sqr
t(b*c)*b^2)
```

mupad [B] time = 0.06, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{b}+\frac{3\,c\,x^2}{2\,b^2}}{c\,x^3+b\,x}-\frac{3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - (1/b + (3*c*x^2)/(2*b^2))/(b*x + c*x^3) - (3*c^(1/2)*atan((c^(1/2)*x)/b^(
1/2)))/(2*b^(5/2))
```

sympy [A] time = 0.32, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{c}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}-\frac{3\sqrt{-\frac{c}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}+\frac{-2b-3cx^2}{2b^3x+2b^2cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 3*sqrt(-c/b**5)*log(-b**3*sqrt(-c/b**5)/c + x)/4 - 3*sqrt(-c/b**5)*log(b**3
*sqrt(-c/b**5)/c + x)/4 + (-2*b - 3*c*x**2)/(2*b**3*x + 2*b**2*c*x**3)
```

3.83 $\int \frac{x}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=49

$$\frac{c\log\left(b+cx^2\right)}{b^3}-\frac{2c\log(x)}{b^3}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{1}{2b^2x^2}$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$-\frac{c}{2b^2\left(b+cx^2\right)}+\frac{c\log\left(b+cx^2\right)}{b^3}-\frac{2c\log(x)}{b^3}-\frac{1}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/(2*b^2*x^2) - c/(2*b^2*(b + c*x^2)) - (2*c*Log[x])/b^3 + (c*Log[b + c*x^
2])/b^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^3\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(b+cx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^2x^2}-\frac{2c}{b^3x}+\frac{c^2}{b^2(b+cx)^2}+\frac{2c^2}{b^3(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{2b^2x^2}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{2c\log(x)}{b^3}+\frac{c\log\left(b+cx^2\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.84

$$-\frac{b\left(\frac{c}{b+cx^2}+\frac{1}{x^2}\right)-2c\log\left(b+cx^2\right)+4c\log(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^2,x]
[Out] -1/2*(b*(x^(-2) + c/(b + c*x^2)) + 4*c*Log[x] - 2*c*Log[b + c*x^2])/b^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(b*x^2 + c*x^4)^2,x]
[Out] IntegrateAlgebraic[x/(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 1.09, size = 73, normalized size = 1.49

$$-\frac{2\,bcx^2+b^2-2\left(c^2x^4+bcx^2\right)\log\left(cx^2+b\right)+4\left(c^2x^4+bcx^2\right)\log(x)}{2\left(b^3cx^4+b^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*x^2 + b^2 - 2*(c^2*x^4 + b*c*x^2)*log(c*x^2 + b) + 4*(c^2*x^4 +
 b*c*x^2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

giac [A] time = 0.17, size = 50, normalized size = 1.02

$$\frac{c\log\left(\left|cx^2+b\right|\right)}{b^3}-\frac{2\,c\log\left(|x|\right)}{b^3}-\frac{2\,cx^2+b}{2\left(cx^4+bx^2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] c*log(abs(c*x^2 + b))/b^3 - 2*c*log(abs(x))/b^3 - 1/2*(2*c*x^2 + b)/((c*x^4
 + b*x^2)*b^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{c}{2\left(c\,x^2+b\right)b^2}-\frac{2c\ln(x)}{b^3}+\frac{c\ln\left(c\,x^2+b\right)}{b^3}-\frac{1}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2/x^2-1/2*c/b^2/(c*x^2+b)-2*c*ln(x)/b^3+c*ln(c*x^2+b)/b^3
```

maxima [A] time = 1.34, size = 52, normalized size = 1.06

$$-\frac{2\,cx^2+b}{2\left(b^2cx^4+b^3x^2\right)}+\frac{c\log\left(cx^2+b\right)}{b^3}-\frac{c\log\left(x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2) + c*log(c*x^2 + b)/b^3 - c*log(x^2
)/b^3
```

mupad [B] time = 4.21, size = 51, normalized size = 1.04

$$\frac{c\ \ln\left(c\,x^2+b\right)}{b^3}-\frac{\frac{1}{2\,b}+\frac{c\,x^2}{b^2}}{c\,x^4+b\,x^2}-\frac{2\,c\ \ln(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (c*log(b + c*x^2))/b^3 - (1/(2*b) + (c*x^2)/b^2)/(b*x^2 + c*x^4) - (2*c*log
(x))/b^3
```

sympy [**A**] time = 0.40, size = 51, normalized size = 1.04

$$\frac{-b-2cx^2}{2b^3x^2+2b^2cx^4}-\frac{2c\log(x)}{b^3}+\frac{c\log\left(\frac{b}{c}+x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-b - 2*c*x**2)/(2*b**3*x**2 + 2*b**2*c*x**4) - 2*c*log(x)/b**3 + c*log(b/c
 + x**2)/b**3
```

3.84 $\int \frac{1}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=68

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5c}{2b^3x}-\frac{5}{6b^2x^3}+\frac{1}{2bx^3\left(b+cx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1593, 290, 325, 205}

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5c}{2b^3x}-\frac{5}{6b^2x^3}+\frac{1}{2bx^3\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -5/(6*b^2*x^3) + (5*c)/(2*b^3*x) + 1/(2*b*x^3*(b + c*x^2)) + (5*c^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^4\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2bx^3\left(b+cx^2\right)} + \frac{5\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{2b}$$
$$= -\frac{5}{6b^2x^3} + \frac{1}{2bx^3\left(b+cx^2\right)} - \frac{(5c)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{2b^2}$$
$$= -\frac{5}{6b^2x^3} + \frac{5c}{2b^3x} + \frac{1}{2bx^3\left(b+cx^2\right)} + \frac{\left(5c^2\right)\int \frac{1}{b+cx^2}\,dx}{2b^3}$$
$$= -\frac{5}{6b^2x^3} + \frac{5c}{2b^3x} + \frac{1}{2bx^3\left(b+cx^2\right)} + \frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.99

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}} + \frac{c^2x}{2b^3\left(b+cx^2\right)} + \frac{2c}{b^3x} - \frac{1}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -1/3*1/(b^2*x^3) + (2*c)/(b^3*x) + (c^2*x)/(2*b^3*(b + c*x^2)) + (5*c^(3/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^(-2), x]
```

fricas [A] time = 1.03, size = 172, normalized size = 2.53

$$\left[\frac{30\,c^2x^4+20\,bcx^2+15\left(c^2x^5+bcx^3\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-4\,b^2}{12\left(b^3cx^5+b^4x^3\right)},\frac{15\,c^2x^4+10\,bcx^2+15\left(c^2x^5+bcx^3\right)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)-2\,b^2}{6\left(b^3cx^5+b^4x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*c^2*x^4 + 20*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(-c/b)*log((c*x
^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 4*b^2)/(b^3*c*x^5 + b^4*x^3), 1/6
*(15*c^2*x^4 + 10*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(c/b)*arctan(x*sqrt(
c/b)) - 2*b^2)/(b^3*c*x^5 + b^4*x^3)]
```

giac [A] time = 0.15, size = 59, normalized size = 0.87

$$\frac{5\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^3}+\frac{c^2x}{2\left(cx^2+b\right)b^3}+\frac{6\,cx^2-b}{3\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/2*c^2*x/((c*x^2 + b)*b^3)
 + 1/3*(6*c*x^2 - b)/(b^3*x^3)
```

maple [A] time = 0.01, size = 59, normalized size = 0.87

$$\frac{c^2x}{2\left(c\,x^2+b\right)b^3}+\frac{5c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,b^3}+\frac{2c}{b^3x}-\frac{1}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b^3*c^2*x/(c*x^2+b)+5/2/b^3*c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1
/3/b^2/x^3+2*c/b^3/x
```

maxima [A] time = 2.96, size = 64, normalized size = 0.94

$$\frac{15\,c^2x^4+10\,bcx^2-2\,b^2}{6\left(b^3cx^5+b^4x^3\right)}+\frac{5\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*c^2*x^4 + 10*b*c*x^2 - 2*b^2)/(b^3*c*x^5 + b^4*x^3) + 5/2*c^2*arcta
n(c*x/sqrt(b*c))/(sqrt(b*c)*b^3)
```

mupad [B] time = 4.17, size = 58, normalized size = 0.85

$$\frac{\frac{5\,c\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{5\,c^2\,x^4}{2\,b^3}}{c\,x^5+b\,x^3}+\frac{5\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((5*c*x^2)/(3*b^2) - 1/(3*b) + (5*c^2*x^4)/(2*b^3))/(b*x^3 + c*x^5) + (5*c^
(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*b^(7/2))
```

sympy [A] time = 0.39, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{-2b^2+10bcx^2+15c^2x^4}{6b^4x^3+6b^3cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**2,x)
```

```
[Out] -5*sqrt(-c**3/b**7)*log(-b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + 5*sqrt(-c**3/b
**7)*log(b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + (-2*b**2 + 10*b*c*x**2 + 15*c*
*2*x**4)/(6*b**4*x**3 + 6*b**3*c*x**5)
```

3.85 $\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}+\frac{3c^2\log(x)}{b^4}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$\frac{c^2}{2b^3\left(b+cx^2\right)}-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}+\frac{3c^2\log(x)}{b^4}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/(4*b^2*x^4) + c/(b^3*x^2) + c^2/(2*b^3*(b + c*x^2)) + (3*c^2*Log[x])/b^4
 - (3*c^2*Log[b + c*x^2])/(2*b^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^5\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^2x^3}-\frac{2c}{b^3x^2}+\frac{3c^2}{b^4x}-\frac{c^3}{b^3(b+cx)^2}-\frac{3c^3}{b^4(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{1}{4b^2x^4}+\frac{c}{b^3x^2}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{3c^2\log(x)}{b^4}-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.86

$$
\frac{-6c^2\log\left(b+cx^2\right)+b\left(\frac{2c^2}{b+cx^2}-\frac{b}{x^4}+\frac{4c}{x^2}\right)+12c^2\log(x)}{4b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (b*(-(b/x^4) + (4*c)/x^2 + (2*c^2)/(b + c*x^2)) + 12*c^2*Log[x] - 6*c^2*Log
[b + c*x^2])/(4*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(b*x^2 + c*x^4)^2), x]
```

fricas [A] time = 0.75, size = 90, normalized size = 1.36

$$
\frac{6\,bc^2x^4+3\,b^2cx^2-b^3-6\left(c^3x^6+bc^2x^4\right)\log\left(cx^2+b\right)+12\left(c^3x^6+bc^2x^4\right)\log(x)}{4\left(b^4cx^6+b^5x^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*c^2*x^4 + 3*b^2*c*x^2 - b^3 - 6*(c^3*x^6 + b*c^2*x^4)*log(c*x^2 +
b) + 12*(c^3*x^6 + b*c^2*x^4)*log(x))/(b^4*c*x^6 + b^5*x^4)
```

giac [A] time = 0.15, size = 86, normalized size = 1.30

$$\frac{3\,c^2\log\left(x^2\right)}{2\,b^4} - \frac{3\,c^2\log\left(\left|cx^2 + b\right|\right)}{2\,b^4} + \frac{3\,c^3x^2 + 4\,bc^2}{2\left(cx^2 + b\right)b^4} - \frac{9\,c^2x^4 - 4\,bcx^2 + b^2}{4\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*c^2*log(x^2)/b^4 - 3/2*c^2*log(abs(c*x^2 + b))/b^4 + 1/2*(3*c^3*x^2 + 4
*b*c^2)/((c*x^2 + b)*b^4) - 1/4*(9*c^2*x^4 - 4*b*c*x^2 + b^2)/(b^4*x^4)
```

maple [A] time = 0.01, size = 61, normalized size = 0.92

$$\frac{c^2}{2\left(c\,x^2 + b\right)b^3} + \frac{3c^2\ln(x)}{b^4} - \frac{3c^2\ln\left(c\,x^2 + b\right)}{2b^4} + \frac{c}{b^3x^2} - \frac{1}{4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/4/b^2/x^4+c/b^3/x^2+1/2*c^2/b^3/(c*x^2+b)+3*c^2*ln(x)/b^4-3/2*c^2*ln(c*x
^2+b)/b^4
```

maxima [A] time = 1.35, size = 70, normalized size = 1.06

$$\frac{6\,c^2x^4 + 3\,bcx^2 - b^2}{4\left(b^3cx^6 + b^4x^4\right)} - \frac{3\,c^2\log\left(cx^2 + b\right)}{2\,b^4} + \frac{3\,c^2\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*c^2*x^4 + 3*b*c*x^2 - b^2)/(b^3*c*x^6 + b^4*x^4) - 3/2*c^2*log(c*x^2
 + b)/b^4 + 3/2*c^2*log(x^2)/b^4
```

mupad [B] time = 4.17, size = 67, normalized size = 1.02

$$\frac{\frac{3\,c\,x^2}{4\,b^2} - \frac{1}{4\,b} + \frac{3\,c^2\,x^4}{2\,b^3}}{c\,x^6 + b\,x^4} - \frac{3\,c^2\,\ln\left(c\,x^2 + b\right)}{2\,b^4} + \frac{3\,c^2\,\ln(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((3*c*x^2)/(4*b^2) - 1/(4*b) + (3*c^2*x^4)/(2*b^3))/(b*x^4 + c*x^6) - (3*c^
2*log(b + c*x^2))/(2*b^4) + (3*c^2*log(x))/b^4
```

sympy [A] time = 0.49, size = 68, normalized size = 1.03

$$\frac{-b^2 + 3bcx^2 + 6c^2x^4}{4b^4x^4 + 4b^3cx^6} + \frac{3c^2\log{(x)}}{b^4} - \frac{3c^2\log{\left(\frac{b}{c} + x^2\right)}}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-b**2 + 3*b*c*x**2 + 6*c**2*x**4)/(4*b**4*x**4 + 4*b**3*c*x**6) + 3*c**2*l
og(x)/b**4 - 3*c**2*log(b/c + x**2)/(2*b**4)
```

3.86 $\int \frac{1}{x^2(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{7c^2}{2b^4x}+\frac{7c}{6b^3x^3}-\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}$$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$-\frac{7c^2}{2b^4x}-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}+\frac{7c}{6b^3x^3}-\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -7/(10*b^2*x^5) + (7*c)/(6*b^3*x^3) - (7*c^2)/(2*b^4*x) + 1/(2*b*x^5*(b + c
*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^6\left(b+cx^2\right)^2}\,dx\\
&= \frac{1}{2bx^5\left(b+cx^2\right)}+\frac{7\int \frac{1}{x^6\left(b+cx^2\right)}\,dx}{2b}\\
&= -\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{(7c)\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{2b^2}\\
&= -\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}+\frac{1}{2bx^5\left(b+cx^2\right)}+\frac{\left(7c^2\right)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{2b^3}\\
&= -\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}-\frac{7c^2}{2b^4x}+\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{\left(7c^3\right)\int \frac{1}{b+cx^2}\,dx}{2b^4}\\
&= -\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}-\frac{7c^2}{2b^4x}+\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.99

$$-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{c^3x}{2b^4\left(b+cx^2\right)}-\frac{3c^2}{b^4x}+\frac{2c}{3b^3x^3}-\frac{1}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/5*1/(b^2*x^5) + (2*c)/(3*b^3*x^3) - (3*c^2)/(b^4*x) - (c^3*x)/(2*b^4*(b
+ c*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(bx^2 + cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(b*x^2 + c*x^4)^2), x]
```

fricas [A] time = 0.66, size = 198, normalized size = 2.44

$$\left[-\frac{210\,c^3x^6 + 140\,bc^2x^4 - 28\,b^2cx^2 + 12\,b^3 - 105\,\left(c^3x^7 + bc^2x^5\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2 - 2\,bx\sqrt{-\frac{c}{b}} - b}{cx^2 + b}\right)}{60\,\left(b^4cx^7 + b^5x^5\right)}, -\frac{105\,c^3x^6 + 70\,bc^2x^4 - 14\,b^2cx^2 + 6\,b^3 + 105\,\left(c^3x^7 + bc^2x^5\right)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)}{30\,\left(b^4cx^7 + b^5x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(210*c^3*x^6 + 140*b*c^2*x^4 - 28*b^2*c*x^2 + 12*b^3 - 105*(c^3*x^7
+ b*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b
^4*c*x^7 + b^5*x^5), -1/30*(105*c^3*x^6 + 70*b*c^2*x^4 - 14*b^2*c*x^2 + 6*b
^3 + 105*(c^3*x^7 + b*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c*x^7 +
b^5*x^5)]
```

giac [A] time = 0.18, size = 70, normalized size = 0.86

$$-\frac{7\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^4} - \frac{c^3x}{2\,\left(cx^2 + b\right)b^4} - \frac{45\,c^2x^4 - 10\,bcx^2 + 3\,b^2}{15\,b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/2*c^3*x/((c*x^2 + b)*b^4
) - 1/15*(45*c^2*x^4 - 10*b*c*x^2 + 3*b^2)/(b^4*x^5)
```

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$-\frac{c^3x}{2\left(c\,x^2 + b\right)b^4} - \frac{7c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b^4} - \frac{3c^2}{b^4x} + \frac{2c}{3b^3x^3} - \frac{1}{5b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^4*c^3*x/(c*x^2+b)-7/2/b^4*c^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-
1/5/b^2/x^5-3*c^2/b^4/x+2/3*c/b^3/x^3
```

maxima [A] time = 3.03, size = 75, normalized size = 0.93

$$-\frac{105\,c^{3}x^{6}+70\,bc^{2}x^{4}-14\,b^{2}cx^{2}+6\,b^{3}}{30\,\left(b^{4}cx^{7}+b^{5}x^{5}\right)}-\frac{7\,c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(105*c^3*x^6 + 70*b*c^2*x^4 - 14*b^2*c*x^2 + 6*b^3)/(b^4*c*x^7 + b^5*
x^5) - 7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

mupad [B] time = 4.28, size = 70, normalized size = 0.86

$$-\frac{\frac{1}{5\,b}-\frac{7\,c\,x^{2}}{15\,b^{2}}+\frac{7\,c^{2}\,x^{4}}{3\,b^{3}}+\frac{7\,c^{3}\,x^{6}}{2\,b^{4}}}{c\,x^{7}+b\,x^{5}}-\frac{7\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^2),x)
```

```
[Out] - (1/(5*b) - (7*c*x^2)/(15*b^2) + (7*c^2*x^4)/(3*b^3) + (7*c^3*x^6)/(2*b^4)
)/(b*x^5 + c*x^7) - (7*c^(5/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*b^(9/2))
```

sympy [A] time = 0.43, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{c^{5}}{b^{9}}}\log\left(-\frac{b^{5}\sqrt{-\frac{c^{5}}{b^{9}}}}{c^{3}}+x\right)}{4}-\frac{7\sqrt{-\frac{c^{5}}{b^{9}}}\log\left(\frac{b^{5}\sqrt{-\frac{c^{5}}{b^{9}}}}{c^{3}}+x\right)}{4}+\frac{-6b^{3}+14b^{2}cx^{2}-70bc^{2}x^{4}-105c^{3}x^{6}}{30b^{5}x^{5}+30b^{4}cx^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 7*sqrt(-c**5/b**9)*log(-b**5*sqrt(-c**5/b**9)/c**3 + x)/4 - 7*sqrt(-c**5/b*
*9)*log(b**5*sqrt(-c**5/b**9)/c**3 + x)/4 + (-6*b**3 + 14*b**2*c*x**2 - 70*
b*c**2*x**4 - 105*c**3*x**6)/(30*b**5*x**5 + 30*b**4*c*x**7)
```

3.87 $\int \frac{x^{14}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=85

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{35bx}{8c^4}-\frac{7x^5}{8c^2\left(b+cx^2\right)}-\frac{x^7}{4c\left(b+cx^2\right)^2}+\frac{35x^3}{24c^3}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 302, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{7x^5}{8c^2\left(b+cx^2\right)}-\frac{35bx}{8c^4}-\frac{x^7}{4c\left(b+cx^2\right)^2}+\frac{35x^3}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-35*b*x)/(8*c^4) + (35*x^3)/(24*c^3) - x^7/(4*c*(b + c*x^2)^2) - (7*x^5)/(
8*c^2*(b + c*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^8}{\left(b+cx^2\right)^3}\,dx\\
&= -\frac{x^7}{4c\left(b+cx^2\right)^2}+\frac{7\int \frac{x^6}{\left(b+cx^2\right)^2}\,dx}{4c}\\
&= -\frac{x^7}{4c\left(b+cx^2\right)^2}-\frac{7x^5}{8c^2\left(b+cx^2\right)}+\frac{35\int \frac{x^4}{b+cx^2}\,dx}{8c^2}\\
&= -\frac{x^7}{4c\left(b+cx^2\right)^2}-\frac{7x^5}{8c^2\left(b+cx^2\right)}+\frac{35\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right)dx}{8c^2}\\
&= -\frac{35bx}{8c^4}+\frac{35x^3}{24c^3}-\frac{x^7}{4c\left(b+cx^2\right)^2}-\frac{7x^5}{8c^2\left(b+cx^2\right)}+\frac{\left(35b^2\right)\int \frac{1}{b+cx^2}\,dx}{8c^4}\\
&= -\frac{35bx}{8c^4}+\frac{35x^3}{24c^3}-\frac{x^7}{4c\left(b+cx^2\right)^2}-\frac{7x^5}{8c^2\left(b+cx^2\right)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.91

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{105b^3x+175b^2cx^3+56bc^2x^5-8c^3x^7}{24c^4\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/24*(105*b^3*x + 175*b^2*c*x^3 + 56*b*c^2*x^5 - 8*c^3*x^7)/(c^4*(b + c*x^
2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^14/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.39, size = 230, normalized size = 2.71

$$\left[\frac{16\,c^3x^7-112\,bc^2x^5-350\,b^2cx^3-210\,b^3x+105\left(bc^2x^4+2\,b^2cx^2+b^3\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{48\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)},\ \frac{8\,c^3x^7-56\,bc^2x^5-175\,b^2cx^3-105\,b^3x+105\left(bc^2x^4+2\,b^2cx^2+b^3\right)\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)}{24\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^3*x^7 - 112*b*c^2*x^5 - 350*b^2*c*x^3 - 210*b^3*x + 105*(b*c^2*
x^4 + 2*b^2*c*x^2 + b^3)*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x
^2 + b)))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1/24*(8*c^3*x^7 - 56*b*c^2*x^5
 - 175*b^2*c*x^3 - 105*b^3*x + 105*(b*c^2*x^4 + 2*b^2*c*x^2 + b^3)*sqrt(b/c
)*arctan(c*x*sqrt(b/c)/b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

giac [A] time = 0.16, size = 73, normalized size = 0.86

$$\frac{35\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^4}-\frac{13\,b^2cx^3+11\,b^3x}{8\left(cx^2+b\right)^2c^4}+\frac{c^6x^3-9\,bc^5x}{3\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*b^2*c*x^3 + 11*b^3
*x)/((c*x^2 + b)^2*c^4) + 1/3*(c^6*x^3 - 9*b*c^5*x)/c^9
```

maple [A] time = 0.01, size = 77, normalized size = 0.91

$$-\frac{13b^2x^3}{8\left(c\,x^2+b\right)^2c^3}-\frac{11b^3x}{8\left(c\,x^2+b\right)^2c^4}+\frac{x^3}{3c^3}+\frac{35b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,c^4}-\frac{3bx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/3*x^3/c^3-3*b*x/c^4-13/8/c^3*b^2/(c*x^2+b)^2*x^3-11/8/c^4*b^3/(c*x^2+b)^2
*x+35/8/c^4*b^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.94, size = 82, normalized size = 0.96

$$-\frac{13\,b^2cx^3+11\,b^3x}{8\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}+\frac{35\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^4}+\frac{cx^3-9\,bx}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(13*b^2*c*x^3 + 11*b^3*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 35/8*b^2
*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/3*(c*x^3 - 9*b*x)/c^4
```

mupad [B] time = 4.21, size = 77, normalized size = 0.91

$$\frac{x^3}{3\,c^3}-\frac{\frac{11\,b^3\,x}{8}+\frac{13\,c\,b^2\,x^3}{8}}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,c^{9/2}}-\frac{3\,b\,x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^3/(3*c^3) - ((11*b^3*x)/8 + (13*b^2*c*x^3)/8)/(b^2*c^4 + c^6*x^4 + 2*b*c^
5*x^2) + (35*b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*c^(9/2)) - (3*b*x)/c^4
```

sympy [A] time = 0.50, size = 133, normalized size = 1.56

$$-\frac{3bx}{c^4}-\frac{35\sqrt{-\frac{b^3}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16}+\frac{35\sqrt{-\frac{b^3}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16}+\frac{-11b^3x-13b^2cx^3}{8b^2c^4+16bc^5x^2+8c^6x^4}+\frac{x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*x/c**4 - 35*sqrt(-b**3/c**9)*log(x - c**4*sqrt(-b**3/c**9)/b)/16 + 35*
sqrt(-b**3/c**9)*log(x + c**4*sqrt(-b**3/c**9)/b)/16 + (-11*b**3*x - 13*b**
2*c*x**3)/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4) + x**3/(3*c**3)
```

3.88 $\int \frac{x^{13}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=65

$$\frac{b^3}{4c^4\left(b+cx^2\right)^2}-\frac{3b^2}{2c^4\left(b+cx^2\right)}-\frac{3b\log\left(b+cx^2\right)}{2c^4}+\frac{x^2}{2c^3}$$

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^3}{4c^4\left(b+cx^2\right)^2}-\frac{3b^2}{2c^4\left(b+cx^2\right)}-\frac{3b\log\left(b+cx^2\right)}{2c^4}+\frac{x^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^2/(2*c^3) + b^3/(4*c^4*(b + c*x^2)^2) - (3*b^2)/(2*c^4*(b + c*x^2)) - (3*
b*Log[b + c*x^2])/(2*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^7}{\left(b+cx^2\right)^3}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{c^3} - \frac{b^3}{c^3(b+cx)^3} + \frac{3b^2}{c^3(b+cx)^2} - \frac{3b}{c^3(b+cx)}\right)dx, x, x^2\right)$$
$$= \frac{x^2}{2c^3} + \frac{b^3}{4c^4\left(b+cx^2\right)^2} - \frac{3b^2}{2c^4\left(b+cx^2\right)} - \frac{3b\log\left(b+cx^2\right)}{2c^4}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.74

$$-\frac{\frac{b^2\left(5b+6cx^2\right)}{\left(b+cx^2\right)^2} + 6b\log\left(b+cx^2\right) - 2cx^2}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(b*x^2 + c*x^4)^3,x]
[Out] -1/4*(-2*c*x^2 + (b^2*(5*b + 6*c*x^2))/(b + c*x^2)^2 + 6*b*Log[b + c*x^2])/
c^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^13/(b*x^2 + c*x^4)^3,x]
[Out] IntegrateAlgebraic[x^13/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.53, size = 91, normalized size = 1.40

$$\frac{2\,c^3x^6 + 4\,bc^2x^4 - 4\,b^2cx^2 - 5\,b^3 - 6\left(bc^2x^4 + 2\,b^2cx^2 + b^3\right)\log\left(cx^2+b\right)}{4\left(c^6x^4 + 2\,bc^5x^2 + b^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*x^6 + 4*b*c^2*x^4 - 4*b^2*c*x^2 - 5*b^3 - 6*(b*c^2*x^4 + 2*b^2*c
*x^2 + b^3)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

giac [A] time = 0.19, size = 62, normalized size = 0.95

$$\frac{x^2}{2\,c^3} - \frac{3\,b\log\left(\left|cx^2+b\right|\right)}{2\,c^4} + \frac{9\,bc^2x^4 + 12\,b^2cx^2 + 4\,b^3}{4\left(cx^2+b\right)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^3 - 3/2*b*log(abs(c*x^2 + b))/c^4 + 1/4*(9*b*c^2*x^4 + 12*b^2*c*x
^2 + 4*b^3)/((c*x^2 + b)^2*c^4)
```

maple [A] time = 0.01, size = 58, normalized size = 0.89

$$\frac{b^3}{4\left(c\,x^2+b\right)^2c^4} + \frac{x^2}{2c^3} - \frac{3b^2}{2\left(c\,x^2+b\right)c^4} - \frac{3b\ln\left(c\,x^2+b\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/2*x^2/c^3+1/4*b^3/c^4/(c*x^2+b)^2-3/2*b^2/c^4/(c*x^2+b)-3/2*b*ln(c*x^2+b)
/c^4
```

maxima [A] time = 1.36, size = 66, normalized size = 1.02

$$-\frac{6\,b^2cx^2 + 5\,b^3}{4\left(c^6x^4 + 2\,bc^5x^2 + b^2c^4\right)} + \frac{x^2}{2\,c^3} - \frac{3\,b\log\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*c*x^2 + 5*b^3)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 1/2*x^2/c^3
- 3/2*b*log(c*x^2 + b)/c^4
```

mupad [B] time = 4.26, size = 68, normalized size = 1.05

$$\frac{x^2}{2\,c^3} - \frac{\frac{5\,b^3}{4\,c} + \frac{3\,b^2\,x^2}{2}}{b^2\,c^3 + 2\,b\,c^4\,x^2 + c^5\,x^4} - \frac{3\,b\,\ln\left(c\,x^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^2/(2*c^3) - ((5*b^3)/(4*c) + (3*b^2*x^2)/2)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*
x^2) - (3*b*log(b + c*x^2))/(2*c^4)
```

sympy [A] time = 0.43, size = 68, normalized size = 1.05

$$-\frac{3b\log\left(b+cx^2\right)}{2c^4}+\frac{-5b^3-6b^2cx^2}{4b^2c^4+8bc^5x^2+4c^6x^4}+\frac{x^2}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*log(b + c*x**2)/(2*c**4) + (-5*b**3 - 6*b**2*c*x**2)/(4*b**2*c**4 + 8*
b*c**5*x**2 + 4*c**6*x**4) + x**2/(2*c**3)
```

3.89 $\int \frac{x^{12}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=74

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{15x}{8c^3}$$

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 321, 205}

$$-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}-\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{15x}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*x)/(8*c^3) - x^5/(4*c*(b + c*x^2)^2) - (5*x^3)/(8*c^2*(b + c*x^2)) - (1
5*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^6}{\left(b+cx^2\right)^3}\,dx\\
&= -\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{5\int \frac{x^4}{\left(b+cx^2\right)^2}\,dx}{4c}\\
&= -\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}+\frac{15\int \frac{x^2}{b+cx^2}\,dx}{8c^2}\\
&= \frac{15x}{8c^3}-\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{(15b)\int \frac{1}{b+cx^2}\,dx}{8c^3}\\
&= \frac{15x}{8c^3}-\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.89

$$\frac{15b^2x+25bcx^3+8c^2x^5}{8c^3\left(b+cx^2\right)^2}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*b^2*x + 25*b*c*x^3 + 8*c^2*x^5)/(8*c^3*(b + c*x^2)^2) - (15*Sqrt[b]*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^12/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.71, size = 202, normalized size = 2.73

$$\left[\frac{16\,c^2x^5 + 50\,bcx^3 + 30\,b^2x + 15\left(c^2x^4 + 2\,bcx^2 + b^2\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2 - 2\,cx\sqrt{-\frac{b}{c}} - b}{cx^2 + b}\right)}{16\left(c^5x^4 + 2\,bc^4x^2 + b^2c^3\right)}, \frac{8\,c^2x^5 + 25\,bcx^3 + 15\,b^2x - 15\left(c^2x^4 + 2\,bcx^2 + b^2\right)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)}{8\left(c^5x^4 + 2\,bc^4x^2 + b^2c^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*c^2*x^5 + 50*b*c*x^3 + 30*b^2*x + 15*(c^2*x^4 + 2*b*c*x^2 + b^2)*
sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^5*x^4 + 2*b*
c^4*x^2 + b^2*c^3), 1/8*(8*c^2*x^5 + 25*b*c*x^3 + 15*b^2*x - 15*(c^2*x^4 +
2*b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^4 + 2*b*c^4*x^2
+ b^2*c^3)]
```

giac [A] time = 0.17, size = 54, normalized size = 0.73

$$-\frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^3} + \frac{x}{c^3} + \frac{9\,bcx^3 + 7\,b^2x}{8\left(cx^2 + b\right)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3 + 1/8*(9*b*c*x^3 + 7*
b^2*x)/((c*x^2 + b)^2*c^3)
```

maple [A] time = 0.01, size = 63, normalized size = 0.85

$$\frac{9b\,x^3}{8\left(c\,x^2 + b\right)^2c^2} + \frac{7b^2x}{8\left(c\,x^2 + b\right)^2c^3} - \frac{15b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;c^3} + \frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(c*x^4+b*x^2)^3,x)
```

```
[Out] x/c^3+9/8/c^2*b/(c*x^2+b)^2*x^3+7/8/c^3*b^2/(c*x^2+b)^2*x-15/8/c^3*b/(b*c)^
(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.82, size = 68, normalized size = 0.92

$$\frac{9\,bcx^3+7\,b^2x}{8\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)}-\frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^3}+\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(9*b*c*x^3 + 7*b^2*x)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) - 15/8*b*arctan
(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3
```

mupad [B] time = 4.25, size = 64, normalized size = 0.86

$$\frac{\frac{7\,b^2\,x}{8}+\frac{9\,c\,b\,x^3}{8}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{x}{c^3}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((7*b^2*x)/8 + (9*b*c*x^3)/8)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + x/c^3 - (
15*b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*c^(7/2))
```

sympy [A] time = 0.46, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{b}{c^7}}\,\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{c^7}}\,\log\left(c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}+\frac{7b^2x+9bcx^3}{8b^2c^3+16bc^4x^2+8c^5x^4}+\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-b/c**7)*log(-c**3*sqrt(-b/c**7) + x)/16 - 15*sqrt(-b/c**7)*log(c**
3*sqrt(-b/c**7) + x)/16 + (7*b**2*x + 9*b*c*x**3)/(8*b**2*c**3 + 16*b*c**4*
x**2 + 8*c**5*x**4) + x/c**3
```

3.90 $\int \frac{x^{11}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -b^2/(4*c^3*(b + c*x^2)^2) + b/(c^3*(b + c*x^2)) + Log[b + c*x^2]/(2*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^5}{\left(b+cx^2\right)^3}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2}{c^2(b+cx)^3} - \frac{2b}{c^2(b+cx)^2} + \frac{1}{c^2(b+cx)}\right)dx, x, x^2\right)$$
$$= -\frac{b^2}{4c^3\left(b+cx^2\right)^2} + \frac{b}{c^3\left(b+cx^2\right)} + \frac{\log\left(b+cx^2\right)}{2c^3}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.80

$$\frac{\frac{b\left(3b+4cx^2\right)}{\left(b+cx^2\right)^2} + 2\log\left(b+cx^2\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(b*x^2 + c*x^4)^3,x]
[Out] ((b*(3*b + 4*c*x^2))/(b + c*x^2)^2 + 2*Log[b + c*x^2])/(4*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(b*x^2 + c*x^4)^3,x]
[Out] IntegrateAlgebraic[x^11/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.75, size = 69, normalized size = 1.41

$$\frac{4\,bcx^2 + 3\,b^2 + 2\left(c^2x^4 + 2\,bcx^2 + b^2\right)\log\left(cx^2+b\right)}{4\left(c^5x^4 + 2\,bc^4x^2 + b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*c*x^2 + 3*b^2 + 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b))/(c^5
*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

giac [A] time = 0.18, size = 42, normalized size = 0.86

$$\frac{\log\left(\left|cx^2+b\right|\right)}{2\,c^3}-\frac{3\,cx^4+2\,bx^2}{4\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^2 + b))/c^3 - 1/4*(3*c*x^4 + 2*b*x^2)/((c*x^2 + b)^2*c^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{b^2}{4\left(c\,x^2+b\right)^2c^3}+\frac{b}{\left(c\,x^2+b\right)c^3}+\frac{\ln\left(c\,x^2+b\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4*b^2/c^3/(c*x^2+b)^2+b/c^3/(c*x^2+b)+1/2*ln(c*x^2+b)/c^3
```

maxima [A] time = 1.35, size = 55, normalized size = 1.12

$$\frac{4\,bcx^2+3\,b^2}{4\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)}+\frac{\log\left(cx^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*b*c*x^2 + 3*b^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) + 1/2*log(c*x^2 +
 b)/c^3
```

mupad [B] time = 4.18, size = 52, normalized size = 1.06

$$\frac{\frac{3\,b^2}{4\,c^3}+\frac{b\,x^2}{c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{\ln\left(c\,x^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*b^2)/(4*c^3) + (b*x^2)/c^2)/(b^2 + c^2*x^4 + 2*b*c*x^2) + log(b + c*x^2
)/(2*c^3)
```

sympy [A] time = 0.37, size = 53, normalized size = 1.08

$$\frac{3b^2 + 4bcx^2}{4b^2c^3 + 8bc^4x^2 + 4c^5x^4} + \frac{\log\left(b + cx^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b**2 + 4*b*c*x**2)/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4) + log(b +
 c*x**2)/(2*c**3)
```

3.91 $\int \frac{x^{10}}{\left(bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=64

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}-\frac{3x}{8c^2\left(b+cx^2\right)}-\frac{x^3}{4c\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 288, 205}

$$-\frac{3x}{8c^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}-\frac{x^3}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^3/(4*c*(b + c*x^2)^2) - (3*x)/(8*c^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]*x
)/Sqrt[b]])/(8*Sqrt[b]*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{10}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^4}{\left(b+cx^2\right)^3}\,dx$$
$$= -\frac{x^3}{4c\left(b+cx^2\right)^2} + \frac{3\int \frac{x^2}{\left(b+cx^2\right)^2}\,dx}{4c}$$
$$= -\frac{x^3}{4c\left(b+cx^2\right)^2} - \frac{3x}{8c^2\left(b+cx^2\right)} + \frac{3\int \frac{1}{b+cx^2}\,dx}{8c^2}$$
$$= -\frac{x^3}{4c\left(b+cx^2\right)^2} - \frac{3x}{8c^2\left(b+cx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.86

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}} - \frac{3bx+5cx^3}{8c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*(3*b*x + 5*c*x^3)/(c^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]]
)/(8*Sqrt[b]*c^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^10/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.78, size = 188, normalized size = 2.94

$$\left[-\frac{10\,bc^2x^3+6\,b^2cx+3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{16\left(bc^5x^4+2\,b^2c^4x^2+b^3c^3\right)}, -\frac{5\,bc^2x^3+3\,b^2cx-3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{8\left(bc^5x^4+2\,b^2c^4x^2+b^3c^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*b*c^2*x^3 + 6*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)
*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b*c^5*x^4 + 2*b^2*c^4*x^2
+ b^3*c^3), -1/8*(5*b*c^2*x^3 + 3*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*s
qrt(b*c)*arctan(sqrt(b*c)*x/b))/(b*c^5*x^4 + 2*b^2*c^4*x^2 + b^3*c^3)]
```

giac [A] time = 0.16, size = 45, normalized size = 0.70

$$\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,c^2} - \frac{5\,cx^3 + 3\,bx}{8\left(cx^2 + b\right)^2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) - 1/8*(5*c*x^3 + 3*b*x)/((c*x^2 +
 b)^2*c^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;c^2} + \frac{-\frac{5x^3}{8c} - \frac{3bx}{8c^2}}{\left(c\,x^2 + b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2)^3,x)
```

```
[Out] (-5/8/c*x^3-3/8*b/c^2*x)/(c*x^2+b)^2+3/8/c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/
2)*c*x)
```

maxima [A] time = 2.91, size = 59, normalized size = 0.92

$$-\frac{5\,cx^3 + 3\,bx}{8\left(c^4x^4 + 2\,bc^3x^2 + b^2c^2\right)} + \frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(5*c*x^3 + 3*b*x)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 3/8*arctan(c*x/s
qrt(b*c))/(sqrt(b*c)*c^2)
```

mupad [B] time = 4.23, size = 56, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,\sqrt{b}\,c^{5/2}}-\frac{\frac{5\,x^3}{8\,c}+\frac{3\,b\,x}{8\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*atan((c^(1/2)*x)/b^(1/2)))/(8*b^(1/2)*c^(5/2)) - ((5*x^3)/(8*c) + (3*b*x
)/(8*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

sympy [A] time = 0.37, size = 110, normalized size = 1.72

$$-\frac{3\sqrt{-\frac{1}{bc^5}}\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{bc^5}}\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}+\frac{-3bx-5cx^3}{8b^2c^2+16bc^3x^2+8c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b*c**5))*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/16 + 3*sqrt(-1/(b*c
**5))*log(b*c**2*sqrt(-1/(b*c**5)) + x)/16 + (-3*b*x - 5*c*x**3)/(8*b**2*c*
*2 + 16*b*c**3*x**2 + 8*c**4*x**4)
```

3.92 $\int \frac{x^9}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^4}{4b\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$\frac{x^4}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^4/(4*b*(b + c*x^2)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^3}{\left(b+cx^2\right)^3}\,dx\\ &= \frac{x^4}{4b\left(b+cx^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{b+2cx^2}{4c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^3,x]
[Out] -1/4*(b + 2*c*x^2)/(c^2*(b + c*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(b*x^2 + c*x^4)^3,x]
[Out] IntegrateAlgebraic[x^9/(b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.81, size = 36, normalized size = 1.89

$$-\frac{2\,cx^2+b}{4\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

giac [A] time = 0.16, size = 22, normalized size = 1.16

$$-\frac{2\,cx^2+b}{4\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="giac")
[Out] -1/4*(2*c*x^2 + b)/((c*x^2 + b)^2*c^2)
```

maple [A] time = 0.01, size = 31, normalized size = 1.63

$$\frac{b}{4\left(c\,x^{2}+b\right)^{2}c^{2}}-\frac{1}{2\left(c\,x^{2}+b\right)c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*b/c^2/(c*x^2+b)^2-1/2/c^2/(c*x^2+b)
```

maxima [B] time = 1.33, size = 36, normalized size = 1.89

$$-\frac{2\,cx^{2}+b}{4\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

mupad [B] time = 4.18, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{4\,c^{2}}+\frac{x^{2}}{2\,c}}{b^{2}+2\,b\,c\,x^{2}+c^{2}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4)^3,x)
```

```
[Out] -(b/(4*c^2) + x^2/(2*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

sympy [B] time = 0.31, size = 36, normalized size = 1.89

$$\frac{-b-2cx^{2}}{4b^{2}c^{2}+8bc^{3}x^{2}+4c^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-b - 2*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

3.93 $\int \frac{x^8}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}+\frac{x}{8bc\left(b+cx^2\right)}-\frac{x}{4c\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}+\frac{x}{8bc\left(b+cx^2\right)}-\frac{x}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x/(4*c*(b + c*x^2)^2) + x/(8*b*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]
]/(8*b^(3/2)*c^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^2}{\left(b+cx^2\right)^3}\,dx\\ &= -\frac{x}{4c\left(b+cx^2\right)^2}+\frac{\int \frac{1}{\left(b+cx^2\right)^2}\,dx}{4c}\\ &= -\frac{x}{4c\left(b+cx^2\right)^2}+\frac{x}{8bc\left(b+cx^2\right)}+\frac{\int \frac{1}{b+cx^2}\,dx}{8bc}\\ &= -\frac{x}{4c\left(b+cx^2\right)^2}+\frac{x}{8bc\left(b+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{b}\,\sqrt{c}\,x\left(cx^2-b\right)}{\left(b+cx^2\right)^2}+\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*Sqrt[c]*x*(-b + c*x^2))/(b + c*x^2)^2 + ArcTan[(Sqrt[c]*x)/Sqrt[b
]])/(8*b^(3/2)*c^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^8/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 1.92, size = 190, normalized size = 2.92

$$\left[\frac{2\,bc^{2}x^{3}-2\,b^{2}cx-\left(c^{2}x^{4}+2\,bcx^{2}+b^{2}\right)\sqrt{-bc}\,\log\left(\frac{cx^{2}-2\,\sqrt{-bc}\,x-b}{cx^{2}+b}\right)}{16\,\left(b^{2}c^{4}x^{4}+2\,b^{3}c^{3}x^{2}+b^{4}c^{2}\right)},\ \frac{bc^{2}x^{3}-b^{2}cx+\left(c^{2}x^{4}+2\,bcx^{2}+b^{2}\right)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{8\,\left(b^{2}c^{4}x^{4}+2\,b^{3}c^{3}x^{2}+b^{4}c^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^3 - 2*b^2*c*x - (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)*log
((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2 +
b^4*c^2), 1/8*(b*c^2*x^3 - b^2*c*x + (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(b*c)*
arctan(sqrt(b*c)*x/b))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2 + b^4*c^2)]
```

giac [A] time = 0.16, size = 50, normalized size = 0.77

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,bc}+\frac{cx^{3}-bx}{8\,\left(cx^{2}+b\right)^{2}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) + 1/8*(c*x^3 - b*x)/((c*x^2 + b)^
2*b*c)
```

maple [A] time = 0.01, size = 49, normalized size = 0.75

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;bc}+\frac{\frac{x^{3}}{8b}-\frac{x}{8c}}{\left(c\,x^{2}+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2)^3,x)
```

```
[Out] (1/8/b*x^3-1/8/c*x)/(c*x^2+b)^2+1/8/c/b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*
x)
```

maxima [A] time = 2.86, size = 62, normalized size = 0.95

$$\frac{cx^{3}-bx}{8\,\left(bc^{3}x^{4}+2\,b^{2}c^{2}x^{2}+b^{3}c\right)}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^3 - b*x)/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 1/8*arctan(c*x/sqrt
(b*c))/(sqrt(b*c)*b*c)
```

mupad [B] time = 4.23, size = 55, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{3/2}\,c^{3/2}}-\frac{\frac{x}{8\,c}-\frac{x^3}{8\,b}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2 + c*x^4)^3,x)
```

```
[Out] atan((c^(1/2)*x)/b^(1/2))/(8*b^(3/2)*c^(3/2)) - (x/(8*c) - x^3/(8*b))/(b^2
+ c^2*x^4 + 2*b*c*x^2)
```

sympy [B] time = 0.35, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16}+\frac{-bx+cx^3}{8b^3c+16b^2c^2x^2+8bc^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**3*c**3))*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + sqrt(-1/(b
**3*c**3))*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + (-b*x + c*x**3)/(8*b**
3*c + 16*b**2*c**2*x**2 + 8*b*c**3*x**4)
```

3.94 $\int \frac{x^7}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4c\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$-\frac{1}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(4*c*(b + c*x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x}{\left(b+cx^2\right)^3}\,dx\\ &= -\frac{1}{4c\left(b+cx^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^3,x]
[Out] -1/4*1/(c*(b + c*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(b*x^2 + c*x^4)^3,x]
[Out] IntegrateAlgebraic[x^7/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.80, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(c^3x^4+2\,bc^2x^2+b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(cx^2+b\right)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="giac")
[Out] -1/4/((c*x^2 + b)^2*c)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{4\left(c\,x^{2}+b\right)^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4/c/(c*x^2+b)^2
```

maxima [A] time = 1.27, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(c^{3}x^{4}+2\,bc^{2}x^{2}+b^{2}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

mupad [B] time = 0.03, size = 28, normalized size = 1.75

$$-\frac{1}{4\,b^{2}\,c+8\,b\,c^{2}\,x^{2}+4\,c^{3}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^3,x)
```

```
[Out] -1/(4*b^2*c + 4*c^3*x^4 + 8*b*c^2*x^2)
```

sympy [A] time = 0.27, size = 27, normalized size = 1.69

$$-\frac{1}{4b^{2}c+8bc^{2}x^{2}+4c^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**3,x)
```

```
[Out] -1/(4*b**2*c + 8*b*c**2*x**2 + 4*c**3*x**4)
```

3.95 $\int \frac{x^6}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=62

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{x}{4b\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 199, 205}

$$\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{x}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x/(4*b*(b + c*x^2)^2) + (3*x)/(8*b^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[b]])/(8*b^(5/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{\left(b+cx^2\right)^3}\,dx$$
$$= \frac{x}{4b\left(b+cx^2\right)^2} + \frac{3\int \frac{1}{\left(b+cx^2\right)^2}\,dx}{4b}$$
$$= \frac{x}{4b\left(b+cx^2\right)^2} + \frac{3x}{8b^2\left(b+cx^2\right)} + \frac{3\int \frac{1}{b+cx^2}\,dx}{8b^2}$$
$$= \frac{x}{4b\left(b+cx^2\right)^2} + \frac{3x}{8b^2\left(b+cx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.89

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}} + \frac{5bx+3cx^3}{8b^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*b*x + 3*c*x^3)/(8*b^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(
8*b^(5/2)*Sqrt[c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^6/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 1.19, size = 188, normalized size = 3.03

$$\left[\frac{6\,bc^2x^3+10\,b^2cx-3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{16\left(b^3c^3x^4+2\,b^4c^2x^2+b^5c\right)}, \frac{3\,bc^2x^3+5\,b^2cx+3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{8\left(b^3c^3x^4+2\,b^4c^2x^2+b^5c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*b*c^2*x^3 + 10*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)*
log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^3*c^3*x^4 + 2*b^4*c^2*x^2
 + b^5*c), 1/8*(3*b*c^2*x^3 + 5*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqr
t(b*c)*arctan(sqrt(b*c)*x/b))/(b^3*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)]
```

giac [A] time = 0.20, size = 45, normalized size = 0.73

$$\frac{3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8 \sqrt{bc}\, b^2} + \frac{3\, cx^3 + 5\, bx}{8 \left(cx^2 + b\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/8*(3*c*x^3 + 5*b*x)/((c*x^2 +
 b)^2*b^2)
```

maple [A] time = 0.01, size = 51, normalized size = 0.82

$$\frac{x}{4 \left(c\, x^2 + b\right)^2 b} + \frac{3x}{8 \left(c\, x^2 + b\right) b^2} + \frac{3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\ b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x/b/(c*x^2+b)^2+3/8*x/b^2/(c*x^2+b)+3/8/b^2/(b*c)^(1/2)*arctan(1/(b*c)^
(1/2)*c*x)
```

maxima [A] time = 2.99, size = 58, normalized size = 0.94

$$\frac{3\, cx^3 + 5\, bx}{8 \left(b^2c^2x^4 + 2\, b^3cx^2 + b^4\right)} + \frac{3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8 \sqrt{bc}\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c*x^3 + 5*b*x)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 3/8*arctan(c*x/sq
rt(b*c))/(sqrt(b*c)*b^2)
```

mupad [B] time = 4.21, size = 55, normalized size = 0.89

$$\frac{\frac{5x}{8b}+\frac{3cx^3}{8b^2}}{b^2+2bcx^2+c^2x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{5/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((5*x)/(8*b) + (3*c*x^3)/(8*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) + (3*atan((c^
(1/2)*x)/b^(1/2)))/(8*b^(5/2)*c^(1/2))
```

sympy [A] time = 0.36, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(-b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{5bx+3cx^3}{8b^4+16b^3cx^2+8b^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b**5*c))*log(-b**3*sqrt(-1/(b**5*c)) + x)/16 + 3*sqrt(-1/(b**5*
c))*log(b**3*sqrt(-1/(b**5*c)) + x)/16 + (5*b*x + 3*c*x**3)/(8*b**4 + 16*b*
*3*c*x**2 + 8*b**2*c**2*x**4)
```

3.96 $\int \frac{x^5}{\left(bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=54

$$-\frac{\log\left(b+cx^2\right)}{2b^3}+\frac{\log(x)}{b^3}+\frac{1}{2b^2\left(b+cx^2\right)}+\frac{1}{4b\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$\frac{1}{2b^2\left(b+cx^2\right)}-\frac{\log\left(b+cx^2\right)}{2b^3}+\frac{\log(x)}{b^3}+\frac{1}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] 1/(4*b*(b + c*x^2)^2) + 1/(2*b^2*(b + c*x^2)) + Log[x]/b^3 - Log[b + c*x^2]
/(2*b^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x\left(b+cx^2\right)^3}\,dx\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x(b+cx)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{1}{b^3x}-\frac{c}{b(b+cx)^3}-\frac{c}{b^2(b+cx)^2}-\frac{c}{b^3(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{1}{4b\left(b+cx^2\right)^2}+\frac{1}{2b^2\left(b+cx^2\right)}+\frac{\log(x)}{b^3}-\frac{\log\left(b+cx^2\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.80

$$\frac{\frac{b\left(3b+2cx^2\right)}{\left(b+cx^2\right)^2}-2\log\left(b+cx^2\right)+4\log(x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(3*b + 2*c*x^2))/(b + c*x^2)^2 + 4*Log[x] - 2*Log[b + c*x^2])/(4*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^5/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 2.31, size = 90, normalized size = 1.67

$$\frac{2\,bcx^2+3\,b^2-2\left(c^2x^4+2\,bcx^2+b^2\right)\log\left(cx^2+b\right)+4\left(c^2x^4+2\,bcx^2+b^2\right)\log(x)}{4\left(b^3c^2x^4+2\,b^4cx^2+b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^2 + 3*b^2 - 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b) + 4*(
c^2*x^4 + 2*b*c*x^2 + b^2)*log(x))/(b^3*c^2*x^4 + 2*b^4*c*x^2 + b^5)
```

giac [A] time = 0.16, size = 59, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,b^3} - \frac{\log\left(\left|cx^2+b\right|\right)}{2\,b^3} + \frac{3\,c^2x^4 + 8\,bcx^2 + 6\,b^2}{4\left(cx^2+b\right)^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/b^3 - 1/2*log(abs(c*x^2 + b))/b^3 + 1/4*(3*c^2*x^4 + 8*b*c*x^2
 + 6*b^2)/((c*x^2 + b)^2*b^3)
```

maple [A] time = 0.01, size = 49, normalized size = 0.91

$$\frac{1}{4\left(c\,x^2+b\right)^2 b} + \frac{1}{2\left(c\,x^2+b\right)b^2} + \frac{\ln(x)}{b^3} - \frac{\ln\left(c\,x^2+b\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4/b/(c*x^2+b)^2+1/2/b^2/(c*x^2+b)+ln(x)/b^3-1/2*ln(c*x^2+b)/b^3
```

maxima [A] time = 1.38, size = 60, normalized size = 1.11

$$\frac{2\,cx^2 + 3\,b}{4\left(b^2c^2x^4 + 2\,b^3cx^2 + b^4\right)} - \frac{\log\left(cx^2+b\right)}{2\,b^3} + \frac{\log\left(x^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*c*x^2 + 3*b)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) - 1/2*log(c*x^2 + b)/
b^3 + 1/2*log(x^2)/b^3
```

mupad [B] time = 0.06, size = 56, normalized size = 1.04

$$\frac{\ln(x)}{b^3} + \frac{\frac{3}{4\,b} + \frac{c\,x^2}{2\,b^2}}{b^2 + 2\,b\,c\,x^2 + c^2\,x^4} - \frac{\ln\left(c\,x^2+b\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^3,x)
```

```
[Out] log(x)/b^3 + (3/(4*b) + (c*x^2)/(2*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - log(
b + c*x^2)/(2*b^3)
```

sympy **[A]** time = 0.45, size = 56, normalized size = 1.04

$$\frac{3b+2cx^2}{4b^4+8b^3cx^2+4b^2c^2x^4}+\frac{\log(x)}{b^3}-\frac{\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b + 2*c*x**2)/(4*b**4 + 8*b**3*c*x**2 + 4*b**2*c**2*x**4) + log(x)/b**3
- log(b/c + x**2)/(2*b**3)
```

3.97 $\int \frac{x^4}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=76

$$-\frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{15}{8b^3x}+\frac{5}{8b^2x\left(b+cx^2\right)}+\frac{1}{4bx\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{15}{8b^3x}+\frac{1}{4bx\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -15/(8*b^3*x) + 1/(4*b*x*(b + c*x^2)^2) + 5/(8*b^2*x*(b + c*x^2)) - (15*Sqr
t[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^2\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5\int \frac{1}{x^2\left(b+cx^2\right)^2}\,dx}{4b}\\
&= \frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5}{8b^2x\left(b+cx^2\right)}+\frac{15\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{8b^2}\\
&= -\frac{15}{8b^3x}+\frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{(15c)\int \frac{1}{b+cx^2}\,dx}{8b^3}\\
&= -\frac{15}{8b^3x}+\frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.89

$$
-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{8b^2+25bcx^2+15c^2x^4}{8b^3x\left(b+cx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*(8*b^2 + 25*b*c*x^2 + 15*c^2*x^4)/(b^3*x*(b + c*x^2)^2) - (15*Sqrt[c]*
ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^4}{\left(bx^2+cx^4\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^4/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 2.47, size = 202, normalized size = 2.66

$$\left[-\frac{30\,c^2x^4 + 50\,bcx^2 - 15\left(c^2x^5 + 2\,bcx^3 + b^2x\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2 - 2\,bx\sqrt{-\frac{c}{b}} - b}{cx^2 + b}\right) + 16\,b^2}{16\left(b^3c^2x^5 + 2\,b^4cx^3 + b^5x\right)}, -\frac{15\,c^2x^4 + 25\,bcx^2 + 15\left(c^2x^5 + 2\,bcx^3 + b^2x\right)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right) + 8\,b^2}{8\left(b^3c^2x^5 + 2\,b^4cx^3 + b^5x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*c^2*x^4 + 50*b*c*x^2 - 15*(c^2*x^5 + 2*b*c*x^3 + b^2*x)*sqrt(-c/
b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) + 16*b^2)/(b^3*c^2*x^5 +
 2*b^4*c*x^3 + b^5*x), -1/8*(15*c^2*x^4 + 25*b*c*x^2 + 15*(c^2*x^5 + 2*b*c*
x^3 + b^2*x)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 8*b^2)/(b^3*c^2*x^5 + 2*b^4*c*
x^3 + b^5*x)]
```

giac [A] time = 0.17, size = 57, normalized size = 0.75

$$-\frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^3} - \frac{7\,c^2x^3 + 9\,bcx}{8\left(cx^2 + b\right)^2b^3} - \frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/8*(7*c^2*x^3 + 9*b*c*x)/(
(c*x^2 + b)^2*b^3) - 1/(b^3*x)
```

maple [A] time = 0.01, size = 66, normalized size = 0.87

$$-\frac{7c^2x^3}{8\left(c\,x^2 + b\right)^2b^3} - \frac{9cx}{8\left(c\,x^2 + b\right)^2b^2} - \frac{15c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,b^3} - \frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^3,x)
```

```
[Out] -7/8/b^3*c^2/(c*x^2+b)^2*x^3-9/8/b^2*c/(c*x^2+b)^2*x-15/8/b^3*c/(b*c)^(1/2)
*arctan(1/(b*c)^(1/2)*c*x)-1/b^3/x
```

maxima [A] time = 3.00, size = 71, normalized size = 0.93

$$-\frac{15\,c^2x^4+25\,bcx^2+8\,b^2}{8\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)}-\frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*c^2*x^4 + 25*b*c*x^2 + 8*b^2)/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)
- 15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3)
```

mupad [B] time = 4.26, size = 66, normalized size = 0.87

$$-\frac{\frac{1}{b}+\frac{25\,c\,x^2}{8\,b^2}+\frac{15\,c^2\,x^4}{8\,b^3}}{b^2\,x+2\,b\,c\,x^3+c^2\,x^5}-\frac{15\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (1/b + (25*c*x^2)/(8*b^2) + (15*c^2*x^4)/(8*b^3))/(b^2*x + c^2*x^5 + 2*b*
c*x^3) - (15*c^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*b^(7/2))
```

sympy [A] time = 0.45, size = 116, normalized size = 1.53

$$\frac{15\sqrt{-\frac{c}{b^7}}\,\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16}-\frac{15\sqrt{-\frac{c}{b^7}}\,\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16}+\frac{-8b^2-25bcx^2-15c^2x^4}{8b^5x+16b^4cx^3+8b^3c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-c/b**7)*log(-b**4*sqrt(-c/b**7)/c + x)/16 - 15*sqrt(-c/b**7)*log(b
**4*sqrt(-c/b**7)/c + x)/16 + (-8*b**2 - 25*b*c*x**2 - 15*c**2*x**4)/(8*b**
5*x + 16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

3.98 $\int \frac{x^3}{\left(bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=67

$$\frac{3c\log\left(b+cx^2\right)}{2b^4}-\frac{3c\log(x)}{b^4}-\frac{c}{b^3\left(b+cx^2\right)}-\frac{1}{2b^3x^2}-\frac{c}{4b^2\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{c}{b^3\left(b+cx^2\right)}-\frac{c}{4b^2\left(b+cx^2\right)^2}+\frac{3c\log\left(b+cx^2\right)}{2b^4}-\frac{3c\log(x)}{b^4}-\frac{1}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(2*b^3*x^2) - c/(4*b^2*(b + c*x^2)^2) - c/(b^3*(b + c*x^2)) - (3*c*Log[x
])/b^4 + (3*c*Log[b + c*x^2])/(2*b^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^3\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(b+cx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^3x^2}-\frac{3c}{b^4x}+\frac{c^2}{b^2(b+cx)^3}+\frac{2c^2}{b^3(b+cx)^2}+\frac{3c^2}{b^4(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{1}{2b^3x^2}-\frac{c}{4b^2\left(b+cx^2\right)^2}-\frac{c}{b^3\left(b+cx^2\right)}-\frac{3c\log(x)}{b^4}+\frac{3c\log\left(b+cx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.88

$$
-\frac{\frac{b\left(2b^2+9bcx^2+6c^2x^4\right)}{x^2\left(b+cx^2\right)^2}-6c\log\left(b+cx^2\right)+12c\log(x)}{4b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*(2*b^2 + 9*b*c*x^2 + 6*c^2*x^4))/(x^2*(b + c*x^2)^2) + 12*c*Log[x]
 - 6*c*Log[b + c*x^2])/b^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3}{\left(bx^2+cx^4\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^3/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.79, size = 119, normalized size = 1.78

$$
-\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3-6\left(c^3x^6+2\,bc^2x^4+b^2cx^2\right)\log\left(cx^2+b\right)+12\left(c^3x^6+2\,bc^2x^4+b^2cx^2\right)\log(x)}{4\left(b^4c^2x^6+2\,b^5cx^4+b^6x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3 - 6*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c*
x^2)*log(c*x^2 + b) + 12*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c*x^2)*log(x))/(b^4*c
^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)
```

giac [A] time = 0.17, size = 66, normalized size = 0.99

$$\frac{3\,c\log\left(\left|cx^2+b\right|\right)}{2\,b^4}-\frac{3\,c\log\left(|x|\right)}{b^4}-\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3}{4\left(cx^2+b\right)^2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/2*c*log(abs(c*x^2 + b))/b^4 - 3*c*log(abs(x))/b^4 - 1/4*(6*b*c^2*x^4 + 9*
b^2*c*x^2 + 2*b^3)/((c*x^2 + b)^2*b^4*x^2)
```

maple [A] time = 0.02, size = 62, normalized size = 0.93

$$-\frac{c}{4\left(c\,x^2+b\right)^2b^2}-\frac{c}{\left(c\,x^2+b\right)b^3}-\frac{3c\ln(x)}{b^4}+\frac{3c\ln\left(c\,x^2+b\right)}{2b^4}-\frac{1}{2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2/b^3/x^2-1/4*c/b^2/(c*x^2+b)^2-c/b^3/(c*x^2+b)-3*c*ln(x)/b^4+3/2*c*ln(c
*x^2+b)/b^4
```

maxima [A] time = 1.38, size = 77, normalized size = 1.15

$$-\frac{6\,c^2x^4+9\,bcx^2+2\,b^2}{4\left(b^3c^2x^6+2\,b^4cx^4+b^5x^2\right)}+\frac{3\,c\log\left(cx^2+b\right)}{2\,b^4}-\frac{3\,c\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*c^2*x^4 + 9*b*c*x^2 + 2*b^2)/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)
+ 3/2*c*log(c*x^2 + b)/b^4 - 3/2*c*log(x^2)/b^4
```

mupad [B] time = 0.06, size = 75, normalized size = 1.12

$$\frac{3\,c\,\ln\left(c\,x^2+b\right)}{2\,b^4}-\frac{\frac{1}{2\,b}+\frac{9\,c\,x^2}{4\,b^2}+\frac{3\,c^2\,x^4}{2\,b^3}}{b^2\,x^2+2\,b\,c\,x^4+c^2\,x^6}-\frac{3\,c\,\ln(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*c*log(b + c*x^2))/(2*b^4) - (1/(2*b) + (9*c*x^2)/(4*b^2) + (3*c^2*x^4)/(
2*b^3))/(b^2*x^2 + c^2*x^6 + 2*b*c*x^4) - (3*c*log(x))/b^4
```

sympy **[A]** time = 0.63, size = 80, normalized size = 1.19

$$\frac{-2b^2 - 9bcx^2 - 6c^2x^4}{4b^5x^2 + 8b^4cx^4 + 4b^3c^2x^6} - \frac{3c\log(x)}{b^4} + \frac{3c\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*b**2 - 9*b*c*x**2 - 6*c**2*x**4)/(4*b**5*x**2 + 8*b**4*c*x**4 + 4*b**3*
c**2*x**6) - 3*c*log(x)/b**4 + 3*c*log(b/c + x**2)/(2*b**4)
```

3.99 $\int \frac{x^2}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=87

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{35c}{8b^4x}-\frac{35}{24b^3x^3}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{1}{4bx^3\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{35c}{8b^4x}-\frac{35}{24b^3x^3}+\frac{1}{4bx^3\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -35/(24*b^3*x^3) + (35*c)/(8*b^4*x) + 1/(4*b*x^3*(b + c*x^2)^2) + 7/(8*b^2*
x^3*(b + c*x^2)) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^4\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7\int \frac{1}{x^4\left(b+cx^2\right)^2}\,dx}{4b}\\
&= \frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{35\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{8b^2}\\
&= -\frac{35}{24b^3x^3}+\frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7}{8b^2x^3\left(b+cx^2\right)}-\frac{(35c)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{8b^3}\\
&= -\frac{35}{24b^3x^3}+\frac{35c}{8b^4x}+\frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{\left(35c^2\right)\int \frac{1}{b+cx^2}\,dx}{8b^4}\\
&= -\frac{35}{24b^3x^3}+\frac{35c}{8b^4x}+\frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.91

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^4x^3\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-8*b^3 + 56*b^2*c*x^2 + 175*b*c^2*x^4 + 105*c^3*x^6)/(24*b^4*x^3*(b + c*x^
2)^2) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 1.17, size = 238, normalized size = 2.74

$$\left[\frac{210\,c^3x^6+350\,bc^2x^4+112\,b^2cx^2-16\,b^3+105\,\left(c^3x^7+2\,bc^2x^5+b^2cx^3\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{48\,\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)},\ \frac{105\,c^3x^6+175\,bc^2x^4+56\,b^2cx^2-8\,b^3+105\,\left(c^3x^7+2\,bc^2x^5+b^2cx^3\right)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)}{24\,\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(210*c^3*x^6 + 350*b*c^2*x^4 + 112*b^2*c*x^2 - 16*b^3 + 105*(c^3*x^7
+ 2*b*c^2*x^5 + b^2*c*x^3)*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c
*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3), 1/24*(105*c^3*x^6 + 175*
b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3 + 105*(c^3*x^7 + 2*b*c^2*x^5 + b^2*c*x^3)*
sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

giac [A] time = 0.18, size = 71, normalized size = 0.82

$$\frac{35\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^4}+\frac{11\,c^3x^3+13\,bc^2x}{8\,\left(cx^2+b\right)^2b^4}+\frac{9\,cx^2-b}{3\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/8*(11*c^3*x^3 + 13*b*c^2
*x)/((c*x^2 + b)^2*b^4) + 1/3*(9*c*x^2 - b)/(b^4*x^3)
```

maple [A] time = 0.01, size = 79, normalized size = 0.91

$$\frac{11c^3x^3}{8\left(c\,x^2+b\right)^2b^4}+\frac{13c^2x}{8\left(c\,x^2+b\right)^2b^3}+\frac{35c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;b^4}+\frac{3c}{b^4x}-\frac{1}{3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^3,x)
```

```
[Out] 11/8/b^4*c^3/(c*x^2+b)^2*x^3+13/8/b^3*c^2/(c*x^2+b)^2*x+35/8/b^4*c^2/(b*c)^
(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/3/b^3/x^3+3*c/b^4/x
```

maxima [A] time = 2.93, size = 86, normalized size = 0.99

$$\frac{105\,c^3x^6 + 175\,bc^2x^4 + 56\,b^2cx^2 - 8\,b^3}{24\,\left(b^4c^2x^7 + 2\,b^5cx^5 + b^6x^3\right)} + \frac{35\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(105*c^3*x^6 + 175*b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3)/(b^4*c^2*x^7 + 2*
b^5*c*x^5 + b^6*x^3) + 35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

mupad [B] time = 4.26, size = 80, normalized size = 0.92

$$\frac{\frac{7\,c\,x^2}{3\,b^2} - \frac{1}{3\,b} + \frac{175\,c^2\,x^4}{24\,b^3} + \frac{35\,c^3\,x^6}{8\,b^4}}{b^2\,x^3 + 2\,b\,c\,x^5 + c^2\,x^7} + \frac{35\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((7*c*x^2)/(3*b^2) - 1/(3*b) + (175*c^2*x^4)/(24*b^3) + (35*c^3*x^6)/(8*b^4
))/(b^2*x^3 + c^2*x^7 + 2*b*c*x^5) + (35*c^(3/2)*atan((c^(1/2)*x)/b^(1/2)))
/(8*b^(9/2))
```

sympy [A] time = 0.50, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2} + x\right)}{16} + \frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2} + x\right)}{16} + \frac{-8b^3 + 56b^2cx^2 + 175bc^2x^4 + 105c^3x^6}{24b^6x^3 + 48b^5cx^5 + 24b^4c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**3,x)
```

```
[Out] -35*sqrt(-c**3/b**9)*log(-b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + 35*sqrt(-c**
3/b**9)*log(b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + (-8*b**3 + 56*b**2*c*x**2
+ 175*b*c**2*x**4 + 105*c**3*x**6)/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*b**4
*c**2*x**7)
```

3.100 $\int \frac{x}{\left(bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=86

$$-\frac{3c^2\log\left(b+cx^2\right)}{b^5}+\frac{6c^2\log(x)}{b^5}+\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}-\frac{1}{4b^3x^4}$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}-\frac{3c^2\log\left(b+cx^2\right)}{b^5}+\frac{6c^2\log(x)}{b^5}+\frac{3c}{2b^4x^2}-\frac{1}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(4*b^3*x^4) + (3*c)/(2*b^4*x^2) + c^2/(4*b^3*(b + c*x^2)^2) + (3*c^2)/(2
*b^4*(b + c*x^2)) + (6*c^2*Log[x])/b^5 - (3*c^2*Log[b + c*x^2])/b^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^5\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(b+cx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^3x^3}-\frac{3c}{b^4x^2}+\frac{6c^2}{b^5x}-\frac{c^3}{b^3(b+cx)^3}-\frac{3c^3}{b^4(b+cx)^2}-\frac{6c^3}{b^5(b+cx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{4b^3x^4}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}+\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{6c^2\log(x)}{b^5}-\frac{3c^2\log\left(b+cx^2\right)}{b^5}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.86

$$
\frac{\frac{b\left(-b^3+4b^2cx^2+18bc^2x^4+12c^3x^6\right)}{x^4\left(b+cx^2\right)^2}-12c^2\log\left(b+cx^2\right)+24c^2\log(x)}{4b^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(-b^3 + 4*b^2*c*x^2 + 18*b*c^2*x^4 + 12*c^3*x^6))/(x^4*(b + c*x^2)^2) +
 24*c^2*Log[x] - 12*c^2*Log[b + c*x^2])/(4*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x}{\left(bx^2+cx^4\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x/(b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.73, size = 134, normalized size = 1.56

$$
\frac{12\,bc^3x^6+18\,b^2c^2x^4+4\,b^3cx^2-b^4-12\left(c^4x^8+2\,bc^3x^6+b^2c^2x^4\right)\log\left(cx^2+b\right)+24\left(c^4x^8+2\,bc^3x^6+b^2c^2x^4\right)\log(x)}{4\left(b^5c^2x^8+2\,b^6cx^6+b^7x^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*b*c^3*x^6 + 18*b^2*c^2*x^4 + 4*b^3*c*x^2 - b^4 - 12*(c^4*x^8 + 2*b*
c^3*x^6 + b^2*c^2*x^4)*log(c*x^2 + b) + 24*(c^4*x^8 + 2*b*c^3*x^6 + b^2*c^2
*x^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

giac [A] time = 0.16, size = 79, normalized size = 0.92

$$-\frac{3\,c^2\log\left(\left|cx^2+b\right|\right)}{b^5}+\frac{6\,c^2\log\left(|x|\right)}{b^5}+\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2-b^3}{4\left(cx^4+bx^2\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*c^2*log(abs(c*x^2 + b))/b^5 + 6*c^2*log(abs(x))/b^5 + 1/4*(12*c^3*x^6 +
18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/((c*x^4 + b*x^2)^2*b^4)
```

maple [A] time = 0.01, size = 79, normalized size = 0.92

$$\frac{c^2}{4\left(c\,x^2+b\right)^2b^3}+\frac{3c^2}{2\left(c\,x^2+b\right)b^4}+\frac{6c^2\ln(x)}{b^5}-\frac{3c^2\ln\left(c\,x^2+b\right)}{b^5}+\frac{3c}{2b^4x^2}-\frac{1}{4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4/b^3/x^4+3/2*c/b^4/x^2+1/4*c^2/b^3/(c*x^2+b)^2+3/2*c^2/b^4/(c*x^2+b)+6*
c^2*ln(x)/b^5-3*c^2*ln(c*x^2+b)/b^5
```

maxima [A] time = 1.35, size = 92, normalized size = 1.07

$$\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2-b^3}{4\left(b^4c^2x^8+2\,b^5cx^6+b^6x^4\right)}-\frac{3\,c^2\log\left(cx^2+b\right)}{b^5}+\frac{3\,c^2\log\left(x^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/(b^4*c^2*x^8 + 2*b^5*c*
x^6 + b^6*x^4) - 3*c^2*log(c*x^2 + b)/b^5 + 3*c^2*log(x^2)/b^5
```

mupad [B] time = 4.25, size = 88, normalized size = 1.02

$$\frac{\frac{c\,x^2}{b^2}-\frac{1}{4\,b}+\frac{9\,c^2\,x^4}{2\,b^3}+\frac{3\,c^3\,x^6}{b^4}}{b^2\,x^4+2\,b\,c\,x^6+c^2\,x^8}-\frac{3\,c^2\,\ln\left(c\,x^2+b\right)}{b^5}+\frac{6\,c^2\,\ln(x)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((c*x^2)/b^2 - 1/(4*b) + (9*c^2*x^4)/(2*b^3) + (3*c^3*x^6)/b^4)/(b^2*x^4 +
c^2*x^8 + 2*b*c*x^6) - (3*c^2*log(b + c*x^2))/b^5 + (6*c^2*log(x))/b^5
```

sympy [A] time = 0.57, size = 90, normalized size = 1.05

$$\frac{-b^3+4b^2cx^2+18bc^2x^4+12c^3x^6}{4b^6x^4+8b^5cx^6+4b^4c^2x^8}+\frac{6c^2\log(x)}{b^5}-\frac{3c^2\log\left(\frac{b}{c}+x^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-b**3 + 4*b**2*c*x**2 + 18*b*c**2*x**4 + 12*c**3*x**6)/(4*b**6*x**4 + 8*b*
*5*c*x**6 + 4*b**4*c**2*x**8) + 6*c**2*log(x)/b**5 - 3*c**2*log(b/c + x**2)
/b**5
```

3.101 $\int \frac{1}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{63c^2}{8b^5x}+\frac{21c}{8b^4x^3}-\frac{63}{40b^3x^5}+\frac{9}{8b^2x^5\left(b+cx^2\right)}+\frac{1}{4bx^5\left(b+cx^2\right)^2}$$

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1593, 290, 325, 205}

$$-\frac{63c^2}{8b^5x}-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}+\frac{21c}{8b^4x^3}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{63}{40b^3x^5}+\frac{1}{4bx^5\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] -63/(40*b^3*x^5) + (21*c)/(8*b^4*x^3) - (63*c^2)/(8*b^5*x) + 1/(4*b*x^5*(b
+ c*x^2)^2) + 9/(8*b^2*x^5*(b + c*x^2)) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(8*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^6\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^5\left(b+cx^2\right)^2} + \frac{9\int \frac{1}{x^6\left(b+cx^2\right)^2}\,dx}{4b}$$

$$= \frac{1}{4bx^5\left(b+cx^2\right)^2} + \frac{9}{8b^2x^5\left(b+cx^2\right)} + \frac{63\int \frac{1}{x^6\left(b+cx^2\right)}\,dx}{8b^2}$$

$$= -\frac{63}{40b^3x^5} + \frac{1}{4bx^5\left(b+cx^2\right)^2} + \frac{9}{8b^2x^5\left(b+cx^2\right)} - \frac{(63c)\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{8b^3}$$

$$= -\frac{63}{40b^3x^5} + \frac{21c}{8b^4x^3} + \frac{1}{4bx^5\left(b+cx^2\right)^2} + \frac{9}{8b^2x^5\left(b+cx^2\right)} + \frac{\left(63c^2\right)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{8b^4}$$

$$= -\frac{63}{40b^3x^5} + \frac{21c}{8b^4x^3} - \frac{63c^2}{8b^5x} + \frac{1}{4bx^5\left(b+cx^2\right)^2} + \frac{9}{8b^2x^5\left(b+cx^2\right)} - \frac{\left(63c^3\right)\int \frac{1}{b+cx^2}\,dx}{8b^5}$$

$$= -\frac{63}{40b^3x^5} + \frac{21c}{8b^4x^3} - \frac{63c^2}{8b^5x} + \frac{1}{4bx^5\left(b+cx^2\right)^2} + \frac{9}{8b^2x^5\left(b+cx^2\right)} - \frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.90

$$-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}} - \frac{8b^4-24b^3cx^2+168b^2c^2x^4+525bc^3x^6+315c^4x^8}{40b^5x^5\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] -1/40*(8*b^4 - 24*b^3*c*x^2 + 168*b^2*c^2*x^4 + 525*b*c^3*x^6 + 315*c^4*x^8
)/(b^5*x^5*(b + c*x^2)^2) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(
11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(b*x^2 + c*x^4)^(-3), x]
```

fricas [A] time = 2.16, size = 264, normalized size = 2.64

$$\left[-\frac{630\,c^4x^8+1050\,bc^3x^6+336\,b^2c^2x^4-48\,b^3cx^2+16\,b^4-315\left(c^4x^9+2\,bc^3x^7+b^2c^2x^5\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{80\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)},\ -\frac{315\,c^4x^8+525\,bc^3x^6+168\,b^2c^2x^4-24\,b^3cx^2+8\,b^4+315\left(c^4x^9+2\,bc^3x^7+b^2c^2x^5\right)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)}{40\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(630*c^4*x^8 + 1050*b*c^3*x^6 + 336*b^2*c^2*x^4 - 48*b^3*c*x^2 + 16*
b^4 - 315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b
*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5), -1/
40*(315*c^4*x^8 + 525*b*c^3*x^6 + 168*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^4 +
315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b
^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)]
```

giac [A] time = 0.18, size = 80, normalized size = 0.80

$$-\frac{63\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^5}-\frac{15\,c^4x^3+17\,bc^3x}{8\left(cx^2+b\right)^2b^5}-\frac{30\,c^2x^4-5\,bcx^2+b^2}{5\,b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -63/8*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) - 1/8*(15*c^4*x^3 + 17*b*c^
3*x)/((c*x^2 + b)^2*b^5) - 1/5*(30*c^2*x^4 - 5*b*c*x^2 + b^2)/(b^5*x^5)
```

maple [A] time = 0.02, size = 89, normalized size = 0.89

$$-\frac{15c^4x^3}{8\left(c\,x^2+b\right)^2 b^5}-\frac{17c^3x}{8\left(c\,x^2+b\right)^2 b^4}-\frac{63c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;b^5}-\frac{6c^2}{b^5x}+\frac{c}{b^4x^3}-\frac{1}{5b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^3,x)
```

```
[Out] -15/8/b^5*c^4/(c*x^2+b)^2*x^3-17/8/b^4*c^3/(c*x^2+b)^2*x-63/8/b^5*c^3/(b*c)
^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/5/b^3/x^5-6*c^2/b^5/x+c/b^4/x^3
```

maxima [A] time = 2.97, size = 97, normalized size = 0.97

$$-\frac{315\,c^4x^8+525\,bc^3x^6+168\,b^2c^2x^4-24\,b^3cx^2+8\,b^4}{40\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)}-\frac{63\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(315*c^4*x^8 + 525*b*c^3*x^6 + 168*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^4
)/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5) - 63/8*c^3*arctan(c*x/sqrt(b*c))/(s
qrt(b*c)*b^5)
```

mupad [B] time = 4.24, size = 92, normalized size = 0.92

$$-\frac{\frac{1}{5\,b}-\frac{3\,c\,x^2}{5\,b^2}+\frac{21\,c^2\,x^4}{5\,b^3}+\frac{105\,c^3\,x^6}{8\,b^4}+\frac{63\,c^4\,x^8}{8\,b^5}}{b^2\,x^5+2\,b\,c\,x^7+c^2\,x^9}-\frac{63\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (1/(5*b) - (3*c*x^2)/(5*b^2) + (21*c^2*x^4)/(5*b^3) + (105*c^3*x^6)/(8*b^
4) + (63*c^4*x^8)/(8*b^5))/(b^2*x^5 + c^2*x^9 + 2*b*c*x^7) - (63*c^(5/2)*at
an((c^(1/2)*x)/b^(1/2)))/(8*b^(11/2))
```

sympy [A] time = 0.58, size = 150, normalized size = 1.50

$$\frac{63\sqrt{-\frac{c^5}{b^{11}}}\,\log\left(-\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16}-\frac{63\sqrt{-\frac{c^5}{b^{11}}}\,\log\left(\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16}+\frac{-8b^4+24b^3cx^2-168b^2c^2x^4-525bc^3x^6-315c^4x^8}{40b^7x^5+80b^6cx^7+40b^5c^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**3,x)
```

```
[Out] 63*sqrt(-c**5/b**11)*log(-b**6*sqrt(-c**5/b**11)/c**3 + x)/16 - 63*sqrt(-c*
*5/b**11)*log(b**6*sqrt(-c**5/b**11)/c**3 + x)/16 + (-8*b**4 + 24*b**3*c*x*
*2 - 168*b**2*c**2*x**4 - 525*b*c**3*x**6 - 315*c**4*x**8)/(40*b**7*x**5 +
80*b**6*c*x**7 + 40*b**5*c**2*x**9)
```

3.102 $\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=95

$$\frac{5c^3\log\left(b+cx^2\right)}{b^6}-\frac{10c^3\log(x)}{b^6}-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{3c^2}{b^5x^2}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}+\frac{3c}{4b^4x^4}-\frac{1}{6b^3x^6}$$

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}-\frac{3c^2}{b^5x^2}+\frac{5c^3\log\left(b+cx^2\right)}{b^6}-\frac{10c^3\log(x)}{b^6}+\frac{3c}{4b^4x^4}-\frac{1}{6b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/(6*b^3*x^6) + (3*c)/(4*b^4*x^4) - (3*c^2)/(b^5*x^2) - c^3/(4*b^4*(b + c*
x^2)^2) - (2*c^3)/(b^5*(b + c*x^2)) - (10*c^3*Log[x])/b^6 + (5*c^3*Log[b +
c*x^2])/b^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^7\left(b+cx^2\right)^3}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^3x^4} - \frac{3c}{b^4x^3} + \frac{6c^2}{b^5x^2} - \frac{10c^3}{b^6x} + \frac{c^4}{b^4(b+cx)^3} + \frac{4c^4}{b^5(b+cx)^2} + \frac{10c^4}{b^6(b+cx)}\right) dx, x$$
$$= -\frac{1}{6b^3x^6} + \frac{3c}{4b^4x^4} - \frac{3c^2}{b^5x^2} - \frac{c^3}{4b^4\left(b+cx^2\right)^2} - \frac{2c^3}{b^5\left(b+cx^2\right)} - \frac{10c^3\log(x)}{b^6} + \frac{5c^3\log\left(b+cx^2\right.}{b^6}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.89

$$-\frac{\frac{b\left(2b^4-5b^3cx^2+20b^2c^2x^4+90bc^3x^6+60c^4x^8\right)}{x^6\left(b+cx^2\right)^2} - 60c^3\log\left(b+cx^2\right) + 120c^3\log(x)}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/12*((b*(2*b^4 - 5*b^3*c*x^2 + 20*b^2*c^2*x^4 + 90*b*c^3*x^6 + 60*c^4*x^8
))/(x^6*(b + c*x^2)^2) + 120*c^3*Log[x] - 60*c^3*Log[b + c*x^2])/b^6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(b*x^2 + c*x^4)^3), x]
```

fricas [A] time = 1.92, size = 145, normalized size = 1.53

$$-\frac{60\,bc^4x^8 + 90\,b^2c^3x^6 + 20\,b^3c^2x^4 - 5\,b^4cx^2 + 2\,b^5 - 60\left(c^5x^{10} + 2\,bc^4x^8 + b^2c^3x^6\right)\log\left(cx^2+b\right) + 120\left(c^5x^{10} + 2\,bc^4x^8 + b^2c^3x^6\right)\log(x)}{12\left(b^6c^2x^{10} + 2\,b^7cx^8 + b^8x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*b*c^4*x^8 + 90*b^2*c^3*x^6 + 20*b^3*c^2*x^4 - 5*b^4*c*x^2 + 2*b^5
 - 60*(c^5*x^10 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(c*x^2 + b) + 120*(c^5*x^10
 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(x))/(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*x^6
)
```

giac [A] time = 0.15, size = 110, normalized size = 1.16

$$-\frac{5\,c^3\log\left(x^2\right)}{b^6}+\frac{5\,c^3\log\left(\left|cx^2+b\right|\right)}{b^6}-\frac{30\,c^5x^4+68\,bc^4x^2+39\,b^2c^3}{4\left(cx^2+b\right)^2b^6}+\frac{110\,c^3x^6-36\,bc^2x^4+9\,b^2cx^2-2\,b^3}{12\,b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -5*c^3*log(x^2)/b^6 + 5*c^3*log(abs(c*x^2 + b))/b^6 - 1/4*(30*c^5*x^4 + 68*
b*c^4*x^2 + 39*b^2*c^3)/((c*x^2 + b)^2*b^6) + 1/12*(110*c^3*x^6 - 36*b*c^2*
x^4 + 9*b^2*c*x^2 - 2*b^3)/(b^6*x^6)
```

maple [A] time = 0.02, size = 90, normalized size = 0.95

$$-\frac{c^3}{4\left(c\,x^2+b\right)^2b^4}-\frac{2c^3}{\left(c\,x^2+b\right)b^5}-\frac{10c^3\ln(x)}{b^6}+\frac{5c^3\ln\left(c\,x^2+b\right)}{b^6}-\frac{3c^2}{b^5x^2}+\frac{3c}{4b^4x^4}-\frac{1}{6b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/6/b^3/x^6+3/4*c/b^4/x^4-3*c^2/b^5/x^2-1/4*c^3/b^4/(c*x^2+b)^2-2*c^3/b^5/
(c*x^2+b)-10*c^3*ln(x)/b^6+5*c^3*ln(c*x^2+b)/b^6
```

maxima [A] time = 1.36, size = 103, normalized size = 1.08

$$-\frac{60\,c^4x^8+90\,bc^3x^6+20\,b^2c^2x^4-5\,b^3cx^2+2\,b^4}{12\left(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6\right)}+\frac{5\,c^3\log\left(cx^2+b\right)}{b^6}-\frac{5\,c^3\log\left(x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*c^4*x^8 + 90*b*c^3*x^6 + 20*b^2*c^2*x^4 - 5*b^3*c*x^2 + 2*b^4)/(b
^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6) + 5*c^3*log(c*x^2 + b)/b^6 - 5*c^3*log
(x^2)/b^6
```

mupad [B] time = 0.10, size = 101, normalized size = 1.06

$$\frac{5\,c^3\,\ln\left(c\,x^2+b\right)}{b^6}-\frac{\frac{1}{6\,b}-\frac{5\,c\,x^2}{12\,b^2}+\frac{5\,c^2\,x^4}{3\,b^3}+\frac{15\,c^3\,x^6}{2\,b^4}+\frac{5\,c^4\,x^8}{b^5}}{b^2\,x^6+2\,b\,c\,x^8+c^2\,x^{10}}-\frac{10\,c^3\,\ln(x)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (5*c^3*log(b + c*x^2))/b^6 - (1/(6*b) - (5*c*x^2)/(12*b^2) + (5*c^2*x^4)/(3
*b^3) + (15*c^3*x^6)/(2*b^4) + (5*c^4*x^8)/b^5)/(b^2*x^6 + c^2*x^10 + 2*b*c
*x^8) - (10*c^3*log(x))/b^6
```

sympy [A] time = 0.65, size = 104, normalized size = 1.09

$$\frac{-2b^4 + 5b^3cx^2 - 20b^2c^2x^4 - 90bc^3x^6 - 60c^4x^8}{12b^7x^6 + 24b^6cx^8 + 12b^5c^2x^{10}} - \frac{10c^3\log(x)}{b^6} + \frac{5c^3\log\left(\frac{b}{c} + x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*b**4 + 5*b**3*c*x**2 - 20*b**2*c**2*x**4 - 90*b*c**3*x**6 - 60*c**4*x**
8)/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) - 10*c**3*log(x)/b*
*6 + 5*c**3*log(b/c + x**2)/b**6
```

3.103 $\int x^5\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=119

$$-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}$$

Rubi [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 670, 640, 612, 620, 206}

$$\frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - (5*b*(b*x^2 + c*x^4)^
(3/2))/(48*c^2) + (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*b^4*ArcTanh[(Sqrt[
c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int x^5\sqrt{bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{(5b)\,\text{Subst}\left(\int x\sqrt{bx+cx^2}\,dx,x,x^2\right)}{16c}\\
&= -\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}+\frac{\left(5b^2\right)\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{32c^2}\\
&= \frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{\left(5b^4\right)\text{Subst}\left(\int\frac{1}{\sqrt{bx+\cdots}}\cdots\right)}{256c^3}\\
&= \frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{\left(5b^4\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\cdots\right)}{128c^3}\\
&= \frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 114, normalized size = 0.96

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\left(15b^3-10b^2cx^2+8bc^2x^4+48c^3x^6\right)-15b^4\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{384c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(15*b^3 - 10*b^2*c*x^2 + 8*b*
c^2*x^4 + 48*c^3*x^6) - 15*b^4*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(384*c^
(7/2)*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.24, size = 98, normalized size = 0.82

$$\frac{5b^4\log\left(-2\sqrt{c}\sqrt{bx^2+cx^4}+b+2cx^2\right)}{256c^{7/2}}+\frac{\sqrt{bx^2+cx^4}\left(15b^3-10b^2cx^2+8bc^2x^4+48c^3x^6\right)}{384c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(15*b^3 - 10*b^2*c*x^2 + 8*b*c^2*x^4 + 48*c^3*x^6))/(3
84*c^3) + (5*b^4*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[b*x^2 + c*x^4]])/(256*c^(
7/2))
```

fricas [A] time = 1.04, size = 188, normalized size = 1.58

$$\left[\frac{15\,b^4\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\left(48\,c^4x^6+8\,bc^3x^4-10\,b^2c^2x^2+15\,b^3c\right)\sqrt{cx^4+bx^2}}{768\,c^4},\ \frac{15\,b^4\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\left(48\,c^4x^6+8\,bc^3x^4-10\,b^2c^2x^2+15\,b^3c\right)\sqrt{cx^4+bx^2}}{384\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*b^4*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
2*(48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 + b*x^2
))/c^4, 1/384*(15*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 +
 b)) + (48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 +
b*x^2))/c^4]
```

giac [A] time = 0.19, size = 101, normalized size = 0.85

$$\frac{1}{384}\left(2\left(4\left(6\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x^2-\frac{5\,b^2\mathrm{sgn}(x)}{c^2}\right)x^2+\frac{15\,b^3\mathrm{sgn}(x)}{c^3}\right)\sqrt{cx^2+b}\,x+\frac{5\,b^4\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{128\,c^{\frac{7}{2}}}-\frac{5\,b^4\log\left(|b|\right)\mathrm{sgn}(x)}{256\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 5*b^2*sgn(x)/c^2)*x^2 + 15*b^
3*sgn(x)/c^3)*sqrt(c*x^2 + b)*x + 5/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + b)))*sgn(x)/c^(7/2) - 5/256*b^4*log(abs(b))*sgn(x)/c^(7/2)
```

maple [A] time = 0.02, size = 124, normalized size = 1.04

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(48\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^5-40\left(c\,x^2+b\right)^{\frac{3}{2}}b\,c^{\frac{3}{2}}x^3-15b^4\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-15\sqrt{c\,x^2+b}\,b^3\sqrt{c}\,x+30\left(c\,x^2+b\right)^{\frac{3}{2}}b^2\sqrt{c}\,x\right)}{384\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(1/2)*(48*x^5*(c*x^2+b)^(3/2)*c^(5/2)-40*(c*x^2+b)^(3/2
)*c^(3/2)*x^3*b+30*(c*x^2+b)^(3/2)*c^(1/2)*x*b^2-15*(c*x^2+b)^(1/2)*c^(1/2)
*x*b^3-15*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^4)/x/(c*x^2+b)^(1/2)/c^(7/2)
```

maxima [A] time = 1.46, size = 121, normalized size = 1.02

$$\frac{5\sqrt{cx^4+bx^2}\,b^2x^2}{64\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}{8\,c}-\frac{5\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{256\,c^{\frac{7}{2}}}+\frac{5\,\sqrt{cx^4+bx^2}\,b^3}{128\,c^3}-\frac{5\left(cx^4+bx^2\right)^{\frac{3}{2}}b}{48\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 1/8*(c*x^4 + b*x^2)^(3/2)*x^2/c - 5/
256*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 5/128*sq
rt(c*x^4 + b*x^2)*b^3/c^3 - 5/48*(c*x^4 + b*x^2)^(3/2)*b/c^2
```

mupad [B] time = 4.68, size = 105, normalized size = 0.88

$$\frac{x^2\left(c\,x^4+b\,x^2\right)^{3/2}}{8\,c}-\frac{5\,b\left(\frac{b^3\,\ln\left(\frac{2\,c\,x^2+b}{\sqrt{c}}+2\,\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*b*((b^3*log((b + 2*c*x^2)/c^(1/2) +
2*(b*x^2 + c*x^4)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4
- 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(x**2*(b + c*x**2)), x)
```

3.104 $\int x^3\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=91

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}$$

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 640, 612, 620, 206}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c^2) + (b*x^2 + c*x^4)^(3/2)/(6*
c) + (b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int x^3\sqrt{bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{b\operatorname{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^3\operatorname{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{32c^2}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^3\operatorname{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^2}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 103, normalized size = 1.13

$$
\frac{x\sqrt{b+cx^2}\left(3b^3\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)+\sqrt{c}\,x\sqrt{b+cx^2}\left(-3b^2+2bcx^2+8c^2x^4\right)\right)}{48c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-3*b^2 + 2*b*c*x^2 + 8*c^2*x
^4) + 3*b^3*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(48*c^(5/2)*Sqrt[x^2*(b +
c*x^2)])
```

IntegrateAlgebraic [A] time = 0.21, size = 93, normalized size = 1.02

$$\frac{\sqrt{bx^2+cx^4}\,(-3b^2+2bcx^2+8c^2x^4)}{48c^2}-\frac{b^3\log\left(-2c^{5/2}\sqrt{bx^2+cx^4}+bc^2+2c^3x^2\right)}{32c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-3*b^2 + 2*b*c*x^2 + 8*c^2*x^4))/(48*c^2) - (b^3*Log[
b*c^2 + 2*c^3*x^2 - 2*c^(5/2)*Sqrt[b*x^2 + c*x^4]])/(32*c^(5/2))
```

fricas [A] time = 0.82, size = 167, normalized size = 1.84

$$\left[\frac{3\,b^3\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\left(8\,c^3x^4+2\,bc^2x^2-3\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^3},\,-\frac{3\,b^3\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-\left(8\,c^3x^4+2\,bc^2x^2-3\,b^2c\right)\sqrt{cx^4+bx^2}}{48\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*b^3*
sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*c^3*x^4 + 2*
b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3]
```

giac [A] time = 0.19, size = 85, normalized size = 0.93

$$\frac{1}{48}\left(2\left(4\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x^2-\frac{3\,b^2\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}\,x-\frac{b^3\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}}+\frac{b^3\log\left(|b|\right)\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 3*b^2*sgn(x)/c^2)*sqrt(c*x^2 + b)
*x - 1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 1/32*
b^3*log(abs(b))*sgn(x)/c^(5/2)
```

maple [A] time = 0.01, size = 104, normalized size = 1.14

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(8\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x^3+3b^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+3\sqrt{c\,x^2+b}\,b^2\sqrt{c}\,x-6\left(c\,x^2+b\right)^{\frac{3}{2}}b\sqrt{c}\,x\right)}{48\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(1/2)*(8*x^3*(c*x^2+b)^(3/2)*c^(3/2)-6*c^(1/2)*(c*x^2+b)
^(3/2)*x*b+3*c^(1/2)*(c*x^2+b)^(1/2)*x*b^2+3*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*
b^3)/x/(c*x^2+b)^(1/2)/c^(5/2)
```

maxima [A] time = 1.45, size = 97, normalized size = 1.07

$$-\frac{\sqrt{cx^4+bx^2}\,bx^2}{8\,c}+\frac{b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{32\,c^{\frac{5}{2}}}-\frac{\sqrt{cx^4+bx^2}\,b^2}{16\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(c*x^4 + b*x^2)*b*x^2/c + 1/32*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4
+ b*x^2)*sqrt(c))/c^(5/2) - 1/16*sqrt(c*x^4 + b*x^2)*b^2/c^2 + 1/6*(c*x^4 +
 b*x^2)^(3/2)/c
```

mupad [B] time = 4.36, size = 77, normalized size = 0.85

$$\frac{b^3\,\ln\left(\frac{2\,c\,x^2+b}{\sqrt{c}}+2\,\sqrt{c\,x^4+b\,x^2}\right)}{32\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{48\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b^3*log((b + 2*c*x^2)/c^(1/2) + 2*(b*x^2 + c*x^4)^(1/2)))/(32*c^(5/2)) + (
(b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x**2*(b + c*x**2)), x)
```

3.105 $\int x\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=68

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2013, 612, 620, 206}

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt
[b*x^2 + c*x^4]])/(8*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
```

```
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int x\sqrt{bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c}$$
$$= \frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c}$$
$$= \frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c} - \frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 1.32

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\left(b+2cx^2\right)-b^2\log\left(\sqrt{c}\,\sqrt{b+cx^2}+cx\right)\right)}{8c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(b + 2*c*x^2) - b^2*Log[c*x +
 Sqrt[c]*Sqrt[b + c*x^2]]))/(8*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.19, size = 78, normalized size = 1.15

$$\frac{b^2\log\left(-2c^{3/2}\sqrt{bx^2+cx^4}+bc+2c^2x^2\right)}{16c^{3/2}}+\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c) + (b^2*Log[b*c + 2*c^2*x^2 - 2*c^
(3/2)*Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

fricas [A] time = 0.87, size = 140, normalized size = 2.06

$$\left[\frac{b^2\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(2\,c^2x^2+bc\right)}{16\,c^2},\,\frac{b^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\left(2\,c^2x^2+bc\right)}{8\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(b^2*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sq
rt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/c^2, 1/8*(b^2*sqrt(-c)*arctan(sqrt(c*x
^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/
c^2]
```

giac [A] time = 0.19, size = 69, normalized size = 1.01

$$\frac{1}{8}\,\sqrt{cx^2+b}\left(2\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x+\frac{b^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{8\,c^{\frac{3}{2}}}-\frac{b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + b)*(2*x^2*sgn(x) + b*sgn(x)/c)*x + 1/8*b^2*log(abs(-sqrt(c
)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/16*b^2*log(abs(b))*sgn(x)/c^(3/2
)
```

maple [A] time = 0.01, size = 84, normalized size = 1.24

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-b^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-\sqrt{c\,x^2+b}\,b\sqrt{c}\,x+2\left(c\,x^2+b\right)^{\frac{3}{2}}\sqrt{c}\,x\right)}{8\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(2*x*(c*x^2+b)^(3/2)*c^(1/2)-c^(1/2)*(c*x^2+b)^(1/2
)*x*b-ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2)/x/(c*x^2+b)^(1/2)/c^(3/2)
```

maxima [A] time = 1.43, size = 73, normalized size = 1.07

$$\frac{1}{4}\,\sqrt{cx^4+bx^2}\,x^2-\frac{b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{16\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}\,b}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^4 + b*x^2)*x^2 - 1/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x
^2)*sqrt(c))/c^(3/2) + 1/8*sqrt(c*x^4 + b*x^2)*b/c
```

mupad [B] time = 4.37, size = 64, normalized size = 0.94

$$\frac{\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2}}{2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2 - (b^2*log((b/2 + c*x^2)/c^(1/2
) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(b + c*x**2)), x)
```

3.106 $\int \frac{\sqrt{bx^2+cx^4}}{x} dx$

Optimal. Leaf size=55

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 664, 620, 206}

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/2 + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*S
qrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{1}{4}b\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.16

$$\frac{1}{2}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{b\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)}{\sqrt{c}\,x\sqrt{b+cx^2}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x,x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(1 + (b*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]])/(Sqrt[c]
*x*Sqrt[b + c*x^2])))/2
```

IntegrateAlgebraic [A] time = 0.16, size = 61, normalized size = 1.11

$$\frac{1}{2}\sqrt{bx^2+cx^4}-\frac{b\log\left(-2\sqrt{c}\sqrt{bx^2+cx^4}+b+2cx^2\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/2 - (b*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[b*x^2 + c*x^4]]
)/(4*Sqrt[c])
```

fricas [A] time = 1.10, size = 115, normalized size = 2.09

$$\left[\frac{b\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\,c}{4\,c}, -\frac{b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}\,c}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c, -1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(
c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*c)/c]
```

giac [A] time = 0.17, size = 52, normalized size = 0.95

$$\frac{b\log\left(|b|\right)\mathrm{sgn}(x)}{4\,\sqrt{c}}+\frac{1}{2}\left(\sqrt{cx^2+b}\,x-\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{\sqrt{c}}\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(b))*sgn(x)/sqrt(c) + 1/2*(sqrt(c*x^2 + b)*x - b*log(abs(-sqrt
(c)*x + sqrt(c*x^2 + b)))/sqrt(c))*sgn(x)
```

maple [A] time = 0.00, size = 64, normalized size = 1.16

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+\sqrt{c\,x^2+b}\,\sqrt{c}\,x\right)}{2\sqrt{c\,x^2+b}\,\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(1/2)+b*ln(c^(1/2)*x+(c*x^2+b)
^(1/2)))/x/(c*x^2+b)^(1/2)/c^(1/2)
```

maxima [A] time = 1.43, size = 49, normalized size = 0.89

$$\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{4\,\sqrt{c}}+\frac{1}{2}\,\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 1/2*sqrt(c
*x^4 + b*x^2)
```

mupad [B] time = 4.21, size = 50, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{2}+\frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x,x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/2 + (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1
/2)))/(4*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x, x)
```

3.107 $\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx$

Optimal. Leaf size=52

$$\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 662, 620, 206}

$$\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/x^2) + Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{bx^2+cx^4}}{x^2} + \frac{1}{2}c\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{bx^2+cx^4}}{x^2} + c\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{\sqrt{bx^2+cx^4}}{x^2} + \sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 1.15

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\frac{\sqrt{c}\,x\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{cx^2}{b}+1}}-1\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-1 + (Sqrt[c]*x*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt
[b]*Sqrt[1 + (c*x^2)/b])))/x^2
```

IntegrateAlgebraic [A] time = 0.13, size = 61, normalized size = 1.17

$$-\frac{\sqrt{bx^2+cx^4}}{x^2}-\frac{1}{2}\sqrt{c}\,\log\left(-2\sqrt{c}\,\sqrt{bx^2+cx^4}\,+b+2cx^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/x^2) - (Sqrt[c]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[b*x^
2 + c*x^4]])/2
```

fricas [A] time = 0.85, size = 115, normalized size = 2.21

$$\left[\frac{\sqrt{c}\,x^2\log\left(-2\,cx^2 - b - 2\,\sqrt{cx^4 + bx^2}\,\sqrt{c}\right) - 2\,\sqrt{cx^4 + bx^2}}{2\,x^2}, -\frac{\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right) + \sqrt{cx^4 + bx^2}}{x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqr
t(c*x^4 + b*x^2))/x^2, -(sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(
c*x^2 + b)) + sqrt(c*x^4 + b*x^2))/x^2]
```

giac [A] time = 0.27, size = 61, normalized size = 1.17

$$-\frac{1}{2}\,\sqrt{c}\,\log\left(\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^2\right)\operatorname{sgn}(x) + \frac{2\,b\sqrt{c}\operatorname{sgn}(x)}{\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^2 - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2*b*sqrt(c)*sgn(
x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)
```

maple [A] time = 0.01, size = 84, normalized size = 1.62

$$\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(bcx\ln\left(\sqrt{c}\,x + \sqrt{c\,x^2 + b}\right) + \sqrt{c\,x^2 + b}\,c^{\frac{3}{2}}x^2 - \left(c\,x^2 + b\right)^{\frac{3}{2}}\sqrt{c}\right)}{\sqrt{c\,x^2 + b}\,b\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^3,x)
```

```
[Out] (c*x^4+b*x^2)^(1/2)*(c^(3/2)*(c*x^2+b)^(1/2)*x^2-(c*x^2+b)^(3/2)*c^(1/2)+ln
(c^(1/2)*x+(c*x^2+b)^(1/2))*x*b*c)/x^2/(c*x^2+b)^(1/2)/b/c^(1/2)
```

maxima [A] time = 1.39, size = 51, normalized size = 0.98

$$\frac{1}{2}\,\sqrt{c}\,\log\left(2\,cx^2 + b + 2\,\sqrt{cx^4 + bx^2}\,\sqrt{c}\right) - \frac{\sqrt{cx^4 + bx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - sqrt(c*x^4 +
 b*x^2)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^3,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**3, x)
```

3.108 $\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx$

Optimal. Leaf size=25

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(3*b*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx=-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -1/3*(x^2*(b + c*x^2))^(3/2)/(b*x^6)
```

IntegrateAlgebraic [A] time = 0.11, size = 35, normalized size = 1.40

$$\frac{\left(-b - cx^2\right)\sqrt{bx^2 + cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^5,x]
[Out] ((-b - c*x^2)*Sqrt[b*x^2 + c*x^4])/(3*b*x^4)
```

fricas [A] time = 0.84, size = 28, normalized size = 1.12

$$-\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)}{3\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(b*x^4)
```

giac [B] time = 0.23, size = 63, normalized size = 2.52

$$\frac{2\left(3\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^4 c^{\frac{3}{2}}\mathrm{sgn}(x) + b^2c^{\frac{3}{2}}\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^2 - b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*c^(3/2)*sgn(x) + b^2*c^(3/2)*sgn(x))
/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$-\frac{\left(c\,x^2 + b\right)\sqrt{c\,x^4 + b\,x^2}}{3b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^5,x)
[Out] -1/3/x^4*(c*x^2+b)/b*(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.44, size = 41, normalized size = 1.64

$$-\frac{\sqrt{cx^4 + bx^2}\, c}{3\, bx^2} - \frac{\sqrt{cx^4 + bx^2}}{3\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
[Out] -1/3*sqrt(c*x^4 + b*x^2)*c/(b*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/x^4
```

mupad [B] time = 4.15, size = 28, normalized size = 1.12

$$-\frac{\left(c\, x^2 + b\right)\, \sqrt{c\, x^4 + b\, x^2}}{3\, b\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^5,x)
[Out] -((b + c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**5,x)
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**5, x)
```

3.109 $\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx$

Optimal. Leaf size=52

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(5*b*x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}-\frac{(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{5b}\\ &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(2cx^2-3b\right)}{15b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^7,x]
[Out] ((x^2*(b + c*x^2))^(3/2)*(-3*b + 2*c*x^2))/(15*b^2*x^8)
```

IntegrateAlgebraic [A] time = 0.13, size = 46, normalized size = 0.88

$$\frac{\sqrt{bx^2+cx^4}\left(-3b^2-bcx^2+2c^2x^4\right)}{15b^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^7,x]
[Out] (Sqrt[b*x^2 + c*x^4]*(-3*b^2 - b*c*x^2 + 2*c^2*x^4))/(15*b^2*x^6)
```

fricas [A] time = 0.58, size = 42, normalized size = 0.81

$$\frac{\left(2\,c^2x^4-bcx^2-3\,b^2\right)\sqrt{cx^4+bx^2}}{15\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")
[Out] 1/15*(2*c^2*x^4 - b*c*x^2 - 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^6)
```

giac [B] time = 0.22, size = 120, normalized size = 2.31

$$\frac{4\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6c^{\frac{5}{2}}\mathrm{sgn}(x)+5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4bc^{\frac{5}{2}}\mathrm{sgn}(x)+5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{5}{2}}\mathrm{sgn}(x)-b^3c^{\frac{5}{2}}\mathrm{sgn}(x)\right)}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")
[Out] 4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^6*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - sq
rt(c*x^2 + b))^4*b*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^2*c
^(5/2)*sgn(x) - b^3*c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

maple [A] time = 0.00, size = 39, normalized size = 0.75

$$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+3b\right)\sqrt{c\,x^{4}+b\,x^{2}}}{15b^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^7,x)
```

```
[Out] -1/15*(c*x^2+b)*(-2*c*x^2+3*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^6
```

maxima [A] time = 1.42, size = 65, normalized size = 1.25

$$\frac{2\sqrt{cx^{4}+bx^{2}}\,c^{2}}{15\,b^{2}x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}\,c}{15\,bx^{4}}-\frac{\sqrt{cx^{4}+bx^{2}}}{5\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 2/15*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - 1/15*sqrt(c*x^4 + b*x^2)*c/(b*x^4)
 - 1/5*sqrt(c*x^4 + b*x^2)/x^6
```

mupad [B] time = 4.26, size = 41, normalized size = 0.79

$$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(3\,b^{2}+b\,c\,x^{2}-2\,c^{2}\,x^{4}\right)}{15\,b^{2}\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^7,x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*b^2 - 2*c^2*x^4 + b*c*x^2))/(15*b^2*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^{2}\left(b+cx^{2}\right)}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**7, x)
```

3.110 $\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=80

$$-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}+\frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

Rubi [A] time = 0.12, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}+\frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(7*b*x^10) + (4*c*(b*x^2 + c*x^4)^(3/2))/(35*b^2*x^8
) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}} - \frac{(4c)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{7b} \\ &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}} + \frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8} + \frac{\left(8c^2\right)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{35b^2} \\ &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}} + \frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8} - \frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.58

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(15b^2-12bcx^2+8c^2x^4\right)}{105b^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -1/105*((x^2*(b + c*x^2))^(3/2)*(15*b^2 - 12*b*c*x^2 + 8*c^2*x^4))/(b^3*x^1
0)
```

IntegrateAlgebraic [A] time = 0.14, size = 57, normalized size = 0.71

$$\frac{\sqrt{bx^2+cx^4}\left(-15b^3-3b^2cx^2+4bc^2x^4-8c^3x^6\right)}{105b^3x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-15*b^3 - 3*b^2*c*x^2 + 4*b*c^2*x^4 - 8*c^3*x^6))/(10
5*b^3*x^8)
```

fricas [A] time = 0.59, size = 53, normalized size = 0.66

$$-\frac{\left(8\,c^3x^6-4\,bc^2x^4+3\,b^2cx^2+15\,b^3\right)\sqrt{cx^4+bx^2}}{105\,b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/105*(8*c^3*x^6 - 4*b*c^2*x^4 + 3*b^2*c*x^2 + 15*b^3)*sqrt(c*x^4 + b*x^2)
/(b^3*x^8)
```

giac [B] time = 0.24, size = 148, normalized size = 1.85

$$\frac{16\left(70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8c^{\frac{7}{2}}\mathrm{sgn}(x)+35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6bc^{\frac{7}{2}}\mathrm{sgn}(x)+21\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4b^2c^{\frac{7}{2}}\mathrm{sgn}(x)-7\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2b^3c^{\frac{7}{2}}\mathrm{sgn}(x)+b^4c^{\frac{7}{2}}\mathrm{sgn}(x)\right)}{105\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(7/2)*sgn(x) + 35*(sqrt(c)*x -
 sqrt(c*x^2 + b))^6*b*c^(7/2)*sgn(x) + 21*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b
^2*c^(7/2)*sgn(x) - 7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^3*c^(7/2)*sgn(x) +
b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

maple [A] time = 0.00, size = 50, normalized size = 0.62

$$-\frac{\left(c\,x^2+b\right)\left(8c^2x^4-12bc\,x^2+15b^2\right)\sqrt{c\,x^4+b\,x^2}}{105b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^9,x)
```

```
[Out] -1/105*(c*x^2+b)*(8*c^2*x^4-12*b*c*x^2+15*b^2)*(c*x^4+b*x^2)^(1/2)/x^8/b^3
```

maxima [A] time = 1.43, size = 89, normalized size = 1.11

$$-\frac{8\sqrt{cx^4+bx^2}\,c^3}{105\,b^3x^2}+\frac{4\sqrt{cx^4+bx^2}\,c^2}{105\,b^2x^4}-\frac{\sqrt{cx^4+bx^2}\,c}{35\,bx^6}-\frac{\sqrt{cx^4+bx^2}}{7\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -8/105*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2) + 4/105*sqrt(c*x^4 + b*x^2)*c^2/(b
^2*x^4) - 1/35*sqrt(c*x^4 + b*x^2)*c/(b*x^6) - 1/7*sqrt(c*x^4 + b*x^2)/x^8
```

mupad [B] time = 4.34, size = 89, normalized size = 1.11

$$\frac{4\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^4}-\frac{c\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^6}-\frac{\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{8\,c^3\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^9,x)
```

```
[Out] (4*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (c*(b*x^2 + c*x^4)^(1/2))/(35
*b*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*x^8) - (8*c^3*(b*x^2 + c*x^4)^(1/2))/(10
5*b^3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^9} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**9, x)
```

3.111 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

Rubi [A] time = 0.16, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(9*b*x^12) + (2*c*(b*x^2 + c*x^4)^(3/2))/(21*b^2*x^1
0) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^8) + (16*c^3*(b*x^2 + c*x^4)^
(3/2))/(315*b^4*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} - \frac{(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx}{3b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} + \frac{\left(8c^2\right)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{21b^2}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} - \frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8} - \frac{\left(16c^3\right)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{105b^3}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} - \frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8} + \frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.53

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(-35b^3+30b^2cx^2-24bc^2x^4+16c^3x^6\right)}{315b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-35*b^3 + 30*b^2*c*x^2 - 24*b*c^2*x^4 + 16*c^3*x^
6))/(315*b^4*x^12)
```

IntegrateAlgebraic [A] time = 0.16, size = 68, normalized size = 0.63

$$\frac{\sqrt{bx^2+cx^4}\left(-35b^4-5b^3cx^2+6b^2c^2x^4-8bc^3x^6+16c^4x^8\right)}{315b^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-35*b^4 - 5*b^3*c*x^2 + 6*b^2*c^2*x^4 - 8*b*c^3*x^6 +
 16*c^4*x^8))/(315*b^4*x^10)
```

fricas [A] time = 0.96, size = 64, normalized size = 0.59

$$\frac{\left(16\,c^4x^8-8\,bc^3x^6+6\,b^2c^2x^4-5\,b^3cx^2-35\,b^4\right)\sqrt{cx^4+bx^2}}{315\,b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/315*(16*c^4*x^8 - 8*b*c^3*x^6 + 6*b^2*c^2*x^4 - 5*b^3*c*x^2 - 35*b^4)*sqr
t(c*x^4 + b*x^2)/(b^4*x^10)
```

giac [A] time = 0.29, size = 178, normalized size = 1.65

$$\frac{32\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}c^{\frac{9}{2}}\mathrm{sgn}(x)+189\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}bc^{\frac{9}{2}}\mathrm{sgn}(x)+84\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}b^2c^{\frac{9}{2}}\mathrm{sgn}(x)-36\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}b^3c^{\frac{9}{2}}\mathrm{sgn}(x)+9\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}b^4c^{\frac{9}{2}}\mathrm{sgn}(x)-b^5c^{\frac{9}{2}}\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(9/2)*sgn(x) + 189*(sqrt(c)*
x - sqrt(c*x^2 + b))^8*b*c^(9/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c*x^2 + b))^
6*b^2*c^(9/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(9/2)*sgn(x
) + 9*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(9/2)*sgn(x) - b^5*c^(9/2)*sgn(
x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

maple [A] time = 0.01, size = 61, normalized size = 0.56

$$-\frac{\left(c\,x^2+b\right)\left(-16c^3x^6+24b\,c^2x^4-30b^2c\,x^2+35b^3\right)\sqrt{c\,x^4+b\,x^2}}{315b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^11,x)
```

```
[Out] -1/315*(c*x^2+b)*(-16*c^3*x^6+24*b*c^2*x^4-30*b^2*c*x^2+35*b^3)*(c*x^4+b*x^
2)^(1/2)/x^10/b^4
```

maxima [A] time = 1.42, size = 113, normalized size = 1.05

$$\frac{16\sqrt{cx^4+bx^2}\,c^4}{315\,b^4x^2}-\frac{8\sqrt{cx^4+bx^2}\,c^3}{315\,b^3x^4}+\frac{2\sqrt{cx^4+bx^2}\,c^2}{105\,b^2x^6}-\frac{\sqrt{cx^4+bx^2}\,c}{63\,bx^8}-\frac{\sqrt{cx^4+bx^2}}{9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 16/315*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8/315*sqrt(c*x^4 + b*x^2)*c^3/(b
^3*x^4) + 2/105*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 1/63*sqrt(c*x^4 + b*x^2
)*c/(b*x^8) - 1/9*sqrt(c*x^4 + b*x^2)/x^10
```

mupad [B] time = 4.50, size = 113, normalized size = 1.05

$$\frac{2\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^6}-\frac{c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8}-\frac{\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{8\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^4}+\frac{16\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^11,x)
```

```
[Out] (2*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (c*(b*x^2 + c*x^4)^(1/2))/(63
*b*x^8) - (b*x^2 + c*x^4)^(1/2)/(9*x^10) - (8*c^3*(b*x^2 + c*x^4)^(1/2))/(3
15*b^3*x^4) + (16*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^{11}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**11, x)
```

3.112 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

Optimal. Leaf size=136

$$-\frac{128c^4\left(bx^2+cx^4\right)^{3/2}}{3465b^5x^6}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

Rubi [A] time = 0.21, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{128c^4\left(bx^2+cx^4\right)^{3/2}}{3465b^5x^6}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(11*b*x^14) + (8*c*(b*x^2 + c*x^4)^(3/2))/(99*b^2*x^
12) - (16*c^2*(b*x^2 + c*x^4)^(3/2))/(231*b^3*x^10) + (64*c^3*(b*x^2 + c*x^
4)^(3/2))/(1155*b^4*x^8) - (128*c^4*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} - \frac{(8c)\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx}{11b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} + \frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} + \frac{\left(16c^2\right)\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx}{33b^2}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} + \frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} - \frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}} - \frac{\left(64c^3\right)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{231b^3}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} + \frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} - \frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}} + \frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8} + \frac{\left(128c^4\right)}{}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} + \frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} - \frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}} + \frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8} - \frac{128c^4\left(b\right.}{34}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.50

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(315b^4-280b^3cx^2+240b^2c^2x^4-192bc^3x^6+128c^4x^8\right)}{3465b^5x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] -1/3465*((x^2*(b + c*x^2))^(3/2)*(315*b^4 - 280*b^3*c*x^2 + 240*b^2*c^2*x^4
 - 192*b*c^3*x^6 + 128*c^4*x^8))/(b^5*x^14)
```

IntegrateAlgebraic [A] time = 0.16, size = 79, normalized size = 0.58

$$\frac{\sqrt{bx^2+cx^4}\left(-315b^5-35b^4cx^2+40b^3c^2x^4-48b^2c^3x^6+64bc^4x^8-128c^5x^{10}\right)}{3465b^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-315*b^5 - 35*b^4*c*x^2 + 40*b^3*c^2*x^4 - 48*b^2*c^3
*x^6 + 64*b*c^4*x^8 - 128*c^5*x^10))/(3465*b^5*x^12)
```

fricas [A] time = 0.94, size = 75, normalized size = 0.55

$$-\frac{\left(128\,c^5x^{10}-64\,bc^4x^8+48\,b^2c^3x^6-40\,b^3c^2x^4+35\,b^4cx^2+315\,b^5\right)\sqrt{cx^4+bx^2}}{3465\,b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/3465*(128*c^5*x^10 - 64*b*c^4*x^8 + 48*b^2*c^3*x^6 - 40*b^3*c^2*x^4 + 35
*b^4*c*x^2 + 315*b^5)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

giac [A] time = 0.25, size = 206, normalized size = 1.51

$$\frac{256\left(1386\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}c^{\frac{11}{2}}\operatorname{sgn}(x)+924\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{11}{2}}\operatorname{sgn}(x)+330\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{11}{2}}\operatorname{sgn}(x)-165\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}b^3c^{\frac{11}{2}}\operatorname{sgn}(x)+55\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}b^4c^{\frac{11}{2}}\operatorname{sgn}(x)-11\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}b^5c^{\frac{11}{2}}\operatorname{sgn}(x)+b^6c^{\frac{11}{2}}\operatorname{sgn}(x)\right)}{3465\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] 256/3465*(1386*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(11/2)*sgn(x) + 924*(sqrt
(c)*x - sqrt(c*x^2 + b))^10*b*c^(11/2)*sgn(x) + 330*(sqrt(c)*x - sqrt(c*x^2
 + b))^8*b^2*c^(11/2)*sgn(x) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c^(1
1/2)*sgn(x) + 55*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(11/2)*sgn(x) - 11*(
sqrt(c)*x - sqrt(c*x^2 + b))^2*b^5*c^(11/2)*sgn(x) + b^6*c^(11/2)*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

maple [A] time = 0.01, size = 72, normalized size = 0.53

$$-\frac{\left(c\,x^2+b\right)\left(128c^4x^8-192c^3x^6b+240c^2x^4b^2-280c\,x^2b^3+315b^4\right)\sqrt{c\,x^4+b\,x^2}}{3465b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^13,x)
```

```
[Out] -1/3465*(c*x^2+b)*(128*c^4*x^8-192*b*c^3*x^6+240*b^2*c^2*x^4-280*b^3*c*x^2+
315*b^4)*(c*x^4+b*x^2)^(1/2)/x^12/b^5
```

maxima [A] time = 1.53, size = 137, normalized size = 1.01

$$-\frac{128\sqrt{cx^4+bx^2}\,c^5}{3465\,b^5x^2}+\frac{64\sqrt{cx^4+bx^2}\,c^4}{3465\,b^4x^4}-\frac{16\sqrt{cx^4+bx^2}\,c^3}{1155\,b^3x^6}+\frac{8\sqrt{cx^4+bx^2}\,c^2}{693\,b^2x^8}-\frac{\sqrt{cx^4+bx^2}\,c}{99\,bx^{10}}-\frac{\sqrt{cx^4+bx^2}}{11\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] -128/3465*sqrt(c*x^4 + b*x^2)*c^5/(b^5*x^2) + 64/3465*sqrt(c*x^4 + b*x^2)*c
^4/(b^4*x^4) - 16/1155*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^6) + 8/693*sqrt(c*x^4
 + b*x^2)*c^2/(b^2*x^8) - 1/99*sqrt(c*x^4 + b*x^2)*c/(b*x^10) - 1/11*sqrt(c
*x^4 + b*x^2)/x^12
```

mupad [B] time = 4.62, size = 137, normalized size = 1.01

$$\frac{8\,c^2\,\sqrt{c\,x^4+b\,x^2}}{693\,b^2\,x^8}-\frac{c\,\sqrt{c\,x^4+b\,x^2}}{99\,b\,x^{10}}-\frac{\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{16\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^6}+\frac{64\,c^4\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^4\,x^4}-\frac{128\,c^5\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^13,x)
```

```
[Out] (8*c^2*(b*x^2 + c*x^4)^(1/2))/(693*b^2*x^8) - (c*(b*x^2 + c*x^4)^(1/2))/(99
*b*x^10) - (b*x^2 + c*x^4)^(1/2)/(11*x^12) - (16*c^3*(b*x^2 + c*x^4)^(1/2))
/(1155*b^3*x^6) + (64*c^4*(b*x^2 + c*x^4)^(1/2))/(3465*b^4*x^4) - (128*c^5*
(b*x^2 + c*x^4)^(1/2))/(3465*b^5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**13, x)
```

3.113 $\int x^4\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=78

$$\frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - (4*b*(b*x^2 + c*x^4)^(3/2))/(
35*c^2*x) + (x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4\sqrt{bx^2+cx^4}\,dx &= \frac{x\left(bx^2+cx^4\right)^{3/2}}{7c} - \frac{(4b)\int x^2\sqrt{bx^2+cx^4}\,dx}{7c} \\
&= -\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x} + \frac{x\left(bx^2+cx^4\right)^{3/2}}{7c} + \frac{\left(8b^2\right)\int \sqrt{bx^2+cx^4}\,dx}{35c^2} \\
&= \frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3} - \frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x} + \frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.59

$$
\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(8b^2-12bcx^2+15c^2x^4\right)}{105c^3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(8*b^2 - 12*b*c*x^2 + 15*c^2*x^4))/(105*c^3*x^3)
```

IntegrateAlgebraic [A] time = 0.06, size = 57, normalized size = 0.73

$$
\frac{\sqrt{bx^2+cx^4}\left(8b^3-4b^2cx^2+3bc^2x^4+15c^3x^6\right)}{105c^3x}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(8*b^3 - 4*b^2*c*x^2 + 3*b*c^2*x^4 + 15*c^3*x^6))/(105
*c^3*x)
```

fricas [A] time = 1.31, size = 53, normalized size = 0.68

$$
\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{105\,c^3x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2)/
(c^3*x)
```

giac [A] time = 0.16, size = 60, normalized size = 0.77

$$-\frac{8\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{105\,c^3}+\frac{15\left(cx^2+b\right)^{\frac{7}{2}}\mathrm{sgn}(x)-42\left(cx^2+b\right)^{\frac{5}{2}}b\mathrm{sgn}(x)+35\left(cx^2+b\right)^{\frac{3}{2}}b^2\mathrm{sgn}(x)}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -8/105*b^(7/2)*sgn(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7/2)*sgn(x) - 42*(c*x^2
+ b)^(5/2)*b*sgn(x) + 35*(c*x^2 + b)^(3/2)*b^2*sgn(x))/c^3
```

maple [A] time = 0.00, size = 50, normalized size = 0.64

$$\frac{\left(c\,x^2+b\right)\left(15c^2x^4-12bc\,x^2+8b^2\right)\sqrt{c\,x^4+b\,x^2}}{105c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/105*(c*x^2+b)*(15*c^2*x^4-12*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(1/2)/c^3/x
```

maxima [A] time = 1.41, size = 46, normalized size = 0.59

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^2+b}}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)/c^3
```

mupad [B] time = 4.23, size = 53, normalized size = 0.68

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(8\,b^3-4\,b^2\,c\,x^2+3\,b\,c^2\,x^4+15\,c^3\,x^6\right)}{105\,c^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(8*b^3 + 15*c^3*x^6 - 4*b^2*c*x^2 + 3*b*c^2*x^4))/(1
05*c^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(x**2*(b + c*x**2)), x)
```

3.114 $\int x^2\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=52

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (b*x^2 + c*x^4)^(3/2)/(5*c*x)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{bx^2+cx^4}\,dx&=\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{(2b)\int\sqrt{bx^2+cx^4}\,dx}{5c}\\&=-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}+\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(3cx^2-2b\right)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^2 + c*x^4],x]
[Out] ((x^2*(b + c*x^2))^(3/2)*(-2*b + 3*c*x^2))/(15*c^2*x^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 45, normalized size = 0.87

$$\frac{\sqrt{bx^2+cx^4}\left(-2b^2+bcx^2+3c^2x^4\right)}{15c^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[b*x^2 + c*x^4],x]
[Out] (Sqrt[b*x^2 + c*x^4]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4))/(15*c^2*x)
```

fricas [A] time = 0.93, size = 41, normalized size = 0.79

$$\frac{\left(3\,c^2x^4+bcx^2-2\,b^2\right)\sqrt{cx^4+bx^2}}{15\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

giac [A] time = 0.16, size = 44, normalized size = 0.85

$$\frac{2\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,c^2}+\frac{3\left(cx^2+b\right)^{\frac{5}{2}}\mathrm{sgn}(x)-5\left(cx^2+b\right)^{\frac{3}{2}}b\mathrm{sgn}(x)}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 2/15*b^(5/2)*sgn(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2)*sgn(x) - 5*(c*x^2 + b)^
(3/2)*b*sgn(x))/c^2
```

maple [A] time = 0.00, size = 39, normalized size = 0.75

$$-\frac{\left(c\,x^{2}+b\right)\left(-3c\,x^{2}+2b\right)\sqrt{c\,x^{4}+b\,x^{2}}}{15c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2)^(1/2),x)
[Out] -1/15*(c*x^2+b)*(-3*c*x^2+2*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

maxima [A] time = 1.39, size = 34, normalized size = 0.65

$$\frac{\left(3\,c^{2}x^{4}+bcx^{2}-2\,b^{2}\right)\sqrt{cx^{2}+b}}{15\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)/c^2
```

mupad [B] time = 4.14, size = 41, normalized size = 0.79

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(-2\,b^{2}+b\,c\,x^{2}+3\,c^{2}\,x^{4}\right)}{15\,c^{2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2 + c*x^4)^(1/2),x)
[Out] ((b*x^2 + c*x^4)^(1/2)*(3*c^2*x^4 - 2*b^2 + b*c*x^2))/(15*c^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{x^{2}\left(b+cx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**2*sqrt(x**2*(b + c*x**2)), x)
```

3.115 $\int \sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=25

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (b*x^2 + c*x^4)^(3/2)/(3*c*x^3)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \sqrt{bx^2+cx^4}\,dx=\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x^2*(b + c*x^2))^(3/2)/(3*c*x^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 25, normalized size = 1.00

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4],x]
[Out] (b*x^2 + c*x^4)^(3/2)/(3*c*x^3)
```

fricas [A] time = 0.99, size = 28, normalized size = 1.12

$$\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{3\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(c*x)
```

giac [A] time = 0.15, size = 27, normalized size = 1.08

$$\frac{\left(cx^2+b\right)^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,c}-\frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(c*x^2 + b)^(3/2)*sgn(x)/c - 1/3*b^(3/2)*sgn(x)/c
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$\frac{\left(c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2}}{3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2),x)
[Out] 1/3*(c*x^2+b)/c/x*(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 14, normalized size = 0.56

$$\frac{\left(cx^2+b\right)^{\frac{3}{2}}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^2 + b)^(3/2)/c
```

mupad [B] time = 4.14, size = 29, normalized size = 1.16

$$\frac{\left(\frac{b}{3\,c}+\frac{x^2}{3}\right)\,\sqrt{c\,x^4+b\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b/(3*c) + x^2/3)*(b*x^2 + c*x^4)^(1/2))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + c*x**4), x)
```

3.116 $\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx &= \frac{\sqrt{bx^2+cx^4}}{x} + b\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx \\ &= \frac{\sqrt{bx^2+cx^4}}{x} - b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right) \\ &= \frac{\sqrt{bx^2+cx^4}}{x} - \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.20

$$\frac{x\left(-\sqrt{b}\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)+b+cx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^2,x]
[Out] (x*(b + c*x^2 - Sqrt[b]*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/
Sqrt[x^2*(b + c*x^2)]
```

IntegrateAlgebraic [A] time = 0.08, size = 50, normalized size = 1.00

$$\frac{\sqrt{bx^2+cx^4}}{x} - \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^2,x]
[Out] Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

fricas [A] time = 0.98, size = 117, normalized size = 2.34

$$\left[\frac{\sqrt{b}\,x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}}{2\,x}, \frac{\sqrt{-b}\,x\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) +
 2*sqrt(c*x^4 + b*x^2))/x, (sqrt(-b)*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/
(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2))/x]
```

giac [A] time = 0.18, size = 69, normalized size = 1.38

$$\frac{b\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\sqrt{cx^2+b}\,\operatorname{sgn}(x)-\frac{\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\,\sqrt{b}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + sqrt(c*x^2 + b)*sgn(x)
 - (b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sgn(x)/sqrt(-b)
```

maple [A] time = 0.01, size = 65, normalized size = 1.30

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-\sqrt{c\,x^2+b}\right)}{\sqrt{c\,x^2+b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^2,x)
```

```
[Out] -(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)-(c*x^2+b)
^(1/2))/x/(c*x^2+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^4+bx^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^2, x)
```

mupad [B] time = 4.31, size = 68, normalized size = 1.36

$$\frac{\sqrt{c\,x^4+b\,x^2}}{x}+\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c\,x^4+b\,x^2}\,1\mathrm{i}}{\sqrt{c}\,x^2\,\sqrt{\frac{b}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^2,x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/x + (b^(1/2)*asin((b^(1/2)*1i)/(c^(1/2)*x))*(b*x^2 +
c*x^4)^(1/2)*1i)/(c^(1/2)*x^2*(b/(c*x^2) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**2, x)
```

3.117 $\int \frac{\sqrt{bx^2+cx^4}}{x^4} dx$

Optimal. Leaf size=56

$$-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*x^3) - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])
/(2*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx = -\frac{\sqrt{bx^2+cx^4}}{2x^3} + \frac{1}{2}c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{\sqrt{bx^2+cx^4}}{2x^3} - \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)$$
$$= -\frac{\sqrt{bx^2+cx^4}}{2x^3} - \frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.12

$$-\frac{cx^2\sqrt{\frac{cx^2}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)+b+cx^2}{2x\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^4,x]
[Out] -1/2*(b + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/b]*ArcTanh[Sqrt[1 + (c*x^2)/b]])/(
x*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.12, size = 56, normalized size = 1.00

$$-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt{bx^2+cx^4}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^4,x]
[Out] -1/2*Sqrt[b*x^2 + c*x^4]/x^3 - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])
/(2*Sqrt[b])
```

fricas [A] time = 3.90, size = 134, normalized size = 2.39

$$\left[\frac{\sqrt{b}\,cx^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\,b}{4\,bx^3}, \frac{\sqrt{-b}\,cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)-\sqrt{cx^4+bx^2}\,b}{2\,bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b*x^3), 1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c*x^
4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*b)/(b*x^3)]
```

giac [A] time = 0.20, size = 50, normalized size = 0.89

$$\frac{\frac{c^2 \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} - \frac{\sqrt{cx^2+b}\, c\,\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - sqrt(c*x^2 + b)
*c*sgn(x)/x^2)/c
```

maple [A] time = 0.01, size = 85, normalized size = 1.52

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\sqrt{b}\;c\,x^2 \ln\left(\frac{2b+2\sqrt{c\,x^2+b}\;\sqrt{b}}{x}\right)-\sqrt{c\,x^2+b}\;c\,x^2+\left(c\,x^2+b\right)^{\frac{3}{2}}\right)}{2\sqrt{c\,x^2+b}\;b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2*c
-(c*x^2+b)^(1/2)*x^2*c+(c*x^2+b)^(3/2))/x^3/(c*x^2+b)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^4,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**4, x)
```

3.118 $\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=84

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}$$

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}-\frac{\sqrt{bx^2+cx^4}}{4x^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*x^5) - (c*Sqrt[b*x^2 + c*x^4])/(8*b*x^3) + (c^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}+\frac{1}{4}c\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}-\frac{c^2\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.55

$$-\frac{c^2\left(x^2\left(b+cx^2\right)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^2}{b}+1\right)}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -1/3*(c^2*(x^2*(b + c*x^2))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x^2
)/b])/(b^3*x^3)
```

IntegrateAlgebraic [A] time = 0.13, size = 71, normalized size = 0.85

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}+\frac{\left(-2b-cx^2\right)\sqrt{bx^2+cx^4}}{8bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] ((-2*b - c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*b*x^5) + (c^2*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(8*b^(3/2))
```

fricas [A] time = 1.36, size = 159, normalized size = 1.89

$$\left[\frac{\sqrt{b}\,c^2x^5\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\left(bcx^2+2\,b^2\right)}{16\,b^2x^5},-\frac{\sqrt{-b}\,c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\left(bcx^2+2\,b^2\right)}{8\,b^2x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))
/x^3) - 2*sqrt(c*x^4 + b*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5), -1/8*(sqrt(-b)*
c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5)]
```

giac [A] time = 0.21, size = 78, normalized size = 0.93

$$-\frac{\frac{c^3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b}+\frac{\left(cx^2+b\right)^{\frac{3}{2}}c^3\mathrm{sgn}(x)+\sqrt{cx^2+b}\,bc^3\mathrm{sgn}(x)}{bc^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/8*(c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + ((c*x^2 +
b)^(3/2)*c^3*sgn(x) + sqrt(c*x^2 + b)*b*c^3*sgn(x))/(b*c^2*x^4))/c
```

maple [A] time = 0.01, size = 106, normalized size = 1.26

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\sqrt{b}\,c^2x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-\sqrt{c\,x^2+b}\,c^2x^4+\left(c\,x^2+b\right)^{\frac{3}{2}}c\,x^2-2\left(c\,x^2+b\right)^{\frac{3}{2}}b\right)}{8\sqrt{c\,x^2+b}\,b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^6,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*c^
2-(c*x^2+b)^(1/2)*x^4*c^2+(c*x^2+b)^(3/2)*x^2*c-2*(c*x^2+b)^(3/2)*b)/x^5/(c
*x^2+b)^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^2}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^6,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**6, x)
```

3.119 $\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx$

Optimal. Leaf size=112

$$-\frac{c^3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}$$

Rubi [A] time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}-\frac{\sqrt{bx^2+cx^4}}{6x^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(6*x^7) - (c*Sqrt[b*x^2 + c*x^4])/(24*b*x^5) + (c^2*Sq
rt[b*x^2 + c*x^4])/(16*b^2*x^3) - (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]])/(16*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{6x^7}+\frac{1}{6}c\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}-\frac{c^2\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{8b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}+\frac{c^3\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{16b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{16b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.41

$$\frac{c^3\left(x^2\left(b+cx^2\right)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{cx^2}{b}+1\right)}{3b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] (c^3*(x^2*(b + c*x^2))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (c*x^2)/b])
/(3*b^4*x^3)
```

IntegrateAlgebraic [A] time = 0.17, size = 82, normalized size = 0.73

$$\frac{\sqrt{bx^2+cx^4}\left(-8b^2-2bcx^2+3c^2x^4\right)}{48b^2x^7}-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-8*b^2 - 2*b*c*x^2 + 3*c^2*x^4))/(48*b^2*x^7) - (c^3*
ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(16*b^(5/2))
```

fricas [A] time = 1.13, size = 185, normalized size = 1.65

$$\left[\frac{3\,\sqrt{b}c^3x^7\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(3\,bc^2x^4-2\,b^2cx^2-8\,b^3\right)\sqrt{cx^4+bx^2}}{96\,b^3x^7},\frac{3\,\sqrt{-b}\,c^3x^7\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\left(3\,bc^2x^4-2\,b^2cx^2-8\,b^3\right)\sqrt{cx^4+bx^2}}{48\,b^3x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*(3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3*x
^7), 1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 +
b*x)) + (3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3*x^7)]
```

giac [A] time = 0.27, size = 100, normalized size = 0.89

$$\frac{\frac{3\,c^4\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b^2}+\frac{3\left(cx^2+b\right)^{\frac{5}{2}}c^4\mathrm{sgn}(x)-8\left(cx^2+b\right)^{\frac{3}{2}}bc^4\mathrm{sgn}(x)-3\,\sqrt{cx^2+b}\,b^2c^4\mathrm{sgn}(x)}{b^2c^3x^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/48*(3*c^4*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^2) + (3*(c*
x^2 + b)^(5/2)*c^4*sgn(x) - 8*(c*x^2 + b)^(3/2)*b*c^4*sgn(x) - 3*sqrt(c*x^2
 + b)*b^2*c^4*sgn(x))/(b^2*c^3*x^6))/c
```

maple [A] time = 0.01, size = 128, normalized size = 1.14

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\sqrt{b}\,c^3x^6\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^2+b}\,c^3x^6+3\left(c\,x^2+b\right)^{\frac{3}{2}}c^2x^4-6\left(c\,x^2+b\right)^{\frac{3}{2}}bc\,x^2+8\left(c\,x^2+b\right)^{\frac{3}{2}}b^2\right)}{48\sqrt{c\,x^2+b}\,b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^8,x)
```

```
[Out] -1/48*(c*x^4+b*x^2)^(1/2)*(3*b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^
6*c^3-3*(c*x^2+b)^(1/2)*x^6*c^3+3*(c*x^2+b)^(3/2)*x^4*c^2-6*(c*x^2+b)^(3/2)
*x^2*b*c+8*(c*x^2+b)^(3/2)*b^2)/x^7/(c*x^2+b)^(1/2)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^8,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**8,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**8, x)
```

3.120 $\int x^3 \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=124

$$-\frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}$$

Rubi [A] time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 640, 612, 620, 206}

$$\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b^3*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - (b*(b + 2*c*x^2)*(b*x
^2 + c*x^4)^(3/2))/(32*c^2) + (b*x^2 + c*x^4)^(5/2)/(10*c) - (3*b^5*ArcTanh
[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(bx^2+cx^4\right)^{3/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\left(bx+cx^2\right)^{3/2} dx, x, x^2\right) \\
&= \frac{\left(bx^2+cx^4\right)^{5/2}}{10c} - \frac{b\operatorname{Subst}\left(\int \left(bx+cx^2\right)^{3/2} dx, x, x^2\right)}{4c} \\
&= -\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2} + \frac{\left(bx^2+cx^4\right)^{5/2}}{10c} + \frac{\left(3b^3\right)\operatorname{Subst}\left(\int \sqrt{bx+cx^2}\, dx, x, x^2\right)}{64c^2} \\
&= \frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3} - \frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2} + \frac{\left(bx^2+cx^4\right)^{5/2}}{10c} - \frac{\left(3b^5\right)\operatorname{Sul}}{} \\
&= \frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3} - \frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2} + \frac{\left(bx^2+cx^4\right)^{5/2}}{10c} - \frac{\left(3b^5\right)\operatorname{Sul}}{} \\
&= \frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3} - \frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2} + \frac{\left(bx^2+cx^4\right)^{5/2}}{10c} - \frac{3b^5\tanh}{25}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 126, normalized size = 1.02

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(15b^4-10b^3cx^2+8b^2c^2x^4+176bc^3x^6+128c^4x^8\right)-15b^{9/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\right)}{1280c^{7/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(15*b^4 - 10*b^3*c*x^
2 + 8*b^2*c^2*x^4 + 176*b*c^3*x^6 + 128*c^4*x^8) - 15*b^(9/2)*ArcSinh[(Sqrt
[c]*x)/Sqrt[b]]))/(1280*c^(7/2)*x*Sqrt[1 + (c*x^2)/b])
```

IntegrateAlgebraic [A] time = 0.34, size = 109, normalized size = 0.88

$$\frac{3b^5\log\left(-2\sqrt{c}\sqrt{bx^2+cx^4}+b+2cx^2\right)}{512c^{7/2}}+\frac{\sqrt{bx^2+cx^4}\left(15b^4-10b^3cx^2+8b^2c^2x^4+176bc^3x^6+128c^4x^8\right)}{1280c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(15*b^4 - 10*b^3*c*x^2 + 8*b^2*c^2*x^4 + 176*b*c^3*x^6
 + 128*c^4*x^8))/(1280*c^3) + (3*b^5*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[b*x^2
 + c*x^4]])/(512*c^(7/2))
```

fricas [A] time = 2.61, size = 210, normalized size = 1.69

$$\left[\frac{15\,b^5\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\left(128\,c^5x^8+176\,bc^4x^6+8\,b^2c^3x^4-10\,b^3c^2x^2+15\,b^4c\right)\sqrt{cx^4+bx^2}}{2560\,c^4},\ \frac{15\,b^5\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\left(128\,c^5x^8+176\,bc^4x^6+8\,b^2c^3x^4-10\,b^3c^2x^2+15\,b^4c\right)\sqrt{cx^4+bx^2}}{1280\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(15*b^5*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 - 10*b^3*c^2*x^2 + 15*b^4*c
)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*b^5*sqrt(-c)*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-c)/(c*x^2 + b)) + (128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 -
10*b^3*c^2*x^2 + 15*b^4*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

giac [A] time = 0.28, size = 115, normalized size = 0.93

$$\frac{3\,b^5\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{256\,c^{\frac{7}{2}}}-\frac{3\,b^5\log\left(|b|\right)\mathrm{sgn}(x)}{512\,c^{\frac{7}{2}}}+\frac{1}{1280}\left(2\left(4\left(2\left(8\,cx^2\mathrm{sgn}(x)+11\,b\mathrm{sgn}(x)\right)x^2+\frac{b^2\mathrm{sgn}(x)}{c}\right)x^2-\frac{5\,b^3\mathrm{sgn}(x)}{c^2}\right)x^2+\frac{15\,b^4\mathrm{sgn}(x)}{c^3}\right)\sqrt{cx^2+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/256*b^5*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(7/2) - 3/512*b^5
*log(abs(b))*sgn(x)/c^(7/2) + 1/1280*(2*(4*(2*(8*c*x^2*sgn(x) + 11*b*sgn(x)
)*x^2 + b^2*sgn(x)/c)*x^2 - 5*b^3*sgn(x)/c^2)*x^2 + 15*b^4*sgn(x)/c^3)*sqrt
(c*x^2 + b)*x
```

maple [A] time = 0.01, size = 142, normalized size = 1.15

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(128\left(c\,x^2+b\right)^{\frac{5}{2}}c^{\frac{5}{2}}x^5-15b^5\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-15\sqrt{c\,x^2+b}\,b^4\sqrt{c}\,x-80\left(c\,x^2+b\right)^{\frac{5}{2}}b\,c^{\frac{3}{2}}x^3-10\left(c\,x^2+b\right)^{\frac{3}{2}}b^3\sqrt{c}\,x+40\left(c\,x^2+b\right)^{\frac{5}{2}}b^2\sqrt{c}\,x\right)}{1280\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(128*x^5*(c*x^2+b)^(5/2)*c^(5/2)-80*c^(3/2)*(c*x
^2+b)^(5/2)*x^3*b+40*c^(1/2)*(c*x^2+b)^(5/2)*x*b^2-10*c^(1/2)*(c*x^2+b)^(3/
2)*x*b^3-15*c^(1/2)*(c*x^2+b)^(1/2)*x*b^4-15*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*
b^5)/x^3/(c*x^2+b)^(3/2)/c^(7/2)
```

maxima [A] time = 1.39, size = 142, normalized size = 1.15

$$\frac{3\,\sqrt{cx^4+bx^2}\,b^3x^2}{128\,c^2}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}bx^2}{16\,c}-\frac{3\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{512\,c^{\frac{7}{2}}}+\frac{3\,\sqrt{cx^4+bx^2}\,b^4}{256\,c^3}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}b^2}{32\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{5}{2}}}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/128*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^2 - 1/16*(c*x^4 + b*x^2)^(3/2)*b*x^2/c
- 3/512*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 3/25
6*sqrt(c*x^4 + b*x^2)*b^4/c^3 - 1/32*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 1/10*(
c*x^4 + b*x^2)^(5/2)/c
```

mupad [B] time = 4.35, size = 134, normalized size = 1.08

$$\frac{\left(c\,x^4+b\,x^2\right)^{5/2}}{10\,c}-\frac{b\,\left(\frac{x^2\,\left(c\,x^4+b\,x^2\right)^{3/2}}{4}-\frac{3\,b^2\,\left(\frac{\left(2\,c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2}}{4\,c}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{16\,c}+\frac{b\,\left(c\,x^4+b\,x^2\right)^{3/2}}{8\,c}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(5/2)/(10*c) - (b*((x^2*(b*x^2 + c*x^4)^(3/2))/4 - (3*b^2*(
((b + 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(4*c) - (b^2*log((b/2 + c*x^2)/c^(1/2
) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))))/(16*c) + (b*(b*x^2 + c*x^4)^(3/2)
)/(8*c)))/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3*(x**2*(b + c*x**2))**(3/2), x)
```

3.121 $\int x\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}$$

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2013, 612, 620, 206}

$$-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) + ((b + 2*c*x^2)*(b*x^
2 + c*x^4)^(3/2))/(16*c) + (3*b^4*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]
])/(128*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int x\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$
$$=\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}-\frac{\left(3b^2\right)\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{32c}$$
$$=-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3b^4\right)\text{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x}{256c^2}$$
$$=-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3b^4\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{}{\surd}\right.}{128c^2}$$
$$=-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}$$

Mathematica [A] time = 0.10, size = 115, normalized size = 1.14

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3b^{7/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)+\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(-3b^3+2b^2cx^2+24bc^2x^4+16c^3x^6\right)\right)}{128c^{5/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-3*b^3 + 2*b^2*c*x^2
 + 24*b*c^2*x^4 + 16*c^3*x^6) + 3*b^(7/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(1
28*c^(5/2)*x*Sqrt[1 + (c*x^2)/b])
```

IntegrateAlgebraic [A] time = 0.29, size = 98, normalized size = 0.97

$$\frac{\sqrt{bx^2+cx^4}\left(-3b^3+2b^2cx^2+24bc^2x^4+16c^3x^6\right)}{128c^2}-\frac{3b^4\log\left(-2\sqrt{c}\sqrt{bx^2+cx^4}+b+2cx^2\right)}{256c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-3*b^3 + 2*b^2*c*x^2 + 24*b*c^2*x^4 + 16*c^3*x^6))/(1
28*c^2) - (3*b^4*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[b*x^2 + c*x^4]])/(256*c^(
5/2))
```

fricas [A] time = 1.14, size = 189, normalized size = 1.87

$$\left[\frac{3\,b^{4}\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\,\left(16\,c^{4}x^{6}+24\,bc^{3}x^{4}+2\,b^{2}c^{2}x^{2}-3\,b^{3}c\right)\sqrt{cx^{4}+bx^{2}}}{256\,c^{3}},\,-\frac{3\,b^{4}\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)-\left(16\,c^{4}x^{6}+24\,bc^{3}x^{4}+2\,b^{2}c^{2}x^{2}-3\,b^{3}c\right)\sqrt{cx^{4}+bx^{2}}}{128\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*b^4*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x^2))
/c^3, -1/128*(3*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b
)) - (16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x
^2))/c^3]
```

giac [A] time = 0.19, size = 99, normalized size = 0.98

$$-\frac{3\,b^{4}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^{2}+b}\right|\right)\mathrm{sgn}\left(x\right)}{128\,c^{\frac{5}{2}}}+\frac{3\,b^{4}\log\left(\left|b\right|\right)\mathrm{sgn}\left(x\right)}{256\,c^{\frac{5}{2}}}+\frac{1}{128}\left(2\left(4\left(2\,cx^{2}\mathrm{sgn}\left(x\right)+3\,b\mathrm{sgn}\left(x\right)\right)x^{2}+\frac{b^{2}\mathrm{sgn}\left(x\right)}{c}\right)x^{2}-\frac{3\,b^{3}\mathrm{sgn}\left(x\right)}{c^{2}}\right)\sqrt{cx^{2}+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 3/256*b^
4*log(abs(b))*sgn(x)/c^(5/2) + 1/128*(2*(4*(2*c*x^2*sgn(x) + 3*b*sgn(x))*x^
2 + b^2*sgn(x)/c)*x^2 - 3*b^3*sgn(x)/c^2)*sqrt(c*x^2 + b)*x
```

maple [A] time = 0.01, size = 122, normalized size = 1.21

$$\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(3b^{4}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)+3\sqrt{c\,x^{2}+b}\,b^{3}\sqrt{c}\,x+16\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x^{3}+2\left(c\,x^{2}+b\right)^{\frac{3}{2}}b^{2}\sqrt{c}\,x-8\left(c\,x^{2}+b\right)^{\frac{5}{2}}b\sqrt{c}\,x\right)}{128\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/128*(c*x^4+b*x^2)^(3/2)*(16*x^3*(c*x^2+b)^(5/2)*c^(3/2)-8*c^(1/2)*(c*x^2+
b)^(5/2)*x*b+2*(c*x^2+b)^(3/2)*b^2*c^(1/2)*x+3*(c*x^2+b)^(1/2)*b^3*c^(1/2)*
x+3*b^4*ln(c^(1/2)*x+(c*x^2+b)^(1/2)))/x^3/(c*x^2+b)^(3/2)/c^(5/2)
```

maxima [A] time = 1.47, size = 118, normalized size = 1.17

$$\frac{1}{8}\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2-\frac{3\sqrt{cx^4+bx^2}\,b^2x^2}{64\,c}+\frac{3\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{256\,c^{\frac{5}{2}}}-\frac{3\sqrt{cx^4+bx^2}\,b^3}{128\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}b}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^4 + b*x^2)^(3/2)*x^2 - 3/64*sqrt(c*x^4 + b*x^2)*b^2*x^2/c + 3/256*
b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) - 3/128*sqrt(c
*x^4 + b*x^2)*b^3/c^2 + 1/16*(c*x^4 + b*x^2)^(3/2)*b/c
```

mupad [B] time = 4.44, size = 99, normalized size = 0.98

$$\frac{\left(c\,x^4+b\,x^2\right)^{3/2}\,\left(c\,x^2+\frac{b}{2}\right)}{8\,c}-\frac{3\,b^2\,\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{32\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(3/2)*(b/2 + c*x^2))/(8*c) - (3*b^2*((b/(4*c) + x^2/2)*(b*
x^2 + c*x^4)^(1/2) - (b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)
))/(8*c^(3/2))))/(32*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(b + c*x**2))**(3/2), x)
```

3.122 $\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=88

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}$$

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 664, 612, 620, 206}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + (b*x^2 + c*x^4)^(3/2)/6 - (b
^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{6}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{4}b\operatorname{Subst}\left(\int \sqrt{bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{32c}\\
&= \frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c}\\
&= \frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 104, normalized size = 1.18

$$
\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(3b^2+14bcx^2+8c^2x^4\right)-3b^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\right)}{48c^{3/2}x\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x,x]

[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(3*b^2 + 14*b*c*x^2 +
 8*c^2*x^4) - 3*b^(5/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(48*c^(3/2)*x*Sqrt[1
 + (c*x^2)/b])
```

IntegrateAlgebraic [A] time = 0.29, size = 91, normalized size = 1.03

$$\frac{b^3\log\left(-2c^{3/2}\sqrt{bx^2+cx^4}+bc+2c^2x^2\right)}{32c^{3/2}}+\frac{\sqrt{bx^2+cx^4}\left(3b^2+14bcx^2+8c^2x^4\right)}{48c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(3*b^2 + 14*b*c*x^2 + 8*c^2*x^4))/(48*c) + (b^3*Log[b*
c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[b*x^2 + c*x^4]])/(32*c^(3/2))
```

fricas [A] time = 1.02, size = 166, normalized size = 1.89

$$\left[\frac{3\,b^3\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\left(8\,c^3x^4+14\,bc^2x^2+3\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^2},\ \frac{3\,b^3\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\left(8\,c^3x^4+14\,bc^2x^2+3\,b^2c\right)\sqrt{cx^4+bx^2}}{48\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*b^3*
sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^4 + 14
*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2]
```

giac [A] time = 0.21, size = 84, normalized size = 0.95

$$\frac{b^3\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\operatorname{sgn}(x)}{16\,c^{\frac{3}{2}}}-\frac{b^3\log\left(|b|\right)\operatorname{sgn}(x)}{32\,c^{\frac{3}{2}}}+\frac{1}{48}\left(2\left(4\,cx^2\operatorname{sgn}(x)+7\,b\operatorname{sgn}(x)\right)x^2+\frac{3\,b^2\operatorname{sgn}(x)}{c}\right)\sqrt{cx^2+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/32*b^3*l
og(abs(b))*sgn(x)/c^(3/2) + 1/48*(2*(4*c*x^2*sgn(x) + 7*b*sgn(x))*x^2 + 3*b
^2*sgn(x)/c)*sqrt(c*x^2 + b)*x
```

maple [A] time = 0.01, size = 102, normalized size = 1.16

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-3b^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-3\sqrt{c\,x^2+b}\,b^2\sqrt{c}\,x-2\left(c\,x^2+b\right)^{\frac{3}{2}}b\sqrt{c}\,x+8\left(c\,x^2+b\right)^{\frac{5}{2}}\sqrt{c}\,x\right)}{48\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(8*x*(c*x^2+b)^(5/2)*c^(1/2)-2*(c*x^2+b)^(3/2)*b*c
^(1/2)*x-3*(c*x^2+b)^(1/2)*b^2*c^(1/2)*x-3*b^3*ln(c^(1/2)*x+(c*x^2+b)^(1/2)
))/x^3/(c*x^2+b)^(3/2)/c^(3/2)
```

maxima [A] time = 1.42, size = 91, normalized size = 1.03

$$\frac{1}{8}\sqrt{cx^4+bx^2}\,bx^2 - \frac{b^3\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{32\,c^{\frac{3}{2}}} + \frac{1}{6}\left(cx^4+bx^2\right)^{\frac{3}{2}} + \frac{\sqrt{cx^4+bx^2}\,b^2}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*b*x^2 - 1/32*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b
*x^2)*sqrt(c))/c^(3/2) + 1/6*(c*x^4 + b*x^2)^(3/2) + 1/16*sqrt(c*x^4 + b*x^
2)*b^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x, x)
```

3.123 $\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}$$

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 664, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*b*Sqrt[b*x^2 + c*x^4])/8 + (b*x^2 + c*x^4)^(3/2)/(4*x^2) + (3*b^2*ArcTan
h[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= \frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{1}{16}\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{1}{8}\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 71, normalized size = 0.89

$$\frac{1}{8}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{3b^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}}+5b+2cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(5*b + 2*c*x^2 + (3*b^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt
[b]])/(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b])))/8
```

IntegrateAlgebraic [A] time = 0.29, size = 73, normalized size = 0.91

$$\frac{1}{8}\left(5b+2cx^2\right)\sqrt{bx^2+cx^4}-\frac{3b^2\log\left(-2\sqrt{c}\sqrt{bx^2+cx^4}+b+2cx^2\right)}{16\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] ((5*b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/8 - (3*b^2*Log[b + 2*c*x^2 - 2*Sqrt[c
]*Sqrt[b*x^2 + c*x^4]])/(16*Sqrt[c])
```

fricas [A] time = 1.58, size = 145, normalized size = 1.81

$$\left[\frac{3\,b^2\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(2\,c^2x^2+5\,bc\right)}{16\,c},-\frac{3\,b^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}\left(2\,c^2x^2+5\,bc\right)}{8\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5*b*c))/c, -1/8*(3*b^2*sqrt(-c)*arctan(sqr
t(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5
*b*c))/c]
```

giac [A] time = 0.25, size = 68, normalized size = 0.85

$$-\frac{3\,b^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{8\,\sqrt{c}}+\frac{3\,b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,\sqrt{c}}+\frac{1}{8}\left(2\,cx^2\mathrm{sgn}(x)+5\,b\mathrm{sgn}(x)\right)\sqrt{cx^2+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/8*b^2*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/sqrt(c) + 3/16*b^2*l
og(abs(b))*sgn(x)/sqrt(c) + 1/8*(2*c*x^2*sgn(x) + 5*b*sgn(x))*sqrt(c*x^2 +
b)*x
```

maple [A] time = 0.00, size = 84, normalized size = 1.05

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+3\sqrt{c\,x^2+b}\,b\sqrt{c}\,x+2\left(c\,x^2+b\right)^{\frac{3}{2}}\sqrt{c}\,x\right)}{8\left(c\,x^2+b\right)^{\frac{3}{2}}\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^3,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(3/2)*(2*(c*x^2+b)^(3/2)*c^(1/2)*x+3*(c*x^2+b)^(1/2)*b*c^
(1/2)*x+3*b^2*ln(c^(1/2)*x+(c*x^2+b)^(1/2)))/x^3/(c*x^2+b)^(3/2)/c^(1/2)
```

maxima [A] time = 1.44, size = 70, normalized size = 0.88

$$\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{16\,\sqrt{c}}+\frac{3}{8}\,\sqrt{cx^4+bx^2}\,b+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 3/8*sqr
t(c*x^4 + b*x^2)*b + 1/4*(c*x^4 + b*x^2)^(3/2)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^3,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**3, x)
```

3.124 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=76

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{3}{2}c\sqrt{bx^2+cx^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 662, 664, 620, 206}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{3}{2}c\sqrt{bx^2+cx^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/2 - (b*x^2 + c*x^4)^(3/2)/x^4 + (3*b*Sqrt[c]*ArcT
anh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{1}{2}(3c)\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{1}{4}(3bc)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{1}{2}(3bc)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{3}{2}b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.71

$$-\frac{b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^2}{b}\right)}{x^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] -((b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/b)]
)/(x^2*Sqrt[1 + (c*x^2)/b]))
```

IntegrateAlgebraic [A] time = 0.29, size = 73, normalized size = 0.96

$$\frac{\left(cx^2-2b\right)\sqrt{bx^2+cx^4}}{2x^2}-\frac{3}{4}b\sqrt{c}\,\log\left(-2\sqrt{c}\,\sqrt{bx^2+cx^4}\,+b+2cx^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] ((-2*b + c*x^2)*Sqrt[b*x^2 + c*x^4])/(2*x^2) - (3*b*Sqrt[c]*Log[b + 2*c*x^2
 - 2*Sqrt[c]*Sqrt[b*x^2 + c*x^4]])/4
```

fricas [A] time = 1.46, size = 139, normalized size = 1.83

$$\left[\frac{3\,b\sqrt{c}\,x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(cx^2-2\,b\right)}{4\,x^2},-\frac{3\,b\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}\left(cx^2-2\,b\right)}{2\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/x^2, -1/2*(3*b*sqrt(-c)*x^2*arctan(sqrt
(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/
x^2]
```

giac [A] time = 0.27, size = 79, normalized size = 1.04

$$\frac{1}{2}\,\sqrt{cx^2+b}\,cx\mathrm{sgn}(x)-\frac{3}{4}\,b\sqrt{c}\,\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}(x)+\frac{2\,b^2\sqrt{c}\,\mathrm{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*c*x*sgn(x) - 3/4*b*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2
+ b))^2)*sgn(x) + 2*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b
)
```

maple [A] time = 0.01, size = 107, normalized size = 1.41

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^2cx\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+3\sqrt{c\,x^2+b}\,b\,c^{\frac{3}{2}}x^2+2\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x^2-2\left(c\,x^2+b\right)^{\frac{5}{2}}\sqrt{c}\right)}{2\left(c\,x^2+b\right)^{\frac{3}{2}}b\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^5,x)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(3/2)*(2*c^(3/2)*(c*x^2+b)^(3/2)*x^2+3*c^(3/2)*(c*x^2+b)^
(1/2)*x^2*b-2*(c*x^2+b)^(5/2)*c^(1/2)+3*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x*b^2
*c)/x^4/(c*x^2+b)^(3/2)/b/c^(1/2)
```

maxima [A] time = 1.44, size = 71, normalized size = 0.93

$$\frac{3}{4}\,b\sqrt{c}\,\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{3\,\sqrt{cx^4+bx^2}\,b}{2\,x^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 3/2*sqrt(c
*x^4 + b*x^2)*b/x^2 + 1/2*(c*x^4 + b*x^2)^(3/2)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^5,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**5, x)
```

3.125 $\int \frac{(bx^2+cx^4)^{3/2}}{x^7} dx$

Optimal. Leaf size=75

$$c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{(bx^2+cx^4)^{3/2}}{3x^6}$$

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 662, 620, 206}

$$c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{(bx^2+cx^4)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -((c*Sqrt[b*x^2 + c*x^4])/x^2) - (b*x^2 + c*x^4)^(3/2)/(3*x^6) + c^(3/2)*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^4}\,dx, x, x^2\right) \\
&= -\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6} + \frac{1}{2}c\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{c\sqrt{bx^2+cx^4}}{x^2} - \frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6} + \frac{1}{2}c^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right) \\
&= -\frac{c\sqrt{bx^2+cx^4}}{x^2} - \frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6} + c^2\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right) \\
&= -\frac{c\sqrt{bx^2+cx^4}}{x^2} - \frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6} + c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.75

$$
-\frac{b\sqrt{x^2\left(b+cx^2\right)}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx^2}{b}\right)}{3x^4\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/3*(b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((c*x^2)
/b)])/(x^4*Sqrt[1 + (c*x^2)/b])
```

IntegrateAlgebraic [A] time = 0.24, size = 73, normalized size = 0.97

$$
\frac{\left(-b-4cx^2\right)\sqrt{bx^2+cx^4}}{3x^4} - \frac{1}{2}c^{3/2}\log\left(-2\sqrt{c}\,\sqrt{bx^2+cx^4}+b+2cx^2\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] ((-b - 4*c*x^2)*Sqrt[b*x^2 + c*x^4])/(3*x^4) - (c^(3/2)*Log[b + 2*c*x^2 - 2
*Sqrt[c]*Sqrt[b*x^2 + c*x^4]])/2
```

fricas [A] time = 1.35, size = 135, normalized size = 1.80

$$\left[\frac{3\,c^{\frac{3}{2}}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\left(4\,cx^2+b\right)}{6\,x^4},\,-\frac{3\,\sqrt{-c}\,cx^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\left(4\,cx^2+b\right)}{3\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/6*(3*c^(3/2)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*s
qrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^4, -1/3*(3*sqrt(-c)*c*x^4*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^
4]
```

giac [A] time = 0.44, size = 122, normalized size = 1.63

$$-\frac{1}{2}\,c^{\frac{3}{2}}\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}(x)+\frac{4\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4bc^{\frac{3}{2}}\mathrm{sgn}(x)-3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{3}{2}}\mathrm{sgn}(x)+2\,b^3c^{\frac{3}{2}}\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 4/3*(3*(sqrt(c)*
x - sqrt(c*x^2 + b))^4*b*c^(3/2)*sgn(x) - 3*(sqrt(c)*x - sqrt(c*x^2 + b))^2
*b^2*c^(3/2)*sgn(x) + 2*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^
2 - b)^3
```

maple [B] time = 0.01, size = 129, normalized size = 1.72

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^2c^2x^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+3\sqrt{c\,x^2+b}\,b\,c^{\frac{5}{2}}x^4+2\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^4-2\left(c\,x^2+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x^2-\left(c\,x^2+b\right)^{\frac{5}{2}}b\sqrt{c}\right)}{3\left(c\,x^2+b\right)^{\frac{3}{2}}b^2\sqrt{c}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^7,x)
```

```
[Out] 1/3*(c*x^4+b*x^2)^(3/2)*(2*c^(5/2)*(c*x^2+b)^(3/2)*x^4+3*c^(5/2)*(c*x^2+b)^
(1/2)*x^4*b-2*c^(3/2)*(c*x^2+b)^(5/2)*x^2+3*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x
^3*b^2*c^2-(c*x^2+b)^(5/2)*b*c^(1/2))/x^6/(c*x^2+b)^(3/2)/b^2/c^(1/2)
```

maxima [A] time = 1.51, size = 89, normalized size = 1.19

$$\frac{1}{2}\,c^{\frac{3}{2}}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-\frac{7\,\sqrt{cx^{4}+bx^{2}}\,c}{6\,x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}\,b}{6\,x^{4}}-\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7/6*sqrt(c*x
^4 + b*x^2)*c/x^2 - 1/6*sqrt(c*x^4 + b*x^2)*b/x^4 - 1/6*(c*x^4 + b*x^2)^(3/
2)/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^7,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**7, x)
```

3.126 $\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=25

$$-\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(5*b*x^10)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/5*(x^2*(b + c*x^2))^(5/2)/(b*x^10)
```

IntegrateAlgebraic [A] time = 0.22, size = 46, normalized size = 1.84

$$\frac{\sqrt{bx^2+cx^4}\left(-b^2-2bcx^2-c^2x^4\right)}{5bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-b^2 - 2*b*c*x^2 - c^2*x^4))/(5*b*x^6)
```

fricas [A] time = 2.06, size = 39, normalized size = 1.56

$$-\frac{\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{cx^4+bx^2}}{5\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

giac [B] time = 0.30, size = 92, normalized size = 3.68

$$\frac{2\left(5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^8c^{\frac{5}{2}}\mathrm{sgn}(x)+10\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4b^2c^{\frac{5}{2}}\mathrm{sgn}(x)+b^4c^{\frac{5}{2}}\mathrm{sgn}(x)\right)}{5\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(5/2)*sgn(x) + 10*(sqrt(c)*x - sqr
t(c*x^2 + b))^4*b^2*c^(5/2)*sgn(x) + b^4*c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt
(c*x^2 + b))^2 - b)^5
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$-\frac{\left(c\,x^2+b\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{5b\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^9,x)
```

```
[Out] -1/5/x^8*(c*x^2+b)/b*(c*x^4+b*x^2)^(3/2)
```

maxima [B] time = 1.41, size = 81, normalized size = 3.24

$$-\frac{\sqrt{cx^4+bx^2}\,c^2}{5\,bx^2}+\frac{\sqrt{cx^4+bx^2}\,c}{10\,x^4}+\frac{3\,\sqrt{cx^4+bx^2}\,b}{10\,x^6}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{2\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) + 1/10*sqrt(c*x^4 + b*x^2)*c/x^4 + 3/1
0*sqrt(c*x^4 + b*x^2)*b/x^6 - 1/2*(c*x^4 + b*x^2)^(3/2)/x^8
```

mupad [B] time = 4.38, size = 30, normalized size = 1.20

$$-\frac{\left(c\,x^2+b\right)^2\,\sqrt{c\,x^4+b\,x^2}}{5\,b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^9,x)
```

```
[Out] -((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2))/(5*b*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**9, x)
```

3.127 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=52

$$\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}}-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}}-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(7*b*x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x^1
0)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx = -\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}} - \frac{(2c)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^9}\,dx}{7b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(2cx^2-5b\right)}{35b^2x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^11,x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*b + 2*c*x^2))/(35*b^2*x^12)
```

IntegrateAlgebraic [A] time = 0.23, size = 57, normalized size = 1.10

$$\frac{\sqrt{bx^2+cx^4}\left(-5b^3-8b^2cx^2-bc^2x^4+2c^3x^6\right)}{35b^2x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^11,x]
[Out] (Sqrt[b*x^2 + c*x^4]*(-5*b^3 - 8*b^2*c*x^2 - b*c^2*x^4 + 2*c^3*x^6))/(35*b^
2*x^8)
```

fricas [A] time = 0.78, size = 53, normalized size = 1.02

$$\frac{\left(2\,c^3x^6-bc^2x^4-8\,b^2cx^2-5\,b^3\right)\sqrt{cx^4+bx^2}}{35\,b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
[Out] 1/35*(2*c^3*x^6 - b*c^2*x^4 - 8*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/(b^2
*x^8)
```

giac [B] time = 0.25, size = 178, normalized size = 3.42

$$\frac{4\left(35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}c^{\frac{7}{2}}\mathrm{sgn}(x)+35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}bc^{\frac{7}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}b^2c^{\frac{7}{2}}\mathrm{sgn}(x)+14\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}b^3c^{\frac{7}{2}}\mathrm{sgn}(x)+7\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}b^4c^{\frac{7}{2}}\mathrm{sgn}(x)-b^5c^{\frac{7}{2}}\mathrm{sgn}(x)\right)}{35\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] 4/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(7/2)*sgn(x) + 35*(sqrt(c)*x -
sqrt(c*x^2 + b))^8*b*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^
2*c^(7/2)*sgn(x) + 14*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(7/2)*sgn(x) +
7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(7/2)*sgn(x) - b^5*c^(7/2)*sgn(x))/
((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

maple [A] time = 0.00, size = 39, normalized size = 0.75

$$-\frac{\left(c\,x^2+b\right)\left(-2c\,x^2+5b\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{35b^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^11,x)
```

```
[Out] -1/35*(c*x^2+b)*(-2*c*x^2+5*b)*(c*x^4+b*x^2)^(3/2)/x^10/b^2
```

maxima [B] time = 1.46, size = 105, normalized size = 2.02

$$\frac{2\sqrt{cx^4+bx^2}\,c^3}{35\,b^2x^2}-\frac{\sqrt{cx^4+bx^2}\,c^2}{35\,bx^4}+\frac{3\sqrt{cx^4+bx^2}\,c}{140\,x^6}+\frac{3\sqrt{cx^4+bx^2}\,b}{28\,x^8}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{4\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] 2/35*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 1/35*sqrt(c*x^4 + b*x^2)*c^2/(b*x^
4) + 3/140*sqrt(c*x^4 + b*x^2)*c/x^6 + 3/28*sqrt(c*x^4 + b*x^2)*b/x^8 - 1/4
*(c*x^4 + b*x^2)^(3/2)/x^10
```

mupad [B] time = 4.58, size = 87, normalized size = 1.67

$$\frac{2\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2}-\frac{8\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6}-\frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4}-\frac{b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^11,x)
```

```
[Out] (2*c^3*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^2) - (8*c*(b*x^2 + c*x^4)^(1/2))/(3
5*x^6) - (c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) - (b*(b*x^2 + c*x^4)^(1/2))
/(7*x^8)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**11,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**11, x)
```

3.128 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=80

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

Rubi [**A**] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(9*b*x^14) + (4*c*(b*x^2 + c*x^4)^(5/2))/(63*b^2*x^1
2) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x^10)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx = -\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}} - \frac{(4c)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx}{9b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}} + \frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}} + \frac{\left(8c^2\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^9}\,dx}{63b^2}$$
$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}} + \frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}} - \frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.58

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(35b^2-20bcx^2+8c^2x^4\right)}{315b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -1/315*((x^2*(b + c*x^2))^(5/2)*(35*b^2 - 20*b*c*x^2 + 8*c^2*x^4))/(b^3*x^1
4)
```

IntegrateAlgebraic [A] time = 0.25, size = 68, normalized size = 0.85

$$\frac{\sqrt{bx^2+cx^4}\left(-35b^4-50b^3cx^2-3b^2c^2x^4+4bc^3x^6-8c^4x^8\right)}{315b^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-35*b^4 - 50*b^3*c*x^2 - 3*b^2*c^2*x^4 + 4*b*c^3*x^6
- 8*c^4*x^8))/(315*b^3*x^10)
```

fricas [A] time = 1.12, size = 64, normalized size = 0.80

$$-\frac{\left(8\,c^4x^8-4\,bc^3x^6+3\,b^2c^2x^4+50\,b^3cx^2+35\,b^4\right)\sqrt{cx^4+bx^2}}{315\,b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/315*(8*c^4*x^8 - 4*b*c^3*x^6 + 3*b^2*c^2*x^4 + 50*b^3*c*x^2 + 35*b^4)*sq
rt(c*x^4 + b*x^2)/(b^3*x^10)
```

giac [B] time = 0.27, size = 206, normalized size = 2.58

$$\frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}c^{\frac{9}{2}}\mathrm{sgn}(x)+315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{9}{2}}\mathrm{sgn}(x)+441\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{9}{2}}\mathrm{sgn}(x)+126\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}b^3c^{\frac{9}{2}}\mathrm{sgn}(x)+36\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}b^4c^{\frac{9}{2}}\mathrm{sgn}(x)-9\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}b^5c^{\frac{9}{2}}\mathrm{sgn}(x)+b^6c^{\frac{9}{2}}\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(9/2)*sgn(x) + 315*(sqrt(c)*
x - sqrt(c*x^2 + b))^10*b*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + b)
)^8*b^2*c^(9/2)*sgn(x) + 126*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c^(9/2)*sg
n(x) + 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(9/2)*sgn(x) - 9*(sqrt(c)*x
 - sqrt(c*x^2 + b))^2*b^5*c^(9/2)*sgn(x) + b^6*c^(9/2)*sgn(x))/((sqrt(c)*x
- sqrt(c*x^2 + b))^2 - b)^9
```

maple [A] time = 0.00, size = 50, normalized size = 0.62

$$-\frac{\left(c\,x^2+b\right)\left(8c^2x^4-20bc\,x^2+35b^2\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{315b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^13,x)
```

```
[Out] -1/315*(c*x^2+b)*(8*c^2*x^4-20*b*c*x^2+35*b^2)*(c*x^4+b*x^2)^(3/2)/x^12/b^3
```

maxima [A] time = 1.52, size = 129, normalized size = 1.61

$$-\frac{8\sqrt{cx^4+bx^2}\,c^4}{315\,b^3x^2}+\frac{4\sqrt{cx^4+bx^2}\,c^3}{315\,b^2x^4}-\frac{\sqrt{cx^4+bx^2}\,c^2}{105\,bx^6}+\frac{\sqrt{cx^4+bx^2}\,c}{126\,x^8}+\frac{\sqrt{cx^4+bx^2}\,b}{18\,x^{10}}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{6\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -8/315*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) + 4/315*sqrt(c*x^4 + b*x^2)*c^3/(b
^2*x^4) - 1/105*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) + 1/126*sqrt(c*x^4 + b*x^2)
*c/x^8 + 1/18*sqrt(c*x^4 + b*x^2)*b/x^10 - 1/6*(c*x^4 + b*x^2)^(3/2)/x^12
```

mupad [B] time = 4.72, size = 111, normalized size = 1.39

$$\frac{4\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4}-\frac{10\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8}-\frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6}-\frac{b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{8\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^13,x)
```

```
[Out] (4*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (10*c*(b*x^2 + c*x^4)^(1/2))/
(63*x^8) - (c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) - (b*(b*x^2 + c*x^4)^(1/
2))/(9*x^10) - (8*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**13,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**13, x)
```

3.129 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\left(bx^2+cx^4\right)^{5/2}}{1155b^4x^{10}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{231b^3x^{12}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}}{33b^2x^{14}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

Rubi [A] time = 0.18, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16c^3\left(bx^2+cx^4\right)^{5/2}}{1155b^4x^{10}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{231b^3x^{12}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}}{33b^2x^{14}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(11*b*x^16) + (2*c*(b*x^2 + c*x^4)^(5/2))/(33*b^2*x^
14) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(231*b^3*x^12) + (16*c^3*(b*x^2 + c*x^4
)^(5/2))/(1155*b^4*x^10)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(6c)\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx}{11b}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} + \frac{(8c^2)\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx}{33b^2}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} - \frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}} - \frac{(16c^3)\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx}{231b^3}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} - \frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}} + \frac{16c^3(bx^2+cx^4)^{5/2}}{1155b^4x^{10}}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.53

$$\frac{(x^2(b+cx^2))^{5/2}(-105b^3+70b^2cx^2-40bc^2x^4+16c^3x^6)}{1155b^4x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-105*b^3 + 70*b^2*c*x^2 - 40*b*c^2*x^4 + 16*c^3*x
^6))/(1155*b^4*x^16)
```

IntegrateAlgebraic [A] time = 0.26, size = 79, normalized size = 0.73

$$\frac{\sqrt{bx^2+cx^4}\,(-105b^5-140b^4cx^2-5b^3c^2x^4+6b^2c^3x^6-8bc^4x^8+16c^5x^{10})}{1155b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-105*b^5 - 140*b^4*c*x^2 - 5*b^3*c^2*x^4 + 6*b^2*c^3*
x^6 - 8*b*c^4*x^8 + 16*c^5*x^10))/(1155*b^4*x^12)
```

fricas [A] time = 1.65, size = 75, normalized size = 0.69

$$\frac{(16\,c^5x^{10}-8\,bc^4x^8+6\,b^2c^3x^6-5\,b^3c^2x^4-140\,b^4cx^2-105\,b^5)\sqrt{cx^4+bx^2}}{1155\,b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")

[Out] 1/1155*(16*c^5*x^10 - 8*b*c^4*x^8 + 6*b^2*c^3*x^6 - 5*b^3*c^2*x^4 - 140*b^4*c*x^2 - 105*b^5)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)

giac [B] time = 0.28, size = 236, normalized size = 2.19

$$\frac{32\left(1155\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}c^{\frac{11}{2}}\operatorname{sgn}(x)+2079\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}bc^{\frac{11}{2}}\operatorname{sgn}(x)+2541\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}b^2c^{\frac{11}{2}}\operatorname{sgn}(x)+825\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}b^3c^{\frac{11}{2}}\operatorname{sgn}(x)+165\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}b^4c^{\frac{11}{2}}\operatorname{sgn}(x)-55\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}b^5c^{\frac{11}{2}}\operatorname{sgn}(x)+11\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}b^6c^{\frac{11}{2}}\operatorname{sgn}(x)-b^7c^{\frac{11}{2}}\operatorname{sgn}(x)\right)}{1155\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")

[Out] 32/1155*(1155*(sqrt(c)*x - sqrt(c*x^2 + b))^14*c^(11/2)*sgn(x) + 2079*(sqrt(c)*x - sqrt(c*x^2 + b))^12*b*c^(11/2)*sgn(x) + 2541*(sqrt(c)*x - sqrt(c*x^2 + b))^10*b^2*c^(11/2)*sgn(x) + 825*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^3*c^(11/2)*sgn(x) + 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^4*c^(11/2)*sgn(x) - 55*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^5*c^(11/2)*sgn(x) + 11*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^6*c^(11/2)*sgn(x) - b^7*c^(11/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11

maple [A] time = 0.00, size = 61, normalized size = 0.56

$$-\frac{\left(c\,x^2+b\right)\left(-16c^3x^6+40b\,c^2x^4-70b^2c\,x^2+105b^3\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{1155b^4x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2)^(3/2)/x^15,x)

[Out] -1/1155*(c*x^2+b)*(-16*c^3*x^6+40*b*c^2*x^4-70*b^2*c*x^2+105*b^3)*(c*x^4+b*x^2)^(3/2)/x^14/b^4

maxima [A] time = 1.49, size = 153, normalized size = 1.42

$$\frac{16\sqrt{cx^4+bx^2}\,c^5}{1155\,b^4x^2}-\frac{8\sqrt{cx^4+bx^2}\,c^4}{1155\,b^3x^4}+\frac{2\sqrt{cx^4+bx^2}\,c^3}{385\,b^2x^6}-\frac{\sqrt{cx^4+bx^2}\,c^2}{231\,bx^8}+\frac{\sqrt{cx^4+bx^2}\,c}{264\,x^{10}}+\frac{3\sqrt{cx^4+bx^2}\,b}{88\,x^{12}}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{8\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")

[Out] 16/1155*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 8/1155*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^4) + 2/385*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 1/231*sqrt(c*x^4 + b*x^2)*c^2/(b*x^8) + 1/264*sqrt(c*x^4 + b*x^2)*c/x^10 + 3/88*sqrt(c*x^4 + b*x^2)*b/x^12 - 1/8*(c*x^4 + b*x^2)^(3/2)/x^14

mupad [B] time = 4.99, size = 135, normalized size = 1.25

$$\frac{2c^3\sqrt{cx^4+bx^2}}{385b^2x^6}-\frac{4c\sqrt{cx^4+bx^2}}{33x^{10}}-\frac{c^2\sqrt{cx^4+bx^2}}{231bx^8}-\frac{b\sqrt{cx^4+bx^2}}{11x^{12}}-\frac{8c^4\sqrt{cx^4+bx^2}}{1155b^3x^4}+\frac{16c^5\sqrt{cx^4+bx^2}}{1155b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^15,x)
```

```
[Out] (2*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (4*c*(b*x^2 + c*x^4)^(1/2))/(
33*x^10) - (c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*x^8) - (b*(b*x^2 + c*x^4)^(1/
2))/(11*x^12) - (8*c^4*(b*x^2 + c*x^4)^(1/2))/(1155*b^3*x^4) + (16*c^5*(b*x
^2 + c*x^4)^(1/2))/(1155*b^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**15, x)
```

3.130 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=136

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}}{15015b^5x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}}+\frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

Rubi [A] time = 0.26, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}}{15015b^5x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}}+\frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(13*b*x^18) + (8*c*(b*x^2 + c*x^4)^(5/2))/(143*b^2*x
^16) - (16*c^2*(b*x^2 + c*x^4)^(5/2))/(429*b^3*x^14) + (64*c^3*(b*x^2 + c*x
^4)^(5/2))/(3003*b^4*x^12) - (128*c^4*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x^1
0)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(8c)\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx}{13b}$$

$$= -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{(48c^2)\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx}{143b^2}$$

$$= -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}} - \frac{16c^2(bx^2+cx^4)^{5/2}}{429b^3x^{14}} - \frac{(64c^3)\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx}{429b^3}$$

$$= -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}} - \frac{16c^2(bx^2+cx^4)^{5/2}}{429b^3x^{14}} + \frac{64c^3(bx^2+cx^4)^{5/2}}{3003b^4x^{12}} + \frac{(128c}{}$$

$$= -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}} - \frac{16c^2(bx^2+cx^4)^{5/2}}{429b^3x^{14}} + \frac{64c^3(bx^2+cx^4)^{5/2}}{3003b^4x^{12}} - \frac{128c^4}{1}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.50

$$-\frac{(x^2(b+cx^2))^{5/2}(1155b^4-840b^3cx^2+560b^2c^2x^4-320bc^3x^6+128c^4x^8)}{15015b^5x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] -1/15015*((x^2*(b + c*x^2))^(5/2)*(1155*b^4 - 840*b^3*c*x^2 + 560*b^2*c^2*x
^4 - 320*b*c^3*x^6 + 128*c^4*x^8))/(b^5*x^18)
```

IntegrateAlgebraic [A] time = 0.28, size = 90, normalized size = 0.66

$$\frac{\sqrt{bx^2+cx^4}\,(-1155b^6-1470b^5cx^2-35b^4c^2x^4+40b^3c^3x^6-48b^2c^4x^8+64bc^5x^{10}-128c^6x^{12})}{15015b^5x^{14}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-1155*b^6 - 1470*b^5*c*x^2 - 35*b^4*c^2*x^4 + 40*b^3*
c^3*x^6 - 48*b^2*c^4*x^8 + 64*b*c^5*x^10 - 128*c^6*x^12))/(15015*b^5*x^14)
```

fricas [A] time = 1.14, size = 86, normalized size = 0.63

$$-\frac{(128\,c^6x^{12}-64\,bc^5x^{10}+48\,b^2c^4x^8-40\,b^3c^3x^6+35\,b^4c^2x^4+1470\,b^5cx^2+1155\,b^6)\sqrt{cx^4+bx^2}}{15015\,b^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/15015*(128*c^6*x^12 - 64*b*c^5*x^10 + 48*b^2*c^4*x^8 - 40*b^3*c^3*x^6 +
35*b^4*c^2*x^4 + 1470*b^5*c*x^2 + 1155*b^6)*sqrt(c*x^4 + b*x^2)/(b^5*x^14)
```

giac [B] time = 0.29, size = 264, normalized size = 1.94

$$\frac{256\left(6006\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}c^{\frac{13}{2}}\mathrm{sgn}(x)+12012\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}bc^{\frac{13}{2}}\mathrm{sgn}(x)+13728\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}b^2c^{\frac{13}{2}}\mathrm{sgn}(x)+4719\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}b^3c^{\frac{13}{2}}\mathrm{sgn}(x)+715\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}b^4c^{\frac{13}{2}}\mathrm{sgn}(x)-286\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}b^5c^{\frac{13}{2}}\mathrm{sgn}(x)+78\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}b^6c^{\frac{13}{2}}\mathrm{sgn}(x)-13\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}b^7c^{\frac{13}{2}}\mathrm{sgn}(x)+b^8c^{\frac{13}{2}}\mathrm{sgn}(x)\right)}{15015\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] 256/15015*(6006*(sqrt(c)*x - sqrt(c*x^2 + b))^16*c^(13/2)*sgn(x) + 12012*(s
qrt(c)*x - sqrt(c*x^2 + b))^14*b*c^(13/2)*sgn(x) + 13728*(sqrt(c)*x - sqrt(
c*x^2 + b))^12*b^2*c^(13/2)*sgn(x) + 4719*(sqrt(c)*x - sqrt(c*x^2 + b))^10*
b^3*c^(13/2)*sgn(x) + 715*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^4*c^(13/2)*sgn(
x) - 286*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^5*c^(13/2)*sgn(x) + 78*(sqrt(c)*
x - sqrt(c*x^2 + b))^4*b^6*c^(13/2)*sgn(x) - 13*(sqrt(c)*x - sqrt(c*x^2 + b
))^2*b^7*c^(13/2)*sgn(x) + b^8*c^(13/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 +
b))^2 - b)^13
```

maple [A] time = 0.01, size = 72, normalized size = 0.53

$$-\frac{\left(c\,x^2+b\right)\left(128c^4x^8-320c^3x^6b+560c^2x^4b^2-840c\,x^2b^3+1155b^4\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{15015b^5x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^17,x)
```

```
[Out] -1/15015*(c*x^2+b)*(128*c^4*x^8-320*b*c^3*x^6+560*b^2*c^2*x^4-840*b^3*c*x^2
+1155*b^4)*(c*x^4+b*x^2)^(3/2)/x^16/b^5
```

maxima [A] time = 1.47, size = 177, normalized size = 1.30

$$-\frac{128\sqrt{cx^4+bx^2}\,c^6}{15015\,b^5x^2}+\frac{64\sqrt{cx^4+bx^2}\,c^5}{15015\,b^4x^4}-\frac{16\sqrt{cx^4+bx^2}\,c^4}{5005\,b^3x^6}+\frac{8\sqrt{cx^4+bx^2}\,c^3}{3003\,b^2x^8}-\frac{\sqrt{cx^4+bx^2}\,c^2}{429\,bx^{10}}+\frac{3\sqrt{cx^4+bx^2}\,c}{1430\,x^{12}}+\frac{3\sqrt{cx^4+bx^2}\,b}{130\,x^{14}}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{10\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -128/15015*sqrt(c*x^4 + b*x^2)*c^6/(b^5*x^2) + 64/15015*sqrt(c*x^4 + b*x^2)
*c^5/(b^4*x^4) - 16/5005*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^6) + 8/3003*sqrt(c*
```

```
x^4 + b*x^2)*c^3/(b^2*x^8) - 1/429*sqrt(c*x^4 + b*x^2)*c^2/(b*x^10) + 3/143
0*sqrt(c*x^4 + b*x^2)*c/x^12 + 3/130*sqrt(c*x^4 + b*x^2)*b/x^14 - 1/10*(c*x
^4 + b*x^2)^(3/2)/x^16
```

mupad [B] time = 5.17, size = 159, normalized size = 1.17

$$\frac{8\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8}-\frac{14\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}}-\frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}}-\frac{b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}}-\frac{16\,c^4\,\sqrt{c\,x^4+b\,x^2}}{5005\,b^3\,x^6}+\frac{64\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^4}-\frac{128\,c^6\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^17,x)
```

```
[Out] (8*c^3*(b*x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (14*c*(b*x^2 + c*x^4)^(1/2))
/(143*x^12) - (c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) - (b*(b*x^2 + c*x^4)
^(1/2))/(13*x^14) - (16*c^4*(b*x^2 + c*x^4)^(1/2))/(5005*b^3*x^6) + (64*c^5
*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*c^6*(b*x^2 + c*x^4)^(1/2))/(
15015*b^5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**17,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**17, x)
```

3.131 $\int x^6 \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=134

$$\frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

Rubi [A] time = 0.25, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (128*b^4*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (64*b^3*(b*x^2 + c*x^4)^(
5/2))/(3003*c^4*x^3) + (16*b^2*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - (8*b*x*
(b*x^2 + c*x^4)^(5/2))/(143*c^2) + (x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^6 \left(bx^2+cx^4\right)^{3/2} dx = \frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c} - \frac{(8b)\int x^4\left(bx^2+cx^4\right)^{3/2} dx}{13c}$$
$$= -\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2} + \frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c} + \frac{\left(48b^2\right)\int x^2\left(bx^2+cx^4\right)^{3/2} dx}{143c^2}$$
$$= \frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x} - \frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2} + \frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c} - \frac{\left(64b^3\right)\int\left(bx^2+cx^4\right)^{3/}}{429c^3}$$
$$= -\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3} + \frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x} - \frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2} + \frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c} +$$
$$= \frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5} - \frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3} + \frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x} - \frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 0.56

$$\frac{x\left(b+cx^2\right)^3\left(128b^4-320b^3cx^2+560b^2c^2x^4-840bc^3x^6+1155c^4x^8\right)}{15015c^5\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(b + c*x^2)^3*(128*b^4 - 320*b^3*c*x^2 + 560*b^2*c^2*x^4 - 840*b*c^3*x^6
 + 1155*c^4*x^8))/(15015*c^5*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.35, size = 68, normalized size = 0.51

$$\frac{\left(bx^2+cx^4\right)^{5/2}\left(128b^4-320b^3cx^2+560b^2c^2x^4-840bc^3x^6+1155c^4x^8\right)}{15015c^5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(b*x^2 + c*x^4)^(3/2),x]
[Out] ((b*x^2 + c*x^4)^(5/2)*(128*b^4 - 320*b^3*c*x^2 + 560*b^2*c^2*x^4 - 840*b*c
^3*x^6 + 1155*c^4*x^8))/(15015*c^5*x^5)
```

fricas [A] time = 1.19, size = 86, normalized size = 0.64

$$\frac{\left(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6\right)\sqrt{cx^4+bx^2}}{15015\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
+ 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

giac [A] time = 0.19, size = 92, normalized size = 0.69

$$-\frac{128\,b^{\frac{13}{2}}\mathrm{sgn}(x)}{15015\,c^5}+\frac{1155\left(cx^2+b\right)^{\frac{13}{2}}\mathrm{sgn}(x)-5460\left(cx^2+b\right)^{\frac{11}{2}}b\mathrm{sgn}(x)+10010\left(cx^2+b\right)^{\frac{9}{2}}b^2\mathrm{sgn}(x)-8580\left(cx^2+b\right)^{\frac{7}{2}}b^3\mathrm{sgn}(x)+3003\left(cx^2+b\right)^{\frac{5}{2}}b^4\mathrm{sgn}(x)}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -128/15015*b^(13/2)*sgn(x)/c^5 + 1/15015*(1155*(c*x^2 + b)^(13/2)*sgn(x) -
5460*(c*x^2 + b)^(11/2)*b*sgn(x) + 10010*(c*x^2 + b)^(9/2)*b^2*sgn(x) - 858
0*(c*x^2 + b)^(7/2)*b^3*sgn(x) + 3003*(c*x^2 + b)^(5/2)*b^4*sgn(x))/c^5
```

maple [A] time = 0.01, size = 72, normalized size = 0.54

$$\frac{\left(c\,x^2+b\right)\left(1155c^4x^8-840c^3x^6b+560c^2x^4b^2-320c\,x^2b^3+128b^4\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{15015c^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/15015*(c*x^2+b)*(1155*c^4*x^8-840*b*c^3*x^6+560*b^2*c^2*x^4-320*b^3*c*x^2
+128*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

maxima [A] time = 1.53, size = 79, normalized size = 0.59

$$\frac{\left(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6\right)\sqrt{cx^2+b}}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
+ 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^2 + b)/c^5
```

mupad [B] time = 4.48, size = 73, normalized size = 0.54

$$\frac{\left(c\,x^2+b\right)^2\,\sqrt{c\,x^4+b\,x^2}\,\left(128\,b^4-320\,b^3\,c\,x^2+560\,b^2\,c^2\,x^4-840\,b\,c^3\,x^6+1155\,c^4\,x^8\right)}{15015\,c^5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2 + c*x^4)^(3/2),x)
[Out] ((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(128*b^4 + 1155*c^4*x^8 - 320*b^3*c*x^
2 - 840*b*c^3*x^6 + 560*b^2*c^2*x^4))/(15015*c^5*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**6*(x**2*(b + c*x**2))**(3/2), x)
```

3.132 $\int x^4 \left(bx^2 + cx^4\right)^{3/2} dx$

Optimal. Leaf size=106

$$-\frac{16b^3 \left(bx^2 + cx^4\right)^{5/2}}{1155c^4x^5} + \frac{8b^2 \left(bx^2 + cx^4\right)^{5/2}}{231c^3x^3} - \frac{2b \left(bx^2 + cx^4\right)^{5/2}}{33c^2x} + \frac{x \left(bx^2 + cx^4\right)^{5/2}}{11c}$$

Rubi [A] time = 0.20, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{16b^3 \left(bx^2 + cx^4\right)^{5/2}}{1155c^4x^5} + \frac{8b^2 \left(bx^2 + cx^4\right)^{5/2}}{231c^3x^3} - \frac{2b \left(bx^2 + cx^4\right)^{5/2}}{33c^2x} + \frac{x \left(bx^2 + cx^4\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-16*b^3*(b*x^2 + c*x^4)^(5/2))/(1155*c^4*x^5) + (8*b^2*(b*x^2 + c*x^4)^(5/
2))/(231*c^3*x^3) - (2*b*(b*x^2 + c*x^4)^(5/2))/(33*c^2*x) + (x*(b*x^2 + c*
x^4)^(5/2))/(11*c)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(bx^2+cx^4\right)^{3/2}dx &= \frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{(6b)\int x^2\left(bx^2+cx^4\right)^{3/2}dx}{11c}\\ &= -\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}+\frac{\left(8b^2\right)\int\left(bx^2+cx^4\right)^{3/2}dx}{33c^2}\\ &= \frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{\left(16b^3\right)\int\frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}dx}{231c^3}\\ &= -\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.60

$$\frac{x\left(b+cx^2\right)^3\left(-16b^3+40b^2cx^2-70bc^2x^4+105c^3x^6\right)}{1155c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(-16*b^3 + 40*b^2*c*x^2 - 70*b*c^2*x^4 + 105*c^3*x^6))/(11
55*c^4*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.34, size = 79, normalized size = 0.75

$$\frac{\sqrt{bx^2+cx^4}\left(-16b^5+8b^4cx^2-6b^3c^2x^4+5b^2c^3x^6+140bc^4x^8+105c^5x^{10}\right)}{1155c^4x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-16*b^5 + 8*b^4*c*x^2 - 6*b^3*c^2*x^4 + 5*b^2*c^3*x^6
 + 140*b*c^4*x^8 + 105*c^5*x^10))/(1155*c^4*x)
```

fricas [A] time = 0.76, size = 75, normalized size = 0.71

$$\frac{\left(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5\right)\sqrt{cx^4+bx^2}}{1155\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

giac [A] time = 0.16, size = 76, normalized size = 0.72

$$\frac{16\,b^{\frac{11}{2}}\operatorname{sgn}(x)}{1155\,c^4}+\frac{105\left(cx^2+b\right)^{\frac{11}{2}}\operatorname{sgn}(x)-385\left(cx^2+b\right)^{\frac{9}{2}}b\operatorname{sgn}(x)+495\left(cx^2+b\right)^{\frac{7}{2}}b^2\operatorname{sgn}(x)-231\left(cx^2+b\right)^{\frac{5}{2}}b^3\operatorname{sgn}(x)}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 16/1155*b^(11/2)*sgn(x)/c^4 + 1/1155*(105*(c*x^2 + b)^(11/2)*sgn(x) - 385*(
c*x^2 + b)^(9/2)*b*sgn(x) + 495*(c*x^2 + b)^(7/2)*b^2*sgn(x) - 231*(c*x^2 +
 b)^(5/2)*b^3*sgn(x))/c^4
```

maple [A] time = 0.01, size = 61, normalized size = 0.58

$$-\frac{\left(c\,x^2+b\right)\left(-105c^3x^6+70b\,c^2x^4-40b^2c\,x^2+16b^3\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{1155c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/1155*(c*x^2+b)*(-105*c^3*x^6+70*b*c^2*x^4-40*b^2*c*x^2+16*b^3)*(c*x^4+b*
x^2)^(3/2)/c^4/x^3
```

maxima [A] time = 1.50, size = 68, normalized size = 0.64

$$\frac{\left(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5\right)\sqrt{cx^2+b}}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)/c^4
```

mupad [B] time = 4.30, size = 62, normalized size = 0.58

$$-\frac{\left(c\,x^2+b\right)^2\,\sqrt{c\,x^4+b\,x^2}\,\left(16\,b^3-40\,b^2\,c\,x^2+70\,b\,c^2\,x^4-105\,c^3\,x^6\right)}{1155\,c^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(16*b^3 - 105*c^3*x^6 - 40*b^2*c*x^2
+ 70*b*c^2*x^4))/(1155*c^4*x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(x**2*(b + c*x**2))**(3/2), x)
```

3.133 $\int x^2 \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=80

$$\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - (4*b*(b*x^2 + c*x^4)^(5/2))/(
63*c^2*x^3) + (b*x^2 + c*x^4)^(5/2)/(9*c*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{(4b)\int\left(bx^2+cx^4\right)^{3/2}\,dx}{9c}\\
&= -\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}+\frac{\left(8b^2\right)\int\frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx}{63c^2}\\
&= \frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.66

$$
\frac{x\left(b+cx^2\right)^3\left(8b^2-20bcx^2+35c^2x^4\right)}{315c^3\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(8*b^2 - 20*b*c*x^2 + 35*c^2*x^4))/(315*c^3*Sqrt[x^2*(b +
c*x^2)])
```

IntegrateAlgebraic [A] time = 0.31, size = 68, normalized size = 0.85

$$
\frac{\sqrt{bx^2+cx^4}\left(8b^4-4b^3cx^2+3b^2c^2x^4+50bc^3x^6+35c^4x^8\right)}{315c^3x}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(8*b^4 - 4*b^3*c*x^2 + 3*b^2*c^2*x^4 + 50*b*c^3*x^6 +
35*c^4*x^8))/(315*c^3*x)
```

fricas [A] time = 1.02, size = 64, normalized size = 0.80

$$
\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^4+bx^2}}{315\,c^3x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^4 + b*x^2)/(c^3*x)
```

giac [A] time = 0.16, size = 60, normalized size = 0.75

$$-\frac{8\,b^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,c^{3}}+\frac{35\left(cx^{2}+b\right)^{\frac{9}{2}}\mathrm{sgn}(x)-90\left(cx^{2}+b\right)^{\frac{7}{2}}b\mathrm{sgn}(x)+63\left(cx^{2}+b\right)^{\frac{5}{2}}b^{2}\mathrm{sgn}(x)}{315\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/315*b^(9/2)*sgn(x)/c^3 + 1/315*(35*(c*x^2 + b)^(9/2)*sgn(x) - 90*(c*x^2
+ b)^(7/2)*b*sgn(x) + 63*(c*x^2 + b)^(5/2)*b^2*sgn(x))/c^3
```

maple [A] time = 0.01, size = 50, normalized size = 0.62

$$\frac{\left(c\,x^{2}+b\right)\left(35c^{2}x^{4}-20bc\,x^{2}+8b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/315*(c*x^2+b)*(35*c^2*x^4-20*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(3/2)/c^3/x^3
```

maxima [A] time = 1.45, size = 57, normalized size = 0.71

$$\frac{\left(35\,c^{4}x^{8}+50\,bc^{3}x^{6}+3\,b^{2}c^{2}x^{4}-4\,b^{3}cx^{2}+8\,b^{4}\right)\sqrt{cx^{2}+b}}{315\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)/c^3
```

mupad [B] time = 4.19, size = 51, normalized size = 0.64

$$\frac{{\left(c\,x^2+b\right)}^2\,\sqrt{c\,x^4+b\,x^2}\,\left(8\,b^2-20\,b\,c\,x^2+35\,c^2\,x^4\right)}{315\,c^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(8*b^2 + 35*c^2*x^4 - 20*b*c*x^2))/(31
5*c^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(b + c*x**2))**(3/2), x)
```

3.134 $\int \left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=52

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{(2b)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx}{7c}\\
&= -\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}+\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.81

$$\frac{x\left(b+cx^2\right)^3\left(5cx^2-2b\right)}{35c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(-2*b + 5*c*x^2))/(35*c^2*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.30, size = 56, normalized size = 1.08

$$\frac{\sqrt{bx^2+cx^4}\left(-2b^3+b^2cx^2+8bc^2x^4+5c^3x^6\right)}{35c^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-2*b^3 + b^2*c*x^2 + 8*b*c^2*x^4 + 5*c^3*x^6))/(35*c^
2*x)
```

fricas [A] time = 1.04, size = 52, normalized size = 1.00

$$\frac{\left(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3\right)\sqrt{cx^4+bx^2}}{35\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^4 + b*x^2)/(c^2
*x)
```

giac [A] time = 0.16, size = 44, normalized size = 0.85

$$\frac{2\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,c^2}+\frac{5\left(cx^2+b\right)^{\frac{7}{2}}\mathrm{sgn}(x)-7\left(cx^2+b\right)^{\frac{5}{2}}b\mathrm{sgn}(x)}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/35*b^(7/2)*sgn(x)/c^2 + 1/35*(5*(c*x^2 + b)^(7/2)*sgn(x) - 7*(c*x^2 + b)^
(5/2)*b*sgn(x))/c^2
```

maple [A] time = 0.01, size = 39, normalized size = 0.75

$$-\frac{\left(c\,x^{2}+b\right)\left(-5c\,x^{2}+2b\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{35c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2),x)
[Out] -1/35*(c*x^2+b)*(-5*c*x^2+2*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

maxima [A] time = 1.50, size = 45, normalized size = 0.87

$$\frac{\left(5\,c^{3}x^{6}+8\,bc^{2}x^{4}+b^{2}cx^{2}-2\,b^{3}\right)\sqrt{cx^{2}+b}}{35\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)/c^2
```

mupad [B] time = 4.16, size = 40, normalized size = 0.77

$$-\frac{\left(c\,x^{2}+b\right)^{2}\sqrt{c\,x^{4}+b\,x^{2}}\,\left(2\,b-5\,c\,x^{2}\right)}{35\,c^{2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2),x)
[Out] -((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(2*b - 5*c*x^2))/(35*c^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^{2}+cx^{4}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2),x)
[Out] Integral((b*x**2 + c*x**4)**(3/2), x)
```

3.135 $\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=25

$$\frac{(bx^2+cx^4)^{5/2}}{5cx^5}$$

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{(bx^2+cx^4)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (b*x^2 + c*x^4)^(5/2)/(5*c*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{(bx^2+cx^4)^{5/2}}{5cx^5}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{(x^2(b+cx^2))^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (x^2*(b + c*x^2))^(5/2)/(5*c*x^5)
```

IntegrateAlgebraic [A] time = 0.27, size = 25, normalized size = 1.00

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^2,x]
[Out] (b*x^2 + c*x^4)^(5/2)/(5*c*x^5)
```

fricas [A] time = 0.96, size = 39, normalized size = 1.56

$$\frac{\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{cx^4+bx^2}}{5\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(c*x)
```

giac [A] time = 0.16, size = 27, normalized size = 1.08

$$\frac{\left(cx^2+b\right)^{\frac{5}{2}}\operatorname{sgn}(x)}{5\,c}-\frac{b^{\frac{5}{2}}\operatorname{sgn}(x)}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")
[Out] 1/5*(c*x^2 + b)^(5/2)*sgn(x)/c - 1/5*b^(5/2)*sgn(x)/c
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$\frac{\left(c\,x^2+b\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{5c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^2,x)
[Out] 1/5*(c*x^2+b)/c/x^3*(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.48, size = 32, normalized size = 1.28

$$\frac{\left(c^2x^4 + 2\,bcx^2 + b^2\right)\sqrt{cx^2 + b}}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)/c
```

mupad [B] time = 4.15, size = 30, normalized size = 1.20

$$\frac{\left(c\,x^2 + b\right)^2\,\sqrt{c\,x^4 + b\,x^2}}{5\,c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^2,x)
```

```
[Out] ((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2))/(5*c*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**2, x)
```

3.136 $\int \frac{(bx^2+cx^4)^{3/2}}{x^4} dx$

Optimal. Leaf size=73

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (b*Sqrt[b*x^2 + c*x^4])/x + (b*x^2 + c*x^4)^(3/2)/(3*x^3) - b^(3/2)*ArcTanh
[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx = \frac{(bx^2+cx^4)^{3/2}}{3x^3} + b\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx$$
$$= \frac{b\sqrt{bx^2+cx^4}}{x} + \frac{(bx^2+cx^4)^{3/2}}{3x^3} + b^2\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{b\sqrt{bx^2+cx^4}}{x} + \frac{(bx^2+cx^4)^{3/2}}{3x^3} - b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)$$
$$= \frac{b\sqrt{bx^2+cx^4}}{x} + \frac{(bx^2+cx^4)^{3/2}}{3x^3} - b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Mathematica [A] time = 0.05, size = 76, normalized size = 1.04

$$\frac{x\left(-3b^{3/2}\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)+4b^2+5bcx^2+c^2x^4\right)}{3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*(4*b^2 + 5*b*c*x^2 + c^2*x^4 - 3*b^(3/2)*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b
+ c*x^2]/Sqrt[b]]))/(3*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.38, size = 62, normalized size = 0.85

$$\frac{\left(4b+cx^2\right)\sqrt{bx^2+cx^4}}{3x} - b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] ((4*b + c*x^2)*Sqrt[b*x^2 + c*x^4])/(3*x) - b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqr
t[b*x^2 + c*x^4]]
```

fricas [A] time = 0.91, size = 140, normalized size = 1.92

$$\left[\frac{3\,b^{\frac{3}{2}}x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(cx^2+4\,b\right)}{6\,x}, \frac{3\,\sqrt{-b}\,bx\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\left(cx^2+4\,b\right)}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^(3/2)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)
 + 2*sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))/x, 1/3*(3*sqrt(-b)*b*x*arctan(sqrt(
c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))
/x]
```

giac [A] time = 0.17, size = 89, normalized size = 1.22

$$\frac{b^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\frac{1}{3}\left(cx^2+b\right)^{\frac{3}{2}}\operatorname{sgn}(x)+\sqrt{cx^2+b}\,b\operatorname{sgn}(x)-\frac{\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\,\sqrt{-b}\,b^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 1/3*(c*x^2 + b)^(3/2
)*sgn(x) + sqrt(c*x^2 + b)*b*sgn(x) - 1/3*(3*b^2*arctan(sqrt(b)/sqrt(-b)) +
 4*sqrt(-b)*b^(3/2))*sgn(x)/sqrt(-b)
```

maple [A] time = 0.01, size = 78, normalized size = 1.07

$$-\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^2+b}\,b-\left(c\,x^2+b\right)^{\frac{3}{2}}\right)}{3\left(c\,x^2+b\right)^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^4,x)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)-(c*
x^2+b)^(3/2)-3*(c*x^2+b)^(1/2)*b)/x^3/(c*x^2+b)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^4,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**4, x)
```

3.137 $\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=79

$$\frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)-\frac{(bx^2+cx^4)^{3/2}}{2x^5}$$

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2021, 2008, 206}

$$-\frac{(bx^2+cx^4)^{3/2}}{2x^5}+\frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/(2*x) - (b*x^2 + c*x^4)^(3/2)/(2*x^5) - (3*Sqrt[b
]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}+\frac{1}{2}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx\\ &= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}+\frac{1}{2}(3bc)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}-\frac{1}{2}(3bc)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}-\frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.56

$$\frac{c\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (c*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x^2)/b])/(
5*b^2*x^5)
```

IntegrateAlgebraic [A] time = 0.48, size = 66, normalized size = 0.84

$$\frac{\left(2cx^2-b\right)\sqrt{bx^2+cx^4}}{2x^3}-\frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] ((-b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(2*x^3) - (3*Sqrt[b]*c*ArcTanh[(Sqrt[b
]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

fricas [A] time = 2.62, size = 147, normalized size = 1.86

$$\left[\frac{3\,\sqrt{b}cx^{3}\log\left(-\frac{cx^{3}+2\,bx-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\,\sqrt{cx^{4}+bx^{2}}\left(2\,cx^{2}-b\right)}{4\,x^{3}},\ \frac{3\,\sqrt{-b}cx^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+\sqrt{cx^{4}+bx^{2}}\left(2\,cx^{2}-b\right)}{2\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/
x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b))/x^3, 1/2*(3*sqrt(-b)*c*x^3*arct
an(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(2*c*x
^2 - b))/x^3]
```

giac [A] time = 0.20, size = 69, normalized size = 0.87

$$\frac{\frac{3\,bc^{2}\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}+2\,\sqrt{cx^{2}+b}\,c^{2}\mathrm{sgn}(x)-\frac{\sqrt{cx^{2}+b}\,bc\mathrm{sgn}(x)}{x^{2}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*sqrt(c*x^
2 + b)*c^2*sgn(x) - sqrt(c*x^2 + b)*b*c*sgn(x)/x^2)/c
```

maple [A] time = 0.01, size = 102, normalized size = 1.29

$$-\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}c\,x^{2}\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^{2}+b}\,bc\,x^{2}-\left(c\,x^{2}+b\right)^{\frac{3}{2}}c\,x^{2}+\left(c\,x^{2}+b\right)^{\frac{5}{2}}\right)}{2\left(c\,x^{2}+b\right)^{\frac{3}{2}}b\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(3/2)*(3*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*b^(3/2)*x^2
*c-(c*x^2+b)^(3/2)*c*x^2+(c*x^2+b)^(5/2)-3*(c*x^2+b)^(1/2)*x^2*b*c)/x^5/(c*
x^2+b)^(3/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^6,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**6, x)
```

3.138 $\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=81

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}$$

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(8*x^3) - (b*x^2 + c*x^4)^(3/2)/(4*x^7) - (3*c^2
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}+\frac{1}{4}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx$$
$$= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}+\frac{1}{8}\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{1}{8}\left(3c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)$$
$$= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.99

$$-\frac{2b^2+3c^2x^4\sqrt{\frac{cx^2}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)+7bcx^2+5c^2x^4}{8x^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/8*(2*b^2 + 7*b*c*x^2 + 5*c^2*x^4 + 3*c^2*x^4*Sqrt[1 + (c*x^2)/b]*ArcTanh
[Sqrt[1 + (c*x^2)/b]])/(x^3*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.54, size = 68, normalized size = 0.84

$$\frac{\left(-2b-5cx^2\right)\sqrt{bx^2+cx^4}}{8x^5}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] ((-2*b - 5*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*x^5) - (3*c^2*ArcTanh[(Sqrt[b]*x)
/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

fricas [A] time = 0.71, size = 164, normalized size = 2.02

$$\left[\frac{3\sqrt{b}c^2x^5\log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\sqrt{cx^4+bx^2}\left(5bcx^2+2b^2\right)}{16bx^5},\frac{3\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)-\sqrt{cx^4+bx^2}\left(5bcx^2+2b^2\right)}{8bx^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5), 1/8*(3*sqrt(-
b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4
+ b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5)]
```

giac [A] time = 0.24, size = 76, normalized size = 0.94

$$\frac{\frac{3\,c^3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}-\frac{5\left(cx^2+b\right)^{\frac{3}{2}}c^3\operatorname{sgn}(x)-3\sqrt{cx^2+b}\,bc^3\operatorname{sgn}(x)}{c^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*(3*c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (5*(c*x^2 + b
)^(3/2)*c^3*sgn(x) - 3*sqrt(c*x^2 + b)*b*c^3*sgn(x))/(c^2*x^4))/c
```

maple [A] time = 0.01, size = 125, normalized size = 1.54

$$-\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}c^2x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^2+b}\;b\,c^2x^4-\left(c\,x^2+b\right)^{\frac{3}{2}}c^2x^4+\left(c\,x^2+b\right)^{\frac{5}{2}}c\,x^2+2\left(c\,x^2+b\right)^{\frac{5}{2}}b\right)}{8\left(c\,x^2+b\right)^{\frac{3}{2}}b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^8,x)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4
*c^2-(c*x^2+b)^(3/2)*c^2*x^4+(c*x^2+b)^(5/2)*x^2*c-3*(c*x^2+b)^(1/2)*x^4*b*
c^2+2*(c*x^2+b)^(5/2)*b)/x^7/(c*x^2+b)^(3/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^8,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**8, x)
```

3.139 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=109

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}-\frac{c\sqrt{bx^2+cx^4}}{8x^5}$$

Rubi [A] time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(8*x^5) - (c^2*Sqrt[b*x^2 + c*x^4])/(16*b*x^3) - (
b*x^2 + c*x^4)^(3/2)/(6*x^9) + (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(16*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{1}{2}c\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx\\ &= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{1}{8}c^2\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}-\frac{c^3\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{16b}\\ &= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{c^3\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{16b}\\ &= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.42

$$\frac{c^3\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] (c^3*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (c*x^2)/b])
/(5*b^4*x^5)
```

IntegrateAlgebraic [A] time = 0.64, size = 82, normalized size = 0.75

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}+\frac{\sqrt{bx^2+cx^4}\left(-8b^2-14bcx^2-3c^2x^4\right)}{48bx^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-8*b^2 - 14*b*c*x^2 - 3*c^2*x^4))/(48*b*x^7) + (c^3*A
rcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(16*b^(3/2))
```

fricas [A] time = 1.28, size = 185, normalized size = 1.70

$$\left[\frac{3\,\sqrt{b}\,c^{3}x^{7}\log\left(-\frac{cx^{3}+2\,bx+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)-2\,\left(3\,bc^{2}x^{4}+14\,b^{2}cx^{2}+8\,b^{3}\right)\sqrt{cx^{4}+bx^{2}}}{96\,b^{2}x^{7}},\,-\frac{3\,\sqrt{-b}\,c^{3}x^{7}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+\left(3\,bc^{2}x^{4}+14\,b^{2}cx^{2}+8\,b^{3}\right)\sqrt{cx^{4}+bx^{2}}}{48\,b^{2}x^{7}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) - 2*(3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^2*
x^7), -1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3
+ b*x)) + (3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^2*x^
7)]
```

giac [A] time = 0.24, size = 100, normalized size = 0.92

$$-\frac{\frac{3\,c^{4}\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b}+\frac{3\,\left(cx^{2}+b\right)^{\frac{5}{2}}c^{4}\mathrm{sgn}(x)+8\,\left(cx^{2}+b\right)^{\frac{3}{2}}bc^{4}\mathrm{sgn}(x)-3\,\sqrt{cx^{2}+b}\,b^{2}c^{4}\mathrm{sgn}(x)}{bc^{3}x^{6}}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/48*(3*c^4*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (3*(c*x
^2 + b)^(5/2)*c^4*sgn(x) + 8*(c*x^2 + b)^(3/2)*b*c^4*sgn(x) - 3*sqrt(c*x^2
+ b)*b^2*c^4*sgn(x))/(b*c^3*x^6))/c
```

maple [A] time = 0.01, size = 145, normalized size = 1.33

$$\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}c^{3}x^{6}\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^{2}+b}\,b\,c^{3}x^{6}-\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{3}x^{6}+\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{2}x^{4}+2\left(c\,x^{2}+b\right)^{\frac{5}{2}}bc\,x^{2}-8\left(c\,x^{2}+b\right)^{\frac{5}{2}}b^{2}\right)}{48\left(c\,x^{2}+b\right)^{\frac{3}{2}}b^{3}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^10,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^6
*c^3-(c*x^2+b)^(3/2)*x^6*c^3+(c*x^2+b)^(5/2)*x^4*c^2-3*(c*x^2+b)^(1/2)*x^6*
```

```
b*c^3+2*(c*x^2+b)^(5/2)*x^2*b*c-8*(c*x^2+b)^(5/2)*b^2)/x^9/(c*x^2+b)^(3/2)/
b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^10, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^10,x)
[Out] int((b*x^2 + c*x^4)^(3/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**10,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**10, x)
```

3.140 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=137

$$-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{c\sqrt{bx^2+cx^4}}{16x^7}$$

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(16*x^7) - (c^2*Sqrt[b*x^2 + c*x^4])/(64*b*x^5) +
(3*c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - (b*x^2 + c*x^4)^(3/2)/(8*x^11)
- (3*c^4*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{12}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}+\frac{1}{8}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}+\frac{1}{16}c^2\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{\left(3c^3\right)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{64b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}+\frac{\left(3c^4\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,d}{128b^2}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{\left(3c^4\right)\mathrm{Subst}\left(\int \frac{1}{1-b}\right.}{128}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+c}}\right.}{128b^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.34

$$
-\frac{c^4\left(x^2\left(b+cx^2\right)\right)^{5/2}{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^5x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] -1/5*(c^4*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (c*x^2
)/b])/(b^5*x^5)
```

IntegrateAlgebraic [A] time = 0.72, size = 93, normalized size = 0.68

$$
\frac{\sqrt{bx^2+cx^4}\left(-16b^3-24b^2cx^2-2bc^2x^4+3c^3x^6\right)}{128b^2x^9}-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-16*b^3 - 24*b^2*c*x^2 - 2*b*c^2*x^4 + 3*c^3*x^6))/(1
28*b^2*x^9) - (3*c^4*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2)
)
```

fricas [A] time = 0.76, size = 207, normalized size = 1.51

$$\left[\frac{3\sqrt{b}c^{4}x^{9}\log\left(-\frac{cx^{3}+2bx-2\sqrt{cx^{4}+bx^{2}}\sqrt{b}}{x^{3}}\right)+2\left(3bc^{3}x^{6}-2b^{2}c^{2}x^{4}-24b^{3}cx^{2}-16b^{4}\right)\sqrt{cx^{4}+bx^{2}}}{256b^{3}x^{9}}, \frac{3\sqrt{-b}c^{4}x^{9}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\sqrt{-b}}{cx^{3}+bx}\right)+\left(3bc^{3}x^{6}-2b^{2}c^{2}x^{4}-24b^{3}cx^{2}-16b^{4}\right)\sqrt{cx^{4}+bx^{2}}}{128b^{3}x^{9}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] [1/256*(3*sqrt(b)*c^4*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) + 2*(3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 16*b^4)*sqrt(c*x
^4 + b*x^2))/(b^3*x^9), 1/128*(3*sqrt(-b)*c^4*x^9*arctan(sqrt(c*x^4 + b*x^2
)*sqrt(-b)/(c*x^3 + b*x)) + (3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 1
6*b^4)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]
```

giac [A] time = 0.23, size = 119, normalized size = 0.87

$$\frac{\frac{3c^{5}\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}b^{2}}+\frac{3\left(cx^{2}+b\right)^{\frac{7}{2}}c^{5}\operatorname{sgn}(x)-11\left(cx^{2}+b\right)^{\frac{5}{2}}bc^{5}\operatorname{sgn}(x)-11\left(cx^{2}+b\right)^{\frac{3}{2}}b^{2}c^{5}\operatorname{sgn}(x)+3\sqrt{cx^{2}+b}b^{3}c^{5}\operatorname{sgn}(x)}{b^{2}c^{4}x^{8}}}{128c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/128*(3*c^5*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^2) + (3*(c
*x^2 + b)^(7/2)*c^5*sgn(x) - 11*(c*x^2 + b)^(5/2)*b*c^5*sgn(x) - 11*(c*x^2
+ b)^(3/2)*b^2*c^5*sgn(x) + 3*sqrt(c*x^2 + b)*b^3*c^5*sgn(x))/(b^2*c^4*x^8)
)/c
```

maple [A] time = 0.02, size = 165, normalized size = 1.20

$$-\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}c^{4}x^{8}\ln\left(\frac{2b+2\sqrt{cx^{2}+b}\sqrt{b}}{x}\right)-3\sqrt{cx^{2}+b}\,bc^{4}x^{8}-\left(cx^{2}+b\right)^{\frac{3}{2}}c^{4}x^{8}+\left(cx^{2}+b\right)^{\frac{5}{2}}c^{3}x^{6}+2\left(cx^{2}+b\right)^{\frac{5}{2}}bc^{2}x^{4}-8\left(cx^{2}+b\right)^{\frac{5}{2}}b^{2}cx^{2}+16\left(cx^{2}+b\right)^{\frac{5}{2}}b^{3}\right)}{128\left(cx^{2}+b\right)^{\frac{3}{2}}b^{4}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^12,x)
```

```
[Out] -1/128*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x
^8*c^4-(c*x^2+b)^(3/2)*x^8*c^4+(c*x^2+b)^(5/2)*x^6*c^3-3*(c*x^2+b)^(1/2)*x^
8*b*c^4+2*(c*x^2+b)^(5/2)*x^4*b*c^2-8*(c*x^2+b)^(5/2)*x^2*b^2*c+16*(c*x^2+b
)^(5/2)*b^3)/x^11/(c*x^2+b)^(3/2)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^12, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^12,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^12, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**12,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**12, x)
```

3.141 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=165

$$\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}$$

Rubi [A] time = 0.25, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}+\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(80*x^9) - (c^2*Sqrt[b*x^2 + c*x^4])/(160*b*x^7)
 + (c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^5) - (3*c^4*Sqrt[b*x^2 + c*x^4])/(2
56*b^3*x^3) - (b*x^2 + c*x^4)^(3/2)/(10*x^13) + (3*c^5*ArcTanh[(Sqrt[b]*x)/
Sqrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{14}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+\frac{1}{10}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^{10}}\,dx \\
&= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+\frac{1}{80}\left(3c^2\right)\int \frac{1}{x^6\sqrt{bx^2+cx^4}}\,dx \\
&= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}-\frac{c^3\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{32b} \\
&= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+\frac{\left(3c^4\right)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}}{128b^2} \\
&= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}- \\
&= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+ \\
&= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+
\end{aligned}$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.28

$$\frac{c^5\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] (c^5*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, 1 + (c*x^2)/b])
/(5*b^6*x^5)
```

IntegrateAlgebraic [A] time = 0.80, size = 104, normalized size = 0.63

$$\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}+\frac{\sqrt{bx^2+cx^4}\,\left(-128b^4-176b^3cx^2-8b^2c^2x^4+10bc^3x^6-15c^4x^8\right)}{1280b^3x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-128*b^4 - 176*b^3*c*x^2 - 8*b^2*c^2*x^4 + 10*b*c^3*x
^6 - 15*c^4*x^8))/(1280*b^3*x^11) + (3*c^5*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 +
 c*x^4]])/(256*b^(7/2))
```

fricas [A] time = 1.17, size = 229, normalized size = 1.39

$$\left[\frac{15\sqrt{b}c^5x^{11}\log\left(-\frac{cx^3+2bx+2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\left(15bc^4x^8-10b^2c^3x^6+8b^3c^2x^4+176b^4cx^2+128b^5\right)\sqrt{cx^4+bx^2}}{2560b^4x^{11}},-\frac{15\sqrt{-b}c^5x^{11}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\left(15bc^4x^8-10b^2c^3x^6+8b^3c^2x^4+176b^4cx^2+128b^5\right)\sqrt{cx^4+bx^2}}{1280b^4x^{11}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*sqrt(b)*c^5*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sq
rt(b))/x^3) - 2*(15*b*c^4*x^8 - 10*b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*c*
x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), -1/1280*(15*sqrt(-b)*c^5*x^
11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*b*c^4*x^8 - 10*
b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*c*x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))
/(b^4*x^11)]
```

giac [A] time = 0.38, size = 138, normalized size = 0.84

$$-\frac{\frac{15c^6\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b^3}+\frac{15\left(cx^2+b\right)^{\frac{9}{2}}c^6\mathrm{sgn}(x)-70\left(cx^2+b\right)^{\frac{7}{2}}bc^6\mathrm{sgn}(x)+128\left(cx^2+b\right)^{\frac{5}{2}}b^2c^6\mathrm{sgn}(x)+70\left(cx^2+b\right)^{\frac{3}{2}}b^3c^6\mathrm{sgn}(x)-15\sqrt{cx^2+b}\,b^4c^6\mathrm{sgn}(x)}{b^3c^5x^{10}}}{1280c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1280*(15*c^6*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^3) + (1
5*(c*x^2 + b)^(9/2)*c^6*sgn(x) - 70*(c*x^2 + b)^(7/2)*b*c^6*sgn(x) + 128*(c
*x^2 + b)^(5/2)*b^2*c^6*sgn(x) + 70*(c*x^2 + b)^(3/2)*b^3*c^6*sgn(x) - 15*s
qrt(c*x^2 + b)*b^4*c^6*sgn(x))/(b^3*c^5*x^10))/c
```

maple [A] time = 0.03, size = 186, normalized size = 1.13

$$\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(15b^{\frac{3}{2}}c^5x^{10}\ln\left(\frac{2b+2\sqrt{cx^2+b}\,\sqrt{b}}{x}\right)-15\sqrt{cx^2+b}\,bc^5x^{10}-5\left(cx^2+b\right)^{\frac{3}{2}}c^5x^{10}+5\left(cx^2+b\right)^{\frac{5}{2}}c^4x^8+10\left(cx^2+b\right)^{\frac{5}{2}}bc^3x^6-40\left(cx^2+b\right)^{\frac{5}{2}}b^2c^2x^4+80\left(cx^2+b\right)^{\frac{5}{2}}b^3cx^2-128\left(cx^2+b\right)^{\frac{5}{2}}b^4\right)}{1280\left(cx^2+b\right)^{\frac{3}{2}}b^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^14,x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(-5*(c*x^2+b)^(3/2)*x^10*c^5+15*b^(3/2)*ln(2*(b+
(c*x^2+b)^(1/2)*b^(1/2))/x)*x^10*c^5+5*(c*x^2+b)^(5/2)*x^8*c^4-15*(c*x^2+b)
^(1/2)*x^10*b*c^5+10*(c*x^2+b)^(5/2)*x^6*b*c^3-40*(c*x^2+b)^(5/2)*x^4*b^2*c
^2+80*(c*x^2+b)^(5/2)*x^2*b^3*c-128*(c*x^2+b)^(5/2)*b^4)/x^13/(c*x^2+b)^(3/
2)/b^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^14, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^14,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^14, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**14,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**14, x)
```

3.142 $\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=114

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

Rubi [A] time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 670, 640, 620, 206}

$$\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*Sqrt[b*x^2 + c*x^4])/(16*c^3) - (5*b*x^2*Sqrt[b*x^2 + c*x^4])/(24*c^
2) + (x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (5*b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*
x^2 + c*x^4]])/(16*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{(5b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{12c}\\
&= -\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{32c^3}\\
&= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^3}\\
&= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 100, normalized size = 0.88

$$\frac{x\left(\sqrt{c}\,x\left(15b^3+5b^2cx^2-2bc^2x^4+8c^3x^6\right)-15b^3\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{48c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(15*b^3 + 5*b^2*c*x^2 - 2*b*c^2*x^4 + 8*c^3*x^6) - 15*b^3*Sqr
t[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(48*c^(7/2)*Sqrt[x^2*(b
 + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.24, size = 93, normalized size = 0.82

$$\frac{5b^3\log\left(-2c^{7/2}\sqrt{bx^2+cx^4}+bc^3+2c^4x^2\right)}{32c^{7/2}}+\frac{\sqrt{bx^2+cx^4}\left(15b^2-10bcx^2+8c^2x^4\right)}{48c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(15*b^2 - 10*b*c*x^2 + 8*c^2*x^4))/(48*c^3) + (5*b^3*L
og[b*c^3 + 2*c^4*x^2 - 2*c^(7/2)*Sqrt[b*x^2 + c*x^4]])/(32*c^(7/2))
```

fricas [A] time = 0.81, size = 166, normalized size = 1.46

$$\left[\frac{15\,b^3\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\left(8\,c^3x^4-10\,bc^2x^2+15\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^4},\ \frac{15\,b^3\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\left(8\,c^3x^4-10\,bc^2x^2+15\,b^2c\right)\sqrt{cx^4+bx^2}}{48\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(15*b
^3*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^4 -
 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

giac [A] time = 0.22, size = 87, normalized size = 0.76

$$\frac{1}{48}\,\sqrt{cx^4+bx^2}\left(2\,x^2\left(\frac{4\,x^2}{c}-\frac{5\,b}{c^2}\right)+\frac{15\,b^2}{c^3}\right)+\frac{5\,b^3\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{32\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2)*(2*x^2*(4*x^2/c - 5*b/c^2) + 15*b^2/c^3) + 5/32*b^
3*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(7/2)
```

maple [A] time = 0.01, size = 105, normalized size = 0.92

$$\frac{\sqrt{c\,x^2+b}\,\left(8\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}x^5-10\sqrt{c\,x^2+b}\,b\,c^{\frac{5}{2}}x^3-15b^3c\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+15\sqrt{c\,x^2+b}\,b^2c^{\frac{3}{2}}x\right)x}{48\sqrt{c\,x^4+b\,x^2}\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*x*(c*x^2+b)^(1/2)*(8*x^5*(c*x^2+b)^(1/2)*c^(7/2)-10*c^(5/2)*(c*x^2+b)^
(1/2)*x^3*b+15*c^(3/2)*(c*x^2+b)^(1/2)*x*b^2-15*ln(c^(1/2)*x+(c*x^2+b)^(1/2
))*b^3*c)/(c*x^4+b*x^2)^(1/2)/c^(9/2)
```

maxima [A] time = 1.47, size = 100, normalized size = 0.88

$$\frac{\sqrt{cx^4+bx^2}\,x^4}{6\,c}-\frac{5\,\sqrt{cx^4+bx^2}\,bx^2}{24\,c^2}-\frac{5\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{32\,c^{\frac{7}{2}}}+\frac{5\,\sqrt{cx^4+bx^2}\,b^2}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(c*x^4 + b*x^2)*x^4/c - 5/24*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 5/32*b
^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 5/16*sqrt(c*x
^4 + b*x^2)*b^2/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^7/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(x**2*(b + c*x**2)), x)
```

3.143 $\int \frac{x^5}{\sqrt{bx^2+cx^4}}\, dx$

Optimal. Leaf size=86

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 670, 640, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-3*b*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[b*x^2 + c*x^4])/(4*c) + (3*b
^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
```

```
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{x^2\sqrt{bx^2+cx^4}}{4c} - \frac{(3b)\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c} + \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c} + \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 1.03

$$\frac{x\left(\sqrt{c}\,x\left(-3b^2-bcx^2+2c^2x^4\right)+3b^2\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{8c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(-3*b^2 - b*c*x^2 + 2*c^2*x^4) + 3*b^2*Sqrt[b + c*x^2]*ArcTan
h[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(8*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.22, size = 82, normalized size = 0.95

$$\frac{(2cx^2-3b)\sqrt{bx^2+cx^4}}{8c^2}-\frac{3b^2\log\left(-2c^{5/2}\sqrt{bx^2+cx^4}+bc^2+2c^3x^2\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((-3*b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c^2) - (3*b^2*Log[b*c^2 + 2*c^3*x
^2 - 2*c^(5/2)*Sqrt[b*x^2 + c*x^4]])/(16*c^(5/2))
```

fricas [A] time = 0.78, size = 145, normalized size = 1.69

$$\left[\frac{3\,b^2\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(2\,c^2x^2-3\,bc\right)}{16\,c^3},-\frac{3\,b^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}\left(2\,c^2x^2-3\,bc\right)}{8\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 - 3*b*c))/c^3, -1/8*(3*b^2*sqrt(-c)*arctan(s
qrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 -
 3*b*c))/c^3]
```

giac [A] time = 0.22, size = 73, normalized size = 0.85

$$\frac{1}{8}\sqrt{cx^4+bx^2}\left(\frac{2\,x^2}{c}-\frac{3\,b}{c^2}\right)-\frac{3\,b^2\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*(2*x^2/c - 3*b/c^2) - 3/16*b^2*log(abs(-2*(sqrt(c)*
x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(5/2)
```

maple [A] time = 0.01, size = 85, normalized size = 0.99

$$\frac{\sqrt{c\,x^2+b}\,\left(2\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}x^3+3b^2c\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-3\sqrt{c\,x^2+b}\,b\,c^{\frac{3}{2}}x\right)x}{8\sqrt{c\,x^4+b\,x^2}\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/8*x*(c*x^2+b)^(1/2)*(2*x^3*(c*x^2+b)^(1/2)*c^(5/2)-3*c^(3/2)*(c*x^2+b)^(1
/2)*x*b+3*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b*x^2)^(1/2)/c^(7/2)
```

maxima [A] time = 1.45, size = 76, normalized size = 0.88

$$\frac{\sqrt{cx^4+bx^2}\,x^2}{4\,c}+\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{16\,c^{\frac{5}{2}}}-\frac{3\,\sqrt{cx^4+bx^2}\,b}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^4 + b*x^2)*x^2/c + 3/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b
*x^2)*sqrt(c))/c^(5/2) - 3/8*sqrt(c*x^4 + b*x^2)*b/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^5/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(x**2*(b + c*x**2)), x)
```

3.144 $\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c}\\ &= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c}\\ &= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.26

$$\frac{x\left(\sqrt{c}\,x\left(b+cx^2\right)-b\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) - b*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b +
c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.19, size = 68, normalized size = 1.17

$$\frac{b\log\left(-2c^{3/2}\sqrt{bx^2+cx^4}+bc+2c^2x^2\right)}{4c^{3/2}}+\frac{\sqrt{bx^2+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) + (b*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[b*x^2 +
 c*x^4]])/(4*c^(3/2))
```

fricas [A] time = 0.92, size = 114, normalized size = 1.97

$$\left[\frac{b\sqrt{c}\log\left(-2\,cx^2 - b + 2\,\sqrt{cx^4 + bx^2}\,\sqrt{c}\right) + 2\,\sqrt{cx^4 + bx^2}\,c}{4\,c^2}, \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right) + \sqrt{cx^4 + bx^2}\,c}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/
(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

giac [A] time = 0.20, size = 59, normalized size = 1.02

$$\frac{b\log\left(\left|-2\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} - b\right|\right)}{4\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(3/2)
+ 1/2*sqrt(c*x^4 + b*x^2)/c
```

maple [A] time = 0.01, size = 64, normalized size = 1.10

$$-\frac{\sqrt{c\,x^2 + b}\,\left(bc\ln\left(\sqrt{c}\,x + \sqrt{c\,x^2 + b}\right) - \sqrt{c\,x^2 + b}\,c^{\frac{3}{2}}x\right)x}{2\sqrt{c\,x^4 + b\,x^2}\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2*x*(c*x^2+b)^(1/2)*(-x*(c*x^2+b)^(1/2)*c^(3/2)+b*ln(c^(1/2)*x+(c*x^2+b)
^(1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

maxima [A] time = 1.45, size = 52, normalized size = 0.90

$$-\frac{b\log\left(2\,cx^2 + b + 2\,\sqrt{cx^4 + bx^2}\,\sqrt{c}\right)}{4\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 1/2*sqrt(
c*x^4 + b*x^2)/c
```

mupad [B] time = 4.30, size = 53, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{2\,c}-\frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4
)^(1/2)))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

3.145 $\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2013, 620, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\,\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2*
(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.13, size = 40, normalized size = 1.29

$$-\frac{\log\left(-2\sqrt{c}\,\sqrt{bx^2+cx^4}\,+b+2cx^2\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

fricas [A] time = 0.88, size = 74, normalized size = 2.39

$$\left[\frac{\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

giac [A] time = 0.19, size = 39, normalized size = 1.26

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/sqrt(c)
```

maple [A] time = 0.00, size = 44, normalized size = 1.42

$$\frac{\sqrt{c\,x^2+b}\;x\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)}{\sqrt{c\,x^4+b\,x^2}\;\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))/c^(1/
2)
```

maxima [A] time = 1.46, size = 32, normalized size = 1.03

$$\frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c)
```

mupad [B] time = 4.36, size = 33, normalized size = 1.06

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2 (b + cx^2)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

3.146 $\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^2 + c*x^4]),x]
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^2 + c*x^4]),x]
[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

IntegrateAlgebraic [A] time = 0.13, size = 23, normalized size = 1.00

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[b*x^2 + c*x^4]),x]
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

fricas [A] time = 1.87, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

giac [A] time = 0.18, size = 25, normalized size = 1.09

$$\frac{1}{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))
```

maple [A] time = 0.00, size = 26, normalized size = 1.13

$$-\frac{c\,x^2+b}{\sqrt{c\,x^4+b\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(1/2),x)
[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.47, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

mupad [B] time = 4.21, size = 21, normalized size = 0.91

$$-\frac{\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -(b*x^2 + c*x^4)^(1/2)/(b*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

3.147 $\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{3bx^4}-\frac{(2c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{3bx^4}+\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(2cx^2-b\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

IntegrateAlgebraic [A] time = 0.15, size = 35, normalized size = 0.67

$$\frac{\left(2cx^2-b\right)\sqrt{bx^2+cx^4}}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
[Out] ((-b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^4)
```

fricas [A] time = 0.72, size = 31, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\left(2\,cx^2-b\right)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

giac [A] time = 0.19, size = 57, normalized size = 1.10

$$\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}\,+b}{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) + b)/(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2))^3
```

maple [A] time = 0.00, size = 37, normalized size = 0.71

$$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+b\right)}{3\sqrt{c\,x^{4}+b\,x^{2}}\;b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 44, normalized size = 0.85

$$\frac{2\,\sqrt{cx^{4}+bx^{2}}\,c}{3\,b^{2}x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}}{3\,bx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/(b*x^4)
```

mupad [B] time = 4.26, size = 29, normalized size = 0.56

$$-\frac{\left(b-2\,c\,x^{2}\right)\,\sqrt{c\,x^{4}+b\,x^{2}}}{3\,b^{2}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
[Out] -((b - 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b^2*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{3}\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

3.148 $\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(5*b*x^6) + (4*c*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^4) - (
8*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{5bx^6} - \frac{(4c)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{5b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{5bx^6} + \frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4} + \frac{\left(8c^2\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{5bx^6} + \frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4} - \frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.58

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3b^2-4bcx^2+8c^2x^4\right)}{15b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
[Out] -1/15*(Sqrt[x^2*(b + c*x^2)]*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4))/(b^3*x^6)
```

IntegrateAlgebraic [A] time = 0.16, size = 46, normalized size = 0.58

$$\frac{\sqrt{bx^2+cx^4}\left(-3b^2+4bcx^2-8c^2x^4\right)}{15b^3x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
[Out] (Sqrt[b*x^2 + c*x^4]*(-3*b^2 + 4*b*c*x^2 - 8*c^2*x^4))/(15*b^3*x^6)
```

fricas [A] time = 1.09, size = 42, normalized size = 0.52

$$-\frac{\left(8\,c^2x^4-4\,bcx^2+3\,b^2\right)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/15*(8*c^2*x^4 - 4*b*c*x^2 + 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^6)
```

giac [A] time = 0.21, size = 90, normalized size = 1.12

$$\frac{20 \left(\sqrt{c}\, x^2 - \sqrt{cx^4 + bx^2}\right)^2 c + 15 \left(\sqrt{c}\, x^2 - \sqrt{cx^4 + bx^2}\right) b \sqrt{c} + 3\, b^2}{15 \left(\sqrt{c}\, x^2 - \sqrt{cx^4 + bx^2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(20*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^2*c + 15*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2))*b*sqrt(c) + 3*b^2)/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^5
```

maple [A] time = 0.00, size = 50, normalized size = 0.62

$$-\frac{\left(c\, x^2 + b\right) \left(8 c^2 x^4 - 4 b c\, x^2 + 3 b^2\right)}{15 \sqrt{c\, x^4 + b\, x^2}\; b^3 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(8*c^2*x^4-4*b*c*x^2+3*b^2)/x^4/b^3/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 68, normalized size = 0.85

$$-\frac{8 \sqrt{cx^4 + bx^2}\, c^2}{15\, b^3 x^2} + \frac{4 \sqrt{cx^4 + bx^2}\, c}{15\, b^2 x^4} - \frac{\sqrt{cx^4 + bx^2}}{5\, bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/15*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) + 4/15*sqrt(c*x^4 + b*x^2)*c/(b^2*x
^4) - 1/5*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

mupad [B] time = 4.33, size = 42, normalized size = 0.52

$$-\frac{\sqrt{c\, x^4 + b\, x^2}\; \left(3\, b^2 - 4\, b\, c\, x^2 + 8\, c^2\, x^4\right)}{15\, b^3\, x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*b^2 + 8*c^2*x^4 - 4*b*c*x^2))/(15*b^3*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

3.149 $\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

Rubi [A] time = 0.17, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(7*b*x^8) + (6*c*Sqrt[b*x^2 + c*x^4])/(35*b^2*x^6) - (
8*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^4) + (16*c^3*Sqrt[b*x^2 + c*x^4])/(35*
b^4*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(6c)\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{7b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{\left(24c^2\right)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{35b^2}\\ &= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4} - \frac{\left(16c^3\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{35b^3}\\ &= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4} + \frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.53

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-5b^3+6b^2cx^2-8bc^2x^4+16c^3x^6\right)}{35b^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16*c^3*x^6))/(
35*b^4*x^8)
```

IntegrateAlgebraic [A] time = 0.16, size = 57, normalized size = 0.53

$$\frac{\sqrt{bx^2+cx^4}\left(-5b^3+6b^2cx^2-8bc^2x^4+16c^3x^6\right)}{35b^4x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16*c^3*x^6))/(35
*b^4*x^8)
```

fricas [A] time = 1.26, size = 53, normalized size = 0.49

$$\frac{\left(16\,c^3x^6-8\,bc^2x^4+6\,b^2cx^2-5\,b^3\right)\sqrt{cx^4+bx^2}}{35\,b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*c^3*x^6 - 8*b*c^2*x^4 + 6*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/(
b^4*x^8)
```

giac [A] time = 0.20, size = 123, normalized size = 1.14

$$\frac{70\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3c^{\frac{3}{2}}+84\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^2bc+35\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)b^2\sqrt{c}+5\,b^3}{35\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/35*(70*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^3*c^(3/2) + 84*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2))^2*b*c + 35*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*b^2*sqr
t(c) + 5*b^3)/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^7
```

maple [A] time = 0.01, size = 61, normalized size = 0.56

$$-\frac{\left(c\,x^2+b\right)\left(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3\right)}{35\sqrt{c\,x^4+b\,x^2}\;b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-16*c^3*x^6+8*b*c^2*x^4-6*b^2*c*x^2+5*b^3)/x^6/b^4/(c*x^4+
b*x^2)^(1/2)
```

maxima [A] time = 1.41, size = 92, normalized size = 0.85

$$\frac{16\sqrt{cx^4+bx^2}\,c^3}{35\,b^4x^2}-\frac{8\sqrt{cx^4+bx^2}\,c^2}{35\,b^3x^4}+\frac{6\sqrt{cx^4+bx^2}\,c}{35\,b^2x^6}-\frac{\sqrt{cx^4+bx^2}}{7\,bx^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 16/35*sqrt(c*x^4 + b*x^2)*c^3/(b^4*x^2) - 8/35*sqrt(c*x^4 + b*x^2)*c^2/(b^3
*x^4) + 6/35*sqrt(c*x^4 + b*x^2)*c/(b^2*x^6) - 1/7*sqrt(c*x^4 + b*x^2)/(b*x
^8)
```

mupad [B] time = 4.27, size = 92, normalized size = 0.85

$$\frac{6\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^6}-\frac{\sqrt{c\,x^4+b\,x^2}}{7\,b\,x^8}-\frac{8\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b^3\,x^4}+\frac{16\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (6*c*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*b*x^8)
- (8*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b^3*x^4) + (16*c^3*(b*x^2 + c*x^4)^(1/2
))/(35*b^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \sqrt{x^2 \left(b + cx^2\right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

3.150 $\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=50

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx&=\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{(2b)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}\\&=-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}+\frac{x\sqrt{bx^2+cx^4}}{3c}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.68

$$\frac{\left(cx^2-2b\right)\sqrt{x^2\left(b+cx^2\right)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^2 + c*x^4],x]
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

IntegrateAlgebraic [A] time = 0.06, size = 34, normalized size = 0.68

$$\frac{\left(cx^2-2b\right)\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[b*x^2 + c*x^4],x]
[Out] ((-2*b + c*x^2)*Sqrt[b*x^2 + c*x^4])/(3*c^2*x)
```

fricas [A] time = 1.59, size = 30, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\left(cx^2-2\,b\right)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.00, size = 37, normalized size = 0.74

$$-\frac{\left(c\,x^2+b\right)\left(-c\,x^2+2b\right)x}{3\sqrt{c\,x^4+b\,x^2}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(1/2),x)
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.50, size = 34, normalized size = 0.68

$$\frac{c^2x^4 - bcx^2 - 2\,b^2}{3\,\sqrt{cx^2 + b}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

mupad [B] time = 4.25, size = 33, normalized size = 0.66

$$-\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(\frac{2\,b}{3\,c^2} - \frac{x^2}{3\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^(1/2),x)
[Out] -((b*x^2 + c*x^4)^(1/2)*((2*b)/(3*c^2) - x^2/(3*c)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

3.151 $\int \frac{x^2}{\sqrt{bx^2+cx^4}} \, dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2 + cx^4}}{cx}$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{\sqrt{bx^2 + cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{bx^2 + cx^4}} \, dx = \frac{\sqrt{bx^2 + cx^4}}{cx}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\sqrt{x^2 \left(b + cx^2\right)}}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

IntegrateAlgebraic [A] time = 0.04, size = 22, normalized size = 1.00

$$\frac{\sqrt{bx^2 + cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[b*x^2 + c*x^4],x]
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

fricas [A] time = 1.06, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^4 + bx^2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)/(c*x)
```

giac [A] time = 0.19, size = 31, normalized size = 1.41

$$-\frac{2\sqrt{b}}{\left(\sqrt{c + \frac{b}{x^2}} - \frac{\sqrt{b}}{x}\right)^2 - c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(b)/((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)
```

maple [A] time = 0.00, size = 26, normalized size = 1.18

$$\frac{\left(c\,x^2 + b\right)x}{\sqrt{c\,x^4 + b\,x^2}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(1/2),x)
[Out] (c*x^2+b)/c*x/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.42, size = 13, normalized size = 0.59

$$\frac{\sqrt{cx^2+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + b)/c
```

mupad [B] time = 4.26, size = 20, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(c*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

3.152 $\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.73

$$-\frac{x\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(b
 + c*x^2)]))
```

IntegrateAlgebraic [A] time = 0.06, size = 30, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

fricas [A] time = 0.98, size = 80, normalized size = 2.67

$$\left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sqr
t(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))/b]
```

giac [A] time = 0.17, size = 46, normalized size = 1.53

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b)
)/(sqrt(-b)*sgn(x))
```

maple [B] time = 0.00, size = 50, normalized size = 1.67

$$-\frac{\sqrt{c\,x^{2}+b}\;x\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\;\sqrt{b}}{x}\right)}{\sqrt{c\,x^{4}+b\,x^{2}}\;\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^
(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{4}+bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{c\,x^{4}+b\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(1/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

3.153 $\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{2bx^3} - \frac{c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{2bx^3} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{2bx^3} + \frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 1.15

$$\frac{c\sqrt{x^2\left(b+cx^2\right)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{2\sqrt{\frac{cx^2}{b}+1}} - \frac{b}{2cx^2}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (c*Sqrt[x^2*(b + c*x^2)]*(-1/2*b/(c*x^2) + ArcTanh[Sqrt[1 + (c*x^2)/b]]/(2*
Sqrt[1 + (c*x^2)/b])))/(b^2*x)
```

IntegrateAlgebraic [A] time = 0.12, size = 59, normalized size = 1.00

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}} - \frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^
4]])/(2*b^(3/2))
```

fricas [A] time = 4.01, size = 133, normalized size = 2.25

$$\left[\frac{\sqrt{b}\,cx^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right) - 2\,\sqrt{cx^4+bx^2}\,b}{4\,b^2x^3}, -\frac{\sqrt{-b}\,cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right) + \sqrt{cx^4+bx^2}\,b}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]-1/2/b/x*sqrt(b*(1/x)^2+c)-2*c/4/b/sqrt(
b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 73, normalized size = 1.24

$$-\frac{\sqrt{c\,x^{2}+b}\,\left(-b c\,x^{2} \ln \left(\frac{2 b+2 \sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)+\sqrt{c\,x^{2}+b}\,b^{\frac{3}{2}}\right)}{2 \sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{5}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2*b+(c*x^2
+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

mupad [B] time = 4.47, size = 76, normalized size = 1.29

$$-\frac{\left(\frac{\sqrt{c}\,x^2\,\sqrt{c+\frac{b}{x^2}}}{2\,b}+\frac{c^{3/2}\,x^3\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)1\mathrm{i}}{2\,b^{3/2}}\right)\sqrt{\frac{b}{c\,x^2}+1}}{x\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -(((c^(1/2)*x^2*(c + b/x^2)^(1/2))/(2*b) + (c^(3/2)*x^3*asin((b^(1/2)*1i)/(
c^(1/2)*x))*1i)/(2*b^(3/2)))*(b/(c*x^2) + 1)^(1/2))/(x*(b*x^2 + c*x^4)^(1/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

3.154 $\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) - (3
*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(3c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} + \frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} + \frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} + \frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 0.51

$$-\frac{c^2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{cx^2}{b}+1\right)}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -((c^2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (c*x^2)/b])
/(b^3*x))
```

IntegrateAlgebraic [A] time = 0.14, size = 71, normalized size = 0.82

$$\frac{\left(3cx^2-2b\right)\sqrt{bx^2+cx^4}}{8b^2x^5} - \frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*b + 3*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^5) - (3*c^2*ArcTanh[(Sqrt[b
]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

fricas [A] time = 1.79, size = 163, normalized size = 1.87

$$\left[\frac{3\sqrt{b}\,c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(3\,bcx^2-2\,b^2\right)}{16\,b^3x^5}, \frac{3\,\sqrt{-b}\,c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\left(3\,bcx^2-2\,b^2\right)}{8\,b^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sqrt
(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^
4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]2*(-2*b^2/16/b^3/x/x+3*b*c/16/b^3)/x*sqr
t(b*(1/x)^2+c)+6*c^2/16/b^2/sqrt(b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 94, normalized size = 1.08

$$-\frac{\sqrt{c\,x^{2}+b}\,\left(3 b\,c^{2} x^{4} \ln\left(\frac{2 b+2 \sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-3 \sqrt{c\,x^{2}+b}\,b^{\frac{3}{2}} c\,x^{2}+2 \sqrt{c\,x^{2}+b}\,b^{\frac{5}{2}}\right)}{8 \sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{7}{2}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b)^(1/2)*(3*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*b*c^2-3*b^(
3/2)*(c*x^2+b)^(1/2)*x^2*c+2*(c*x^2+b)^(1/2)*b^(5/2))/x^3/(c*x^4+b*x^2)^(1/
2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

3.155 $\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}-\frac{x^6}{c\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}-\frac{x^6}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^6/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(8*c^3) + (5*x^
2*Sqrt[b*x^2 + c*x^4])/(4*c^2) + (15*b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(8*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} - \frac{(15b)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c^2}$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^3}$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+\cdots}}\right)}{8c^3}$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.81

$$\frac{x\left(15b^{5/2}\sqrt{\frac{cx^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)+\sqrt{c}\,x\left(-15b^2-5bcx^2+2c^2x^4\right)\right)}{8c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(-15*b^2 - 5*b*c*x^2 + 2*c^2*x^4) + 15*b^(5/2)*Sqrt[1 + (c*x^
2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(8*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.45, size = 102, normalized size = 0.94

$$\frac{\sqrt{bx^2+cx^4}\,\left(-15b^2-5bcx^2+2c^2x^4\right)}{8c^3\left(b+cx^2\right)} - \frac{15b^2\log\left(-2c^{7/2}\sqrt{bx^2+cx^4}+bc^3+2c^4x^2\right)}{16c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-15*b^2 - 5*b*c*x^2 + 2*c^2*x^4))/(8*c^3*(b + c*x^2))
 - (15*b^2*Log[b*c^3 + 2*c^4*x^2 - 2*c^(7/2)*Sqrt[b*x^2 + c*x^4]])/(16*c^(7
/2))
```

fricas [A] time = 0.73, size = 209, normalized size = 1.92

$$\left[\frac{15\left(b^{2}cx^{2}+b^{3}\right)\sqrt{c}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\left(2\,c^{3}x^{4}-5\,bc^{2}x^{2}-15\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{16\left(c^{5}x^{2}+bc^{4}\right)},-\frac{15\left(b^{2}cx^{2}+b^{3}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)-\left(2\,c^{3}x^{4}-5\,bc^{2}x^{2}-15\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{8\left(c^{5}x^{2}+bc^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c*x^2 + b^3)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2
)*sqrt(c)) + 2*(2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^4 + b*x^2))/(c
^5*x^2 + b*c^4), -1/8*(15*(b^2*c*x^2 + b^3)*sqrt(-c)*arctan(sqrt(c*x^4 + b*
x^2)*sqrt(-c)/(c*x^2 + b)) - (2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^
4 + b*x^2))/(c^5*x^2 + b*c^4)]
```

giac [A] time = 0.27, size = 114, normalized size = 1.05

$$\frac{1}{8}\,\sqrt{cx^{4}+bx^{2}}\left(\frac{2\,x^{2}}{c^{2}}-\frac{7\,b}{c^{3}}\right)-\frac{15\,b^{2}\log\left(\left|-2\left(\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{7}{2}}}-\frac{b^{3}}{\left(\left(\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}}\right)c+b\sqrt{c}\right)c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*(2*x^2/c^2 - 7*b/c^3) - 15/16*b^2*log(abs(-2*(sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(7/2) - b^3/(((sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2))*c + b*sqrt(c))*c^3)
```

maple [A] time = 0.01, size = 87, normalized size = 0.80

$$\frac{\left(c\,x^{2}+b\right)\left(2c^{\frac{7}{2}}x^{5}-5b\,c^{\frac{5}{2}}x^{3}-15b^{2}c^{\frac{3}{2}}x+15\sqrt{c\,x^{2}+b}\;b^{2}c\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+b}\right)\right)x^{3}}{8\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/8*x^3*(c*x^2+b)*(2*x^5*c^(7/2)-5*c^(5/2)*x^3*b-15*c^(3/2)*x*b^2+15*ln(c^(
1/2)*x+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*b^2*c)/(c*x^4+b*x^2)^(3/2)/c^(9/2)
```

maxima [A] time = 1.48, size = 103, normalized size = 0.94

$$\frac{x^6}{4\sqrt{cx^4+bx^2}\,c}-\frac{5\,bx^4}{8\sqrt{cx^4+bx^2}\,c^2}-\frac{15\,b^2x^2}{8\sqrt{cx^4+bx^2}\,c^3}+\frac{15\,b^2\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{16\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^6/(sqrt(c*x^4 + b*x^2)*c) - 5/8*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) - 15/
8*b^2*x^2/(sqrt(c*x^4 + b*x^2)*c^3) + 15/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*
x^4 + b*x^2)*sqrt(c))/c^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^9/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**9/(x**2*(b + c*x**2))**(3/2), x)
```

3.156 $\int \frac{x^7}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{x^4}{c\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 668, 640, 620, 206}

$$\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}-\frac{x^4}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^4/(c*Sqrt[b*x^2 + c*x^4])) + (3*Sqrt[b*x^2 + c*x^4])/(2*c^2) - (3*b*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
```

```
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{2c}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{(3b)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4c^2}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{(3b)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^2}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.94

$$
\frac{x\left(\sqrt{c}\,x\left(3b+cx^2\right)-3b^{3/2}\sqrt{\frac{cx^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\right)}{2c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(3*b + c*x^2) - 3*b^(3/2)*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[c
]*x)/Sqrt[b]]))/(2*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.39, size = 88, normalized size = 1.09

$$\frac{\sqrt{bx^2+cx^4}\left(3b+cx^2\right)}{2c^2\left(b+cx^2\right)}+\frac{3b\log\left(-2c^{5/2}\sqrt{bx^2+cx^4}+bc^2+2c^3x^2\right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((3*b + c*x^2)*Sqrt[b*x^2 + c*x^4])/(2*c^2*(b + c*x^2)) + (3*b*Log[b*c^2 +
2*c^3*x^2 - 2*c^(5/2)*Sqrt[b*x^2 + c*x^4]])/(4*c^(5/2))
```

fricas [A] time = 1.09, size = 180, normalized size = 2.22

$$\left[\frac{3\left(bcx^2+b^2\right)\sqrt{c}\log\left(-2cx^2-b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\sqrt{cx^4+bx^2}\left(c^2x^2+3bc\right)}{4\left(c^4x^2+bc^3\right)},\frac{3\left(bcx^2+b^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\left(c^2x^2+3bc\right)}{2\left(c^4x^2+bc^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b*c*x^2 + b^2)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sq
rt(c)) + 2*sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3), 1/2*(3
*(b*c*x^2 + b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))
+ sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3)]
```

giac [A] time = 0.24, size = 99, normalized size = 1.22

$$\frac{3\,b\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{5}{2}}}+\frac{b^2}{\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)c+b\sqrt{c}\right)c^2}+\frac{\sqrt{cx^4+bx^2}}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(5/2)
+ b^2/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*c + b*sqrt(c))*c^2) + 1/2*sqrt(
c*x^4 + b*x^2)/c^2
```

maple [A] time = 0.01, size = 73, normalized size = 0.90

$$\frac{\left(c\,x^2+b\right)\left(c^{\frac{5}{2}}x^3+3b\,c^{\frac{3}{2}}x-3\sqrt{c\,x^2+b}\;bc\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)\right)x^3}{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2*x^3*(c*x^2+b)*(x^3*c^(5/2)+3*c^(3/2)*x*b-3*ln(c^(1/2)*x+(c*x^2+b)^(1/2)
)*(c*x^2+b)^(1/2)*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

maxima [A] time = 1.51, size = 77, normalized size = 0.95

$$\frac{x^4}{2\sqrt{cx^4+bx^2}\,c}+\frac{3\,bx^2}{2\sqrt{cx^4+bx^2}\,c^2}-\frac{3\,b\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{4\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 3/2*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3/4
*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**7/(x**2*(b + c*x**2))**(3/2), x)
```

3.157 $\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2}{c\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 652, 620, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^2/(c*Sqrt[b*x^2 + c*x^4])) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
/c^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{x^2}{c\sqrt{bx^2+cx^4}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}\\
&= -\frac{x^2}{c\sqrt{bx^2+cx^4}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{c}\\
&= -\frac{x^2}{c\sqrt{bx^2+cx^4}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 1.20

$$\frac{\sqrt{b}\,x\sqrt{\frac{cx^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)-\sqrt{c}\,x^2}{c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[c]*x^2) + Sqrt[b]*x*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]
])/(c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.31, size = 74, normalized size = 1.35

$$-\frac{\log\left(-2c^{3/2}\sqrt{bx^2+cx^4}+bc+2c^2x^2\right)}{2c^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(c*(b + c*x^2))) - Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sq
rt[b*x^2 + c*x^4]]/(2*c^(3/2))
```

fricas [A] time = 1.65, size = 150, normalized size = 2.73

$$\left[\frac{\left(cx^2+b\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\,c}{2\,\left(c^3x^2+bc^2\right)},\,-\frac{\left(cx^2+b\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\,c}{c^3x^2+bc^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^2 + b)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
 - 2*sqrt(c*x^4 + b*x^2)*c)/(c^3*x^2 + b*c^2), -((c*x^2 + b)*sqrt(-c)*arcta
n(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/(c^3*x
^2 + b*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{-2,[1]%%%},[2,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,3]%
%%}+%%%{-2,[0,4]%%%} / %%%{%%%{1,[2]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1]%%%},[0,2]%%%} Error: Bad
Argument Value
```

maple [A] time = 0.01, size = 63, normalized size = 1.15

$$-\frac{\left(c\,x^2+b\right)\left(c^{\frac{3}{2}}x-\sqrt{c\,x^2+b}\;c\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)\right)x^3}{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -x^3*(c*x^2+b)*(x*c^(3/2)-ln(c^(1/2)*x+(c*x^2+b)^(1/2))*c*(c*x^2+b)^(1/2))/
(c*x^4+b*x^2)^(3/2)/c^(5/2)
```

maxima [A] time = 1.47, size = 54, normalized size = 0.98

$$-\frac{x^2}{\sqrt{cx^4+bx^2}\,c}+\frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/(sqrt(c*x^4 + b*x^2)*c) + 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*
sqrt(c))/c^(3/2)
```

mupad [B] time = 4.33, size = 55, normalized size = 1.00

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,c^{3/2}}-\frac{x^2}{c\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(3/2)) - x^2/(c*(b*
x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(b + c*x**2))**(3/2), x)
```

3.158 $\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^2/(b*Sqrt[b*x^2 + c*x^4])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{x^2}{b\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x^2}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^2/(b*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.27, size = 28, normalized size = 1.27

$$\frac{\sqrt{bx^2+cx^4}}{b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(b*x^2 + c*x^4)^(3/2),x]
[Out] Sqrt[b*x^2 + c*x^4]/(b*(b + c*x^2))
```

fricas [A] time = 1.91, size = 26, normalized size = 1.18

$$\frac{\sqrt{cx^4+bx^2}}{bcx^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)/(b*c*x^2 + b^2)
```

giac [A] time = 0.18, size = 35, normalized size = 1.59

$$\frac{1}{\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}\,+b\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] 1/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) + b)*sqrt(c))
```

maple [A] time = 0.00, size = 28, normalized size = 1.27

$$\frac{\left(c\,x^2+b\right)x^4}{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(3/2),x)
[Out] (c*x^2+b)/b*x^4/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.45, size = 20, normalized size = 0.91

$$\frac{x^2}{\sqrt{cx^4 + bx^2}\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(c*x^4 + b*x^2)*b)
```

mupad [B] time = 4.13, size = 26, normalized size = 1.18

$$\frac{\sqrt{c\,x^4 + b\,x^2}}{b\,\left(c\,x^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(b*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2*(b + c*x**2))**(3/2), x)
```

3.159 $\int \frac{x}{(bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=28

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2013, 613}

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b + 2*c*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(bx^2+cx^4)^{3/2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(bx+cx^2)^{3/2}} \, dx, x, x^2\right) \\ &= -\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{-b-2cx^2}{b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^(3/2),x]
[Out] (-b - 2*c*x^2)/(b^2*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.26, size = 41, normalized size = 1.46

$$\frac{\left(-b-2cx^2\right)\sqrt{bx^2+cx^4}}{b^2x^2\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(b*x^2 + c*x^4)^(3/2),x]
[Out] ((-b - 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(b^2*x^2*(b + c*x^2))
```

fricas [A] time = 2.48, size = 41, normalized size = 1.46

$$-\frac{\sqrt{cx^4+bx^2}\left(2\,cx^2+b\right)}{b^2cx^4+b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4 + b*x^2)*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2)
```

giac [A] time = 0.19, size = 28, normalized size = 1.00

$$-\frac{\frac{2\,cx^2}{b^2}+\frac{1}{b}}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -(2*c*x^2/b^2 + 1/b)/sqrt(c*x^4 + b*x^2)
```

maple [A] time = 0.00, size = 37, normalized size = 1.32

$$-\frac{\left(c\,x^{2}+b\right)\left(2c\,x^{2}+b\right)x^{2}}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(3/2),x)
[Out] -x^2*(c*x^2+b)*(2*c*x^2+b)/b^2/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.47, size = 41, normalized size = 1.46

$$-\frac{2\,cx^{2}}{\sqrt{cx^{4}+bx^{2}}\,b^{2}}-\frac{1}{\sqrt{cx^{4}+bx^{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] -2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) - 1/(sqrt(c*x^4 + b*x^2)*b)
```

mupad [B] time = 4.13, size = 26, normalized size = 0.93

$$-\frac{2\,c\,x^{2}+b}{b^{2}\,\sqrt{c\,x^{4}+b\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^(3/2),x)
[Out] -(b + 2*c*x^2)/(b^2*(b*x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x/(x**2*(b + c*x**2))**(3/2), x)
```

3.160 $\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{1}{bx^2\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{1}{bx^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^2*Sqrt[b*x^2 + c*x^4]) - (4*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^4) + (8*c*
Sqrt[b*x^2 + c*x^4])/(3*b^3*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}+\frac{4\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}-\frac{(8c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b^2}\\ &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.65

$$-\frac{\left(b+cx^2\right)\left(b^2-4bcx^2-8c^2x^4\right)}{3b^3\left(x^2\left(b+cx^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/3*((b + c*x^2)*(b^2 - 4*b*c*x^2 - 8*c^2*x^4))/(b^3*(x^2*(b + c*x^2))^(3/
2))
```

IntegrateAlgebraic [A] time = 0.28, size = 55, normalized size = 0.74

$$\frac{\sqrt{bx^2+cx^4}\,\left(-b^2+4bcx^2+8c^2x^4\right)}{3b^3x^4\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-b^2 + 4*b*c*x^2 + 8*c^2*x^4))/(3*b^3*x^4*(b + c*x^2)
)
```

fricas [A] time = 3.97, size = 54, normalized size = 0.73

$$\frac{\left(8\,c^2x^4+4\,bcx^2-b^2\right)\sqrt{cx^4+bx^2}}{3\left(b^3cx^6+b^4x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*c^2*x^4 + 4*b*c*x^2 - b^2)*sqrt(c*x^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x), x)
```

maple [A] time = 0.01, size = 45, normalized size = 0.61

$$-\frac{\left(c\,x^2 + b\right)\left(-8c^2x^4 - 4bc\,x^2 + b^2\right)}{3\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-8*c^2*x^4-4*b*c*x^2+b^2)/b^3/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.49, size = 65, normalized size = 0.88

$$\frac{8\,c^2x^2}{3\,\sqrt{cx^4 + bx^2}\,b^3} + \frac{4\,c}{3\,\sqrt{cx^4 + bx^2}\,b^2} - \frac{1}{3\,\sqrt{cx^4 + bx^2}\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4/3*c/(sqrt(c*x^4 + b*x^2)*b^2) - 1
/3/(sqrt(c*x^4 + b*x^2)*b*x^2)
```

mupad [B] time = 4.24, size = 51, normalized size = 0.69

$$\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(-b^2 + 4\,b\,c\,x^2 + 8\,c^2\,x^4\right)}{3\,b^3\,x^4\,\left(c\,x^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - b^2 + 4*b*c*x^2))/(3*b^3*x^4*(b + c*x^2
))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

3.161 $\int \frac{1}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{1}{bx^4\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.18, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{1}{bx^4\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (6*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^6) + (8*c*
Sqrt[b*x^2 + c*x^4])/(5*b^3*x^4) - (16*c^2*Sqrt[b*x^2 + c*x^4])/(5*b^4*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^4\sqrt{bx^2+cx^4}}+\frac{6\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{b}$$
$$= \frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}-\frac{(24c)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{5b^2}$$
$$= \frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}+\frac{\left(16c^2\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{5b^3}$$
$$= \frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.56

$$\frac{-b^3+2b^2cx^2-8bc^2x^4-16c^3x^6}{5b^4x^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
[Out] (-b^3 + 2*b^2*c*x^2 - 8*b*c^2*x^4 - 16*c^3*x^6)/(5*b^4*x^4*Sqrt[x^2*(b + c*
x^2)])
```

IntegrateAlgebraic [A] time = 0.30, size = 66, normalized size = 0.65

$$\frac{\sqrt{bx^2+cx^4}\left(-b^3+2b^2cx^2-8bc^2x^4-16c^3x^6\right)}{5b^4x^6\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
[Out] (Sqrt[b*x^2 + c*x^4]*(-b^3 + 2*b^2*c*x^2 - 8*b*c^2*x^4 - 16*c^3*x^6))/(5*b^
4*x^6*(b + c*x^2))
```

fricas [A] time = 2.28, size = 63, normalized size = 0.62

$$-\frac{\left(16\,c^{3}x^{6}+8\,bc^{2}x^{4}-2\,b^{2}cx^{2}+b^{3}\right)\sqrt{cx^{4}+bx^{2}}}{5\left(b^{4}cx^{8}+b^{5}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(16*c^3*x^6 + 8*b*c^2*x^4 - 2*b^2*c*x^2 + b^3)*sqrt(c*x^4 + b*x^2)/(b^
4*c*x^8 + b^5*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^3), x)
```

maple [A] time = 0.00, size = 59, normalized size = 0.58

$$-\frac{\left(c\,x^{2}+b\right)\left(16c^{3}x^{6}+8b\,c^{2}x^{4}-2b^{2}c\,x^{2}+b^{3}\right)}{5\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/5*(c*x^2+b)*(16*c^3*x^6+8*b*c^2*x^4-2*b^2*c*x^2+b^3)/x^2/b^4/(c*x^4+b*x^
2)^(3/2)
```

maxima [A] time = 1.51, size = 89, normalized size = 0.87

$$-\frac{16\,c^{3}x^{2}}{5\sqrt{cx^{4}+bx^{2}}\,b^{4}}-\frac{8\,c^{2}}{5\sqrt{cx^{4}+bx^{2}}\,b^{3}}+\frac{2\,c}{5\sqrt{cx^{4}+bx^{2}}\,b^{2}x^{2}}-\frac{1}{5\sqrt{cx^{4}+bx^{2}}\,bx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/5*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b^4) - 8/5*c^2/(sqrt(c*x^4 + b*x^2)*b^3)
 + 2/5*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) - 1/5/(sqrt(c*x^4 + b*x^2)*b*x^4)
```

mupad [B] time = 4.31, size = 60, normalized size = 0.59

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(b^3-2\,b^2\,c\,x^2+8\,b\,c^2\,x^4+16\,c^3\,x^6\right)}{5\,b^4\,x^6\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(b^3 + 16*c^3*x^6 - 2*b^2*c*x^2 + 8*b*c^2*x^4))/(5*
b^4*x^6*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

3.162 $\int \frac{1}{x^5(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{1}{bx^6\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.23, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{1}{bx^6\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (8*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^8) + (48*c
*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^6) - (64*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^4*x
^4) + (128*c^3*Sqrt[b*x^2 + c*x^4])/(35*b^5*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^6\sqrt{bx^2+cx^4}} + \frac{8\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx}{b}$$

$$= \frac{1}{bx^6\sqrt{bx^2+cx^4}} - \frac{8\sqrt{bx^2+cx^4}}{7b^2x^8} - \frac{(48c)\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{7b^2}$$

$$= \frac{1}{bx^6\sqrt{bx^2+cx^4}} - \frac{8\sqrt{bx^2+cx^4}}{7b^2x^8} + \frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6} + \frac{\left(192c^2\right)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{35b^3}$$

$$= \frac{1}{bx^6\sqrt{bx^2+cx^4}} - \frac{8\sqrt{bx^2+cx^4}}{7b^2x^8} + \frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6} - \frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4} - \frac{\left(128c^3\right)\int \frac{}{x\sqrt{}}}{35b}$$

$$= \frac{1}{bx^6\sqrt{bx^2+cx^4}} - \frac{8\sqrt{bx^2+cx^4}}{7b^2x^8} + \frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6} - \frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4} + \frac{128c^3\sqrt{bx^2+}}{35b^5x^2}$$

Mathematica [A] time = 0.01, size = 68, normalized size = 0.52

$$\frac{-5b^4+8b^3cx^2-16b^2c^2x^4+64bc^3x^6+128c^4x^8}{35b^5x^6\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*b^4 + 8*b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6 + 128*c^4*x^8)/(35*b^
5*x^6*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.30, size = 77, normalized size = 0.59

$$\frac{\sqrt{bx^2+cx^4}\left(-5b^4+8b^3cx^2-16b^2c^2x^4+64bc^3x^6+128c^4x^8\right)}{35b^5x^8\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-5*b^4 + 8*b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6
+ 128*c^4*x^8))/(35*b^5*x^8*(b + c*x^2))
```

fricas [A] time = 2.05, size = 76, normalized size = 0.58

$$\frac{\left(128\,c^{4}x^{8}+64\,bc^{3}x^{6}-16\,b^{2}c^{2}x^{4}+8\,b^{3}cx^{2}-5\,b^{4}\right)\sqrt{cx^{4}+bx^{2}}}{35\,\left(b^{5}cx^{10}+b^{6}x^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(128*c^4*x^8 + 64*b*c^3*x^6 - 16*b^2*c^2*x^4 + 8*b^3*c*x^2 - 5*b^4)*sq
rt(c*x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^5), x)
```

maple [A] time = 0.01, size = 72, normalized size = 0.55

$$-\frac{\left(c\,x^{2}+b\right)\left(-128c^{4}x^{8}-64c^{3}x^{6}b+16c^{2}x^{4}b^{2}-8c\,x^{2}b^{3}+5b^{4}\right)}{35\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-128*c^4*x^8-64*b*c^3*x^6+16*b^2*c^2*x^4-8*b^3*c*x^2+5*b^4
)/x^4/b^5/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.47, size = 113, normalized size = 0.87

$$\frac{128\,c^{4}x^{2}}{35\,\sqrt{cx^{4}+bx^{2}}\,b^{5}}+\frac{64\,c^{3}}{35\,\sqrt{cx^{4}+bx^{2}}\,b^{4}}-\frac{16\,c^{2}}{35\,\sqrt{cx^{4}+bx^{2}}\,b^{3}x^{2}}+\frac{8\,c}{35\,\sqrt{cx^{4}+bx^{2}}\,b^{2}x^{4}}-\frac{1}{7\,\sqrt{cx^{4}+bx^{2}}\,bx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 128/35*c^4*x^2/(sqrt(c*x^4 + b*x^2)*b^5) + 64/35*c^3/(sqrt(c*x^4 + b*x^2)*b
^4) - 16/35*c^2/(sqrt(c*x^4 + b*x^2)*b^3*x^2) + 8/35*c/(sqrt(c*x^4 + b*x^2)
*b^2*x^4) - 1/7/(sqrt(c*x^4 + b*x^2)*b*x^6)
```

mupad [B] time = 4.41, size = 114, normalized size = 0.88

$$\frac{13\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b^3\,x^6}-\frac{\sqrt{c\,x^4+b\,x^2}}{7\,b^2\,x^8}-\frac{29\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^4}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{93\,c^3}{35\,b^4}+\frac{128\,c^4\,x^2}{35\,b^5}\right)}{x^2\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] (13*c*(b*x^2 + c*x^4)^(1/2))/(35*b^3*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*b^2*x^
8) - (29*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b^4*x^4) + ((b*x^2 + c*x^4)^(1/2)*(
(93*c^3)/(35*b^4) + (128*c^4*x^2)/(35*b^5)))/(x^2*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

3.163 $\int \frac{x^6}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{bx^2+cx^4}}{c^2x}-\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 1588}

$$\frac{2\sqrt{bx^2+cx^4}}{c^2x}-\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^3/(c*Sqrt[b*x^2 + c*x^4])) + (2*Sqrt[b*x^2 + c*x^4])/(c^2*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^3}{c\sqrt{bx^2+cx^4}} + \frac{2\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{c}$$
$$= -\frac{x^3}{c\sqrt{bx^2+cx^4}} + \frac{2\sqrt{bx^2+cx^4}}{c^2x}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.62

$$\frac{x\left(2b+cx^2\right)}{c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(2*b + c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.39, size = 40, normalized size = 0.85

$$\frac{\left(2b+cx^2\right)\sqrt{bx^2+cx^4}}{c^2x\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(b*x^2 + c*x^4)^(3/2),x]
[Out] ((2*b + c*x^2)*Sqrt[b*x^2 + c*x^4])/(c^2*x*(b + c*x^2))
```

fricas [A] time = 0.69, size = 39, normalized size = 0.83

$$\frac{\sqrt{cx^4+bx^2}\left(cx^2+2\,b\right)}{c^3x^3+bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)*(c*x^2 + 2*b)/(c^3*x^3 + b*c^2*x)
```

giac [A] time = 0.21, size = 52, normalized size = 1.11

$$-\frac{2\sqrt{b}}{\left(\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c\right)c}+\frac{b}{\sqrt{c+\frac{b}{x^2}}\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b)/(((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)*c) + b/(sqrt(c + b/x^2)*c
^2*x)
```

maple [A] time = 0.00, size = 37, normalized size = 0.79

$$\frac{(c\,x^2+b)\,(c\,x^2+2b)\,x^3}{(c\,x^4+b\,x^2)^{\frac{3}{2}}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] (c*x^2+b)*(c*x^2+2*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.48, size = 22, normalized size = 0.47

$$\frac{cx^2+2\,b}{\sqrt{cx^2+b}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (c*x^2 + 2*b)/(sqrt(c*x^2 + b)*c^2)
```

mupad [B] time = 4.23, size = 38, normalized size = 0.81

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(c\,x^2+2\,b\right)}{c^2\,x\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(2*b + c*x^2))/(c^2*x*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(b + c*x**2))**(3/2), x)
```

3.164 $\int \frac{x^4}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x/(c*Sqrt[b*x^2 + c*x^4]))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x}{c\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{x}{c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x/(c*Sqrt[x^2*(b + c*x^2)]))
```

IntegrateAlgebraic [A] time = 0.35, size = 32, normalized size = 1.52

$$-\frac{\sqrt{bx^2+cx^4}}{cx\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(b*x^2 + c*x^4)^(3/2),x]
[Out] -(Sqrt[b*x^2 + c*x^4]/(c*x*(b + c*x^2)))
```

fricas [A] time = 1.08, size = 29, normalized size = 1.38

$$-\frac{\sqrt{cx^4+bx^2}}{c^2x^3+bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4 + b*x^2)/(c^2*x^3 + b*c*x)
```

giac [A] time = 0.21, size = 17, normalized size = 0.81

$$-\frac{1}{\sqrt{c+\frac{b}{x^2}}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -1/(sqrt(c + b/x^2)*c*x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.38

$$-\frac{\left(c\,x^2+b\right)x^3}{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(3/2),x)
[Out] -(c*x^2+b)/c*x^3/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.44, size = 14, normalized size = 0.67

$$-\frac{1}{\sqrt{cx^2+b}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(c*x^2 + b)*c)
```

mupad [B] time = 4.15, size = 30, normalized size = 1.43

$$-\frac{\sqrt{c\,x^4+b\,x^2}}{c\,x\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -(b*x^2 + c*x^4)^(1/2)/(c*x*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(b + c*x**2))**(3/2), x)
```

3.165 $\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2023, 2008, 206}

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x/(b*Sqrt[b*x^2 + c*x^4]) - ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/b^(3/2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rubi steps

$$\int \frac{x^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{x}{b\sqrt{bx^2+cx^4}} + \frac{\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{b}$$
$$= \frac{x}{b\sqrt{bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{b}$$
$$= \frac{x}{b\sqrt{bx^2+cx^4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.75

$$\frac{x\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{b}+1\right)}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/b])/(b*Sqrt[x^2*(b + c*x^2)]
)
```

IntegrateAlgebraic [A] time = 0.42, size = 62, normalized size = 1.22

$$\frac{\sqrt{bx^2+cx^4}}{bx\left(b+cx^2\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(b*x*(b + c*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*
x^4]]/b^(3/2)
```

fricas [A] time = 1.68, size = 162, normalized size = 3.18

$$\left[\frac{\left(cx^3+bx\right)\sqrt{b}\,\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,b}{2\left(b^2cx^3+b^3x\right)}, \frac{\left(cx^3+bx\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,b}{b^2cx^3+b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^3 + b*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqr
t(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*c*x^3 + b^3*x), ((c*x^3 + b*x)*s
qrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*b)/(b^2*c*x^3 + b^3*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]1/b/x*sqrt(b*(1/x)^2+c)/(b*(1/x)^2+c)+1/
b/sqrt(b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 65, normalized size = 1.27

$$\frac{\left(c\,x^{2}+b\right)\left(-\sqrt{c\,x^{2}+b}\;b\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\;\sqrt{b}}{x}\right)+b^{\frac{3}{2}}\right)x^{3}}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b)*(b^(3/2)-ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*b*(c*x^2+b)^(1/2
))/(c*x^4+b*x^2)^(3/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^2/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**2/(x**2*(b + c*x**2))**(3/2), x)
```

3.166 $\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2006, 2025, 2008, 206}

$$-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] 1/(b*x*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(2*b^2*x^3) + (3*c*Ar
cTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx\sqrt{bx^2+cx^4}}+\frac{3\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}-\frac{(3c)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}\\ &= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b^2}\\ &= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.49

$$-\frac{cx\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{b}+1\right)}{b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-3/2),x]
[Out] -((c*x*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x^2)/b])/(b^2*Sqrt[x^2*(b + c
*x^2)]))
```

IntegrateAlgebraic [A] time = 0.49, size = 78, normalized size = 0.96

$$\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}+\frac{\sqrt{bx^2+cx^4}\left(-b-3cx^2\right)}{2b^2x^3\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] ((-b - 3*c*x^2)*Sqrt[b*x^2 + c*x^4])/(2*b^2*x^3*(b + c*x^2)) + (3*c*ArcTanh
[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

fricas [A] time = 0.84, size = 199, normalized size = 2.46

$$\left[\frac{3\left(c^2x^5+bcx^3\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\left(3\,bcx^2+b^2\right)}{4\left(b^3cx^5+b^4x^3\right)},-\frac{3\left(c^2x^5+bcx^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\left(3\,bcx^2+b^2\right)}{2\left(b^3cx^5+b^4x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(c^2*x^5 + b*c*x^3)*sqrt(b)*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*
x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5 +
b^4*x^3), -1/2*(3*(c^2*x^5 + b*c*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*s
qrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5
+ b^4*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 77, normalized size = 0.95

$$-\frac{\left(c\,x^2+b\right)\left(-3\sqrt{c\,x^2+b}\;bc\,x^2\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\;\sqrt{b}}{x}\right)+3b^{\frac{3}{2}}c\,x^2+b^{\frac{5}{2}}\right)x}{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2*x*(c*x^2+b)*(3*b^(3/2)*x^2*c-3*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*(c*
x^2+b)^(1/2)*x^2*b*c+b^(5/2))/(c*x^4+b*x^2)^(3/2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(-3/2), x)
```

mupad [B] time = 4.34, size = 42, normalized size = 0.52

$$-\frac{x\left(\frac{b}{c\,x^2}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{b}{c\,x^2}\right)}{5\left(c\,x^4+b\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -(x*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -b/(c*x^2)))/(5*(b*x^2
 + c*x^4)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((b*x**2 + c*x**4)**(-3/2), x)
```

3.167 $\int \frac{1}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

Rubi [A] time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2008, 206}

$$-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^3*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(4*b^2*x^5) + (15*c
*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) - (15*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2
+ c*x^4]])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^3\sqrt{bx^2+cx^4}}+\frac{5\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}-\frac{(15c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b^2}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}+\frac{\left(15c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^3}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{\left(15c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^3}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.38

$$\frac{c^2x\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{cx^2}{b}+1\right)}{b^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (c^2*x*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (c*x^2)/b])/(b^3*Sqrt[x^2*(b + c
*x^2)])
```

IntegrateAlgebraic [A] time = 0.59, size = 91, normalized size = 0.83

$$\frac{\sqrt{bx^2+cx^4}\left(-2b^2+5bcx^2+15c^2x^4\right)}{8b^3x^5\left(b+cx^2\right)}-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[b*x^2 + c*x^4]*(-2*b^2 + 5*b*c*x^2 + 15*c^2*x^4))/(8*b^3*x^5*(b + c*x
^2)) - (15*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(7/2))
```

fricas [A] time = 0.61, size = 229, normalized size = 2.10

$$\left[\frac{15\left(c^3x^7+bc^2x^5\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(15\,bc^2x^4+5\,b^2cx^2-2\,b^3\right)\sqrt{cx^4+bx^2}}{16\left(b^4cx^7+b^5x^5\right)},\frac{15\left(c^3x^7+bc^2x^5\right)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\left(15\,bc^2x^4+5\,b^2cx^2-2\,b^3\right)\sqrt{cx^4+bx^2}}{8\left(b^4cx^7+b^5x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
+ b*x^2)*sqrt(b))/x^3) + 2*(15*b*c^2*x^4 + 5*b^2*c*x^2 - 2*b^3)*sqrt(c*x^4
+ b*x^2))/(b^4*c*x^7 + b^5*x^5), 1/8*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(-b)*arc
tan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*b*c^2*x^4 + 5*b^2*c*x
^2 - 2*b^3)*sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

maple [A] time = 0.01, size = 94, normalized size = 0.86

$$-\frac{\left(c\,x^2+b\right)\left(15\sqrt{c\,x^2+b}\;b\,c^2x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\;\sqrt{b}}{x}\right)-15b^{\frac{3}{2}}c^2x^4-5b^{\frac{5}{2}}c\,x^2+2b^{\frac{7}{2}}\right)}{8\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^{\frac{9}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/8/x*(c*x^2+b)*(15*(c*x^2+b)^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^
4*b*c^2-15*b^(3/2)*x^4*c^2-5*b^(5/2)*x^2*c+2*b^(7/2))/(c*x^4+b*x^2)^(3/2)/b
^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

mupad [B] time = 4.64, size = 44, normalized size = 0.40

$$-\frac{\left(\frac{b}{c\,x^2}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{b}{c\,x^2}\right)}{7\,x\,\left(c\,x^4+b\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -b/(c*x^2)))/(7*x*(b*x^2
 + c*x^4)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

3.168 $\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)-\frac{1}{8}\sqrt{3x^2-4x^4}$$

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 619, 216}

$$-\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

```
[Out] -Sqrt[3*x^2 - 4*x^4]/8 - (3*ArcSin[1 - (8*x^2)/3])/32
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{3x-4x^2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{8}\sqrt{3x^2-4x^4} + \frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3x-4x^2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{8}\sqrt{3x^2-4x^4} - \frac{1}{32}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 3-8x^2\right)$$
$$= -\frac{1}{8}\sqrt{3x^2-4x^4} - \frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.68

$$\frac{x\left(8x^3+3\sqrt{4x^2-3}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-3}}\right)-6x\right)}{16\sqrt{3x^2-4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

```
[Out] (x*(-6*x + 8*x^3 + 3*Sqrt[-3 + 4*x^2]*ArcTanh[(2*x)/Sqrt[-3 + 4*x^2]]))/(16
*Sqrt[3*x^2 - 4*x^4])
```

IntegrateAlgebraic [C] time = 0.12, size = 55, normalized size = 1.62

$$-\frac{1}{8}\sqrt{3x^2-4x^4} + \frac{3}{32}i\log\left(-8ix^2+4\sqrt{3x^2-4x^4}+3i\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

```
[Out] -1/8*Sqrt[3*x^2 - 4*x^4] + ((3*I)/32)*Log[3*I - (8*I)*x^2 + 4*Sqrt[3*x^2 -
4*x^4]]
```

fricas [A] time = 0.78, size = 37, normalized size = 1.09

$$-\frac{1}{8}\sqrt{-4x^4+3x^2} - \frac{3}{16}\arctan\left(\frac{\sqrt{-4x^4+3x^2}}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/16*arctan(1/2*sqrt(-4*x^4 + 3*x^2)/x^2)
```

giac [A] time = 0.18, size = 26, normalized size = 0.76

$$-\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}+\frac{3}{32}\arcsin\left(\frac{8}{3}\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) + 3/32*arcsin(8/3*x^2 - 1)
```

maple [A] time = 0.01, size = 48, normalized size = 1.41

$$\frac{\sqrt{-4x^2+3}\,\left(-2\sqrt{-4x^2+3}\,x+3\arcsin\left(\frac{2\sqrt{3}\,x}{3}\right)\right)x}{16\sqrt{-4x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/16*x*(-4*x^2+3)^(1/2)*(-2*x*(-4*x^2+3)^(1/2)+3*arcsin(2/3*3^(1/2)*x))/(-4
*x^4+3*x^2)^(1/2)
```

maxima [A] time = 3.03, size = 26, normalized size = 0.76

$$-\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}-\frac{3}{32}\arcsin\left(-\frac{8}{3}\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/32*arcsin(-8/3*x^2 + 1)
```

mupad [B] time = 4.33, size = 42, normalized size = 1.24

$$-\frac{\sqrt{3\,x^2-4\,x^4}}{8}-\frac{\ln\left(x^2-\frac{3}{8}-\frac{\sqrt{3-4\,x^2}\,\sqrt{x^2}\,1\mathrm{i}}{2}\right)\,3\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2 - 4*x^4)^(1/2),x)
```

```
[Out] - (log(x^2 - ((3 - 4*x^2)^(1/2)*(x^2)^(1/2)*1i)/2 - 3/8)*3i)/32 - (3*x^2 -
4*x^4)^(1/2)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-x^2\left(4x^2-3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 - 3)), x)
```

3.169 $\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{3}{32}\sin^{-1}\left(\frac{8x^2}{3}+1\right)-\frac{1}{8}\sqrt{-4x^4-3x^2}$$

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 619, 216}

$$-\frac{1}{8}\sqrt{-4x^4-3x^2}-\frac{3}{32}\sin^{-1}\left(\frac{8x^2}{3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

```
[Out] -Sqrt[-3*x^2 - 4*x^4]/8 - (3*ArcSin[1 + (8*x^2)/3])/32
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{-3x-4x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{8}\sqrt{-3x^2-4x^4} - \frac{3}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{-3x-4x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{8}\sqrt{-3x^2-4x^4} + \frac{1}{32}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, -3-8x^2\right)\\ &= -\frac{1}{8}\sqrt{-3x^2-4x^4} - \frac{3}{32}\sin^{-1}\left(1+\frac{8x^2}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.53

$$\frac{x\left(8x^3-3\sqrt{4x^2+3}\,\sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+6x\right)}{16\sqrt{-x^2\left(4x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

```
[Out] (x*(6*x + 8*x^3 - 3*Sqrt[3 + 4*x^2]*ArcSinh[(2*x)/Sqrt[3]]))/(16*Sqrt[-(x^2
*(3 + 4*x^2))])
```

IntegrateAlgebraic [A] time = 0.12, size = 51, normalized size = 1.50

$$\frac{3}{16}\tan^{-1}\left(\frac{2\sqrt{-4x^4-3x^2}}{4x^2+3}\right)-\frac{1}{8}\sqrt{-4x^4-3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

```
[Out] -1/8*Sqrt[-3*x^2 - 4*x^4] + (3*ArcTan[(2*Sqrt[-3*x^2 - 4*x^4])/(3 + 4*x^2)]
)/16
```

fricas [C] time = 0.91, size = 59, normalized size = 1.74

$$-\frac{1}{8}\sqrt{-4\,x^2-3}\,x-\frac{3}{32}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-3}}{x}\right)+\frac{3}{32}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 - 3)*x - 3/32*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 3))/x) + 3/3
2*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 3))/x)
```

giac [A] time = 0.21, size = 27, normalized size = 0.79

$$-\frac{1}{8}\sqrt{4x^4+3x^2}\,i-\frac{3}{32}\arcsin\left(\frac{8}{3}x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(4*x^4 + 3*x^2)*i - 3/32*arcsin(8/3*x^2 + 1)
```

maple [B] time = 0.01, size = 54, normalized size = 1.59

$$-\frac{\sqrt{-4x^2-3}\,\left(2\sqrt{-4x^2-3}\,x+3\arctan\left(\frac{2x}{\sqrt{-4x^2-3}}\right)\right)x}{16\sqrt{-4x^4-3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^4-3*x^2)^(1/2),x)
```

```
[Out] -1/16*x*(-4*x^2-3)^(1/2)*(2*x*(-4*x^2-3)^(1/2)+3*arctan(2*x/(-4*x^2-3)^(1/2
)))/(-4*x^4-3*x^2)^(1/2)
```

maxima [A] time = 2.98, size = 26, normalized size = 0.76

$$-\frac{1}{8}\sqrt{-4x^4-3x^2}+\frac{3}{32}\arcsin\left(-\frac{8}{3}x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 - 3*x^2) + 3/32*arcsin(-8/3*x^2 - 1)
```

mupad [B] time = 4.36, size = 41, normalized size = 1.21

$$-\frac{\sqrt{-4x^4-3x^2}}{8}+\frac{\ln\left(\frac{\sqrt{4x^2+3}\,\sqrt{x^2}}{2}+x^2+\frac{3}{8}\right)3\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(- 3*x^2 - 4*x^4)^(1/2),x)
```

```
[Out] (log(((4*x^2 + 3)^(1/2)*(x^2)^(1/2))/2 + x^2 + 3/8)*3i)/32 - (- 3*x^2 - 4*x
^4)^(1/2)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-x^2 \left(4x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**4-3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 + 3)), x)
```

3.170 $\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{4x^4+3x^2}-\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 620, 206}

$$\frac{1}{8}\sqrt{4x^4+3x^2}-\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[3*x^2 + 4*x^4]/8 - (3*ArcTanh[(2*x^2)/Sqrt[3*x^2 + 4*x^4]])/16
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{3x+4x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{3x+4x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x^2}{\sqrt{3x^2+4x^4}}\right)\\ &= \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{3x^2+4x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.13

$$\frac{x\left(8x^3-3\sqrt{4x^2+3}\,\sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+6x\right)}{16\sqrt{x^2\left(4x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[3*x^2 + 4*x^4],x]
[Out] (x*(6*x + 8*x^3 - 3*Sqrt[3 + 4*x^2]*ArcSinh[(2*x)/Sqrt[3]]))/(16*Sqrt[x^2*(
3 + 4*x^2)])
```

IntegrateAlgebraic [A] time = 0.12, size = 49, normalized size = 1.09

$$\frac{1}{8}\sqrt{4x^4+3x^2}+\frac{3}{32}\log\left(-8x^2+4\sqrt{4x^4+3x^2}-3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[3*x^2 + 4*x^4],x]
[Out] Sqrt[3*x^2 + 4*x^4]/8 + (3*Log[-3 - 8*x^2 + 4*Sqrt[3*x^2 + 4*x^4]])/32
```

fricas [A] time = 0.70, size = 45, normalized size = 1.00

$$\frac{1}{8}\sqrt{4\,x^4+3\,x^2}+\frac{3}{16}\log\left(-\frac{2\,x^2-\sqrt{4\,x^4+3\,x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) + 3/16*log(-(2*x^2 - sqrt(4*x^4 + 3*x^2))/x)
```

giac [A] time = 0.17, size = 41, normalized size = 0.91

$$\frac{1}{8}\sqrt{4x^4+3x^2}+\frac{3}{32}\log\left(8x^2-4\sqrt{4x^4+3x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) + 3/32*log(8*x^2 - 4*sqrt(4*x^4 + 3*x^2) + 3)
```

maple [A] time = 0.01, size = 48, normalized size = 1.07

$$-\frac{\sqrt{4x^2+3}\left(-2\sqrt{4x^2+3}\,x+3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,x}{3}\right)\right)x}{16\sqrt{4x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4+3*x^2)^(1/2),x)
```

```
[Out] -1/16*x*(4*x^2+3)^(1/2)*(-2*x*(4*x^2+3)^(1/2)+3*arcsinh(2/3*3^(1/2)*x))/(4*
x^4+3*x^2)^(1/2)
```

maxima [A] time = 3.05, size = 41, normalized size = 0.91

$$\frac{1}{8}\sqrt{4x^4+3x^2}-\frac{3}{32}\log\left(8x^2+4\sqrt{4x^4+3x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) - 3/32*log(8*x^2 + 4*sqrt(4*x^4 + 3*x^2) + 3)
```

mupad [B] time = 4.40, size = 40, normalized size = 0.89

$$\frac{\sqrt{4x^4+3x^2}}{8}-\frac{3\,\ln\left(\frac{\sqrt{4x^2+3}\,\sqrt{x^2}}{2}+x^2+\frac{3}{8}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2 + 4*x^4)^(1/2),x)
```

```
[Out] (3*x^2 + 4*x^4)^(1/2)/8 - (3*log(((4*x^2 + 3)^(1/2)*(x^2)^(1/2))/2 + x^2 +
3/8))/32
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2 \left(4x^2+3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(4*x**2 + 3)), x)
```

3.171 $\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 620, 206}

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[-3*x^2 + 4*x^4]/8 + (3*ArcTanh[(2*x^2)/Sqrt[-3*x^2 + 4*x^4]])/16
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{-3x+4x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{-3x+4x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{x^2}{\sqrt{-3x^2+4x^4}}\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{-3x^2+4x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.27

$$\frac{x\left(8x^3+3\sqrt{4x^2-3}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-3}}\right)-6x\right)}{16\sqrt{x^2\left(4x^2-3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

```
[Out] (x*(-6*x + 8*x^3 + 3*Sqrt[-3 + 4*x^2]*ArcTanh[(2*x)/Sqrt[-3 + 4*x^2]]))/(16
*Sqrt[x^2*(-3 + 4*x^2)])
```

IntegrateAlgebraic [A] time = 0.13, size = 51, normalized size = 1.13

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2\sqrt{4x^4-3x^2}}{4x^2-3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[-3*x^2 + 4*x^4]/8 + (3*ArcTanh[(2*Sqrt[-3*x^2 + 4*x^4])/(-3 + 4*x^2)])
/16
```

fricas [A] time = 0.86, size = 45, normalized size = 1.00

$$\frac{1}{8}\sqrt{4x^4-3x^2}-\frac{3}{16}\log\left(-\frac{2x^2-\sqrt{4x^4-3x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(4*x^4 - 3*x^2) - 3/16*log(-(2*x^2 - sqrt(4*x^4 - 3*x^2))/x)
```

giac [A] time = 0.17, size = 42, normalized size = 0.93

$$\frac{1}{8}\sqrt{4x^4-3x^2}-\frac{3}{32}\log\left(\left|-8x^2+4\sqrt{4x^4-3x^2}+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(4*x^4 - 3*x^2) - 3/32*log(abs(-8*x^2 + 4*sqrt(4*x^4 - 3*x^2) + 3))
```

maple [A] time = 0.01, size = 60, normalized size = 1.33

$$\frac{\sqrt{4x^2-3}\,\left(4\sqrt{4x^2-3}\,x+3\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-3}\right)\right)x}{32\sqrt{4x^4-3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4-3*x^2)^(1/2),x)
[Out] 1/32*x*(4*x^2-3)^(1/2)*(3*ln(x*4^(1/2)+(4*x^2-3)^(1/2))*4^(1/2)+4*x*(4*x^2-
3)^(1/2))/(4*x^4-3*x^2)^(1/2)
```

maxima [A] time = 3.04, size = 41, normalized size = 0.91

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{32}\log\left(8x^2+4\sqrt{4x^4-3x^2}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/8*sqrt(4*x^4 - 3*x^2) + 3/32*log(8*x^2 + 4*sqrt(4*x^4 - 3*x^2) - 3)
```

mupad [B] time = 4.46, size = 40, normalized size = 0.89

$$\frac{3\ln\left(\frac{\sqrt{4x^2-3}\,\sqrt{x^2}}{2}+x^2-\frac{3}{8}\right)}{32}+\frac{\sqrt{4x^4-3x^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4 - 3*x^2)^(1/2),x)
```

```
[Out] (3*log(((4*x^2 - 3)^(1/2)*(x^2)^(1/2))/2 + x^2 - 3/8))/32 + (4*x^4 - 3*x^2)
^(1/2)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2 \left(4x^2 - 3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**4-3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(4*x**2 - 3)), x)
```

3.172 $\int \frac{x^3}{\sqrt{ax^2+bx^4}} \, dx$

Optimal. Leaf size=58

$$\frac{\sqrt{ax^2+bx^4}}{2b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 620, 206}

$$\frac{\sqrt{ax^2+bx^4}}{2b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^4]/(2*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^2 + b*x^4]])/
(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{ax+bx^2}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{ax^2+bx^4}}{2b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\sqrt{ax+bx^2}}\,dx, x, x^2\right)}{4b}\\
&= \frac{\sqrt{ax^2+bx^4}}{2b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^2+bx^4}}\right)}{2b}\\
&= \frac{\sqrt{ax^2+bx^4}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.26

$$\frac{x\left(\sqrt{b}\,x\left(a+bx^2\right)-a\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^4],x]
[Out] (x*(Sqrt[b]*x*(a + b*x^2) - a*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]]))/(2*b^(3/2)*Sqrt[x^2*(a + b*x^2)])
```

IntegrateAlgebraic [A] time = 0.20, size = 68, normalized size = 1.17

$$\frac{a\log\left(-2b^{3/2}\sqrt{ax^2+bx^4}+ab+2b^2x^2\right)}{4b^{3/2}}+\frac{\sqrt{ax^2+bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a*x^2 + b*x^4],x]
[Out] Sqrt[a*x^2 + b*x^4]/(2*b) + (a*Log[a*b + 2*b^2*x^2 - 2*b^(3/2)*Sqrt[a*x^2 +
 b*x^4]])/(4*b^(3/2))
```

fricas [A] time = 0.88, size = 114, normalized size = 1.97

$$\left[\frac{a\sqrt{b}\log\left(-2\,bx^{2}-a+2\,\sqrt{bx^{4}+ax^{2}}\,\sqrt{b}\right)+2\,\sqrt{bx^{4}+ax^{2}}\,b}{4\,b^{2}},\ \frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^{4}+ax^{2}}\,\sqrt{-b}}{bx^{2}+a}\right)+\sqrt{bx^{4}+ax^{2}}\,b}{2\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-2*b*x^2 - a + 2*sqrt(b*x^4 + a*x^2)*sqrt(b)) + 2*sqrt(
b*x^4 + a*x^2)*b)/b^2, 1/2*(a*sqrt(-b)*arctan(sqrt(b*x^4 + a*x^2)*sqrt(-b)/
(b*x^2 + a)) + sqrt(b*x^4 + a*x^2)*b)/b^2]
```

giac [A] time = 0.20, size = 59, normalized size = 1.02

$$\frac{a\log\left(\left|-2\left(\sqrt{b}\,x^{2}-\sqrt{bx^{4}+ax^{2}}\right)\sqrt{b}\,-a\right|\right)}{4\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^{4}+ax^{2}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*a*log(abs(-2*(sqrt(b)*x^2 - sqrt(b*x^4 + a*x^2))*sqrt(b) - a))/b^(3/2)
+ 1/2*sqrt(b*x^4 + a*x^2)/b
```

maple [A] time = 0.01, size = 64, normalized size = 1.10

$$\frac{\sqrt{b\,x^{2}+a}\,\left(-ab\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)+\sqrt{b\,x^{2}+a}\,b^{\frac{3}{2}}x\right)x}{2\sqrt{b\,x^{4}+a\,x^{2}}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)*(x*(b*x^2+a)^(1/2)*b^(3/2)-a*ln(b^(1/2)*x+(b*x^2+a)^(
1/2))*b)/(b*x^4+a*x^2)^(1/2)/b^(5/2)
```

maxima [A] time = 1.42, size = 52, normalized size = 0.90

$$-\frac{a\log\left(2\,bx^{2}+a+2\,\sqrt{bx^{4}+ax^{2}}\,\sqrt{b}\right)}{4\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^{4}+ax^{2}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*log(2*b*x^2 + a + 2*sqrt(b*x^4 + a*x^2)*sqrt(b))/b^(3/2) + 1/2*sqrt(
b*x^4 + a*x^2)/b
```

mupad [B] time = 4.71, size = 53, normalized size = 0.91

$$\frac{\sqrt{b\,x^4+a\,x^2}}{2\,b}-\frac{a\,\ln\left(\frac{b\,x^2+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^4+a\,x^2}\right)}{4\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^4)^(1/2),x)
```

```
[Out] (a*x^2 + b*x^4)^(1/2)/(2*b) - (a*log((a/2 + b*x^2)/b^(1/2) + (a*x^2 + b*x^4
)^(1/2)))/(4*b^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x**2)), x)
```

3.173 $\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2018, 640, 620, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] -Sqrt[a*x^2 - b*x^4]/(2*b) + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a*x^2 - b*x^4]])/
(2*b^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{ax-bx^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{ax-bx^2}}\,dx,x,x^2\right)}{4b}\\ &= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{x^2}{\sqrt{ax^2-bx^4}}\right)}{2b}\\ &= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 1.28

$$\frac{x\left(\sqrt{b}\,x\left(bx^2-a\right)+a\sqrt{a-bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a-bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] (x*(Sqrt[b]*x*(-a + b*x^2) + a*Sqrt[a - b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(2*b^(3/2)*Sqrt[x^2*(a - b*x^2)])
```

IntegrateAlgebraic [B] time = 0.30, size = 146, normalized size = 2.43

$$\frac{a\sqrt{-b}\,\log\left(a^2+4abx^2-8\sqrt{-b}\,bx^2\sqrt{ax^2-bx^4}-8b^2x^4\right)}{8b^2}-\frac{a\tan^{-1}\left(\frac{2\sqrt{-b}\,\sqrt{b}\,x^2}{a}-\frac{2\sqrt{b}\,\sqrt{ax^2-bx^4}}{a}\right)}{4b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] -1/2*Sqrt[a*x^2 - b*x^4]/b - (a*ArcTan[(2*Sqrt[-b]*Sqrt[b]*x^2)/a - (2*Sqrt[b]*Sqrt[a*x^2 - b*x^4])/a])/(4*b^(3/2)) + (a*Sqrt[-b]*Log[a^2 + 4*a*b*x^2 - 8*b^2*x^4 - 8*Sqrt[-b]*b*x^2*Sqrt[a*x^2 - b*x^4]])/(8*b^2)
```

fricas [A] time = 0.92, size = 120, normalized size = 2.00

$$\left[-\frac{a\sqrt{-b}\log\left(2\,bx^2-a-2\,\sqrt{-bx^4+ax^2}\,\sqrt{-b}\right)+2\,\sqrt{-bx^4+ax^2}\,b}{4\,b^2},-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx^4+ax^2}\,\sqrt{b}}{bx^2-a}\right)+\sqrt{-bx^4+ax^2}\,b}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*sqrt(-b)*log(2*b*x^2 - a - 2*sqrt(-b*x^4 + a*x^2)*sqrt(-b)) + 2*sq
rt(-b*x^4 + a*x^2)*b)/b^2, -1/2*(a*sqrt(b)*arctan(sqrt(-b*x^4 + a*x^2)*sqrt
(b)/(b*x^2 - a)) + sqrt(-b*x^4 + a*x^2)*b)/b^2]
```

giac [A] time = 0.20, size = 68, normalized size = 1.13

$$-\frac{a\log\left(\left|2\left(\sqrt{-b}\,x^2-\sqrt{-bx^4+ax^2}\right)\sqrt{-b}\,+a\right|\right)}{4\,\sqrt{-b}\,b}-\frac{\sqrt{-bx^4+ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*a*log(abs(2*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a*x^2))*sqrt(-b) + a))/(sqrt
(-b)*b) - 1/2*sqrt(-b*x^4 + a*x^2)/b
```

maple [A] time = 0.01, size = 67, normalized size = 1.12

$$\frac{\sqrt{-b\,x^2+a}\,\left(ab\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+a}}\right)-\sqrt{-b\,x^2+a}\,b^{\frac{3}{2}}x\right)x}{2\sqrt{-b\,x^4+a\,x^2}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^4+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(-b*x^2+a)^(1/2)*(-x*(-b*x^2+a)^(1/2)*b^(3/2)+a*arctan(b^(1/2)*x/(-b*
x^2+a)^(1/2))*b)/(-b*x^4+a*x^2)^(1/2)/b^(5/2)
```

maxima [A] time = 3.03, size = 42, normalized size = 0.70

$$-\frac{a\arcsin\left(-\frac{2\,bx^2-a}{a}\right)}{4\,b^{\frac{3}{2}}}-\frac{\sqrt{-bx^4+ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*arcsin(-(2*b*x^2 - a)/a)/b^(3/2) - 1/2*sqrt(-b*x^4 + a*x^2)/b
```

mupad [B] time = 4.62, size = 60, normalized size = 1.00

$$-\frac{\sqrt{a\,x^2-b\,x^4}}{2\,b}-\frac{a\,\ln\left(\frac{\frac{a}{2}-b\,x^2}{\sqrt{-b}}+\sqrt{a\,x^2-b\,x^4}\right)}{4\,{\left(-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 - b*x^4)^(1/2),x)
```

```
[Out] - (a*x^2 - b*x^4)^(1/2)/(2*b) - (a*log((a/2 - b*x^2)/(-b)^(1/2) + (a*x^2 -
b*x^4)^(1/2)))/(4*(-b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-x^2\left(-a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(-a + b*x**2)), x)
```

3.174 $\int x^{7/2} \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=21

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(bx^2+cx^4\right)dx&=\int\left(bx^{11/2}+cx^{15/2}\right)dx\\&=\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{221}\left(17bx^{13/2}+13cx^{17/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(b*x^2 + c*x^4),x]
[Out] (2*(17*b*x^(13/2) + 13*c*x^(17/2)))/221
```

fricas [A] time = 1.56, size = 18, normalized size = 0.86

$$\frac{2}{221}\left(13\,cx^8+17\,bx^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/221*(13*c*x^8 + 17*b*x^6)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$\frac{2}{17}\,cx^{\frac{17}{2}}+\frac{2}{13}\,bx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(13c\,x^2+17b\right)x^{\frac{13}{2}}}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2),x)
[Out] 2/221*x^(13/2)*(13*c*x^2+17*b)
```

maxima [A] time = 1.30, size = 13, normalized size = 0.62

$$\frac{2}{17}\,cx^{\frac{17}{2}}+\frac{2}{13}\,bx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

mupad [B] time = 0.04, size = 15, normalized size = 0.71

$$\frac{2\,x^{13/2}\,\left(13\,c\,x^2+17\,b\right)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(13/2)*(17*b + 13*c*x^2))/221
```

sympy [A] time = 11.34, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{13}{2}}}{13}+\frac{2cx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(13/2)/13 + 2*c*x**(17/2)/17
```

3.175 $\int x^{5/2} \left(bx^2 + cx^4\right) dx$

Optimal. Leaf size=21

$$\frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^{5/2} \left(bx^2 + cx^4\right) dx &= \int \left(bx^{9/2} + cx^{13/2}\right) dx \\ &= \frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{165}\left(15bx^{11/2}+11cx^{15/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*(15*b*x^(11/2) + 11*c*x^(15/2)))/165
```

fricas [A] time = 1.02, size = 18, normalized size = 0.86

$$\frac{2}{165}\left(11\,cx^7+15\,bx^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/165*(11*c*x^7 + 15*b*x^5)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(11c\,x^2+15b\right)x^{\frac{11}{2}}}{165}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/165*x^(11/2)*(11*c*x^2+15*b)
```

maxima [A] time = 1.36, size = 13, normalized size = 0.62

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{11/2}\,\left(11\,c\,x^2+15\,b\right)}{165}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4),x)
[Out] (2*x^(11/2)*(15*b + 11*c*x^2))/165
```

sympy [A] time = 5.57, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{11}{2}}}{11}+\frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2),x)
[Out] 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

3.176 $\int x^{3/2} \left(bx^2+cx^4\right) \, dx$

Optimal. Leaf size=21

$$\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2} \left(bx^2+cx^4\right) \, dx &= \int \left(bx^{7/2}+cx^{11/2}\right) \, dx\\ &= \frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{117}\left(13bx^{9/2}+9cx^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(b*x^2 + c*x^4),x]
[Out] (2*(13*b*x^(9/2) + 9*c*x^(13/2)))/117
```

fricas [A] time = 1.84, size = 18, normalized size = 0.86

$$\frac{2}{117}\left(9\,cx^{6}+13\,bx^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/117*(9*c*x^6 + 13*b*x^4)*sqrt(x)
```

giac [A] time = 0.16, size = 13, normalized size = 0.62

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(9c\,x^{2}+13b\right)x^{\frac{9}{2}}}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2),x)
[Out] 2/117*x^(9/2)*(9*c*x^2+13*b)
```

maxima [A] time = 1.34, size = 13, normalized size = 0.62

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{9/2}\,\left(9\,c\,x^2+13\,b\right)}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4),x)
[Out] (2*x^(9/2)*(13*b + 9*c*x^2))/117
```

sympy [A] time = 2.53, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{9}{2}}}{9} + \frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2),x)
[Out] 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

3.177 $\int \sqrt{x} \left(bx^2 + cx^4\right) dx$

Optimal. Leaf size=21

$$\frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x} \left(bx^2 + cx^4\right) dx &= \int \left(bx^{5/2} + cx^{9/2}\right) dx \\ &= \frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{77}\left(11bx^{7/2}+7cx^{11/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(b*x^2 + c*x^4),x]
[Out] (2*(11*b*x^(7/2) + 7*c*x^(11/2)))/77
```

fricas [A] time = 0.58, size = 18, normalized size = 0.86

$$\frac{2}{77}\left(7\,cx^5+11\,bx^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/77*(7*c*x^5 + 11*b*x^3)*sqrt(x)
```

giac [A] time = 0.21, size = 13, normalized size = 0.62

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(7c\,x^2+11b\right)x^{\frac{7}{2}}}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2),x)
[Out] 2/77*x^(7/2)*(7*c*x^2+11*b)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{7/2}\,\left(7\,c\,x^2+11\,b\right)}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(7/2)*(11*b + 7*c*x^2))/77
```

sympy [A] time = 1.73, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

3.178 $\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx &= \int \left(bx^{3/2}+cx^{7/2}\right)\,dx\\ &= \frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{45}\left(9bx^{5/2}+5cx^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/Sqrt[x],x]
[Out] (2*(9*b*x^(5/2) + 5*c*x^(9/2)))/45
```

fricas [A] time = 0.97, size = 18, normalized size = 0.86

$$\frac{2}{45}\left(5\,cx^{4}+9\,bx^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*c*x^4 + 9*b*x^2)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(5c\,x^{2}+9b\right)x^{\frac{5}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(1/2),x)
[Out] 2/45*x^(5/2)*(5*c*x^2+9*b)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{5/2}\,\left(5\,c\,x^2+9\,b\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(1/2),x)
[Out] (2*x^(5/2)*(9*b + 5*c*x^2))/45
```

sympy [A] time = 0.78, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{5}{2}}}{5}+\frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(1/2),x)
[Out] 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

3.179 $\int \frac{bx^2+cx^4}{x^{3/2}} \, dx$

Optimal. Leaf size=21

$$\frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2 + cx^4}{x^{3/2}} \, dx &= \int \left(b\sqrt{x} + cx^{5/2}\right) dx \\ &= \frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{21}\left(7bx^{3/2}+3cx^{7/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^(3/2),x]
[Out] (2*(7*b*x^(3/2) + 3*c*x^(7/2)))/21
```

fricas [A] time = 1.10, size = 16, normalized size = 0.76

$$\frac{2}{21}\left(3\,cx^3+7\,bx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
[Out] 2/21*(3*c*x^3 + 7*b*x)*sqrt(x)
```

giac [A] time = 0.20, size = 13, normalized size = 0.62

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(3c\,x^2+7b\right)x^{\frac{3}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(3/2),x)
[Out] 2/21*x^(3/2)*(3*c*x^2+7*b)
```

maxima [A] time = 1.30, size = 13, normalized size = 0.62

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{3/2}\,\left(3\,c\,x^2+7\,b\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(3/2),x)
```

```
[Out] (2*x^(3/2)*(7*b + 3*c*x^2))/21
```

sympy [A] time = 0.79, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{3}{2}}}{3} + \frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(3/2),x)
```

```
[Out] 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

3.180 $\int \frac{bx^2+cx^4}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^{5/2}}\,dx &= \int \left(\frac{b}{\sqrt{x}}+cx^{3/2}\right)dx\\ &= 2b\sqrt{x}+\frac{2}{5}cx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.05

$$\frac{2}{5}\left(5b\sqrt{x}+cx^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^(5/2),x]
[Out] (2*(5*b*Sqrt[x] + c*x^(5/2)))/5
```

fricas [A] time = 1.33, size = 14, normalized size = 0.74

$$\frac{2}{5}\left(cx^2+5\,b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
[Out] 2/5*(c*x^2 + 5*b)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.79

$$\frac{2\left(c\,x^2+5b\right)\sqrt{x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(5/2),x)
[Out] 2/5*x^(1/2)*(c*x^2+5*b)
```

maxima [A] time = 1.31, size = 13, normalized size = 0.68

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.74

$$\frac{2\sqrt{x}\,\left(c\,x^2+5\,b\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(5/2),x)
```

```
[Out] (2*x^(1/2)*(5*b + c*x^2))/5
```

sympy [A] time = 1.00, size = 17, normalized size = 0.89

$$2b\sqrt{x} + \frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(5/2),x)
```

```
[Out] 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

3.181 $\int \frac{bx^2+cx^4}{x^{7/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2}{3}cx^{3/2}-\frac{2b}{\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{3}cx^{3/2}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(7/2),x]
[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^{7/2}}\,dx &= \int \left(\frac{b}{x^{3/2}}+c\sqrt{x}\right)dx\\ &= -\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{3}cx^{3/2}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(7/2),x]
[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 0.95

$$\frac{2\left(cx^2-3b\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)/x^(7/2),x]
[Out] (2*(-3*b + c*x^2))/(3*Sqrt[x])
```

fricas [A] time = 0.91, size = 14, normalized size = 0.74

$$\frac{2\left(cx^2-3\,b\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
[Out] 2/3*(c*x^2 - 3*b)/sqrt(x)
```

giac [A] time = 0.16, size = 13, normalized size = 0.68

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2\left(-c\,x^2+3b\right)}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(7/2),x)
[Out] -2/3/x^(1/2)*(-c*x^2+3*b)
```

maxima [A] time = 1.34, size = 13, normalized size = 0.68

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{6\,b-2\,c\,x^2}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(7/2),x)
[Out] -(6*b - 2*c*x^2)/(3*x^(1/2))
```

sympy [A] time = 1.80, size = 17, normalized size = 0.89

$$-\frac{2b}{\sqrt{x}}+\frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(7/2),x)
[Out] -2*b/sqrt(x) + 2*c*x**(3/2)/3
```

3.182 $\int x^{7/2} \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(17/2))/17 + (4*b*c*x^(21/2))/21 + (2*c^2*x^(25/2))/25
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx &= \int x^{15/2}\left(b+cx^2\right)^2\,dx\\
&= \int \left(b^2x^{15/2}+2bcx^{19/2}+c^2x^{23/2}\right)dx\\
&= \frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{17/2}\left(525b^2+850bcx^2+357c^2x^4\right)}{8925}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(17/2)*(525*b^2 + 850*b*c*x^2 + 357*c^2*x^4))/8925
```

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{2\left(525b^2x^{17/2} + 850bcx^{21/2} + 357c^2x^{25/2}\right)}{8925}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*(525*b^2*x^(17/2) + 850*b*c*x^(21/2) + 357*c^2*x^(25/2)))/8925
```

fricas [A] time = 2.05, size = 29, normalized size = 0.81

$$\frac{2}{8925}\left(357\,c^2x^{12} + 850\,bcx^{10} + 525\,b^2x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 2/8925*(357*c^2*x^12 + 850*b*c*x^10 + 525*b^2*x^8)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{25}\,c^2x^{\frac{25}{2}} + \frac{4}{21}\,bcx^{\frac{21}{2}} + \frac{2}{17}\,b^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(357c^2x^4 + 850bc\,x^2 + 525b^2\right)x^{\frac{17}{2}}}{8925}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^2,x)
```

[Out] 2/8925*x^(17/2)*(357*c^2*x^4+850*b*c*x^2+525*b^2)

maxima [A] time = 1.28, size = 24, normalized size = 0.67

$$\frac{2}{25}\,c^2x^{\frac{25}{2}} + \frac{4}{21}\,bcx^{\frac{21}{2}} + \frac{2}{17}\,b^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")

[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)

mupad [B] time = 0.05, size = 25, normalized size = 0.69

$$x^{17/2}\left(\frac{2\,b^2}{17}+\frac{4\,b\,c\,x^2}{21}+\frac{2\,c^2\,x^4}{25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(b*x^2 + c*x^4)^2,x)

[Out] x^(17/2)*((2*b^2)/17 + (2*c^2*x^4)/25 + (4*b*c*x^2)/21)

sympy [A] time = 35.14, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{17}{2}}}{17} + \frac{4bcx^{\frac{21}{2}}}{21} + \frac{2c^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(7/2)*(c*x**4+b*x**2)**2,x)

[Out] 2*b**2*x**(17/2)/17 + 4*b*c*x**(21/2)/21 + 2*c**2*x**(25/2)/25

3.183 $\int x^{5/2} \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(15/2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx &= \int x^{13/2}\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2x^{13/2}+2bcx^{17/2}+c^2x^{21/2}\right)dx\\ &= \frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{15/2}\left(437b^2+690bcx^2+285c^2x^4\right)}{6555}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(15/2)*(437*b^2 + 690*b*c*x^2 + 285*c^2*x^4))/6555
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 0.94

$$\frac{2\left(437b^2x^{15/2}+690bcx^{19/2}+285c^2x^{23/2}\right)}{6555}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(437*b^2*x^(15/2) + 690*b*c*x^(19/2) + 285*c^2*x^(23/2)))/6555
```

fricas [A] time = 0.82, size = 29, normalized size = 0.81

$$\frac{2}{6555}\left(285\,c^2x^{11}+690\,bcx^9+437\,b^2x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/6555*(285*c^2*x^11 + 690*b*c*x^9 + 437*b^2*x^7)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(285c^2x^4+690bc\,x^2+437b^2\right)x^{\frac{15}{2}}}{6555}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/6555*x^(15/2)*(285*c^2*x^4+690*b*c*x^2+437*b^2)
```

maxima [A] time = 1.35, size = 24, normalized size = 0.67

$$\frac{2}{23}\,c^{2}x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^{2}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.69

$$x^{15/2}\left(\frac{2\,b^2}{15}+\frac{4\,b\,c\,x^2}{19}+\frac{2\,c^2\,x^4}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4)^2,x)
[Out] x^(15/2)*((2*b^2)/15 + (2*c^2*x^4)/23 + (4*b*c*x^2)/19)
```

sympy [A] time = 20.81, size = 34, normalized size = 0.94

$$\frac{2b^{2}x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^{2}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**2,x)
[Out] 2*b**2*x**(15/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

3.184 $\int x^{3/2} \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(13/2))/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx&=\int x^{11/2}\left(b+cx^2\right)^2\,dx\\&=\int\left(b^2x^{11/2}+2bcx^{15/2}+c^2x^{19/2}\right)dx\\&=\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{13/2}\left(357b^2+546bcx^2+221c^2x^4\right)}{4641}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(13/2)*(357*b^2 + 546*b*c*x^2 + 221*c^2*x^4))/4641
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 0.94

$$\frac{2\left(357b^2x^{13/2}+546bcx^{17/2}+221c^2x^{21/2}\right)}{4641}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(357*b^2*x^(13/2) + 546*b*c*x^(17/2) + 221*c^2*x^(21/2)))/4641
```

fricas [A] time = 2.35, size = 29, normalized size = 0.81

$$\frac{2}{4641}\left(221\,c^2x^{10}+546\,bcx^8+357\,b^2x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/4641*(221*c^2*x^10 + 546*b*c*x^8 + 357*b^2*x^6)*sqrt(x)
```

giac [A] time = 0.16, size = 24, normalized size = 0.67

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(221c^2x^4+546bc\,x^2+357b^2\right)x^{\frac{13}{2}}}{4641}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/4641*x^(13/2)*(221*c^2*x^4+546*b*c*x^2+357*b^2)
```

maxima [A] time = 1.23, size = 24, normalized size = 0.67

$$\frac{2}{21}\,c^2x^{\frac{21}{2}} + \frac{4}{17}\,bcx^{\frac{17}{2}} + \frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

mupad [B] time = 4.27, size = 25, normalized size = 0.69

$$x^{13/2}\left(\frac{2\,b^2}{13}+\frac{4\,b\,c\,x^2}{17}+\frac{2\,c^2\,x^4}{21}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(13/2)*((2*b^2)/13 + (2*c^2*x^4)/21 + (4*b*c*x^2)/17)
```

sympy [A] time = 11.38, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{13}{2}}}{13} + \frac{4bcx^{\frac{17}{2}}}{17} + \frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(13/2)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

3.185 $\int \sqrt{x} \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(11/2))/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(bx^2+cx^4\right)^2 dx &= \int x^{9/2}\left(b+cx^2\right)^2 dx\\ &= \int \left(b^2x^{9/2}+2bcx^{13/2}+c^2x^{17/2}\right) dx\\ &= \frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{11/2}\left(285b^2+418bcx^2+165c^2x^4\right)}{3135}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(11/2)*(285*b^2 + 418*b*c*x^2 + 165*c^2*x^4))/3135
```

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{2\left(285b^2x^{11/2}+418bcx^{15/2}+165c^2x^{19/2}\right)}{3135}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
[Out] (2*(285*b^2*x^(11/2) + 418*b*c*x^(15/2) + 165*c^2*x^(19/2)))/3135
```

fricas [A] time = 1.51, size = 29, normalized size = 0.81

$$\frac{2}{3135}\left(165\,c^2x^9+418\,bcx^7+285\,b^2x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 2/3135*(165*c^2*x^9 + 418*b*c*x^7 + 285*b^2*x^5)*sqrt(x)
```

giac [A] time = 0.14, size = 24, normalized size = 0.67

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(165c^2x^4+418bc\,x^2+285b^2\right)x^{\frac{11}{2}}}{3135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/3135*x^(11/2)*(165*c^2*x^4+418*b*c*x^2+285*b^2)
```

maxima [A] time = 1.36, size = 24, normalized size = 0.67

$$\frac{2}{19}\,c^{2}x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^{2}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.69

$$x^{11/2}\left(\frac{2\,b^2}{11}+\frac{4\,b\,c\,x^2}{15}+\frac{2\,c^2\,x^4}{19}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4)^2,x)
[Out] x^(11/2)*((2*b^2)/11 + (2*c^2*x^4)/19 + (4*b*c*x^2)/15)
```

sympy [A] time = 2.75, size = 34, normalized size = 0.94

$$\frac{2b^{2}x^{\frac{11}{2}}}{11}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^{2}x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**2,x)
[Out] 2*b**2*x**(11/2)/11 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2)/19
```

3.186 $\int \frac{(bx^2+cx^4)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{9}b^2x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{9}b^2x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] (2*b^2*x^(9/2))/9 + (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{\sqrt{x}}\,dx &= \int x^{7/2}\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2x^{7/2}+2bcx^{11/2}+c^2x^{15/2}\right)dx\\ &= \frac{2}{9}b^2x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{9/2}\left(221b^2+306bcx^2+117c^2x^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/Sqrt[x],x]
[Out] (2*x^(9/2)*(221*b^2 + 306*b*c*x^2 + 117*c^2*x^4))/1989
```

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{2\left(221b^2x^{9/2}+306bcx^{13/2}+117c^2x^{17/2}\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/Sqrt[x],x]
[Out] (2*(221*b^2*x^(9/2) + 306*b*c*x^(13/2) + 117*c^2*x^(17/2)))/1989
```

fricas [A] time = 1.31, size = 29, normalized size = 0.81

$$\frac{2}{1989}\left(117\,c^2x^8+306\,bcx^6+221\,b^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/1989*(117*c^2*x^8 + 306*b*c*x^6 + 221*b^2*x^4)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(117c^2x^4+306bc\,x^2+221b^2\right)x^{\frac{9}{2}}}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(1/2),x)
[Out] 2/1989*x^(9/2)*(117*c^2*x^4+306*b*c*x^2+221*b^2)
```

maxima [A] time = 1.25, size = 24, normalized size = 0.67

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.69

$$x^{9/2}\left(\frac{2\,b^2}{9}+\frac{4\,b\,c\,x^2}{13}+\frac{2\,c^2\,x^4}{17}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(1/2),x)
[Out] x^(9/2)*((2*b^2)/9 + (2*c^2*x^4)/17 + (4*b*c*x^2)/13)
```

sympy [A] time = 4.98, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{9}{2}}}{9}+\frac{4bcx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(1/2),x)
[Out] 2*b**2*x**(9/2)/9 + 4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

3.187 $\int \frac{\left(bx^2+cx^4\right)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*b^2*x^(7/2))/7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^2}{x^{3/2}}\,dx &= \int x^{5/2}\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2x^{5/2}+2bcx^{9/2}+c^2x^{13/2}\right)dx\\ &= \frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}\left(165b^2+210bcx^2+77c^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(3/2),x]
[Out] (2*x^(7/2)*(165*b^2 + 210*b*c*x^2 + 77*c^2*x^4))/1155
```

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{2\left(165b^2x^{7/2} + 210bcx^{11/2} + 77c^2x^{15/2}\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^(3/2),x]
[Out] (2*(165*b^2*x^(7/2) + 210*b*c*x^(11/2) + 77*c^2*x^(15/2)))/1155
```

fricas [A] time = 1.59, size = 29, normalized size = 0.81

$$\frac{2}{1155}\left(77\,c^2x^7 + 210\,bcx^5 + 165\,b^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")
[Out] 2/1155*(77*c^2*x^7 + 210*b*c*x^5 + 165*b^2*x^3)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{11}\,bcx^{\frac{11}{2}} + \frac{2}{7}\,b^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(77c^2x^4 + 210bc\,x^2 + 165b^2\right)x^{\frac{7}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(3/2),x)
```

```
[Out] 2/1155*x^(7/2)*(77*c^2*x^4+210*b*c*x^2+165*b^2)
```

maxima [A] time = 1.33, size = 24, normalized size = 0.67

$$\frac{2}{15}\,c^{2}x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^{2}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

mupad [B] time = 4.44, size = 26, normalized size = 0.72

$$\frac{2\,x^{7/2}\,\left(165\,b^2+210\,b\,c\,x^2+77\,c^2\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(3/2),x)
[Out] (2*x^(7/2)*(165*b^2 + 77*c^2*x^4 + 210*b*c*x^2))/1155
```

sympy [A] time = 5.22, size = 34, normalized size = 0.94

$$\frac{2b^{2}x^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{11}{2}}}{11}+\frac{2c^{2}x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(3/2),x)
[Out] 2*b**2*x**(7/2)/7 + 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

3.188 $\int \frac{\left(bx^2+cx^4\right)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (2*b^2*x^(5/2))/5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^2}{x^{5/2}}\,dx &= \int x^{3/2}\left(b+cx^2\right)^2\,dx\\
&= \int \left(b^2x^{3/2}+2bcx^{7/2}+c^2x^{11/2}\right)dx\\
&= \frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2}\left(117b^2+130bcx^2+45c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(5/2),x]
[Out] (2*x^(5/2)*(117*b^2 + 130*b*c*x^2 + 45*c^2*x^4))/585
```

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{2}{585}\left(117b^2x^{5/2}+130bcx^{9/2}+45c^2x^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^(5/2),x]
[Out] (2*(117*b^2*x^(5/2) + 130*b*c*x^(9/2) + 45*c^2*x^(13/2)))/585
```

fricas [A] time = 1.46, size = 29, normalized size = 0.81

$$\frac{2}{585}\left(45\,c^2x^6+130\,bcx^4+117\,b^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
[Out] 2/585*(45*c^2*x^6 + 130*b*c*x^4 + 117*b^2*x^2)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,b^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(45c^2x^4+130bc\,x^2+117b^2\right)x^{\frac{5}{2}}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(5/2),x)
[Out] 2/585*x^(5/2)*(45*c^2*x^4+130*b*c*x^2+117*b^2)
```

maxima [A] time = 1.35, size = 24, normalized size = 0.67

$$\frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\,b^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{5/2}\,\left(117\,b^2+130\,b\,c\,x^2+45\,c^2\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(5/2),x)
[Out] (2*x^(5/2)*(117*b^2 + 45*c^2*x^4 + 130*b*c*x^2))/585
```

sympy [A] time = 5.99, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{5}{2}}}{5} + \frac{4bcx^{\frac{9}{2}}}{9} + \frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(5/2),x)
[Out] 2*b**2*x**(5/2)/5 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

3.189 $\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{3}b^2x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{3}b^2x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*b^2*x^(3/2))/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx &= \int \sqrt{x}\,(b+cx^2)^2\,dx\\ &= \int \left(b^2\sqrt{x}+2bcx^{5/2}+c^2x^{9/2}\right)dx\\ &= \frac{2}{3}b^2x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2}\left(77b^2+66bcx^2+21c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(7/2),x]
[Out] (2*x^(3/2)*(77*b^2 + 66*b*c*x^2 + 21*c^2*x^4))/231
```

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{2}{231}\left(77b^2x^{3/2}+66bcx^{7/2}+21c^2x^{11/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^2/x^(7/2),x]
[Out] (2*(77*b^2*x^(3/2) + 66*b*c*x^(7/2) + 21*c^2*x^(11/2)))/231
```

fricas [A] time = 0.72, size = 27, normalized size = 0.75

$$\frac{2}{231}\left(21\,c^2x^5+66\,bcx^3+77\,b^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
[Out] 2/231*(21*c^2*x^5 + 66*b*c*x^3 + 77*b^2*x)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\,b^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(21c^2x^4+66bc\,x^2+77b^2\right)x^{\frac{3}{2}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(7/2),x)
[Out] 2/231*x^(3/2)*(21*c^2*x^4+66*b*c*x^2+77*b^2)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.67

$$\frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,b^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.72

$$\frac{2\,x^{3/2}\,\left(77\,b^2+66\,b\,c\,x^2+21\,c^2\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(7/2),x)
[Out] (2*x^(3/2)*(77*b^2 + 21*c^2*x^4 + 66*b*c*x^2))/231
```

sympy [A] time = 8.69, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{3}{2}}}{3} + \frac{4bcx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(7/2),x)
[Out] 2*b**2*x**(3/2)/3 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

3.190 $\int x^{7/2} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{25}b^2cx^{25/2}+\frac{2}{21}b^3x^{21/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c^
3*x^(33/2))/33
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{7/2}\left(bx^2+cx^4\right)^3 \, dx &= \int x^{19/2}\left(b+cx^2\right)^3 \, dx \\
&= \int \left(b^3x^{19/2}+3b^2cx^{23/2}+3bc^2x^{27/2}+c^3x^{31/2}\right) dx \\
&= \frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c^
3*x^(33/2))/33
```

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 0.92

$$\frac{2\left(7975b^3x^{21/2}+20097b^2cx^{25/2}+17325bc^2x^{29/2}+5075c^3x^{33/2}\right)}{167475}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(7975*b^3*x^(21/2) + 20097*b^2*c*x^(25/2) + 17325*b*c^2*x^(29/2) + 5075*
c^3*x^(33/2)))/167475
```

fricas [A] time = 1.32, size = 40, normalized size = 0.78

$$\frac{2}{167475}\left(5075\,c^3x^{16}+17325\,bc^2x^{14}+20097\,b^2cx^{12}+7975\,b^3x^{10}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/167475*(5075*c^3*x^16 + 17325*b*c^2*x^14 + 20097*b^2*c*x^12 + 7975*b^3*x^
10)*sqrt(x)
```

giac [A] time = 0.18, size = 35, normalized size = 0.69

$$\frac{2}{33}\,c^3x^{\frac{33}{2}}+\frac{6}{29}\,bc^2x^{\frac{29}{2}}+\frac{6}{25}\,b^2cx^{\frac{25}{2}}+\frac{2}{21}\,b^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x^
(21/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(5075c^{3}x^{6}+17325b\,c^{2}x^{4}+20097b^{2}c\,x^{2}+7975b^{3}\right)x^{\frac{21}{2}}}{167475}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/167475*x^(21/2)*(5075*c^3*x^6+17325*b*c^2*x^4+20097*b^2*c*x^2+7975*b^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{33}\,c^{3}x^{\frac{33}{2}}+\frac{6}{29}\,bc^{2}x^{\frac{29}{2}}+\frac{6}{25}\,b^{2}cx^{\frac{25}{2}}+\frac{2}{21}\,b^{3}x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x^
(21/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^{3}\,x^{21/2}}{21}+\frac{2\,c^{3}\,x^{33/2}}{33}+\frac{6\,b^{2}\,c\,x^{25/2}}{25}+\frac{6\,b\,c^{2}\,x^{29/2}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(21/2))/21 + (2*c^3*x^(33/2))/33 + (6*b^2*c*x^(25/2))/25 + (6*b*c^
2*x^(29/2))/29
```

sympy [A] time = 83.18, size = 49, normalized size = 0.96

$$\frac{2b^{3}x^{\frac{21}{2}}}{21}+\frac{6b^{2}cx^{\frac{25}{2}}}{25}+\frac{6bc^{2}x^{\frac{29}{2}}}{29}+\frac{2c^{3}x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(21/2)/21 + 6*b**2*c*x**(25/2)/25 + 6*b*c**2*x**(29/2)/29 + 2*c**
3*x**(33/2)/33
```

3.191 $\int x^{5/2} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{23}b^2cx^{23/2}+\frac{2}{19}b^3x^{19/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3
*x^(31/2))/31
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx &= \int x^{17/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{17/2}+3b^2cx^{21/2}+3bc^2x^{25/2}+c^3x^{29/2}\right)dx\\ &= \frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3
*x^(31/2))/31
```

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 0.92

$$\frac{2\left(6417b^3x^{19/2}+15903b^2cx^{23/2}+13547bc^2x^{27/2}+3933c^3x^{31/2}\right)}{121923}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(6417*b^3*x^(19/2) + 15903*b^2*c*x^(23/2) + 13547*b*c^2*x^(27/2) + 3933*
c^3*x^(31/2)))/121923
```

fricas [A] time = 0.96, size = 40, normalized size = 0.78

$$\frac{2}{121923}\left(3933\,c^3x^{15}+13547\,bc^2x^{13}+15903\,b^2cx^{11}+6417\,b^3x^9\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/121923*(3933*c^3*x^15 + 13547*b*c^2*x^13 + 15903*b^2*c*x^11 + 6417*b^3*x^
9)*sqrt(x)
```

giac [A] time = 0.17, size = 35, normalized size = 0.69

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^(
19/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(3933c^{3}x^{6}+13547b\,c^{2}x^{4}+15903b^{2}c\,x^{2}+6417b^{3}\right)x^{\frac{19}{2}}}{121923}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/121923*x^(19/2)*(3933*c^3*x^6+13547*b*c^2*x^4+15903*b^2*c*x^2+6417*b^3)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.69

$$\frac{2}{31}\,c^{3}x^{\frac{31}{2}}+\frac{2}{9}\,bc^{2}x^{\frac{27}{2}}+\frac{6}{23}\,b^{2}cx^{\frac{23}{2}}+\frac{2}{19}\,b^{3}x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^(
19/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^{3}\,x^{19/2}}{19}+\frac{2\,c^{3}\,x^{31/2}}{31}+\frac{6\,b^{2}\,c\,x^{23/2}}{23}+\frac{2\,b\,c^{2}\,x^{27/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(19/2))/19 + (2*c^3*x^(31/2))/31 + (6*b^2*c*x^(23/2))/23 + (2*b*c^
2*x^(27/2))/9
```

sympy [A] time = 53.38, size = 49, normalized size = 0.96

$$\frac{2b^{3}x^{\frac{19}{2}}}{19}+\frac{6b^{2}cx^{\frac{23}{2}}}{23}+\frac{2bc^{2}x^{\frac{27}{2}}}{9}+\frac{2c^{3}x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(19/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**3
*x**(31/2)/31
```

3.192 $\int x^{3/2} \left(bx^2 + cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{17}b^3x^{17/2} + \frac{2}{7}b^2cx^{21/2} + \frac{6}{25}bc^2x^{25/2} + \frac{2}{29}c^3x^{29/2}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{7}b^2cx^{21/2} + \frac{2}{17}b^3x^{17/2} + \frac{6}{25}bc^2x^{25/2} + \frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3
*x^(29/2))/29
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2} \left(bx^2 + cx^4\right)^3 \, dx &= \int x^{15/2} \left(b + cx^2\right)^3 \, dx \\
&= \int \left(b^3x^{15/2} + 3b^2cx^{19/2} + 3bc^2x^{23/2} + c^3x^{27/2}\right) dx \\
&= \frac{2}{17}b^3x^{17/2} + \frac{2}{7}b^2cx^{21/2} + \frac{6}{25}bc^2x^{25/2} + \frac{2}{29}c^3x^{29/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$\frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3
*x^(29/2))/29
```

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 0.92

$$\frac{2\left(5075b^3x^{17/2}+12325b^2cx^{21/2}+10353bc^2x^{25/2}+2975c^3x^{29/2}\right)}{86275}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(5075*b^3*x^(17/2) + 12325*b^2*c*x^(21/2) + 10353*b*c^2*x^(25/2) + 2975*
c^3*x^(29/2)))/86275
```

fricas [A] time = 0.94, size = 40, normalized size = 0.78

$$\frac{2}{86275}\left(2975\,c^3x^{14}+10353\,bc^2x^{12}+12325\,b^2cx^{10}+5075\,b^3x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/86275*(2975*c^3*x^14 + 10353*b*c^2*x^12 + 12325*b^2*c*x^10 + 5075*b^3*x^8
)*sqrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^(
17/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(2975c^3x^6+10353b\,c^2x^4+12325b^2c\,x^2+5075b^3\right)x^{\frac{17}{2}}}{86275}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/86275*x^(17/2)*(2975*c^3*x^6+10353*b*c^2*x^4+12325*b^2*c*x^2+5075*b^3)
```

maxima [A] time = 1.31, size = 35, normalized size = 0.69

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^(
17/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{17/2}}{17}+\frac{2\,c^3\,x^{29/2}}{29}+\frac{2\,b^2\,c\,x^{21/2}}{7}+\frac{6\,b\,c^2\,x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*c^3*x^(29/2))/29 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2
*x^(25/2))/25
```

sympy [A] time = 33.34, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(17/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**3
*x**(29/2)/29
```

3.193 $\int \sqrt{x} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{19}b^2cx^{19/2}+\frac{2}{15}b^3x^{15/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(15/2))/15 + (6*b^2*c*x^(19/2))/19 + (6*b*c^2*x^(23/2))/23 + (2*c^
3*x^(27/2))/27
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(bx^2+cx^4\right)^3 dx &= \int x^{13/2}\left(b+cx^2\right)^3 dx\\ &= \int \left(b^3x^{13/2}+3b^2cx^{17/2}+3bc^2x^{21/2}+c^3x^{25/2}\right) dx\\ &= \frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{15/2}\left(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6\right)}{58995}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(15/2)*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6))/58
995
```

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 0.92

$$\frac{2\left(3933b^3x^{15/2}+9315b^2cx^{19/2}+7695bc^2x^{23/2}+2185c^3x^{27/2}\right)}{58995}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(3933*b^3*x^(15/2) + 9315*b^2*c*x^(19/2) + 7695*b*c^2*x^(23/2) + 2185*c^
3*x^(27/2)))/58995
```

fricas [A] time = 1.86, size = 40, normalized size = 0.78

$$\frac{2}{58995}\left(2185\,c^3x^{13}+7695\,bc^2x^{11}+9315\,b^2cx^9+3933\,b^3x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/58995*(2185*c^3*x^13 + 7695*b*c^2*x^11 + 9315*b^2*c*x^9 + 3933*b^3*x^7)*s
qrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x^
(15/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(2185c^{3}x^{6}+7695b\,c^{2}x^{4}+9315b^{2}c\,x^{2}+3933b^{3}\right)x^{\frac{15}{2}}}{58995}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/58995*x^(15/2)*(2185*c^3*x^6+7695*b*c^2*x^4+9315*b^2*c*x^2+3933*b^3)
```

maxima [A] time = 1.33, size = 35, normalized size = 0.69

$$\frac{2}{27}\,c^{3}x^{\frac{27}{2}}+\frac{6}{23}\,bc^{2}x^{\frac{23}{2}}+\frac{6}{19}\,b^{2}cx^{\frac{19}{2}}+\frac{2}{15}\,b^{3}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x^
(15/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^{3}\,x^{15/2}}{15}+\frac{2\,c^{3}\,x^{27/2}}{27}+\frac{6\,b^{2}\,c\,x^{19/2}}{19}+\frac{6\,b\,c^{2}\,x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(15/2))/15 + (2*c^3*x^(27/2))/27 + (6*b^2*c*x^(19/2))/19 + (6*b*c^
2*x^(23/2))/23
```

sympy [A] time = 4.32, size = 49, normalized size = 0.96

$$\frac{2b^{3}x^{\frac{15}{2}}}{15}+\frac{6b^{2}cx^{\frac{19}{2}}}{19}+\frac{6bc^{2}x^{\frac{23}{2}}}{23}+\frac{2c^{3}x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(15/2)/15 + 6*b**2*c*x**(19/2)/19 + 6*b*c**2*x**(23/2)/23 + 2*c**
3*x**(27/2)/27
```

3.194 $\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{17}b^2cx^{17/2}+\frac{2}{13}b^3x^{13/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*b^3*x^(13/2))/13 + (6*b^2*c*x^(17/2))/17 + (2*b*c^2*x^(21/2))/7 + (2*c^3
*x^(25/2))/25
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx &= \int x^{11/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{11/2}+3b^2cx^{15/2}+3bc^2x^{19/2}+c^3x^{23/2}\right)dx\\ &= \frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{13/2}\left(2975b^3+6825b^2cx^2+5525bc^2x^4+1547c^3x^6\right)}{38675}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*x^(13/2)*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6))/38
675
```

IntegrateAlgebraic [A] time = 0.02, size = 47, normalized size = 0.92

$$\frac{2\left(2975b^3x^{13/2}+6825b^2cx^{17/2}+5525bc^2x^{21/2}+1547c^3x^{25/2}\right)}{38675}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*(2975*b^3*x^(13/2) + 6825*b^2*c*x^(17/2) + 5525*b*c^2*x^(21/2) + 1547*c^
3*x^(25/2)))/38675
```

fricas [A] time = 0.69, size = 40, normalized size = 0.78

$$\frac{2}{38675}\left(1547\,c^3x^{12}+5525\,bc^2x^{10}+6825\,b^2cx^8+2975\,b^3x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/38675*(1547*c^3*x^12 + 5525*b*c^2*x^10 + 6825*b^2*c*x^8 + 2975*b^3*x^6)*s
qrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(1547c^3x^6+5525b\,c^2x^4+6825b^2c\,x^2+2975b^3\right)x^{\frac{13}{2}}}{38675}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] 2/38675*x^(13/2)*(1547*c^3*x^6+5525*b*c^2*x^4+6825*b^2*c*x^2+2975*b^3)
```

maxima [A] time = 1.31, size = 35, normalized size = 0.69

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{13/2}}{13}+\frac{2\,c^3\,x^{25/2}}{25}+\frac{6\,b^2\,c\,x^{17/2}}{17}+\frac{2\,b\,c^2\,x^{21/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(1/2),x)
```

```
[Out] (2*b^3*x^(13/2))/13 + (2*c^3*x^(25/2))/25 + (6*b^2*c*x^(17/2))/17 + (2*b*c^
2*x^(21/2))/7
```

sympy [A] time = 17.61, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{17}{2}}}{17}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 2*b**3*x**(13/2)/13 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**3
*x**(25/2)/25
```

3.195 $\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{11}b^3x^{11/2}+\frac{2}{5}b^2cx^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{5}b^2cx^{15/2}+\frac{2}{11}b^3x^{11/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*b^3*x^(11/2))/11 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2*x^(19/2))/19 + (2*c^3
*x^(23/2))/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx &= \int x^{9/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{9/2}+3b^2cx^{13/2}+3bc^2x^{17/2}+c^3x^{21/2}\right)dx\\ &= \frac{2}{11}b^3x^{11/2}+\frac{2}{5}b^2cx^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{11/2}\left(2185b^3+4807b^2cx^2+3795bc^2x^4+1045c^3x^6\right)}{24035}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*x^(11/2)*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^6))/24
035
```

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 0.92

$$\frac{2\left(2185b^3x^{11/2}+4807b^2cx^{15/2}+3795bc^2x^{19/2}+1045c^3x^{23/2}\right)}{24035}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*(2185*b^3*x^(11/2) + 4807*b^2*c*x^(15/2) + 3795*b*c^2*x^(19/2) + 1045*c^
3*x^(23/2)))/24035
```

fricas [A] time = 0.69, size = 40, normalized size = 0.78

$$\frac{2}{24035}\left(1045\,c^3x^{11}+3795\,bc^2x^9+4807\,b^2cx^7+2185\,b^3x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/24035*(1045*c^3*x^11 + 3795*b*c^2*x^9 + 4807*b^2*c*x^7 + 2185*b^3*x^5)*sq
rt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\,b^2cx^{\frac{15}{2}}+\frac{2}{11}\,b^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^(
11/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(1045c^{3}x^{6}+3795b\,c^{2}x^{4}+4807b^{2}c\,x^{2}+2185b^{3}\right)x^{\frac{11}{2}}}{24035}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(3/2),x)
```

```
[Out] 2/24035*x^(11/2)*(1045*c^3*x^6+3795*b*c^2*x^4+4807*b^2*c*x^2+2185*b^3)
```

maxima [A] time = 1.31, size = 35, normalized size = 0.69

$$\frac{2}{23}\,c^{3}x^{\frac{23}{2}}+\frac{6}{19}\,bc^{2}x^{\frac{19}{2}}+\frac{2}{5}\,b^{2}cx^{\frac{15}{2}}+\frac{2}{11}\,b^{3}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^(
11/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^{3}\,x^{11/2}}{11}+\frac{2\,c^{3}\,x^{23/2}}{23}+\frac{2\,b^{2}\,c\,x^{15/2}}{5}+\frac{6\,b\,c^{2}\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(3/2),x)
```

```
[Out] (2*b^3*x^(11/2))/11 + (2*c^3*x^(23/2))/23 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2
*x^(19/2))/19
```

sympy [A] time = 19.94, size = 49, normalized size = 0.96

$$\frac{2b^{3}x^{\frac{11}{2}}}{11}+\frac{2b^{2}cx^{\frac{15}{2}}}{5}+\frac{6bc^{2}x^{\frac{19}{2}}}{19}+\frac{2c^{3}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(3/2),x)
```

```
[Out] 2*b**3*x**(11/2)/11 + 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**3
*x**(23/2)/23
```

3.196 $\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{9}b^3x^{9/2}+\frac{6}{13}b^2cx^{13/2}+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{13}b^2cx^{13/2}+\frac{2}{9}b^3x^{9/2}+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*b^3*x^(9/2))/9 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*x^(17/2))/17 + (2*c^3*
x^(21/2))/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx &= \int x^{7/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{7/2}+3b^2cx^{11/2}+3bc^2x^{15/2}+c^3x^{19/2}\right)dx\\ &= \frac{2}{9}b^3x^{9/2}+\frac{6}{13}b^2cx^{13/2}+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{9/2}\left(1547b^3+3213b^2cx^2+2457bc^2x^4+663c^3x^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*x^(9/2)*(1547*b^3 + 3213*b^2*c*x^2 + 2457*b*c^2*x^4 + 663*c^3*x^6))/1392
3
```

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 0.92

$$\frac{2\left(1547b^3x^{9/2}+3213b^2cx^{13/2}+2457bc^2x^{17/2}+663c^3x^{21/2}\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*(1547*b^3*x^(9/2) + 3213*b^2*c*x^(13/2) + 2457*b*c^2*x^(17/2) + 663*c^3*
x^(21/2)))/13923
```

fricas [A] time = 0.86, size = 40, normalized size = 0.78

$$\frac{2}{13923}\left(663\,c^3x^{10}+2457\,bc^2x^8+3213\,b^2cx^6+1547\,b^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*c^3*x^10 + 2457*b*c^2*x^8 + 3213*b^2*c*x^6 + 1547*b^3*x^4)*sqr
t(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^(
9/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3\right)x^{\frac{9}{2}}}{13923}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(5/2),x)
```

```
[Out] 2/13923*x^(9/2)*(663*c^3*x^6+2457*b*c^2*x^4+3213*b^2*c*x^2+1547*b^3)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.69

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^(
9/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{9/2}}{9}+\frac{2\,c^3\,x^{21/2}}{21}+\frac{6\,b^2\,c\,x^{13/2}}{13}+\frac{6\,b\,c^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(5/2),x)
```

```
[Out] (2*b^3*x^(9/2))/9 + (2*c^3*x^(21/2))/21 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*
x^(17/2))/17
```

sympy [A] time = 22.20, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*b**3*x**(9/2)/9 + 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3*
x**(21/2)/21
```

3.197 $\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{11}b^2cx^{11/2}+\frac{2}{7}b^3x^{7/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (2*b^3*x^(7/2))/7 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*x^(15/2))/5 + (2*c^3*x
^(19/2))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx &= \int x^{5/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{5/2}+3b^2cx^{9/2}+3bc^2x^{13/2}+c^3x^{17/2}\right)dx\\ &= \frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{7/2}\left(1045b^3+1995b^2cx^2+1463bc^2x^4+385c^3x^6\right)}{7315}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(7/2),x]
[Out] (2*x^(7/2)*(1045*b^3 + 1995*b^2*c*x^2 + 1463*b*c^2*x^4 + 385*c^3*x^6))/7315
```

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 0.92

$$\frac{2\left(1045b^3x^{7/2}+1995b^2cx^{11/2}+1463bc^2x^{15/2}+385c^3x^{19/2}\right)}{7315}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + c*x^4)^3/x^(7/2),x]
[Out] (2*(1045*b^3*x^(7/2) + 1995*b^2*c*x^(11/2) + 1463*b*c^2*x^(15/2) + 385*c^3*
x^(19/2)))/7315
```

fricas [A] time = 1.72, size = 40, normalized size = 0.78

$$\frac{2}{7315}\left(385\,c^3x^9+1463\,bc^2x^7+1995\,b^2cx^5+1045\,b^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")
[Out] 2/7315*(385*c^3*x^9 + 1463*b*c^2*x^7 + 1995*b^2*c*x^5 + 1045*b^3*x^3)*sqrt(
x)
```

giac [A] time = 0.18, size = 35, normalized size = 0.69

$$\frac{2}{19}c^3x^{\frac{19}{2}}+\frac{2}{5}bc^2x^{\frac{15}{2}}+\frac{6}{11}b^2cx^{\frac{11}{2}}+\frac{2}{7}b^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(7
/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(385c^3x^6+1463b\,c^2x^4+1995b^2c\,x^2+1045b^3\right)x^{\frac{7}{2}}}{7315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(7/2),x)
```

```
[Out] 2/7315*x^(7/2)*(385*c^3*x^6+1463*b*c^2*x^4+1995*b^2*c*x^2+1045*b^3)
```

maxima [A] time = 1.37, size = 35, normalized size = 0.69

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(7
/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{7/2}}{7}+\frac{2\,c^3\,x^{19/2}}{19}+\frac{6\,b^2\,c\,x^{11/2}}{11}+\frac{2\,b\,c^2\,x^{15/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(7/2),x)
```

```
[Out] (2*b^3*x^(7/2))/7 + (2*c^3*x^(19/2))/19 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*
x^(15/2))/5
```

sympy [A] time = 26.50, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(7/2),x)
```

```
[Out] 2*b**3*x**(7/2)/7 + 6*b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*x
**(19/2)/19
```

3.198 $\int \frac{x^{13/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=217

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}}+\frac{b^{7/4}}{}$$

Rubi [A] time = 0.23, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{11/4}}-\frac{2bx^{3/2}}{3c^2}+\frac{2x^{7/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*b*x^(3/2))/(3*c^2) + (2*x^(7/2))/(7*c) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*Log[Sqrt[b] - Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(7/4)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(1
1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}}{bx^2+cx^4}\,dx = \int \frac{x^{9/2}}{b+cx^2}\,dx$$
$$= \frac{2x^{7/2}}{7c} - \frac{b\int \frac{x^{5/2}}{b+cx^2}\,dx}{c}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^2\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^2}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} - \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^3} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{}\right.}{2c^3}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}} - \frac{b^{7/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{2\sqrt{2}\,c^{11/4}}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}} + \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}} + \frac{b^{7/4}\log\left(\sqrt{b}-\sqrt{2}\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.41

$$\frac{2c^{3/4}x^{3/2}\left(3cx^2-7b\right)+21(-b)^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+21b(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{21c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*c^(3/4)*x^(3/2)*(-7*b + 3*c*x^2) + 21*(-b)^(7/4)*ArcTan[(c^(1/4)*Sqrt[x]
)/(-b)^(1/4)] + 21*(-b)^(3/4)*b*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(21*
c^(11/4))
```

IntegrateAlgebraic [A] time = 0.20, size = 138, normalized size = 0.64

$$-\frac{b^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{11/4}}-\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,c^{11/4}}+\frac{2\left(3cx^{7/2}-7bx^{3/2}\right)}{21c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*(-7*b*x^(3/2) + 3*c*x^(7/2)))/(21*c^2) - (b^(7/4)*ArcTan[(b^(1/4)/(Sqrt[
2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(Sqrt[2]*c^(11/4)) -
 (b^(7/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])
/(Sqrt[2]*c^(11/4))
```

fricas [A] time = 0.87, size = 182, normalized size = 0.84

$$-\frac{84\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^5c^3\sqrt{x}\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}-\sqrt{-b^7c^5\sqrt{-\frac{b^7}{c^{11}}}+b^{10}x}\,c^3\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}}{b^7}\right)-21\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}}+b^5\sqrt{x}\right)+21\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(-c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}}+b^5\sqrt{x}\right)-4\left(3\,cx^3-7\,bx\right)\sqrt{x}}{42\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/42*(84*c^2*(-b^7/c^11)^(1/4)*arctan(-(b^5*c^3*sqrt(x)*(-b^7/c^11)^(1/4)
- sqrt(-b^7*c^5*sqrt(-b^7/c^11) + b^10*x)*c^3*(-b^7/c^11)^(1/4))/b^7) - 21*
c^2*(-b^7/c^11)^(1/4)*log(c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) + 21*c^2*(-b
^7/c^11)^(1/4)*log(-c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) - 4*(3*c*x^3 - 7*b
*x)*sqrt(x))/c^2
```

giac [A] time = 0.17, size = 197, normalized size = 0.91

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^5}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^5}+\frac{2\left(3\,c^6x^{\frac{7}{2}}-7\,bc^5x^{\frac{3}{2}}\right)}{21\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*(b*c^3)^(3/4
)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*(b*c
```

```
^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 2/21*(3
*c^6*x^(7/2) - 7*b*c^5*x^(3/2))/c^7
```

maple [A] time = 0.01, size = 158, normalized size = 0.73

$$\frac{2x^{\frac{7}{2}}}{7c}-\frac{2b\,x^{\frac{3}{2}}}{3c^{2}}+\frac{\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}+\frac{\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}+\frac{\sqrt{2}\,b^{2}\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/7*x^(7/2)/c-2/3*b*x^(3/2)/c^2+1/4*b^2/c^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2)))+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.01, size = 198, normalized size = 0.91

$$\frac{b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,c^{2}}+\frac{2\left(3\,c x^{\frac{7}{2}}-7\,b x^{\frac{3}{2}}\right)}{21\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^
2 + 2/21*(3*c*x^(7/2) - 7*b*x^(3/2))/c^2
```

mupad [B] time = 0.11, size = 66, normalized size = 0.30

$$\frac{2\,x^{7/2}}{7\,c}-\frac{2\,b\,x^{3/2}}{3\,c^{2}}+\frac{{\left(-b\right)}^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{c^{11/4}}+\frac{{\left(-b\right)}^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,1{}\mathrm{i}}{c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(7/2))/(7*c) - (2*b*x^(3/2))/(3*c^2) + ((-b)^(7/4)*atan((c^(1/4)*x^(1/
2))/(-b)^(1/4)))/c^(11/4) + ((-b)^(7/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4
))*1i)/c^(11/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.199 $\int \frac{x^{11/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=215

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{b^{5}}{}$$

Rubi [A] time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{9/4}}-\frac{2b\sqrt{x}}{c^2}+\frac{2x^{5/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*b*Sqrt[x])/c^2 + (2*x^(5/2))/(5*c) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) - (b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}}{bx^2+cx^4}\,dx = \int \frac{x^{7/2}}{b+cx^2}\,dx$$
$$= \frac{2x^{5/2}}{5c} - \frac{b\int \frac{x^{3/2}}{b+cx^2}\,dx}{c}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^2\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{c^2}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2} + \frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}} + \frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x\right.}{2c^{5/2}}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} - \frac{b^{5/4}\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}} + \frac{b^{5/4}\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \right.}{2\sqrt{2}\,c^{9/4}}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} - \frac{b^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}} + \frac{b^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}} - \frac{b^{5/4}\log\left(\sqrt{b} - \sqrt{2}\right.}{2\sqrt{}}$$

Mathematica [A] time = 0.06, size = 203, normalized size = 0.94

$$\frac{-5\sqrt{2}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+5\sqrt{2}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-10\sqrt{2}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+10\sqrt{2}b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)-40b\sqrt[4]{c}\sqrt{x}+8c^{5/4}x^{5/2}}{20c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-40*b*c^(1/4)*Sqrt[x] + 8*c^(5/4)*x^(5/2) - 10*Sqrt[2]*b^(5/4)*ArcTan[1 -
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 10*Sqrt[2]*b^(5/4)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)] - 5*Sqrt[2]*b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 5*Sqrt[2]*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(20*c^(9/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 134, normalized size = 0.62

$$-\frac{b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,c^{9/4}}+\frac{2\sqrt{x}\left(cx^2-5b\right)}{5c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x]*(-5*b + c*x^2))/(5*c^2) - (b^(5/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1
/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(Sqrt[2]*c^(9/4)) + (b^(5/4
)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2
]*c^(9/4))
```

fricas [A] time = 1.69, size = 170, normalized size = 0.79

$$\frac{20\,c^2\left(-\frac{b^5}{c^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc^7\sqrt{x}\left(-\frac{b^5}{c^9}\right)^{\frac{3}{4}}-\sqrt{c^4\sqrt{-\frac{b^5}{c^9}}+b^2x}\,c^7\left(-\frac{b^5}{c^9}\right)^{\frac{3}{4}}}{b^5}\right)+5\,c^2\left(-\frac{b^5}{c^9}\right)^{\frac{1}{4}}\log\left(c^2\left(-\frac{b^5}{c^9}\right)^{\frac{1}{4}}+b\sqrt{x}\right)-5\,c^2\left(-\frac{b^5}{c^9}\right)^{\frac{1}{4}}\log\left(-c^2\left(-\frac{b^5}{c^9}\right)^{\frac{1}{4}}+b\sqrt{x}\right)+4\left(cx^2-5\,b\right)\sqrt{x}}{10\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*c^2*(-b^5/c^9)^(1/4)*arctan(-(b*c^7*sqrt(x)*(-b^5/c^9)^(3/4) - sqr
t(c^4*sqrt(-b^5/c^9) + b^2*x)*c^7*(-b^5/c^9)^(3/4))/b^5) + 5*c^2*(-b^5/c^9)
^(1/4)*log(c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) - 5*c^2*(-b^5/c^9)^(1/4)*log(-
c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) + 4*(c*x^2 - 5*b)*sqrt(x))/c^2
```

giac [A] time = 0.17, size = 196, normalized size = 0.91

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^3}+\frac{2\left(c^4x^{\frac{5}{2}}-5\,bc^3\sqrt{x}\right)}{5\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*(b*c^3)^(1/4
)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*(b*c
```

```
^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 2/5*(c^
4*x^(5/2) - 5*b*c^3*sqrt(x))/c^5
```

maple [A] time = 0.01, size = 152, normalized size = 0.71

$$\frac{2x^{\frac{5}{2}}}{5c}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2c^{2}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c^{2}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4c^{2}}-\frac{2b\sqrt{x}}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/5*x^(5/2)/c-2*b*x^(1/2)/c^2+1/4*b/c^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/
4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))
)+1/2*b/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*b
/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.97, size = 194, normalized size = 0.90

$$\frac{2\left(cx^{\frac{5}{2}}-5\,b\sqrt{x}\right)}{5\,c^{2}}+\frac{\frac{2\,\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,b^{\frac{5}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}\,b^{\frac{5}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}}{4\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/5*(c*x^(5/2) - 5*b*sqrt(x))/c^2 + 1/4*(2*sqrt(2)*b^(3/2)*arctan(1/2*sqrt(
2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqr
t(sqrt(b)*sqrt(c)) + 2*sqrt(2)*b^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)
*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c))
+ sqrt(2)*b^(5/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b)
)/c^(1/4) - sqrt(2)*b^(5/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*
x + sqrt(b))/c^(1/4))/c^2
```

mupad [B] time = 4.49, size = 67, normalized size = 0.31

$$\frac{2\,x^{5/2}}{5\,c}-\frac{2\,b\,\sqrt{x}}{c^2}-\frac{{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{c^{9/4}}+\frac{{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,1{}\mathrm{i}}{c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(5/2))/(5*c) - (2*b*x^(1/2))/c^2 - ((-b)^(5/4)*atan((c^(1/4)*x^(1/2))/
(-b)^(1/4)))/c^(9/4) + ((-b)^(5/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*1i
)/c^(9/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.200 $\int \frac{x^{9/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=204

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}}-$$

Rubi [A] time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{7/4}}+\frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*c^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(Sqrt[2]*c^(7/4)) - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4)) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = \int \frac{x^{5/2}}{b+cx^2}\,dx$$
$$= \frac{2x^{3/2}}{3c} - \frac{b\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c}$$
$$= \frac{2x^{3/2}}{3c} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2x^{3/2}}{3c} + \frac{b\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}}$$
$$= \frac{2x^{3/2}}{3c} - \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2} - \frac{b^{3/4}\,\text{Su}}{}$$
$$= \frac{2x^{3/2}}{3c} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}} + \frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}} - \frac{b^{3/4}}{}$$
$$= \frac{2x^{3/2}}{3c} + \frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}} - \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{2\sqrt{2}\,c^{7/4}}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.38

$$\frac{(-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{7/4}} - \frac{(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{7/4}} + \frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) + ((-b)^(3/4)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(7/
4) - ((-b)^(3/4)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(7/4)
```

IntegrateAlgebraic [A] time = 0.18, size = 124, normalized size = 0.61

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{7/4}} + \frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,c^{7/4}} + \frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(9/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) + (b^(3/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x
)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(Sqrt[2]*c^(7/4)) + (b^(3/4)*ArcTanh[(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(7/4))
```

fricas [A] time = 0.86, size = 165, normalized size = 0.81

$$\frac{12\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^2c^2\sqrt{x}\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}-\sqrt{-b^3c^3\sqrt{-\frac{b^3}{c^7}}+b^4x}\,c^2\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}}{b^3}\right)-3\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(c^5\left(-\frac{b^3}{c^7}\right)^{\frac{3}{4}}+b^2\sqrt{x}\right)+3\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(-c^5\left(-\frac{b^3}{c^7}\right)^{\frac{3}{4}}+b^2\sqrt{x}\right)+4\,x^{\frac{3}{2}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*(-b^3/c^7)^(1/4)*arctan(-(b^2*c^2*sqrt(x)*(-b^3/c^7)^(1/4) - sqrt
(-b^3*c^3*sqrt(-b^3/c^7) + b^4*x)*c^2*(-b^3/c^7)^(1/4))/b^3) - 3*c*(-b^3/c^
7)^(1/4)*log(c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) + 3*c*(-b^3/c^7)^(1/4)*log
(-c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) + 4*x^(3/2))/c
```

giac [A] time = 0.18, size = 178, normalized size = 0.87

$$\frac{2\,x^{\frac{3}{2}}}{3\,c}-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^4}-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^4}+\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^4}-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/c - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/4*sqrt(2)*
(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/4*sq
rt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4
```

maple [A] time = 0.01, size = 143, normalized size = 0.70

$$\frac{2x^{\frac{3}{2}}}{3c}-\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,b\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/3*x^(3/2)/c-1/4*b/c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/
2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2*b/c^2/(b/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*b/c^2/(b/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.12, size = 186, normalized size = 0.91

$$-\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4c}+\frac{2x^{\frac{3}{2}}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c +
 2/3*x^(3/2)/c
```

mupad [B] time = 4.36, size = 54, normalized size = 0.26

$$\frac{2\,x^{3/2}}{3\,c}+\frac{{(-b)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{c^{7/4}}-\frac{{(-b)}^{3/4}\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(3/2))/(3*c) + ((-b)^(3/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(7/4)
 - ((-b)^(3/4)*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(7/4)
```

sympy [A] time = 166.49, size = 180, normalized size = 0.88

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } b = 0 \wedge c = 0 \\ \frac{2x^{\frac{7}{2}}}{7b} & \text{for } c = 0 \\ \frac{2x^{\frac{3}{2}}}{3c} & \text{for } b = 0 \\ \frac{(-1)^{\frac{3}{4}}b^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c^2\sqrt[4]{\frac{1}{c}}} - \frac{(-1)^{\frac{3}{4}}b^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c^2\sqrt[4]{\frac{1}{c}}} - \frac{(-1)^{\frac{3}{4}}b^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{c^2\sqrt[4]{\frac{1}{c}}} + \frac{2x^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(b, 0) & Eq(c, 0)), (2*x**(7/2)/(7*b), Eq(c, 0))
, (2*x**(3/2)/(3*c), Eq(b, 0)), ((-1)**(3/4)*b**(3/4)*log(-(-1)**(1/4)*b**(
1/4)*(1/c)**(1/4) + sqrt(x))/(2*c**2*(1/c)**(1/4)) - (-1)**(3/4)*b**(3/4)*l
og((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c**2*(1/c)**(1/4)) - (-1
)**(3/4)*b**(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(c**2*(
1/c)**(1/4)) + 2*x**(3/2)/(3*c), True))
```

3.201 $\int \frac{x^{7/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}}{}$$

Rubi [A] time = 0.18, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{5/4}}+\frac{2\sqrt{x}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + (b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sq
rt[2]*c^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*c^(5/4)) + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4)) - (b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx = \int \frac{x^{3/2}}{b+cx^2}\,dx$$
$$= \frac{2\sqrt{x}}{c} - \frac{b\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{c}$$
$$= \frac{2\sqrt{x}}{c} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/2}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/2}} + \ldots$$
$$= \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}} - \ldots$$
$$= \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} \ldots\right.}{2\sqrt{2}\,c^{5/4}}$$

Mathematica [A] time = 0.04, size = 189, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)+2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)+8\sqrt[4]{c}\sqrt{x}}{4c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (8*c^(1/4)*Sqrt[x] + 2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)] - 2*Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
 + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(4*c^(5/4))
```

IntegrateAlgebraic [A] time = 0.18, size = 123, normalized size = 0.61

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,c^{5/4}}+\frac{2\sqrt{x}}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + (b^(1/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(S
qrt[2]*b^(1/4)))/Sqrt[x]])/(Sqrt[2]*c^(5/4)) - (b^(1/4)*ArcTanh[(Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(5/4))
```

fricas [A] time = 0.90, size = 124, normalized size = 0.61

$$-\frac{4\,c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^2\sqrt{-\frac{b}{c^5}}+x}\,c^4\left(-\frac{b}{c^5}\right)^{\frac{3}{4}}-c^4\sqrt{x}\left(-\frac{b}{c^5}\right)^{\frac{3}{4}}}{b}\right)+c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\sqrt{x}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*c*(-b/c^5)^(1/4)*arctan((sqrt(c^2*sqrt(-b/c^5) + x)*c^4*(-b/c^5)^(3
/4) - c^4*sqrt(x)*(-b/c^5)^(3/4))/b) + c*(-b/c^5)^(1/4)*log(c*(-b/c^5)^(1/4
) + sqrt(x)) - c*(-b/c^5)^(1/4)*log(-c*(-b/c^5)^(1/4) + sqrt(x)) - 4*sqrt(x
))/c
```

giac [A] time = 0.16, size = 178, normalized size = 0.88

$$-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^2}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/c^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^2 - 1/4*sqrt(2)*(b*c^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 2*sqrt(x)/c
```

maple [A] time = 0.01, size = 140, normalized size = 0.69

$$-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4c}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2*x^(1/2)/c-1/4/c*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/
c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2/c*(b/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/c*(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.05, size = 185, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}b^{\frac{1}{4}}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}}}{4c}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqr
t(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sqrt
(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(
sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(1/4)*log(sqrt(2)*b^(1/
4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(1/4)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c + 2*sqrt(x)
/c
```

mupad [B] time = 4.36, size = 55, normalized size = 0.27

$$\frac{2\sqrt{x}}{c}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{c^{5/4}}-\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(1/2))/c - ((-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(5/4) - (
(-b)^(1/4)*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(5/4)
```

sympy [A] time = 77.50, size = 172, normalized size = 0.85

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } b = 0 \wedge c = 0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } c = 0 \\ \frac{2\sqrt{x}}{c} & \text{for } b = 0 \\ \frac{\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c} - \frac{\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c} + \frac{\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{c} + \frac{2\sqrt{x}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(5/2)/(5*b), Eq(c, 0)),
 (2*sqrt(x)/c, Eq(b, 0)), ((-1)**(1/4)*b**(1/4)*(1/c)**(1/4)*log(-(-1)**(1/
4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c) - (-1)**(1/4)*b**(1/4)*(1/c)**(1/
4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c) + (-1)**(1/4)*b**
(1/4)*(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/c + 2*
sqrt(x)/c, True))
```

3.202 $\int \frac{x^{5/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}$$

Rubi [A] time = 0.14, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1584, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)) +
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1
/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(1/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx = \int \frac{\sqrt{x}}{b+cx^2}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}}{}}\right.}{2\sqrt{2}\sqrt[4]{}}$$
$$= \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right.}{\sqrt{2}\sqrt[4]{b}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}} - \frac{\log\left(\sqrt{\right.}{}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.28

$$\frac{b\left(\tan^{-1}\left(\frac{b\sqrt[4]{c}\sqrt{x}}{(-b)^{5/4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)\right)}{(-b)^{5/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(ArcTan[(b*c^(1/4)*Sqrt[x])/(-b)^(5/4)] + ArcTanh[(c^(1/4)*Sqrt[x])/(-b)
^(1/4)]))/((-b)^(5/4)*c^(3/4))
```

IntegrateAlgebraic [A] time = 0.15, size = 114, normalized size = 0.59

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x
]]/(Sqrt[2]*b^(1/4)*c^(3/4))) - ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(
Sqrt[b] + Sqrt[c]*x)]/(Sqrt[2]*b^(1/4)*c^(3/4))
```

fricas [A] time = 1.02, size = 126, normalized size = 0.66

$$-2\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc\sqrt{-\frac{1}{bc^3}}+x}\,c\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}-c\sqrt{x}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(-bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -2*(-1/(b*c^3))^(1/4)*arctan(sqrt(-b*c*sqrt(-1/(b*c^3)) + x)*c*(-1/(b*c^3))
^(1/4) - c*sqrt(x)*(-1/(b*c^3))^(1/4)) + 1/2*(-1/(b*c^3))^(1/4)*log(b*c^2*(
-1/(b*c^3))^(3/4) + sqrt(x)) - 1/2*(-1/(b*c^3))^(1/4)*log(-b*c^2*(-1/(b*c^3
))^(3/4) + sqrt(x))
```

giac [A] time = 0.19, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c^3) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/4*sqrt(2)*(b*c^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/4*sqrt(2
)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3)
```

maple [A] time = 0.00, size = 132, normalized size = 0.69

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4/c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x
+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2/c/(b/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.00, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{4\,b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{4\,b^{\frac{1}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*
sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - 1/4*sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + 1/4*sqrt(2)*log
(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4))
```

mupad [B] time = 0.08, size = 38, normalized size = 0.20

$$\frac{\operatorname{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)-\operatorname{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{(-b)^{1/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (atan((c^(1/4)*x^(1/2))/(-b)^(1/4)) - atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/
((-b)^(1/4)*c^(3/4))
```

sympy [A] time = 48.27, size = 165, normalized size = 0.86

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } b = 0 \wedge c = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } c = 0 \\ -\frac{2}{c\sqrt{x}} & \text{for } b = 0 \\ -\frac{(-1)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2\sqrt[4]{b}\,c\,\sqrt[4]{\frac{1}{c}}} + \frac{(-1)^{\frac{3}{4}} \log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2\sqrt[4]{b}\,c\,\sqrt[4]{\frac{1}{c}}} + \frac{(-1)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}}\right)}{\sqrt[4]{b}\,c\,\sqrt[4]{\frac{1}{c}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(3/2)/(3*b), Eq(c, 0)),
 (-2/(c*sqrt(x)), Eq(b, 0)), (-(-1)**(3/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)*
*(1/4) + sqrt(x))/(2*b**(1/4)*c*(1/c)**(1/4)) + (-1)**(3/4)*log((-1)**(1/4)
*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(1/4)*c*(1/c)**(1/4)) + (-1)**(3/4)
*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(1/4)*c*(1/c)**(1/4)
), True))
```

3.203 $\int \frac{x^{3/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,b^{3/4}\sqrt[4]{}}$$

Rubi [A] time = 0.14, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1584, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)) -
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(3
/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(3/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = \int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\sqrt{c}} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} \cdots}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2} \cdots}{\cdots}}\cdots\right)}{2\sqrt{2}\,b^{3 \cdots}}$$

$$= -\frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx \cdots\right)}{\sqrt{2}\,b^{3 \cdots}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{\cdots}\right.}{\cdots}$$

Mathematica [A] time = 0.03, size = 146, normalized size = 0.76

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)+\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x] + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2
*Sqrt[2]*b^(3/4)*c^(1/4))
```

IntegrateAlgebraic [A] time = 0.15, size = 113, normalized size = 0.59

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}} - \frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x
]]/(Sqrt[2]*b^(3/4)*c^(1/4))) + ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(
Sqrt[b] + Sqrt[c]*x)]/(Sqrt[2]*b^(3/4)*c^(1/4))
```

fricas [A] time = 1.02, size = 126, normalized size = 0.66

$$2\left(-\frac{1}{b^{3}c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^{2}\sqrt{-\frac{1}{b^{3}c}}+x}\,b^{2}c\left(-\frac{1}{b^{3}c}\right)^{\frac{3}{4}}-b^{2}c\sqrt{x}\left(-\frac{1}{b^{3}c}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{b^{3}c}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^{3}c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{b^{3}c}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b^{3}c}\right)^{\frac{1}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(b^3*c))^(1/4)*arctan(sqrt(b^2*sqrt(-1/(b^3*c)) + x)*b^2*c*(-1/(b^3*c
))^(3/4) - b^2*c*sqrt(x)*(-1/(b^3*c))^(3/4)) + 1/2*(-1/(b^3*c))^(1/4)*log(b
*(-1/(b^3*c))^(1/4) + sqrt(x)) - 1/2*(-1/(b^3*c))^(1/4)*log(-b*(-1/(b^3*c))
^(1/4) + sqrt(x))
```

giac [A] time = 0.16, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc}-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c) + 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c) + 1/4*sqrt(2)*(b*c^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c) - 1/4*sqrt(2)*(b*c
^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c)
```

maple [A] time = 0.01, size = 132, normalized size = 0.69

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*(b/c)^(1/4)/b*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x
-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2*(b/c)^(1/4)/b*2^(1/2)*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.02, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4b^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*
sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 1/4*sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - 1/4*sqrt(2)*log
(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4))
```

mupad [B] time = 4.43, size = 37, normalized size = 0.19

$$-\frac{\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)+\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{(-b)^{3/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x^(1/2))/(-b)^(1/4)) + atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))
/((-b)^(3/4)*c^(1/4))
```

sympy [A] time = 27.22, size = 160, normalized size = 0.83

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ b=0 \wedge c=0 \\ -\frac{2}{3cx^{\frac{3}{2}}} & \text{for}\ b=0 \\ \frac{2\sqrt{x}}{b} & \text{for}\ c=0 \\ -\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{3}{4}}}+\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{c}}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{3}{4}}}-\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{c}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(b, 0) & Eq(c, 0)), (-2/(3*c*x**(3/2)), Eq(b, 0)
), (2*sqrt(x)/b, Eq(c, 0)), (-(-1)**(1/4)*(1/c)**(1/4)*log(-(-1)**(1/4)*b**
(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)) + (-1)**(1/4)*(1/c)**(1/4)*log((
-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)) - (-1)**(1/4)*(1/c
)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(3/4), True))
```

3.204 $\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{c}}{}$$

Rubi [A] time = 0.16, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{5/4}}-\frac{2}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4),x]
```

```
[Out] -2/(b*Sqrt[x]) + (c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*b^(5/4)) - (c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(
Sqrt[2]*b^(5/4)) - (c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)) + (c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx = \int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2}{b\sqrt{x}} - \frac{c\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b}$$
$$= -\frac{2}{b\sqrt{x}} - \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2}{b\sqrt{x}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2}{b\sqrt{x}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} - \frac{\sqrt[4]{c}\,\text{Subs}}{}$$
$$= -\frac{2}{b\sqrt{x}} - \frac{\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}} + \frac{\sqrt[4]{c}\,\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{}}{}$$
$$= -\frac{2}{b\sqrt{x}} + \frac{\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{2\sqrt{2}\,b^{5/4}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{cx^2}{b}\right)}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4),x]
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -((c*x^2)/b)])/(b*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.18, size = 122, normalized size = 0.60

$$\frac{\sqrt[4]{c}\,\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{5/4}} + \frac{\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{5/4}} - \frac{2}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(b*x^2 + c*x^4),x]
```

```
[Out] -2/(b*Sqrt[x]) + (c^(1/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(
Sqrt[2]*b^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(5/4)) + (c^(1/4)*ArcTanh[(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(5/4))
```

fricas [A] time = 1.04, size = 142, normalized size = 0.70

$$\frac{4\,bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc\sqrt{x}\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}-\sqrt{-b^3c\sqrt{-\frac{c}{b^5}}+c^2x}\,b\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}}{c}\right)-bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{x}\right)+bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(-b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{x}\right)-4\sqrt{x}}{2\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*x*(-c/b^5)^(1/4)*arctan(-(b*c*sqrt(x)*(-c/b^5)^(1/4) - sqrt(-b^3*c
*sqrt(-c/b^5) + c^2*x)*b*(-c/b^5)^(1/4))/c) - b*x*(-c/b^5)^(1/4)*log(b^4*(-
c/b^5)^(3/4) + c*sqrt(x)) + b*x*(-c/b^5)^(1/4)*log(-b^4*(-c/b^5)^(3/4) + c*
sqrt(x)) - 4*sqrt(x))/(b*x)
```

giac [A] time = 0.17, size = 190, normalized size = 0.94

$$-\frac{2}{b\sqrt{x}}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -2/(b*sqrt(x)) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) +
 1/4*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b^2*c^2) - 1/4*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/(b^2*c^2)
```

maple [A] time = 0.01, size = 140, normalized size = 0.69

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b}-\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}b}-\frac{2}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/4/b/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(
x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2/b/(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/b/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
b/c)^(1/4)*x^(1/2)-1)-2/b/x^(1/2)
```

maxima [A] time = 2.92, size = 186, normalized size = 0.92

$$-\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4b}-\frac{2}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b -
 2/(b*sqrt(x))
```

mupad [B] time = 4.53, size = 54, normalized size = 0.27

$$\frac{(-c)^{1/4}\operatorname{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{5/4}}-\frac{(-c)^{1/4}\operatorname{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{5/4}}-\frac{2}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4),x)
```

```
[Out] ((-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(5/4) - ((-c)^(1/4)*atan
(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(5/4) - 2/(b*x^(1/2))
```

sympy [A] time = 18.32, size = 170, normalized size = 0.84

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{5cx^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } c = 0 \\ -\frac{2}{b\sqrt{x}} + \frac{(-1)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{5}{4}}\sqrt[4]{\frac{1}{c}}} - \frac{(-1)^{\frac{3}{4}} \log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{5}{4}}\sqrt[4]{\frac{1}{c}}} - \frac{(-1)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{5}{4}}\sqrt[4]{\frac{1}{c}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(b, 0) & Eq(c, 0)), (-2/(5*c*x**(5/2)), Eq(b, 0)
), (-2/(b*sqrt(x)), Eq(c, 0)), (-2/(b*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(1/
4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)*(1/c)**(1/4)) - (-1)**(3/4)
*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)*(1/c)**(1/4))
 - (-1)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(5/4)*
(1/c)**(1/4)), True))
```

3.205 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=204

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\,\mathrm{t}}{}$$

Rubi [A] time = 0.17, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{7/4}}-\frac{2}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*b*x^(3/2)) + (c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/
(Sqrt[2]*b^(7/4)) - (c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*b^(7/4)) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)) - (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{3bx^{3/2}} - \frac{c\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b}$$

$$= -\frac{2}{3bx^{3/2}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= -\frac{2}{3bx^{3/2}} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}$$

$$= -\frac{2}{3bx^{3/2}} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{}\right.}{2b^{3/2}}$$

$$= -\frac{2}{3bx^{3/2}} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\right.}{2\sqrt{2}\,b^{7/4}}$$

$$= -\frac{2}{3bx^{3/2}} + \frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{}\right.}{2\sqrt{2}\,b}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.14

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{cx^2}{b}\right)}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^2)/b)])/(3*b*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 125, normalized size = 0.61

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{7/4}} - \frac{2}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*b*x^(3/2)) + (c^(3/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)
/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(7/4)) - (c^(3/4)*ArcTanh[(Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(7/4))
```

fricas [A] time = 0.74, size = 167, normalized size = 0.82

$$-\frac{12\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^5c\sqrt{x}\left(-\frac{c^3}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{c^3}{b^7}}+c^2x}\,b^5\left(-\frac{c^3}{b^7}\right)^{\frac{3}{4}}}{c^3}\right)+3\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}+c\sqrt{x}\right)-3\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log\left(-b^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}+c\sqrt{x}\right)+4\sqrt{x}}{6\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*b*x^2*(-c^3/b^7)^(1/4)*arctan(-(b^5*c*sqrt(x)*(-c^3/b^7)^(3/4) - s
qrt(b^4*sqrt(-c^3/b^7) + c^2*x)*b^5*(-c^3/b^7)^(3/4))/c^3) + 3*b*x^2*(-c^3/
b^7)^(1/4)*log(b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) - 3*b*x^2*(-c^3/b^7)^(1/4)
*log(-b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) + 4*sqrt(x))/(b*x^2)
```

giac [A] time = 0.16, size = 178, normalized size = 0.87

$$-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2}-\frac{2}{3\,bx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^2 - 1/4*sqrt(2)*(b*c^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 - 2/3/(b*x^(3/2)
)
```

maple [A] time = 0.01, size = 143, normalized size = 0.70

$$-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b^2}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^2}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b^2}-\frac{2}{3b\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] -1/4*c/b^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2
))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2*c/b^2*(b/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*c/b^2*(b/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/3/b/x^(3/2)
```

maxima [A] time = 2.99, size = 187, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}}{4b}-\frac{2}{3bx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*
c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sq
rt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(
2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(3/4)
*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b - 2
/3/(b*x^(3/2))
```

mupad [B] time = 0.10, size = 53, normalized size = 0.26

$$\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{7/4}}-\frac{2}{3\,b\,x^{3/2}}+\frac{(-c)^{3/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(7/4) - 2/(3*b*x^(3/2)) +
 ((-c)^(3/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(7/4)
```

sympy [A] time = 27.74, size = 178, normalized size = 0.87

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } c = 0 \\ -\frac{2}{7cx^{\frac{7}{2}}} & \text{for } b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} + \frac{\sqrt[4]{-1}\,c\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{7}{4}}} - \frac{\sqrt[4]{-1}\,c\sqrt[4]{\frac{1}{c}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{7}{4}}} + \frac{\sqrt[4]{-1}\,c\sqrt[4]{\frac{1}{c}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(b, 0) & Eq(c, 0)), (-2/(3*b*x**(3/2)), Eq(c, 0)
), (-2/(7*c*x**(7/2)), Eq(b, 0)), (-2/(3*b*x**(3/2)) + (-1)**(1/4)*c*(1/c)*
*(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/4)) - (-1
)**(1/4)*c*(1/c)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2
*b**(7/4)) + (-1)**(1/4)*c*(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*
(1/c)**(1/4)))/b**(7/4), True))
```

3.206 $\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=215

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{c^{5/4}\,\mathrm{t}}{}$$

Rubi [A] time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{9/4}}+\frac{2c}{b^2\sqrt{x}}-\frac{2}{5bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(5*b*x^(5/2)) + (2*c)/(b^2*Sqrt[x]) - (c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) - (c^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2}{5bx^{5/2}} - \frac{c\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{b}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c^2\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^2}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} - \frac{c^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{\right.}}{b^2}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^2} + \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx\right.}{2b^2}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c^{5/4}\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}} - \frac{c^{5/4}\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{2\sqrt{2}\,b^{9/4}}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} - \frac{c^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}} + \frac{c^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}} + \frac{c^{5/4}\log\left(\sqrt{\right.}}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{cx^2}{b}\right)}{5bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^2)/b)])/(5*b*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 134, normalized size = 0.62

$$-\frac{c^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{9/4}}-\frac{2\left(b-5cx^2\right)}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(b - 5*c*x^2))/(5*b^2*x^(5/2)) - (c^(5/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1
/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(9/4)) - (c^(5/4
)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2
]*b^(9/4))
```

fricas [A] time = 1.05, size = 193, normalized size = 0.90

$$-\frac{20\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^2c^4\sqrt{x}\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}-\sqrt{-b^5c^5\sqrt{-\frac{c^5}{b^9}}+c^8x}\,b^2\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}}{c^5}\right)-5\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^7\left(-\frac{c^5}{b^9}\right)^{\frac{3}{4}}+c^4\sqrt{x}\right)+5\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\log\left(-b^7\left(-\frac{c^5}{b^9}\right)^{\frac{3}{4}}+c^4\sqrt{x}\right)-4\left(5\,cx^2-b\right)\sqrt{x}}{10\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*x^3*(-c^5/b^9)^(1/4)*arctan(-(b^2*c^4*sqrt(x)*(-c^5/b^9)^(1/4
) - sqrt(-b^5*c^5*sqrt(-c^5/b^9) + c^8*x)*b^2*(-c^5/b^9)^(1/4))/c^5) - 5*b^
2*x^3*(-c^5/b^9)^(1/4)*log(b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) + 5*b^2*x^3*
(-c^5/b^9)^(1/4)*log(-b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) - 4*(5*c*x^2 - b)
*sqrt(x))/(b^2*x^3)
```

giac [A] time = 0.17, size = 200, normalized size = 0.93

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c}+\frac{2\left(5\,cx^2-b\right)}{5\,b^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b^3*c) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) - 1/4*sqrt(2)*(b*c^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/4*sqrt(2
```

```
)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) +
 2/5*(5*c*x^2 - b)/(b^2*x^(5/2))
```

maple [A] time = 0.01, size = 152, normalized size = 0.71

$$\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2}}+\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2}}+\frac{\sqrt{2}\, c \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4 \left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2}}+\frac{2c}{b^{2} \sqrt{x}}-\frac{2}{5 b\, x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*c/b^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
)/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2*c/b^2/(b/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c/b^2/(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/5/b/x^(5/2)+2*c/b^2/x^(1/2)
```

maxima [A] time = 3.13, size = 198, normalized size = 0.92

$$\frac{c^{2} \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}}+2 \sqrt{c} \sqrt{x}\right)}{2 \sqrt{\sqrt{b} \sqrt{c}}}\right)}{\sqrt{\sqrt{b} \sqrt{c}} \sqrt{c}}+\frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}}-2 \sqrt{c} \sqrt{x}\right)}{2 \sqrt{\sqrt{b} \sqrt{c}}}\right)}{\sqrt{\sqrt{b} \sqrt{c}} \sqrt{c}}-\frac{\sqrt{2} \log\left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}+\sqrt{c} x+\sqrt{b}\right)}{b^{\frac{1}{4}} c^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x}+\sqrt{c} x+\sqrt{b}\right)}{b^{\frac{1}{4}} c^{\frac{3}{4}}}\right)}{4 b^{2}}+\frac{2 \left(5 c x^{2}-b\right)}{5 b^{2} x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^
2 + 2/5*(5*c*x^2 - b)/(b^2*x^(5/2))
```

mupad [B] time = 0.09, size = 66, normalized size = 0.31

$$\frac{(-c)^{5/4} \operatorname{atanh}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right)}{b^{9/4}}-\frac{(-c)^{5/4} \operatorname{atan}\left(\frac{(-c)^{1/4} \sqrt{x}}{b^{1/4}}\right)}{b^{9/4}}-\frac{\frac{2}{5\,b}-\frac{2\,c\,x^2}{b^2}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(9/4) - ((-c)^(5/4)*atan
(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(9/4) - (2/(5*b) - (2*c*x^2)/b^2)/x^(5/2)
```

sympy [A] time = 48.98, size = 190, normalized size = 0.88

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } c = 0 \\ -\frac{2}{9cx^{\frac{9}{2}}} & \text{for } b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} + \frac{2c}{b^2\sqrt{x}} - \frac{(-1)^{\frac{3}{4}} c \log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{9}{4}}\sqrt[4]{\frac{1}{c}}} + \frac{(-1)^{\frac{3}{4}} c \log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{9}{4}}\sqrt[4]{\frac{1}{c}}} + \frac{(-1)^{\frac{3}{4}} c \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{9}{4}}\sqrt[4]{\frac{1}{c}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(b, 0) & Eq(c, 0)), (-2/(5*b*x**(5/2)), Eq(c, 0)
), (-2/(9*c*x**(9/2)), Eq(b, 0)), (-2/(5*b*x**(5/2)) + 2*c/(b**2*sqrt(x)) -
 (-1)**(3/4)*c*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(9/4
)*(1/c)**(1/4)) + (-1)**(3/4)*c*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqr
t(x))/(2*b**(9/4)*(1/c)**(1/4)) + (-1)**(3/4)*c*atan((-1)**(3/4)*sqrt(x)/(b
**(1/4)*(1/c)**(1/4)))/(b**(9/4)*(1/c)**(1/4)), True))
```

3.207 $\int \frac{1}{x^{5/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=217

$$-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}+\cdots$$

Rubi [A] time = 0.18, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{11/4}}+\frac{2c}{3b^2x^{3/2}}-\frac{2}{7bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(7*b*x^(7/2)) + (2*c)/(3*b^2*x^(3/2)) - (c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) + (c^(7/4)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(7/4)*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) + (c^(7/4)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/
4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2}{7bx^{7/2}} - \frac{c\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{b}$$
$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{c^2\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b^2}$$
$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{c^2\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}} + \frac{c^2\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{b^{5/2}}$$
$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}\right.}{2b^{5/2}}$$
$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} - \frac{c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}} + \frac{c^{7/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{2\sqrt{2}\,b^{11/4}}$$
$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} - \frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}} + \frac{c^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}} - \frac{c^{7/4}\log\left(\right.}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\frac{cx^2}{b}\right)}{7bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 1, -3/4, -((c*x^2)/b)])/(7*b*x^(7/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 135, normalized size = 0.62

$$-\frac{c^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{11/4}}-\frac{2\left(3b-7cx^2\right)}{21b^2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(3*b - 7*c*x^2))/(21*b^2*x^(7/2)) - (c^(7/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c
^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(11/4)) + (c^
(7/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sq
rt[2]*b^(11/4))
```

fricas [A] time = 1.55, size = 189, normalized size = 0.87

$$\frac{84\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^8c^2\sqrt{x}\left(-\frac{c^7}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^6\sqrt{-\frac{c^7}{b^{11}}}+c^4x}\,b^8\left(-\frac{c^7}{b^{11}}\right)^{\frac{3}{4}}}{c^7}\right)+21\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}+c^2\sqrt{x}\right)-21\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\log\left(-b^3\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}+c^2\sqrt{x}\right)+4\left(7\,cx^2-3\,b\right)\sqrt{x}}{42\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*x^4*(-c^7/b^11)^(1/4)*arctan(-(b^8*c^2*sqrt(x)*(-c^7/b^11)^(3/
4) - sqrt(b^6*sqrt(-c^7/b^11) + c^4*x)*b^8*(-c^7/b^11)^(3/4))/c^7) + 21*b^2
*x^4*(-c^7/b^11)^(1/4)*log(b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x)) - 21*b^2*x^
4*(-c^7/b^11)^(1/4)*log(-b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x)) + 4*(7*c*x^2
- 3*b)*sqrt(x))/(b^2*x^4)
```

giac [A] time = 0.20, size = 192, normalized size = 0.88

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3}+\frac{2\left(7\,cx^2-3\,b\right)}{21\,b^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/b^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 + 1/4*sqrt(2)*(b*c^3)^(1/4
)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/4*sqrt(2)*(b*c
```

```
^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 2/21*(7
*c*x^2 - 3*b)/(b^2*x^(7/2))
```

maple [A] time = 0.01, size = 158, normalized size = 0.73

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b^{3}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^{3}}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b^{3}}+\frac{2c}{3b^{2}x^{\frac{3}{2}}}-\frac{2}{7b\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*c^2/b^3*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/
2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2*c^2/b^3*(b/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c^2/b^3*(b/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/7/b/x^(7/2)+2/3*c/b^2/x^(3/2)
```

maxima [A] time = 3.12, size = 201, normalized size = 0.93

$$\frac{\frac{2\sqrt{2}c^{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}c^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{7}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}c^{\frac{7}{4}}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}}{4b^{2}}+\frac{2\left(7cx^{2}-3b\right)}{21b^{2}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)
*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7
/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^
2 + 2/21*(7*c*x^2 - 3*b)/(b^2*x^(7/2))
```

mupad [B] time = 4.39, size = 65, normalized size = 0.30

$$\frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{11/4}}-\frac{\frac{2}{7\,b}-\frac{2\,c\,x^2}{3\,b^2}}{x^{7/2}}+\frac{(-c)^{7/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x^2 + c*x^4)),x)
```

[Out] ((-c)^(7/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(11/4) - (2/(7*b) - (2*c*x^2)/(3*b^2))/x^(7/2) + ((-c)^(7/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(11/4)

sympy [A] time = 106.89, size = 197, normalized size = 0.91

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } c = 0 \\ -\frac{2}{11cx^{\frac{11}{2}}} & \text{for } b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} + \frac{2c}{3b^2x^{\frac{3}{2}}} - \frac{\sqrt[4]{-1}\,c^2\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{11}{4}}} + \frac{\sqrt[4]{-1}\,c^2\sqrt[4]{\frac{1}{c}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{11}{4}}} - \frac{\sqrt[4]{-1}\,c^2\sqrt[4]{\frac{1}{c}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{11}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(5/2)/(c*x**4+b*x**2),x)

[Out] Piecewise((zoo/x**(11/2), Eq(b, 0) & Eq(c, 0)), (-2/(7*b*x**(7/2)), Eq(c, 0)), (-2/(11*c*x**(11/2)), Eq(b, 0)), (-2/(7*b*x**(7/2)) + 2*c/(3*b**2*x**(3/2)) - (-1)**(1/4)*c**2*(1/c)**(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(11/4)) + (-1)**(1/4)*c**2*(1/c)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(11/4)) - (-1)**(1/4)*c**2*(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(11/4), True))

3.208 $\int \frac{1}{x^{7/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=230

$$-\frac{c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}-\frac{c^{9/}}{}$$

Rubi [A] time = 0.22, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1584, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2c^2}{b^3\sqrt{x}}-\frac{c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}-\frac{c^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{13/4}}+\frac{2c}{5b^2x^{5/2}}-\frac{2}{9bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(9*b*x^(9/2)) + (2*c)/(5*b^2*x^(5/2)) - (2*c^2)/(b^3*Sqrt[x]) + (c^(9/4)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(9/
4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(
9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]
*b^(13/4)) + (c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{9bx^{9/2}} - \frac{c\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{b}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} + \frac{c^2\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{b^2}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} + \frac{c^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3} - \frac{c^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}}{b+cx^4}\right.}{b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^3} - \frac{c^2\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{2}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^{9/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}} + \frac{c^{9/4}\log\left(\sqrt{b}+\right.}{2}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} + \frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}} - \frac{c^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}} - \frac{c}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{9}{4},1;-\frac{5}{4};-\frac{cx^2}{b}\right)}{9bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
[Out] (-2*Hypergeometric2F1[-9/4, 1, -5/4, -((c*x^2)/b)])/(9*b*x^(9/2))
```

IntegrateAlgebraic [A] time = 0.22, size = 145, normalized size = 0.63

$$\frac{c^{9/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{13/4}}-\frac{2\left(5b^2-9bcx^2+45c^2x^4\right)}{45b^3x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(5*b^2 - 9*b*c*x^2 + 45*c^2*x^4))/(45*b^3*x^(9/2)) + (c^(9/4)*ArcTan[(b
^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(Sqrt[2
]*b^(13/4)) + (c^(9/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] +
 Sqrt[c]*x)])/(Sqrt[2]*b^(13/4))
```

fricas [A] time = 0.78, size = 204, normalized size = 0.89

$$\frac{180\,b^3x^5\left(-\frac{c^9}{b^{13}}\right)^{\frac14}\arctan\left(-\frac{b^3c^7\sqrt{x}\left(-\frac{c^9}{b^{13}}\right)^{\frac14}-\sqrt{-b^7c^9\sqrt{-\frac{c^9}{b^{13}}}+c^{14}x}\,b^3\left(-\frac{c^9}{b^{13}}\right)^{\frac14}}{c^9}\right)-45\,b^3x^5\left(-\frac{c^9}{b^{13}}\right)^{\frac14}\log\left(b^{10}\left(-\frac{c^9}{b^{13}}\right)^{\frac34}+c^7\sqrt{x}\right)+45\,b^3x^5\left(-\frac{c^9}{b^{13}}\right)^{\frac14}\log\left(-b^{10}\left(-\frac{c^9}{b^{13}}\right)^{\frac34}+c^7\sqrt{x}\right)-4\left(45\,c^2x^4-9\,bcx^2+5\,b^2\right)\sqrt{x}}{90\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*x^5*(-c^9/b^13)^(1/4)*arctan(-(b^3*c^7*sqrt(x)*(-c^9/b^13)^(1
/4) - sqrt(-b^7*c^9*sqrt(-c^9/b^13) + c^14*x)*b^3*(-c^9/b^13)^(1/4))/c^9) -
 45*b^3*x^5*(-c^9/b^13)^(1/4)*log(b^10*(-c^9/b^13)^(3/4) + c^7*sqrt(x)) + 4
5*b^3*x^5*(-c^9/b^13)^(1/4)*log(-b^10*(-c^9/b^13)^(3/4) + c^7*sqrt(x)) - 4*
(45*c^2*x^4 - 9*b*c*x^2 + 5*b^2)*sqrt(x))/(b^3*x^5)
```

giac [A] time = 0.18, size = 199, normalized size = 0.87

$$-\frac{\sqrt{2}\left(bc^3\right)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{2\,b^4}-\frac{\sqrt{2}\left(bc^3\right)^{\frac34}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{2\,b^4}+\frac{\sqrt{2}\left(bc^3\right)^{\frac34}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4}-\frac{\sqrt{2}\left(bc^3\right)^{\frac34}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4}-\frac{2\left(45\,c^2x^4-9\,bcx^2+5\,b^2\right)}{45\,b^3x^{\frac92}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*(b*c^3)^(3/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/4*sqrt(2)*(b*c^3)^(
```

```
3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 2/45*(45*c^2*x
^4 - 9*b*c*x^2 + 5*b^2)/(b^3*x^(9/2))
```

maple [A] time = 0.01, size = 169, normalized size = 0.73

$$-\frac{\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3}-\frac{\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3}-\frac{\sqrt{2}\,c^2\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}b^3}-\frac{2c^2}{b^3\sqrt{x}}+\frac{2c}{5b^2x^{\frac{5}{2}}}-\frac{2}{9b\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/4*c^2/b^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1
/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2*c^2/b^3/(b/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*c^2/b^3/(b/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/9/b/x^(9/2)-2*c^2/b^3/x^(1/2)+2/5
*c/b^2/x^(5/2)
```

maxima [A] time = 3.09, size = 209, normalized size = 0.91

$$-\frac{c^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,b^3}-\frac{2\left(45\,c^2x^4-9\,bcx^2+5\,b^2\right)}{45\,b^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)
*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sq
rt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/
4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b
^3 - 2/45*(45*c^2*x^4 - 9*b*c*x^2 + 5*b^2)/(b^3*x^(9/2))
```

mupad [B] time = 4.47, size = 77, normalized size = 0.33

$$\frac{(-c)^{9/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{13/4}}-\frac{(-c)^{9/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{13/4}}-\frac{\frac{2}{9\,b}-\frac{2\,c\,x^2}{5\,b^2}+\frac{2\,c^2\,x^4}{b^3}}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(13/4) - ((-c)^(9/4)*ata
n(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(13/4) - (2/(9*b) - (2*c*x^2)/(5*b^2) +
(2*c^2*x^4)/b^3)/x^(9/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.209 $\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=243

$$-\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}$$

Rubi [A] time = 0.21, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{13/4}}-\frac{9b\sqrt{x}}{2c^3}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}+\frac{9x^{5/2}}{10c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-9*b*Sqrt[x])/(2*c^3) + (9*x^(5/2))/(10*c^2) - x^(9/2)/(2*c*(b + c*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*c^(13/4)) - (9*b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*c^(13/4)) + (9*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{11/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{9\int \frac{x^{7/2}}{b+cx^2}\,dx}{4c}$$

$$= \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} - \frac{(9b)\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^3}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^3}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^3} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^{7/2}} + \frac{\left(9b^{3/2}\right)\text{Sub}}{}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} - \frac{9b^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}} + \frac{9b^{5/4}\log\left(\sqrt{b}\right.}{}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} - \frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}} + \frac{9b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}}$$

Mathematica [A] time = 0.21, size = 220, normalized size = 0.91

$$\frac{-45\sqrt{2}\,b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)+45\sqrt{2}\,b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-90\sqrt{2}\,b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)+90\sqrt{2}\,b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)+\frac{8\sqrt[4]{c}\,\sqrt{x}\left(-45b^2-36bcx^2+4c^2x^4\right)}{b+cx^2}}{80c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((8*c^(1/4)*Sqrt[x]*(-45*b^2 - 36*b*c*x^2 + 4*c^2*x^4))/(b + c*x^2) - 90*Sq
rt[2]*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 90*Sqrt[2]*b^
(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 45*Sqrt[2]*b^(5/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 45*Sqrt[2]*b^(5/
4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(80*c^(13/4)
)
```

IntegrateAlgebraic [A] time = 0.38, size = 245, normalized size = 1.01

$$\frac{\left(-\frac{9b^{5/4}x^2}{4\sqrt{2}\,c^{9/4}}-\frac{9b^{9/4}}{4\sqrt{2}\,c^{13/4}}\right)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+\frac{9b^{5/4}x^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}\,c^{9/4}}+\frac{9b^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}\,c^{13/4}}-\frac{9b^2\sqrt{x}}{2c^3}-\frac{18bx^{5/2}}{5c^2}+\frac{2x^{9/2}}{5c}}{b+cx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-9*b^2*Sqrt[x])/(2*c^3) - (18*b*x^(5/2))/(5*c^2) + (2*x^(9/2))/(5*c) + ((
-9*b^(9/4))/(4*Sqrt[2]*c^(13/4)) - (9*b^(5/4)*x^2)/(4*Sqrt[2]*c^(9/4)))*Arc
Tan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + (9*b^(9/4)*A
rcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(4*Sqrt[2]
*c^(13/4)) + (9*b^(5/4)*x^2*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt
[b] + Sqrt[c]*x)])/(4*Sqrt[2]*c^(9/4)))/(b + c*x^2)
```

fricas [A] time = 0.96, size = 227, normalized size = 0.93

$$\frac{180\left(c^4x^2+bc^3\right)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc^{10}\sqrt{x}\left(-\frac{b^5}{c^{13}}\right)^{\frac{3}{4}}-\sqrt{c^6\sqrt{-\frac{b^5}{c^{13}}}+b^2x}\,c^{10}\left(-\frac{b^5}{c^{13}}\right)^{\frac{3}{4}}}{b^5}\right)+45\left(c^4x^2+bc^3\right)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\log\left(9c^3\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}+9b\sqrt{x}\right)-45\left(c^4x^2+bc^3\right)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\log\left(-9c^3\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}+9b\sqrt{x}\right)+4\left(4c^2x^4-36bcx^2-45b^2\right)\sqrt{x}}{40\left(c^4x^2+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(180*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*arctan(-(b*c^10*sqrt(x)*(-b^5
/c^13)^(3/4) - sqrt(c^6*sqrt(-b^5/c^13) + b^2*x)*c^10*(-b^5/c^13)^(3/4))/b^
5) + 45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(9*c^3*(-b^5/c^13)^(1/4) + 9
*b*sqrt(x)) - 45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(-9*c^3*(-b^5/c^13)
```

```
^(1/4) + 9*b*sqrt(x)) + 4*(4*c^2*x^4 - 36*b*c*x^2 - 45*b^2)*sqrt(x))/(c^4*x
^2 + b*c^3)
```

giac [A] time = 0.20, size = 216, normalized size = 0.89

$$\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{4}}+\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{4}}+\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{4}}-\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{4}}-\frac{b^{2}\sqrt{x}}{2\left(cx^{2}+b\right)c^{3}}+\frac{2\left(c^{8}x^{\frac{5}{2}}-10bc^{7}\sqrt{x}\right)}{5c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^4 + 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 9/16*sqrt(2)*(b*c^3)^(1/
4)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 9/16*sqrt(2)*(b
*c^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*b
^2*sqrt(x)/((c*x^2 + b)*c^3) + 2/5*(c^8*x^(5/2) - 10*b*c^7*sqrt(x))/c^10
```

maple [A] time = 0.01, size = 172, normalized size = 0.71

$$\frac{2x^{\frac{5}{2}}}{5c^{2}}-\frac{b^{2}\sqrt{x}}{2\left(c\,x^{2}+b\right)c^{3}}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8c^{3}}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8c^{3}}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16c^{3}}-\frac{4b\sqrt{x}}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/5*x^(5/2)/c^2-4*b*x^(1/2)/c^3-1/2/c^3*b^2*x^(1/2)/(c*x^2+b)+9/16/c^3*b*(b
/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(
1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+9/8/c^3*b*(b/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8/c^3*b*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.02, size = 217, normalized size = 0.89

$$-\frac{b^{2}\sqrt{x}}{2\left(c^{4}x^{2}+bc^{3}\right)}+\frac{2\left(cx^{\frac{5}{2}}-10b\sqrt{x}\right)}{5c^{3}}+\frac{9\left(\frac{2\sqrt{2}b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}b^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}b^{\frac{5}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}b^{\frac{5}{4}}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}}\right)}{16c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*sqrt(x)/(c^4*x^2 + b*c^3) + 2/5*(c*x^(5/2) - 10*b*sqrt(x))/c^3 + 9
/16*(2*sqrt(2)*b^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*b^(3/
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(5/4)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(5/4)*log(-sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c^3
```

mupad [B] time = 0.10, size = 92, normalized size = 0.38

$$\frac{2\,x^{5/2}}{5\,c^2}-\frac{b^2\,\sqrt{x}}{2\,\left(c^4\,x^2+b\,c^3\right)}-\frac{4\,b\,\sqrt{x}}{c^3}-\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,c^{13/4}}+\frac{{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,9{}\mathrm{i}}{4\,c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*x^(5/2))/(5*c^2) - (b^2*x^(1/2))/(2*(b*c^3 + c^4*x^2)) - (4*b*x^(1/2))/c
^3 - (9*(-b)^(5/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*c^(13/4)) + ((-b)
^(5/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*9i)/(4*c^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.210 $\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}+\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}}$$

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}+\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}}-\frac{7b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,c^{11/4}}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}+\frac{7x^{3/2}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (7*x^(3/2))/(6*c^2) - x^(7/2)/(2*c*(b + c*x^2)) + (7*b^(3/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(11/
4)) + (7*b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(8*Sqrt[2]*c^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{9/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{7/2}}{2c\left(b+cx^2\right)} + \frac{7\int \frac{x^{5/2}}{b+cx^2}\,dx}{4c}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{(7b)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c^2}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{(7b)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} + \frac{(7b)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{5/2}} - \frac{(7b)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \right.}{4c^{5/2}}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{(7b)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^3} - \frac{(7b)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}\right.}{8c^3}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{7b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}} + \frac{7b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\right.}{8\sqrt{2}\,c^{11/4}}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} + \frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}} - \frac{7b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}} - \frac{7b^{3/4}\log}{}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.25

$$-\frac{2x^{3/2}\left(7\left(b+cx^2\right){}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)-7b-cx^2\right)}{3c^2\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*x^(3/2)*(-7*b - c*x^2 + 7*(b + c*x^2)*Hypergeometric2F1[3/4, 2, 7/4, -(
(c*x^2)/b)]))/(3*c^2*(b + c*x^2))
```

IntegrateAlgebraic [A] time = 0.34, size = 230, normalized size = 1.00

$$\frac{\left(\frac{7b^{3/4}x^2}{4\sqrt{2}c^{7/4}}+\frac{7b^{7/4}}{4\sqrt{2}c^{11/4}}\right)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\frac{7b^{3/4}x^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}c^{7/4}}+\frac{7b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}c^{11/4}}+\frac{7bx^{3/2}}{6c^2}+\frac{2x^{7/2}}{3c}}{b+cx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((7*b*x^(3/2))/(6*c^2) + (2*x^(7/2))/(3*c) + ((7*b^(7/4))/(4*Sqrt[2]*c^(11/
4)) + (7*b^(3/4)*x^2)/(4*Sqrt[2]*c^(7/4)))*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + (7*b^(7/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(4*Sqrt[2]*c^(11/4)) + (7*b^(3/4)*x^2*A
rcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(4*Sqrt[2]
*c^(7/4)))/(b + c*x^2)
```

fricas [A] time = 0.86, size = 229, normalized size = 1.00

$$\frac{84\left(c^3x^2+bc^2\right)\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{343b^2c^3\sqrt{x}\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}-\sqrt{-117649b^3c^5\sqrt{-\frac{b^3}{c^{11}}}+117649b^4xc^3}\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}}{343b^3}\right)-21\left(c^3x^2+bc^2\right)\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}\log\left(343c^8\left(-\frac{b^3}{c^{11}}\right)^{\frac{3}{4}}+343b^2\sqrt{x}\right)+21\left(c^3x^2+bc^2\right)\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}\log\left(-343c^8\left(-\frac{b^3}{c^{11}}\right)^{\frac{3}{4}}+343b^2\sqrt{x}\right)+4\left(4cx^3+7bx\right)\sqrt{x}}{24\left(c^3x^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(84*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*arctan(-1/343*(343*b^2*c^3*sqr
t(x)*(-b^3/c^11)^(1/4) - sqrt(-117649*b^3*c^5*sqrt(-b^3/c^11) + 117649*b^4*
x)*c^3*(-b^3/c^11)^(1/4))/b^3) - 21*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*log
(343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) + 21*(c^3*x^2 + b*c^2)*(-b^3/
c^11)^(1/4)*log(-343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) + 4*(4*c*x^3
+ 7*b*x)*sqrt(x))/(c^3*x^2 + b*c^2)
```

giac [A] time = 0.18, size = 196, normalized size = 0.85

$$\frac{bx^{\frac{3}{2}}}{2\left(cx^{2}+b\right)c^{2}}+\frac{2\,x^{\frac{3}{2}}}{3\,c^{2}}-\frac{7\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{5}}-\frac{7\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{5}}+\frac{7\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^{5}}-\frac{7\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^(3/2)/((c*x^2 + b)*c^2) + 2/3*x^(3/2)/c^2 - 7/8*sqrt(2)*(b*c^3)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 -
7/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^5 + 7/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^5 - 7/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt
(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5
```

maple [A] time = 0.01, size = 161, normalized size = 0.70

$$\frac{b\,x^{\frac{3}{2}}}{2\left(c\,x^{2}+b\right)c^{2}}+\frac{2x^{\frac{3}{2}}}{3c^{2}}-\frac{7\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}-\frac{7\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}-\frac{7\sqrt{2}\;b\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/3*x^(3/2)/c^2+1/2*b/c^2*x^(3/2)/(c*x^2+b)-7/16*b/c^3/(b/c)^(1/4)*2^(1/2)*
ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/
2)+(b/c)^(1/2)))-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-
1)
```

maxima [A] time = 3.05, size = 207, normalized size = 0.90

$$\frac{bx^{\frac{3}{2}}}{2\left(c^{3}x^{2}+bc^{2}\right)}-\frac{7\,b\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,c^{2}}+\frac{2\,x^{\frac{3}{2}}}{3\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^(3/2)/(c^3*x^2 + b*c^2) - 7/16*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqr
t(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt
(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1
/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt
(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/(b^(1/4)*c^(3/4)))/c^2 + 2/3*x^(3/2)/c^2
```

mupad [B] time = 0.11, size = 80, normalized size = 0.35

$$\frac{2\,x^{3/2}}{3\,c^2}+\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,c^{11/4}}+\frac{b\,x^{3/2}}{2\,\left(c^3\,x^2+b\,c^2\right)}+\frac{{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,7{}\mathrm{i}}{4\,c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*x^(3/2))/(3*c^2) + (7*(-b)^(3/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*
c^(11/4)) + ((-b)^(3/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*7i)/(4*c^(11/
4)) + (b*x^(3/2))/(2*(b*c^2 + c^3*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.211 $\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=230

$$\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}}-$$

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{9/4}}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}+\frac{5\sqrt{x}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (5*Sqrt[x])/(2*c^2) - x^(5/2)/(2*c*(b + c*x^2)) + (5*b^(1/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) - (5*b^(1/4)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) + (5*b^(1/4)*Log[
Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(9/4))
 - (5*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
8*Sqrt[2]*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{7/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{5/2}}{2c\left(b+cx^2\right)} + \frac{5\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} - \frac{(5b)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^2}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^2} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,d\right.}{4c^2}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^{5/2}} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+\cdots}\right.}{8c^{5/2}}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} + \frac{5\sqrt[4]{b}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{9/4}} - \frac{5\sqrt[4]{b}\,\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\right.}{8\sqrt{2}\,c^{9/4}}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} + \frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}} - \frac{5\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}} + \frac{5\sqrt[4]{b}\,\log}{}$$

Mathematica [A] time = 0.10, size = 221, normalized size = 0.96

$$\frac{\frac{32c^{5/4}x^{5/2}}{b+cx^2}+\frac{40b\sqrt[4]{c}\sqrt{x}}{b+cx^2}+5\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-5\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+10\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-10\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{16c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((40*b*c^(1/4)*Sqrt[x])/(b + c*x^2) + (32*c^(5/4)*x^(5/2))/(b + c*x^2) + 10
*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 10*Sqrt[2]
*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 5*Sqrt[2]*b^(1/4)*
Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 5*Sqrt[2]*b^(1
/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(16*c^(9/4)
)
```

IntegrateAlgebraic [A] time = 0.34, size = 151, normalized size = 0.66

$$\frac{5\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}c^{9/4}}+\frac{5b\sqrt{x}+4cx^{5/2}}{2c^2\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (5*b*Sqrt[x] + 4*c*x^(5/2))/(2*c^2*(b + c*x^2)) + (5*b^(1/4)*ArcTan[(b^(1/4
)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*c
^(9/4)) - (5*b^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + S
qrt[c]*x)])/(4*Sqrt[2]*c^(9/4))
```

fricas [A] time = 0.81, size = 192, normalized size = 0.83

$$-\frac{20\left(c^3x^2+bc^2\right)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^4\sqrt{-\frac{b}{c^9}}+x}\,c^7\left(-\frac{b}{c^9}\right)^{\frac{3}{4}}-c^7\sqrt{x}\left(-\frac{b}{c^9}\right)^{\frac{3}{4}}}{b}\right)+5\left(c^3x^2+bc^2\right)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\log\left(5c^2\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}+5\sqrt{x}\right)-5\left(c^3x^2+bc^2\right)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\log\left(-5c^2\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}+5\sqrt{x}\right)-4\left(4cx^2+5b\right)\sqrt{x}}{8\left(c^3x^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*(c^3*x^2 + b*c^2)*(-b/c^9)^(1/4)*arctan((sqrt(c^4*sqrt(-b/c^9) + x
)*c^7*(-b/c^9)^(3/4) - c^7*sqrt(x)*(-b/c^9)^(3/4))/b) + 5*(c^3*x^2 + b*c^2)
*(-b/c^9)^(1/4)*log(5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) - 5*(c^3*x^2 + b*c^2)
*(-b/c^9)^(1/4)*log(-5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) - 4*(4*c*x^2 + 5*b)*
sqrt(x))/(c^3*x^2 + b*c^2)
```

giac [A] time = 0.20, size = 196, normalized size = 0.85

$$-\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{3}}-\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{3}}-\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{3}}+\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{3}}+\frac{b\sqrt{x}}{2\left(cx^{2}+b\right)c^{2}}+\frac{2\sqrt{x}}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/c^3 - 5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 - 5/16*sqrt(2)*(b*c^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 5/16*sqrt(2)*(b*c^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 1/2*b*sqrt(x
)/((c*x^2 + b)*c^2) + 2*sqrt(x)/c^2
```

maple [A] time = 0.01, size = 158, normalized size = 0.69

$$\frac{b\sqrt{x}}{2\left(c\,x^{2}+b\right)c^{2}}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8c^{2}}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8c^{2}}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16c^{2}}+\frac{2\sqrt{x}}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2*x^(1/2)/c^2+1/2*b/c^2*x^(1/2)/(c*x^2+b)-5/16/c^2*(b/c)^(1/4)*2^(1/2)*ln((
x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(
b/c)^(1/2)))-5/8/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)
+1)-5/8/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.06, size = 206, normalized size = 0.90

$$\frac{b\sqrt{x}}{2\left(c^{3}x^{2}+bc^{2}\right)}-\frac{5\left(\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}b^{\frac{1}{4}}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}}\right)}{16c^{2}}+\frac{2\sqrt{x}}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*sqrt(x)/(c^3*x^2 + b*c^2) - 5/16*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2
)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt
```

```
(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) +
 sqrt(2)*b^(1/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))
/c^(1/4) - sqrt(2)*b^(1/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/c^(1/4))/c^2 + 2*sqrt(x)/c^2
```

mupad [B] time = 4.32, size = 80, normalized size = 0.35

$$\frac{2\sqrt{x}}{c^2}-\frac{5\,{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{4\,c^{9/4}}+\frac{b\,\sqrt{x}}{2\,\left(c^3\,x^2+b\,c^2\right)}+\frac{{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{{(-b)}^{1/4}}\right)\,5\mathrm{i}}{4\,c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*x^(1/2))/c^2 - (5*(-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*c^(9
/4)) + ((-b)^(1/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*5i)/(4*c^(9/4)) +
(b*x^(1/2))/(2*(b*c^2 + c^3*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.212 $\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^2}\,dx$

<u>**Optimal**</u>. Leaf size=218

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\dots$$

<u>**Rubi**</u> **[A]** time = 0.17, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -x^(3/2)/(2*c*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4)) - (3*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4))
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

<u>Rule 288</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

<u>Rule 297</u>

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{5/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} - \frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{3/2}} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{3/2}}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^2} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right.}{8c^2}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}} - \frac{3\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right.}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\right.}{8\sqrt{2}\sqrt[4]{b}\,c}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.20

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{b}-\frac{1}{b+cx^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*(-(b + c*x^2)^(-1) + Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)
]/b))/c
```

IntegrateAlgebraic [A] time = 0.33, size = 139, normalized size = 0.64

$$-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*x^(3/2)/(c*(b + c*x^2)) - (3*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1
/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) - (3*ArcTan
h[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(4*Sqrt[2]*b^(1
/4)*c^(7/4))
```

fricas [A] time = 1.05, size = 185, normalized size = 0.85

$$-\frac{12\left(c^2x^2+bc\right)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc^3\sqrt{-\frac{1}{bc^7}}+x}\,c^2\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}-c^2\sqrt{x}\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\right)-3\left(c^2x^2+bc\right)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\log\left(bc^5\left(-\frac{1}{bc^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)+3\left(c^2x^2+bc\right)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\log\left(-bc^5\left(-\frac{1}{bc^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)+4x^{\frac{3}{2}}}{8\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*arctan(sqrt(-b*c^3*sqrt(-1/(b*c
^7)) + x)*c^2*(-1/(b*c^7))^(1/4) - c^2*sqrt(x)*(-1/(b*c^7))^(1/4)) - 3*(c^2
*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(b*c^5*(-1/(b*c^7))^(3/4) + sqrt(x)) + 3*
(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(-b*c^5*(-1/(b*c^7))^(3/4) + sqrt(x))
 + 4*x^(3/2))/(c^2*x^2 + b*c)
```

giac [A] time = 0.18, size = 199, normalized size = 0.91

$$-\frac{x^{\frac{3}{2}}}{2\left(cx^2+b\right)c}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8bc^4}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8bc^4}-\frac{3\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16bc^4}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((c*x^2 + b)*c) + 3/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 3/8*sqrt(2)*(b*c^
3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))
/(b*c^4) - 3/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
```

```
sqrt(b/c))/(b*c^4) + 3/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)
^(1/4) + x + sqrt(b/c))/(b*c^4)
```

maple [A] time = 0.01, size = 149, normalized size = 0.68

$$-\frac{x^{\frac{3}{2}}}{2\left(c\,x^{2}+b\right)c}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x^(3/2)/c/(c*x^2+b)+3/16/c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(
1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+3/8/
c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/c^2/(b/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.00, size = 195, normalized size = 0.89

$$-\frac{x^{\frac{3}{2}}}{2\left(c^{2}x^{2}+bc\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x^(3/2)/(c^2*x^2 + b*c) + 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*s
qrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) -
 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c))
- sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/
4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sq
rt(b))/(b^(1/4)*c^(3/4)))/c
```

mupad [B] time = 0.09, size = 64, normalized size = 0.29

$$\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{4\,{(-b)}^{1/4}\,c^{7/4}}-\frac{x^{3/2}}{2\,c\,\left(c\,x^2+b\right)}-\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{4\,{(-b)}^{1/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(1/4)*c^(7/4)) - x^(3/2)/(2*
c*(b + c*x^2)) - (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(1/4)*c^(7
/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.213 $\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{3/4}c^{5/4}}$$

Rubi [A] time = 0.16, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -Sqrt[x]/(2*c*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(4*Sqrt[2]*b^(3/4)*c^(5/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
/(4*Sqrt[2]*b^(3/4)*c^(5/4)) - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{3/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} + \frac{\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c}$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c}$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{b}\,c} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{b}\,c}$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{b}\,c^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{b}\,c^{3/2}}$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}} +$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{8\sqrt{2}\,b^{3/4}c^{5/4}}$$

Mathematica [A] time = 0.10, size = 198, normalized size = 0.91

$$\frac{-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}-\frac{8\sqrt[4]{c}\sqrt{x}}{b+cx^2}}{16c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-8*c^(1/4)*Sqrt[x])/(b + c*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/b^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/b^(3/4) - (Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/b^(3/4) + (Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/b^(3/4))/(16*c^(5/4))
```

IntegrateAlgebraic [A] time = 0.32, size = 139, normalized size = 0.64

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}b^{3/4}c^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{3/4}c^{5/4}}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*Sqrt[x]/(c*(b + c*x^2)) - ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)
*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]]/(4*Sqrt[2]*b^(3/4)*c^(5/4)) + ArcTanh[(Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)]/(4*Sqrt[2]*b^(3/4)*c^(5
/4))
```

fricas [A] time = 1.00, size = 187, normalized size = 0.86

$$\frac{4\left(c^2x^2+bc\right)\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x}\,b^2c^4\left(-\frac{1}{b^3c^5}\right)^{\frac{3}{4}}-b^2c^4\sqrt{x}\left(-\frac{1}{b^3c^5}\right)^{\frac{3}{4}}\right)+\left(c^2x^2+bc\right)\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}\log\left(bc\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\left(c^2x^2+bc\right)\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}\log\left(-bc\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\sqrt{x}}{8\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*arctan(sqrt(b^2*c^2*sqrt(-1/(b^
3*c^5)) + x)*b^2*c^4*(-1/(b^3*c^5))^(3/4) - b^2*c^4*sqrt(x)*(-1/(b^3*c^5))^
(3/4)) + (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(b*c*(-1/(b^3*c^5))^(1/4)
+ sqrt(x)) - (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(-b*c*(-1/(b^3*c^5))^(
1/4) + sqrt(x)) - 4*sqrt(x))/(c^2*x^2 + b*c)
```

giac [A] time = 0.17, size = 199, normalized size = 0.91

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8bc^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8bc^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16bc^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16bc^2}-\frac{\sqrt{x}}{2\left(cx^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c^2) + 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) + 1/16*sqrt(2)*(b*c^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2) - 1/16*sqrt
```

```
(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2)
 - 1/2*sqrt(x)/((c*x^2 + b)*c)
```

maple [A] time = 0.01, size = 158, normalized size = 0.72

$$-\frac{\sqrt{x}}{2\left(c\,x^{2}+b\right)c}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8bc}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8bc}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x^(1/2)/c/(c*x^2+b)+1/16/c*(b/c)^(1/4)/b*2^(1/2)*ln((x+(b/c)^(1/4)*2^(
1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/8/
c*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/c*(b/c)^(
1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.10, size = 195, normalized size = 0.89

$$\frac{\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,c}-\frac{\sqrt{x}}{2\left(c^{2}x^{2}+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt
(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c - 1
/2*sqrt(x)/(c^2*x^2 + b*c)
```

mupad [B] time = 4.31, size = 64, normalized size = 0.29

$$-\frac{\sqrt{x}}{2\,c\,\left(c\,x^2+b\right)}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{3/4}\,c^{5/4}}-\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{3/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - x^(1/2)/(2*c*(b + c*x^2)) - atan((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(3
/4)*c^(5/4)) - atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(3/4)*c^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.214 $\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{5/4}c^{3/4}}$$

Rubi [A] time = 0.16, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(
4*Sqrt[2]*b^(5/4)*c^(3/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(4*Sqrt[2]*b^(5/4)*c^(3/4)) + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx$$
$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b}$$
$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b}$$
$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b\sqrt{c}}$$
$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc} +$$
$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}} +$$
$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{8\sqrt{2}\,b^{5/4}c^{3/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^2)
```

IntegrateAlgebraic [A] time = 0.31, size = 139, normalized size = 0.64

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)
/(Sqrt[2]*b^(1/4)))/Sqrt[x]]/(4*Sqrt[2]*b^(5/4)*c^(3/4)) - ArcTanh[(Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)]/(4*Sqrt[2]*b^(5/4)*c^(3/4)
)
```

fricas [A] time = 1.93, size = 182, normalized size = 0.83

$$-\frac{4\left(bcx^2+b^2\right)\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b^3c\sqrt{-\frac{1}{b^5c^3}}+x}\,bc\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}-bc\sqrt{x}\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\right)-\left(bcx^2+b^2\right)\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\log\left(b^4c^2\left(-\frac{1}{b^5c^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)+\left(bcx^2+b^2\right)\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\log\left(-b^4c^2\left(-\frac{1}{b^5c^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4x^{\frac{3}{2}}}{8\left(bcx^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*arctan(sqrt(-b^3*c*sqrt(-1/(b^
5*c^3)) + x)*b*c*(-1/(b^5*c^3))^(1/4) - b*c*sqrt(x)*(-1/(b^5*c^3))^(1/4)) -
 (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(b^4*c^2*(-1/(b^5*c^3))^(3/4) + sq
rt(x)) + (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(-b^4*c^2*(-1/(b^5*c^3))^(
3/4) + sqrt(x)) - 4*x^(3/2))/(b*c*x^2 + b^2)
```

giac [A] time = 0.18, size = 199, normalized size = 0.91

$$\frac{x^{\frac{3}{2}}}{2\left(cx^2+b\right)b}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^2c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^2c^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^2c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((c*x^2 + b)*b) + 1/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/8*sqrt(2)*(b*c
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/(b^2*c^3) - 1/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) +
```

```
x + sqrt(b/c))/(b^2*c^3) + 1/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

maple [A] time = 0.01, size = 158, normalized size = 0.72

$$\frac{x^{\frac{3}{2}}}{2\left(c\,x^{2}+b\right)b}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}bc}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}bc}+\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^(3/2)/b/(c*x^2+b)+1/16/b/c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1
/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/8/b
/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/b/c/(b/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 194, normalized size = 0.89

$$\frac{x^{\frac{3}{2}}}{2\left(bcx^{2}+b^{2}\right)}+\frac{\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^(3/2)/(b*c*x^2 + b^2) + 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b
^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sq
rt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) -
2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) -
 sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4
)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqr
t(b))/(b^(1/4)*c^(3/4)))/b
```

mupad [B] time = 4.34, size = 64, normalized size = 0.29

$$\frac{x^{3/2}}{2\,b\,\left(c\,x^2+b\right)}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{5/4}\,c^{3/4}}+\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{5/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - atan((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(5/4
)*c^(3/4)) + atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(5/4)*c^(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.215 $\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=218

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Rubi [A] time = 0.17, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] Sqrt[x]/(2*b*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) - (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*Log[Sqrt[b] + Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{\sqrt{x}\,\left(b+cx^2\right)^2}\,dx$$
$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} + \frac{3\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b}$$
$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b}$$
$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}}$$
$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{3/2}\sqrt{c}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{3/2}\sqrt{c}}$$
$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} - \frac{3\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{b}+\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}$$
$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\right.}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.09, size = 199, normalized size = 0.91

$$\frac{\frac{8b^{3/4}\sqrt{x}}{b+cx^2} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{\sqrt[4]{c}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{c}}}{16b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((8*b^(3/4)*Sqrt[x])/(b + c*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)])/c^(1/4) - (3*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/c^(1/4) + (3*Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/c^(1/4))/(16*b^(7/4))
```

IntegrateAlgebraic [A] time = 0.30, size = 139, normalized size = 0.64

$$-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] Sqrt[x]/(2*b*(b + c*x^2)) - (3*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)
*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*ArcTanh[(
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(4*Sqrt[2]*b^(7/4)
*c^(1/4))
```

fricas [A] time = 1.08, size = 179, normalized size = 0.82

$$\frac{12\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right)^{\frac14}\arctan\left(\sqrt{b^4\sqrt{-\frac{1}{b^7c}}+x}\,b^5c\left(-\frac{1}{b^7c}\right)^{\frac34}-b^5c\sqrt{x}\left(-\frac{1}{b^7c}\right)^{\frac34}\right)+3\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right)^{\frac14}\log\left(b^2\left(-\frac{1}{b^7c}\right)^{\frac14}+\sqrt{x}\right)-3\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right)^{\frac14}\log\left(-b^2\left(-\frac{1}{b^7c}\right)^{\frac14}+\sqrt{x}\right)+4\sqrt{x}}{8\left(bcx^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(12*(b*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*arctan(sqrt(b^4*sqrt(-1/(b^7*c))
 + x)*b^5*c*(-1/(b^7*c))^(3/4) - b^5*c*sqrt(x)*(-1/(b^7*c))^(3/4)) + 3*(b*c
*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(b^2*(-1/(b^7*c))^(1/4) + sqrt(x)) - 3*(b
*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(-b^2*(-1/(b^7*c))^(1/4) + sqrt(x)) + 4
*sqrt(x))/(b*c*x^2 + b^2)
```

giac [A] time = 0.19, size = 199, normalized size = 0.91

$$\frac{3\sqrt{2}\left(bc^3\right)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{8b^2c}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac14}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{8b^2c}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac14}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{16b^2c}-\frac{3\sqrt{2}\left(bc^3\right)^{\frac14}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{16b^2c}+\frac{\sqrt{x}}{2\left(cx^2+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b^2*c) + 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 3/16*sqrt(2)*(b*c^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 3/16*sqrt
```

```
(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c)
 + 1/2*sqrt(x)/((c*x^2 + b)*b)
```

maple [A] time = 0.01, size = 149, normalized size = 0.68

$$\frac{\sqrt{x}}{2\left(c\,x^{2}+b\right)b}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8b^{2}}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^(1/2)/b/(c*x^2+b)+3/16/b^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1
/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+3/8/b
^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/b^2*(b/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.05, size = 194, normalized size = 0.89

$$\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{16\,b}+\frac{\sqrt{x}}{2\left(bcx^{2}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt
(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b + 1
/2*sqrt(x)/(b*c*x^2 + b^2)
```

mupad [B] time = 0.10, size = 64, normalized size = 0.29

$$\frac{\sqrt{x}}{2\,b\,\left(c\,x^2+b\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(1/2)/(2*b*(b + c*x^2)) + (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^
(7/4)*c^(1/4)) + (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(7/4)*c^(1
/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.216 $\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{5\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}}-$$

Rubi [A] time = 0.19, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5}{2b^2\sqrt{x}}+\frac{1}{2b\sqrt{x}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -5/(2*b^2*Sqrt[x]) + 1/(2*b*Sqrt[x]*(b + c*x^2)) + (5*c^(1/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)
) + (5*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(8*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{3/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} + \frac{5\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{(5c)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{(5c)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} + \frac{\left(5\sqrt{c}\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^2} - \frac{\left(5\sqrt{c}\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}}{b}\right.}{4b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^2} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}\right.}{8b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{5\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}} + \frac{5\sqrt[4]{c}\,\log\left(\sqrt{b}+\right.}{8}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} + \frac{5\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}} - \frac{5\sqrt[4]{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}} - \frac{5}{}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-\frac{cx^2}{b}\right)}{b^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 2, 3/4, -((c*x^2)/b)])/(b^2*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.34, size = 149, normalized size = 0.65

$$\frac{5\sqrt[4]{c}\,\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}\,b^{9/4}}+\frac{-4b-5cx^2}{2b^2\sqrt{x}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-4*b - 5*c*x^2)/(2*b^2*Sqrt[x]*(b + c*x^2)) + (5*c^(1/4)*ArcTan[(b^(1/4)/(
Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*b^(9
/4)) + (5*c^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt
[c]*x)])/(4*Sqrt[2]*b^(9/4))
```

fricas [A] time = 1.19, size = 208, normalized size = 0.90

$$\frac{20\left(b^2cx^3+b^3x\right)\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,b^2c\sqrt{x}\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}-\sqrt{-15625\,b^5c\sqrt{-\frac{c}{b^9}}+15625\,c^2x}\,b^2\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}}{125\,c}\right)-5\left(b^2cx^3+b^3x\right)\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\log\left(125\,b^7\left(-\frac{c}{b^9}\right)^{\frac{3}{4}}+125\,c\sqrt{x}\right)+5\left(b^2cx^3+b^3x\right)\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\log\left(-125\,b^7\left(-\frac{c}{b^9}\right)^{\frac{3}{4}}+125\,c\sqrt{x}\right)-4\left(5\,cx^2+4\,b\right)\sqrt{x}}{8\left(b^2cx^3+b^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(20*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*arctan(-1/125*(125*b^2*c*sqrt(x)
*(-c/b^9)^(1/4) - sqrt(-15625*b^5*c*sqrt(-c/b^9) + 15625*c^2*x)*b^2*(-c/b^9
)^(1/4))/c) - 5*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*log(125*b^7*(-c/b^9)^(3/
4) + 125*c*sqrt(x)) + 5*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*log(-125*b^7*(-c
/b^9)^(3/4) + 125*c*sqrt(x)) - 4*(5*c*x^2 + 4*b)*sqrt(x))/(b^2*c*x^3 + b^3*
x)
```

giac [A] time = 0.17, size = 210, normalized size = 0.91

$$-\frac{5\,cx^{2}+4\,b}{2\,\left(cx^{\frac{5}{2}}+b\sqrt{x}\right)b^{2}}-\frac{5\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{3}c^{2}}-\frac{5\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{3}c^{2}}+\frac{5\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{3}c^{2}}-\frac{5\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*c*x^2 + 4*b)/((c*x^(5/2) + b*sqrt(x))*b^2) - 5/8*sqrt(2)*(b*c^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*
c^2) - 5/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 5/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 5/16*sqrt(2)*(b*c^3)^(3/4
)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2)
```

maple [A] time = 0.02, size = 158, normalized size = 0.69

$$-\frac{c\,x^{\frac{3}{2}}}{2\left(c\,x^{2}+b\right)b^{2}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{2}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{2}}-\frac{5\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{2}}-\frac{2}{b^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*c*x^(3/2)/(c*x^2+b)-5/16/b^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)
*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-
5/8/b^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-5/8/b^2/(
b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/b^2/x^(1/2)
```

maxima [A] time = 2.93, size = 208, normalized size = 0.90

$$-\frac{5\,cx^{2}+4\,b}{2\left(b^{2}cx^{\frac{5}{2}}+b^{3}\sqrt{x}\right)}-\frac{5\,c\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(5*c*x^2 + 4*b)/(b^2*c*x^(5/2) + b^3*sqrt(x)) - 5/16*c*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sq
rt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqr
t(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*
sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^2
```

mupad [B] time = 0.09, size = 77, normalized size = 0.33

$$\frac{5\,{\left(-c\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{9/4}}-\frac{5\,{\left(-c\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{9/4}}-\frac{\frac{2}{b}+\frac{5\,c\,x^2}{2\,b^2}}{b\,\sqrt{x}+c\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (5*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(9/4)) - (5*(-c)^(1
/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(9/4)) - (2/b + (5*c*x^2)/(2*b
^2))/(b*x^(1/2) + c*x^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.217 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=230

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}}-$$

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -7/(6*b^2*x^(3/2)) + 1/(2*b*x^(3/2)*(b + c*x^2)) + (7*c^(3/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) - (7*c^(3/4)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) + (7*c^(3/4)*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11
/4)) - (7*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{5/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{3/2}\left(b+cx^2\right)} + \frac{7\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} - \frac{(7c)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} - \frac{(7c)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} - \frac{(7c)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{5/2}} - \frac{(7c)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{4b^{5/2}}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} - \frac{\left(7\sqrt{c}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{5/2}} - \frac{\left(7\sqrt{c}\right)\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} + \frac{7c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}} - \frac{7c^{3/4}\log\left(\sqrt{b}+\right.}{8}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} + \frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}} - \frac{7c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}} + \cdots$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-\frac{cx^2}{b}\right)}{3b^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 2, 1/4, -((c*x^2)/b)])/(3*b^2*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.34, size = 149, normalized size = 0.65

$$\frac{7c^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}\,b^{11/4}}+\frac{-4b-7cx^2}{6b^2x^{3/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-4*b - 7*c*x^2)/(6*b^2*x^(3/2)*(b + c*x^2)) + (7*c^(3/4)*ArcTan[(b^(1/4)/(
Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*b^(1
1/4)) - (7*c^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqr
t[c]*x)])/(4*Sqrt[2]*b^(11/4))
```

fricas [A] time = 1.33, size = 228, normalized size = 0.99

$$-\frac{84\left(b^2cx^4+b^3x^2\right)\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^8c\sqrt{x}\left(-\frac{c^3}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^6\sqrt{-\frac{c^3}{b^{11}}}+c^2x}\,b^8\left(-\frac{c^3}{b^{11}}\right)^{\frac{3}{4}}}{c^3}\right)+21\left(b^2cx^4+b^3x^2\right)\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}\log\left(7b^3\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}+7c\sqrt{x}\right)-21\left(b^2cx^4+b^3x^2\right)\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}\log\left(-7b^3\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}+7c\sqrt{x}\right)+4\left(7cx^2+4b\right)\sqrt{x}}{24\left(b^2cx^4+b^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(84*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*arctan(-(b^8*c*sqrt(x)*(-
c^3/b^11)^(3/4) - sqrt(b^6*sqrt(-c^3/b^11) + c^2*x)*b^8*(-c^3/b^11)^(3/4))/
c^3) + 21*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(7*b^3*(-c^3/b^11)^(1/
4) + 7*c*sqrt(x)) - 21*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(-7*b^3*(
-c^3/b^11)^(1/4) + 7*c*sqrt(x)) + 4*(7*c*x^2 + 4*b)*sqrt(x))/(b^2*c*x^4 + b
^3*x^2)
```

giac [A] time = 0.18, size = 196, normalized size = 0.85

$$-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^3}-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^3}-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^3}+\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^3}-\frac{c\sqrt{x}}{2\left(cx^2+b\right)b^2}-\frac{2}{3b^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")

[Out] -7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^3 - 7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 - 7/16*sqrt(2)*(b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 7/16*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/2*c*sqrt(x)/((c*x^2 + b)*b^2) - 2/3/(b^2*x^(3/2))

maple [A] time = 0.02, size = 161, normalized size = 0.70

$$-\frac{c\sqrt{x}}{2\left(c\,x^{2}+b\right)b^{2}}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8b^{3}}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^{3}}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16b^{3}}-\frac{2}{3b^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(c*x^4+b*x^2)^2,x)

[Out] -1/2/b^2*c*x^(1/2)/(c*x^2+b)-7/16/b^3*c*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/3/b^2/x^(3/2)

maxima [A] time = 3.03, size = 209, normalized size = 0.91

$$-\frac{7\,cx^{2}+4\,b}{6\left(b^{2}cx^{\frac{7}{2}}+b^{3}x^{\frac{3}{2}}\right)}-\frac{7\left(\frac{2\,\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{16\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")

[Out] -1/6*(7*c*x^2 + 4*b)/(b^2*c*x^(7/2) + b^3*x^(3/2)) - 7/16*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(3/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^2

mupad [B] time = 0.11, size = 77, normalized size = 0.33

$$\frac{7\,{\left(-c\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{11/4}}-\frac{\frac{2}{3\,b}+\frac{7\,c\,x^2}{6\,b^2}}{b\,x^{3/2}+c\,x^{7/2}}+\frac{7\,{\left(-c\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (7*(-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(11/4)) - (2/(3*b) +
 (7*c*x^2)/(6*b^2))/(b*x^(3/2) + c*x^(7/2)) + (7*(-c)^(3/4)*atanh(((-c)^(1/
4)*x^(1/2))/b^(1/4)))/(4*b^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.218 $\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=243

$$\frac{9c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{9c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}}+\cdots$$

Rubi [A] time = 0.22, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{9c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9c}{2b^3\sqrt{x}}-\frac{9}{10b^2x^{5/2}}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -9/(10*b^2*x^(5/2)) + (9*c)/(2*b^3*Sqrt[x]) + 1/(2*b*x^(5/2)*(b + c*x^2)) -
 (9*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13
/4)) + (9*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*b^(13/4)) + (9*c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*b^(13/4)) - (9*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{9\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{(9c)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{\left(9c^2\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^3}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{\left(9c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^3}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{\left(9c^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^3} + \frac{\left(9c^{3/2}\right)\text{S}}{}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{(9c)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{8b^3} + \frac{(9c)\,\text{S}}{}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{9c^{5/4}\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}} - \frac{9c^{5/4}\,\text{l}}{}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{9c^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}} + \frac{9c^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}}{}\right.}{4\sqrt{2}\,b^{13/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-\frac{cx^2}{b}\right)}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 2, -1/4, -((c*x^2)/b)])/(5*b^2*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.35, size = 160, normalized size = 0.66

$$-\frac{9c^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}\,b^{13/4}}-\frac{9c^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{-4b^2+36bcx^2+45c^2x^4}{10b^3x^{5/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-4*b^2 + 36*b*c*x^2 + 45*c^2*x^4)/(10*b^3*x^(5/2)*(b + c*x^2)) - (9*c^(5/4
)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x
]])/(4*Sqrt[2]*b^(13/4)) - (9*c^(5/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x])/(Sqrt[b] + Sqrt[c]*x)])/(4*Sqrt[2]*b^(13/4))
```

fricas [A] time = 1.04, size = 251, normalized size = 1.03

$$-\frac{180\left(b^3cx^5+b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729b^3c^4\sqrt{x}\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-531441b^7c^5\sqrt{-\frac{c^5}{b^{13}}}+531441c^8x}\,b^3\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}}{729c^5}\right)-45\left(b^3cx^5+b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}\log\left(729b^{10}\left(-\frac{c^5}{b^{13}}\right)^{\frac{3}{4}}+729c^4\sqrt{x}\right)+45\left(b^3cx^5+b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}\log\left(-729b^{10}\left(-\frac{c^5}{b^{13}}\right)^{\frac{3}{4}}+729c^4\sqrt{x}\right)-4\left(45c^2x^4+36bcx^2-4b^2\right)\sqrt{x}}{40\left(b^3cx^5+b^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(180*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)^(1/4)*arctan(-1/729*(729*b^3*c
^4*sqrt(x)*(-c^5/b^13)^(1/4) - sqrt(-531441*b^7*c^5*sqrt(-c^5/b^13) + 53144
1*c^8*x)*b^3*(-c^5/b^13)^(1/4))/c^5) - 45*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)
^(1/4)*log(729*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x)) + 45*(b^3*c*x^5 +
b^4*x^3)*(-c^5/b^13)^(1/4)*log(-729*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x
)) - 4*(45*c^2*x^4 + 36*b*c*x^2 - 4*b^2)*sqrt(x))/(b^3*c*x^5 + b^4*x^3)
```

giac [A] time = 0.19, size = 220, normalized size = 0.91

$$\frac{c^2x^{\frac{3}{2}}}{2\left(cx^2+b\right)b^3}+\frac{9\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^4c}+\frac{9\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^4c}-\frac{9\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^4c}+\frac{9\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^4c}+\frac{2\left(10cx^2-b\right)}{5b^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^(3/2)/((c*x^2 + b)*b^3) + 9/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 9/8*sqrt(2)*
(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b^4*c) - 9/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/(b^4*c) + 9/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 2/5*(10*c*x^2 - b)/(b^3*x^(5/2))
```

maple [A] time = 0.02, size = 172, normalized size = 0.71

$$\frac{c^{2} x^{\frac{3}{2}}}{2\left(c\,x^{2}+b\right) b^{3}}+\frac{9\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{3}}+\frac{9\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{3}}+\frac{9\sqrt{2}\,c\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{3}}+\frac{4c}{b^{3}\sqrt{x}}-\frac{2}{5b^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b^3*c^2*x^(3/2)/(c*x^2+b)+9/16/b^3*c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1
/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
))+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8/
b^3*c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/5/b^2/x^(
5/2)+4*c/b^3/x^(1/2)
```

maxima [A] time = 2.97, size = 221, normalized size = 0.91

$$\frac{45\,c^{2}x^{4}+36\,bcx^{2}-4\,b^{2}}{10\left(b^{3}cx^{\frac{9}{2}}+b^{4}x^{\frac{5}{2}}\right)}+\frac{9\,c^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(45*c^2*x^4 + 36*b*c*x^2 - 4*b^2)/(b^3*c*x^(9/2) + b^4*x^(5/2)) + 9/16
*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c
```

```
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt
(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

mupad [B] time = 4.37, size = 87, normalized size = 0.36

$$\frac{\frac{18\,c\,x^2}{5\,b^2}-\frac{2}{5\,b}+\frac{9\,c^2\,x^4}{2\,b^3}}{b\,x^{5/2}+c\,x^{9/2}}-\frac{9\,{\left(-c\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{13/4}}+\frac{9\,{\left(-c\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((18*c*x^2)/(5*b^2) - 2/(5*b) + (9*c^2*x^4)/(2*b^3))/(b*x^(5/2) + c*x^(9/2)
) - (9*(-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(13/4)) + (9*(-c
)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.219 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=243

$$-\frac{11c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}+\frac{11c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}-\frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}$$

Rubi [A] time = 0.21, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{11c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}+\frac{11c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}-\frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}+\frac{11c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{15/4}}+\frac{11c}{6b^3x^{3/2}}-\frac{11}{14b^2x^{7/2}}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -11/(14*b^2*x^(7/2)) + (11*c)/(6*b^3*x^(3/2)) + 1/(2*b*x^(7/2)*(b + c*x^2))
 - (11*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^
(15/4)) + (11*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqr
t[2]*b^(15/4)) - (11*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) + (11*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{11\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{(11c)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^3}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{7/2}} + \frac{(11c}{}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{7/2}}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{11c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}} + \frac{1}{}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}} + \frac{11c^{7/4}\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},2;-\frac{3}{4};-\frac{cx^2}{b}\right)}{7b^2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 2, -3/4, -((c*x^2)/b)])/(7*b^2*x^(7/2))
```

IntegrateAlgebraic [A] time = 0.35, size = 160, normalized size = 0.66

$$-\frac{11c^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}\,b^{15/4}}+\frac{11c^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4\sqrt{2}\,b^{15/4}}+\frac{-12b^2+44bcx^2+77c^2x^4}{42b^3x^{7/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (-12*b^2 + 44*b*c*x^2 + 77*c^2*x^4)/(42*b^3*x^(7/2)*(b + c*x^2)) - (11*c^(7
/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt
[x]])/(4*Sqrt[2]*b^(15/4)) + (11*c^(7/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*S
qrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(4*Sqrt[2]*b^(15/4))
```

fricas [A] time = 0.79, size = 245, normalized size = 1.01

$$\frac{924\left(b^3cx^6+b^4x^4\right)\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{11}c^2\sqrt{x}\left(-\frac{c^7}{b^{15}}\right)^{\frac{3}{4}}-\sqrt{b^8\sqrt{-\frac{c^7}{b^{15}}}+c^4x}\,b^{11}\left(-\frac{c^7}{b^{15}}\right)^{\frac{3}{4}}}{c^7}\right)+231\left(b^3cx^6+b^4x^4\right)\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}\log\left(11\,b^4\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}+11\,c^2\sqrt{x}\right)-231\left(b^3cx^6+b^4x^4\right)\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}\log\left(-11\,b^4\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}+11\,c^2\sqrt{x}\right)+4\left(77\,c^2x^4+44\,bcx^2-12\,b^2\right)\sqrt{x}}{168\left(b^3cx^6+b^4x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/168*(924*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*arctan(-(b^11*c^2*sqrt(x
)*(-c^7/b^15)^(3/4) - sqrt(b^8*sqrt(-c^7/b^15) + c^4*x)*b^11*(-c^7/b^15)^(3
/4))/c^7) + 231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*log(11*b^4*(-c^7/b^
15)^(1/4) + 11*c^2*sqrt(x)) - 231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*l
og(-11*b^4*(-c^7/b^15)^(1/4) + 11*c^2*sqrt(x)) + 4*(77*c^2*x^4 + 44*b*c*x^2
 - 12*b^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

giac [A] time = 0.17, size = 212, normalized size = 0.87

$$\frac{11\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{11\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{11\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4}-\frac{11\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4}+\frac{c^2\sqrt{x}}{2\left(cx^2+b\right)b^3}+\frac{2\left(14\,cx^2-3\,b\right)}{21\,b^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/b^4 + 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 11/16*sqrt(2)*(b*c^3)^
(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 11/16*sqrt(2
)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 1
/2*c^2*sqrt(x)/((c*x^2 + b)*b^3) + 2/21*(14*c*x^2 - 3*b)/(b^3*x^(7/2))
```

maple [A] time = 0.02, size = 178, normalized size = 0.73

$$\frac{c^{2} \sqrt{x}}{2 \left(c \,x^{2}+b \right) b^{3}}+\frac{11 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, c^{2} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8 b^{4}}+\frac{11 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, c^{2} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8 b^{4}}+\frac{11 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, c^{2} \ln \left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16 b^{4}}+\frac{4 c}{3 b^{3} x^{\frac{3}{2}}}-\frac{2}{7 b^{2} x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^2/x^(1/2),x)
```

```
[Out] 1/2/b^3*c^2*x^(1/2)/(c*x^2+b)+11/16/b^4*c^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)
^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1
/2)))+11/8/b^4*c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1
)+11/8/b^4*c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/
7/b^2/x^(7/2)+4/3*c/b^3/x^(3/2)
```

maxima [A] time = 3.16, size = 224, normalized size = 0.92

$$\frac{77 \, c^{2} x^{4} + 44 \, b c x^{2} - 12 \, b^{2}}{42 \, {\left(b^{3} c x^{\frac{11}{2}} + b^{4} x^{\frac{7}{2}}\right)}} + \frac{11 \, {\left(\frac{2 \, \sqrt{2} c^{2} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} + 2 \, \sqrt{c} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{b} \sqrt{c}}}\right)}{\sqrt{b} \sqrt{\sqrt{b} \sqrt{c}}} + \frac{2 \, \sqrt{2} c^{2} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} - 2 \, \sqrt{c} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{b} \sqrt{c}}}\right)}{\sqrt{b} \sqrt{\sqrt{b} \sqrt{c}}} + \frac{\sqrt{2} c^{\frac{7}{4}} \log\left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x} + \sqrt{c} x + \sqrt{b}\right)}{b^{\frac{3}{4}}} - \frac{\sqrt{2} c^{\frac{7}{4}} \log\left(-\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x} + \sqrt{c} x + \sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}}{16 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/42*(77*c^2*x^4 + 44*b*c*x^2 - 12*b^2)/(b^3*c*x^(11/2) + b^4*x^(7/2)) + 11
/16*(2*sqrt(2)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)
*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7
/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^
3
```

mupad [B] time = 0.11, size = 87, normalized size = 0.36

$$\frac{\frac{22\,c\,x^2}{21\,b^2}-\frac{2}{7\,b}+\frac{11\,c^2\,x^4}{6\,b^3}}{b\,x^{7/2}+c\,x^{11/2}}+\frac{11\,{\left(-c\right)}^{7/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{15/4}}+\frac{11\,{\left(-c\right)}^{7/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((22*c*x^2)/(21*b^2) - 2/(7*b) + (11*c^2*x^4)/(6*b^3))/(b*x^(7/2) + c*x^(11
/2)) + (11*(-c)^(7/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(15/4)) + (1
1*(-c)^(7/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] Timed out
```

3.220 $\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=258

$$-\frac{13c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{17/4}}+\frac{13c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{17/4}}+\frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{17/4}}$$

Rubi [A] time = 0.23, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{13c^2}{2b^4\sqrt{x}}-\frac{13c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{17/4}}+\frac{13c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{17/4}}+\frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{17/4}}-\frac{13c^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{17/4}}+\frac{13c}{10b^3x^{5/2}}-\frac{13}{18b^2x^{9/2}}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -13/(18*b^2*x^(9/2)) + (13*c)/(10*b^3*x^(5/2)) - (13*c^2)/(2*b^4*Sqrt[x]) +
 1/(2*b*x^(9/2)*(b + c*x^2)) + (13*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt
[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4)) + (13*c^(9/4
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^
(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{13\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{(13c)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{\left(13c^2\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{\left(13c^3\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{\left(13c^3\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{\left(13c^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \right.}{4b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{\left(13c^2\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, \right.}{8b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{13c^{9/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{8\sqrt{2}\,b^{17/4}}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{17/4}} - \frac{13c}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{9}{4},2;-\frac{5}{4};-\frac{cx^2}{b}\right)}{9b^2x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-9/4, 2, -5/4, -((c*x^2)/b)])/(9*b^2*x^(9/2))
```

IntegrateAlgebraic [A] time = 0.39, size = 171, normalized size = 0.66

$$\frac{13c^{9/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{4\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{17/4}}+\frac{-20b^3+52b^2cx^2-468bc^2x^4-585c^3x^6}{90b^4x^{9/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (-20*b^3 + 52*b^2*c*x^2 - 468*b*c^2*x^4 - 585*c^3*x^6)/(90*b^4*x^(9/2)*(b +
 c*x^2)) + (13*c^(9/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqr
t[2]*b^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*b^(17/4)) + (13*c^(9/4)*ArcTanh[(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(4*Sqrt[2]*b^(17/4))
```

fricas [A] time = 1.16, size = 262, normalized size = 1.02

$$\frac{2340\left(b^4cx^7+b^5x^5\right)\left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{2197\,b^4c^7\sqrt{x}\left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}}-\sqrt{-4826809\,b^9c^9\sqrt{-\frac{c^9}{b^{17}}}+4826809\,c^{14}x}\,b^4\left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}}}{2197\,c^9}\right)-585\left(b^4cx^7+b^5x^5\right)\left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}}\log\left(2197\,b^{13}\left(-\frac{c^9}{b^{17}}\right)^{\frac{3}{4}}+2197\,c^7\sqrt{x}\right)+585\left(b^4cx^7+b^5x^5\right)\left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}}\log\left(-2197\,b^{13}\left(-\frac{c^9}{b^{17}}\right)^{\frac{3}{4}}+2197\,c^7\sqrt{x}\right)-4\left(585\,c^3x^6+468\,bc^2x^4-52\,b^2cx^2+20\,b^3\right)\sqrt{x}}{360\left(b^4cx^7+b^5x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(2340*(b^4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*arctan(-1/2197*(2197*b^
4*c^7*sqrt(x)*(-c^9/b^17)^(1/4) - sqrt(-4826809*b^9*c^9*sqrt(-c^9/b^17) + 4
826809*c^14*x)*b^4*(-c^9/b^17)^(1/4))/c^9) - 585*(b^4*c*x^7 + b^5*x^5)*(-c^
9/b^17)^(1/4)*log(2197*b^13*(-c^9/b^17)^(3/4) + 2197*c^7*sqrt(x)) + 585*(b^
4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*log(-2197*b^13*(-c^9/b^17)^(3/4) + 219
7*c^7*sqrt(x)) - 4*(585*c^3*x^6 + 468*b*c^2*x^4 - 52*b^2*c*x^2 + 20*b^3)*sq
rt(x))/(b^4*c*x^7 + b^5*x^5)
```

giac [A] time = 0.17, size = 219, normalized size = 0.85

$$-\frac{c^3x^{\frac{3}{2}}}{2\left(cx^2+b\right)b^4}-\frac{13\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5}-\frac{13\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5}+\frac{13\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5}-\frac{13\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5}-\frac{2\left(135\,c^2x^4-18\,bcx^2+5\,b^2\right)}{45\,b^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*c^3*x^(3/2)/((c*x^2 + b)*b^4) - 13/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*
sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 13/8*sqrt(2)*(
b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1
/4))/b^5 + 13/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
+ sqrt(b/c))/b^5 - 13/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(
1/4) + x + sqrt(b/c))/b^5 - 2/45*(135*c^2*x^4 - 18*b*c*x^2 + 5*b^2)/(b^4*x^
(9/2))
```

maple [A] time = 0.02, size = 189, normalized size = 0.73

$$-\frac{c^{3}x^{\frac{3}{2}}}{2\left(cx^{2}+b\right)b^{4}}-\frac{13\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{4}}-\frac{13\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{4}}-\frac{13\sqrt{2}\,c^{2}\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{4}}-\frac{6c^{2}}{b^{4}\sqrt{x}}+\frac{4c}{5b^{3}x^{\frac{5}{2}}}-\frac{2}{9b^{2}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^4*c^3*x^(3/2)/(c*x^2+b)-13/16/b^4*c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c
)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(
1/2)))-13/8/b^4*c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+
1)-13/8/b^4*c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2
/9/b^2/x^(9/2)-6*c^2/b^4/x^(1/2)+4/5*c/b^3/x^(5/2)
```

maxima [A] time = 3.01, size = 232, normalized size = 0.90

$$-\frac{585\,c^{3}x^{6}+468\,bc^{2}x^{4}-52\,b^{2}cx^{2}+20\,b^{3}}{90\left(b^{4}cx^{\frac{13}{2}}+b^{5}x^{\frac{9}{2}}\right)}-\frac{13\,c^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/90*(585*c^3*x^6 + 468*b*c^2*x^4 - 52*b^2*c*x^2 + 20*b^3)/(b^4*c*x^(13/2)
 + b^5*x^(9/2)) - 13/16*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*
sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2
)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/
4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(1/4)*c^(3/4)))/b^4
```

mupad [B] time = 4.37, size = 99, normalized size = 0.38

$$\frac{13\,{\left(-c\right)}^{9/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{17/4}}-\frac{13\,{\left(-c\right)}^{9/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{17/4}}-\frac{\frac{2}{9\,b}-\frac{26\,c\,x^2}{45\,b^2}+\frac{26\,c^2\,x^4}{5\,b^3}+\frac{13\,c^3\,x^6}{2\,b^4}}{b\,x^{9/2}+c\,x^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] (13*(-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(17/4)) - (13*(-c)
^(9/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(17/4)) - (2/(9*b) - (26*c*
x^2)/(45*b^2) + (26*c^2*x^4)/(5*b^3) + (13*c^3*x^6)/(2*b^4))/(b*x^(9/2) + c
*x^(13/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.221 $\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=251

$$\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{13/4}}$$

Rubi [A] time = 0.21, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}+\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}c^{13/4}}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}+\frac{45\sqrt{x}}{16c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (45*Sqrt[x])/(16*c^3) - x^(9/2)/(4*c*(b + c*x^2)^2) - (9*x^(5/2))/(16*c^2*(
b + c*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*c^(13/4)) + (45*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{23/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{11/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{9/2}}{4c\left(b+cx^2\right)^2} + \frac{9\int \frac{x^{7/2}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} + \frac{45\int \frac{x^{3/2}}{b+cx^2}\,dx}{32c^2}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} - \frac{(45b)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32c^3}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} - \frac{(45b)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^3}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} - \frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32c^3} - \cdots$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} - \frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64c^{7/2}} - \cdots$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} + \frac{45\sqrt[4]{b}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,c^{13/4}} - \cdots$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} + \frac{45\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,c^{13/4}} - \frac{45\sqrt[4]{b}\,\tan^{-1}\left(1+\cdots\right)}{32\sqrt{2}\,c^{13\cdots}}$$

Mathematica [A] time = 0.25, size = 220, normalized size = 0.88

$$\frac{\frac{8\sqrt[4]{c}\sqrt{x}\left(45b^2+81bcx^2+32c^2x^4\right)}{\left(b+cx^2\right)^2}+45\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-45\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)+90\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-90\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{128c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((8*c^(1/4)*Sqrt[x]*(45*b^2 + 81*b*c*x^2 + 32*c^2*x^4))/(b + c*x^2)^2 + 90*
Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 90*Sqrt[2]*
b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 45*Sqrt[2]*b^(1/4)*
Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 45*Sqrt[2]*b^(
1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(128*c^(13
/4))
```

IntegrateAlgebraic [A] time = 0.55, size = 324, normalized size = 1.29

$$\frac{-\frac{45b^{5/4}x^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{16\sqrt{2}c^{9/4}}+\left(\frac{45b^{5/4}x^2}{16\sqrt{2}c^{9/4}}+\frac{45b^{9/4}}{32\sqrt{2}c^{13/4}}+\frac{45\sqrt[4]{b}x^4}{32\sqrt{2}c^{5/4}}\right)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-\frac{45b^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}c^{13/4}}+\frac{45b^2\sqrt{x}}{16c^3}-\frac{45\sqrt[4]{b}x^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}c^{5/4}}+\frac{81bx^{5/2}}{16c^2}+\frac{2x^{9/2}}{c}}{\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((45*b^2*Sqrt[x])/(16*c^3) + (81*b*x^(5/2))/(16*c^2) + (2*x^(9/2))/c + ((45
*b^(9/4))/(32*Sqrt[2]*c^(13/4)) + (45*b^(5/4)*x^2)/(16*Sqrt[2]*c^(9/4)) + (
45*b^(1/4)*x^4)/(32*Sqrt[2]*c^(5/4)))*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x])] - (45*b^(9/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*S
qrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32*Sqrt[2]*c^(13/4)) - (45*b^(5/4)*x^2*Arc
Tanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(16*Sqrt[2]*
c^(9/4)) - (45*b^(1/4)*x^4*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[
b] + Sqrt[c]*x)])/(32*Sqrt[2]*c^(5/4)))/(b + c*x^2)^2
```

fricas [A] time = 1.00, size = 247, normalized size = 0.98

$$-\frac{180\left(c^5x^4+2bc^4x^2+b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^6\sqrt{-\frac{b}{c^{13}}}+x}\,c^{10}\left(-\frac{b}{c^{13}}\right)^{\frac{3}{4}}-c^{10}\sqrt{x}\left(-\frac{b}{c^{13}}\right)^{\frac{3}{4}}}{b}\right)+45\left(c^5x^4+2bc^4x^2+b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\log\left(45c^3\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}+45\sqrt{x}\right)-45\left(c^5x^4+2bc^4x^2+b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\log\left(-45c^3\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}+45\sqrt{x}\right)-4\left(32c^2x^4+81bcx^2+45b^2\right)\sqrt{x}}{64\left(c^5x^4+2bc^4x^2+b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(180*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*arctan((sqrt(c
^6*sqrt(-b/c^13) + x)*c^10*(-b/c^13)^(3/4) - c^10*sqrt(x)*(-b/c^13)^(3/4))/
b) + 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*log(45*c^3*(-b/c^
13)^(1/4) + 45*sqrt(x)) - 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1
/4)*log(-45*c^3*(-b/c^13)^(1/4) + 45*sqrt(x)) - 4*(32*c^2*x^4 + 81*b*c*x^2
+ 45*b^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

giac [A] time = 0.18, size = 208, normalized size = 0.83

$$-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64c^4}-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64c^4}-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128c^4}+\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128c^4}+\frac{2\sqrt{x}}{c^3}+\frac{17bcx^{\frac{5}{2}}+13b^2\sqrt{x}}{16\left(cx^2+b\right)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/c^4 - 45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 45/128*sqrt(2)*(b*c^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 45/128*sqrt(2)
*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 2*sq
rt(x)/c^3 + 1/16*(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/((c*x^2 + b)^2*c^3)
```

maple [A] time = 0.02, size = 178, normalized size = 0.71

$$\frac{17 b\,x^{\frac{5}{2}}}{16\left(c\,x^{2}+b\right)^{2} c^{2}}+\frac{13 b^{2} \sqrt{x}}{16\left(c\,x^{2}+b\right)^{2} c^{3}}-\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64 c^{3}}-\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64 c^{3}}-\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128 c^{3}}+\frac{2 \sqrt{x}}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*x^(1/2)/c^3+17/16/c^2*b/(c*x^2+b)^2*x^(5/2)+13/16/c^3*b^2/(c*x^2+b)^2*x^(
1/2)-45/128/c^3*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)
^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-45/64/c^3*(b/c)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/c^3*(b/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.95, size = 229, normalized size = 0.91

$$\frac{17\,bcx^{\frac{5}{2}}+13\,b^{2}\sqrt{x}}{16\left(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3}\right)}-\frac{45\left(\frac{2\,\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,\sqrt{b}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}\right)}{128\,c^{3}}+\frac{2\,\sqrt{x}}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) -
45/128*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*s
qrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sq
rt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqr
t(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(1/4)*log(sqrt(2)*b^(
1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(1/4)*log(-
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c^3 + 2*sqr
t(x)/c^3
```

mupad [B] time = 4.39, size = 101, normalized size = 0.40

$$\frac{\frac{13\,b^2\,\sqrt{x}}{16}+\frac{17\,b\,c\,x^{5/2}}{16}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{2\,\sqrt{x}}{c^3}-\frac{45\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,c^{13/4}}+\frac{{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,45{}\mathrm{i}}{32\,c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((13*b^2*x^(1/2))/16 + (17*b*c*x^(5/2))/16)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^
2) + (2*x^(1/2))/c^3 - (45*(-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(
32*c^(13/4)) + ((-b)^(1/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*45i)/(32*c
^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.222 $\int \frac{x^{21/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=239

$$\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{21\tan^{-1}}{32}$$

Rubi [A] time = 0.20, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}+\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{21\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^(7/2)/(4*c*(b + c*x^2)^2) - (7*x^(3/2))/(16*c^2*(b + c*x^2)) - (21*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(11/4)) + (
21*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(11
/4)) + (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*
Sqrt[2]*b^(1/4)*c^(11/4)) - (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{21/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{9/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} + \frac{7\int \frac{x^{5/2}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32c^2}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^2}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} - \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32c^{5/2}} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{32c^{5/2}}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64c^3} + \frac{21\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{11/4}} - \frac{21\log\left(\sqrt{b}+\right.}{64}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{11/4}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{11/4}} + \frac{2}{}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.28

$$\frac{2x^{3/2}\left(7\left(b+cx^2\right)^2 {}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)-b\left(7b+5cx^2\right)\right)}{5bc^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(-(b*(7*b + 5*c*x^2)) + 7*(b + c*x^2)^2*Hypergeometric2F1[3/4, 3
, 7/4, -((c*x^2)/b)]))/(5*b*c^2*(b + c*x^2)^2)
```

IntegrateAlgebraic [A] time = 0.49, size = 311, normalized size = 1.30

$$\frac{-\frac{21b^{3/4}x^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{16\sqrt{2}c^{7/4}}+\left(-\frac{21b^{3/4}x^2}{16\sqrt{2}c^{7/4}}-\frac{21b^{7/4}}{32\sqrt{2}c^{11/4}}-\frac{21x^4}{32\sqrt{2}\sqrt[4]{b}c^{3/4}}\right)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-\frac{21b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}c^{11/4}}-\frac{21x^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}\sqrt[4]{b}c^{3/4}}-\frac{7bx^{3/2}}{16c^2}-\frac{11x^{7/2}}{16c}}{\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-7*b*x^(3/2))/(16*c^2) - (11*x^(7/2))/(16*c) + ((-21*b^(7/4))/(32*Sqrt[2]
*c^(11/4)) - (21*b^(3/4)*x^2)/(16*Sqrt[2]*c^(7/4)) - (21*x^4)/(32*Sqrt[2]*b
^(1/4)*c^(3/4)))*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x])] - (21*b^(7/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sq
rt[c]*x)])/(32*Sqrt[2]*c^(11/4)) - (21*b^(3/4)*x^2*ArcTanh[(Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(16*Sqrt[2]*c^(7/4)) - (21*x^4*Ar
cTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32*Sqrt[2]
*b^(1/4)*c^(3/4)))/(b + c*x^2)^2
```

fricas [A] time = 1.28, size = 248, normalized size = 1.04

$$-\frac{84\left(c^4x^4+2bc^3x^2+b^2c^2\right)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc^5\sqrt{-\frac{1}{bc^{11}}}+x}c^3\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}-c^3\sqrt{x}\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\right)-21\left(c^4x^4+2bc^3x^2+b^2c^2\right)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\log\left(bc^8\left(-\frac{1}{bc^{11}}\right)^{\frac{3}{4}}+\sqrt{x}\right)+21\left(c^4x^4+2bc^3x^2+b^2c^2\right)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\log\left(-bc^8\left(-\frac{1}{bc^{11}}\right)^{\frac{3}{4}}+\sqrt{x}\right)+4\left(11cx^3+7bx\right)\sqrt{x}}{64\left(c^4x^4+2bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(84*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*arctan(sqrt
(-b*c^5*sqrt(-1/(b*c^11)) + x)*c^3*(-1/(b*c^11))^(1/4) - c^3*sqrt(x)*(-1/(b
*c^11))^(1/4)) - 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*l
og(b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) + 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c
^2)*(-1/(b*c^11))^(1/4)*log(-b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) + 4*(11*c
*x^3 + 7*b*x)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

giac [A] time = 0.22, size = 209, normalized size = 0.87

$$-\frac{11cx^{\frac{7}{2}}+7bx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2c^2}+\frac{21\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64bc^5}+\frac{21\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64bc^5}-\frac{21\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128bc^5}+\frac{21\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/((c*x^2 + b)^2*c^2) + 21/64*sqrt(2)*(b*c
^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))
/(b*c^5) + 21/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(
1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 21/128*sqrt(2)*(b*c^3)^(3/4)*log(s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5) + 21/128*sqrt(2)*(b*c^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

maple [A] time = 0.02, size = 161, normalized size = 0.67

$$\frac{21\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} c^{3}}+\frac{21\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} c^{3}}+\frac{21\sqrt{2}\ \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}} c^{3}}+\frac{-\frac{11 x^{\frac{7}{2}}}{16 c}-\frac{7 b x^{\frac{3}{2}}}{16 c^{2}}}{\left(c x^{2}+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(-11/32/c*x^(7/2)-7/32*b/c^2*x^(3/2))/(c*x^2+b)^2+21/128/c^3/(b/c)^(1/4)*
2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/
2)*x^(1/2)+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)+1)+21/64/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)-1)
```

maxima [A] time = 2.95, size = 218, normalized size = 0.91

$$-\frac{11\, c x^{\frac{7}{2}}+7\, b x^{\frac{3}{2}}}{16\left(c^{4} x^{4}+2\, b c^{3} x^{2}+b^{2} c^{2}\right)}+\frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\, b^{\frac{1}{4}} c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\, b^{\frac{1}{4}} c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\, b^{\frac{1}{4}} c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\, x+\sqrt{b}\right)}{b^{\frac{1}{4}} c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\, b^{\frac{1}{4}} c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\, x+\sqrt{b}\right)}{b^{\frac{1}{4}} c^{\frac{3}{4}}}\right)}{128\, c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 21/1
28*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*
sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(
1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^2
```

mupad [B] time = 4.28, size = 87, normalized size = 0.36

$$\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{1/4}\,c^{11/4}}-\frac{\frac{11\,x^{7/2}}{16\,c}+\frac{7\,b\,x^{3/2}}{16\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{1/4}\,c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (21*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(1/4)*c^(11/4)) - ((11*x^(
7/2))/(16*c) + (7*b*x^(3/2))/(16*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (21*at
anh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(1/4)*c^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.223 $\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\right.}{32\sqrt{}}$$

Rubi [A] time = 0.19, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)}-\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^(5/2)/(4*c*(b + c*x^2)^2) - (5*Sqrt[x])/(16*c^2*(b + c*x^2)) - (5*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4)) + (5*
ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4))
 - (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(3/4)*c^(9/4)) + (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{7/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} + \frac{5\int \frac{x^{3/2}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32c^2}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^2}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32\sqrt{b}\,c^2} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{32\sqrt{b}\,c^2}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{b}\,c^{5/2}} + \frac{5\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{64}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} - \frac{5\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}} + \frac{5\log\left(\sqrt{b}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}} - \frac{5\,\text{lo}}{}$$

Mathematica [A] time = 0.10, size = 242, normalized size = 1.01

$$\frac{-\frac{15\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{3/4}} + \frac{15\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{3/4}} - \frac{30\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}} + \frac{30\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}} - \frac{256c^{5/4}x^{5/2}}{\left(b+cx^2\right)^2} + \frac{40\sqrt[4]{c}\sqrt{x}}{b+cx^2} - \frac{160b\sqrt[4]{c}\sqrt{x}}{\left(b+cx^2\right)^2}}{384c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-160*b*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 - (256*c^(5/4)*x^(5/2))/(b + c*x^2)
^2 + (40*c^(1/4)*Sqrt[x])/(b + c*x^2) - (30*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) + (30*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/b^(3/4) - (15*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) + (15*Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4))/(384*c^(9/4))
```

IntegrateAlgebraic [A] time = 0.36, size = 311, normalized size = 1.30

$$\frac{\left(-\frac{5b^{5/4}}{32\sqrt{2}\,c^{9/4}}-\frac{5x^4}{32\sqrt{2}\,b^{3/4}\sqrt[4]{c}}-\frac{5\sqrt[4]{b}\,x^2}{16\sqrt{2}\,c^{5/4}}\right)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\frac{5b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{32\sqrt{2}\,c^{9/4}}+\frac{5x^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{32\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{5\sqrt[4]{b}\,x^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{16\sqrt{2}\,c^{5/4}}-\frac{5b\sqrt{x}}{16c^2}-\frac{9x^{5/2}}{16c}}{\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-5*b*Sqrt[x])/(16*c^2) - (9*x^(5/2))/(16*c) + ((-5*b^(5/4))/(32*Sqrt[2]*c
^(9/4)) - (5*b^(1/4)*x^2)/(16*Sqrt[2]*c^(5/4)) - (5*x^4)/(32*Sqrt[2]*b^(3/4
)*c^(1/4)))*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])]
 + (5*b^(5/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*
x)])/(32*Sqrt[2]*c^(9/4)) + (5*b^(1/4)*x^2*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(16*Sqrt[2]*c^(5/4)) + (5*x^4*ArcTanh[(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32*Sqrt[2]*b^(3/4)*
c^(1/4)))/(b + c*x^2)^2
```

fricas [A] time = 1.34, size = 254, normalized size = 1.06

$$\frac{20\left(c^4x^4+2bc^3x^2+b^2c^2\right)\left(-\frac{1}{b^3c^9}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2c^4\sqrt{-\frac{1}{b^3c^9}}+x}\,b^2c^7\left(-\frac{1}{b^3c^9}\right)^{\frac{3}{4}}-b^2c^7\sqrt{x}\left(-\frac{1}{b^3c^9}\right)^{\frac{3}{4}}\right)+5\left(c^4x^4+2bc^3x^2+b^2c^2\right)\left(-\frac{1}{b^3c^9}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{1}{b^3c^9}\right)^{\frac{1}{4}}+\sqrt{x}\right)-5\left(c^4x^4+2bc^3x^2+b^2c^2\right)\left(-\frac{1}{b^3c^9}\right)^{\frac{1}{4}}\log\left(-bc^2\left(-\frac{1}{b^3c^9}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\left(9cx^2+5b\right)\sqrt{x}}{64\left(c^4x^4+2bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*arctan(sqrt
(b^2*c^4*sqrt(-1/(b^3*c^9)) + x)*b^2*c^7*(-1/(b^3*c^9))^(3/4) - b^2*c^7*sqr
t(x)*(-1/(b^3*c^9))^(3/4)) + 5*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c
^9))^(1/4)*log(b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt(x)) - 5*(c^4*x^4 + 2*b*c^3
*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*log(-b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt
(x)) - 4*(9*c*x^2 + 5*b)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

giac [A] time = 0.18, size = 209, normalized size = 0.87

$$\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^3}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^3}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^3}-\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^3}-\frac{9\,cx^{\frac{5}{2}}+5\,b\sqrt{x}}{16\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b*c^3) + 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) + 5/128*sqrt(2)*(b*c
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) - 5/128*
sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*
c^3) - 1/16*(9*c*x^(5/2) + 5*b*sqrt(x))/((c*x^2 + b)^2*c^2)
```

maple [A] time = 0.02, size = 170, normalized size = 0.71

$$\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b\,c^{2}}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b\,c^{2}}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b\,c^{2}}+\frac{-\frac{9x^{\frac{5}{2}}}{16c}-\frac{5b\sqrt{x}}{16c^{2}}}{\left(c\,x^{2}+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(-9/32/c*x^(5/2)-5/32*b/c^2*x^(1/2))/(c*x^2+b)^2+5/128/c^2*(b/c)^(1/4)/b*
2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/
2)*x^(1/2)+(b/c)^(1/2)))+5/64/c^2*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)+1)+5/64/c^2*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)-1)
```

maxima [A] time = 2.98, size = 218, normalized size = 0.91

$$-\frac{9\,cx^{\frac{5}{2}}+5\,b\sqrt{x}}{16\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)}+\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*c*x^(5/2) + 5*b*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 5/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
rt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/
4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c^2
```

mupad [B] time = 0.10, size = 87, normalized size = 0.36

$$-\frac{\frac{9\,x^{5/2}}{16\,c}+\frac{5\,b\,\sqrt{x}}{16\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{3/4}\,c^{9/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{3/4}\,c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((9*x^(5/2))/(16*c) + (5*b*x^(1/2))/(16*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
 - (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(3/4)*c^(9/4)) - (5*atan
h((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(3/4)*c^(9/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.224 $\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=242

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{2}\,}$$

Rubi [A] time = 0.19, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^(3/2)/(4*c*(b + c*x^2)^2) + (3*x^(3/2))/(16*b*c*(b + c*x^2)) - (3*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4)) + (3*
ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4))
 + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(5/4)*c^(7/4)) - (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{5/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3\int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} + \frac{3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32bc}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32bc^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4} \cdots\right.}{32bc^{3/2}}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64bc^2} + \frac{3\,\text{Subst}\left(\int \frac{\cdots}{\frac{\sqrt{b}}{\sqrt{c}}+\cdots}\right.}{6\cdots}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}} - \frac{3\log\left(\sqrt{b}+\sqrt{2}\cdots\right.}{64\sqrt{2}\cdots}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}} + \frac{3\,\text{lo}\cdots}{\cdots}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.19

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{b^2}-\frac{1}{(b+cx^2)^2}\right)}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(-(b + c*x^2)^(-2) + Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)
]/b^2))/(5*c)
```

IntegrateAlgebraic [A] time = 0.29, size = 310, normalized size = 1.28

$$\frac{\left(-\frac{3b^{3/4}}{32\sqrt{2}c^{7/4}}-\frac{3\sqrt[4]{c}x^4}{32\sqrt{2}b^{5/4}}-\frac{3x^2}{16\sqrt{2}\sqrt[4]{b}c^{3/4}}\right)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-\frac{3b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}c^{7/4}}-\frac{3\sqrt[4]{c}x^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{5/4}}-\frac{3x^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{16\sqrt{2}\sqrt[4]{b}c^{3/4}}+\frac{3x^{7/2}}{16b}-\frac{x^{3/2}}{16c}}{(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-1/16*x^(3/2)/c + (3*x^(7/2))/(16*b) + ((-3*b^(3/4))/(32*Sqrt[2]*c^(7/4))
- (3*x^2)/(16*Sqrt[2]*b^(1/4)*c^(3/4)) - (3*c^(1/4)*x^4)/(32*Sqrt[2]*b^(5/4
)))*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - (3*b^
(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32
*Sqrt[2]*c^(7/4)) - (3*x^2*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[
b] + Sqrt[c]*x)])/(16*Sqrt[2]*b^(1/4)*c^(3/4)) - (3*c^(1/4)*x^4*ArcTanh[(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32*Sqrt[2]*b^(5/4))
)/(b + c*x^2)^2
```

fricas [A] time = 0.96, size = 260, normalized size = 1.07

$$-\frac{12\left(bc^3x^4+2b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b^3c^3\sqrt{-\frac{1}{b^5c^7}}+x}\,bc^2\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}-bc^2\sqrt{x}\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}\right)-3\left(bc^3x^4+2b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}\log\left(b^4c^5\left(-\frac{1}{b^5c^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)+3\left(bc^3x^4+2b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}\log\left(-b^4c^5\left(-\frac{1}{b^5c^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4\left(3cx^3-bx\right)\sqrt{x}}{64\left(bc^3x^4+2b^2c^2x^2+b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*arctan(s
qrt(-b^3*c^3*sqrt(-1/(b^5*c^7)) + x)*b*c^2*(-1/(b^5*c^7))^(1/4) - b*c^2*sqr
t(x)*(-1/(b^5*c^7))^(1/4)) - 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5
*c^7))^(1/4)*log(b^4*c^5*(-1/(b^5*c^7))^(3/4) + sqrt(x)) + 3*(b*c^3*x^4 + 2
*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*log(-b^4*c^5*(-1/(b^5*c^7))^(3/4
```

```
) + sqrt(x)) - 4*(3*c*x^3 - b*x)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*
c)
```

giac [A] time = 0.18, size = 212, normalized size = 0.88

$$\frac{3\,cx^{\frac{7}{2}}-bx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2bc}+\frac{3\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4}+\frac{3\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4}-\frac{3\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^4}+\frac{3\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*c*x^(7/2) - b*x^(3/2))/((c*x^2 + b)^2*b*c) + 3/64*sqrt(2)*(b*c^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2
*c^4) + 3/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) - 3/128*sqrt(2)*(b*c^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 3/128*sqrt(2)*(b*c^3)^
(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4)
```

maple [A] time = 0.02, size = 169, normalized size = 0.70

$$\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b\,c^2}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b\,c^2}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}b\,c^2}+\frac{\frac{3x^{\frac{7}{2}}}{16b}-\frac{x^{\frac{3}{2}}}{16c}}{\left(c\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(3/32/b*x^(7/2)-1/32/c*x^(3/2))/(c*x^2+b)^2+3/128/c^2/b/(b/c)^(1/4)*2^(1/
2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^
(1/2)+(b/c)^(1/2)))+3/64/c^2/b/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/
4)*x^(1/2)+1)+3/64/c^2/b/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)
```

maxima [A] time = 3.09, size = 222, normalized size = 0.92

$$\frac{3\,cx^{\frac{7}{2}}-bx^{\frac{3}{2}}}{16\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)}+\frac{3\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*c*x^(7/2) - b*x^(3/2))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c)
```

mupad [B] time = 0.09, size = 85, normalized size = 0.35

$$\frac{\frac{3\,x^{7/2}}{16\,b}-\frac{x^{3/2}}{16\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{5/4}\,c^{7/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{5/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*x^(7/2))/(16*b) - x^(3/2)/(16*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (3*atan
((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(5/4)*c^(7/4)) + (3*atanh((c^(1/4)
*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(5/4)*c^(7/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.225 $\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\right.}{32\sqrt{}}$$

Rubi [A] time = 0.18, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{\sqrt{x}}{16bc\left(b+cx^2\right)}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -Sqrt[x]/(4*c*(b + c*x^2)^2) + Sqrt[x]/(16*b*c*(b + c*x^2)) - (3*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) + (3*ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) - (
3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b
^(7/4)*c^(5/4)) + (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(64*Sqrt[2]*b^(7/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{3/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} + \frac{3\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32bc}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{3/2}c} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{32b^{3/2}c}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^{3/2}c^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{64}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} - \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}} + \frac{3\log\left(\sqrt{b}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}} - \frac{3\,\text{lo}}{}$$

Mathematica [A] time = 0.11, size = 223, normalized size = 0.92

$$\frac{-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{7/4}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{7/4}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{7/4}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{7/4}}+\frac{8\sqrt[4]{c}\sqrt{x}}{b^2+bcx^2}-\frac{32\sqrt[4]{c}\sqrt{x}}{(b+cx^2)^2}}{128c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-32*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 + (8*c^(1/4)*Sqrt[x])/(b^2 + b*c*x^2)
- (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(7/4) + (6*Sq
rt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(7/4) - (3*Sqrt[2]*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4) + (3*Sqr
t[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4))/(
128*c^(5/4))
```

IntegrateAlgebraic [A] time = 0.44, size = 153, normalized size = 0.63

$$-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{32\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{7/4}c^{5/4}}+\frac{cx^{5/2}-3b\sqrt{x}}{16bc\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*b*Sqrt[x] + c*x^(5/2))/(16*b*c*(b + c*x^2)^2) - (3*ArcTan[(b^(1/4)/(Sqr
t[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*b^(7/4
)*c^(5/4)) + (3*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c
]*x)])/(32*Sqrt[2]*b^(7/4)*c^(5/4))
```

fricas [A] time = 1.26, size = 257, normalized size = 1.06

$$\frac{12\left(bc^3x^4+2b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^4c^2\sqrt{-\frac{1}{b^7c^5}}+x}\,b^5c^4\left(-\frac{1}{b^7c^5}\right)^{\frac{3}{4}}-b^5c^4\sqrt{x}\left(-\frac{1}{b^7c^5}\right)^{\frac{3}{4}}\right)+3\left(bc^3x^4+2b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\log\left(b^2c\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\left(bc^3x^4+2b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\log\left(-b^2c\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\left(cx^2-3b\right)\sqrt{x}}{64\left(bc^3x^4+2b^2c^2x^2+b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*arctan(sq
rt(b^4*c^2*sqrt(-1/(b^7*c^5)) + x)*b^5*c^4*(-1/(b^7*c^5))^(3/4) - b^5*c^4*s
qrt(x)*(-1/(b^7*c^5))^(3/4)) + 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b
^7*c^5))^(1/4)*log(b^2*c*(-1/(b^7*c^5))^(1/4) + sqrt(x)) - 3*(b*c^3*x^4 + 2
```

```
*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*log(-b^2*c*(-1/(b^7*c^5))^(1/4)
+ sqrt(x)) + 4*(c*x^2 - 3*b)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)
```

giac [A] time = 0.18, size = 211, normalized size = 0.87

$$\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^2}-\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^2}+\frac{cx^{\frac{5}{2}}-3\,b\sqrt{x}}{16\left(cx^2+b\right)^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^2*c^2) + 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 3/128*sqrt(2)*
(b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) -
3/128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c
))/(b^2*c^2) + 1/16*(c*x^(5/2) - 3*b*sqrt(x))/((c*x^2 + b)^2*b*c)
```

maple [A] time = 0.02, size = 169, normalized size = 0.70

$$\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^2c}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^2c}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b^2c}+\frac{\frac{x^{\frac{5}{2}}}{16b}-\frac{3\sqrt{x}}{16c}}{\left(c\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32/b*x^(5/2)-3/32/c*x^(1/2))/(c*x^2+b)^2+3/128/c/b^2*(b/c)^(1/4)*2^(1/
2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^
(1/2)+(b/c)^(1/2)))+3/64/c/b^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/
4)*x^(1/2)+1)+3/64/c/b^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)
```

maxima [A] time = 2.96, size = 221, normalized size = 0.91

$$\frac{cx^{\frac{5}{2}}-3\,b\sqrt{x}}{16\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(c*x^(5/2) - 3*b*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/(b*c)
```

mupad [B] time = 0.10, size = 85, normalized size = 0.35

$$\frac{\frac{x^{5/2}}{16\,b}-\frac{3\,\sqrt{x}}{16\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{7/4}\,c^{5/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{7/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(5/2)/(16*b) - (3*x^(1/2))/(16*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) + (3*atan
((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(7/4)*c^(5/4)) + (3*atanh((c^(1/4)
*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(7/4)*c^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.226 $\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=239

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\right.}{32\sqrt{2}\,\ldots}$$

Rubi [A] time = 0.18, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5x^{3/2}}{16b^2\left(b+cx^2\right)}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^(3/2)/(4*b*(b + c*x^2)^2) + (5*x^(3/2))/(16*b^2*(b + c*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4))
+ (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(9/4)*c^(3/4)) - (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{\sqrt{x}}{\left(b+cx^2\right)^3}\,dx$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5\int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^2}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^2\sqrt{c}} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,d\right.}{32b^2\sqrt{c}}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^2c} + \frac{5\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{64}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}} - \frac{5\log\left(\sqrt{b}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}} + \frac{5\log}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^3)

IntegrateAlgebraic [A] time = 0.28, size = 149, normalized size = 0.62

$$-\frac{5\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{x^{3/2}\left(9b+5cx^2\right)}{16b^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(13/2)/(b*x^2 + c*x^4)^3,x]

[Out] (x^(3/2)*(9*b + 5*c*x^2))/(16*b^2*(b + c*x^2)^2) - (5*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*b^(9/4)*c^(3/4)) - (5*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32*Sqrt[2]*b^(9/4)*c^(3/4))

fricas [A] time = 0.85, size = 250, normalized size = 1.05

$$-\frac{20\left(b^2c^2x^4+2b^3cx^2+b^4\right)\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b^5c\sqrt{-\frac{1}{b^9c^3}}+x}\,b^2c\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}-b^2c\sqrt{x}\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}\right)-5\left(b^2c^2x^4+2b^3cx^2+b^4\right)\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}\log\left(b^7c^2\left(-\frac{1}{b^9c^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)+5\left(b^2c^2x^4+2b^3cx^2+b^4\right)\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}\log\left(-b^7c^2\left(-\frac{1}{b^9c^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4\left(5cx^3+9bx\right)\sqrt{x}}{64\left(b^2c^2x^4+2b^3cx^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")

[Out] -1/64*(20*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*arctan(sqrt(-b^5*c*sqrt(-1/(b^9*c^3)) + x)*b^2*c*(-1/(b^9*c^3))^(1/4) - b^2*c*sqrt(x)*(-1/(b^9*c^3))^(1/4)) - 5*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*log(b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt(x)) + 5*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*log(-b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt(x)) - 4*(5*c*x^3 + 9*b*x)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)

giac [A] time = 0.20, size = 209, normalized size = 0.87

$$\frac{5cx^{\frac{7}{2}}+9bx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^2}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3c^3}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3c^3}-\frac{5\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128b^3c^3}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")

[Out] 1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/((c*x^2 + b)^2*b^2) + 5/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b

```
^3*c^3) + 5/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) - 5/128*sqrt(2)*(b*c^3)^(3/4)*log(sq
rt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3) + 5/128*sqrt(2)*(b*c^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

maple [A] time = 0.01, size = 175, normalized size = 0.73

$$\frac{x^{\frac{3}{2}}}{4\left(c\,x^{2}+b\right)^{2} b}+\frac{5x^{\frac{3}{2}}}{16\left(c\,x^{2}+b\right) b^{2}}+\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2} c}+\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2} c}+\frac{5\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x^(3/2)/b/(c*x^2+b)^2+5/16*x^(3/2)/b^2/(c*x^2+b)+5/128/b^2/c/(b/c)^(1/4
)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(
1/2)*x^(1/2)+(b/c)^(1/2)))+5/64/b^2/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)+1)+5/64/b^2/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)-1)
```

maxima [A] time = 3.09, size = 217, normalized size = 0.91

$$\frac{5\,c x^{\frac{7}{2}}+9\,b x^{\frac{3}{2}}}{16\left(b^{2} c^{2} x^{4}+2\,b^{3} c x^{2}+b^{4}\right)}+\frac{5\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}} c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}} c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}} c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}} c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}} c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}} c^{\frac{3}{4}}}\right)}{128\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 5/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^2
```

mupad [B] time = 0.09, size = 86, normalized size = 0.36

$$\frac{\frac{9\,x^{3/2}}{16\,b}+\frac{5\,c\,x^{7/2}}{16\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{9/4}\,c^{3/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{9/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((9*x^(3/2))/(16*b) + (5*c*x^(7/2))/(16*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) +
 (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(9/4)*c^(3/4)) - (5*atanh(
(c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(9/4)*c^(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.227 $\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\tan}{3}$$

Rubi [A] time = 0.19, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] Sqrt[x]/(4*b*(b + c*x^2)^2) + (7*Sqrt[x])/(16*b^2*(b + c*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/
4)) - (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*S
qrt[2]*b^(11/4)*c^(1/4)) + (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{\sqrt{x}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{5/2}} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{32b^{5/2}}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^{5/2}\sqrt{c}} + \frac{21\,\text{Subst}\left(\int \right.}{64}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} - \frac{21\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{b}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}} - \frac{21}{}$$

Mathematica [A] time = 0.08, size = 220, normalized size = 0.92

$$\frac{\frac{32b^{7/4}\sqrt{x}}{\left(b+cx^2\right)^2}+\frac{56b^{3/4}\sqrt{x}}{b+cx^2}-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{\sqrt[4]{c}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{c}}}{128b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((32*b^(7/4)*Sqrt[x])/(b + c*x^2)^2 + (56*b^(3/4)*Sqrt[x])/(b + c*x^2) - (4
2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) + (42*Sqrt
[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) - (21*Sqrt[2]*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) + (21*Sqr
t[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4))/(
128*b^(11/4))
```

IntegrateAlgebraic [A] time = 0.27, size = 149, normalized size = 0.62

$$-\frac{21\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{\sqrt{x}\left(11b+7cx^2\right)}{16b^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (Sqrt[x]*(11*b + 7*c*x^2))/(16*b^2*(b + c*x^2)^2) - (21*ArcTan[(b^(1/4)/(Sq
rt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*b^(11
/4)*c^(1/4)) + (21*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqr
t[c]*x)])/(32*Sqrt[2]*b^(11/4)*c^(1/4))
```

fricas [A] time = 1.10, size = 241, normalized size = 1.01

$$\frac{84\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^6\sqrt{-\frac{1}{b^{11}c}}+x}\,b^8c\left(-\frac{1}{b^{11}c}\right)^{\frac{3}{4}}-b^8c\sqrt{x}\left(-\frac{1}{b^{11}c}\right)^{\frac{3}{4}}\right)+21\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-21\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\log\left(-b^3\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\left(7\,cx^2+11\,b\right)\sqrt{x}}{64\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(84*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*arctan(sqrt(
b^6*sqrt(-1/(b^11*c)) + x)*b^8*c*(-1/(b^11*c))^(3/4) - b^8*c*sqrt(x)*(-1/(b
^11*c))^(3/4)) + 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*l
og(b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) - 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4
)*(-1/(b^11*c))^(1/4)*log(-b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) + 4*(7*c*x^2
+ 11*b)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

giac [A] time = 0.21, size = 209, normalized size = 0.87

$$\frac{21\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c}+\frac{21\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c}+\frac{21\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c}-\frac{21\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c}+\frac{7\,cx^{\frac{5}{2}}+11\,b\sqrt{x}}{16\left(cx^2+b\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/(b^3*c) + 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 21/128*sqrt(2)*(
b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) - 21/
128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b^3*c) + 1/16*(7*c*x^(5/2) + 11*b*sqrt(x))/((c*x^2 + b)^2*b^2)
```

maple [A] time = 0.01, size = 166, normalized size = 0.69

$$\frac{\sqrt{x}}{4\left(c x^{2}+b\right)^{2} b}+\frac{7\sqrt{x}}{16\left(c x^{2}+b\right) b^{2}}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^{3}}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^{3}}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x^(1/2)/b/(c*x^2+b)^2+7/16*x^(1/2)/b^2/(c*x^2+b)+21/128/b^3*(b/c)^(1/4)
*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1
/2)*x^(1/2)+(b/c)^(1/2)))+21/64/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)+1)+21/64/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)-1)
```

maxima [A] time = 2.97, size = 217, normalized size = 0.91

$$\frac{7\,c x^{\frac{5}{2}}+11\,b\sqrt{x}}{16\left(b^{2}c^{2}x^{4}+2\,b^{3}c x^{2}+b^{4}\right)}+\frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(7*c*x^(5/2) + 11*b*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 21/12
8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1
/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt(2)
*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2
```

mupad [B] time = 4.29, size = 86, normalized size = 0.36

$$\frac{\frac{11\sqrt{x}}{16\,b}+\frac{7\,c\,x^{5/2}}{16\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{11/4}\,c^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{11/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((11*x^(1/2))/(16*b) + (7*c*x^(5/2))/(16*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
- (21*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(11/4)*c^(1/4)) - (21*at
anh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(11/4)*c^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.228 $\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=251

$$-\frac{45\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{13/4}}$$

Rubi [A] time = 0.22, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)}-\frac{45\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{13/4}}-\frac{45\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{13/4}}-\frac{45}{16b^3\sqrt{x}}+\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -45/(16*b^3*Sqrt[x]) + 1/(4*b*Sqrt[x]*(b + c*x^2)^2) + 9/(16*b^2*Sqrt[x]*(b
 + c*x^2)) + (45*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32
*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(32*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)) + (45*c^(1/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{3/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9\int \frac{1}{x^{3/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} + \frac{45\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{(45c)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{(45c)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} + \frac{\left(45\sqrt{c}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{45\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{45\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\right.}{64\sqrt{2}\,b^{13/4}}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} + \frac{45\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{13/4}} - \frac{45\sqrt[4]{c}\ \text{t}}{}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-\frac{cx^2}{b}\right)}{b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 3, 3/4, -((c*x^2)/b)])/(b^3*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.49, size = 160, normalized size = 0.64

$$\frac{45\sqrt[4]{c}\,\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\,\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\,\sqrt[4]{b}}}{\sqrt{x}}\right)}{32\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{32\sqrt{2}\,b^{13/4}}+\frac{-32b^2-81bcx^2-45c^2x^4}{16b^3\sqrt{x}\,\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-32*b^2 - 81*b*c*x^2 - 45*c^2*x^4)/(16*b^3*Sqrt[x]*(b + c*x^2)^2) + (45*c^
(1/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sq
rt[x]])/(32*Sqrt[2]*b^(13/4)) + (45*c^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32*Sqrt[2]*b^(13/4))
```

fricas [A] time = 3.19, size = 263, normalized size = 1.05

$$\frac{180\left(b^3c^2x^5+2b^4cx^3+b^5x\right)\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125b^3c\sqrt{x}\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-8303765625b^7c\sqrt{-\frac{c}{b^{13}}}+8303765625c^2x}\,b^3\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}}{91125c}\right)-45\left(b^3c^2x^5+2b^4cx^3+b^5x\right)\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\log\left(91125b^{10}\left(-\frac{c}{b^{13}}\right)^{\frac{3}{4}}+91125c\sqrt{x}\right)+45\left(b^3c^2x^5+2b^4cx^3+b^5x\right)\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\log\left(-91125b^{10}\left(-\frac{c}{b^{13}}\right)^{\frac{3}{4}}+91125c\sqrt{x}\right)-4\left(45c^2x^4+81bcx^2+32b^2\right)\sqrt{x}}{64\left(b^3c^2x^5+2b^4cx^3+b^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(180*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-c/b^13)^(1/4)*arctan(-1/911
25*(91125*b^3*c*sqrt(x)*(-c/b^13)^(1/4) - sqrt(-8303765625*b^7*c*sqrt(-c/b^
13) + 8303765625*c^2*x)*b^3*(-c/b^13)^(1/4))/c) - 45*(b^3*c^2*x^5 + 2*b^4*c
*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(91125*b^10*(-c/b^13)^(3/4) + 91125*c*sqrt
(x)) + 45*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(-91125*b^
10*(-c/b^13)^(3/4) + 91125*c*sqrt(x)) - 4*(45*c^2*x^4 + 81*b*c*x^2 + 32*b^2
)*sqrt(x))/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)
```

giac [A] time = 0.19, size = 220, normalized size = 0.88

$$-\frac{2}{b^3\sqrt{x}}-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^4c^2}-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^4c^2}+\frac{45\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128b^4c^2}-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128b^4c^2}-\frac{13c^2x^{\frac{7}{2}}+17bcx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -2/(b^3*sqrt(x)) - 45/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 45/64*sqrt(2)*(b*c^3)^(3/
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*
c^2) + 45/128*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/(b^4*c^2) - 45/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 1/16*(13*c^2*x^(7/2) + 17*b*c*x^(3/2)
)/((c*x^2 + b)^2*b^3)
```

maple [A] time = 0.02, size = 178, normalized size = 0.71

$$-\frac{13c^2x^{\frac{7}{2}}}{16\left(cx^2+b\right)^2 b^3}-\frac{17cx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2 b^2}-\frac{45\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^3}-\frac{45\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^3}-\frac{45\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}} b^3}-\frac{2}{b^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -13/16*c^2/b^3/(c*x^2+b)^2*x^(7/2)-17/16*c/b^2/(c*x^2+b)^2*x^(3/2)-45/128/b
^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b
/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-45/64/b^3/(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)-1)-2/b^3/x^(1/2)
```

maxima [A] time = 3.14, size = 230, normalized size = 0.92

$$-\frac{45\,c^2x^4+81\,bcx^2+32\,b^2}{16\left(b^3c^2x^{\frac{9}{2}}+2\,b^4cx^{\frac{5}{2}}+b^5\sqrt{x}\right)}-\frac{45\,c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(45*c^2*x^4 + 81*b*c*x^2 + 32*b^2)/(b^3*c^2*x^(9/2) + 2*b^4*c*x^(5/2)
 + b^5*sqrt(x)) - 45/128*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c
^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*s
qrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c
)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)
*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4
)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(1/4)*c^(3/4)))/b^3
```

mupad [B] time = 4.37, size = 99, normalized size = 0.39

$$\frac{45\,(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{13/4}}-\frac{45\,(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{13/4}}-\frac{\frac{2}{b}+\frac{81\,c\,x^2}{16\,b^2}+\frac{45\,c^2\,x^4}{16\,b^3}}{b^2\,\sqrt{x}+c^2\,x^{9/2}+2\,b\,c\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (45*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(13/4)) - (45*(-c
)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(13/4)) - (2/b + (81*c*x^
2)/(16*b^2) + (45*c^2*x^4)/(16*b^3))/(b^2*x^(1/2) + c^2*x^(9/2) + 2*b*c*x^(
5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.229 $\displaystyle\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=251

$$\frac{77c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}-\frac{77c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}+\frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}}$$

Rubi [A] time = 0.21, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}-\frac{77c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}+\frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}}-\frac{77c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{15/4}}+\frac{11}{16b^2x^{3/2}\left(b+cx^2\right)}-\frac{77}{48b^3x^{3/2}}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -77/(48*b^3*x^(3/2)) + 1/(4*b*x^(3/2)*(b + c*x^2)^2) + 11/(16*b^2*x^(3/2)*(
b + c*x^2)) + (77*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(15/4)) + (77*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{5/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11\int \frac{1}{x^{5/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} + \frac{77\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{(77c)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{(77c)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{(77c)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{7/2}}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{\left(77\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,\right.}{64b^{7/2}}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} + \frac{77c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\right.}{64\sqrt{2}\,b^{15/4}}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} + \frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}} - \frac{77c^{3/}}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-\frac{cx^2}{b}\right)}{3b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 3, 1/4, -((c*x^2)/b)])/(3*b^3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.47, size = 160, normalized size = 0.64

$$\frac{77c^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{32\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{15/4}}+\frac{-32b^2-121bcx^2-77c^2x^4}{48b^3x^{3/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-32*b^2 - 121*b*c*x^2 - 77*c^2*x^4)/(48*b^3*x^(3/2)*(b + c*x^2)^2) + (77*c
^(3/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/S
qrt[x]])/(32*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32*Sqrt[2]*b^(15/4))
```

fricas [A] time = 0.56, size = 283, normalized size = 1.13

$$-\frac{924\left(b^3c^2x^6+2b^4cx^4+b^5x^2\right)\left(-\frac{c^3}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{11}c\sqrt{x}\left(-\frac{c^3}{b^{15}}\right)^{\frac{3}{4}}-\sqrt{b^8\sqrt{-\frac{c^3}{b^{15}}}+c^2x}\,b^{11}\left(-\frac{c^3}{b^{15}}\right)^{\frac{3}{4}}}{c^3}\right)+231\left(b^3c^2x^6+2b^4cx^4+b^5x^2\right)\left(-\frac{c^3}{b^{15}}\right)^{\frac{1}{4}}\log\left(77b^4\left(-\frac{c^3}{b^{15}}\right)^{\frac{1}{4}}+77c\sqrt{x}\right)-231\left(b^3c^2x^6+2b^4cx^4+b^5x^2\right)\left(-\frac{c^3}{b^{15}}\right)^{\frac{1}{4}}\log\left(-77b^4\left(-\frac{c^3}{b^{15}}\right)^{\frac{1}{4}}+77c\sqrt{x}\right)+4\left(77c^2x^4+121bcx^2+32b^2\right)\sqrt{x}}{192\left(b^3c^2x^6+2b^4cx^4+b^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(924*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*arctan(
-(b^11*c*sqrt(x)*(-c^3/b^15)^(3/4) - sqrt(b^8*sqrt(-c^3/b^15) + c^2*x)*b^11
*(-c^3/b^15)^(3/4))/c^3) + 231*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/
b^15)^(1/4)*log(77*b^4*(-c^3/b^15)^(1/4) + 77*c*sqrt(x)) - 231*(b^3*c^2*x^6
 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*log(-77*b^4*(-c^3/b^15)^(1/4) +
 77*c*sqrt(x)) + 4*(77*c^2*x^4 + 121*b*c*x^2 + 32*b^2)*sqrt(x))/(b^3*c^2*x^
6 + 2*b^4*c*x^4 + b^5*x^2)
```

giac [A] time = 0.18, size = 208, normalized size = 0.83

$$-\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^4}-\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^4}-\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128b^4}+\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128b^4}-\frac{15c^2x^{\frac{5}{2}}+19bc\sqrt{x}}{16\left(cx^2+b\right)^2b^3}-\frac{2}{3b^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/b^4 - 77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 77/128*sqrt(2)*(b*c^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 77/128*sqrt(2)
*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/16
*(15*c^2*x^(5/2) + 19*b*c*sqrt(x))/((c*x^2 + b)^2*b^3) - 2/3/(b^3*x^(3/2))
```

maple [A] time = 0.02, size = 181, normalized size = 0.72

$$-\frac{15c^{2}x^{\frac{5}{2}}}{16\left(cx^{2}+b\right)^{2}b^{3}}-\frac{19c\sqrt{x}}{16\left(cx^{2}+b\right)^{2}b^{2}}-\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^{4}}-\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^{4}}-\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b^{4}}-\frac{2}{3b^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -15/16*c^2/b^3/(c*x^2+b)^2*x^(5/2)-19/16*c/b^2/(c*x^2+b)^2*x^(1/2)-77/128*c
/b^4*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-
(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-77/64*c/b^4*(b/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-77/64*c/b^4*(b/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/3/b^3/x^(3/2)
```

maxima [A] time = 3.05, size = 231, normalized size = 0.92

$$-\frac{77\,c^{2}x^{4}+121\,bcx^{2}+32\,b^{2}}{48\left(b^{3}c^{2}x^{\frac{11}{2}}+2\,b^{4}cx^{\frac{7}{2}}+b^{5}x^{\frac{3}{2}}\right)}-\frac{77\left(\frac{2\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(77*c^2*x^4 + 121*b*c*x^2 + 32*b^2)/(b^3*c^2*x^(11/2) + 2*b^4*c*x^(7/
2) + b^5*x^(3/2)) - 77/128*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)
*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*
sqrt(c))) + 2*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sq
rt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqr
t(2)*c^(3/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(
3/4) - sqrt(2)*c^(3/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/b^(3/4))/b^3
```

mupad [B] time = 0.13, size = 99, normalized size = 0.39

$$\frac{77\,{\left(-c\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{15/4}}-\frac{\frac{2}{3\,b}+\frac{121\,c\,x^2}{48\,b^2}+\frac{77\,c^2\,x^4}{48\,b^3}}{b^2\,x^{3/2}+c^2\,x^{11/2}+2\,b\,c\,x^{7/2}}+\frac{77\,{\left(-c\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (77*(-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(15/4)) - (2/(3*b)
 + (121*c*x^2)/(48*b^2) + (77*c^2*x^4)/(48*b^3))/(b^2*x^(3/2) + c^2*x^(11/2
) + 2*b*c*x^(7/2)) + (77*(-c)^(3/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(3
2*b^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.230 $\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=264

$$\frac{117c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}-\frac{117c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{17/4}}$$

Rubi [A] time = 0.23, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}-\frac{117c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{17/4}}+\frac{117c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{17/4}}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{117c}{16b^4\sqrt{x}}-\frac{117}{80b^3x^{5/2}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -117/(80*b^3*x^(5/2)) + (117*c)/(16*b^4*Sqrt[x]) + 1/(4*b*x^(5/2)*(b + c*x^
2)^2) + 13/(16*b^2*x^(5/2)*(b + c*x^2)) - (117*c^(5/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17
/4)) - (117*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{1}{x^{7/2}\left(b+cx^2\right)^3}\,dx$$

$$=\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13\int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$=\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{117\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}-\frac{(117c)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{\left(117c^2\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^4}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{\left(117c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx\right.}{16b^4}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}-\frac{\left(117c^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}}{b+cx^4}\right.}{32b^4}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{(117c)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}\right.}{64b^4}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{117c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\right.}{64\sqrt{2}\,b^{17/4}}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{17/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-\frac{cx^2}{b}\right)}{5b^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 3, -1/4, -((c*x^2)/b)])/(5*b^3*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.47, size = 171, normalized size = 0.65

$$-\frac{117c^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{32\sqrt{2}\,b^{17/4}}-\frac{117c^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{32\sqrt{2}\,b^{17/4}}+\frac{-32b^3+416b^2cx^2+1053bc^2x^4+585c^3x^6}{80b^4x^{5/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-32*b^3 + 416*b^2*c*x^2 + 1053*b*c^2*x^4 + 585*c^3*x^6)/(80*b^4*x^(5/2)*(b
 + c*x^2)^2) - (117*c^(5/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)
/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*b^(17/4)) - (117*c^(5/4)*ArcTanh[
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32*Sqrt[2]*b^(17
/4))
```

fricas [A] time = 0.96, size = 306, normalized size = 1.16

$$-\frac{2340\left(b^4c^2x^7+2b^5cx^5+b^6x^3\right)\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613b^4c^4\sqrt{x}\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769b^9c^5\sqrt{-\frac{c^5}{b^{17}}}+2565164201769c^8x}\,b^4\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}}{1601613c^5}\right)-585\left(b^4c^2x^7+2b^5cx^5+b^6x^3\right)\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}\log\left(1601613b^{13}\left(-\frac{c^5}{b^{17}}\right)^{\frac{3}{4}}+1601613c^4\sqrt{x}\right)+585\left(b^4c^2x^7+2b^5cx^5+b^6x^3\right)\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}\log\left(-1601613b^{13}\left(-\frac{c^5}{b^{17}}\right)^{\frac{3}{4}}+1601613c^4\sqrt{x}\right)-4\left(585c^3x^6+1053bc^2x^4+416b^2cx^2-32b^3\right)\sqrt{x}}{320\left(b^4c^2x^7+2b^5cx^5+b^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(2340*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*arctan
(-1/1601613*(1601613*b^4*c^4*sqrt(x)*(-c^5/b^17)^(1/4) - sqrt(-256516420176
9*b^9*c^5*sqrt(-c^5/b^17) + 2565164201769*c^8*x)*b^4*(-c^5/b^17)^(1/4))/c^5
) - 585*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*log(1601613
*b^13*(-c^5/b^17)^(3/4) + 1601613*c^4*sqrt(x)) + 585*(b^4*c^2*x^7 + 2*b^5*c
*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*log(-1601613*b^13*(-c^5/b^17)^(3/4) + 160
1613*c^4*sqrt(x)) - 4*(585*c^3*x^6 + 1053*b*c^2*x^4 + 416*b^2*c*x^2 - 32*b^
3)*sqrt(x))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)
```

giac [A] time = 0.27, size = 232, normalized size = 0.88

$$\frac{117\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{5}c}+\frac{117\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{5}c}-\frac{117\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{5}c}+\frac{117\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{5}c}+\frac{21\,c^{3}x^{\frac{7}{2}}+25\,bc^{2}x^{\frac{3}{2}}}{16\,\left(cx^{2}+b\right)^{2}b^{4}}+\frac{2\,\left(15\,cx^{2}-b\right)}{5\,b^{4}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/(b^5*c) + 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 117/128*sqrt(2
)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) +
117/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/(b^5*c) + 1/16*(21*c^3*x^(7/2) + 25*b*c^2*x^(3/2))/((c*x^2 + b)^2*b^4)
 + 2/5*(15*c*x^2 - b)/(b^4*x^(5/2))
```

maple [A] time = 0.02, size = 192, normalized size = 0.73

$$\frac{21c^{3}x^{\frac{7}{2}}}{16\left(c\,x^{2}+b\right)^{2}b^{4}}+\frac{25c^{2}x^{\frac{3}{2}}}{16\left(c\,x^{2}+b\right)^{2}b^{3}}+\frac{117\sqrt{2}\;c\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{4}}+\frac{117\sqrt{2}\;c\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{4}}+\frac{117\sqrt{2}\;c\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{4}}+\frac{6c}{b^{4}\sqrt{x}}-\frac{2}{5b^{3}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 21/16*c^3/b^4/(c*x^2+b)^2*x^(7/2)+25/16*c^2/b^3/(c*x^2+b)^2*x^(3/2)+117/128
*c/b^4/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(
x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+117/64*c/b^4/(b/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+117/64*c/b^4/(b/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/5/b^3/x^(5/2)+6*c/b^4/x^(1/2)
```

maxima [A] time = 3.14, size = 243, normalized size = 0.92

$$\frac{585\,c^{3}x^{6}+1053\,bc^{2}x^{4}+416\,b^{2}cx^{2}-32\,b^{3}}{80\left(b^{4}c^{2}x^{\frac{13}{2}}+2\,b^{5}cx^{\frac{9}{2}}+b^{6}x^{\frac{5}{2}}\right)}+\frac{117\,c^{2}\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(585*c^3*x^6 + 1053*b*c^2*x^4 + 416*b^2*c*x^2 - 32*b^3)/(b^4*c^2*x^(13
/2) + 2*b^5*c*x^(9/2) + b^6*x^(5/2)) + 117/128*c^2*(2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/
(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^
(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqr
t(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x +
sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
 sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^4
```

mupad [B] time = 0.12, size = 109, normalized size = 0.41

$$\frac{\frac{26\,c\,x^2}{5\,b^2}-\frac{2}{5\,b}+\frac{1053\,c^2\,x^4}{80\,b^3}+\frac{117\,c^3\,x^6}{16\,b^4}}{b^2\,x^{5/2}+c^2\,x^{13/2}+2\,b\,c\,x^{9/2}}-\frac{117\,{\left(-c\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{17/4}}+\frac{117\,{\left(-c\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((26*c*x^2)/(5*b^2) - 2/(5*b) + (1053*c^2*x^4)/(80*b^3) + (117*c^3*x^6)/(16
*b^4))/(b^2*x^(5/2) + c^2*x^(13/2) + 2*b*c*x^(9/2)) - (117*(-c)^(5/4)*atan(
((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(17/4)) + (117*(-c)^(5/4)*atanh(((-c)^
(1/4)*x^(1/2))/b^(1/4)))/(32*b^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.231 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{165c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}+\frac{165c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{19/4}}$$

Rubi [A] time = 0.23, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{165c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}+\frac{165c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{19/4}}+\frac{165c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{19/4}}+\frac{55c}{16b^4x^{3/2}}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{165}{112b^3x^{7/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -165/(112*b^3*x^(7/2)) + (55*c)/(16*b^4*x^(3/2)) + 1/(4*b*x^(7/2)*(b + c*x^
2)^2) + 15/(16*b^2*x^(7/2)*(b + c*x^2)) - (165*c^(7/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) + (165*c^(7/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (165*c^(7/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19
/4)) + (165*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(19/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{1}{x^{9/2}\left(b+cx^2\right)^3}\,dx$$

$$=\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15\int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$=\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{165\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{(165c)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^4}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^2\right)\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\right.}{16b^4}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}-}{b+c}\right.}{32b^{9/2}}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}}\right.}{64b^{9}}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{165c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\right.}{64\sqrt{2}\,b}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{32\sqrt{2}\,b^{19/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},3;-\frac{3}{4};-\frac{cx^2}{b}\right)}{7b^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 3, -3/4, -((c*x^2)/b)])/(7*b^3*x^(7/2))
```

IntegrateAlgebraic [A] time = 0.47, size = 171, normalized size = 0.65

$$-\frac{165c^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{32\sqrt{2}\,b^{19/4}}+\frac{165c^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{32\sqrt{2}\,b^{19/4}}+\frac{-32b^3+160b^2cx^2+605bc^2x^4+385c^3x^6}{112b^4x^{7/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-32*b^3 + 160*b^2*c*x^2 + 605*b*c^2*x^4 + 385*c^3*x^6)/(112*b^4*x^(7/2)*(b
 + c*x^2)^2) - (165*c^(7/4)*ArcTan[(b^(1/4)/(Sqrt[2]*c^(1/4)) - (c^(1/4)*x)
/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*b^(19/4)) + (165*c^(7/4)*ArcTanh[
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(32*Sqrt[2]*b^(19
/4))
```

fricas [A] time = 0.60, size = 300, normalized size = 1.14

$$\frac{4620\left(b^4c^2x^8+2b^5cx^6+b^6x^4\right)\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{14}c^2\sqrt{x}\left(-\frac{c^7}{b^{19}}\right)^{\frac{3}{4}}-\sqrt{b^{10}\sqrt{-\frac{c^7}{b^{19}}}+c^4x}\,b^{14}\left(-\frac{c^7}{b^{19}}\right)^{\frac{3}{4}}}{c^7}\right)+1155\left(b^4c^2x^8+2b^5cx^6+b^6x^4\right)\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}\log\left(165b^5\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}+165c^2\sqrt{x}\right)-1155\left(b^4c^2x^8+2b^5cx^6+b^6x^4\right)\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}\log\left(-165b^5\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}+165c^2\sqrt{x}\right)+4\left(385c^3x^6+605bc^2x^4+160b^2cx^2-32b^3\right)\sqrt{x}}{448\left(b^4c^2x^8+2b^5cx^6+b^6x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/448*(4620*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*arctan(
-(b^14*c^2*sqrt(x)*(-c^7/b^19)^(3/4) - sqrt(b^10*sqrt(-c^7/b^19) + c^4*x)*b
^14*(-c^7/b^19)^(3/4))/c^7) + 1155*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-
c^7/b^19)^(1/4)*log(165*b^5*(-c^7/b^19)^(1/4) + 165*c^2*sqrt(x)) - 1155*(b^
4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*log(-165*b^5*(-c^7/b^1
9)^(1/4) + 165*c^2*sqrt(x)) + 4*(385*c^3*x^6 + 605*b*c^2*x^4 + 160*b^2*c*x^
2 - 32*b^3)*sqrt(x))/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)
```

giac [A] time = 0.21, size = 224, normalized size = 0.85

$$\frac{165\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{5}}+\frac{165\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}} c \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{5}}+\frac{165\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}} c \log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{5}}-\frac{165\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}} c \log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{5}}+\frac{23\,c^{3}x^{\frac{5}{2}}+27\,bc^{2}\sqrt{x}}{16\left(cx^{2}+b\right)^{2}b^{4}}+\frac{2\left(7\,cx^{2}-b\right)}{7\,b^{4}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*
sqrt(x))/(b/c)^(1/4))/b^5 + 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 + 165/128*sqrt(2)*(b
*c^3)^(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 165/12
8*sqrt(2)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/b^5 + 1/16*(23*c^3*x^(5/2) + 27*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4) + 2/7*(
7*c*x^2 - b)/(b^4*x^(7/2))
```

maple [A] time = 0.02, size = 198, normalized size = 0.75

$$\frac{23c^{3}x^{\frac{5}{2}}}{16\left(c\,x^{2}+b\right)^{2}b^{4}}+\frac{27c^{2}\sqrt{x}}{16\left(c\,x^{2}+b\right)^{2}b^{3}}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^{5}}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^{5}}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b^{5}}+\frac{2c}{b^{4}x^{\frac{3}{2}}}-\frac{2}{7b^{3}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 23/16/b^4*c^3/(c*x^2+b)^2*x^(5/2)+27/16/b^3*c^2/(c*x^2+b)^2*x^(1/2)+165/128
/b^5*c^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))
/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+165/64/b^5*c^2*(b/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+165/64/b^5*c^2*(b/c)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/7/b^3/x^(7/2)+2*c/b^4/x^(3/2)
```

maxima [A] time = 2.93, size = 246, normalized size = 0.93

$$\frac{385\,c^{3}x^{6}+605\,bc^{2}x^{4}+160\,b^{2}cx^{2}-32\,b^{3}}{112\left(b^{4}c^{2}x^{\frac{15}{2}}+2\,b^{5}cx^{\frac{11}{2}}+b^{6}x^{\frac{7}{2}}\right)}+\frac{165\left(\frac{2\,\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,c^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{7}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{7}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/112*(385*c^3*x^6 + 605*b*c^2*x^4 + 160*b^2*c*x^2 - 32*b^3)/(b^4*c^2*x^(15
/2) + 2*b^5*c*x^(11/2) + b^6*x^(7/2)) + 165/128*(2*sqrt(2)*c^2*arctan(1/2*s
```

```
qrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))
/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c^2*arctan(-1/2*sqrt(2)*(sqrt(
2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqr
t(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
 sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7/4)*log(-sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^4
```

mupad [B] time = 4.35, size = 109, normalized size = 0.41

$$\frac{\frac{10\,c\,x^2}{7\,b^2}-\frac{2}{7\,b}+\frac{605\,c^2\,x^4}{112\,b^3}+\frac{55\,c^3\,x^6}{16\,b^4}}{b^2\,x^{7/2}+c^2\,x^{15/2}+2\,b\,c\,x^{11/2}}+\frac{165\,{\left(-c\right)}^{7/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{19/4}}+\frac{165\,{\left(-c\right)}^{7/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((10*c*x^2)/(7*b^2) - 2/(7*b) + (605*c^2*x^4)/(112*b^3) + (55*c^3*x^6)/(16*
b^4))/(b^2*x^(7/2) + c^2*x^(15/2) + 2*b*c*x^(11/2)) + (165*(-c)^(7/4)*atan(
((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(19/4)) + (165*(-c)^(7/4)*atanh(((-c)^
(1/4)*x^(1/2))/b^(1/4)))/(32*b^(19/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.232 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=279

$$-\frac{221c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{\ldots}\right.}{32\sqrt{2}b^{21/4}}$$

Rubi [A] time = 0.27, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{221c^2}{16b^5\sqrt{x}}-\frac{221c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}}-\frac{221c^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{21/4}}+\frac{221c}{80b^4x^{5/2}}+\frac{17}{16b^2x^{9/2}\left(b+cx^2\right)}-\frac{221}{144b^3x^{9/2}}+\frac{1}{4bx^{9/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -221/(144*b^3*x^(9/2)) + (221*c)/(80*b^4*x^(5/2)) - (221*c^2)/(16*b^5*Sqrt[
x]) + 1/(4*b*x^(9/2)*(b + c*x^2)^2) + 17/(16*b^2*x^(9/2)*(b + c*x^2)) + (22
1*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/
4)) - (221*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[
2]*b^(21/4)) - (221*c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4)) + (221*c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17\int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{221\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{(221c)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{\left(221c^2\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^4}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{\left(221c^3\right)\int \frac{\sqrt{\ }}{b+\ }}{32b^5}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{\left(221c^3\right)\text{Subs}}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{\left(221c^{5/2}\right)\text{Sul}}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{\left(221c^2\right)\text{Subs}}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{221c^{9/4}\log\left(\right.}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{221c^{9/4}\tan^{-1}}{32\sqrt{}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.10

$$-\frac{2\,{}_2F_1\left(-\frac{9}{4},3;-\frac{5}{4};-\frac{cx^2}{b}\right)}{9b^3x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-9/4, 3, -5/4, -((c*x^2)/b)])/(9*b^3*x^(9/2))
```

IntegrateAlgebraic [A] time = 0.48, size = 182, normalized size = 0.65

$$\frac{221c^{9/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{32\sqrt{2}\,b^{21/4}}+\frac{221c^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{32\sqrt{2}\,b^{21/4}}+\frac{-160b^4+544b^3cx^2-7072b^2c^2x^4-17901bc^3x^6-9945c^4x^8}{720b^5x^{9/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-160*b^4 + 544*b^3*c*x^2 - 7072*b^2*c^2*x^4 - 17901*b*c^3*x^6 - 9945*c^4*x
^8)/(720*b^5*x^(9/2)*(b + c*x^2)^2) + (221*c^(9/4)*ArcTan[(b^(1/4)/(Sqrt[2]
*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*b^(21/4))
+ (221*c^(9/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]
*x)])/(32*Sqrt[2]*b^(21/4))
```

fricas [A] time = 0.87, size = 317, normalized size = 1.14

$$\frac{39780\left(b^5c^2x^9+2b^6cx^7+b^7x^5\right)\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}\arctan\left(-\frac{10793861\,b^5c^7\sqrt{x}\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}-\sqrt{-116507435287321\,b^{11}c^9\sqrt{-\frac{c^9}{b^{21}}}+116507435287321\,c^{14}x}\,b^5\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}}{10793861\,c^9}\right)-9945\left(b^5c^2x^9+2b^6cx^7+b^7x^5\right)\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}\log\left(10793861\,b^{16}\left(-\frac{c^9}{b^{21}}\right)^{\frac{3}{4}}+10793861\,c^7\sqrt{x}\right)+9945\left(b^5c^2x^9+2b^6cx^7+b^7x^5\right)\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}\log\left(-10793861\,b^{16}\left(-\frac{c^9}{b^{21}}\right)^{\frac{3}{4}}+10793861\,c^7\sqrt{x}\right)-4\left(9945c^4x^8+17901bc^3x^6+7072b^2c^2x^4-544b^3cx^2+160b^4\right)\sqrt{x}}{2880\left(b^5c^2x^9+2b^6cx^7+b^7x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2880*(39780*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*arcta
n(-1/10793861*(10793861*b^5*c^7*sqrt(x)*(-c^9/b^21)^(1/4) - sqrt(-116507435
287321*b^11*c^9*sqrt(-c^9/b^21) + 116507435287321*c^14*x)*b^5*(-c^9/b^21)^(
1/4))/c^9) - 9945*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*l
og(10793861*b^16*(-c^9/b^21)^(3/4) + 10793861*c^7*sqrt(x)) + 9945*(b^5*c^2*
x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*log(-10793861*b^16*(-c^9/b^2
1)^(3/4) + 10793861*c^7*sqrt(x)) - 4*(9945*c^4*x^8 + 17901*b*c^3*x^6 + 7072
*b^2*c^2*x^4 - 544*b^3*c*x^2 + 160*b^4)*sqrt(x))/(b^5*c^2*x^9 + 2*b^6*c*x^7
 + b^7*x^5)
```

giac [A] time = 0.19, size = 231, normalized size = 0.83

$$-\frac{221\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{6}}-\frac{221\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{6}}+\frac{221\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{6}}-\frac{221\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{6}}-\frac{29\,c^{4}x^{\frac{7}{2}}+33\,bc^{3}x^{\frac{3}{2}}}{16\left(cx^{2}+b\right)^{2}b^{5}}-\frac{2\left(270\,c^{2}x^{4}-27\,bcx^{2}+5\,b^{2}\right)}{45\,b^{5}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/b^6 - 221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 221/128*sqrt(2)*(b*c^
3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 - 221/128*sqr
t(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 -
1/16*(29*c^4*x^(7/2) + 33*b*c^3*x^(3/2))/((c*x^2 + b)^2*b^5) - 2/45*(270*c^
2*x^4 - 27*b*c*x^2 + 5*b^2)/(b^5*x^(9/2))
```

maple [A] time = 0.02, size = 209, normalized size = 0.75

$$-\frac{29c^{4}x^{\frac{7}{2}}}{16\left(c\,x^{2}+b\right)^{2}b^{5}}-\frac{33c^{3}x^{\frac{3}{2}}}{16\left(c\,x^{2}+b\right)^{2}b^{4}}-\frac{221\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{5}}-\frac{221\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{5}}-\frac{221\sqrt{2}\,c^{2}\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{5}}-\frac{12c^{2}}{b^{5}\sqrt{x}}+\frac{6c}{5b^{4}x^{\frac{5}{2}}}-\frac{2}{9b^{3}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -29/16*c^4/b^5/(c*x^2+b)^2*x^(7/2)-33/16*c^3/b^4/(c*x^2+b)^2*x^(3/2)-221/12
8*c^2/b^5/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
)/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-221/64*c^2/b^5/(b/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-221/64*c^2/b^5/(b/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/9/b^3/x^(9/2)-12*c^2/b^5/x^(1/
2)+6/5*c/b^4/x^(5/2)
```

maxima [A] time = 3.04, size = 254, normalized size = 0.91

$$-\frac{9945\,c^{4}x^{8}+17901\,bc^{3}x^{6}+7072\,b^{2}c^{2}x^{4}-544\,b^{3}cx^{2}+160\,b^{4}}{720\left(b^{5}c^{2}x^{\frac{17}{2}}+2\,b^{6}cx^{\frac{13}{2}}+b^{7}x^{\frac{9}{2}}\right)}-\frac{221\,c^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/720*(9945*c^4*x^8 + 17901*b*c^3*x^6 + 7072*b^2*c^2*x^4 - 544*b^3*c*x^2 +
 160*b^4)/(b^5*c^2*x^(17/2) + 2*b^6*c*x^(13/2) + b^7*x^(9/2)) - 221/128*c^3
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
rt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/
4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^5
```

mupad [B] time = 0.14, size = 121, normalized size = 0.43

$$\frac{221\,(-c)^{9/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{21/4}}-\frac{221\,(-c)^{9/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{21/4}}-\frac{\frac{2}{9\,b}-\frac{34\,c\,x^2}{45\,b^2}+\frac{442\,c^2\,x^4}{45\,b^3}+\frac{1989\,c^3\,x^6}{80\,b^4}+\frac{221\,c^4\,x^8}{16\,b^5}}{b^2\,x^{9/2}+c^2\,x^{17/2}+2\,b\,c\,x^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (221*(-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(21/4)) - (221*(
-c)^(9/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(21/4)) - (2/(9*b) - (3
4*c*x^2)/(45*b^2) + (442*c^2*x^4)/(45*b^3) + (1989*c^3*x^6)/(80*b^4) + (221
*c^4*x^8)/(16*b^5))/(b^2*x^(9/2) + c^2*x^(17/2) + 2*b*c*x^(13/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.233 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=279

$$\frac{285c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}-\frac{285c^{11/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}+\frac{285c^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right.}{32\sqrt{2}\,b^{23/4}}$$

Rubi [A] time = 0.26, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{95c^2}{16b^5x^{3/2}}+\frac{285c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}-\frac{285c^{11/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}+\frac{285c^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{23/4}}-\frac{285c^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{23/4}}+\frac{285c}{112b^4x^{7/2}}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}-\frac{285}{176b^3x^{11/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -285/(176*b^3*x^(11/2)) + (285*c)/(112*b^4*x^(7/2)) - (95*c^2)/(16*b^5*x^(3
/2)) + 1/(4*b*x^(11/2)*(b + c*x^2)^2) + 19/(16*b^2*x^(11/2)*(b + c*x^2)) +
(285*c^(11/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(23/4)) - (285*c^(11/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*
Sqrt[2]*b^(23/4)) + (285*c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4)) - (285*c^(11/4)*Log[Sqrt[b] + Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{13/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19\int \frac{1}{x^{13/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} + \frac{285\int \frac{1}{x^{13/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{(285c)\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} + \frac{\left(285c^2\right)\int \frac{1}{x^{5/2}(\cdots}}{32b^4}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{(28\cdots}{\cdots}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{(28\cdots}{\cdots}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{(28\cdots}{\cdots}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{(28\cdots}{\cdots}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} + \frac{285\cdots}{\cdots}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} + \frac{285\cdots}{\cdots}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.10

$$-\frac{2\,{}_2F_1\left(-\frac{11}{4},3;-\frac{7}{4};-\frac{cx^2}{b}\right)}{11b^3x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-11/4, 3, -7/4, -((c*x^2)/b)])/(11*b^3*x^(11/2))
```

IntegrateAlgebraic [A] time = 0.49, size = 182, normalized size = 0.65

$$\frac{285c^{11/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{b}}}{\sqrt{x}}\right)}{32\sqrt{2}\,b^{23/4}}-\frac{285c^{11/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{32\sqrt{2}\,b^{23/4}}+\frac{-224b^4+608b^3cx^2-3040b^2c^2x^4-11495bc^3x^6-7315c^4x^8}{1232b^5x^{11/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] (-224*b^4 + 608*b^3*c*x^2 - 3040*b^2*c^2*x^4 - 11495*b*c^3*x^6 - 7315*c^4*x
^8)/(1232*b^5*x^(11/2)*(b + c*x^2)^2) + (285*c^(11/4)*ArcTan[(b^(1/4)/(Sqrt
[2]*c^(1/4)) - (c^(1/4)*x)/(Sqrt[2]*b^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*b^(23/4
)) - (285*c^(11/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqr
t[c]*x)])/(32*Sqrt[2]*b^(23/4))
```

fricas [A] time = 2.08, size = 311, normalized size = 1.11

$$-\frac{87780\left(b^5c^2x^{10}+2b^6cx^8+b^7x^6\right)\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{17}c^3\sqrt{x}\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{3}{4}}-\sqrt{b^{12}\sqrt{-\frac{c^{11}}{b^{23}}}+c^6x}\,b^{17}\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{3}{4}}}{c^{11}}\right)+21945\left(b^5c^2x^{10}+2b^6cx^8+b^7x^6\right)\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\log\left(285b^6\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}+285c^3\sqrt{x}\right)-21945\left(b^5c^2x^{10}+2b^6cx^8+b^7x^6\right)\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\log\left(-285b^6\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}+285c^3\sqrt{x}\right)+4\left(7315c^4x^8+11495bc^3x^6+3040b^2c^2x^4-608b^3cx^2+224b^4\right)\sqrt{x}}{4928\left(b^5c^2x^{10}+2b^6cx^8+b^7x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4928*(87780*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*ar
ctan(-(b^17*c^3*sqrt(x)*(-c^11/b^23)^(3/4) - sqrt(b^12*sqrt(-c^11/b^23) + c
^6*x)*b^17*(-c^11/b^23)^(3/4))/c^11) + 21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 +
b^7*x^6)*(-c^11/b^23)^(1/4)*log(285*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x
)) - 21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*log(-2
85*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x)) + 4*(7315*c^4*x^8 + 11495*b*c^
3*x^6 + 3040*b^2*c^2*x^4 - 608*b^3*c*x^2 + 224*b^4)*sqrt(x))/(b^5*c^2*x^10
+ 2*b^6*c*x^8 + b^7*x^6)
```

giac [A] time = 0.17, size = 243, normalized size = 0.87

$$-\frac{285\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{1}{4}}c^{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{6}}-\frac{285\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{1}{4}}c^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{6}}-\frac{285\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{1}{4}}c^{2}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{6}}+\frac{285\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{1}{4}}c^{2}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{6}}-\frac{31\,c^{4}x^{\frac{5}{2}}+35\,bc^{3}\sqrt{x}}{16\left(cx^{2}+b\right)^{2}b^{5}}-\frac{2\left(154\,c^{2}x^{4}-33\,bcx^{2}+7\,b^{2}\right)}{77\,b^{5}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] -285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) +
 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(-1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/128*sqrt(
2)*(b*c^3)^(1/4)*c^2*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 +
 285/128*sqrt(2)*(b*c^3)^(1/4)*c^2*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/b^6 - 1/16*(31*c^4*x^(5/2) + 35*b*c^3*sqrt(x))/((c*x^2 + b)^2*b^5
) - 2/77*(154*c^2*x^4 - 33*b*c*x^2 + 7*b^2)/(b^5*x^(11/2))
```

maple [A] time = 0.02, size = 209, normalized size = 0.75

$$-\frac{31c^{4}x^{\frac{5}{2}}}{16\left(c\,x^{2}+b\right)^{2}b^{5}}-\frac{35c^{3}\sqrt{x}}{16\left(c\,x^{2}+b\right)^{2}b^{4}}-\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^{6}}-\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^{6}}-\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b^{6}}-\frac{4c^{2}}{b^{5}x^{\frac{3}{2}}}+\frac{6c}{7b^{4}x^{\frac{7}{2}}}-\frac{2}{11b^{3}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] -31/16*c^4/b^5/(c*x^2+b)^2*x^(5/2)-35/16*c^3/b^4/(c*x^2+b)^2*x^(1/2)-285/12
8*c^3/b^6*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
)/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-285/64*c^3/b^6*(b/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-285/64*c^3/b^6*(b/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/11/b^3/x^(11/2)-4*c^2/b^5/x^(3
/2)+6/7*c/b^4/x^(7/2)
```

maxima [A] time = 3.06, size = 257, normalized size = 0.92

$$-\frac{7315\,c^{4}x^{8}+11495\,bc^{3}x^{6}+3040\,b^{2}c^{2}x^{4}-608\,b^{3}cx^{2}+224\,b^{4}}{1232\left(b^{5}c^{2}x^{\frac{19}{2}}+2\,b^{6}cx^{\frac{15}{2}}+b^{7}x^{\frac{11}{2}}\right)}-\frac{285\left(\frac{2\,\sqrt{2}c^{3}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}c^{3}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{11}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}c^{\frac{11}{4}}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1232*(7315*c^4*x^8 + 11495*b*c^3*x^6 + 3040*b^2*c^2*x^4 - 608*b^3*c*x^2
+ 224*b^4)/(b^5*c^2*x^(19/2) + 2*b^6*c*x^(15/2) + b^7*x^(11/2)) - 285/128*(
```

```
2*sqrt(2)*c^3*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c^3*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(11/4)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(11/4)
*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^5
```

mupad [B] time = 4.39, size = 121, normalized size = 0.43

$$\frac{285\,{\left(-c\right)}^{11/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{23/4}}-\frac{\frac{2}{11\,b}-\frac{38\,c\,x^2}{77\,b^2}+\frac{190\,c^2\,x^4}{77\,b^3}+\frac{1045\,c^3\,x^6}{112\,b^4}+\frac{95\,c^4\,x^8}{16\,b^5}}{b^2\,x^{11/2}+c^2\,x^{19/2}+2\,b\,c\,x^{15/2}}+\frac{285\,{\left(-c\right)}^{11/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{23/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (285*(-c)^(11/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(23/4)) - (2/(11
*b) - (38*c*x^2)/(77*b^2) + (190*c^2*x^4)/(77*b^3) + (1045*c^3*x^6)/(112*b^
4) + (95*c^4*x^8)/(16*b^5))/(b^2*x^(11/2) + c^2*x^(19/2) + 2*b*c*x^(15/2))
+ (285*(-c)^(11/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(23/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.234 $\int (cx)^m \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=73

$$\frac{b^3x^7(cx)^m}{m+7}+\frac{3b^2cx^9(cx)^m}{m+9}+\frac{3bc^2x^{11}(cx)^m}{m+11}+\frac{c^3x^{13}(cx)^m}{m+13}$$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 270}

$$\frac{3b^2cx^9(cx)^m}{m+9}+\frac{b^3x^7(cx)^m}{m+7}+\frac{3bc^2x^{11}(cx)^m}{m+11}+\frac{c^3x^{13}(cx)^m}{m+13}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (b^3*x^7*(c*x)^m)/(7 + m) + (3*b^2*c*x^9*(c*x)^m)/(9 + m) + (3*b*c^2*x^11*(
c*x)^m)/(11 + m) + (c^3*x^13*(c*x)^m)/(13 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^2+cx^4\right)^3 dx &= (x^{-m}(cx)^m)\,\text{Subst}\left(\int x^m \left(bx^2+cx^4\right)^3 dx, x, x\right)\\ &= (x^{-m}(cx)^m)\,\text{Subst}\left(\int x^{6+m} \left(b+cx^2\right)^3 dx, x, x\right)\\ &= (x^{-m}(cx)^m)\,\text{Subst}\left(\int \left(b^3x^{6+m}+3b^2cx^{8+m}+3bc^2x^{10+m}+c^3x^{12+m}\right) dx, x, x\right)\\ &= \frac{b^3x^7(cx)^m}{7+m}+\frac{3b^2cx^9(cx)^m}{9+m}+\frac{3bc^2x^{11}(cx)^m}{11+m}+\frac{c^3x^{13}(cx)^m}{13+m}\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.81

$$x^7(cx)^m\left(\frac{b^3}{m+7}+\frac{3b^2cx^2}{m+9}+\frac{3bc^2x^4}{m+11}+\frac{c^3x^6}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^7*(c*x)^m*(b^3/(7 + m) + (3*b^2*c*x^2)/(9 + m) + (3*b*c^2*x^4)/(11 + m) +
 (c^3*x^6)/(13 + m))
```

IntegrateAlgebraic [F] time = 0.33, size = 0, normalized size = 0.00

$$\int (cx)^m \left(bx^2+cx^4\right)^3 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(c*x)^m*(b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.66, size = 161, normalized size = 2.21

$$\frac{\left(\left(c^3m^3+27\,c^3m^2+239\,c^3m+693\,c^3\right)x^{13}+3\left(bc^2m^3+29\,bc^2m^2+271\,bc^2m+819\,bc^2\right)x^{11}+3\left(b^2cm^3+31\,b^2cm^2+311\,b^2cm+1001\,b^2c\right)x^9+\left(b^3m^3+33\,b^3m^2+359\,b^3m+1287\,b^3\right)x^7\right)(cx)^m}{m^4+40\,m^3+590\,m^2+3800\,m+9009}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^3 + 27*c^3*m^2 + 239*c^3*m + 693*c^3)*x^13 + 3*(b*c^2*m^3 + 29*b*c^
2*m^2 + 271*b*c^2*m + 819*b*c^2)*x^11 + 3*(b^2*c*m^3 + 31*b^2*c*m^2 + 311*b
^2*c*m + 1001*b^2*c)*x^9 + (b^3*m^3 + 33*b^3*m^2 + 359*b^3*m + 1287*b^3)*x^
7)*(c*x)^m/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

giac [B] time = 0.18, size = 264, normalized size = 3.62

$$\frac{(cx)^m c^3m^3x^{13} + 27\,(cx)^m c^3m^2x^{13} + 3\,(cx)^m bc^2m^3x^{11} + 239\,(cx)^m c^3mx^{13} + 87\,(cx)^m bc^2m^2x^{11} + 693\,(cx)^m c^3x^{13} + 3\,(cx)^m b^2cm^3x^9 + 813\,(cx)^m bc^2mx^{11} + 93\,(cx)^m b^2cm^2x^9 + 2457\,(cx)^m bc^2x^{11} + (cx)^m b^3m^3x^7 + 933\,(cx)^m b^2cmx^9 + 33\,(cx)^m b^3m^2x^7 + 3003\,(cx)^m b^2cx^9 + 359\,(cx)^m b^3mx^7 + 1287\,(cx)^m b^3x^7}{m^4 + 40\,m^3 + 590\,m^2 + 3800\,m + 9009}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] ((c*x)^m*c^3*m^3*x^13 + 27*(c*x)^m*c^3*m^2*x^13 + 3*(c*x)^m*b*c^2*m^3*x^11
+ 239*(c*x)^m*c^3*m*x^13 + 87*(c*x)^m*b*c^2*m^2*x^11 + 693*(c*x)^m*c^3*x^13
 + 3*(c*x)^m*b^2*c*m^3*x^9 + 813*(c*x)^m*b*c^2*m*x^11 + 93*(c*x)^m*b^2*c*m^
2*x^9 + 2457*(c*x)^m*b*c^2*x^11 + (c*x)^m*b^3*m^3*x^7 + 933*(c*x)^m*b^2*c*m
*x^9 + 33*(c*x)^m*b^3*m^2*x^7 + 3003*(c*x)^m*b^2*c*x^9 + 359*(c*x)^m*b^3*m*
x^7 + 1287*(c*x)^m*b^3*x^7)/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

maple [B] time = 0.01, size = 181, normalized size = 2.48

$$\frac{\left(c^3m^3x^6 + 27c^3m^2x^6 + 3b\,c^2m^3x^4 + 239c^3m\,x^6 + 87b\,c^2m^2x^4 + 693c^3x^6 + 3b^2c\,m^3x^2 + 813b\,c^2m\,x^4 + 93b^2c\,m^2x^2 + 2457b\,c^2x^4 + b^3m^3 + 933b^2cm\,x^2 + 33b^3m^2 + 3003b^2c\,x^2 + 359b^3m + 1287b^3\right)x^7\,(cx)^m}{(m+13)\,(m+11)\,(m+9)\,(m+7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2)^3,x)
```

```
[Out] (c*x)^m*(c^3*m^3*x^6+27*c^3*m^2*x^6+3*b*c^2*m^3*x^4+239*c^3*m*x^6+87*b*c^2*
m^2*x^4+693*c^3*x^6+3*b^2*c*m^3*x^2+813*b*c^2*m*x^4+93*b^2*c*m^2*x^2+2457*b
*c^2*x^4+b^3*m^3+933*b^2*c*m*x^2+33*b^3*m^2+3003*b^2*c*x^2+359*b^3*m+1287*b
^3)*x^7/(13+m)/(11+m)/(9+m)/(7+m)
```

maxima [A] time = 1.51, size = 76, normalized size = 1.04

$$\frac{c^{m+3}x^{13}x^m}{m+13} + \frac{3\,bc^{m+2}x^{11}x^m}{m+11} + \frac{3\,b^2c^{m+1}x^9x^m}{m+9} + \frac{b^3c^mx^7x^m}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] c^(m + 3)*x^13*x^m/(m + 13) + 3*b*c^(m + 2)*x^11*x^m/(m + 11) + 3*b^2*c^(m
+ 1)*x^9*x^m/(m + 9) + b^3*c^m*x^7*x^m/(m + 7)
```

mupad [B] time = 4.29, size = 171, normalized size = 2.34

$$(c\,x)^m\,\left(\frac{b^3\,x^7\,\left(m^3+33\,m^2+359\,m+1287\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}+\frac{c^3\,x^{13}\,\left(m^3+27\,m^2+239\,m+693\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}+\frac{3\,b\,c^2\,x^{11}\,\left(m^3+29\,m^2+271\,m+819\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}+\frac{3\,b^2\,c\,x^9\,\left(m^3+31\,m^2+311\,m+1001\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (c*x)^m*((b^3*x^7*(359*m + 33*m^2 + m^3 + 1287))/(3800*m + 590*m^2 + 40*m^3
 + m^4 + 9009) + (c^3*x^13*(239*m + 27*m^2 + m^3 + 693))/(3800*m + 590*m^2
+ 40*m^3 + m^4 + 9009) + (3*b*c^2*x^11*(271*m + 29*m^2 + m^3 + 819))/(3800*
m + 590*m^2 + 40*m^3 + m^4 + 9009) + (3*b^2*c*x^9*(311*m + 31*m^2 + m^3 + 1
001))/(3800*m + 590*m^2 + 40*m^3 + m^4 + 9009))
```

sympy [A] time = 5.28, size = 758, normalized size = 10.38

$$\begin{cases} \frac{-\frac{b^3}{6x^6} - \frac{3b^2c}{4x^4} - \frac{3bc^2}{2x^2} + c^3\log(x)}{c^{13}} & \text{for } m = -13 \\ \frac{-\frac{b^3}{4x^4} - \frac{3b^2c}{2x^2} + 3bc^2\log(x) + \frac{c^3x^2}{2}}{c^{11}} & \text{for } m = -11 \\ \frac{-\frac{b^3}{2x^2} + 3b^2c\log(x) + \frac{3bc^2x^2}{2} + \frac{c^3x^4}{4}}{c^9} & \text{for } m = -9 \\ \frac{b^3\log(x) + \frac{3b^2cx^2}{2} + \frac{3bc^2x^4}{4} + \frac{c^3x^6}{6}}{c^7} & \text{for } m = -7 \\ \frac{b^3c^mm^3x^7x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{33b^3c^mm^2x^7x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{359b^3c^mmx^7x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{1287b^3c^mx^7x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{3b^2cc^mm^3x^9x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{93b^2cc^mm^2x^9x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{933b^2cc^mmx^9x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{3003b^2cc^mx^9x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{3bc^2c^mm^3x^{11}x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{87bc^2c^mm^2x^{11}x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{813bc^2c^mmx^{11}x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{2457bc^2c^mx^{11}x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{c^3c^mm^3x^{13}x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{27c^3c^mm^2x^{13}x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{239c^3c^mmx^{13}x^m}{m^4+40m^3+590m^2+3800m+9009} + \frac{693c^3c^mx^{13}x^m}{m^4+40m^3+590m^2+3800m+9009} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2)**3,x)
```

```
[Out] Piecewise(((-b**3/(6*x**6) - 3*b**2*c/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*l
og(x))/c**13, Eq(m, -13)), ((-b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2*
log(x) + c**3*x**2/2)/c**11, Eq(m, -11)), ((-b**3/(2*x**2) + 3*b**2*c*log(x
) + 3*b*c**2*x**2/2 + c**3*x**4/4)/c**9, Eq(m, -9)), ((b**3*log(x) + 3*b**2
*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/c**7, Eq(m, -7)), (b**3*c**m*m**
3*x**7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 33*b**3*c**m*m**2
*x**7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 359*b**3*c**m*m*x*
*7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 1287*b**3*c**m*x**7*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b**2*c*c**m*m**3*x**9*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 93*b**2*c*c**m*m**2*x**9*
x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 933*b**2*c*c**m*m*x**9*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3003*b**2*c*c**m*x**9*x**
m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b*c**2*c**m*m**3*x**11*x*
*m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 87*b*c**2*c**m*m**2*x**11*
x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 813*b*c**2*c**m*m*x**11*
x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 2457*b*c**2*c**m*x**11*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + c**3*c**m*m**3*x**13*x**m
/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 27*c**3*c**m*m**2*x**13*x**m
/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 239*c**3*c**m*m*x**13*x**m/(
m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 693*c**3*c**m*x**13*x**m/(m**4
 + 40*m**3 + 590*m**2 + 3800*m + 9009), True))
```

3.235 $\int (cx)^m \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=52

$$\frac{b^2x^5(cx)^m}{m+5}+\frac{2bcx^7(cx)^m}{m+7}+\frac{c^2x^9(cx)^m}{m+9}$$

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 270}

$$\frac{b^2x^5(cx)^m}{m+5}+\frac{2bcx^7(cx)^m}{m+7}+\frac{c^2x^9(cx)^m}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5*(c*x)^m)/(5 + m) + (2*b*c*x^7*(c*x)^m)/(7 + m) + (c^2*x^9*(c*x)^m)
/(9 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^2+cx^4\right)^2 dx &= (x^{-m}(cx)^m)\,\text{Subst}\left(\int x^m \left(bx^2+cx^4\right)^2 dx, x, x\right)\\ &= (x^{-m}(cx)^m)\,\text{Subst}\left(\int x^{4+m} \left(b+cx^2\right)^2 dx, x, x\right)\\ &= (x^{-m}(cx)^m)\,\text{Subst}\left(\int \left(b^2x^{4+m}+2bcx^{6+m}+c^2x^{8+m}\right) dx, x, x\right)\\ &= \frac{b^2x^5(cx)^m}{5+m}+\frac{2bcx^7(cx)^m}{7+m}+\frac{c^2x^9(cx)^m}{9+m}\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.83

$$x^5(cx)^m\left(\frac{b^2}{m+5}+\frac{2bcx^2}{m+7}+\frac{c^2x^4}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4)^2,x]
[Out] x^5*(c*x)^m*(b^2/(5 + m) + (2*b*c*x^2)/(7 + m) + (c^2*x^4)/(9 + m))
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int (cx)^m \left(bx^2+cx^4\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x)^m*(b*x^2 + c*x^4)^2,x]
[Out] Defer[IntegrateAlgebraic][(c*x)^m*(b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 0.65, size = 89, normalized size = 1.71

$$\frac{\left(\left(c^2m^2+12\,c^2m+35\,c^2\right)x^9+2\left(bcm^2+14\,bcm+45\,bc\right)x^7+\left(b^2m^2+16\,b^2m+63\,b^2\right)x^5\right)(cx)^m}{m^3+21\,m^2+143\,m+315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] ((c^2*m^2 + 12*c^2*m + 35*c^2)*x^9 + 2*(b*c*m^2 + 14*b*c*m + 45*b*c)*x^7 +
(b^2*m^2 + 16*b^2*m + 63*b^2)*x^5)*(c*x)^m/(m^3 + 21*m^2 + 143*m + 315)
```

giac [B] time = 0.17, size = 141, normalized size = 2.71

$$\frac{(cx)^m c^2m^2x^9+12\,(cx)^m c^2mx^9+2\,(cx)^m bcm^2x^7+35\,(cx)^m c^2x^9+28\,(cx)^m bcmx^7+(cx)^m b^2m^2x^5+90\,(cx)^m bcx^7+16\,(cx)^m b^2mx^5+63\,(cx)^m b^2x^5}{m^3+21\,m^2+143\,m+315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] ((c*x)^m*c^2*m^2*x^9 + 12*(c*x)^m*c^2*m*x^9 + 2*(c*x)^m*b*c*m^2*x^7 + 35*(c
*x)^m*c^2*x^9 + 28*(c*x)^m*b*c*m*x^7 + (c*x)^m*b^2*m^2*x^5 + 90*(c*x)^m*b*c
*x^7 + 16*(c*x)^m*b^2*m*x^5 + 63*(c*x)^m*b^2*x^5)/(m^3 + 21*m^2 + 143*m + 3
15)
```

maple [A] time = 0.01, size = 96, normalized size = 1.85

$$\frac{\left(c^{2}m^{2}x^{4}+12c^{2}m\,x^{4}+2bc\,m^{2}x^{2}+35c^{2}x^{4}+28bcm\,x^{2}+b^{2}m^{2}+90bc\,x^{2}+16b^{2}m+63b^{2}\right)x^{5}\left(cx\right)^{m}}{\left(m+9\right)\left(m+7\right)\left(m+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2)^2,x)
```

```
[Out] (c*x)^m*(c^2*m^2*x^4+12*c^2*m*x^4+2*b*c*m^2*x^2+35*c^2*x^4+28*b*c*m*x^2+b^2
*m^2+90*b*c*x^2+16*b^2*m+63*b^2)*x^5/(m+9)/(m+7)/(5+m)
```

maxima [A] time = 1.45, size = 55, normalized size = 1.06

$$\frac{c^{m+2}x^{9}x^{m}}{m+9}+\frac{2\,bc^{m+1}x^{7}x^{m}}{m+7}+\frac{b^{2}c^{m}x^{5}x^{m}}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] c^(m + 2)*x^9*x^m/(m + 9) + 2*b*c^(m + 1)*x^7*x^m/(m + 7) + b^2*c^m*x^5*x^m
/(m + 5)
```

mupad [B] time = 4.19, size = 97, normalized size = 1.87

$$(c\,x)^{m}\left(\frac{b^{2}\,x^{5}\,\left(m^{2}+16\,m+63\right)}{m^{3}+21\,m^{2}+143\,m+315}+\frac{c^{2}\,x^{9}\,\left(m^{2}+12\,m+35\right)}{m^{3}+21\,m^{2}+143\,m+315}+\frac{2\,b\,c\,x^{7}\,\left(m^{2}+14\,m+45\right)}{m^{3}+21\,m^{2}+143\,m+315}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2 + c*x^4)^2,x)
```

```
[Out] (c*x)^m*((b^2*x^5*(16*m + m^2 + 63))/(143*m + 21*m^2 + m^3 + 315) + (c^2*x^
9*(12*m + m^2 + 35))/(143*m + 21*m^2 + m^3 + 315) + (2*b*c*x^7*(14*m + m^2
+ 45))/(143*m + 21*m^2 + m^3 + 315))
```

sympy [A] time = 2.29, size = 352, normalized size = 6.77

$$\begin{cases} \frac{-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2\log(x)}{c^9} & \text{for } m = -9 \\ \frac{-\frac{b^2}{2x^2} + 2bc\log(x) + \frac{c^2x^2}{2}}{c^7} & \text{for } m = -7 \\ \frac{b^2\log(x) + bcx^2 + \frac{c^2x^4}{4}}{c^5} & \text{for } m = -5 \\ \frac{b^2c^m m^2x^5x^m}{m^3+21m^2+143m+315} + \frac{16b^2c^m mx^5x^m}{m^3+21m^2+143m+315} + \frac{63b^2c^m x^5x^m}{m^3+21m^2+143m+315} + \frac{2bcc^m m^2x^7x^m}{m^3+21m^2+143m+315} + \frac{28bcc^m mx^7x^m}{m^3+21m^2+143m+315} + \frac{90bcc^m x^7x^m}{m^3+21m^2+143m+315} + \frac{c^2c^m m^2x^9x^m}{m^3+21m^2+143m+315} + \frac{12c^2c^m mx^9x^m}{m^3+21m^2+143m+315} + \frac{35c^2c^m x^9x^m}{m^3+21m^2+143m+315} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2)**2,x)
```

```
[Out] Piecewise((((-b**2/(4*x**4) - b*c/x**2 + c**2*log(x))/c**9, Eq(m, -9)), ((-b
**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/c**7, Eq(m, -7)), ((b**2*log(x)
+ b*c*x**2 + c**2*x**4/4)/c**5, Eq(m, -5)), (b**2*c**m*m**2*x**5*x**m/(m**3
 + 21*m**2 + 143*m + 315) + 16*b**2*c**m*m*x**5*x**m/(m**3 + 21*m**2 + 143*
m + 315) + 63*b**2*c**m*x**5*x**m/(m**3 + 21*m**2 + 143*m + 315) + 2*b*c*c*
*m*m**2*x**7*x**m/(m**3 + 21*m**2 + 143*m + 315) + 28*b*c*c**m*m*x**7*x**m/
(m**3 + 21*m**2 + 143*m + 315) + 90*b*c*c**m*x**7*x**m/(m**3 + 21*m**2 + 14
3*m + 315) + c**2*c**m*m**2*x**9*x**m/(m**3 + 21*m**2 + 143*m + 315) + 12*c
**2*c**m*m*x**9*x**m/(m**3 + 21*m**2 + 143*m + 315) + 35*c**2*c**m*x**9*x**
m/(m**3 + 21*m**2 + 143*m + 315), True))
```

3.236 $\int (cx)^m \left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=34

$$\frac{b(cx)^{m+3}}{c^3(m+3)}+\frac{(cx)^{m+5}}{c^4(m+5)}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{b(cx)^{m+3}}{c^3(m+3)}+\frac{(cx)^{m+5}}{c^4(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] (b*(c*x)^(3 + m))/(c^3*(3 + m)) + (c*x)^(5 + m)/(c^4*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^2+cx^4\right)\,dx &= \int \left(\frac{b(cx)^{2+m}}{c^2}+\frac{(cx)^{4+m}}{c^3}\right)dx\\ &= \frac{b(cx)^{3+m}}{c^3(3+m)}+\frac{(cx)^{5+m}}{c^4(5+m)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.79

$$x^3(cx)^m\left(\frac{b}{m+3}+\frac{cx^2}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] x^3*(c*x)^m*(b/(3 + m) + (c*x^2)/(5 + m))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (cx)^m \left(bx^2 + cx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c*x)^m*(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.80, size = 39, normalized size = 1.15

$$\frac{\left((cm + 3\,c)x^5 + (bm + 5\,b)x^3\right)(cx)^m}{m^2 + 8\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] ((c*m + 3*c)*x^5 + (b*m + 5*b)*x^3)*(c*x)^m/(m^2 + 8*m + 15)
```

giac [A] time = 0.20, size = 56, normalized size = 1.65

$$\frac{(cx)^m\,cmx^5 + 3\,(cx)^m\,cx^5 + (cx)^m\,bmx^3 + 5\,(cx)^m\,bx^3}{m^2 + 8\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] ((c*x)^m*c*m*x^5 + 3*(c*x)^m*c*x^5 + (c*x)^m*b*m*x^3 + 5*(c*x)^m*b*x^3)/(m^
2 + 8*m + 15)
```

maple [A] time = 0.00, size = 39, normalized size = 1.15

$$\frac{\left(cm\,x^2 + 3c\,x^2 + bm + 5b\right)x^3\,(cx)^m}{(m+5)\,(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2),x)
```

```
[Out] (c*x)^m*(c*m*x^2+3*c*x^2+b*m+5*b)*x^3/(m+5)/(3+m)
```

maxima [A] time = 1.43, size = 34, normalized size = 1.00

$$\frac{c^{m+1}x^5x^m}{m+5} + \frac{bc^mx^3x^m}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] c^(m + 1)*x^5*x^m/(m + 5) + b*c^m*x^3*x^m/(m + 3)
```

mupad [B] time = 4.15, size = 38, normalized size = 1.12

$$\frac{x^3\,{\left(c\,x\right)}^m\,\left(5\,b+b\,m+3\,c\,x^2+c\,m\,x^2\right)}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2 + c*x^4),x)
```

```
[Out] (x^3*(c*x)^m*(5*b + b*m + 3*c*x^2 + c*m*x^2))/(8*m + m^2 + 15)
```

sympy [A] time = 0.76, size = 119, normalized size = 3.50

$$\begin{cases} \frac{-\frac{b}{2x^2}+c\log(x)}{c^5} & \text{for } m = -5 \\ \frac{b\log(x)+\frac{cx^2}{2}}{c^3} & \text{for } m = -3 \\ \frac{bc^m m x^3 x^m}{m^2+8m+15} + \frac{5bc^m x^3 x^m}{m^2+8m+15} + \frac{cc^m m x^5 x^m}{m^2+8m+15} + \frac{3cc^m x^5 x^m}{m^2+8m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((((-b/(2*x**2) + c*log(x))/c**5, Eq(m, -5)), ((b*log(x) + c*x**2/2
)/c**3, Eq(m, -3)), (b*c**m*m*x**3*x**m/(m**2 + 8*m + 15) + 5*b*c**m*x**3*x
**m/(m**2 + 8*m + 15) + c*c**m*m*x**5*x**m/(m**2 + 8*m + 15) + 3*c*c**m*x**
5*x**m/(m**2 + 8*m + 15), True))
```

3.237 $\int x^3 \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^2+b^2x^4\right)dx &= \int \left(a^2x^3+2abx^5+b^2x^7\right)dx\\ &= \frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a^2 + 2abx^2 + b^2x^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.68, size = 24, normalized size = 0.80

$$\frac{1}{8}x^8b^2 + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{8}b^2x^8 + \frac{1}{3}ab\,x^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

maxima [A] time = 1.36, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2),x)
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

3.238 $\int x^2 \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^2+b^2x^4\right)dx&=\int\left(a^2x^2+2abx^4+b^2x^6\right)dx\\&=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a^2 + 2abx^2 + b^2x^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.47, size = 24, normalized size = 0.80

$$\frac{1}{7}x^7b^2 + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{7}b^2x^7 + \frac{2}{5}ab\,x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^{2}x^{3}}{3}+\frac{2abx^{5}}{5}+\frac{b^{2}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

3.239 $\int x \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + (b^2*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x \left(a^2+2abx^2+b^2x^4\right) \, dx &= \int \left(a^2x+2abx^3+b^2x^5\right) \, dx\\ &= \frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.53

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a^2+2abx^2+b^2x^4\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.80, size = 24, normalized size = 0.80

$$\frac{1}{6}x^6b^2+\frac{1}{2}x^4ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^2 + 1/2*x^4*b*a + 1/2*x^2*a^2
```

giac [A] time = 0.17, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*b^2*x^6
```

maxima [A] time = 1.29, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^2}{2}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^{2}x^{2}}{2}+\frac{abx^{4}}{2}+\frac{b^{2}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

3.240 $\int \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right) dx=a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2abx^2+b^2x^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] IntegrateAlgebraic[a^2 + 2*a*b*x^2 + b^2*x^4, x]
```

fricas [A]   time = 0.41, size = 21, normalized size = 0.84

$$\frac{1}{5}x^5b^2 + \frac{2}{3}x^3ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="fricas")
[Out] 1/5*x^5*b^2 + 2/3*x^3*b*a + x*a^2
```

giac [A]   time = 0.15, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A]   time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}b^2x^5 + \frac{2}{3}ab\,x^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^2*x^4+2*a*b*x^2+a^2,x)
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

maxima [A]   time = 1.35, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

mupad [B]   time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^2 + b^2*x^4 + 2*a*b*x^2,x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

sympy [A] time = 0.07, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**2*x**4+2*a*b*x**2+a**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.241 $\int \frac{a^2+2abx^2+b^2x^4}{x} \, dx$

Optimal. Leaf size=23

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a^2+2abx^2+b^2x^4}{x} \, dx &= \int \left(\frac{a^2}{x}+2abx+b^2x^3\right) \, dx \\ &= abx^2+\frac{b^2x^4}{4}+a^2\log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x, x]
```

fricas [A] time = 0.75, size = 21, normalized size = 0.91

$$\frac{1}{4}\,b^2x^4 + abx^2 + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="fricas")
[Out] 1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

giac [A] time = 0.17, size = 24, normalized size = 1.04

$$\frac{1}{4}\,b^2x^4 + abx^2 + \frac{1}{2}\,a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="giac")
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{b^2x^4}{4} + ab\,x^2 + a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x,x)
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

maxima [A] time = 1.28, size = 24, normalized size = 1.04

$$\frac{1}{4}\,b^2x^4 + abx^2 + \frac{1}{2}\,a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="maxima")
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

mupad [B] time = 4.10, size = 21, normalized size = 0.91

$$a^2\ \ln(x) + \frac{b^2\,x^4}{4} + a\,b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x,x)
[Out] a^2*log(x) + (b^2*x^4)/4 + a*b*x^2
```

sympy [A] time = 0.10, size = 20, normalized size = 0.87

$$a^2 \log{(x)} + abx^2 + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x,x)
[Out] a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

3.242 $\int \frac{a^2+2abx^2+b^2x^4}{x^2}\,dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^2}\,dx &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^2}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2, x]
```

fricas [A] time = 0.84, size = 25, normalized size = 1.04

$$\frac{b^2x^4 + 6\,abx^2 - 3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="fricas")
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

giac [A] time = 0.15, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="giac")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{b^2x^3}{3} + 2abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^2,x)
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

maxima [A] time = 1.31, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

mupad [B] time = 0.03, size = 22, normalized size = 0.92

$$\frac{b^2\,x^3}{3}-\frac{a^2}{x}+2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^2,x)
```

```
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

sympy [A] time = 0.10, size = 19, normalized size = 0.79

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

3.243 $\int \frac{a^2+2abx^2+b^2x^4}{x^3} \, dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
```

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^3} \, dx &= \int \left(\frac{a^2}{x^3}+\frac{2ab}{x}+b^2x\right) dx\\ &= -\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3, x]
```

fricas [A] time = 0.68, size = 27, normalized size = 1.00

$$\frac{b^2x^4 + 4\,abx^2\log(x) - a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="fricas")
[Out] 1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

giac [A] time = 0.19, size = 32, normalized size = 1.19

$$\frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="giac")
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{b^2x^2}{2} + 2ab\ln(x) - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^3,x)
[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.89

$$\frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

mupad [B] time = 0.03, size = 23, normalized size = 0.85

$$\frac{b^2\,x^2}{2}-\frac{a^2}{2\,x^2}+2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^3,x)
```

```
[Out] (b^2*x^2)/2 - a^2/(2*x^2) + 2*a*b*log(x)
```

sympy [A] time = 0.14, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2}+2ab\log\left(x\right)+\frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

3.244 $\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx$

Optimal. Leaf size=23

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx &= \int \left(b^2+\frac{a^2}{x^4}+\frac{2ab}{x^2}\right)dx\\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4, x]
```

fricas [A] time = 0.81, size = 26, normalized size = 1.13

$$\frac{3\,b^2x^4 - 6\,abx^2 - a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

giac [A] time = 0.20, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="giac")
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$b^2x - \frac{2ab}{x} - \frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^4,x)
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

maxima [A] time = 1.33, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="maxima")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

mupad [B] time = 4.11, size = 24, normalized size = 1.04

$$b^2\,x-\frac{\frac{a^2}{3}+2\,b\,a\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^4,x)
```

```
[Out] b^2*x - (a^2/3 + 2*a*b*x^2)/x^3
```

sympy [A] time = 0.14, size = 22, normalized size = 0.96

$$b^{2}x+\frac{-a^{2}-6abx^{2}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**4,x)
```

```
[Out] b**2*x + (-a**2 - 6*a*b*x**2)/(3*x**3)
```

3.245 $\int \frac{a^2+2abx^2+b^2x^4}{x^5}\,dx$

Optimal. Leaf size=24

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^5}\,dx &= \int \left(\frac{a^2}{x^5}+\frac{2ab}{x^3}+\frac{b^2}{x}\right)dx\\ &= -\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5, x]
```

fricas [A] time = 0.85, size = 28, normalized size = 1.17

$$\frac{4\,b^2x^4\log(x) - 4\,abx^2 - a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="fricas")
[Out] 1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

giac [A] time = 0.15, size = 34, normalized size = 1.42

$$\frac{1}{2}\,b^2\log\left(x^2\right) - \frac{3\,b^2x^4 + 4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="giac")
[Out] 1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$b^2\ln(x) - \frac{ab}{x^2} - \frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^5,x)
[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

maxima [A] time = 1.37, size = 26, normalized size = 1.08

$$\frac{1}{2}\,b^2\log\left(x^2\right) - \frac{4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 1.00

$$b^2\,\ln(x)-\frac{\frac{a^2}{4}+b\,a\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^5,x)
```

```
[Out] b^2*log(x) - (a^2/4 + a*b*x^2)/x^4
```

sympy [A] time = 0.17, size = 24, normalized size = 1.00

$$b^2\log\left(x\right)+\frac{-a^2-4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**5,x)
```

```
[Out] b**2*log(x) + (-a**2 - 4*a*b*x**2)/(4*x**4)
```

3.246 $\int \frac{a^2+2abx^2+b^2x^4}{x^6} \, dx$

Optimal. Leaf size=28

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a^2+2abx^2+b^2x^4}{x^6} \, dx &= \int \left(\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2}{x^2}\right) dx \\ &= -\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x} \end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^6}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6, x]
```

fricas **[A]** time = 1.09, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="fricas")
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

giac **[A]** time = 0.17, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="giac")
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

maple **[A]** time = 0.00, size = 25, normalized size = 0.89

$$-\frac{b^2}{x} - \frac{2ab}{3x^3} - \frac{a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^6,x)
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

maxima **[A]** time = 1.34, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$-\frac{\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}+b^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^6,x)
```

```
[Out] -(a^2/5 + b^2*x^4 + (2*a*b*x^2)/3)/x^5
```

sympy [A] time = 0.18, size = 27, normalized size = 0.96

$$\frac{-3a^2-10abx^2-15b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**6,x)
```

```
[Out] (-3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

3.247 $\int \frac{a^2+2abx^2+b^2x^4}{x^7} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^7} \, dx &= \int \left(\frac{a^2}{x^7}+\frac{2ab}{x^5}+\frac{b^2}{x^3}\right) dx\\ &= -\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (a*b)/(2*x^4) - b^2/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^7} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7, x]
```

fricas [A] time = 0.77, size = 24, normalized size = 0.80

$$-\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="fricas")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

giac [A] time = 0.18, size = 24, normalized size = 0.80

$$-\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="giac")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{2x^2} - \frac{ab}{2x^4} - \frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^7,x)
[Out] -1/6*a^2/x^6-1/2*a*b/x^4-1/2*b^2/x^2
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$-\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

mupad [B] time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{6}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^7,x)
```

```
[Out] -(a^2/6 + (b^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

sympy [A] time = 0.20, size = 26, normalized size = 0.87

$$\frac{-a^2-3abx^2-3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**7,x)
```

```
[Out] (-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

3.248 $\int \frac{a^2+2abx^2+b^2x^4}{x^8} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = $\{14\}$

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^8} \, dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right) dx\\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^8}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8, x]
```

fricas [A] time = 0.70, size = 26, normalized size = 0.87

$$-\frac{35\, b^2 x^4 + 42\, abx^2 + 15\, a^2}{105\, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="fricas")
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

giac [A] time = 0.17, size = 26, normalized size = 0.87

$$-\frac{35\, b^2 x^4 + 42\, abx^2 + 15\, a^2}{105\, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="giac")
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{3x^3} - \frac{2ab}{5x^5} - \frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^8,x)
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

maxima [A] time = 1.35, size = 26, normalized size = 0.87

$$-\frac{35\, b^2 x^4 + 42\, abx^2 + 15\, a^2}{105\, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

mupad [B] time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{7}+\frac{2\,a\,b\,x^2}{5}+\frac{b^2\,x^4}{3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^8,x)
```

```
[Out] -(a^2/7 + (b^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

sympy [A] time = 0.21, size = 27, normalized size = 0.90

$$\frac{-15a^2-42abx^2-35b^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**8,x)
```

```
[Out] (-15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)
```

3.249 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{6}{11}a^2b^2x^{11}+\frac{4}{9}a^3bx^9+\frac{a^4x^7}{7}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 + (
b^4*x^15)/15
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx&=\frac{\int x^6\left(ab+b^2x^2\right)^4\,dx}{b^4}\\&=\frac{\int\left(a^4b^4x^6+4a^3b^5x^8+6a^2b^6x^{10}+4ab^7x^{12}+b^8x^{14}\right)dx}{b^4}\\&=\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 + (
b^4*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.68, size = 46, normalized size = 0.82

$$\frac{1}{15}x^{15}b^4+\frac{4}{13}x^{13}b^3a+\frac{6}{11}x^{11}b^2a^2+\frac{4}{9}x^9ba^3+\frac{1}{7}x^7a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^4 + 4/13*x^13*b^3*a + 6/11*x^11*b^2*a^2 + 4/9*x^9*b*a^3 + 1/7*x
^7*a^4
```

giac [A] time = 0.15, size = 46, normalized size = 0.82

$$\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*a
^4*x^7
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$\frac{1}{15}b^4x^{15}+\frac{4}{13}a\,b^3x^{13}+\frac{6}{11}a^2b^2x^{11}+\frac{4}{9}a^3b\,x^9+\frac{1}{7}a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/7*a^4*x^7+4/9*a^3*b*x^9+6/11*a^2*b^2*x^11+4/13*a*b^3*x^13+1/15*b^4*x^15
```

maxima [A] time = 1.33, size = 46, normalized size = 0.82

$$\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*a
^4*x^7
```

mupad [B] time = 0.02, size = 46, normalized size = 0.82

$$\frac{a^4\,x^7}{7}+\frac{4\,a^3\,b\,x^9}{9}+\frac{6\,a^2\,b^2\,x^{11}}{11}+\frac{4\,a\,b^3\,x^{13}}{13}+\frac{b^4\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^7)/7 + (b^4*x^15)/15 + (4*a^3*b*x^9)/9 + (4*a*b^3*x^13)/13 + (6*a^2*
b^2*x^11)/11
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{a^4x^7}{7}+\frac{4a^3bx^9}{9}+\frac{6a^2b^2x^{11}}{11}+\frac{4ab^3x^{13}}{13}+\frac{b^4x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**7/7 + 4*a**3*b*x**9/9 + 6*a**2*b**2*x**11/11 + 4*a*b**3*x**13/13 +
b**4*x**15/15
```

3.250 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^5}{10b^3}+\frac{\left(a+bx^2\right)^7}{14b^3}-\frac{a\left(a+bx^2\right)^6}{6b^3}$$

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^2\left(a+bx^2\right)^5}{10b^3}+\frac{\left(a+bx^2\right)^7}{14b^3}-\frac{a\left(a+bx^2\right)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^2*(a + b*x^2)^5)/(10*b^3) - (a*(a + b*x^2)^6)/(6*b^3) + (a + b*x^2)^7/(1
4*b^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\int x^5\left(ab+b^2x^2\right)^4\,dx}{b^4}$$
$$=\frac{\text{Subst}\left(\int x^2\left(ab+b^2x\right)^4\,dx,x,x^2\right)}{2b^4}$$
$$=\frac{\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^4}{b^2}-\frac{2a\left(ab+b^2x\right)^5}{b^3}+\frac{\left(ab+b^2x\right)^6}{b^4}\right)dx,x,x^2\right)}{2b^4}$$
$$=\frac{a^2\left(a+bx^2\right)^5}{10b^3}-\frac{a\left(a+bx^2\right)^6}{6b^3}+\frac{\left(a+bx^2\right)^7}{14b^3}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.06

$$\frac{a^4x^6}{6}+\frac{1}{2}a^3bx^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}ab^3x^{12}+\frac{b^4x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^6)/6 + (a^3*b*x^8)/2 + (3*a^2*b^2*x^10)/5 + (a*b^3*x^12)/3 + (b^4*x^
14)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.87, size = 46, normalized size = 0.87

$$\frac{1}{14}x^{14}b^4+\frac{1}{3}x^{12}b^3a+\frac{3}{5}x^{10}b^2a^2+\frac{1}{2}x^8ba^3+\frac{1}{6}x^6a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^4 + 1/3*x^12*b^3*a + 3/5*x^10*b^2*a^2 + 1/2*x^8*b*a^3 + 1/6*x^6
*a^4
```

giac [A] time = 0.15, size = 46, normalized size = 0.87

$$\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^4
*x^6
```

maple [A] time = 0.00, size = 47, normalized size = 0.89

$$\frac{1}{14}b^4x^{14}+\frac{1}{3}a\,b^3x^{12}+\frac{3}{5}a^2b^2x^{10}+\frac{1}{2}a^3b\,x^8+\frac{1}{6}a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/14*b^4*x^14+1/3*a*b^3*x^12+3/5*a^2*b^2*x^10+1/2*a^3*b*x^8+1/6*a^4*x^6
```

maxima [A] time = 1.34, size = 46, normalized size = 0.87

$$\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^4
*x^6
```

mupad [B] time = 0.02, size = 46, normalized size = 0.87

$$\frac{a^4\,x^6}{6}+\frac{a^3\,b\,x^8}{2}+\frac{3\,a^2\,b^2\,x^{10}}{5}+\frac{a\,b^3\,x^{12}}{3}+\frac{b^4\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^6)/6 + (b^4*x^14)/14 + (a^3*b*x^8)/2 + (a*b^3*x^12)/3 + (3*a^2*b^2*x
^10)/5
```

sympy [A] time = 0.08, size = 49, normalized size = 0.92

$$\frac{a^4x^6}{6} + \frac{a^3bx^8}{2} + \frac{3a^2b^2x^{10}}{5} + \frac{ab^3x^{12}}{3} + \frac{b^4x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**6/6 + a**3*b*x**8/2 + 3*a**2*b**2*x**10/5 + a*b**3*x**12/3 + b**4*x
**14/14
```

3.251 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{2}{3}a^2b^2x^9+\frac{4}{7}a^3bx^7+\frac{a^4x^5}{5}+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b^
4*x^13)/13
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx &= \frac{\int x^4 \left(ab+b^2x^2\right)^4 \, dx}{b^4}\\
&= \frac{\int \left(a^4b^4x^4+4a^3b^5x^6+6a^2b^6x^8+4ab^7x^{10}+b^8x^{12}\right) dx}{b^4}\\
&= \frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b^
4*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.83, size = 46, normalized size = 0.82

$$\frac{1}{13}x^{13}b^4+\frac{4}{11}x^{11}b^3a+\frac{2}{3}x^9b^2a^2+\frac{4}{7}x^7ba^3+\frac{1}{5}x^5a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^4 + 4/11*x^11*b^3*a + 2/3*x^9*b^2*a^2 + 4/7*x^7*b*a^3 + 1/5*x^5
*a^4
```

giac [A] time = 0.16, size = 46, normalized size = 0.82

$$\frac{1}{13}\,b^4x^{13}+\frac{4}{11}\,ab^3x^{11}+\frac{2}{3}\,a^2b^2x^9+\frac{4}{7}\,a^3bx^7+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^4
*x^5
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$\frac{1}{13}b^4x^{13}+\frac{4}{11}a\,b^3x^{11}+\frac{2}{3}a^2b^2x^9+\frac{4}{7}a^3b\,x^7+\frac{1}{5}a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/5*a^4*x^5+4/7*a^3*b*x^7+2/3*a^2*b^2*x^9+4/11*a*b^3*x^11+1/13*b^4*x^13
```

maxima [A] time = 1.33, size = 46, normalized size = 0.82

$$\frac{1}{13}\,b^{4}x^{13}+\frac{4}{11}\,ab^{3}x^{11}+\frac{2}{3}\,a^{2}b^{2}x^{9}+\frac{4}{7}\,a^{3}bx^{7}+\frac{1}{5}\,a^{4}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^4
*x^5
```

mupad [B] time = 0.02, size = 46, normalized size = 0.82

$$\frac{a^4\,x^5}{5}+\frac{4\,a^3\,b\,x^7}{7}+\frac{2\,a^2\,b^2\,x^9}{3}+\frac{4\,a\,b^3\,x^{11}}{11}+\frac{b^4\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^5)/5 + (b^4*x^13)/13 + (4*a^3*b*x^7)/7 + (4*a*b^3*x^11)/11 + (2*a^2*
b^2*x^9)/3
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{a^{4}x^{5}}{5}+\frac{4a^{3}bx^{7}}{7}+\frac{2a^{2}b^{2}x^{9}}{3}+\frac{4ab^{3}x^{11}}{11}+\frac{b^{4}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**5/5 + 4*a**3*b*x**7/7 + 2*a**2*b**2*x**9/3 + 4*a*b**3*x**11/11 + b*
*4*x**13/13
```

3.252 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^6}{12b^2}-\frac{a\left(a+bx^2\right)^5}{10b^2}$$

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{\left(a+bx^2\right)^6}{12b^2}-\frac{a\left(a+bx^2\right)^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(a*(a + b*x^2)^5)/(10*b^2) + (a + b*x^2)^6/(12*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{\int x^3 \left(ab+b^2x^2\right)^4 dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int x \left(ab+b^2x\right)^4 dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a\left(ab+b^2x\right)^4}{b}+\frac{\left(ab+b^2x\right)^5}{b^2}\right) dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a\left(a+bx^2\right)^5}{10b^2}+\frac{\left(a+bx^2\right)^6}{12b^2}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.65

$$\frac{a^4x^4}{4}+\frac{2}{3}a^3bx^6+\frac{3}{4}a^2b^2x^8+\frac{2}{5}ab^3x^{10}+\frac{b^4x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^4)/4 + (2*a^3*b*x^6)/3 + (3*a^2*b^2*x^8)/4 + (2*a*b^3*x^10)/5 + (b^4
*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.72, size = 46, normalized size = 1.35

$$\frac{1}{12}x^{12}b^4+\frac{2}{5}x^{10}b^3a+\frac{3}{4}x^8b^2a^2+\frac{2}{3}x^6ba^3+\frac{1}{4}x^4a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^4 + 2/5*x^10*b^3*a + 3/4*x^8*b^2*a^2 + 2/3*x^6*b*a^3 + 1/4*x^4*
a^4
```

giac [A] time = 0.15, size = 46, normalized size = 1.35

$$\frac{1}{12}\,b^4x^{12} + \frac{2}{5}\,ab^3x^{10} + \frac{3}{4}\,a^2b^2x^8 + \frac{2}{3}\,a^3bx^6 + \frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4*
x^4
```

maple [A] time = 0.00, size = 47, normalized size = 1.38

$$\frac{1}{12}b^4x^{12} + \frac{2}{5}a\,b^3x^{10} + \frac{3}{4}a^2b^2x^8 + \frac{2}{3}a^3b\,x^6 + \frac{1}{4}a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/12*b^4*x^12+2/5*a*b^3*x^10+3/4*a^2*b^2*x^8+2/3*a^3*b*x^6+1/4*a^4*x^4
```

maxima [A] time = 1.28, size = 46, normalized size = 1.35

$$\frac{1}{12}\,b^4x^{12} + \frac{2}{5}\,ab^3x^{10} + \frac{3}{4}\,a^2b^2x^8 + \frac{2}{3}\,a^3bx^6 + \frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4*
x^4
```

mupad [B] time = 0.02, size = 46, normalized size = 1.35

$$\frac{a^4\,x^4}{4} + \frac{2\,a^3\,b\,x^6}{3} + \frac{3\,a^2\,b^2\,x^8}{4} + \frac{2\,a\,b^3\,x^{10}}{5} + \frac{b^4\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^4)/4 + (b^4*x^12)/12 + (2*a^3*b*x^6)/3 + (2*a*b^3*x^10)/5 + (3*a^2*b
^2*x^8)/4
```

sympy [A] time = 0.08, size = 53, normalized size = 1.56

$$\frac{a^4x^4}{4}+\frac{2a^3bx^6}{3}+\frac{3a^2b^2x^8}{4}+\frac{2ab^3x^{10}}{5}+\frac{b^4x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**4/4 + 2*a**3*b*x**6/3 + 3*a**2*b**2*x**8/4 + 2*a*b**3*x**10/5 + b**
4*x**12/12
```

3.253 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{6}{7}a^2b^2x^7+\frac{4}{5}a^3bx^5+\frac{a^4x^3}{3}+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4*
x^11)/11
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int x^2\left(ab+b^2x^2\right)^4\,dx}{b^4}\\ &= \frac{\int\left(a^4b^4x^2+4a^3b^5x^4+6a^2b^6x^6+4ab^7x^8+b^8x^{10}\right)dx}{b^4}\\ &= \frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4*
x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.43, size = 46, normalized size = 0.82

$$\frac{1}{11}x^{11}b^4+\frac{4}{9}x^9b^3a+\frac{6}{7}x^7b^2a^2+\frac{4}{5}x^5ba^3+\frac{1}{3}x^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^4 + 4/9*x^9*b^3*a + 6/7*x^7*b^2*a^2 + 4/5*x^5*b*a^3 + 1/3*x^3*a
^4
```

giac [A] time = 0.15, size = 46, normalized size = 0.82

$$\frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*x
^3
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$\frac{1}{11}b^4x^{11}+\frac{4}{9}a\,b^3x^9+\frac{6}{7}a^2b^2x^7+\frac{4}{5}a^3b\,x^5+\frac{1}{3}a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/3*a^4*x^3+4/5*a^3*b*x^5+6/7*a^2*b^2*x^7+4/9*a*b^3*x^9+1/11*b^4*x^11
```

maxima [A] time = 1.32, size = 46, normalized size = 0.82

$$\frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*x
^3
```

mupad [B] time = 0.02, size = 46, normalized size = 0.82

$$\frac{a^4\,x^3}{3}+\frac{4\,a^3\,b\,x^5}{5}+\frac{6\,a^2\,b^2\,x^7}{7}+\frac{4\,a\,b^3\,x^9}{9}+\frac{b^4\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^3)/3 + (b^4*x^11)/11 + (4*a^3*b*x^5)/5 + (4*a*b^3*x^9)/9 + (6*a^2*b^
2*x^7)/7
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{a^4x^3}{3}+\frac{4a^3bx^5}{5}+\frac{6a^2b^2x^7}{7}+\frac{4ab^3x^9}{9}+\frac{b^4x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**3/3 + 4*a**3*b*x**5/5 + 6*a**2*b**2*x**7/7 + 4*a*b**3*x**9/9 + b**4
*x**11/11
```

3.254 $\int x \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^5}{10b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$\frac{\left(a+bx^2\right)^5}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a + b*x^2)^5/(10*b)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x \left(a^2+2abx^2+b^2x^4\right)^2 \, dx &= \frac{\int x \left(ab+b^2x^2\right)^4 \, dx}{b^4} \\ &= \frac{\left(a+bx^2\right)^5}{10b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^5}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a + b*x^2)^5/(10*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [B] time = 0.72, size = 44, normalized size = 2.75

$$\frac{1}{10}x^{10}b^4+\frac{1}{2}x^8b^3a+x^6b^2a^2+x^4ba^3+\frac{1}{2}x^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^4 + 1/2*x^8*b^3*a + x^6*b^2*a^2 + x^4*b*a^3 + 1/2*x^2*a^4
```

giac [B] time = 0.15, size = 44, normalized size = 2.75

$$\frac{1}{10}\,b^4x^{10}+\frac{1}{2}\,ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

maple [B] time = 0.00, size = 45, normalized size = 2.81

$$\frac{1}{10}b^4x^{10}+\frac{1}{2}a\,b^3x^8+a^2b^2x^6+a^3b\,x^4+\frac{1}{2}a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/10*b^4*x^10+1/2*a*b^3*x^8+a^2*b^2*x^6+a^3*b*x^4+1/2*a^4*x^2
```

maxima [B] time = 1.33, size = 44, normalized size = 2.75

$$\frac{1}{10}\,b^4x^{10} + \frac{1}{2}\,ab^3x^8 + a^2b^2x^6 + a^3bx^4 + \frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

mupad [B] time = 0.02, size = 44, normalized size = 2.75

$$\frac{a^4\,x^2}{2} + a^3\,b\,x^4 + a^2\,b^2\,x^6 + \frac{a\,b^3\,x^8}{2} + \frac{b^4\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
[Out] (a^4*x^2)/2 + (b^4*x^10)/10 + a^3*b*x^4 + (a*b^3*x^8)/2 + a^2*b^2*x^6
```

sympy [B] time = 0.08, size = 44, normalized size = 2.75

$$\frac{a^4x^2}{2} + a^3bx^4 + a^2b^2x^6 + \frac{ab^3x^8}{2} + \frac{b^4x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
[Out] a**4*x**2/2 + a**3*b*x**4 + a**2*b**2*x**6 + a*b**3*x**8/2 + b**4*x**10/10
```

3.255 $\int \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=51

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 194}

$$\frac{6}{5}a^2b^2x^5+\frac{4}{3}a^3bx^3+a^4x+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/9
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^2+b^2x^4\right)^2 \, dx &= \frac{\int \left(ab+b^2x^2\right)^4 \, dx}{b^4}\\ &= \frac{\int \left(a^4b^4+4a^3b^5x^2+6a^2b^6x^4+4ab^7x^6+b^8x^8\right) dx}{b^4}\\ &= a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 51, normalized size = 1.00

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.65, size = 43, normalized size = 0.84

$$\frac{1}{9}x^9b^4 + \frac{4}{7}x^7b^3a + \frac{6}{5}x^5b^2a^2 + \frac{4}{3}x^3ba^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^4 + 4/7*x^7*b^3*a + 6/5*x^5*b^2*a^2 + 4/3*x^3*b*a^3 + x*a^4
```

giac [A] time = 0.14, size = 43, normalized size = 0.84

$$\frac{1}{9}\,b^4x^9 + \frac{4}{7}\,ab^3x^7 + \frac{6}{5}\,a^2b^2x^5 + \frac{4}{3}\,a^3bx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

maple [A] time = 0.00, size = 44, normalized size = 0.86

$$\frac{1}{9}b^4x^9 + \frac{4}{7}a\,b^3x^7 + \frac{6}{5}a^2b^2x^5 + \frac{4}{3}a^3b\,x^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

maxima [A] time = 1.28, size = 55, normalized size = 1.08

$$\frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{4}{5}\,a^2b^2x^5+a^4x+\frac{2}{15}\left(3\,b^2x^5+10\,abx^3\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 4/5*a^2*b^2*x^5 + a^4*x + 2/15*(3*b^2*x^5 + 1
0*a*b*x^3)*a^2
```

mupad [B] time = 0.02, size = 43, normalized size = 0.84

$$a^4\,x+\frac{4\,a^3\,b\,x^3}{3}+\frac{6\,a^2\,b^2\,x^5}{5}+\frac{4\,a\,b^3\,x^7}{7}+\frac{b^4\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] a^4*x + (b^4*x^9)/9 + (4*a^3*b*x^3)/3 + (4*a*b^3*x^7)/7 + (6*a^2*b^2*x^5)/5
```

sympy [A] time = 0.08, size = 49, normalized size = 0.96

$$a^4x+\frac{4a^3bx^3}{3}+\frac{6a^2b^2x^5}{5}+\frac{4ab^3x^7}{7}+\frac{b^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x + 4*a**3*b*x**3/3 + 6*a**2*b**2*x**5/5 + 4*a*b**3*x**7/7 + b**4*x**9
/9
```

3.256 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x}\,dx$

Optimal. Leaf size=50

$$a^4\log(x)+2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3}{2}a^2b^2x^4+2a^3bx^2+a^4\log(x)+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[x
]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(4a^3b^5+\frac{a^4b^4}{x}+6a^2b^6x+4ab^7x^2+b^8x^3\right)\,dx, x, x^2\right)}{2b^4}$$
$$= 2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}+a^4\log(x)$$

Mathematica [A] time = 0.00, size = 50, normalized size = 1.00

$$a^4\log(x)+2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[x
]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x, x]
```

fricas [A] time = 0.77, size = 44, normalized size = 0.88

$$\frac{1}{8}\,b^4x^8+\frac{2}{3}\,ab^3x^6+\frac{3}{2}\,a^2b^2x^4+2\,a^3bx^2+a^4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4*log(x)
```

giac [A] time = 0.15, size = 47, normalized size = 0.94

$$\frac{1}{8}\,b^{4}x^{8}+\frac{2}{3}\,ab^{3}x^{6}+\frac{3}{2}\,a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+\frac{1}{2}\,a^{4}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(x
^2)
```

maple [A] time = 0.00, size = 45, normalized size = 0.90

$$\frac{b^{4}x^{8}}{8}+\frac{2a\,b^{3}x^{6}}{3}+\frac{3a^{2}b^{2}x^{4}}{2}+2a^{3}b\,x^{2}+a^{4}\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x,x)
```

```
[Out] 2*a^3*b*x^2+3/2*a^2*b^2*x^4+2/3*a*b^3*x^6+1/8*b^4*x^8+a^4*ln(x)
```

maxima [A] time = 1.34, size = 47, normalized size = 0.94

$$\frac{1}{8}\,b^{4}x^{8}+\frac{2}{3}\,ab^{3}x^{6}+\frac{3}{2}\,a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+\frac{1}{2}\,a^{4}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(x
^2)
```

mupad [B] time = 0.03, size = 44, normalized size = 0.88

$$a^{4}\,\ln(x)+\frac{b^{4}\,x^{8}}{8}+2\,a^{3}\,b\,x^{2}+\frac{2\,a\,b^{3}\,x^{6}}{3}+\frac{3\,a^{2}\,b^{2}\,x^{4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x,x)
```

```
[Out] a^4*log(x) + (b^4*x^8)/8 + 2*a^3*b*x^2 + (2*a*b^3*x^6)/3 + (3*a^2*b^2*x^4)/
2
```

sympy [A] time = 0.13, size = 49, normalized size = 0.98

$$a^4 \log(x) + 2a^3bx^2 + \frac{3a^2b^2x^4}{2} + \frac{2ab^3x^6}{3} + \frac{b^4x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x,x)
```

```
[Out] a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x
**8/8
```

3.257 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$2a^2b^2x^3+4a^3bx-\frac{a^4}{x}+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^2}\,dx}{b^4}\\
&= \frac{\int \left(4a^3b^5+\frac{a^4b^4}{x^2}+6a^2b^6x^2+4ab^7x^4+b^8x^6\right)dx}{b^4}\\
&= -\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2, x]
```

fricas [A] time = 1.02, size = 48, normalized size = 1.00

$$\frac{5\,b^4x^8+28\,ab^3x^6+70\,a^2b^2x^4+140\,a^3bx^2-35\,a^4}{35\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="fricas")
[Out] 1/35*(5*b^4*x^8 + 28*a*b^3*x^6 + 70*a^2*b^2*x^4 + 140*a^3*b*x^2 - 35*a^4)/x
```

giac [A] time = 0.15, size = 44, normalized size = 0.92

$$\frac{1}{7}\,b^4x^7+\frac{4}{5}\,ab^3x^5+2\,a^2b^2x^3+4\,a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="giac")
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

maple [A] time = 0.00, size = 45, normalized size = 0.94

$$\frac{b^4x^7}{7}+\frac{4a\,b^3x^5}{5}+2a^2b^2x^3+4a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x)
```

```
[Out] -a^4/x+4*a^3*b*x+2*a^2*b^2*x^3+4/5*a*b^3*x^5+1/7*b^4*x^7
```

maxima [A] time = 1.34, size = 44, normalized size = 0.92

$$\frac{1}{7}\,b^4x^7+\frac{4}{5}\,ab^3x^5+2\,a^2b^2x^3+4\,a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

mupad [B] time = 0.02, size = 44, normalized size = 0.92

$$\frac{b^4\,x^7}{7}-\frac{a^4}{x}+\frac{4\,a\,b^3\,x^5}{5}+2\,a^2\,b^2\,x^3+4\,a^3\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^2,x)
```

```
[Out] (b^4*x^7)/7 - a^4/x + (4*a*b^3*x^5)/5 + 2*a^2*b^2*x^3 + 4*a^3*b*x
```

sympy [A] time = 0.13, size = 44, normalized size = 0.92

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4ab^3x^5}{5}+\frac{b^4x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**2,x)
```

```
[Out] -a**4/x + 4*a**3*b*x + 2*a**2*b**2*x**3 + 4*a*b**3*x**5/5 + b**4*x**7/7
```

3.258 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3} \, dx$

Optimal. Leaf size=48

$$-\frac{a^4}{2x^2} + 4a^3b\log(x) + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6}$$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$3a^2b^2x^2 + 4a^3b\log(x) - \frac{a^4}{2x^2} + ab^3x^4 + \frac{b^4x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

```
[Out] -a^4/(2*x^2) + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^3}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^3}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^2}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(6a^2b^6+\frac{a^4b^4}{x^2}+\frac{4a^3b^5}{x}+4ab^7x+b^8x^2\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{2x^2}+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}+4a^3b\log(x)$$

Mathematica [A] time = 0.00, size = 48, normalized size = 1.00

$$-\frac{a^4}{2x^2}+4a^3b\log(x)+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
[Out] -1/2*a^4/x^2 + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3, x]
```

fricas [A] time = 0.75, size = 49, normalized size = 1.02

$$\frac{b^4x^8+6\,ab^3x^6+18\,a^2b^2x^4+24\,a^3bx^2\log(x)-3\,a^4}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 + 6*a*b^3*x^6 + 18*a^2*b^2*x^4 + 24*a^3*b*x^2*log(x) - 3*a^4)/
x^2
```

giac [A] time = 0.16, size = 56, normalized size = 1.17

$$\frac{1}{6}\,b^4x^6 + ab^3x^4 + 3\,a^2b^2x^2 + 2\,a^3b\log\left(x^2\right) - \frac{4\,a^3bx^2 + a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*(4*a^3*b*x
^2 + a^4)/x^2
```

maple [A] time = 0.01, size = 45, normalized size = 0.94

$$\frac{b^4x^6}{6} + a\,b^3x^4 + 3a^2b^2x^2 + 4a^3b\ln(x) - \frac{a^4}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x)
```

```
[Out] -1/2*a^4/x^2+3*a^2*b^2*x^2+a*b^3*x^4+1/6*b^4*x^6+4*a^3*b*ln(x)
```

maxima [A] time = 1.34, size = 46, normalized size = 0.96

$$\frac{1}{6}\,b^4x^6 + ab^3x^4 + 3\,a^2b^2x^2 + 2\,a^3b\log\left(x^2\right) - \frac{a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*a^4/x^2
```

mupad [B] time = 0.03, size = 44, normalized size = 0.92

$$\frac{b^4\,x^6}{6} - \frac{a^4}{2\,x^2} + a\,b^3\,x^4 + 4\,a^3\,b\,\ln(x) + 3\,a^2\,b^2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^3,x)
```

```
[Out] (b^4*x^6)/6 - a^4/(2*x^2) + a*b^3*x^4 + 4*a^3*b*log(x) + 3*a^2*b^2*x^2
```

sympy [A] time = 0.17, size = 46, normalized size = 0.96

$$-\frac{a^4}{2x^2} + 4a^3b\log(x) + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**3,x)
```

```
[Out] -a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 + b**4*x*
*6/6
```

3.259 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^4}\,dx$

Optimal. Leaf size=50

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^4}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^4}\,dx}{b^4}\\
&= \frac{\int \left(6a^2b^6+\frac{a^4b^4}{x^4}+\frac{4a^3b^5}{x^2}+4ab^7x^2+b^8x^4\right)dx}{b^4}\\
&= -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4, x]
```

fricas [A] time = 0.84, size = 48, normalized size = 0.96

$$\frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="fricas")
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3
```

giac [A] time = 0.17, size = 45, normalized size = 0.90

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="giac")
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

maple [A] time = 0.00, size = 45, normalized size = 0.90

$$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x)
```

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

maxima [A] time = 1.44, size = 45, normalized size = 0.90

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

mupad [B] time = 0.04, size = 47, normalized size = 0.94

$$\frac{b^4\,x^5}{5}-\frac{\frac{a^4}{3}+4\,b\,a^3\,x^2}{x^3}+6\,a^2\,b^2\,x+\frac{4\,a\,b^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^4,x)
```

```
[Out] (b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

sympy [A] time = 0.17, size = 49, normalized size = 0.98

$$6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}+\frac{-a^4-12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(3
*x**3)
```

3.260 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5} dx$

Optimal. Leaf size=49

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+6a^2b^2\log(x)+2ab^3x^2+\frac{b^4x^4}{4}$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$6a^2b^2\log(x)-\frac{2a^3b}{x^2}-\frac{a^4}{4x^4}+2ab^3x^2+\frac{b^4x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

```
[Out] -a^4/(4*x^4) - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^5}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x^3}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(4ab^7+\frac{a^4b^4}{x^3}+\frac{4a^3b^5}{x^2}+\frac{6a^2b^6}{x}+b^8x\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2ab^3x^2+\frac{b^4x^4}{4}+6a^2b^2\log(x)$$

Mathematica [A] time = 0.00, size = 49, normalized size = 1.00

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+6a^2b^2\log(x)+2ab^3x^2+\frac{b^4x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
[Out] -1/4*a^4/x^4 - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5, x]
```

fricas [A] time = 0.81, size = 49, normalized size = 1.00

$$\frac{b^4x^8+8\,ab^3x^6+24\,a^2b^2x^4\log(x)-8\,a^3bx^2-a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^8 + 8*a*b^3*x^6 + 24*a^2*b^2*x^4*log(x) - 8*a^3*b*x^2 - a^4)/x^4
```

giac [A] time = 0.15, size = 59, normalized size = 1.20

$$\frac{1}{4}\,b^4x^4+2\,ab^3x^2+3\,a^2b^2\log\left(x^2\right)-\frac{18\,a^2b^2x^4+8\,a^3bx^2+a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(18*a^2*b^2*x^4 + 8*a^
3*b*x^2 + a^4)/x^4
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$\frac{b^4x^4}{4}+2a\,b^3x^2+6a^2b^2\ln(x)-\frac{2a^3b}{x^2}-\frac{a^4}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x)
```

```
[Out] -1/4*a^4/x^4-2*a^3*b/x^2+2*a*b^3*x^2+1/4*b^4*x^4+6*a^2*b^2*ln(x)
```

maxima [A] time = 1.36, size = 48, normalized size = 0.98

$$\frac{1}{4}\,b^4x^4+2\,ab^3x^2+3\,a^2b^2\log\left(x^2\right)-\frac{8\,a^3bx^2+a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(8*a^3*b*x^2 + a^4)/x^
4
```

mupad [B] time = 0.04, size = 48, normalized size = 0.98

$$\frac{b^4\,x^4}{4}-\frac{\frac{a^4}{4}+2\,b\,a^3\,x^2}{x^4}+2\,a\,b^3\,x^2+6\,a^2\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^5,x)
```

```
[Out] (b^4*x^4)/4 - (a^4/4 + 2*a^3*b*x^2)/x^4 + 2*a*b^3*x^2 + 6*a^2*b^2*log(x)
```

sympy [A] time = 0.22, size = 49, normalized size = 1.00

$$6a^2b^2\log(x) + 2ab^3x^2 + \frac{b^4x^4}{4} + \frac{-a^4 - 8a^3bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**5,x)
```

```
[Out] 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4 + (-a**4 - 8*a**3*b*x**2)/
(4*x**4)
```

3.261 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{x}-\frac{4a^3b}{3x^3}-\frac{a^4}{5x^5}+4ab^3x+\frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
```

```
[Out] -a^4/(5*x^5) - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^6}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^6}\,dx}{b^4}\\
&= \frac{\int \left(4ab^7+\frac{a^4b^4}{x^6}+\frac{4a^3b^5}{x^4}+\frac{6a^2b^6}{x^2}+b^8x^2\right)dx}{b^4}\\
&= -\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
[Out] -1/5*a^4/x^5 - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6, x]
```

fricas [A] time = 0.85, size = 48, normalized size = 0.96

$$\frac{5\,b^4x^8+60\,ab^3x^6-90\,a^2b^2x^4-20\,a^3bx^2-3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="fricas")
[Out] 1/15*(5*b^4*x^8 + 60*a*b^3*x^6 - 90*a^2*b^2*x^4 - 20*a^3*b*x^2 - 3*a^4)/x^5
```

giac [A] time = 0.15, size = 47, normalized size = 0.94

$$\frac{1}{3}\,b^4x^3+4\,ab^3x-\frac{90\,a^2b^2x^4+20\,a^3bx^2+3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="giac")
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

maple [A] time = 0.01, size = 45, normalized size = 0.90

$$\frac{b^4x^3}{3}+4a\,b^3x-\frac{6a^2b^2}{x}-\frac{4a^3b}{3x^3}-\frac{a^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x)
```

```
[Out] -1/5*a^4/x^5-4/3*a^3*b/x^3-6*a^2*b^2/x+4*a*b^3*x+1/3*b^4*x^3
```

maxima [A] time = 1.37, size = 47, normalized size = 0.94

$$\frac{1}{3}\,b^4x^3+4\,ab^3x-\frac{90\,a^2b^2x^4+20\,a^3bx^2+3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

mupad [B] time = 0.04, size = 47, normalized size = 0.94

$$\frac{b^4\,x^3}{3}-\frac{\frac{a^4}{5}+\frac{4\,a^3\,b\,x^2}{3}+6\,a^2\,b^2\,x^4}{x^5}+4\,a\,b^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^6,x)
```

```
[Out] (b^4*x^3)/3 - (a^4/5 + (4*a^3*b*x^2)/3 + 6*a^2*b^2*x^4)/x^5 + 4*a*b^3*x
```

sympy [A] time = 0.23, size = 49, normalized size = 0.98

$$4ab^3x+\frac{b^4x^3}{3}+\frac{-3a^4-20a^3bx^2-90a^2b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**6,x)
```

```
[Out] 4*a*b**3*x + b**4*x**3/3 + (-3*a**4 - 20*a**3*b*x**2 - 90*a**2*b**2*x**4)/(
15*x**5)
```

3.262 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^7} dx$

Optimal. Leaf size=49

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{3a^2b^2}{x^2}-\frac{a^3b}{x^4}-\frac{a^4}{6x^6}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

```
[Out] -a^4/(6*x^6) - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^7}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^7}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^4}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(b^8+\frac{a^4b^4}{x^4}+\frac{4a^3b^5}{x^3}+\frac{6a^2b^6}{x^2}+\frac{4ab^7}{x}\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+\frac{b^4x^2}{2}+4ab^3\log(x)$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
[Out] -1/6*a^4/x^6 - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7, x]
```

fricas [A] time = 0.89, size = 50, normalized size = 1.02

$$\frac{3\,b^4x^8+24\,ab^3x^6\log(x)-18\,a^2b^2x^4-6\,a^3bx^2-a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 24*a*b^3*x^6*log(x) - 18*a^2*b^2*x^4 - 6*a^3*b*x^2 - a^4)/
x^6
```

giac [A] time = 0.17, size = 57, normalized size = 1.16

$$\frac{1}{2}b^4x^2 + 2ab^3\log\left(x^2\right) - \frac{22ab^3x^6 + 18a^2b^2x^4 + 6a^3bx^2 + a^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(22*a*b^3*x^6 + 18*a^2*b^2*x^4 + 6*a^3
*b*x^2 + a^4)/x^6
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$\frac{b^4x^2}{2} + 4ab^3\ln(x) - \frac{3a^2b^2}{x^2} - \frac{a^3b}{x^4} - \frac{a^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x)
```

```
[Out] -1/6*a^4/x^6-a^3*b/x^4-3*a^2*b^2/x^2+1/2*b^4*x^2+4*a*b^3*ln(x)
```

maxima [A] time = 1.31, size = 48, normalized size = 0.98

$$\frac{1}{2}b^4x^2 + 2ab^3\log\left(x^2\right) - \frac{18a^2b^2x^4 + 6a^3bx^2 + a^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(18*a^2*b^2*x^4 + 6*a^3*b*x^2 + a^4)/x
^6
```

mupad [B] time = 0.04, size = 47, normalized size = 0.96

$$\frac{b^4x^2}{2} - \frac{\frac{a^4}{6} + a^3bx^2 + 3a^2b^2x^4}{x^6} + 4ab^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^7,x)
```

```
[Out] (b^4*x^2)/2 - (a^4/6 + a^3*b*x^2 + 3*a^2*b^2*x^4)/x^6 + 4*a*b^3*log(x)
```

sympy [A] time = 0.30, size = 49, normalized size = 1.00

$$4ab^3\log(x) + \frac{b^4x^2}{2} + \frac{-a^4 - 6a^3bx^2 - 18a^2b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**7,x)
```

```
[Out] 4*a*b**3*log(x) + b**4*x**2/2 + (-a**4 - 6*a**3*b*x**2 - 18*a**2*b**2*x**4)
/(6*x**6)
```

3.263 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^8}\,dx$

Optimal. Leaf size=47

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{2a^2b^2}{x^3}-\frac{4a^3b}{5x^5}-\frac{a^4}{7x^7}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
```

```
[Out] -a^4/(7*x^7) - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^8}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^8}\,dx}{b^4}\\
&= \frac{\int \left(b^8+\frac{a^4b^4}{x^8}+\frac{4a^3b^5}{x^6}+\frac{6a^2b^6}{x^4}+\frac{4ab^7}{x^2}\right)dx}{b^4}\\
&= -\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x
\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.00

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
[Out] -1/7*a^4/x^7 - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8, x]
```

fricas [A] time = 0.71, size = 48, normalized size = 1.02

$$\frac{35\,b^4x^8-140\,ab^3x^6-70\,a^2b^2x^4-28\,a^3bx^2-5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="fricas")
[Out] 1/35*(35*b^4*x^8 - 140*a*b^3*x^6 - 70*a^2*b^2*x^4 - 28*a^3*b*x^2 - 5*a^4)/x
^7
```

giac [A] time = 0.16, size = 46, normalized size = 0.98

$$b^4x-\frac{140\,ab^3x^6+70\,a^2b^2x^4+28\,a^3bx^2+5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="giac")
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

maple [A] time = 0.01, size = 44, normalized size = 0.94

$$b^4x-\frac{4a\,b^3}{x}-\frac{2a^2b^2}{x^3}-\frac{4a^3b}{5x^5}-\frac{a^4}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x)
```

```
[Out] -1/7*a^4/x^7-4/5*a^3*b/x^5-2*a^2*b^2/x^3-4*a*b^3/x+b^4*x
```

maxima [A] time = 1.45, size = 46, normalized size = 0.98

$$b^4x - \frac{140\,ab^3x^6 + 70\,a^2b^2x^4 + 28\,a^3bx^2 + 5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

mupad [B] time = 4.19, size = 46, normalized size = 0.98

$$b^4\,x - \frac{\frac{a^4}{7} + \frac{4\,a^3\,b\,x^2}{5} + 2\,a^2\,b^2\,x^4 + 4\,a\,b^3\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^8,x)
```

```
[Out] b^4*x - (a^4/7 + (4*a^3*b*x^2)/5 + 4*a*b^3*x^6 + 2*a^2*b^2*x^4)/x^7
```

sympy [A] time = 0.30, size = 48, normalized size = 1.02

$$b^4x + \frac{-5a^4 - 28a^3bx^2 - 70a^2b^2x^4 - 140ab^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**8,x)
```

```
[Out] b**4*x + (-5*a**4 - 28*a**3*b*x**2 - 70*a**2*b**2*x**4 - 140*a*b**3*x**6)/(
35*x**7)
```

3.264 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9}\,dx$

Optimal. Leaf size=50

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{3a^2b^2}{2x^4}-\frac{2a^3b}{3x^6}-\frac{a^4}{8x^8}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] -a^4/(8*x^8) - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b^
4*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^9}\,dx=\frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^9}\,dx}{b^4}$$
$$=\frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^5}\,dx,x,x^2\right)}{2b^4}$$
$$=\frac{\text{Subst}\left(\int \left(\frac{a^4b^4}{x^5}+\frac{4a^3b^5}{x^4}+\frac{6a^2b^6}{x^3}+\frac{4ab^7}{x^2}+\frac{b^8}{x}\right)dx,x,x^2\right)}{2b^4}$$
$$=-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Mathematica [A] time = 0.00, size = 50, normalized size = 1.00

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] -1/8*a^4/x^8 - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b^
4*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9, x]
```

fricas [A] time = 0.82, size = 50, normalized size = 1.00

$$\frac{24\,b^4x^8\log(x)-48\,ab^3x^6-36\,a^2b^2x^4-16\,a^3bx^2-3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^4*x^8*log(x) - 48*a*b^3*x^6 - 36*a^2*b^2*x^4 - 16*a^3*b*x^2 - 3*
a^4)/x^8
```

giac [A] time = 0.15, size = 58, normalized size = 1.16

$$\frac{1}{2}\,b^4\log\left(x^2\right)-\frac{25\,b^4x^8+48\,ab^3x^6+36\,a^2b^2x^4+16\,a^3bx^2+3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(x^2) - 1/24*(25*b^4*x^8 + 48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^
3*b*x^2 + 3*a^4)/x^8
```

maple [A] time = 0.00, size = 45, normalized size = 0.90

$$b^4\ln(x)-\frac{2a\,b^3}{x^2}-\frac{3a^2b^2}{2x^4}-\frac{2a^3b}{3x^6}-\frac{a^4}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x)
```

```
[Out] -1/8*a^4/x^8-2/3*a^3*b/x^6-3/2*a^2*b^2/x^4-2*a*b^3/x^2+b^4*ln(x)
```

maxima [A] time = 1.38, size = 50, normalized size = 1.00

$$\frac{1}{2}\,b^4\log\left(x^2\right)-\frac{48\,ab^3x^6+36\,a^2b^2x^4+16\,a^3bx^2+3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(x^2) - 1/24*(48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^3*b*x^2 + 3*a
^4)/x^8
```

mupad [B] time = 0.05, size = 47, normalized size = 0.94

$$b^4\,\ln(x)-\frac{\frac{a^4}{8}+\frac{2\,a^3\,b\,x^2}{3}+\frac{3\,a^2\,b^2\,x^4}{2}+2\,a\,b^3\,x^6}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^9,x)
```

```
[Out] b^4*log(x) - (a^4/8 + (2*a^3*b*x^2)/3 + 2*a*b^3*x^6 + (3*a^2*b^2*x^4)/2)/x^
8
```

sympy [**A**] time = 0.37, size = 49, normalized size = 0.98

$$b^4 \log(x) + \frac{-3a^4 - 16a^3bx^2 - 36a^2b^2x^4 - 48ab^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**9,x)
```

```
[Out] b**4*log(x) + (-3*a**4 - 16*a**3*b*x**2 - 36*a**2*b**2*x**4 - 48*a*b**3*x**
6)/(24*x**8)
```

3.265 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{5x^5}-\frac{4a^3b}{7x^7}-\frac{a^4}{9x^9}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
- b^4/x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{10}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{10}}\,dx}{b^4}\\
&= \frac{\int \left(\frac{a^4b^4}{x^{10}}+\frac{4a^3b^5}{x^8}+\frac{6a^2b^6}{x^6}+\frac{4ab^7}{x^4}+\frac{b^8}{x^2}\right)dx}{b^4}\\
&= -\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -1/9*a^4/x^9 - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
- b^4/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10, x]
```

fricas [A] time = 0.70, size = 48, normalized size = 0.89

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

giac [A] time = 0.17, size = 48, normalized size = 0.89

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

maple [A] time = 0.01, size = 47, normalized size = 0.87

$$-\frac{b^4}{x}-\frac{4a\,b^3}{3x^3}-\frac{6a^2b^2}{5x^5}-\frac{4a^3b}{7x^7}-\frac{a^4}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x)
```

```
[Out] -1/9*a^4/x^9-4/7*a^3*b/x^7-6/5*a^2*b^2/x^5-4/3*a*b^3/x^3-b^4/x
```

maxima [A] time = 1.43, size = 48, normalized size = 0.89

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

mupad [B] time = 0.03, size = 47, normalized size = 0.87

$$-\frac{\frac{a^4}{9}+\frac{4\,a^3\,b\,x^2}{7}+\frac{6\,a^2\,b^2\,x^4}{5}+\frac{4\,a\,b^3\,x^6}{3}+b^4\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^10,x)
```

```
[Out] -(a^4/9 + b^4*x^8 + (4*a^3*b*x^2)/7 + (4*a*b^3*x^6)/3 + (6*a^2*b^2*x^4)/5)/
x^9
```

sympy [A] time = 0.37, size = 51, normalized size = 0.94

$$\frac{-35a^4-180a^3bx^2-378a^2b^2x^4-420ab^3x^6-315b^4x^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**10,x)
```

```
[Out] (-35*a**4 - 180*a**3*b*x**2 - 378*a**2*b**2*x**4 - 420*a*b**3*x**6 - 315*b*
*4*x**8)/(315*x**9)
```

3.266 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{11}} \, dx$

Optimal. Leaf size=19

$$-\frac{\left(a+bx^2\right)^5}{10ax^{10}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$-\frac{\left(a+bx^2\right)^5}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

```
[Out] -(a + b*x^2)^5/(10*a*x^10)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{11}} \, dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{11}} \, dx}{b^4}\\ &= -\frac{\left(a+bx^2\right)^5}{10ax^{10}}\end{aligned}$$

Mathematica [B] time = 0.00, size = 52, normalized size = 2.74

$$-\frac{a^4}{10x^{10}}-\frac{a^3b}{2x^8}-\frac{a^2b^2}{x^6}-\frac{ab^3}{x^4}-\frac{b^4}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
[Out] -1/10*a^4/x^10 - (a^3*b)/(2*x^8) - (a^2*b^2)/x^6 - (a*b^3)/x^4 - b^4/(2*x^2
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11, x]
```

fricas [B] time = 0.76, size = 46, normalized size = 2.42

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="fricas")
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

giac [B] time = 0.18, size = 46, normalized size = 2.42

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="giac")
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

maple [B] time = 0.00, size = 47, normalized size = 2.47

$$-\frac{b^4}{2x^2}-\frac{a\,b^3}{x^4}-\frac{a^2b^2}{x^6}-\frac{a^3b}{2x^8}-\frac{a^4}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x)
```

```
[Out] -1/2*b^4/x^2-a^2*b^2/x^6-1/2*a^3*b/x^8-1/10*a^4/x^10-a*b^3/x^4
```

maxima [B] time = 1.35, size = 46, normalized size = 2.42

$$-\frac{5 b^4 x^8 + 10 a b^3 x^6 + 10 a^2 b^2 x^4 + 5 a^3 b x^2 + a^4}{10 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

mupad [B] time = 0.03, size = 46, normalized size = 2.42

$$-\frac{\frac{a^4}{10} + \frac{a^3 b x^2}{2} + a^2 b^2 x^4 + a b^3 x^6 + \frac{b^4 x^8}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^11,x)
```

```
[Out] -(a^4/10 + (b^4*x^8)/2 + (a^3*b*x^2)/2 + a*b^3*x^6 + a^2*b^2*x^4)/x^10
```

sympy [B] time = 0.39, size = 49, normalized size = 2.58

$$\frac{-a^4 - 5a^3bx^2 - 10a^2b^2x^4 - 10ab^3x^6 - 5b^4x^8}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**11,x)
```

```
[Out] (-a**4 - 5*a**3*b*x**2 - 10*a**2*b**2*x**4 - 10*a*b**3*x**6 - 5*b**4*x**8)/
(10*x**10)
```

3.267 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{7x^7}-\frac{4a^3b}{9x^9}-\frac{a^4}{11x^{11}}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -a^4/(11*x^11) - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^5
) - b^4/(3*x^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^{12}}\,dx}{b^4}\\
&= \frac{\int \left(\frac{a^4b^4}{x^{12}}+\frac{4a^3b^5}{x^{10}}+\frac{6a^2b^6}{x^8}+\frac{4ab^7}{x^6}+\frac{b^8}{x^4}\right)dx}{b^4}\\
&= -\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -1/11*a^4/x^11 - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^5
) - b^4/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12, x]
```

fricas [A] time = 0.66, size = 48, normalized size = 0.86

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

giac [A] time = 0.16, size = 48, normalized size = 0.86

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

maple [A] time = 0.01, size = 47, normalized size = 0.84

$$-\frac{b^4}{3x^3}-\frac{4a\,b^3}{5x^5}-\frac{6a^2b^2}{7x^7}-\frac{4a^3b}{9x^9}-\frac{a^4}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x)
```

```
[Out] -1/11*a^4/x^11-4/9*a^3*b/x^9-6/7*a^2*b^2/x^7-4/5*a*b^3/x^5-1/3*b^4/x^3
```

maxima [A] time = 1.30, size = 48, normalized size = 0.86

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

mupad [B] time = 4.84, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{11}+\frac{4\,a^3\,b\,x^2}{9}+\frac{6\,a^2\,b^2\,x^4}{7}+\frac{4\,a\,b^3\,x^6}{5}+\frac{b^4\,x^8}{3}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^12,x)
```

```
[Out] -(a^4/11 + (b^4*x^8)/3 + (4*a^3*b*x^2)/9 + (4*a*b^3*x^6)/5 + (6*a^2*b^2*x^4
)/7)/x^11
```

sympy [A] time = 0.40, size = 51, normalized size = 0.91

$$\frac{-315a^4-1540a^3bx^2-2970a^2b^2x^4-2772ab^3x^6-1155b^4x^8}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**12,x)
```

```
[Out] (-315*a**4 - 1540*a**3*b*x**2 - 2970*a**2*b**2*x**4 - 2772*a*b**3*x**6 - 11
55*b**4*x**8)/(3465*x**11)
```

3.268 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{13}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^5}{60a^2x^{10}}-\frac{\left(a+bx^2\right)^5}{12ax^{12}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{b\left(a+bx^2\right)^5}{60a^2x^{10}}-\frac{\left(a+bx^2\right)^5}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

```
[Out] -(a + b*x^2)^5/(12*a*x^12) + (b*(a + b*x^2)^5)/(60*a^2*x^10)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{13}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{13}}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^7}\,dx, x, x^2\right)}{2b^4}$$
$$= -\frac{\left(a+bx^2\right)^5}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^6}\,dx, x, x^2\right)}{12ab^3}$$
$$= -\frac{\left(a+bx^2\right)^5}{12ax^{12}} + \frac{b\left(a+bx^2\right)^5}{60a^2x^{10}}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.40

$$-\frac{a^4}{12x^{12}} - \frac{2a^3b}{5x^{10}} - \frac{3a^2b^2}{4x^8} - \frac{2ab^3}{3x^6} - \frac{b^4}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
[Out] -1/12*a^4/x^12 - (2*a^3*b)/(5*x^10) - (3*a^2*b^2)/(4*x^8) - (2*a*b^3)/(3*x^
6) - b^4/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13, x]
```

fricas [A] time = 0.82, size = 48, normalized size = 1.20

$$-\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

giac [A] time = 0.15, size = 48, normalized size = 1.20

$$-\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

maple [A] time = 0.00, size = 47, normalized size = 1.18

$$-\frac{b^4}{4x^4}-\frac{2a\,b^3}{3x^6}-\frac{3a^2b^2}{4x^8}-\frac{2a^3b}{5x^{10}}-\frac{a^4}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x)
```

```
[Out] -2/3*a*b^3/x^6-2/5*a^3*b/x^10-1/12*a^4/x^12-3/4*a^2*b^2/x^8-1/4*b^4/x^4
```

maxima [A] time = 1.35, size = 48, normalized size = 1.20

$$-\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

mupad [B] time = 4.22, size = 48, normalized size = 1.20

$$-\frac{\frac{a^4}{12}+\frac{2\,a^3\,b\,x^2}{5}+\frac{3\,a^2\,b^2\,x^4}{4}+\frac{2\,a\,b^3\,x^6}{3}+\frac{b^4\,x^8}{4}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^13,x)
```

```
[Out] -(a^4/12 + (b^4*x^8)/4 + (2*a^3*b*x^2)/5 + (2*a*b^3*x^6)/3 + (3*a^2*b^2*x^4
)/4)/x^12
```

sympy [A] time = 0.43, size = 51, normalized size = 1.28

$$\frac{-5a^4-24a^3bx^2-45a^2b^2x^4-40ab^3x^6-15b^4x^8}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**13,x)
```

```
[Out] (-5*a**4 - 24*a**3*b*x**2 - 45*a**2*b**2*x**4 - 40*a*b**3*x**6 - 15*b**4*x*
*8)/(60*x**12)
```

3.269 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{14}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{2a^2b^2}{3x^9}-\frac{4a^3b}{11x^{11}}-\frac{a^4}{13x^{13}}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

```
[Out] -a^4/(13*x^13) - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*x
^7) - b^4/(5*x^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{14}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{14}}\,dx}{b^4}\\ &= \frac{\int \left(\frac{a^4b^4}{x^{14}}+\frac{4a^3b^5}{x^{12}}+\frac{6a^2b^6}{x^{10}}+\frac{4ab^7}{x^8}+\frac{b^8}{x^6}\right)dx}{b^4}\\ &= -\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
[Out] -1/13*a^4/x^13 - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*x
^7) - b^4/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14, x]
```

fricas [A] time = 0.66, size = 48, normalized size = 0.86

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="fricas")
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

giac [A] time = 0.21, size = 48, normalized size = 0.86

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="giac")
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$-\frac{b^4}{5x^5}-\frac{4a\,b^3}{7x^7}-\frac{2a^2b^2}{3x^9}-\frac{4a^3b}{11x^{11}}-\frac{a^4}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x)
```

```
[Out] -1/13*a^4/x^13-4/11*a^3*b/x^11-2/3*a^2*b^2/x^9-4/7*a*b^3/x^7-1/5*b^4/x^5
```

maxima [A] time = 1.46, size = 48, normalized size = 0.86

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

mupad [B] time = 0.04, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{13}+\frac{4\,a^3\,b\,x^2}{11}+\frac{2\,a^2\,b^2\,x^4}{3}+\frac{4\,a\,b^3\,x^6}{7}+\frac{b^4\,x^8}{5}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^14,x)
```

```
[Out] -(a^4/13 + (b^4*x^8)/5 + (4*a^3*b*x^2)/11 + (4*a*b^3*x^6)/7 + (2*a^2*b^2*x^
4)/3)/x^13
```

sympy [A] time = 0.44, size = 51, normalized size = 0.91

$$\frac{-1155a^4-5460a^3bx^2-10010a^2b^2x^4-8580ab^3x^6-3003b^4x^8}{15015x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**14,x)
```

```
[Out] (-1155*a**4 - 5460*a**3*b*x**2 - 10010*a**2*b**2*x**4 - 8580*a*b**3*x**6 -
3003*b**4*x**8)/(15015*x**13)
```

3.270 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{3a^2b^2}{5x^{10}}-\frac{a^3b}{3x^{12}}-\frac{a^4}{14x^{14}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -a^4/(14*x^14) - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
- b^4/(6*x^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^{15}}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x^8}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4b^4}{x^8}+\frac{4a^3b^5}{x^7}+\frac{6a^2b^6}{x^6}+\frac{4ab^7}{x^5}+\frac{b^8}{x^4}\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -1/14*a^4/x^14 - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
- b^4/(6*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15, x]
```

fricas [A] time = 0.59, size = 48, normalized size = 0.86

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="fricas")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

giac [A] time = 0.15, size = 48, normalized size = 0.86

$$-\frac{35\,b^4x^8 + 105\,ab^3x^6 + 126\,a^2b^2x^4 + 70\,a^3bx^2 + 15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="giac")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

maple [A] time = 0.01, size = 47, normalized size = 0.84

$$-\frac{b^4}{6x^6} - \frac{a\,b^3}{2x^8} - \frac{3a^2b^2}{5x^{10}} - \frac{a^3b}{3x^{12}} - \frac{a^4}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x)
```

```
[Out] -1/14*a^4/x^14-1/3*a^3*b/x^12-3/5*a^2*b^2/x^10-1/2*a*b^3/x^8-1/6*b^4/x^6
```

maxima [A] time = 1.43, size = 48, normalized size = 0.86

$$-\frac{35\,b^4x^8 + 105\,ab^3x^6 + 126\,a^2b^2x^4 + 70\,a^3bx^2 + 15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="maxima")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

mupad [B] time = 4.33, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{14} + \frac{a^3\,b\,x^2}{3} + \frac{3\,a^2\,b^2\,x^4}{5} + \frac{a\,b^3\,x^6}{2} + \frac{b^4\,x^8}{6}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^15,x)
```

```
[Out] -(a^4/14 + (b^4*x^8)/6 + (a^3*b*x^2)/3 + (a*b^3*x^6)/2 + (3*a^2*b^2*x^4)/5)
/x^14
```

sympy [A] time = 0.45, size = 51, normalized size = 0.91

$$\frac{-15a^4 - 70a^3bx^2 - 126a^2b^2x^4 - 105ab^3x^6 - 35b^4x^8}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**15,x)
```

```
[Out] (-15*a**4 - 70*a**3*b*x**2 - 126*a**2*b**2*x**4 - 105*a*b**3*x**6 - 35*b**4
*x**8)/(210*x**14)
```

3.271 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{16}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{11x^{11}}-\frac{4a^3b}{13x^{13}}-\frac{a^4}{15x^{15}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -a^4/(15*x^15) - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4*a*b^3)/(9
*x^9) - b^4/(7*x^7)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{16}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{16}}\,dx}{b^4}\\
&= \frac{\int \left(\frac{a^4b^4}{x^{16}}+\frac{4a^3b^5}{x^{14}}+\frac{6a^2b^6}{x^{12}}+\frac{4ab^7}{x^{10}}+\frac{b^8}{x^8}\right)dx}{b^4}\\
&= -\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -1/15*a^4/x^15 - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4*a*b^3)/(9
*x^9) - b^4/(7*x^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16, x]
```

fricas [A] time = 0.87, size = 48, normalized size = 0.86

$$-\frac{6435\,b^4x^8+20020\,ab^3x^6+24570\,a^2b^2x^4+13860\,a^3bx^2+3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

giac [A] time = 0.15, size = 48, normalized size = 0.86

$$-\frac{6435\,b^4x^8+20020\,ab^3x^6+24570\,a^2b^2x^4+13860\,a^3bx^2+3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$-\frac{b^4}{7x^7}-\frac{4a\,b^3}{9x^9}-\frac{6a^2b^2}{11x^{11}}-\frac{4a^3b}{13x^{13}}-\frac{a^4}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x)
```

```
[Out] -1/15*a^4/x^15-4/13*a^3*b/x^13-6/11*a^2*b^2/x^11-4/9*a*b^3/x^9-1/7*b^4/x^7
```

maxima [A] time = 1.38, size = 48, normalized size = 0.86

$$-\frac{6435\,b^4x^8+20020\,ab^3x^6+24570\,a^2b^2x^4+13860\,a^3bx^2+3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

mupad [B] time = 4.35, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{15}+\frac{4\,a^3\,b\,x^2}{13}+\frac{6\,a^2\,b^2\,x^4}{11}+\frac{4\,a\,b^3\,x^6}{9}+\frac{b^4\,x^8}{7}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^16,x)
```

```
[Out] -(a^4/15 + (b^4*x^8)/7 + (4*a^3*b*x^2)/13 + (4*a*b^3*x^6)/9 + (6*a^2*b^2*x^
4)/11)/x^15
```

sympy [A] time = 0.46, size = 51, normalized size = 0.91

$$\frac{-3003a^4-13860a^3bx^2-24570a^2b^2x^4-20020ab^3x^6-6435b^4x^8}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**16,x)
```

```
[Out] (-3003*a**4 - 13860*a**3*b*x**2 - 24570*a**2*b**2*x**4 - 20020*a*b**3*x**6
- 6435*b**4*x**8)/(45045*x**15)
```

3.272 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{17}a^2b^4x^{17}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{13}a^4b^2x^{13}+\frac{6}{11}a^5bx^{11}+\frac{a^6x^9}{9}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3
 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx &= \frac{\int x^8 \left(ab+b^2x^2\right)^6 \, dx}{b^6} \\
&= \frac{\int \left(a^6b^6x^8+6a^5b^7x^{10}+15a^4b^8x^{12}+20a^3b^9x^{14}+15a^2b^{10}x^{16}+6ab^{11}x^{18}+b^{12}x^2\right.}{b^6} \\
&= \frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.00

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3
 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.71, size = 68, normalized size = 0.83

$$\frac{1}{21}x^{21}b^6+\frac{6}{19}x^{19}b^5a+\frac{15}{17}x^{17}b^4a^2+\frac{4}{3}x^{15}b^3a^3+\frac{15}{13}x^{13}b^2a^4+\frac{6}{11}x^{11}ba^5+\frac{1}{9}x^9a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^6 + 6/19*x^19*b^5*a + 15/17*x^17*b^4*a^2 + 4/3*x^15*b^3*a^3 + 1
5/13*x^13*b^2*a^4 + 6/11*x^11*b*a^5 + 1/9*x^9*a^6
```

giac [A] time = 0.16, size = 68, normalized size = 0.83

$$\frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 + 1
5/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

maple [A] time = 0.00, size = 69, normalized size = 0.84

$$\frac{1}{21}b^6x^{21}+\frac{6}{19}a\,b^5x^{19}+\frac{15}{17}a^2b^4x^{17}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{13}a^4b^2x^{13}+\frac{6}{11}a^5b\,x^{11}+\frac{1}{9}a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/9*a^6*x^9+6/11*a^5*b*x^11+15/13*a^4*b^2*x^13+4/3*a^3*b^3*x^15+15/17*a^2*b
^4*x^17+6/19*a*b^5*x^19+1/21*b^6*x^21
```

maxima [A] time = 1.35, size = 68, normalized size = 0.83

$$\frac{1}{21}\,b^{6}x^{21}+\frac{6}{19}\,ab^{5}x^{19}+\frac{15}{17}\,a^{2}b^{4}x^{17}+\frac{4}{3}\,a^{3}b^{3}x^{15}+\frac{15}{13}\,a^{4}b^{2}x^{13}+\frac{6}{11}\,a^{5}bx^{11}+\frac{1}{9}\,a^{6}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 + 1
5/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

mupad [B] time = 0.03, size = 68, normalized size = 0.83

$$\frac{a^6\,x^9}{9}+\frac{6\,a^5\,b\,x^{11}}{11}+\frac{15\,a^4\,b^2\,x^{13}}{13}+\frac{4\,a^3\,b^3\,x^{15}}{3}+\frac{15\,a^2\,b^4\,x^{17}}{17}+\frac{6\,a\,b^5\,x^{19}}{19}+\frac{b^6\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^9)/9 + (b^6*x^21)/21 + (6*a^5*b*x^11)/11 + (6*a*b^5*x^19)/19 + (15*a
^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3 + (15*a^2*b^4*x^17)/17
```

sympy [A] time = 0.09, size = 80, normalized size = 0.98

$$\frac{a^{6}x^{9}}{9}+\frac{6a^{5}bx^{11}}{11}+\frac{15a^{4}b^{2}x^{13}}{13}+\frac{4a^{3}b^{3}x^{15}}{3}+\frac{15a^{2}b^{4}x^{17}}{17}+\frac{6ab^{5}x^{19}}{19}+\frac{b^{6}x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**9/9 + 6*a**5*b*x**11/11 + 15*a**4*b**2*x**13/13 + 4*a**3*b**3*x**15
/3 + 15*a**2*b**4*x**17/17 + 6*a*b**5*x**19/19 + b**6*x**21/21
```

3.273 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 dx$

Optimal. Leaf size=72

$$-\frac{a^3\left(a+bx^2\right)^7}{14b^4}+\frac{3a^2\left(a+bx^2\right)^8}{16b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}-\frac{a\left(a+bx^2\right)^9}{6b^4}$$

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3a^2\left(a+bx^2\right)^8}{16b^4}-\frac{a^3\left(a+bx^2\right)^7}{14b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}-\frac{a\left(a+bx^2\right)^9}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^3*(a + b*x^2)^7)/(14*b^4) + (3*a^2*(a + b*x^2)^8)/(16*b^4) - (a*(a + b*
x^2)^9)/(6*b^4) + (a + b*x^2)^10/(20*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{\int x^7 \left(ab+b^2x^2\right)^6 \, dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int x^3 \left(ab+b^2x\right)^6 \, dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a^3\left(ab+b^2x\right)^6}{b^3}+\frac{3a^2\left(ab+b^2x\right)^7}{b^4}-\frac{3a\left(ab+b^2x\right)^8}{b^5}+\frac{\left(ab+b^2x\right)^9}{b^6}\right) dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^3\left(a+bx^2\right)^7}{14b^4}+\frac{3a^2\left(a+bx^2\right)^8}{16b^4}-\frac{a\left(a+bx^2\right)^9}{6b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.14

$$\frac{a^6x^8}{8}+\frac{3}{5}a^5bx^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}ab^5x^{18}+\frac{b^6x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^8)/8 + (3*a^5*b*x^10)/5 + (5*a^4*b^2*x^12)/4 + (10*a^3*b^3*x^14)/7 +
 (15*a^2*b^4*x^16)/16 + (a*b^5*x^18)/3 + (b^6*x^20)/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.65, size = 68, normalized size = 0.94

$$\frac{1}{20}x^{20}b^6+\frac{1}{3}x^{18}b^5a+\frac{15}{16}x^{16}b^4a^2+\frac{10}{7}x^{14}b^3a^3+\frac{5}{4}x^{12}b^2a^4+\frac{3}{5}x^{10}ba^5+\frac{1}{8}x^8a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^6 + 1/3*x^18*b^5*a + 15/16*x^16*b^4*a^2 + 10/7*x^14*b^3*a^3 + 5
/4*x^12*b^2*a^4 + 3/5*x^10*b*a^5 + 1/8*x^8*a^6
```

giac [A] time = 0.15, size = 68, normalized size = 0.94

$$\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 + 5
/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

maple [A] time = 0.00, size = 69, normalized size = 0.96

$$\frac{1}{20}b^6x^{20}+\frac{1}{3}a\,b^5x^{18}+\frac{15}{16}a^2b^4x^{16}+\frac{10}{7}a^3b^3x^{14}+\frac{5}{4}a^4b^2x^{12}+\frac{3}{5}a^5b\,x^{10}+\frac{1}{8}a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/20*b^6*x^20+1/3*a*b^5*x^18+15/16*a^2*b^4*x^16+10/7*a^3*b^3*x^14+5/4*a^4*b
^2*x^12+3/5*a^5*b*x^10+1/8*a^6*x^8
```

maxima [A] time = 1.43, size = 68, normalized size = 0.94

$$\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 + 5
/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

mupad [B] time = 0.03, size = 68, normalized size = 0.94

$$\frac{a^6\,x^8}{8}+\frac{3\,a^5\,b\,x^{10}}{5}+\frac{5\,a^4\,b^2\,x^{12}}{4}+\frac{10\,a^3\,b^3\,x^{14}}{7}+\frac{15\,a^2\,b^4\,x^{16}}{16}+\frac{a\,b^5\,x^{18}}{3}+\frac{b^6\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^8)/8 + (b^6*x^20)/20 + (3*a^5*b*x^10)/5 + (a*b^5*x^18)/3 + (5*a^4*b^
2*x^12)/4 + (10*a^3*b^3*x^14)/7 + (15*a^2*b^4*x^16)/16
```

sympy [A] time = 0.09, size = 78, normalized size = 1.08

$$\frac{a^6x^8}{8} + \frac{3a^5bx^{10}}{5} + \frac{5a^4b^2x^{12}}{4} + \frac{10a^3b^3x^{14}}{7} + \frac{15a^2b^4x^{16}}{16} + \frac{ab^5x^{18}}{3} + \frac{b^6x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**8/8 + 3*a**5*b*x**10/5 + 5*a**4*b**2*x**12/4 + 10*a**3*b**3*x**14/7
 + 15*a**2*b**4*x**16/16 + a*b**5*x**18/3 + b**6*x**20/20
```

3.274 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=79

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$a^2b^4x^{15}+\frac{20}{13}a^3b^3x^{13}+\frac{15}{11}a^4b^2x^{11}+\frac{2}{3}a^5bx^9+\frac{a^6x^7}{7}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13
 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx &= \frac{\int x^6 \left(ab+b^2x^2\right)^6 \, dx}{b^6}\\ &= \frac{\int \left(a^6b^6x^6+6a^5b^7x^8+15a^4b^8x^{10}+20a^3b^9x^{12}+15a^2b^{10}x^{14}+6ab^{11}x^{16}+b^{12}x\right.}{b^6}\\ &= \frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}\end{aligned}$$

Mathematica [A] time = 0.00, size = 79, normalized size = 1.00

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13
 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.91, size = 67, normalized size = 0.85

$$\frac{1}{19}x^{19}b^6+\frac{6}{17}x^{17}b^5a+x^{15}b^4a^2+\frac{20}{13}x^{13}b^3a^3+\frac{15}{11}x^{11}b^2a^4+\frac{2}{3}x^9ba^5+\frac{1}{7}x^7a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^6 + 6/17*x^17*b^5*a + x^15*b^4*a^2 + 20/13*x^13*b^3*a^3 + 15/11
*x^11*b^2*a^4 + 2/3*x^9*b*a^5 + 1/7*x^7*a^6
```

giac [A] time = 0.15, size = 67, normalized size = 0.85

$$\frac{1}{19}\,b^6x^{19}+\frac{6}{17}\,ab^5x^{17}+a^2b^4x^{15}+\frac{20}{13}\,a^3b^3x^{13}+\frac{15}{11}\,a^4b^2x^{11}+\frac{2}{3}\,a^5bx^9+\frac{1}{7}\,a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/11
*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

maple [A] time = 0.00, size = 68, normalized size = 0.86

$$\frac{1}{19}b^6x^{19}+\frac{6}{17}a\,b^5x^{17}+a^2b^4x^{15}+\frac{20}{13}a^3b^3x^{13}+\frac{15}{11}a^4b^2x^{11}+\frac{2}{3}a^5b\,x^9+\frac{1}{7}a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/7*a^6*x^7+2/3*a^5*b*x^9+15/11*a^4*b^2*x^11+20/13*a^3*b^3*x^13+a^2*b^4*x^1
5+6/17*a*b^5*x^17+1/19*b^6*x^19
```

maxima [A] time = 1.39, size = 67, normalized size = 0.85

$$\frac{1}{19}\,b^6x^{19}+\frac{6}{17}\,ab^5x^{17}+a^2b^4x^{15}+\frac{20}{13}\,a^3b^3x^{13}+\frac{15}{11}\,a^4b^2x^{11}+\frac{2}{3}\,a^5bx^9+\frac{1}{7}\,a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/11
*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

mupad [B] time = 0.03, size = 67, normalized size = 0.85

$$\frac{a^6\,x^7}{7}+\frac{2\,a^5\,b\,x^9}{3}+\frac{15\,a^4\,b^2\,x^{11}}{11}+\frac{20\,a^3\,b^3\,x^{13}}{13}+a^2\,b^4\,x^{15}+\frac{6\,a\,b^5\,x^{17}}{17}+\frac{b^6\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^7)/7 + (b^6*x^19)/19 + (2*a^5*b*x^9)/3 + (6*a*b^5*x^17)/17 + (15*a^4
*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13 + a^2*b^4*x^15
```

sympy [A] time = 0.09, size = 76, normalized size = 0.96

$$\frac{a^6x^7}{7}+\frac{2a^5bx^9}{3}+\frac{15a^4b^2x^{11}}{11}+\frac{20a^3b^3x^{13}}{13}+a^2b^4x^{15}+\frac{6ab^5x^{17}}{17}+\frac{b^6x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**7/7 + 2*a**5*b*x**9/3 + 15*a**4*b**2*x**11/11 + 20*a**3*b**3*x**13/
13 + a**2*b**4*x**15 + 6*a*b**5*x**17/17 + b**6*x**19/19
```

3.275 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=53

$$\frac{a^2 \left(a+bx^2\right)^7}{14b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}-\frac{a \left(a+bx^2\right)^8}{8b^3}$$

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^2 \left(a+bx^2\right)^7}{14b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}-\frac{a \left(a+bx^2\right)^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^2*(a + b*x^2)^7)/(14*b^3) - (a*(a + b*x^2)^8)/(8*b^3) + (a + b*x^2)^9/(1
8*b^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{\int x^5 \left(ab+b^2x^2\right)^6 \, dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int x^2 \left(ab+b^2x\right)^6 \, dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2\left(ab+b^2x\right)^6}{b^2} - \frac{2a\left(ab+b^2x\right)^7}{b^3} + \frac{\left(ab+b^2x\right)^8}{b^4}\right) dx, x, x^2\right)}{2b^6}$$
$$= \frac{a^2\left(a+bx^2\right)^7}{14b^3} - \frac{a\left(a+bx^2\right)^8}{8b^3} + \frac{\left(a+bx^2\right)^9}{18b^3}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.55

$$\frac{a^6x^6}{6} + \frac{3}{4}a^5bx^8 + \frac{3}{2}a^4b^2x^{10} + \frac{5}{3}a^3b^3x^{12} + \frac{15}{14}a^2b^4x^{14} + \frac{3}{8}ab^5x^{16} + \frac{b^6x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^6)/6 + (3*a^5*b*x^8)/4 + (3*a^4*b^2*x^10)/2 + (5*a^3*b^3*x^12)/3 + (
15*a^2*b^4*x^14)/14 + (3*a*b^5*x^16)/8 + (b^6*x^18)/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.59, size = 68, normalized size = 1.28

$$\frac{1}{18}x^{18}b^6 + \frac{3}{8}x^{16}b^5a + \frac{15}{14}x^{14}b^4a^2 + \frac{5}{3}x^{12}b^3a^3 + \frac{3}{2}x^{10}b^2a^4 + \frac{3}{4}x^8ba^5 + \frac{1}{6}x^6a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^6 + 3/8*x^16*b^5*a + 15/14*x^14*b^4*a^2 + 5/3*x^12*b^3*a^3 + 3/
2*x^10*b^2*a^4 + 3/4*x^8*b*a^5 + 1/6*x^6*a^6
```

giac [A] time = 0.15, size = 68, normalized size = 1.28

$$\frac{1}{18}\,b^6x^{18}+\frac{3}{8}\,ab^5x^{16}+\frac{15}{14}\,a^2b^4x^{14}+\frac{5}{3}\,a^3b^3x^{12}+\frac{3}{2}\,a^4b^2x^{10}+\frac{3}{4}\,a^5bx^8+\frac{1}{6}\,a^6x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3/
2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

maple [A] time = 0.00, size = 69, normalized size = 1.30

$$\frac{1}{18}b^6x^{18}+\frac{3}{8}a\,b^5x^{16}+\frac{15}{14}a^2b^4x^{14}+\frac{5}{3}a^3b^3x^{12}+\frac{3}{2}a^4b^2x^{10}+\frac{3}{4}a^5b\,x^8+\frac{1}{6}a^6x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/18*b^6*x^18+3/8*a*b^5*x^16+15/14*a^2*b^4*x^14+5/3*a^3*b^3*x^12+3/2*a^4*b^
2*x^10+3/4*a^5*b*x^8+1/6*a^6*x^6
```

maxima [A] time = 1.36, size = 68, normalized size = 1.28

$$\frac{1}{18}\,b^6x^{18}+\frac{3}{8}\,ab^5x^{16}+\frac{15}{14}\,a^2b^4x^{14}+\frac{5}{3}\,a^3b^3x^{12}+\frac{3}{2}\,a^4b^2x^{10}+\frac{3}{4}\,a^5bx^8+\frac{1}{6}\,a^6x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3/
2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

mupad [B] time = 0.03, size = 68, normalized size = 1.28

$$\frac{a^6\,x^6}{6}+\frac{3\,a^5\,b\,x^8}{4}+\frac{3\,a^4\,b^2\,x^{10}}{2}+\frac{5\,a^3\,b^3\,x^{12}}{3}+\frac{15\,a^2\,b^4\,x^{14}}{14}+\frac{3\,a\,b^5\,x^{16}}{8}+\frac{b^6\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^6)/6 + (b^6*x^18)/18 + (3*a^5*b*x^8)/4 + (3*a*b^5*x^16)/8 + (3*a^4*b
^2*x^10)/2 + (5*a^3*b^3*x^12)/3 + (15*a^2*b^4*x^14)/14
```

sympy [A] time = 0.09, size = 80, normalized size = 1.51

$$\frac{a^6x^6}{6} + \frac{3a^5bx^8}{4} + \frac{3a^4b^2x^{10}}{2} + \frac{5a^3b^3x^{12}}{3} + \frac{15a^2b^4x^{14}}{14} + \frac{3ab^5x^{16}}{8} + \frac{b^6x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**6/6 + 3*a**5*b*x**8/4 + 3*a**4*b**2*x**10/2 + 5*a**3*b**3*x**12/3 +
 15*a**2*b**4*x**14/14 + 3*a*b**5*x**16/8 + b**6*x**18/18
```

3.276 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{13}a^2b^4x^{13}+\frac{20}{11}a^3b^3x^{11}+\frac{5}{3}a^4b^2x^9+\frac{6}{7}a^5bx^7+\frac{a^6x^5}{5}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
(15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx &= \frac{\int x^4 \left(ab+b^2x^2\right)^6 \, dx}{b^6}\\
&= \frac{\int \left(a^6b^6x^4+6a^5b^7x^6+15a^4b^8x^8+20a^3b^9x^{10}+15a^2b^{10}x^{12}+6ab^{11}x^{14}+b^{12}x^{16}\right)}{b^6}\\
&= \frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.00

$$\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
(15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.77, size = 68, normalized size = 0.83

$$\frac{1}{17}x^{17}b^6+\frac{2}{5}x^{15}b^5a+\frac{15}{13}x^{13}b^4a^2+\frac{20}{11}x^{11}b^3a^3+\frac{5}{3}x^9b^2a^4+\frac{6}{7}x^7ba^5+\frac{1}{5}x^5a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^6 + 2/5*x^15*b^5*a + 15/13*x^13*b^4*a^2 + 20/11*x^11*b^3*a^3 +
5/3*x^9*b^2*a^4 + 6/7*x^7*b*a^5 + 1/5*x^5*a^6
```

giac [A] time = 0.17, size = 68, normalized size = 0.83

$$\frac{1}{17}\,b^6x^{17}+\frac{2}{5}\,ab^5x^{15}+\frac{15}{13}\,a^2b^4x^{13}+\frac{20}{11}\,a^3b^3x^{11}+\frac{5}{3}\,a^4b^2x^9+\frac{6}{7}\,a^5bx^7+\frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

maple [A] time = 0.00, size = 69, normalized size = 0.84

$$\frac{1}{17}b^6x^{17}+\frac{2}{5}a\,b^5x^{15}+\frac{15}{13}a^2b^4x^{13}+\frac{20}{11}a^3b^3x^{11}+\frac{5}{3}a^4b^2x^9+\frac{6}{7}a^5b\,x^7+\frac{1}{5}a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/5*a^6*x^5+6/7*a^5*b*x^7+5/3*a^4*b^2*x^9+20/11*a^3*b^3*x^11+15/13*a^2*b^4*
x^13+2/5*a*b^5*x^15+1/17*b^6*x^17
```

maxima [A] time = 1.29, size = 68, normalized size = 0.83

$$\frac{1}{17}\,b^6x^{17}+\frac{2}{5}\,ab^5x^{15}+\frac{15}{13}\,a^2b^4x^{13}+\frac{20}{11}\,a^3b^3x^{11}+\frac{5}{3}\,a^4b^2x^9+\frac{6}{7}\,a^5bx^7+\frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

mupad [B] time = 0.03, size = 68, normalized size = 0.83

$$\frac{a^6\,x^5}{5}+\frac{6\,a^5\,b\,x^7}{7}+\frac{5\,a^4\,b^2\,x^9}{3}+\frac{20\,a^3\,b^3\,x^{11}}{11}+\frac{15\,a^2\,b^4\,x^{13}}{13}+\frac{2\,a\,b^5\,x^{15}}{5}+\frac{b^6\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^5)/5 + (b^6*x^17)/17 + (6*a^5*b*x^7)/7 + (2*a*b^5*x^15)/5 + (5*a^4*b
^2*x^9)/3 + (20*a^3*b^3*x^11)/11 + (15*a^2*b^4*x^13)/13
```

sympy [A] time = 0.09, size = 80, normalized size = 0.98

$$\frac{a^6x^5}{5}+\frac{6a^5bx^7}{7}+\frac{5a^4b^2x^9}{3}+\frac{20a^3b^3x^{11}}{11}+\frac{15a^2b^4x^{13}}{13}+\frac{2ab^5x^{15}}{5}+\frac{b^6x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**5/5 + 6*a**5*b*x**7/7 + 5*a**4*b**2*x**9/3 + 20*a**3*b**3*x**11/11
+ 15*a**2*b**4*x**13/13 + 2*a*b**5*x**15/5 + b**6*x**17/17
```

3.277 $\int x^3\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^8}{16b^2}-\frac{a\left(a+bx^2\right)^7}{14b^2}$$

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{\left(a+bx^2\right)^8}{16b^2}-\frac{a\left(a+bx^2\right)^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a*(a + b*x^2)^7)/(14*b^2) + (a + b*x^2)^8/(16*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{\int x^3\left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$=\frac{\operatorname{Subst}\left(\int x\left(ab+b^2x\right)^6\,dx,x,x^2\right)}{2b^6}$$
$$=\frac{\operatorname{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^6}{b}+\frac{\left(ab+b^2x\right)^7}{b^2}\right)dx,x,x^2\right)}{2b^6}$$
$$=-\frac{a\left(a+bx^2\right)^7}{14b^2}+\frac{\left(a+bx^2\right)^8}{16b^2}$$

Mathematica [B] time = 0.00, size = 77, normalized size = 2.26

$$\frac{a^6x^4}{4}+a^5bx^6+\frac{15}{8}a^4b^2x^8+2a^3b^3x^{10}+\frac{5}{4}a^2b^4x^{12}+\frac{3}{7}ab^5x^{14}+\frac{b^6x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^4)/4 + a^5*b*x^6 + (15*a^4*b^2*x^8)/8 + 2*a^3*b^3*x^10 + (5*a^2*b^4*
x^12)/4 + (3*a*b^5*x^14)/7 + (b^6*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [B] time = 0.66, size = 67, normalized size = 1.97

$$\frac{1}{16}x^{16}b^6+\frac{3}{7}x^{14}b^5a+\frac{5}{4}x^{12}b^4a^2+2x^{10}b^3a^3+\frac{15}{8}x^8b^2a^4+x^6ba^5+\frac{1}{4}x^4a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^6 + 3/7*x^14*b^5*a + 5/4*x^12*b^4*a^2 + 2*x^10*b^3*a^3 + 15/8*x
^8*b^2*a^4 + x^6*b*a^5 + 1/4*x^4*a^6
```

giac [B] time = 0.17, size = 67, normalized size = 1.97

$$\frac{1}{16}\,b^{6}x^{16} + \frac{3}{7}\,ab^{5}x^{14} + \frac{5}{4}\,a^{2}b^{4}x^{12} + 2\,a^{3}b^{3}x^{10} + \frac{15}{8}\,a^{4}b^{2}x^{8} + a^{5}bx^{6} + \frac{1}{4}\,a^{6}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*a
^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

maple [B] time = 0.00, size = 68, normalized size = 2.00

$$\frac{1}{16}b^{6}x^{16} + \frac{3}{7}a\,b^{5}x^{14} + \frac{5}{4}a^{2}b^{4}x^{12} + 2a^{3}b^{3}x^{10} + \frac{15}{8}a^{4}b^{2}x^{8} + a^{5}b\,x^{6} + \frac{1}{4}a^{6}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/16*b^6*x^16+3/7*a*b^5*x^14+5/4*a^2*b^4*x^12+2*a^3*b^3*x^10+15/8*a^4*b^2*x
^8+a^5*b*x^6+1/4*a^6*x^4
```

maxima [B] time = 1.36, size = 67, normalized size = 1.97

$$\frac{1}{16}\,b^{6}x^{16} + \frac{3}{7}\,ab^{5}x^{14} + \frac{5}{4}\,a^{2}b^{4}x^{12} + 2\,a^{3}b^{3}x^{10} + \frac{15}{8}\,a^{4}b^{2}x^{8} + a^{5}bx^{6} + \frac{1}{4}\,a^{6}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*a
^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

mupad [B] time = 0.03, size = 67, normalized size = 1.97

$$\frac{a^{6}\,x^{4}}{4} + a^{5}\,b\,x^{6} + \frac{15\,a^{4}\,b^{2}\,x^{8}}{8} + 2\,a^{3}\,b^{3}\,x^{10} + \frac{5\,a^{2}\,b^{4}\,x^{12}}{4} + \frac{3\,a\,b^{5}\,x^{14}}{7} + \frac{b^{6}\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^4)/4 + (b^6*x^16)/16 + a^5*b*x^6 + (3*a*b^5*x^14)/7 + (15*a^4*b^2*x^
8)/8 + 2*a^3*b^3*x^10 + (5*a^2*b^4*x^12)/4
```

sympy [B] time = 0.09, size = 75, normalized size = 2.21

$$\frac{a^6x^4}{4} + a^5bx^6 + \frac{15a^4b^2x^8}{8} + 2a^3b^3x^{10} + \frac{5a^2b^4x^{12}}{4} + \frac{3ab^5x^{14}}{7} + \frac{b^6x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**4/4 + a**5*b*x**6 + 15*a**4*b**2*x**8/8 + 2*a**3*b**3*x**10 + 5*a**
2*b**4*x**12/4 + 3*a*b**5*x**14/7 + b**6*x**16/16
```

3.278 $\int x^2\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=82

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{11}a^2b^4x^{11}+\frac{20}{9}a^3b^3x^9+\frac{15}{7}a^4b^2x^7+\frac{6}{5}a^5bx^5+\frac{a^6x^3}{3}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (
15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int x^2\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int\left(a^6b^6x^2+6a^5b^7x^4+15a^4b^8x^6+20a^3b^9x^8+15a^2b^{10}x^{10}+6ab^{11}x^{12}+b^{12}x^{14}\right.}{b^6}\\ &= \frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.00

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (
15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.76, size = 68, normalized size = 0.83

$$\frac{1}{15}x^{15}b^6+\frac{6}{13}x^{13}b^5a+\frac{15}{11}x^{11}b^4a^2+\frac{20}{9}x^9b^3a^3+\frac{15}{7}x^7b^2a^4+\frac{6}{5}x^5ba^5+\frac{1}{3}x^3a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^6 + 6/13*x^13*b^5*a + 15/11*x^11*b^4*a^2 + 20/9*x^9*b^3*a^3 + 1
5/7*x^7*b^2*a^4 + 6/5*x^5*b*a^5 + 1/3*x^3*a^6
```

giac [A] time = 0.15, size = 68, normalized size = 0.83

$$\frac{1}{15}\,b^6x^{15}+\frac{6}{13}\,ab^5x^{13}+\frac{15}{11}\,a^2b^4x^{11}+\frac{20}{9}\,a^3b^3x^9+\frac{15}{7}\,a^4b^2x^7+\frac{6}{5}\,a^5bx^5+\frac{1}{3}\,a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 + 1
5/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

maple [A] time = 0.00, size = 69, normalized size = 0.84

$$\frac{1}{15}b^6x^{15}+\frac{6}{13}a\,b^5x^{13}+\frac{15}{11}a^2b^4x^{11}+\frac{20}{9}a^3b^3x^9+\frac{15}{7}a^4b^2x^7+\frac{6}{5}a^5b\,x^5+\frac{1}{3}a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/3*a^6*x^3+6/5*a^5*b*x^5+15/7*a^4*b^2*x^7+20/9*a^3*b^3*x^9+15/11*a^2*b^4*x
^11+6/13*a*b^5*x^13+1/15*b^6*x^15
```

maxima [A] time = 1.36, size = 68, normalized size = 0.83

$$\frac{1}{15}\,b^{6}x^{15}+\frac{6}{13}\,ab^{5}x^{13}+\frac{15}{11}\,a^{2}b^{4}x^{11}+\frac{20}{9}\,a^{3}b^{3}x^{9}+\frac{15}{7}\,a^{4}b^{2}x^{7}+\frac{6}{5}\,a^{5}bx^{5}+\frac{1}{3}\,a^{6}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 + 1
5/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

mupad [B] time = 0.03, size = 68, normalized size = 0.83

$$\frac{a^6\,x^3}{3}+\frac{6\,a^5\,b\,x^5}{5}+\frac{15\,a^4\,b^2\,x^7}{7}+\frac{20\,a^3\,b^3\,x^9}{9}+\frac{15\,a^2\,b^4\,x^{11}}{11}+\frac{6\,a\,b^5\,x^{13}}{13}+\frac{b^6\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^3)/3 + (b^6*x^15)/15 + (6*a^5*b*x^5)/5 + (6*a*b^5*x^13)/13 + (15*a^4
*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (15*a^2*b^4*x^11)/11
```

sympy [A] time = 0.09, size = 80, normalized size = 0.98

$$\frac{a^{6}x^{3}}{3}+\frac{6a^{5}bx^{5}}{5}+\frac{15a^{4}b^{2}x^{7}}{7}+\frac{20a^{3}b^{3}x^{9}}{9}+\frac{15a^{2}b^{4}x^{11}}{11}+\frac{6ab^{5}x^{13}}{13}+\frac{b^{6}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**3/3 + 6*a**5*b*x**5/5 + 15*a**4*b**2*x**7/7 + 20*a**3*b**3*x**9/9 +
 15*a**2*b**4*x**11/11 + 6*a*b**5*x**13/13 + b**6*x**15/15
```

3.279 $\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a + b*x^2)^7/(14*b)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int x\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\left(a+bx^2\right)^7}{14b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a + b*x^2)^7/(14*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [B] time = 0.69, size = 68, normalized size = 4.25

$$\frac{1}{14}x^{14}b^6+\frac{1}{2}x^{12}b^5a+\frac{3}{2}x^{10}b^4a^2+\frac{5}{2}x^8b^3a^3+\frac{5}{2}x^6b^2a^4+\frac{3}{2}x^4ba^5+\frac{1}{2}x^2a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^6 + 1/2*x^12*b^5*a + 3/2*x^10*b^4*a^2 + 5/2*x^8*b^3*a^3 + 5/2*x
^6*b^2*a^4 + 3/2*x^4*b*a^5 + 1/2*x^2*a^6
```

giac [B] time = 0.15, size = 68, normalized size = 4.25

$$\frac{1}{14}\,b^6x^{14}+\frac{1}{2}\,ab^5x^{12}+\frac{3}{2}\,a^2b^4x^{10}+\frac{5}{2}\,a^3b^3x^8+\frac{5}{2}\,a^4b^2x^6+\frac{3}{2}\,a^5bx^4+\frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*a
^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

maple [B] time = 0.00, size = 69, normalized size = 4.31

$$\frac{1}{14}b^6x^{14}+\frac{1}{2}a\,b^5x^{12}+\frac{3}{2}a^2b^4x^{10}+\frac{5}{2}a^3b^3x^8+\frac{5}{2}a^4b^2x^6+\frac{3}{2}a^5b\,x^4+\frac{1}{2}a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/14*b^6*x^14+1/2*a*b^5*x^12+3/2*a^2*b^4*x^10+5/2*a^3*b^3*x^8+5/2*a^4*b^2*x
^6+3/2*a^5*b*x^4+1/2*a^6*x^2
```

maxima [B] time = 1.38, size = 68, normalized size = 4.25

$$\frac{1}{14}\,b^{6}x^{14}+\frac{1}{2}\,ab^{5}x^{12}+\frac{3}{2}\,a^{2}b^{4}x^{10}+\frac{5}{2}\,a^{3}b^{3}x^{8}+\frac{5}{2}\,a^{4}b^{2}x^{6}+\frac{3}{2}\,a^{5}bx^{4}+\frac{1}{2}\,a^{6}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*a
^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

mupad [B] time = 0.03, size = 68, normalized size = 4.25

$$\frac{a^{6}\,x^{2}}{2}+\frac{3\,a^{5}\,b\,x^{4}}{2}+\frac{5\,a^{4}\,b^{2}\,x^{6}}{2}+\frac{5\,a^{3}\,b^{3}\,x^{8}}{2}+\frac{3\,a^{2}\,b^{4}\,x^{10}}{2}+\frac{a\,b^{5}\,x^{12}}{2}+\frac{b^{6}\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^2)/2 + (b^6*x^14)/14 + (3*a^5*b*x^4)/2 + (a*b^5*x^12)/2 + (5*a^4*b^2
*x^6)/2 + (5*a^3*b^3*x^8)/2 + (3*a^2*b^4*x^10)/2
```

sympy [B] time = 0.09, size = 78, normalized size = 4.88

$$\frac{a^{6}x^{2}}{2}+\frac{3a^{5}bx^{4}}{2}+\frac{5a^{4}b^{2}x^{6}}{2}+\frac{5a^{3}b^{3}x^{8}}{2}+\frac{3a^{2}b^{4}x^{10}}{2}+\frac{ab^{5}x^{12}}{2}+\frac{b^{6}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**2/2 + 3*a**5*b*x**4/2 + 5*a**4*b**2*x**6/2 + 5*a**3*b**3*x**8/2 + 3
*a**2*b**4*x**10/2 + a*b**5*x**12/2 + b**6*x**14/14
```

3.280 $\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=73

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 194}

$$\frac{5}{3}a^2b^4x^9+\frac{20}{7}a^3b^3x^7+3a^4b^2x^5+2a^5bx^3+a^6x+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/
3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int \left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6+6a^5b^7x^2+15a^4b^8x^4+20a^3b^9x^6+15a^2b^{10}x^8+6ab^{11}x^{10}+b^{12}x^{12}\right)dx}{b^6}\\ &= a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.00

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/
3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.65, size = 65, normalized size = 0.89

$$\frac{1}{13}x^{13}b^6+\frac{6}{11}x^{11}b^5a+\frac{5}{3}x^9b^4a^2+\frac{20}{7}x^7b^3a^3+3x^5b^2a^4+2x^3ba^5+xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^6 + 6/11*x^11*b^5*a + 5/3*x^9*b^4*a^2 + 20/7*x^7*b^3*a^3 + 3*x^
5*b^2*a^4 + 2*x^3*b*a^5 + x*a^6
```

giac [A] time = 0.16, size = 65, normalized size = 0.89

$$\frac{1}{13}\,b^6x^{13}+\frac{6}{11}\,ab^5x^{11}+\frac{5}{3}\,a^2b^4x^9+\frac{20}{7}\,a^3b^3x^7+3\,a^4b^2x^5+2\,a^5bx^3+a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 5/3*a^2*b^4*x^9 + 20/7*a^3*b^3*x^7 + 3*a^
4*b^2*x^5 + 2*a^5*b*x^3 + a^6*x
```

maple [A] time = 0.00, size = 66, normalized size = 0.90

$$\frac{1}{13}b^6x^{13}+\frac{6}{11}a\,b^5x^{11}+\frac{5}{3}a^2b^4x^9+\frac{20}{7}a^3b^3x^7+3a^4b^2x^5+2a^5b\,x^3+a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] a^6*x+2*a^5*b*x^3+3*a^4*b^2*x^5+20/7*a^3*b^3*x^7+5/3*a^2*b^4*x^9+6/11*a*b^5
*x^11+1/13*b^6*x^13
```

maxima [A] time = 1.34, size = 100, normalized size = 1.37

$$\frac{1}{13}\,b^6x^{13} + \frac{6}{11}\,ab^5x^{11} + \frac{4}{3}\,a^2b^4x^9 + \frac{8}{7}\,a^3b^3x^7 + a^6x + \frac{1}{5}\left(3\,b^2x^5 + 10\,abx^3\right)a^4 + \frac{1}{105}\left(35\,b^4x^9 + 180\,ab^3x^7 + 252\,a^2b^2x^5\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 4/3*a^2*b^4*x^9 + 8/7*a^3*b^3*x^7 + a^6*x
 + 1/5*(3*b^2*x^5 + 10*a*b*x^3)*a^4 + 1/105*(35*b^4*x^9 + 180*a*b^3*x^7 + 2
52*a^2*b^2*x^5)*a^2
```

mupad [B] time = 0.03, size = 65, normalized size = 0.89

$$a^6\,x + 2\,a^5\,b\,x^3 + 3\,a^4\,b^2\,x^5 + \frac{20\,a^3\,b^3\,x^7}{7} + \frac{5\,a^2\,b^4\,x^9}{3} + \frac{6\,a\,b^5\,x^{11}}{11} + \frac{b^6\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] a^6*x + (b^6*x^13)/13 + 2*a^5*b*x^3 + (6*a*b^5*x^11)/11 + 3*a^4*b^2*x^5 + (
20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/3
```

sympy [A] time = 0.08, size = 73, normalized size = 1.00

$$a^6x + 2a^5bx^3 + 3a^4b^2x^5 + \frac{20a^3b^3x^7}{7} + \frac{5a^2b^4x^9}{3} + \frac{6ab^5x^{11}}{11} + \frac{b^6x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x + 2*a**5*b*x**3 + 3*a**4*b**2*x**5 + 20*a**3*b**3*x**7/7 + 5*a**2*b*
*4*x**9/3 + 6*a*b**5*x**11/11 + b**6*x**13/13
```

3.281 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x} dx$

Optimal. Leaf size=76

$$a^6 \log(x) + 3a^5bx^2 + \frac{15}{4}a^4b^2x^4 + \frac{10}{3}a^3b^3x^6 + \frac{15}{8}a^2b^4x^8 + \frac{3}{5}ab^5x^{10} + \frac{b^6x^{12}}{12}$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{15}{8}a^2b^4x^8 + \frac{10}{3}a^3b^3x^6 + \frac{15}{4}a^4b^2x^4 + 3a^5bx^2 + a^6 \log(x) + \frac{3}{5}ab^5x^{10} + \frac{b^6x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
+ (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(6a^5b^7+\frac{a^6b^6}{x}+15a^4b^8x+20a^3b^9x^2+15a^2b^{10}x^3+6ab^{11}x^4+b^{12}x^5\right) dx$$
$$= 3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}+a^6\log(x)$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.00

$$a^6\log(x)+3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
+ (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x, x]
```

fricas [A] time = 0.79, size = 66, normalized size = 0.87

$$\frac{1}{12}\,b^6x^{12}+\frac{3}{5}\,ab^5x^{10}+\frac{15}{8}\,a^2b^4x^8+\frac{10}{3}\,a^3b^3x^6+\frac{15}{4}\,a^4b^2x^4+3\,a^5bx^2+a^6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6*log(x)
```

giac [A] time = 0.15, size = 69, normalized size = 0.91

$$\frac{1}{12}\,b^6x^{12}+\frac{3}{5}\,ab^5x^{10}+\frac{15}{8}\,a^2b^4x^8+\frac{10}{3}\,a^3b^3x^6+\frac{15}{4}\,a^4b^2x^4+3\,a^5bx^2+\frac{1}{2}\,a^6\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

maple [A] time = 0.00, size = 67, normalized size = 0.88

$$\frac{b^6x^{12}}{12}+\frac{3a\,b^5x^{10}}{5}+\frac{15a^2b^4x^8}{8}+\frac{10a^3b^3x^6}{3}+\frac{15a^4b^2x^4}{4}+3a^5b\,x^2+a^6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x,x)
```

```
[Out] 3*a^5*b*x^2+15/4*a^4*b^2*x^4+10/3*a^3*b^3*x^6+15/8*a^2*b^4*x^8+3/5*a*b^5*x^
10+1/12*b^6*x^12+a^6*ln(x)
```

maxima [A] time = 1.40, size = 69, normalized size = 0.91

$$\frac{1}{12}\,b^6x^{12}+\frac{3}{5}\,ab^5x^{10}+\frac{15}{8}\,a^2b^4x^8+\frac{10}{3}\,a^3b^3x^6+\frac{15}{4}\,a^4b^2x^4+3\,a^5bx^2+\frac{1}{2}\,a^6\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

mupad [B] time = 0.04, size = 66, normalized size = 0.87

$$a^6\,\ln(x)+\frac{b^6\,x^{12}}{12}+3\,a^5\,b\,x^2+\frac{3\,a\,b^5\,x^{10}}{5}+\frac{15\,a^4\,b^2\,x^4}{4}+\frac{10\,a^3\,b^3\,x^6}{3}+\frac{15\,a^2\,b^4\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x,x)
```

```
[Out] a^6*log(x) + (b^6*x^12)/12 + 3*a^5*b*x^2 + (3*a*b^5*x^10)/5 + (15*a^4*b^2*x
^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
```

sympy [A] time = 0.17, size = 76, normalized size = 1.00

$$a^6 \log(x) + 3a^5bx^2 + \frac{15a^4b^2x^4}{4} + \frac{10a^3b^3x^6}{3} + \frac{15a^2b^4x^8}{8} + \frac{3ab^5x^{10}}{5} + \frac{b^6x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x,x)
```

```
[Out] a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**4/4 + 10*a**3*b**3*x**6/3 + 1
5*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x**12/12
```

3.282 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{7}a^2b^4x^7+4a^3b^3x^5+5a^4b^2x^3+6a^5bx-\frac{a^6}{x}+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7 +
 (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^2}\,dx}{b^6}\\ &= \frac{\int \left(6a^5b^7+\frac{a^6b^6}{x^2}+15a^4b^8x^2+20a^3b^9x^4+15a^2b^{10}x^6+6ab^{11}x^8+b^{12}x^{10}\right)dx}{b^6}\\ &= -\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 72, normalized size = 1.00

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7 +
 (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2, x]
```

fricas [A] time = 0.91, size = 70, normalized size = 0.97

$$\frac{21\,b^6x^{12}+154\,ab^5x^{10}+495\,a^2b^4x^8+924\,a^3b^3x^6+1155\,a^4b^2x^4+1386\,a^5bx^2-231\,a^6}{231\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/231*(21*b^6*x^12 + 154*a*b^5*x^10 + 495*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 1
155*a^4*b^2*x^4 + 1386*a^5*b*x^2 - 231*a^6)/x
```

giac [A] time = 0.16, size = 66, normalized size = 0.92

$$\frac{1}{11}\,b^6x^{11}+\frac{2}{3}\,ab^5x^9+\frac{15}{7}\,a^2b^4x^7+4\,a^3b^3x^5+5\,a^4b^2x^3+6\,a^5bx-\frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b^
2*x^3 + 6*a^5*b*x - a^6/x
```

maple [A] time = 0.00, size = 67, normalized size = 0.93

$$\frac{b^6x^{11}}{11}+\frac{2a\,b^5x^9}{3}+\frac{15a^2b^4x^7}{7}+4a^3b^3x^5+5a^4b^2x^3+6a^5bx-\frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x)
```

```
[Out] -a^6/x+6*a^5*b*x+5*a^4*b^2*x^3+4*a^3*b^3*x^5+15/7*a^2*b^4*x^7+2/3*a*b^5*x^9
+1/11*b^6*x^11
```

maxima [A] time = 1.35, size = 66, normalized size = 0.92

$$\frac{1}{11}\,b^6x^{11}+\frac{2}{3}\,ab^5x^9+\frac{15}{7}\,a^2b^4x^7+4\,a^3b^3x^5+5\,a^4b^2x^3+6\,a^5bx-\frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b^
2*x^3 + 6*a^5*b*x - a^6/x
```

mupad [B] time = 0.03, size = 66, normalized size = 0.92

$$\frac{b^6\,x^{11}}{11}-\frac{a^6}{x}+\frac{2\,a\,b^5\,x^9}{3}+5\,a^4\,b^2\,x^3+4\,a^3\,b^3\,x^5+\frac{15\,a^2\,b^4\,x^7}{7}+6\,a^5\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^2,x)
```

```
[Out] (b^6*x^11)/11 - a^6/x + (2*a*b^5*x^9)/3 + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (
15*a^2*b^4*x^7)/7 + 6*a^5*b*x
```

sympy [A] time = 0.16, size = 70, normalized size = 0.97

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15a^2b^4x^7}{7}+\frac{2ab^5x^9}{3}+\frac{b^6x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**2,x)
```

```
[Out] -a**6/x + 6*a**5*b*x + 5*a**4*b**2*x**3 + 4*a**3*b**3*x**5 + 15*a**2*b**4*x
**7/7 + 2*a*b**5*x**9/3 + b**6*x**11/11
```

3.283 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx$

Optimal. Leaf size=77

$$-\frac{a^6}{2x^2}+6a^5b\log(x)+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{5}{2}a^2b^4x^6+5a^3b^3x^4+\frac{15}{2}a^4b^2x^2+6a^5b\log(x)-\frac{a^6}{2x^2}+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -a^6/(2*x^2) + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3*
a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^3}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^2}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(15a^4b^8+\frac{a^6b^6}{x^2}+\frac{6a^5b^7}{x}+20a^3b^9x+15a^2b^{10}x^2+6ab^{11}x^3+b^{12}x^4\right) dx, x, x\right.}{2b^6}$$
$$= -\frac{a^6}{2x^2}+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}+6a^5b\log(x)$$

Mathematica [A] time = 0.01, size = 77, normalized size = 1.00

$$-\frac{a^6}{2x^2}+6a^5b\log(x)+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -1/2*a^6/x^2 + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3*
a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3, x]
```

fricas [A] time = 0.69, size = 72, normalized size = 0.94

$$\frac{2\,b^6x^{12}+15\,ab^5x^{10}+50\,a^2b^4x^8+100\,a^3b^3x^6+150\,a^4b^2x^4+120\,a^5bx^2\log(x)-10\,a^6}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^6*x^12 + 15*a*b^5*x^10 + 50*a^2*b^4*x^8 + 100*a^3*b^3*x^6 + 150*a
^4*b^2*x^4 + 120*a^5*b*x^2*log(x) - 10*a^6)/x^2
```

giac [A] time = 0.15, size = 79, normalized size = 1.03

$$\frac{1}{10}\,b^{6}x^{10}+\frac{3}{4}\,ab^{5}x^{8}+\frac{5}{2}\,a^{2}b^{4}x^{6}+5\,a^{3}b^{3}x^{4}+\frac{15}{2}\,a^{4}b^{2}x^{2}+3\,a^{5}b\log\left(x^{2}\right)-\frac{6\,a^{5}bx^{2}+a^{6}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4*
b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*(6*a^5*b*x^2 + a^6)/x^2
```

maple [A] time = 0.01, size = 68, normalized size = 0.88

$$\frac{b^{6}x^{10}}{10}+\frac{3a\,b^{5}x^{8}}{4}+\frac{5a^{2}b^{4}x^{6}}{2}+5a^{3}b^{3}x^{4}+\frac{15a^{4}b^{2}x^{2}}{2}+6a^{5}b\ln(x)-\frac{a^{6}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x)
```

```
[Out] -1/2*a^6/x^2+15/2*a^4*b^2*x^2+5*a^3*b^3*x^4+5/2*a^2*b^4*x^6+3/4*a*b^5*x^8+1
/10*b^6*x^10+6*a^5*b*ln(x)
```

maxima [A] time = 1.43, size = 69, normalized size = 0.90

$$\frac{1}{10}\,b^{6}x^{10}+\frac{3}{4}\,ab^{5}x^{8}+\frac{5}{2}\,a^{2}b^{4}x^{6}+5\,a^{3}b^{3}x^{4}+\frac{15}{2}\,a^{4}b^{2}x^{2}+3\,a^{5}b\log\left(x^{2}\right)-\frac{a^{6}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4*
b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*a^6/x^2
```

mupad [B] time = 0.04, size = 67, normalized size = 0.87

$$\frac{b^6\,x^{10}}{10}-\frac{a^6}{2\,x^2}+\frac{3\,a\,b^5\,x^8}{4}+6\,a^5\,b\,\ln(x)+\frac{15\,a^4\,b^2\,x^2}{2}+5\,a^3\,b^3\,x^4+\frac{5\,a^2\,b^4\,x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^3,x)
```

```
[Out] (b^6*x^10)/10 - a^6/(2*x^2) + (3*a*b^5*x^8)/4 + 6*a^5*b*log(x) + (15*a^4*b^
2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2
```

sympy [A] time = 0.20, size = 76, normalized size = 0.99

$$-\frac{a^6}{2x^2} + 6a^5b\log(x) + \frac{15a^4b^2x^2}{2} + 5a^3b^3x^4 + \frac{5a^2b^4x^6}{2} + \frac{3ab^5x^8}{4} + \frac{b^6x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**3,x)
```

```
[Out] -a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**2*x**2/2 + 5*a**3*b**3*x**4 +
 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x**10/10
```

3.284 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4} dx$

Optimal. Leaf size=74

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$3a^2b^4x^5+\frac{20}{3}a^3b^3x^3+15a^4b^2x-\frac{6a^5b}{x}-\frac{a^6}{3x^3}+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -a^6/(3*x^3) - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*
x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4} dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^4} dx}{b^6}\\
&= \frac{\int \left(15a^4b^8+\frac{a^6b^6}{x^4}+\frac{6a^5b^7}{x^2}+20a^3b^9x^2+15a^2b^{10}x^4+6ab^{11}x^6+b^{12}x^8\right) dx}{b^6}\\
&= -\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 74, normalized size = 1.00

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -1/3*a^6/x^3 - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*
x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4, x]
```

fricas [A] time = 0.70, size = 70, normalized size = 0.95

$$\frac{7\,b^6x^{12}+54\,ab^5x^{10}+189\,a^2b^4x^8+420\,a^3b^3x^6+945\,a^4b^2x^4-378\,a^5bx^2-21\,a^6}{63\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^6*x^12 + 54*a*b^5*x^10 + 189*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 945*
a^4*b^2*x^4 - 378*a^5*b*x^2 - 21*a^6)/x^3
```

giac [A] time = 0.18, size = 67, normalized size = 0.91

$$\frac{1}{9}\,b^6x^9+\frac{6}{7}\,ab^5x^7+3\,a^2b^4x^5+\frac{20}{3}\,a^3b^3x^3+15\,a^4b^2x-\frac{18\,a^5bx^2+a^6}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^2
*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

maple [A] time = 0.00, size = 67, normalized size = 0.91

$$\frac{b^6x^9}{9}+\frac{6a\,b^5x^7}{7}+3a^2b^4x^5+\frac{20a^3b^3x^3}{3}+15a^4b^2x-\frac{6a^5b}{x}-\frac{a^6}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x)
```

```
[Out] -1/3*a^6/x^3-6*a^5*b/x+15*a^4*b^2*x+20/3*a^3*b^3*x^3+3*a^2*b^4*x^5+6/7*a*b^
5*x^7+1/9*b^6*x^9
```

maxima [A] time = 1.29, size = 67, normalized size = 0.91

$$\frac{1}{9}\,b^6x^9+\frac{6}{7}\,ab^5x^7+3\,a^2b^4x^5+\frac{20}{3}\,a^3b^3x^3+15\,a^4b^2x-\frac{18\,a^5bx^2+a^6}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^2
*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

mupad [B] time = 0.03, size = 69, normalized size = 0.93

$$\frac{b^6\,x^9}{9}-\frac{\frac{a^6}{3}+6\,b\,a^5\,x^2}{x^3}+15\,a^4\,b^2\,x+\frac{6\,a\,b^5\,x^7}{7}+\frac{20\,a^3\,b^3\,x^3}{3}+3\,a^2\,b^4\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^4,x)
```

```
[Out] (b^6*x^9)/9 - (a^6/3 + 6*a^5*b*x^2)/x^3 + 15*a^4*b^2*x + (6*a*b^5*x^7)/7 +
(20*a^3*b^3*x^3)/3 + 3*a^2*b^4*x^5
```

sympy [A] time = 0.21, size = 75, normalized size = 1.01

$$15a^4b^2x+\frac{20a^3b^3x^3}{3}+3a^2b^4x^5+\frac{6ab^5x^7}{7}+\frac{b^6x^9}{9}+\frac{-a^6-18a^5bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**4,x)
```

```
[Out] 15*a**4*b**2*x + 20*a**3*b**3*x**3/3 + 3*a**2*b**4*x**5 + 6*a*b**5*x**7/7 +
 b**6*x**9/9 + (-a**6 - 18*a**5*b*x**2)/(3*x**3)
```

3.285 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^5}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+15a^4b^2\log(x)+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}$$

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{15}{4}a^2b^4x^4+10a^3b^3x^2+15a^4b^2\log(x)-\frac{3a^5b}{x^2}-\frac{a^6}{4x^4}+ab^5x^6+\frac{b^6x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -a^6/(4*x^4) - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5*
x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^5}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^5}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^3}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(20a^3b^9+\frac{a^6b^6}{x^3}+\frac{6a^5b^7}{x^2}+\frac{15a^4b^8}{x}+15a^2b^{10}x+6ab^{11}x^2+b^{12}x^3\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}+15a^4b^2\log(x)$$

Mathematica [A] time = 0.00, size = 72, normalized size = 1.00

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+15a^4b^2\log(x)+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -1/4*a^6/x^4 - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5*
x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5, x]
```

fricas [A] time = 0.76, size = 71, normalized size = 0.99

$$\frac{b^6x^{12}+8\,ab^5x^{10}+30\,a^2b^4x^8+80\,a^3b^3x^6+120\,a^4b^2x^4\log(x)-24\,a^5bx^2-2\,a^6}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(b^6*x^12 + 8*a*b^5*x^10 + 30*a^2*b^4*x^8 + 80*a^3*b^3*x^6 + 120*a^4*b^
2*x^4*log(x) - 24*a^5*b*x^2 - 2*a^6)/x^4
```

giac [A] time = 0.16, size = 80, normalized size = 1.11

$$\frac{1}{8}\,b^6x^8 + ab^5x^6 + \frac{15}{4}\,a^2b^4x^4 + 10\,a^3b^3x^2 + \frac{15}{2}\,a^4b^2\log\left(x^2\right) - \frac{45\,a^4b^2x^4 + 12\,a^5bx^2 + a^6}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2*
log(x^2) - 1/4*(45*a^4*b^2*x^4 + 12*a^5*b*x^2 + a^6)/x^4
```

maple [A] time = 0.01, size = 67, normalized size = 0.93

$$\frac{b^6x^8}{8} + a\,b^5x^6 + \frac{15a^2b^4x^4}{4} + 10a^3b^3x^2 + 15a^4b^2\ln(x) - \frac{3a^5b}{x^2} - \frac{a^6}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x)
```

```
[Out] -1/4*a^6/x^4-3*a^5*b/x^2+10*a^3*b^3*x^2+15/4*a^2*b^4*x^4+a*b^5*x^6+1/8*b^6*
x^8+15*a^4*b^2*ln(x)
```

maxima [A] time = 1.37, size = 69, normalized size = 0.96

$$\frac{1}{8}\,b^6x^8 + ab^5x^6 + \frac{15}{4}\,a^2b^4x^4 + 10\,a^3b^3x^2 + \frac{15}{2}\,a^4b^2\log\left(x^2\right) - \frac{12\,a^5bx^2 + a^6}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2*
log(x^2) - 1/4*(12*a^5*b*x^2 + a^6)/x^4
```

mupad [B] time = 0.04, size = 69, normalized size = 0.96

$$\frac{b^6\,x^8}{8} - \frac{\frac{a^6}{4} + 3\,b\,a^5\,x^2}{x^4} + a\,b^5\,x^6 + 10\,a^3\,b^3\,x^2 + \frac{15\,a^2\,b^4\,x^4}{4} + 15\,a^4\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^5,x)
```

```
[Out] (b^6*x^8)/8 - (a^6/4 + 3*a^5*b*x^2)/x^4 + a*b^5*x^6 + 10*a^3*b^3*x^2 + (15*
a^2*b^4*x^4)/4 + 15*a^4*b^2*log(x)
```

sympy [A] time = 0.25, size = 73, normalized size = 1.01

$$15a^4b^2\log(x) + 10a^3b^3x^2 + \frac{15a^2b^4x^4}{4} + ab^5x^6 + \frac{b^6x^8}{8} + \frac{-a^6 - 12a^5bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**5,x)
```

```
[Out] 15*a**4*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**6
 + b**6*x**8/8 + (-a**6 - 12*a**5*b*x**2)/(4*x**4)
```

3.286 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^6}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$5a^2b^4x^3+20a^3b^3x-\frac{15a^4b^2}{x}-\frac{2a^5b}{x^3}-\frac{a^6}{5x^5}+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] -a^6/(5*x^5) - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x^
3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^6}\,dx&=\frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^6}\,dx}{b^6}\\&=\frac{\int \left(20a^3b^9+\frac{a^6b^6}{x^6}+\frac{6a^5b^7}{x^4}+\frac{15a^4b^8}{x^2}+15a^2b^{10}x^2+6ab^{11}x^4+b^{12}x^6\right)dx}{b^6}\\&=-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 72, normalized size = 1.00

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] -1/5*a^6/x^5 - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x^
3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6, x]
```

fricas [A] time = 0.78, size = 70, normalized size = 0.97

$$\frac{5\,b^6x^{12}+42\,ab^5x^{10}+175\,a^2b^4x^8+700\,a^3b^3x^6-525\,a^4b^2x^4-70\,a^5bx^2-7\,a^6}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^6*x^12 + 42*a*b^5*x^10 + 175*a^2*b^4*x^8 + 700*a^3*b^3*x^6 - 525*
a^4*b^2*x^4 - 70*a^5*b*x^2 - 7*a^6)/x^5
```

giac [A] time = 0.16, size = 67, normalized size = 0.93

$$\frac{1}{7}\,b^6x^7+\frac{6}{5}\,ab^5x^5+5\,a^2b^4x^3+20\,a^3b^3x-\frac{75\,a^4b^2x^4+10\,a^5bx^2+a^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b^
2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

maple [A] time = 0.01, size = 67, normalized size = 0.93

$$\frac{b^6x^7}{7}+\frac{6a\,b^5x^5}{5}+5a^2b^4x^3+20a^3b^3x-\frac{15a^4b^2}{x}-\frac{2a^5b}{x^3}-\frac{a^6}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x)
```

```
[Out] -1/5*a^6/x^5-2*a^5*b/x^3-15*a^4*b^2/x+20*a^3*b^3*x+5*a^2*b^4*x^3+6/5*a*b^5*
x^5+1/7*b^6*x^7
```

maxima [A] time = 1.31, size = 67, normalized size = 0.93

$$\frac{1}{7}\,b^6x^7+\frac{6}{5}\,ab^5x^5+5\,a^2b^4x^3+20\,a^3b^3x-\frac{75\,a^4b^2x^4+10\,a^5bx^2+a^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b^
2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

mupad [B] time = 0.03, size = 69, normalized size = 0.96

$$\frac{b^6\,x^7}{7}-\frac{\frac{a^6}{5}+2\,a^5\,b\,x^2+15\,a^4\,b^2\,x^4}{x^5}+20\,a^3\,b^3\,x+\frac{6\,a\,b^5\,x^5}{5}+5\,a^2\,b^4\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^6,x)
```

```
[Out] (b^6*x^7)/7 - (a^6/5 + 2*a^5*b*x^2 + 15*a^4*b^2*x^4)/x^5 + 20*a^3*b^3*x + (
6*a*b^5*x^5)/5 + 5*a^2*b^4*x^3
```

sympy [A] time = 0.26, size = 73, normalized size = 1.01

$$20a^3b^3x+5a^2b^4x^3+\frac{6ab^5x^5}{5}+\frac{b^6x^7}{7}+\frac{-a^6-10a^5bx^2-75a^4b^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**6,x)
```

```
[Out] 20*a**3*b**3*x + 5*a**2*b**4*x**3 + 6*a*b**5*x**5/5 + b**6*x**7/7 + (-a**6
- 10*a**5*b*x**2 - 75*a**4*b**2*x**4)/(5*x**5)
```

3.287 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^7}\,dx$

Optimal. Leaf size=79

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+20a^3b^3\log(x)+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{15a^4b^2}{2x^2}+\frac{15}{2}a^2b^4x^2+20a^3b^3\log(x)-\frac{3a^5b}{2x^4}-\frac{a^6}{6x^6}+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -a^6/(6*x^6) - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)/
2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^7}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^4}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(15a^2b^{10}+\frac{a^6b^6}{x^4}+\frac{6a^5b^7}{x^3}+\frac{15a^4b^8}{x^2}+\frac{20a^3b^9}{x}+6ab^{11}x+b^{12}x^2\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}+20a^3b^3\log(x)$$

Mathematica [A] time = 0.01, size = 79, normalized size = 1.00

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+20a^3b^3\log(x)+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -1/6*a^6/x^6 - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)/
2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7, x]
```

fricas [A] time = 0.75, size = 71, normalized size = 0.90

$$\frac{b^6x^{12}+9\,ab^5x^{10}+45\,a^2b^4x^8+120\,a^3b^3x^6\log(x)-45\,a^4b^2x^4-9\,a^5bx^2-a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(b^6*x^12 + 9*a*b^5*x^10 + 45*a^2*b^4*x^8 + 120*a^3*b^3*x^6*log(x) - 45
*a^4*b^2*x^4 - 9*a^5*b*x^2 - a^6)/x^6
```

giac [A] time = 0.15, size = 81, normalized size = 1.03

$$\frac{1}{6}\,b^6x^6+\frac{3}{2}\,ab^5x^4+\frac{15}{2}\,a^2b^4x^2+10\,a^3b^3\log\left(x^2\right)-\frac{110\,a^3b^3x^6+45\,a^4b^2x^4+9\,a^5bx^2+a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6*
(110*a^3*b^3*x^6 + 45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

maple [A] time = 0.01, size = 68, normalized size = 0.86

$$\frac{b^6x^6}{6}+\frac{3a\,b^5x^4}{2}+\frac{15a^2b^4x^2}{2}+20a^3b^3\ln(x)-\frac{15a^4b^2}{2x^2}-\frac{3a^5b}{2x^4}-\frac{a^6}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x)
```

```
[Out] -1/6*a^6/x^6-3/2*a^5*b/x^4-15/2*a^4*b^2/x^2+15/2*a^2*b^4*x^2+3/2*a*b^5*x^4+
1/6*b^6*x^6+20*a^3*b^3*ln(x)
```

maxima [A] time = 1.39, size = 70, normalized size = 0.89

$$\frac{1}{6}\,b^6x^6+\frac{3}{2}\,ab^5x^4+\frac{15}{2}\,a^2b^4x^2+10\,a^3b^3\log\left(x^2\right)-\frac{45\,a^4b^2x^4+9\,a^5bx^2+a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6*
(45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

mupad [B] time = 4.34, size = 70, normalized size = 0.89

$$\frac{b^6\,x^6}{6}-\frac{\frac{a^6}{6}+\frac{3\,a^5\,b\,x^2}{2}+\frac{15\,a^4\,b^2\,x^4}{2}}{x^6}+\frac{3\,a\,b^5\,x^4}{2}+\frac{15\,a^2\,b^4\,x^2}{2}+20\,a^3\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^7,x)
```

```
[Out] (b^6*x^6)/6 - (a^6/6 + (3*a^5*b*x^2)/2 + (15*a^4*b^2*x^4)/2)/x^6 + (3*a*b^5
*x^4)/2 + (15*a^2*b^4*x^2)/2 + 20*a^3*b^3*log(x)
```

sympy [A] time = 0.32, size = 76, normalized size = 0.96

$$20a^3b^3\log(x) + \frac{15a^2b^4x^2}{2} + \frac{3ab^5x^4}{2} + \frac{b^6x^6}{6} + \frac{-a^6 - 9a^5bx^2 - 45a^4b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**7,x)
```

```
[Out] 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2 + b**6*x**6/6 +
 (-a**6 - 9*a**5*b*x**2 - 45*a**4*b**2*x**4)/(6*x**6)
```

3.288 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^8}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x-\frac{6a^5b}{5x^5}-\frac{a^6}{7x^7}+2ab^5x^3+\frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -a^6/(7*x^7) - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a^
2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^8}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^8}\,dx}{b^6}\\ &= \frac{\int \left(15a^2b^{10}+\frac{a^6b^6}{x^8}+\frac{6a^5b^7}{x^6}+\frac{15a^4b^8}{x^4}+\frac{20a^3b^9}{x^2}+6ab^{11}x^2+b^{12}x^4\right)dx}{b^6}\\ &= -\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 72, normalized size = 1.00

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -1/7*a^6/x^7 - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a^
2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8, x]
```

fricas [A] time = 0.75, size = 70, normalized size = 0.97

$$\frac{7\,b^6x^{12}+70\,ab^5x^{10}+525\,a^2b^4x^8-700\,a^3b^3x^6-175\,a^4b^2x^4-42\,a^5bx^2-5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(7*b^6*x^12 + 70*a*b^5*x^10 + 525*a^2*b^4*x^8 - 700*a^3*b^3*x^6 - 175*
a^4*b^2*x^4 - 42*a^5*b*x^2 - 5*a^6)/x^7
```

giac [A] time = 0.15, size = 69, normalized size = 0.96

$$\frac{1}{5}\,b^6x^5+2\,ab^5x^3+15\,a^2b^4x-\frac{700\,a^3b^3x^6+175\,a^4b^2x^4+42\,a^5bx^2+5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4*
b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

maple [A] time = 0.01, size = 67, normalized size = 0.93

$$\frac{b^6x^5}{5}+2a\,b^5x^3+15a^2b^4x-\frac{20a^3b^3}{x}-\frac{5a^4b^2}{x^3}-\frac{6a^5b}{5x^5}-\frac{a^6}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x)
```

```
[Out] -1/7*a^6/x^7-6/5*a^5*b/x^5-5*a^4*b^2/x^3-20*a^3*b^3/x+15*a^2*b^4*x+2*a*b^5*
x^3+1/5*b^6*x^5
```

maxima [A] time = 1.38, size = 69, normalized size = 0.96

$$\frac{1}{5}\,b^6x^5+2\,ab^5x^3+15\,a^2b^4x-\frac{700\,a^3b^3x^6+175\,a^4b^2x^4+42\,a^5bx^2+5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4*
b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

mupad [B] time = 0.05, size = 69, normalized size = 0.96

$$\frac{b^6\,x^5}{5}-\frac{\frac{a^6}{7}+\frac{6\,a^5\,b\,x^2}{5}+5\,a^4\,b^2\,x^4+20\,a^3\,b^3\,x^6}{x^7}+15\,a^2\,b^4\,x+2\,a\,b^5\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^8,x)
```

```
[Out] (b^6*x^5)/5 - (a^6/7 + (6*a^5*b*x^2)/5 + 5*a^4*b^2*x^4 + 20*a^3*b^3*x^6)/x^
7 + 15*a^2*b^4*x + 2*a*b^5*x^3
```

sympy [A] time = 0.34, size = 73, normalized size = 1.01

$$15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}+\frac{-5a^6-42a^5bx^2-175a^4b^2x^4-700a^3b^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**8,x)
```

```
[Out] 15*a**2*b**4*x + 2*a*b**5*x**3 + b**6*x**5/5 + (-5*a**6 - 42*a**5*b*x**2 -
175*a**4*b**2*x**4 - 700*a**3*b**3*x**6)/(35*x**7)
```

3.289 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9} dx$

Optimal. Leaf size=73

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)+3ab^5x^2+\frac{b^6x^4}{4}$$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)-\frac{a^5b}{x^6}-\frac{a^6}{8x^8}+3ab^5x^2+\frac{b^6x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -a^6/(8*x^8) - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a*
b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^9}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^5}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(6ab^{11}+\frac{a^6b^6}{x^5}+\frac{6a^5b^7}{x^4}+\frac{15a^4b^8}{x^3}+\frac{20a^3b^9}{x^2}+\frac{15a^2b^{10}}{x}+b^{12}x\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+3ab^5x^2+\frac{b^6x^4}{4}+15a^2b^4\log(x)$$

Mathematica [A] time = 0.01, size = 73, normalized size = 1.00

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)+3ab^5x^2+\frac{b^6x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -1/8*a^6/x^8 - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a*
b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9, x]
```

fricas [A] time = 0.78, size = 72, normalized size = 0.99

$$\frac{2\,b^6x^{12}+24\,ab^5x^{10}+120\,a^2b^4x^8\log(x)-80\,a^3b^3x^6-30\,a^4b^2x^4-8\,a^5bx^2-a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="fricas")
```

[Out] 1/8*(2*b^6*x^12 + 24*a*b^5*x^10 + 120*a^2*b^4*x^8*log(x) - 80*a^3*b^3*x^6 - 30*a^4*b^2*x^4 - 8*a^5*b*x^2 - a^6)/x^8

giac [A] time = 0.16, size = 81, normalized size = 1.11

$$\frac{1}{4}\,b^6x^4 + 3\,ab^5x^2 + \frac{15}{2}\,a^2b^4\log\left(x^2\right) - \frac{125\,a^2b^4x^8 + 80\,a^3b^3x^6 + 30\,a^4b^2x^4 + 8\,a^5bx^2 + a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="giac")

[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(125*a^2*b^4*x^8 + 80*a^3*b^3*x^6 + 30*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8

maple [A] time = 0.01, size = 68, normalized size = 0.93

$$\frac{b^6x^4}{4} + 3a\,b^5x^2 + 15a^2b^4\ln(x) - \frac{10a^3b^3}{x^2} - \frac{15a^4b^2}{4x^4} - \frac{a^5b}{x^6} - \frac{a^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x)

[Out] -1/8*a^6/x^8-a^5*b/x^6-15/4*a^4*b^2/x^4-10*a^3*b^3/x^2+3*a*b^5*x^2+1/4*b^6*x^4+15*a^2*b^4*ln(x)

maxima [A] time = 1.31, size = 70, normalized size = 0.96

$$\frac{1}{4}\,b^6x^4 + 3\,ab^5x^2 + \frac{15}{2}\,a^2b^4\log\left(x^2\right) - \frac{80\,a^3b^3x^6 + 30\,a^4b^2x^4 + 8\,a^5bx^2 + a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="maxima")

[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(80*a^3*b^3*x^6 + 30*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8

mupad [B] time = 0.05, size = 69, normalized size = 0.95

$$\frac{b^6\,x^4}{4} - \frac{\frac{a^6}{8} + a^5\,b\,x^2 + \frac{15\,a^4\,b^2\,x^4}{4} + 10\,a^3\,b^3\,x^6}{x^8} + 3\,a\,b^5\,x^2 + 15\,a^2\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^9,x)

```
[Out] (b^6*x^4)/4 - (a^6/8 + a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + 10*a^3*b^3*x^6)/x^8
 + 3*a*b^5*x^2 + 15*a^2*b^4*log(x)
```

sympy **[A]** time = 0.42, size = 73, normalized size = 1.00

$$15a^2b^4\log(x) + 3ab^5x^2 + \frac{b^6x^4}{4} + \frac{-a^6 - 8a^5bx^2 - 30a^4b^2x^4 - 80a^3b^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**9,x)
```

```
[Out] 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6*x**4/4 + (-a**6 - 8*a**5*b*x**2
- 30*a**4*b**2*x**4 - 80*a**3*b**3*x**6)/(8*x**8)
```

3.290 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{10}}\,dx$

Optimal. Leaf size=74

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}-\frac{6a^5b}{7x^7}-\frac{a^6}{9x^9}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -a^6/(9*x^9) - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3) -
 (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{10}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{10}}\,dx}{b^6}\\
&= \frac{\int \left(6ab^{11}+\frac{a^6b^6}{x^{10}}+\frac{6a^5b^7}{x^8}+\frac{15a^4b^8}{x^6}+\frac{20a^3b^9}{x^4}+\frac{15a^2b^{10}}{x^2}+b^{12}x^2\right)dx}{b^6}\\
&= -\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 74, normalized size = 1.00

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -1/9*a^6/x^9 - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3) -
 (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10, x]
```

fricas [A] time = 0.82, size = 70, normalized size = 0.95

$$\frac{21\,b^6x^{12}+378\,ab^5x^{10}-945\,a^2b^4x^8-420\,a^3b^3x^6-189\,a^4b^2x^4-54\,a^5bx^2-7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(21*b^6*x^12 + 378*a*b^5*x^10 - 945*a^2*b^4*x^8 - 420*a^3*b^3*x^6 - 18
9*a^4*b^2*x^4 - 54*a^5*b*x^2 - 7*a^6)/x^9
```

giac [A] time = 0.24, size = 69, normalized size = 0.93

$$\frac{1}{3}\,b^6x^3+6\,ab^5x-\frac{945\,a^2b^4x^8+420\,a^3b^3x^6+189\,a^4b^2x^4+54\,a^5bx^2+7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^4
*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

maple [A] time = 0.01, size = 67, normalized size = 0.91

$$\frac{b^6x^3}{3}+6a\,b^5x-\frac{15a^2b^4}{x}-\frac{20a^3b^3}{3x^3}-\frac{3a^4b^2}{x^5}-\frac{6a^5b}{7x^7}-\frac{a^6}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x)
```

```
[Out] -1/9*a^6/x^9-6/7*a^5*b/x^7-3*a^4*b^2/x^5-20/3*a^3*b^3/x^3-15*a^2*b^4/x+6*a*
b^5*x+1/3*b^6*x^3
```

maxima [A] time = 1.41, size = 69, normalized size = 0.93

$$\frac{1}{3}\,b^6x^3+6\,ab^5x-\frac{945\,a^2b^4x^8+420\,a^3b^3x^6+189\,a^4b^2x^4+54\,a^5bx^2+7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^4
*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

mupad [B] time = 0.05, size = 70, normalized size = 0.95

$$-\frac{\frac{a^6}{9}+\frac{6\,a^5\,b\,x^2}{7}+3\,a^4\,b^2\,x^4+\frac{20\,a^3\,b^3\,x^6}{3}+15\,a^2\,b^4\,x^8-6\,a\,b^5\,x^{10}-\frac{b^6\,x^{12}}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^10,x)
```

```
[Out] -(a^6/9 - (b^6*x^12)/3 + (6*a^5*b*x^2)/7 - 6*a*b^5*x^10 + 3*a^4*b^2*x^4 + (
20*a^3*b^3*x^6)/3 + 15*a^2*b^4*x^8)/x^9
```

sympy [A] time = 0.44, size = 73, normalized size = 0.99

$$6ab^5x+\frac{b^6x^3}{3}+\frac{-7a^6-54a^5bx^2-189a^4b^2x^4-420a^3b^3x^6-945a^2b^4x^8}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**10,x)
```

```
[Out] 6*a*b**5*x + b**6*x**3/3 + (-7*a**6 - 54*a**5*b*x**2 - 189*a**4*b**2*x**4 -
 420*a**3*b**3*x**6 - 945*a**2*b**4*x**8)/(63*x**9)
```

3.291 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx$

Optimal. Leaf size=77

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}-\frac{3a^5b}{4x^8}-\frac{a^6}{10x^{10}}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -a^6/(10*x^10) - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
- (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{11}}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^6}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(b^{12}+\frac{a^6b^6}{x^6}+\frac{6a^5b^7}{x^5}+\frac{15a^4b^8}{x^4}+\frac{20a^3b^9}{x^3}+\frac{15a^2b^{10}}{x^2}+\frac{6ab^{11}}{x}\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+\frac{b^6x^2}{2}+6ab^5\log(x)$$

Mathematica [A] time = 0.01, size = 77, normalized size = 1.00

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -1/10*a^6/x^10 - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
- (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11, x]
```

fricas [A] time = 0.53, size = 72, normalized size = 0.94

$$\frac{10\,b^6x^{12}+120\,ab^5x^{10}\log(x)-150\,a^2b^4x^8-100\,a^3b^3x^6-50\,a^4b^2x^4-15\,a^5bx^2-2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 120*a*b^5*x^10*log(x) - 150*a^2*b^4*x^8 - 100*a^3*b^3*x
^6 - 50*a^4*b^2*x^4 - 15*a^5*b*x^2 - 2*a^6)/x^10
```

giac [A] time = 0.15, size = 81, normalized size = 1.05

$$\frac{1}{2}\,b^{6}x^{2}+3\,ab^{5}\log\left(x^{2}\right)-\frac{137\,ab^{5}x^{10}+150\,a^{2}b^{4}x^{8}+100\,a^{3}b^{3}x^{6}+50\,a^{4}b^{2}x^{4}+15\,a^{5}bx^{2}+2\,a^{6}}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(137*a*b^5*x^10 + 150*a^2*b^4*x^8 + 1
00*a^3*b^3*x^6 + 50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

maple [A] time = 0.01, size = 68, normalized size = 0.88

$$\frac{b^{6}x^{2}}{2}+6a\,b^{5}\ln(x)-\frac{15a^{2}b^{4}}{2x^{2}}-\frac{5a^{3}b^{3}}{x^{4}}-\frac{5a^{4}b^{2}}{2x^{6}}-\frac{3a^{5}b}{4x^{8}}-\frac{a^{6}}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x)
```

```
[Out] -1/10*a^6/x^10-3/4*a^5*b/x^8-5/2*a^4*b^2/x^6-5*a^3*b^3/x^4-15/2*a^2*b^4/x^2
+1/2*b^6*x^2+6*a*b^5*ln(x)
```

maxima [A] time = 1.38, size = 72, normalized size = 0.94

$$\frac{1}{2}\,b^{6}x^{2}+3\,ab^{5}\log\left(x^{2}\right)-\frac{150\,a^{2}b^{4}x^{8}+100\,a^{3}b^{3}x^{6}+50\,a^{4}b^{2}x^{4}+15\,a^{5}bx^{2}+2\,a^{6}}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(150*a^2*b^4*x^8 + 100*a^3*b^3*x^6 +
50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

mupad [B] time = 4.40, size = 70, normalized size = 0.91

$$\frac{b^{6}\,x^{2}}{2}-\frac{\frac{a^{6}}{10}+\frac{3\,a^{5}\,b\,x^{2}}{4}+\frac{5\,a^{4}\,b^{2}\,x^{4}}{2}+5\,a^{3}\,b^{3}\,x^{6}+\frac{15\,a^{2}\,b^{4}\,x^{8}}{2}}{x^{10}}+6\,a\,b^{5}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^11,x)
```

```
[Out] (b^6*x^2)/2 - (a^6/10 + (3*a^5*b*x^2)/4 + (5*a^4*b^2*x^4)/2 + 5*a^3*b^3*x^6
 + (15*a^2*b^4*x^8)/2)/x^10 + 6*a*b^5*log(x)
```

sympy [A] time = 0.63, size = 75, normalized size = 0.97

$$6ab^5\log(x) + \frac{b^6x^2}{2} + \frac{-2a^6 - 15a^5bx^2 - 50a^4b^2x^4 - 100a^3b^3x^6 - 150a^2b^4x^8}{20x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**11,x)
```

```
[Out] 6*a*b**5*log(x) + b**6*x**2/2 + (-2*a**6 - 15*a**5*b*x**2 - 50*a**4*b**2*x*
*4 - 100*a**3*b**3*x**6 - 150*a**2*b**4*x**8)/(20*x**10)
```

3.292 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{12}}\,dx$

Optimal. Leaf size=71

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{2a^5b}{3x^9}-\frac{a^6}{11x^{11}}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -a^6/(11*x^11) - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^5
 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{12}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{12}}\,dx}{b^6}\\
&= \frac{\int \left(b^{12}+\frac{a^6b^6}{x^{12}}+\frac{6a^5b^7}{x^{10}}+\frac{15a^4b^8}{x^8}+\frac{20a^3b^9}{x^6}+\frac{15a^2b^{10}}{x^4}+\frac{6ab^{11}}{x^2}\right)dx}{b^6}\\
&= -\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x
\end{aligned}$$

Mathematica [A] time = 0.01, size = 71, normalized size = 1.00

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -1/11*a^6/x^11 - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^5
 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12, x]
```

fricas [A] time = 0.63, size = 70, normalized size = 0.99

$$\frac{231\,b^6x^{12}-1386\,ab^5x^{10}-1155\,a^2b^4x^8-924\,a^3b^3x^6-495\,a^4b^2x^4-154\,a^5bx^2-21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/231*(231*b^6*x^12 - 1386*a*b^5*x^10 - 1155*a^2*b^4*x^8 - 924*a^3*b^3*x^6
- 495*a^4*b^2*x^4 - 154*a^5*b*x^2 - 21*a^6)/x^11
```

giac [A] time = 0.15, size = 68, normalized size = 0.96

$$b^6x-\frac{1386\,ab^5x^{10}+1155\,a^2b^4x^8+924\,a^3b^3x^6+495\,a^4b^2x^4+154\,a^5bx^2+21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="giac")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*a
^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

maple [A] time = 0.01, size = 66, normalized size = 0.93

$$b^6x-\frac{6a\,b^5}{x}-\frac{5a^2b^4}{x^3}-\frac{4a^3b^3}{x^5}-\frac{15a^4b^2}{7x^7}-\frac{2a^5b}{3x^9}-\frac{a^6}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x)
```

```
[Out] -1/11*a^6/x^11-2/3*a^5*b/x^9-15/7*a^4*b^2/x^7-4*a^3*b^3/x^5-5*a^2*b^4/x^3-6
*a*b^5/x+b^6*x
```

maxima [A] time = 1.31, size = 68, normalized size = 0.96

$$b^6x-\frac{1386\,ab^5x^{10}+1155\,a^2b^4x^8+924\,a^3b^3x^6+495\,a^4b^2x^4+154\,a^5bx^2+21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*a
^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

mupad [B] time = 4.30, size = 68, normalized size = 0.96

$$b^6\,x-\frac{\frac{a^6}{11}+\frac{2\,a^5\,b\,x^2}{3}+\frac{15\,a^4\,b^2\,x^4}{7}+4\,a^3\,b^3\,x^6+5\,a^2\,b^4\,x^8+6\,a\,b^5\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^12,x)
```

```
[Out] b^6*x - (a^6/11 + (2*a^5*b*x^2)/3 + 6*a*b^5*x^10 + (15*a^4*b^2*x^4)/7 + 4*a
^3*b^3*x^6 + 5*a^2*b^4*x^8)/x^11
```

sympy [A] time = 0.52, size = 71, normalized size = 1.00

$$b^6x+\frac{-21a^6-154a^5bx^2-495a^4b^2x^4-924a^3b^3x^6-1155a^2b^4x^8-1386ab^5x^{10}}{231x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**12,x)
```

```
[Out] b**6*x + (-21*a**6 - 154*a**5*b*x**2 - 495*a**4*b**2*x**4 - 924*a**3*b**3*x
**6 - 1155*a**2*b**4*x**8 - 1386*a*b**5*x**10)/(231*x**11)
```

3.293 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{13}}\,dx$

Optimal. Leaf size=76

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3a^5b}{5x^{10}}-\frac{a^6}{12x^{12}}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] -a^6/(12*x^12) - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/(
3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{13}}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^7}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^6b^6}{x^7}+\frac{6a^5b^7}{x^6}+\frac{15a^4b^8}{x^5}+\frac{20a^3b^9}{x^4}+\frac{15a^2b^{10}}{x^3}+\frac{6ab^{11}}{x^2}+\frac{b^{12}}{x}\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.00

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] -1/12*a^6/x^12 - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/(
3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13, x]
```

fricas [A] time = 0.77, size = 72, normalized size = 0.95

$$\frac{120\,b^6x^{12}\log(x)-360\,ab^5x^{10}-450\,a^2b^4x^8-400\,a^3b^3x^6-225\,a^4b^2x^4-72\,a^5bx^2-10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^6*x^12*log(x) - 360*a*b^5*x^10 - 450*a^2*b^4*x^8 - 400*a^3*b^3
*x^6 - 225*a^4*b^2*x^4 - 72*a^5*b*x^2 - 10*a^6)/x^12
```

giac [A] time = 0.15, size = 80, normalized size = 1.05

$$\frac{1}{2}\,b^6\log\left(x^2\right)-\frac{147\,b^6x^{12}+360\,ab^5x^{10}+450\,a^2b^4x^8+400\,a^3b^3x^6+225\,a^4b^2x^4+72\,a^5bx^2+10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*b^6*log(x^2) - 1/120*(147*b^6*x^12 + 360*a*b^5*x^10 + 450*a^2*b^4*x^8 +
 400*a^3*b^3*x^6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

maple [A] time = 0.01, size = 67, normalized size = 0.88

$$b^6\ln(x)-\frac{3a\,b^5}{x^2}-\frac{15a^2b^4}{4x^4}-\frac{10a^3b^3}{3x^6}-\frac{15a^4b^2}{8x^8}-\frac{3a^5b}{5x^{10}}-\frac{a^6}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x)
```

```
[Out] -1/12*a^6/x^12-3/5*a^5*b/x^10-15/8*a^4*b^2/x^8-10/3*a^3*b^3/x^6-15/4*a^2*b^
4/x^4-3*a*b^5/x^2+b^6*ln(x)
```

maxima [A] time = 1.40, size = 72, normalized size = 0.95

$$\frac{1}{2}\,b^6\log\left(x^2\right)-\frac{360\,ab^5x^{10}+450\,a^2b^4x^8+400\,a^3b^3x^6+225\,a^4b^2x^4+72\,a^5bx^2+10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*log(x^2) - 1/120*(360*a*b^5*x^10 + 450*a^2*b^4*x^8 + 400*a^3*b^3*x^
6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

mupad [B] time = 0.07, size = 69, normalized size = 0.91

$$b^6\,\ln(x)-\frac{\frac{a^6}{12}+\frac{3\,a^5\,b\,x^2}{5}+\frac{15\,a^4\,b^2\,x^4}{8}+\frac{10\,a^3\,b^3\,x^6}{3}+\frac{15\,a^2\,b^4\,x^8}{4}+3\,a\,b^5\,x^{10}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^13,x)
```

```
[Out] b^6*log(x) - (a^6/12 + (3*a^5*b*x^2)/5 + 3*a*b^5*x^10 + (15*a^4*b^2*x^4)/8
+ (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/4)/x^12
```

sympy [A] time = 0.62, size = 73, normalized size = 0.96

$$b^6 \log(x) + \frac{-10a^6 - 72a^5bx^2 - 225a^4b^2x^4 - 400a^3b^3x^6 - 450a^2b^4x^8 - 360ab^5x^{10}}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**13,x)
```

```
[Out] b**6*log(x) + (-10*a**6 - 72*a**5*b*x**2 - 225*a**4*b**2*x**4 - 400*a**3*b*
*3*x**6 - 450*a**2*b**4*x**8 - 360*a*b**5*x**10)/(120*x**12)
```

3.294 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx$

Optimal. Leaf size=76

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{6a^5b}{11x^{11}}-\frac{a^6}{13x^{13}}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -a^6/(13*x^13) - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/(
7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{14}}\,dx}{b^6}\\
&= \frac{\int \left(\frac{a^6b^6}{x^{14}}+\frac{6a^5b^7}{x^{12}}+\frac{15a^4b^8}{x^{10}}+\frac{20a^3b^9}{x^8}+\frac{15a^2b^{10}}{x^6}+\frac{6ab^{11}}{x^4}+\frac{b^{12}}{x^2}\right)dx}{b^6}\\
&= -\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 76, normalized size = 1.00

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -1/13*a^6/x^13 - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/(
7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14, x]
```

fricas [A] time = 0.80, size = 70, normalized size = 0.92

$$-\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

giac [A] time = 0.18, size = 70, normalized size = 0.92

$$-\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="giac")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

maple [A] time = 0.00, size = 69, normalized size = 0.91

$$-\frac{b^{6}}{x}-\frac{2 a\,b^{5}}{x^{3}}-\frac{3 a^{2} b^{4}}{x^{5}}-\frac{20 a^{3} b^{3}}{7 x^{7}}-\frac{5 a^{4} b^{2}}{3 x^{9}}-\frac{6 a^{5} b}{11 x^{11}}-\frac{a^{6}}{13 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x)
```

```
[Out] -1/13*a^6/x^13-6/11*a^5*b/x^11-5/3*a^4*b^2/x^9-20/7*a^3*b^3/x^7-3*a^2*b^4/x
^5-2*a*b^5/x^3-b^6/x
```

maxima [A] time = 1.34, size = 70, normalized size = 0.92

$$-\frac{3003\,b^{6}x^{12}+6006\,ab^{5}x^{10}+9009\,a^{2}b^{4}x^{8}+8580\,a^{3}b^{3}x^{6}+5005\,a^{4}b^{2}x^{4}+1638\,a^{5}bx^{2}+231\,a^{6}}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

mupad [B] time = 0.05, size = 69, normalized size = 0.91

$$-\frac{\frac{a^6}{13}+\frac{6\,a^5\,b\,x^2}{11}+\frac{5\,a^4\,b^2\,x^4}{3}+\frac{20\,a^3\,b^3\,x^6}{7}+3\,a^2\,b^4\,x^8+2\,a\,b^5\,x^{10}+b^6\,x^{12}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^14,x)
```

```
[Out] -(a^6/13 + b^6*x^12 + (6*a^5*b*x^2)/11 + 2*a*b^5*x^10 + (5*a^4*b^2*x^4)/3 +
 (20*a^3*b^3*x^6)/7 + 3*a^2*b^4*x^8)/x^13
```

sympy [A] time = 0.60, size = 75, normalized size = 0.99

$$\frac{-231a^{6}-1638a^{5}bx^{2}-5005a^{4}b^{2}x^{4}-8580a^{3}b^{3}x^{6}-9009a^{2}b^{4}x^{8}-6006ab^{5}x^{10}-3003b^{6}x^{12}}{3003x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**14,x)
```

```
[Out] (-231*a**6 - 1638*a**5*b*x**2 - 5005*a**4*b**2*x**4 - 8580*a**3*b**3*x**6 -
 9009*a**2*b**4*x**8 - 6006*a*b**5*x**10 - 3003*b**6*x**12)/(3003*x**13)
```

3.295 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}} dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^7}{14ax^{14}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$-\frac{(a+bx^2)^7}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] -(a + b*x^2)^7/(14*a*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}} dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^{15}} dx}{b^6}\\ &= -\frac{(a+bx^2)^7}{14ax^{14}}\end{aligned}$$

Mathematica [B] time = 0.01, size = 82, normalized size = 4.32

$$-\frac{a^6}{14x^{14}}-\frac{a^5b}{2x^{12}}-\frac{3a^4b^2}{2x^{10}}-\frac{5a^3b^3}{2x^8}-\frac{5a^2b^4}{2x^6}-\frac{3ab^5}{2x^4}-\frac{b^6}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] -1/14*a^6/x^14 - (a^5*b)/(2*x^12) - (3*a^4*b^2)/(2*x^10) - (5*a^3*b^3)/(2*x
^8) - (5*a^2*b^4)/(2*x^6) - (3*a*b^5)/(2*x^4) - b^6/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15, x]
```

fricas [B] time = 0.81, size = 68, normalized size = 3.58

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

giac [B] time = 0.16, size = 68, normalized size = 3.58

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="giac")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

maple [B] time = 0.01, size = 69, normalized size = 3.63

$$-\frac{b^6}{2x^2}-\frac{3a\,b^5}{2x^4}-\frac{5a^2b^4}{2x^6}-\frac{5a^3b^3}{2x^8}-\frac{3a^4b^2}{2x^{10}}-\frac{a^5b}{2x^{12}}-\frac{a^6}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x)
```

```
[Out] -1/2*b^6/x^2-3/2*a^4*b^2/x^10-1/14*a^6/x^14-1/2*a^5*b/x^12-5/2*a^2*b^4/x^6-
5/2*a^3*b^3/x^8-3/2*a*b^5/x^4
```

maxima [B] time = 1.38, size = 68, normalized size = 3.58

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

mupad [B] time = 4.36, size = 70, normalized size = 3.68

$$-\frac{\frac{a^6}{14}+\frac{a^5\,b\,x^2}{2}+\frac{3\,a^4\,b^2\,x^4}{2}+\frac{5\,a^3\,b^3\,x^6}{2}+\frac{5\,a^2\,b^4\,x^8}{2}+\frac{3\,a\,b^5\,x^{10}}{2}+\frac{b^6\,x^{12}}{2}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^15,x)
```

```
[Out] -(a^6/14 + (b^6*x^12)/2 + (a^5*b*x^2)/2 + (3*a*b^5*x^10)/2 + (3*a^4*b^2*x^4
)/2 + (5*a^3*b^3*x^6)/2 + (5*a^2*b^4*x^8)/2)/x^14
```

sympy [B] time = 0.63, size = 73, normalized size = 3.84

$$\frac{-a^6-7a^5bx^2-21a^4b^2x^4-35a^3b^3x^6-35a^2b^4x^8-21ab^5x^{10}-7b^6x^{12}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**15,x)
```

```
[Out] (-a**6 - 7*a**5*b*x**2 - 21*a**4*b**2*x**4 - 35*a**3*b**3*x**6 - 35*a**2*b*
*4*x**8 - 21*a*b**5*x**10 - 7*b**6*x**12)/(14*x**14)
```

3.296 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{16}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6a^5b}{13x^{13}}-\frac{a^6}{15x^{15}}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -a^6/(15*x^15) - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^3
)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{16}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{16}}\,dx}{b^6}\\
&= \frac{\int \left(\frac{a^6b^6}{x^{16}}+\frac{6a^5b^7}{x^{14}}+\frac{15a^4b^8}{x^{12}}+\frac{20a^3b^9}{x^{10}}+\frac{15a^2b^{10}}{x^8}+\frac{6ab^{11}}{x^6}+\frac{b^{12}}{x^4}\right)dx}{b^6}\\
&= -\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -1/15*a^6/x^15 - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^3
)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16, x]
```

fricas [A] time = 1.07, size = 70, normalized size = 0.85

$$-\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

giac [A] time = 0.15, size = 70, normalized size = 0.85

$$-\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

maple [A] time = 0.00, size = 69, normalized size = 0.84

$$-\frac{b^{6}}{3x^{3}}-\frac{6a\,b^{5}}{5x^{5}}-\frac{15a^{2}b^{4}}{7x^{7}}-\frac{20a^{3}b^{3}}{9x^{9}}-\frac{15a^{4}b^{2}}{11x^{11}}-\frac{6a^{5}b}{13x^{13}}-\frac{a^{6}}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x)
```

```
[Out] -1/15*a^6/x^15-6/13*a^5*b/x^13-15/11*a^4*b^2/x^11-20/9*a^3*b^3/x^9-15/7*a^2
*b^4/x^7-6/5*a*b^5/x^5-1/3*b^6/x^3
```

maxima [A] time = 1.37, size = 70, normalized size = 0.85

$$-\frac{15015\,b^{6}x^{12}+54054\,ab^{5}x^{10}+96525\,a^{2}b^{4}x^{8}+100100\,a^{3}b^{3}x^{6}+61425\,a^{4}b^{2}x^{4}+20790\,a^{5}bx^{2}+3003\,a^{6}}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

mupad [B] time = 0.05, size = 70, normalized size = 0.85

$$-\frac{\frac{a^{6}}{15}+\frac{6\,a^{5}\,b\,x^{2}}{13}+\frac{15\,a^{4}\,b^{2}\,x^{4}}{11}+\frac{20\,a^{3}\,b^{3}\,x^{6}}{9}+\frac{15\,a^{2}\,b^{4}\,x^{8}}{7}+\frac{6\,a\,b^{5}\,x^{10}}{5}+\frac{b^{6}\,x^{12}}{3}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^16,x)
```

```
[Out] -(a^6/15 + (b^6*x^12)/3 + (6*a^5*b*x^2)/13 + (6*a*b^5*x^10)/5 + (15*a^4*b^2
*x^4)/11 + (20*a^3*b^3*x^6)/9 + (15*a^2*b^4*x^8)/7)/x^15
```

sympy [A] time = 0.62, size = 75, normalized size = 0.91

$$\frac{-3003a^{6}-20790a^{5}bx^{2}-61425a^{4}b^{2}x^{4}-100100a^{3}b^{3}x^{6}-96525a^{2}b^{4}x^{8}-54054ab^{5}x^{10}-15015b^{6}x^{12}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**16,x)
```

```
[Out] (-3003*a**6 - 20790*a**5*b*x**2 - 61425*a**4*b**2*x**4 - 100100*a**3*b**3*x
**6 - 96525*a**2*b**4*x**8 - 54054*a*b**5*x**10 - 15015*b**6*x**12)/(45045*
x**15)
```

3.297 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{17}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^7}{112a^2x^{14}}-\frac{\left(a+bx^2\right)^7}{16ax^{16}}$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{b\left(a+bx^2\right)^7}{112a^2x^{14}}-\frac{\left(a+bx^2\right)^7}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -(a + b*x^2)^7/(16*a*x^16) + (b*(a + b*x^2)^7)/(112*a^2*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{17}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{17}}\,dx}{b^6}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^9}\,dx, x, x^2\right)}{2b^6}$$
$$= -\frac{\left(a+bx^2\right)^7}{16ax^{16}} - \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^8}\,dx, x, x^2\right)}{16ab^5}$$
$$= -\frac{\left(a+bx^2\right)^7}{16ax^{16}} + \frac{b\left(a+bx^2\right)^7}{112a^2x^{14}}$$

Mathematica [A] time = 0.00, size = 78, normalized size = 1.95

$$-\frac{a^6}{16x^{16}} - \frac{3a^5b}{7x^{14}} - \frac{5a^4b^2}{4x^{12}} - \frac{2a^3b^3}{x^{10}} - \frac{15a^2b^4}{8x^8} - \frac{ab^5}{x^6} - \frac{b^6}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -1/16*a^6/x^16 - (3*a^5*b)/(7*x^14) - (5*a^4*b^2)/(4*x^12) - (2*a^3*b^3)/x^
10 - (15*a^2*b^4)/(8*x^8) - (a*b^5)/x^6 - b^6/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17, x]
```

fricas [A] time = 0.85, size = 70, normalized size = 1.75

$$-\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

giac [A] time = 0.17, size = 70, normalized size = 1.75

$$-\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

maple [A] time = 0.01, size = 69, normalized size = 1.72

$$-\frac{b^6}{4x^4}-\frac{a\,b^5}{x^6}-\frac{15a^2b^4}{8x^8}-\frac{2a^3b^3}{x^{10}}-\frac{5a^4b^2}{4x^{12}}-\frac{3a^5b}{7x^{14}}-\frac{a^6}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x)
```

```
[Out] -2*a^3*b^3/x^10-1/16*a^6/x^16-5/4*a^4*b^2/x^12-a*b^5/x^6-15/8*a^2*b^4/x^8-3
/7*a^5*b/x^14-1/4*b^6/x^4
```

maxima [A] time = 1.39, size = 70, normalized size = 1.75

$$-\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

mupad [B] time = 4.37, size = 69, normalized size = 1.72

$$-\frac{\frac{a^6}{16}+\frac{3\,a^5\,b\,x^2}{7}+\frac{5\,a^4\,b^2\,x^4}{4}+2\,a^3\,b^3\,x^6+\frac{15\,a^2\,b^4\,x^8}{8}+a\,b^5\,x^{10}+\frac{b^6\,x^{12}}{4}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^17,x)
```

```
[Out] -(a^6/16 + (b^6*x^12)/4 + (3*a^5*b*x^2)/7 + a*b^5*x^10 + (5*a^4*b^2*x^4)/4
+ 2*a^3*b^3*x^6 + (15*a^2*b^4*x^8)/8)/x^16
```

sympy [B] time = 0.68, size = 75, normalized size = 1.88

$$\frac{-7a^6-48a^5bx^2-140a^4b^2x^4-224a^3b^3x^6-210a^2b^4x^8-112ab^5x^{10}-28b^6x^{12}}{112x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**17,x)
```

```
[Out] (-7*a**6 - 48*a**5*b*x**2 - 140*a**4*b**2*x**4 - 224*a**3*b**3*x**6 - 210*a
**2*b**4*x**8 - 112*a*b**5*x**10 - 28*b**6*x**12)/(112*x**16)
```

3.298 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{18}} dx$

Optimal. Leaf size=82

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{2a^5b}{5x^{15}}-\frac{a^6}{17x^{17}}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -a^6/(17*x^17) - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (20*a^3*b^3)
/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) - b^6/(5*x^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{18}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{18}}\,dx}{b^6}\\
&= \frac{\int \left(\frac{a^6b^6}{x^{18}}+\frac{6a^5b^7}{x^{16}}+\frac{15a^4b^8}{x^{14}}+\frac{20a^3b^9}{x^{12}}+\frac{15a^2b^{10}}{x^{10}}+\frac{6ab^{11}}{x^8}+\frac{b^{12}}{x^6}\right)dx}{b^6}\\
&= -\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -1/17*a^6/x^17 - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (20*a^3*b^3)
/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) - b^6/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{18}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18, x]
```

fricas [A] time = 0.51, size = 70, normalized size = 0.85

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="fricas")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

giac [A] time = 0.17, size = 70, normalized size = 0.85

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="giac")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

maple [A] time = 0.01, size = 69, normalized size = 0.84

$$-\frac{b^{6}}{5x^{5}}-\frac{6a\,b^{5}}{7x^{7}}-\frac{5a^{2}b^{4}}{3x^{9}}-\frac{20a^{3}b^{3}}{11x^{11}}-\frac{15a^{4}b^{2}}{13x^{13}}-\frac{2a^{5}b}{5x^{15}}-\frac{a^{6}}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x)
```

```
[Out] -1/17*a^6/x^17-2/5*a^5*b/x^15-15/13*a^4*b^2/x^13-20/11*a^3*b^3/x^11-5/3*a^2
*b^4/x^9-6/7*a*b^5/x^7-1/5*b^6/x^5
```

maxima [A] time = 1.37, size = 70, normalized size = 0.85

$$-\frac{51051\,b^{6}x^{12}+218790\,ab^{5}x^{10}+425425\,a^{2}b^{4}x^{8}+464100\,a^{3}b^{3}x^{6}+294525\,a^{4}b^{2}x^{4}+102102\,a^{5}bx^{2}+15015\,a^{6}}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="maxima")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

mupad [B] time = 0.05, size = 70, normalized size = 0.85

$$-\frac{\frac{a^{6}}{17}+\frac{2\,a^{5}\,b\,x^{2}}{5}+\frac{15\,a^{4}\,b^{2}\,x^{4}}{13}+\frac{20\,a^{3}\,b^{3}\,x^{6}}{11}+\frac{5\,a^{2}\,b^{4}\,x^{8}}{3}+\frac{6\,a\,b^{5}\,x^{10}}{7}+\frac{b^{6}\,x^{12}}{5}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^18,x)
```

```
[Out] -(a^6/17 + (b^6*x^12)/5 + (2*a^5*b*x^2)/5 + (6*a*b^5*x^10)/7 + (15*a^4*b^2*
x^4)/13 + (20*a^3*b^3*x^6)/11 + (5*a^2*b^4*x^8)/3)/x^17
```

sympy [A] time = 0.66, size = 75, normalized size = 0.91

$$\frac{-15015a^{6}-102102a^{5}bx^{2}-294525a^{4}b^{2}x^{4}-464100a^{3}b^{3}x^{6}-425425a^{2}b^{4}x^{8}-218790ab^{5}x^{10}-51051b^{6}x^{12}}{255255x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**18,x)
```

```
[Out] (-15015*a**6 - 102102*a**5*b*x**2 - 294525*a**4*b**2*x**4 - 464100*a**3*b**
3*x**6 - 425425*a**2*b**4*x**8 - 218790*a*b**5*x**10 - 51051*b**6*x**12)/(2
55255*x**17)
```

3.299 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2(a+bx^2)^7}{504a^3x^{14}}+\frac{b(a+bx^2)^7}{72a^2x^{16}}-\frac{(a+bx^2)^7}{18ax^{18}}$$

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$-\frac{b^2(a+bx^2)^7}{504a^3x^{14}}+\frac{b(a+bx^2)^7}{72a^2x^{16}}-\frac{(a+bx^2)^7}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -(a + b*x^2)^7/(18*a*x^18) + (b*(a + b*x^2)^7)/(72*a^2*x^16) - (b^2*(a + b*
x^2)^7)/(504*a^3*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{19}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{19}}\,dx}{b^6} \\
&= \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^{10}}\,dx,x,x^2\right)}{2b^6} \\
&= -\frac{\left(a+bx^2\right)^7}{18ax^{18}} - \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^9}\,dx,x,x^2\right)}{9ab^5} \\
&= -\frac{\left(a+bx^2\right)^7}{18ax^{18}} + \frac{b\left(a+bx^2\right)^7}{72a^2x^{16}} + \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^8}\,dx,x,x^2\right)}{72a^2b^4} \\
&= -\frac{\left(a+bx^2\right)^7}{18ax^{18}} + \frac{b\left(a+bx^2\right)^7}{72a^2x^{16}} - \frac{b^2\left(a+bx^2\right)^7}{504a^3x^{14}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.32

$$-\frac{a^6}{18x^{18}}-\frac{3a^5b}{8x^{16}}-\frac{15a^4b^2}{14x^{14}}-\frac{5a^3b^3}{3x^{12}}-\frac{3a^2b^4}{2x^{10}}-\frac{3ab^5}{4x^8}-\frac{b^6}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -1/18*a^6/x^18 - (3*a^5*b)/(8*x^16) - (15*a^4*b^2)/(14*x^14) - (5*a^3*b^3)/
(3*x^12) - (3*a^2*b^4)/(2*x^10) - (3*a*b^5)/(4*x^8) - b^6/(6*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{19}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19, x]
```

fricas [A] time = 0.64, size = 70, normalized size = 1.13

$$-\frac{84\,b^{6}x^{12}+378\,ab^{5}x^{10}+756\,a^{2}b^{4}x^{8}+840\,a^{3}b^{3}x^{6}+540\,a^{4}b^{2}x^{4}+189\,a^{5}bx^{2}+28\,a^{6}}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

giac [A] time = 0.17, size = 70, normalized size = 1.13

$$-\frac{84\,b^{6}x^{12}+378\,ab^{5}x^{10}+756\,a^{2}b^{4}x^{8}+840\,a^{3}b^{3}x^{6}+540\,a^{4}b^{2}x^{4}+189\,a^{5}bx^{2}+28\,a^{6}}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

maple [A] time = 0.01, size = 69, normalized size = 1.11

$$-\frac{b^{6}}{6x^{6}}-\frac{3a\,b^{5}}{4x^{8}}-\frac{3a^{2}b^{4}}{2x^{10}}-\frac{5a^{3}b^{3}}{3x^{12}}-\frac{15a^{4}b^{2}}{14x^{14}}-\frac{3a^{5}b}{8x^{16}}-\frac{a^{6}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x)
```

```
[Out] -3/2*a^2*b^4/x^10-1/6*b^6/x^6-15/14*a^4*b^2/x^14-3/4*a*b^5/x^8-3/8*a^5*b/x^
16-1/18*a^6/x^18-5/3*a^3*b^3/x^12
```

maxima [A] time = 1.34, size = 70, normalized size = 1.13

$$-\frac{84\,b^{6}x^{12}+378\,ab^{5}x^{10}+756\,a^{2}b^{4}x^{8}+840\,a^{3}b^{3}x^{6}+540\,a^{4}b^{2}x^{4}+189\,a^{5}bx^{2}+28\,a^{6}}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

mupad [B] time = 4.31, size = 70, normalized size = 1.13

$$-\frac{\frac{a^6}{18}+\frac{3\,a^5\,b\,x^2}{8}+\frac{15\,a^4\,b^2\,x^4}{14}+\frac{5\,a^3\,b^3\,x^6}{3}+\frac{3\,a^2\,b^4\,x^8}{2}+\frac{3\,a\,b^5\,x^{10}}{4}+\frac{b^6\,x^{12}}{6}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^19,x)
```

```
[Out] -(a^6/18 + (b^6*x^12)/6 + (3*a^5*b*x^2)/8 + (3*a*b^5*x^10)/4 + (15*a^4*b^2*
x^4)/14 + (5*a^3*b^3*x^6)/3 + (3*a^2*b^4*x^8)/2)/x^18
```

sympy [A] time = 0.74, size = 75, normalized size = 1.21

$$\frac{-28a^6-189a^5bx^2-540a^4b^2x^4-840a^3b^3x^6-756a^2b^4x^8-378ab^5x^{10}-84b^6x^{12}}{504x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**19,x)
```

```
[Out] (-28*a**6 - 189*a**5*b*x**2 - 540*a**4*b**2*x**4 - 840*a**3*b**3*x**6 - 756
*a**2*b**4*x**8 - 378*a*b**5*x**10 - 84*b**6*x**12)/(504*x**18)
```

3.300 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{20}}\,dx$

Optimal. Leaf size=80

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{6a^5b}{17x^{17}}-\frac{a^6}{19x^{19}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -a^6/(19*x^19) - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b^3)/(13*x^
13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/(7*x^7)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{20}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{20}}\,dx}{b^6}\\
&= \frac{\int \left(\frac{a^6b^6}{x^{20}}+\frac{6a^5b^7}{x^{18}}+\frac{15a^4b^8}{x^{16}}+\frac{20a^3b^9}{x^{14}}+\frac{15a^2b^{10}}{x^{12}}+\frac{6ab^{11}}{x^{10}}+\frac{b^{12}}{x^8}\right)dx}{b^6}\\
&= -\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 80, normalized size = 1.00

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -1/19*a^6/x^19 - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b^3)/(13*x^
13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/(7*x^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{20}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20, x]
```

fricas [A] time = 0.74, size = 70, normalized size = 0.88

$$-\frac{138567\,b^6x^{12}+646646\,ab^5x^{10}+1322685\,a^2b^4x^8+1492260\,a^3b^3x^6+969969\,a^4b^2x^4+342342\,a^5bx^2+51051\,a^6}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="fricas")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

giac [A] time = 0.18, size = 70, normalized size = 0.88

$$-\frac{138567\,b^6x^{12}+646646\,ab^5x^{10}+1322685\,a^2b^4x^8+1492260\,a^3b^3x^6+969969\,a^4b^2x^4+342342\,a^5bx^2+51051\,a^6}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="giac")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

maple [A] time = 0.01, size = 69, normalized size = 0.86

$$-\frac{b^{6}}{7x^{7}}-\frac{2a\,b^{5}}{3x^{9}}-\frac{15a^{2}b^{4}}{11x^{11}}-\frac{20a^{3}b^{3}}{13x^{13}}-\frac{a^{4}b^{2}}{x^{15}}-\frac{6a^{5}b}{17x^{17}}-\frac{a^{6}}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x)
```

```
[Out] -1/19*a^6/x^19-6/17*a^5*b/x^17-a^4*b^2/x^15-20/13*a^3*b^3/x^13-15/11*a^2*b^
4/x^11-2/3*a*b^5/x^9-1/7*b^6/x^7
```

maxima [A] time = 1.35, size = 70, normalized size = 0.88

$$-\frac{138567\,b^{6}x^{12}+646646\,ab^{5}x^{10}+1322685\,a^{2}b^{4}x^{8}+1492260\,a^{3}b^{3}x^{6}+969969\,a^{4}b^{2}x^{4}+342342\,a^{5}bx^{2}+51051\,a^{6}}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="maxima")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

mupad [B] time = 0.05, size = 69, normalized size = 0.86

$$-\frac{\frac{a^{6}}{19}+\frac{6\,a^{5}\,b\,x^{2}}{17}+a^{4}\,b^{2}\,x^{4}+\frac{20\,a^{3}\,b^{3}\,x^{6}}{13}+\frac{15\,a^{2}\,b^{4}\,x^{8}}{11}+\frac{2\,a\,b^{5}\,x^{10}}{3}+\frac{b^{6}\,x^{12}}{7}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^20,x)
```

```
[Out] -(a^6/19 + (b^6*x^12)/7 + (6*a^5*b*x^2)/17 + (2*a*b^5*x^10)/3 + a^4*b^2*x^4
 + (20*a^3*b^3*x^6)/13 + (15*a^2*b^4*x^8)/11)/x^19
```

sympy [A] time = 0.71, size = 75, normalized size = 0.94

$$\frac{-51051a^{6}-342342a^{5}bx^{2}-969969a^{4}b^{2}x^{4}-1492260a^{3}b^{3}x^{6}-1322685a^{2}b^{4}x^{8}-646646ab^{5}x^{10}-138567b^{6}x^{12}}{969969x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**20,x)
```

```
[Out] (-51051*a**6 - 342342*a**5*b*x**2 - 969969*a**4*b**2*x**4 - 1492260*a**3*b*
*3*x**6 - 1322685*a**2*b**4*x**8 - 646646*a*b**5*x**10 - 138567*b**6*x**12)
/(969969*x**19)
```

3.301 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{21}}\,dx$

Optimal. Leaf size=84

$$\frac{b^3\left(a+bx^2\right)^7}{1680a^4x^{14}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{\left(a+bx^2\right)^7}{20ax^{20}}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{b^3\left(a+bx^2\right)^7}{1680a^4x^{14}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{\left(a+bx^2\right)^7}{20ax^{20}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -(a + b*x^2)^7/(20*a*x^20) + (b*(a + b*x^2)^7)/(60*a^2*x^18) - (b^2*(a + b*
x^2)^7)/(240*a^3*x^16) + (b^3*(a + b*x^2)^7)/(1680*a^4*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{21}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{21}}\,dx}{b^6}\\
&= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^{11}}\,dx,x,x^2\right)}{2b^6}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}-\frac{3\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^{10}}\,dx,x,x^2\right)}{20ab^5}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}+\frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^9}\,dx,x,x^2\right)}{30a^2b^4}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}-\frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^8}\,dx,x,x^2\right)}{240a^3b^3}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}+\frac{b^3\left(a+bx^2\right)^7}{1680a^4x^{14}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 0.98

$$-\frac{a^6}{20x^{20}}-\frac{a^5b}{3x^{18}}-\frac{15a^4b^2}{16x^{16}}-\frac{10a^3b^3}{7x^{14}}-\frac{5a^2b^4}{4x^{12}}-\frac{3ab^5}{5x^{10}}-\frac{b^6}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -1/20*a^6/x^20 - (a^5*b)/(3*x^18) - (15*a^4*b^2)/(16*x^16) - (10*a^3*b^3)/(
7*x^14) - (5*a^2*b^4)/(4*x^12) - (3*a*b^5)/(5*x^10) - b^6/(8*x^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{21}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21, x]
```

fricas [A] time = 0.83, size = 70, normalized size = 0.83

$$-\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="fricas")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

giac [A] time = 0.15, size = 70, normalized size = 0.83

$$-\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="giac")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

maple [A] time = 0.01, size = 69, normalized size = 0.82

$$-\frac{b^6}{8x^8}-\frac{3a\,b^5}{5x^{10}}-\frac{5a^2b^4}{4x^{12}}-\frac{10a^3b^3}{7x^{14}}-\frac{15a^4b^2}{16x^{16}}-\frac{a^5b}{3x^{18}}-\frac{a^6}{20x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x)
```

```
[Out] -3/5*a*b^5/x^10-1/3*a^5*b/x^18-1/20*a^6/x^20-1/8*b^6/x^8-15/16*a^4*b^2/x^16
-10/7*a^3*b^3/x^14-5/4*a^2*b^4/x^12
```

maxima [A] time = 1.42, size = 70, normalized size = 0.83

$$-\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="maxima")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

mupad [B] time = 0.05, size = 70, normalized size = 0.83

$$-\frac{\frac{a^6}{20}+\frac{a^5\,b\,x^2}{3}+\frac{15\,a^4\,b^2\,x^4}{16}+\frac{10\,a^3\,b^3\,x^6}{7}+\frac{5\,a^2\,b^4\,x^8}{4}+\frac{3\,a\,b^5\,x^{10}}{5}+\frac{b^6\,x^{12}}{8}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^21,x)
```

```
[Out] -(a^6/20 + (b^6*x^12)/8 + (a^5*b*x^2)/3 + (3*a*b^5*x^10)/5 + (15*a^4*b^2*x^
4)/16 + (10*a^3*b^3*x^6)/7 + (5*a^2*b^4*x^8)/4)/x^20
```

sympy [A] time = 0.86, size = 75, normalized size = 0.89

$$\frac{-84a^6-560a^5bx^2-1575a^4b^2x^4-2400a^3b^3x^6-2100a^2b^4x^8-1008ab^5x^{10}-210b^6x^{12}}{1680x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**21,x)
```

```
[Out] (-84*a**6 - 560*a**5*b*x**2 - 1575*a**4*b**2*x**4 - 2400*a**3*b**3*x**6 - 2
100*a**2*b**4*x**8 - 1008*a*b**5*x**10 - 210*b**6*x**12)/(1680*x**20)
```

3.302 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{22}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6a^5b}{19x^{19}}-\frac{a^6}{21x^{21}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -a^6/(21*x^21) - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (4*a^3*b^3)
/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11) - b^6/(9*x^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{22}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{22}}\,dx}{b^6}\\
&= \frac{\int \left(\frac{a^6b^6}{x^{22}}+\frac{6a^5b^7}{x^{20}}+\frac{15a^4b^8}{x^{18}}+\frac{20a^3b^9}{x^{16}}+\frac{15a^2b^{10}}{x^{14}}+\frac{6ab^{11}}{x^{12}}+\frac{b^{12}}{x^{10}}\right)dx}{b^6}\\
&= -\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -1/21*a^6/x^21 - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (4*a^3*b^3)
/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11) - b^6/(9*x^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{22}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22, x]
```

fricas [A] time = 0.85, size = 70, normalized size = 0.85

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="fricas")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

giac [A] time = 0.17, size = 70, normalized size = 0.85

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="giac")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

maple [A] time = 0.01, size = 69, normalized size = 0.84

$$-\frac{b^6}{9x^9}-\frac{6a\,b^5}{11x^{11}}-\frac{15a^2b^4}{13x^{13}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^4b^2}{17x^{17}}-\frac{6a^5b}{19x^{19}}-\frac{a^6}{21x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x)
```

```
[Out] -1/21*a^6/x^21-6/19*a^5*b/x^19-15/17*a^4*b^2/x^17-4/3*a^3*b^3/x^15-15/13*a^
2*b^4/x^13-6/11*a*b^5/x^11-1/9*b^6/x^9
```

maxima [A] time = 1.34, size = 70, normalized size = 0.85

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="maxima")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

mupad [B] time = 0.05, size = 70, normalized size = 0.85

$$-\frac{\frac{a^6}{21}+\frac{6\,a^5\,b\,x^2}{19}+\frac{15\,a^4\,b^2\,x^4}{17}+\frac{4\,a^3\,b^3\,x^6}{3}+\frac{15\,a^2\,b^4\,x^8}{13}+\frac{6\,a\,b^5\,x^{10}}{11}+\frac{b^6\,x^{12}}{9}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^22,x)
```

```
[Out] -(a^6/21 + (b^6*x^12)/9 + (6*a^5*b*x^2)/19 + (6*a*b^5*x^10)/11 + (15*a^4*b^
2*x^4)/17 + (4*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/13)/x^21
```

sympy [A] time = 0.76, size = 75, normalized size = 0.91

$$\frac{-138567a^6-918918a^5bx^2-2567565a^4b^2x^4-3879876a^3b^3x^6-3357585a^2b^4x^8-1587222ab^5x^{10}-323323b^6x^{12}}{2909907x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**22,x)
```

```
[Out] (-138567*a**6 - 918918*a**5*b*x**2 - 2567565*a**4*b**2*x**4 - 3879876*a**3*
b**3*x**6 - 3357585*a**2*b**4*x**8 - 1587222*a*b**5*x**10 - 323323*b**6*x**
12)/(2909907*x**21)
```

3.303 $\int \frac{x^{11}}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=83

$$\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3a^2x^4}{4b^4}-\frac{2a^3x^2}{b^5}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^{11}}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(-\frac{4a^3}{b^7}+\frac{3a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{b^4}-\frac{a^5}{b^7(a+bx)^2}+\frac{5a^4}{b^7(a+bx)}\right)dx,x,x^2\right)$$
$$= -\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.87

$$\frac{\frac{12a^5}{a+bx^2}+60a^4\log\left(a+bx^2\right)-48a^3bx^2+18a^2b^2x^4-8ab^3x^6+3b^4x^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a + b
*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.48, size = 93, normalized size = 1.12

$$\frac{3\,b^5x^{10}-5\,ab^4x^8+10\,a^2b^3x^6-30\,a^3b^2x^4-48\,a^4bx^2+12\,a^5+60\left(a^4bx^2+a^5\right)\log\left(bx^2+a\right)}{24\left(b^7x^2+ab^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*b
*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

giac [A] time = 0.16, size = 92, normalized size = 1.11

$$\frac{5\,a^4\log\left(\left|bx^2+a\right|\right)}{2\,b^6}-\frac{5\,a^4bx^2+4\,a^5}{2\left(bx^2+a\right)b^6}+\frac{3\,b^6x^8-8\,ab^5x^6+18\,a^2b^4x^4-48\,a^3b^3x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b^
6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

maple [A] time = 0.01, size = 74, normalized size = 0.89

$$\frac{x^8}{8b^2}-\frac{a\,x^6}{3b^3}+\frac{3a^2x^4}{4b^4}-\frac{2a^3x^2}{b^5}+\frac{a^5}{2\left(b\,x^2+a\right)b^6}+\frac{5a^4\ln\left(b\,x^2+a\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.37, size = 77, normalized size = 0.93

$$\frac{a^5}{2\left(b^7x^2+ab^6\right)}+\frac{5\,a^4\log\left(bx^2+a\right)}{2\,b^6}+\frac{3\,b^3x^8-8\,ab^2x^6+18\,a^2bx^4-48\,a^3x^2}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

mupad [B] time = 4.36, size = 79, normalized size = 0.95

$$\frac{x^8}{8\,b^2}+\frac{a^5}{2\,b\,\left(b^6\,x^2+a\,b^5\right)}-\frac{a\,x^6}{3\,b^3}+\frac{5\,a^4\,\ln\left(b\,x^2+a\right)}{2\,b^6}+\frac{3\,a^2\,x^4}{4\,b^4}-\frac{2\,a^3\,x^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^8/(8*b^2) + a^5/(2*b*(a*b^5 + b^6*x^2)) - (a*x^6)/(3*b^3) + (5*a^4*log(a
+ b*x^2))/(2*b^6) + (3*a^2*x^4)/(4*b^4) - (2*a^3*x^2)/b^5
```

sympy [A] time = 0.32, size = 80, normalized size = 0.96

$$\frac{a^5}{2ab^6 + 2b^7x^2} + \frac{5a^4 \log\left(a + bx^2\right)}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x*
*2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

3.304 $\int \frac{x^9}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=70

$$-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3a^2x^2}{2b^4}-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2
)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^9}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{3a^2}{b^6}-\frac{2ax}{b^5}+\frac{x^2}{b^4}+\frac{a^4}{b^6(a+bx)^2}-\frac{4a^3}{b^6(a+bx)}\right)dx,x,x^2\right)$$
$$= \frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.86

$$\frac{-\frac{3a^4}{a+bx^2}-12a^3\log\left(a+bx^2\right)+9a^2bx^2-3ab^2x^4+b^3x^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a + 
 b*x^2])/(6*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.59, size = 81, normalized size = 1.16

$$\frac{b^4x^8-2\,ab^3x^6+6\,a^2b^2x^4+9\,a^3bx^2-3\,a^4-12\left(a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{6\left(b^6x^2+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3*
b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

giac [A] time = 0.17, size = 80, normalized size = 1.14

$$-\frac{2\,a^{3}\log\left(\left|bx^{2}+a\right|\right)}{b^{5}}+\frac{b^{4}x^{6}-3\,ab^{3}x^{4}+9\,a^{2}b^{2}x^{2}}{6\,b^{6}}+\frac{4\,a^{3}bx^{2}+3\,a^{4}}{2\left(bx^{2}+a\right)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^2
)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

maple [A] time = 0.01, size = 63, normalized size = 0.90

$$\frac{x^{6}}{6b^{2}}-\frac{a\,x^{4}}{2b^{3}}+\frac{3a^{2}x^{2}}{2b^{4}}-\frac{a^{4}}{2\left(b\,x^{2}+a\right)b^{5}}-\frac{2a^{3}\ln\left(b\,x^{2}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*
x^2+a)/b^5
```

maxima [A] time = 1.35, size = 65, normalized size = 0.93

$$-\frac{a^{4}}{2\left(b^{6}x^{2}+ab^{5}\right)}-\frac{2\,a^{3}\log\left(bx^{2}+a\right)}{b^{5}}+\frac{b^{2}x^{6}-3\,abx^{4}+9\,a^{2}x^{2}}{6\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a*
b*x^4 + 9*a^2*x^2)/b^4
```

mupad [B] time = 0.04, size = 68, normalized size = 0.97

$$\frac{x^{6}}{6\,b^{2}}-\frac{a^{4}}{2\,b\,\left(b^{5}\,x^{2}+a\,b^{4}\right)}-\frac{a\,x^{4}}{2\,b^{3}}-\frac{2\,a^{3}\,\ln\left(b\,x^{2}+a\right)}{b^{5}}+\frac{3\,a^{2}\,x^{2}}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^6/(6*b^2) - a^4/(2*b*(a*b^4 + b^5*x^2)) - (a*x^4)/(2*b^3) - (2*a^3*log(a
+ b*x^2))/b^5 + (3*a^2*x^2)/(2*b^4)
```

sympy **[A]** time = 0.29, size = 66, normalized size = 0.94

$$-\frac{a^4}{2ab^5+2b^6x^2}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2/
(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

3.305 $\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=57

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^7}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\operatorname{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\operatorname{Subst}\left(\int \left(-\frac{2a}{b^5}+\frac{x}{b^4}-\frac{a^3}{b^5(a+bx)^2}+\frac{3a^2}{b^5(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{\frac{2a^3}{a+bx^2}+6a^2\log\left(a+bx^2\right)-4abx^2+b^2x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] IntegrateAlgebraic[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.75, size = 70, normalized size = 1.23

$$\frac{b^3x^6-3\,ab^2x^4-4\,a^2bx^2+2\,a^3+6\left(a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log(
b*x^2 + a))/(b^5*x^2 + a*b^4)
```

giac [A] time = 0.16, size = 67, normalized size = 1.18

$$\frac{3\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^4}+\frac{b^2x^4-4\,abx^2}{4\,b^4}-\frac{3\,a^2bx^2+2\,a^3}{2\left(bx^2+a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a^
2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

maple [A] time = 0.01, size = 52, normalized size = 0.91

$$\frac{x^4}{4b^2}-\frac{a\,x^2}{b^3}+\frac{a^3}{2\left(b\,x^2+a\right)b^4}+\frac{3a^2\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

maxima [A] time = 1.35, size = 54, normalized size = 0.95

$$\frac{a^3}{2\left(b^5x^2+ab^4\right)}+\frac{3\,a^2\log\left(bx^2+a\right)}{2\,b^4}+\frac{bx^4-4\,ax^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*x
^2)/b^3
```

mupad [B] time = 0.05, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,b^2}+\frac{a^3}{2\,b\,\left(b^4\,x^2+a\,b^3\right)}-\frac{a\,x^2}{b^3}+\frac{3\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^4/(4*b^2) + a^3/(2*b*(a*b^3 + b^4*x^2)) - (a*x^2)/b^3 + (3*a^2*log(a + b*
x^2))/(2*b^4)
```

sympy [A] time = 0.27, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4 + 2b^5x^2} + \frac{3a^2\log\left(a + bx^2\right)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b*
*3 + x**4/(4*b**2)
```

3.306 $\int \frac{x^5}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=44

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^5}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{b^4}+\frac{a^2}{b^4(a+bx)^2}-\frac{2a}{b^4(a+bx)}\right)dx,x,x^2\right)$$
$$= \frac{x^2}{2b^2}-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.86

$$\frac{-\frac{a^2}{a+bx^2}-2a\log\left(a+bx^2\right)+bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.75, size = 56, normalized size = 1.27

$$\frac{b^2x^4+abx^2-a^2-2\left(abx^2+a^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2 +
 a*b^3)
```

giac [A] time = 0.17, size = 49, normalized size = 1.11

$$\frac{x^2}{2\,b^2} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{b^3} + \frac{2\,abx^2 + a^2}{2\left(bx^2 + a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 + a
)*b^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.93

$$\frac{x^2}{2b^2} - \frac{a^2}{2\left(b\,x^2 + a\right)b^3} - \frac{a\ln\left(b\,x^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

maxima [A] time = 1.30, size = 43, normalized size = 0.98

$$-\frac{a^2}{2\left(b^4x^2 + ab^3\right)} + \frac{x^2}{2\,b^2} - \frac{a\log\left(bx^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

mupad [B] time = 0.05, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,b^2} - \frac{a^2}{2\,\left(b^4\,x^2 + a\,b^3\right)} - \frac{a\,\ln\left(b\,x^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^2/(2*b^2) - a^2/(2*(a*b^3 + b^4*x^2)) - (a*log(a + b*x^2))/b^3
```

sympy [A] time = 0.25, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3 + 2b^4x^2} - \frac{a\log\left(a + bx^2\right)}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
[Out] -a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

3.307 $\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=33

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^3}{\left(ab+b^2x^2\right)^2}\,dx \\
&= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(-\frac{a}{b^3(a+bx)^2}+\frac{1}{b^3(a+bx)}\right)dx,x,x^2\right) \\
&= \frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$
\frac{\frac{a}{a+bx^2}+\log\left(a+bx^2\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.85, size = 35, normalized size = 1.06

$$
\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)+a}{2\left(b^3x^2+ab^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.16, size = 30, normalized size = 0.91

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^2}+\frac{a}{2\left(bx^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))/b^2 + 1/2*a/((b*x^2 + a)*b^2)
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{a}{2\left(b\,x^2+a\right)b^2}+\frac{\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.41, size = 32, normalized size = 0.97

$$\frac{a}{2\left(b^3x^2+ab^2\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] 1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.05, size = 29, normalized size = 0.88

$$\frac{\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{a}{2\,b^2\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] log(a + b*x^2)/(2*b^2) + a/(2*b^2*(a + b*x^2))
```

sympy [A] time = 0.21, size = 29, normalized size = 0.88

$$\frac{a}{2ab^2 + 2b^3x^2} + \frac{\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

3.308 $\int \frac{x}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/(2*b*(a + b*x^2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a^2+2abx^2+b^2x^4} \, dx &= b^2 \int \frac{x}{\left(ab+b^2x^2\right)^2} \, dx \\ &= -\frac{1}{2b\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*1/(b*(a + b*x^2))
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a^2 + 2abx^2 + b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A]   time = 0.87, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b^2x^2 + ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

giac [A]   time = 0.17, size = 14, normalized size = 0.88

$$-\frac{1}{2\left(bx^2 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

maple [A]   time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b\,x^2 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/b/(b*x^2+a)
```

maxima [A] time = 1.38, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] -1/2/(b^2*x^2 + a*b)
```

mupad [B] time = 4.32, size = 14, normalized size = 0.88

$$-\frac{1}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] -1/(2*b*(a + b*x^2))
```

sympy [A] time = 0.17, size = 15, normalized size = 0.94

$$-\frac{1}{2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2),x)
[Out] -1/(2*a*b + 2*b**2*x**2)
```

3.309 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{x\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{a^2b^2x}-\frac{1}{ab(a+bx)^2}-\frac{1}{a^2b(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2}-\log\left(a+bx^2\right)+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)), x]
```

fricas [A] time = 0.91, size = 47, normalized size = 1.24

$$-\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)-2\left(bx^2+a\right)\log(x)-a}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a^3)

giac [A] time = 0.15, size = 47, normalized size = 1.24

$$\frac{\log\left(x^2\right)}{2\,a^2} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a^2} + \frac{bx^2 + 2\,a}{2\left(bx^2 + a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")

[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2 + a)*a^2)

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{2\left(b\,x^2 + a\right)a} + \frac{\ln(x)}{a^2} - \frac{\ln\left(b\,x^2 + a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2),x)

[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2

maxima [A] time = 1.36, size = 37, normalized size = 0.97

$$\frac{1}{2\left(abx^2 + a^2\right)} - \frac{\log\left(bx^2 + a\right)}{2\,a^2} + \frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")

[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2

mupad [B] time = 4.39, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2} + \frac{1}{2\,a\,\left(b\,x^2 + a\right)} - \frac{\ln\left(b\,x^2 + a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)

```
[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

sympy [**A**] time = 0.32, size = 34, normalized size = 0.89

$$\frac{1}{2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

3.310 $\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=49

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{x^3\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{a^2b^2x^2}-\frac{2}{a^3bx}+\frac{1}{a^2(a+bx)^2}+\frac{2}{a^3(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)), x]
```

fricas [A] time = 1.01, size = 73, normalized size = 1.49

$$-\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log(x)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

giac [A] time = 0.16, size = 51, normalized size = 1.04

$$-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{b}{2\left(b\,x^2+a\right)a^2}-\frac{2b\ln(x)}{a^3}+\frac{b\ln\left(b\,x^2+a\right)}{a^3}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.36, size = 52, normalized size = 1.06

$$-\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^2
)/a^3
```

mupad [B] time = 0.08, size = 51, normalized size = 1.04

$$\frac{b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{1}{2\,a}+\frac{b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

sympy [A] time = 0.39, size = 51, normalized size = 1.04

$$\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

3.311 $\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=66

$$-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{b^2}{2a^3\left(a+bx^2\right)}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5 \left(a^2+2abx^2+b^2x^4\right)} dx = b^2 \int \frac{1}{x^5 \left(ab+b^2x^2\right)^2} dx$$
$$= \frac{1}{2} b^2 \operatorname{Subst}\left(\int \frac{1}{x^3 \left(ab+b^2x\right)^2} dx, x, x^2\right)$$
$$= \frac{1}{2} b^2 \operatorname{Subst}\left(\int \left(\frac{1}{a^2b^2x^3} - \frac{2}{a^3bx^2} + \frac{3}{a^4x} - \frac{b}{a^3(a+bx)^2} - \frac{3b}{a^4(a+bx)}\right) dx, x, x^2\right)$$
$$= -\frac{1}{4a^2x^4} + \frac{b}{a^3x^2} + \frac{b^2}{2a^3\left(a+bx^2\right)} + \frac{3b^2\log(x)}{a^4} - \frac{3b^2\log\left(a+bx^2\right)}{2a^4}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.86

$$\frac{-6b^2\log\left(a+bx^2\right)+a\left(\frac{2b^2}{a+bx^2}-\frac{a}{x^4}+\frac{4b}{x^2}\right)+12b^2\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Log
[a + b*x^2])/(4*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a^2+2abx^2+b^2x^4\right)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)), x]
```

fricas [A] time = 1.20, size = 90, normalized size = 1.36

$$\frac{6\,ab^2x^4+3\,a^2bx^2-a^3-6\left(b^3x^6+ab^2x^4\right)\log\left(bx^2+a\right)+12\left(b^3x^6+ab^2x^4\right)\log(x)}{4\left(a^4bx^6+a^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

giac [A] time = 0.17, size = 86, normalized size = 1.30

$$\frac{3\,b^2 \log\left(x^2\right)}{2\,a^4} - \frac{3\,b^2 \log\left(\left|bx^2 + a\right|\right)}{2\,a^4} + \frac{3\,b^3x^2 + 4\,ab^2}{2\left(bx^2 + a\right)a^4} - \frac{9\,b^2x^4 - 4\,abx^2 + a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 + 4
*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

maple [A] time = 0.01, size = 61, normalized size = 0.92

$$\frac{b^2}{2\left(b\,x^2 + a\right)a^3} + \frac{3b^2 \ln(x)}{a^4} - \frac{3b^2 \ln\left(b\,x^2 + a\right)}{2a^4} + \frac{b}{a^3x^2} - \frac{1}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

maxima [A] time = 1.41, size = 70, normalized size = 1.06

$$\frac{6\,b^2x^4 + 3\,abx^2 - a^2}{4\left(a^3bx^6 + a^4x^4\right)} - \frac{3\,b^2 \log\left(bx^2 + a\right)}{2\,a^4} + \frac{3\,b^2 \log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^2
 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

mupad [B] time = 0.07, size = 67, normalized size = 1.02

$$\frac{\frac{3\,b\,x^2}{4\,a^2} - \frac{1}{4\,a} + \frac{3\,b^2\,x^4}{2\,a^3}}{b\,x^6 + a\,x^4} - \frac{3\,b^2\,\ln\left(b\,x^2 + a\right)}{2\,a^4} + \frac{3\,b^2\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] ((3*b*x^2)/(4*a^2) - 1/(4*a) + (3*b^2*x^4)/(2*a^3))/(a*x^4 + b*x^6) - (3*b^
2*log(a + b*x^2))/(2*a^4) + (3*b^2*log(x))/a^4
```

sympy [A] time = 0.47, size = 68, normalized size = 1.03

$$\frac{-a^2+3abx^2+6b^2x^4}{4a^4x^4+4a^3bx^6}+\frac{3b^2\log\left(x\right)}{a^4}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*l
og(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

3.312 $\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=92

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

Rubi [A] time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{3a^2x^3}{2b^4}-\frac{9a^3x}{2b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^{10}}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9}{2}\int \frac{x^8}{ab+b^2x^2}\,dx\\
&= -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9}{2}\int \left(-\frac{a^3}{b^5}+\frac{a^2x^2}{b^4}-\frac{ax^4}{b^3}+\frac{x^6}{b^2}+\frac{a^4}{b^4\left(ab+b^2x^2\right)}\right)dx\\
&= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{\left(9a^4\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^4}\\
&= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.89

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{x\left(-\frac{35a^4}{a+bx^2}-280a^3+70a^2bx^2-28ab^2x^4+10b^3x^6\right)}{70b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*x
^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

[Out] IntegrateAlgebraic[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4), x]

fricas [A] time = 0.87, size = 212, normalized size = 2.30

$$\left[\frac{20\,b^4x^9 - 36\,ab^3x^7 + 84\,a^2b^2x^5 - 420\,a^3bx^3 - 630\,a^4x + 315\left(a^3bx^2 + a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{140\left(b^6x^2 + ab^5\right)}, \frac{10\,b^4x^9 - 18\,ab^3x^7 + 42\,a^2b^2x^5 - 210\,a^3bx^3 - 315\,a^4x + 315\left(a^3bx^2 + a^4\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{70\left(b^6x^2 + ab^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a^
4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2*b^
2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

giac [A] time = 0.15, size = 84, normalized size = 0.91

$$\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5} - \frac{a^4x}{2\left(bx^2+a\right)b^5} + \frac{5\,b^{12}x^7 - 14\,ab^{11}x^5 + 35\,a^2b^{10}x^3 - 140\,a^3b^9x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5)
 + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^14
```

maple [A] time = 0.01, size = 78, normalized size = 0.85

$$\frac{x^7}{7b^2} - \frac{2a\,x^5}{5b^3} + \frac{a^2x^3}{b^4} - \frac{a^4x}{2\left(b\,x^2+a\right)b^5} + \frac{9a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^5} - \frac{4a^3x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/7*x^7/b^2-2/5*a*x^5/b^3+a^2*x^3/b^4-4*a^3*x/b^5-1/2/b^5*a^4*x/(b*x^2+a)+9
/2/b^5*a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

maxima [A] time = 2.99, size = 82, normalized size = 0.89

$$-\frac{a^4x}{2\left(b^6x^2+ab^5\right)} + \frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5} + \frac{5\,b^3x^7 - 14\,ab^2x^5 + 35\,a^2bx^3 - 140\,a^3x}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^4*x/(b^6*x^2 + a*b^5) + 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5
) + 1/35*(5*b^3*x^7 - 14*a*b^2*x^5 + 35*a^2*b*x^3 - 140*a^3*x)/b^5
```

mupad [B] time = 0.04, size = 77, normalized size = 0.84

$$\frac{x^7}{7\,b^2}-\frac{2\,a\,x^5}{5\,b^3}-\frac{4\,a^3\,x}{b^5}+\frac{9\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{11/2}}+\frac{a^2\,x^3}{b^4}-\frac{a^4\,x}{2\,\left(b^6\,x^2+a\,b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^7/(7*b^2) - (2*a*x^5)/(5*b^3) - (4*a^3*x)/b^5 + (9*a^(7/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(11/2)) + (a^2*x^3)/b^4 - (a^4*x)/(2*(a*b^5 + b^6*x^2))
```

sympy [A] time = 0.35, size = 134, normalized size = 1.46

$$-\frac{a^4x}{2ab^5+2b^6x^2}-\frac{4a^3x}{b^5}+\frac{a^2x^3}{b^4}-\frac{2ax^5}{5b^3}-\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x**
5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 + 9
*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2)
```

3.313 $\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{7a^2x}{2b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^8}{\left(ab+b^2x^2\right)^2}\,dx \\
&= -\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7}{2}\int \frac{x^6}{ab+b^2x^2}\,dx \\
&= -\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7}{2}\int \left(\frac{a^2}{b^4}-\frac{ax^2}{b^3}+\frac{x^4}{b^2}-\frac{a^3}{b^3\left(ab+b^2x^2\right)}\right)dx \\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\left(a+bx^2\right)}-\frac{\left(7a^3\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^3} \\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\left(a+bx^2\right)}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.90

$$\frac{x\left(\frac{15a^3}{a+bx^2}+90a^2-20abx^2+6b^2x^4\right)}{30b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7*
a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.89, size = 190, normalized size = 2.41

$$\left[\frac{12\,b^{3}x^{7} - 28\,ab^{2}x^{5} + 140\,a^{2}bx^{3} + 210\,a^{3}x + 105\,\left(a^{2}bx^{2} + a^{3}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2} - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^{2} + a}\right)}{60\,\left(b^{5}x^{2} + ab^{4}\right)}, \frac{6\,b^{3}x^{7} - 14\,ab^{2}x^{5} + 70\,a^{2}bx^{3} + 105\,a^{3}x - 105\,\left(a^{2}bx^{2} + a^{3}\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{30\,\left(b^{5}x^{2} + ab^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*x
^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*
x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x - 1
05*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^4)]
```

giac [A] time = 0.15, size = 73, normalized size = 0.92

$$-\frac{7\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{4}} + \frac{a^{3}x}{2\,\left(bx^{2} + a\right)b^{4}} + \frac{3\,b^{8}x^{5} - 10\,ab^{7}x^{3} + 45\,a^{2}b^{6}x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

maple [A] time = 0.01, size = 68, normalized size = 0.86

$$\frac{x^{5}}{5b^{2}} - \frac{2a\,x^{3}}{3b^{3}} + \frac{a^{3}x}{2\left(b\,x^{2} + a\right)b^{4}} - \frac{7a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\ b^{4}} + \frac{3a^{2}x}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/5*x^5/b^2-2/3*a*x^3/b^3+3*a^2*x/b^4+1/2/b^4*a^3*x/(b*x^2+a)-7/2/b^4*a^3/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.95, size = 71, normalized size = 0.90

$$\frac{a^{3}x}{2\,\left(b^{5}x^{2} + ab^{4}\right)} - \frac{7\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{4}} + \frac{3\,b^{2}x^{5} - 10\,abx^{3} + 45\,a^{2}x}{15\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x/(b^5*x^2 + a*b^4) - 7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/15*(3*b^2*x^5 - 10*a*b*x^3 + 45*a^2*x)/b^4
```

mupad [B] time = 4.27, size = 66, normalized size = 0.84

$$\frac{x^5}{5\,b^2}-\frac{2\,a\,x^3}{3\,b^3}+\frac{3\,a^2\,x}{b^4}-\frac{7\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{9/2}}+\frac{a^3\,x}{2\,\left(b^5\,x^2+a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^5/(5*b^2) - (2*a*x^3)/(3*b^3) + (3*a^2*x)/b^4 - (7*a^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(9/2)) + (a^3*x)/(2*(a*b^4 + b^5*x^2))
```

sympy [A] time = 0.32, size = 124, normalized size = 1.57

$$\frac{a^3x}{2ab^4+2b^5x^2}+\frac{3a^2x}{b^4}-\frac{2ax^3}{3b^3}+\frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}-\frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}+\frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sqr
t(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*lo
g(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

3.314 $\int \frac{x^6}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=66

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^6}{\left(ab+b^2x^2\right)^2}\,dx\\ &= -\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5}{2}\int \frac{x^4}{ab+b^2x^2}\,dx\\ &= -\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5}{2}\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2\left(ab+b^2x^2\right)}\right)dx\\ &= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{\left(5a^2\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^2}\\ &= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.91

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x\left(-\frac{3a^2}{a+bx^2}-12a+2bx^2\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

[Out] IntegrateAlgebraic[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4), x]

fricas [A] time = 0.66, size = 164, normalized size = 2.48

$$\left[\frac{4\,b^{2}x^{5}-20\,abx^{3}-30\,a^{2}x+15\left(abx^{2}+a^{2}\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)}{12\left(b^{4}x^{2}+ab^{3}\right)},\ \frac{2\,b^{2}x^{5}-10\,abx^{3}-15\,a^{2}x+15\left(abx^{2}+a^{2}\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{6\left(b^{4}x^{2}+ab^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")

[Out] [1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*b^2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^4*x^2 + a*b^3)]

giac [A] time = 0.16, size = 61, normalized size = 0.92

$$\frac{5\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{3}}-\frac{a^{2}x}{2\left(bx^{2}+a\right)b^{3}}+\frac{b^{4}x^{3}-6\,ab^{3}x}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")

[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3) + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{x^{3}}{3b^{2}}-\frac{a^{2}x}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{5a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{3}}-\frac{2ax}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2),x)

[Out] 1/3*x^3/b^2-2*a*x/b^3-1/2/b^3*a^2*x/(b*x^2+a)+5/2/b^3*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)

maxima [A] time = 3.13, size = 59, normalized size = 0.89

$$-\frac{a^{2}x}{2\left(b^{4}x^{2}+ab^{3}\right)}+\frac{5\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{3}}+\frac{bx^{3}-6\,ax}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x/(b^4*x^2 + a*b^3) + 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3
) + 1/3*(b*x^3 - 6*a*x)/b^3
```

mupad [B] time = 0.06, size = 56, normalized size = 0.85

$$\frac{x^3}{3\,b^2}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{7/2}}-\frac{a^2\,x}{2\,\left(b^4\,x^2+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^3/(3*b^2) + (5*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(7/2)) - (a^2*x)/(
2*(a*b^3 + b^4*x^2)) - (2*a*x)/b^3
```

sympy [A] time = 0.30, size = 107, normalized size = 1.62

$$-\frac{a^2x}{2ab^3+2b^4x^2}-\frac{2ax}{b^3}-\frac{5\sqrt{-\frac{a^3}{b^7}}\,\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{5\sqrt{-\frac{a^3}{b^7}}\,\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b**
7)/a)/4 + x**3/(3*b**2)
```

3.315 $\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 321, 205}

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^4}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3}{2}\int \frac{x^2}{ab+b^2x^2}\,dx\\
&= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{(3a)\int \frac{1}{ab+b^2x^2}\,dx}{2b}\\
&= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}+\frac{ax}{2b^2\left(a+bx^2\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.85, size = 136, normalized size = 2.47

$$\left[\frac{4\,bx^3 + 3\left(bx^2 + a\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 6\,ax}{4\left(b^3x^2 + ab^2\right)}, \frac{2\,bx^3 - 3\left(bx^2 + a\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 3\,ax}{2\left(b^3x^2 + ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

giac [A] time = 0.15, size = 42, normalized size = 0.76

$$-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2} + \frac{ax}{2\left(bx^2 + a\right)b^2} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
x/b^2
```

maple [A] time = 0.01, size = 43, normalized size = 0.78

$$\frac{ax}{2\left(b\,x^2 + a\right)b^2} - \frac{3a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] x/b^2+1/2/b^2*a*x/(b*x^2+a)-3/2/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.08, size = 45, normalized size = 0.82

$$\frac{ax}{2\left(b^3x^2 + ab^2\right)} - \frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*x/(b^3*x^2 + a*b^2) - 3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + x
/b^2
```

mupad [B] time = 4.29, size = 43, normalized size = 0.78

$$\frac{x}{b^2} + \frac{a\,x}{2\,\left(b^3\,x^2 + a\,b^2\right)} - \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x/b^2 + (a*x)/(2*(a*b^2 + b^3*x^2)) - (3*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))
/(2*b^(5/2))
```

sympy [A] time = 0.27, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2 + 2b^3x^2} + \frac{3\sqrt{-\frac{a}{b^5}}\,\log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{4} - \frac{3\sqrt{-\frac{a}{b^5}}\,\log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{4} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x)
/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

3.316 $\int \frac{x^2}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^2}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= -\frac{x}{2b\left(a+bx^2\right)} + \frac{1}{2}\int \frac{1}{ab+b^2x^2}\,dx$$
$$= -\frac{x}{2b\left(a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}} - \frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] -1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 0.82, size = 120, normalized size = 2.67

$$\left[-\frac{2\,abx + \left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(ab^3x^2+a^2b^2\right)},\, -\frac{abx - \left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(ab^3x^2+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

giac [A] time = 0.20, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b} - \frac{x}{2\left(bx^2 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(b\,x^2 + a\right)b} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] -1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.00, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(b^2x^2 + ab\right)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] -1/2*x/(b^2*x^2 + a*b) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 0.05, size = 33, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\;b^{3/2}} - \frac{x}{2\,b\,\left(b\,x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2)) - x/(2*b*(a + b*x^2))
```

sympy [B] time = 0.22, size = 78, normalized size = 1.73

$$-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) + x
)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

3.317 $\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{x}{2a\left(a+bx^2\right)} + \frac{b\int \frac{1}{ab+b^2x^2}\,dx}{2a}$$
$$= \frac{x}{2a\left(a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1), x]
```

fricas [A] time = 0.81, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx-\left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)}, \frac{abx+\left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

giac [A] time = 0.15, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}+\frac{x}{2\left(bx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$\frac{x}{2\left(b\,x^2+a\right)a}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.85, size = 35, normalized size = 0.78

$$\frac{x}{2\left(abx^2+a^2\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(b\,x^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

sympy [B] time = 0.22, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

3.318 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)} dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{1}{2ax\left(a+bx^2\right)}+\frac{(3b)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a}\\ &= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{\left(3b^2\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^2}\\ &= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

[Out] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)), x]

fricas [A] time = 1.87, size = 136, normalized size = 2.39

$$\left[-\frac{6\,bx^2-3\left(bx^3+ax\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\left(a^2bx^3+a^3x\right)},-\frac{3\,bx^2+3\left(bx^3+ax\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\left(a^2bx^3+a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

giac [A] time = 0.18, size = 47, normalized size = 0.82

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bx^2+2\,a}{2\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.81

$$-\frac{bx}{2\left(b\,x^2+a\right)a^2}-\frac{3b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\,a^2}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/a^2/x-1/2/a^2*b*x/(b*x^2+a)-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)
```

maxima [A] time = 2.95, size = 49, normalized size = 0.86

$$-\frac{3\,bx^2+2\,a}{2\left(a^2bx^3+a^3x\right)}-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

mupad [B] time = 4.49, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

sympy [A] time = 0.32, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

3.319 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{1}{2ax^3\left(a+bx^2\right)}+\frac{(5b)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{2a}\\ &= -\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}-\frac{\left(5b^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a^2}\\ &= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{\left(5b^3\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^3}\\ &= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.99

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)), x]
```

fricas [A] time = 0.84, size = 172, normalized size = 2.53

$$\left[\frac{30\,b^2x^4+20\,abx^2+15\left(b^2x^5+abx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\left(a^3bx^5+a^4x^3\right)}, \frac{15\,b^2x^4+10\,abx^2+15\left(b^2x^5+abx^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-2\,a^2}{6\left(a^3bx^5+a^4x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

giac [A] time = 0.15, size = 59, normalized size = 0.87

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}+\frac{b^2x}{2\left(bx^2+a\right)a^3}+\frac{6\,bx^2-a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

maple [A] time = 0.01, size = 59, normalized size = 0.87

$$\frac{b^2x}{2\left(b\,x^2+a\right)a^3}+\frac{5b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^3}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/3/a^2/x^3+2*b/a^3/x+1/2/a^3*b^2*x/(b*x^2+a)+5/2/a^3*b^2/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.02, size = 64, normalized size = 0.94

$$\frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\left(a^3bx^5+a^4x^3\right)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.43, size = 58, normalized size = 0.85

$$\frac{\frac{5\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

sympy [A] time = 0.36, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\,\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\,\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.320 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=81

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{7b^2}{2a^4x}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{7b^2}{2a^4x}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^6\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ax^5\left(a+bx^2\right)}+\frac{(7b)\int \frac{1}{x^6\left(ab+b^2x^2\right)}\,dx}{2a}\\
&= -\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{\left(7b^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{2a^2}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}+\frac{1}{2ax^5\left(a+bx^2\right)}+\frac{\left(7b^3\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a^3}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{\left(7b^4\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^4}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.99

$$
-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{b^3x}{2a^4\left(a+bx^2\right)}-\frac{3b^2}{a^4x}+\frac{2b}{3a^3x^3}-\frac{1}{5a^2x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/5*1/(a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a
+ b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)), x]
```

fricas [A] time = 0.82, size = 198, normalized size = 2.44

$$\left[-\frac{210\,b^3x^6+140\,ab^2x^4-28\,a^2bx^2+12\,a^3-105\left(b^3x^7+ab^2x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{60\left(a^4bx^7+a^5x^5\right)},\,-\frac{105\,b^3x^6+70\,ab^2x^4-14\,a^2bx^2+6\,a^3+105\left(b^3x^7+ab^2x^5\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{30\left(a^4bx^7+a^5x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
+ a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a
^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a
^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^7 +
a^5*x^5)]
```

giac [A] time = 0.15, size = 70, normalized size = 0.86

$$-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}-\frac{b^3x}{2\left(bx^2+a\right)a^4}-\frac{45\,b^2x^4-10\,abx^2+3\,a^2}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^4
) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$-\frac{b^3x}{2\left(b\,x^2+a\right)a^4}-\frac{7b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^4}-\frac{3b^2}{a^4x}+\frac{2b}{3a^3x^3}-\frac{1}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/5/a^2/x^5-3*b^2/a^4/x+2/3*b/a^3/x^3-1/2/a^4*b^3*x/(b*x^2+a)-7/2/a^4*b^3/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.08, size = 75, normalized size = 0.93

$$-\frac{105\,b^3x^6+70\,ab^2x^4-14\,a^2bx^2+6\,a^3}{30\left(a^4bx^7+a^5x^5\right)}-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3)/(a^4*b*x^7 + a^5*
x^5) - 7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 4.71, size = 70, normalized size = 0.86

$$-\frac{\frac{1}{5\,a}-\frac{7\,b\,x^2}{15\,a^2}+\frac{7\,b^2\,x^4}{3\,a^3}+\frac{7\,b^3\,x^6}{2\,a^4}}{b\,x^7+a\,x^5}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] - (1/(5*a) - (7*b*x^2)/(15*a^2) + (7*b^2*x^4)/(3*a^3) + (7*b^3*x^6)/(2*a^4)
)/(a*x^5 + b*x^7) - (7*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(9/2))
```

sympy [A] time = 0.43, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^5}{a^9}}\,\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}-\frac{7\sqrt{-\frac{b^5}{a^9}}\,\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}+\frac{-6a^3+14a^2bx^2-70ab^2x^4-105b^3x^6}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a*
*9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 + (-6*a**3 + 14*a**2*b*x**2 - 70*
a*b**2*x**4 - 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

3.321 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}$$

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-2*a*x^2)/b^5 + x^4/(4*b^4) + a^5/(6*b^6*(a + b*x^2)^3) - (5*a^4)/(4*b^6*(
a + b*x^2)^2) + (5*a^3)/(b^6*(a + b*x^2)) + (5*a^2*Log[a + b*x^2])/b^6
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^{11}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$

$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(-\frac{4a}{b^9}+\frac{x}{b^8}-\frac{a^5}{b^9(a+bx)^4}+\frac{5a^4}{b^9(a+bx)^3}-\frac{10a^3}{b^9(a+bx)^2}+\frac{10a^2}{b^9(a+bx)}\right.\right.$$

$$= -\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}+\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.86

$$\frac{\frac{2a^5}{\left(a+bx^2\right)^3}-\frac{15a^4}{\left(a+bx^2\right)^2}+\frac{60a^3}{a+bx^2}+60a^2\log\left(a+bx^2\right)-24abx^2+3b^2x^4}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-24*a*b*x^2 + 3*b^2*x^4 + (2*a^5)/(a + b*x^2)^3 - (15*a^4)/(a + b*x^2)^2 +
 (60*a^3)/(a + b*x^2) + 60*a^2*Log[a + b*x^2])/(12*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.84, size = 137, normalized size = 1.51

$$\frac{3\,b^5x^{10}-15\,ab^4x^8-63\,a^2b^3x^6-9\,a^3b^2x^4+81\,a^4bx^2+47\,a^5+60\left(a^2b^3x^6+3\,a^3b^2x^4+3\,a^4bx^2+a^5\right)\log\left(bx^2+a\right)}{12\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 - 15*a*b^4*x^8 - 63*a^2*b^3*x^6 - 9*a^3*b^2*x^4 + 81*a^4*b
*x^2 + 47*a^5 + 60*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*log(b*
x^2 + a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)
```

giac [A] time = 0.17, size = 91, normalized size = 1.00

$$\frac{5\,a^2\log\left(\left|bx^2+a\right|\right)}{b^6}+\frac{b^4x^4-8\,ab^3x^2}{4\,b^8}-\frac{110\,a^2b^3x^6+270\,a^3b^2x^4+225\,a^4bx^2+63\,a^5}{12\,\left(bx^2+a\right)^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*log(abs(b*x^2 + a))/b^6 + 1/4*(b^4*x^4 - 8*a*b^3*x^2)/b^8 - 1/12*(110
*a^2*b^3*x^6 + 270*a^3*b^2*x^4 + 225*a^4*b*x^2 + 63*a^5)/((b*x^2 + a)^3*b^6
)
```

maple [A] time = 0.01, size = 86, normalized size = 0.95

$$\frac{x^4}{4b^4}+\frac{a^5}{6\left(b\,x^2+a\right)^3b^6}-\frac{5a^4}{4\left(b\,x^2+a\right)^2b^6}-\frac{2a\,x^2}{b^5}+\frac{5a^3}{\left(b\,x^2+a\right)b^6}+\frac{5a^2\ln\left(b\,x^2+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2*a*x^2/b^5+1/4*x^4/b^4+1/6*a^5/b^6/(b*x^2+a)^3-5/4*a^4/b^6/(b*x^2+a)^2+5*
a^3/b^6/(b*x^2+a)+5*a^2*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.39, size = 99, normalized size = 1.09

$$\frac{60\,a^3b^2x^4+105\,a^4bx^2+47\,a^5}{12\,\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)}+\frac{5\,a^2\log\left(bx^2+a\right)}{b^6}+\frac{bx^4-8\,ax^2}{4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*a^3*b^2*x^4 + 105*a^4*b*x^2 + 47*a^5)/(b^9*x^6 + 3*a*b^8*x^4 + 3*a
^2*b^7*x^2 + a^3*b^6) + 5*a^2*log(b*x^2 + a)/b^6 + 1/4*(b*x^4 - 8*a*x^2)/b^
5
```

mupad [B] time = 4.48, size = 98, normalized size = 1.08

$$\frac{\frac{47\,a^5}{12\,b}+\frac{35\,a^4\,x^2}{4}+5\,a^3\,b\,x^4}{a^3\,b^5+3\,a^2\,b^6\,x^2+3\,a\,b^7\,x^4+b^8\,x^6}+\frac{x^4}{4\,b^4}-\frac{2\,a\,x^2}{b^5}+\frac{5\,a^2\,\ln\left(b\,x^2+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((47*a^5)/(12*b) + (35*a^4*x^2)/4 + 5*a^3*b*x^4)/(a^3*b^5 + b^8*x^6 + 3*a*b
^7*x^4 + 3*a^2*b^6*x^2) + x^4/(4*b^4) - (2*a*x^2)/b^5 + (5*a^2*log(a + b*x^
2))/b^6
```

sympy [A] time = 0.63, size = 100, normalized size = 1.10

$$\frac{5a^2\log\left(a+bx^2\right)}{b^6}-\frac{2ax^2}{b^5}+\frac{47a^5+105a^4bx^2+60a^3b^2x^4}{12a^3b^6+36a^2b^7x^2+36ab^8x^4+12b^9x^6}+\frac{x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 5*a**2*log(a + b*x**2)/b**6 - 2*a*x**2/b**5 + (47*a**5 + 105*a**4*b*x**2 +
60*a**3*b**2*x**4)/(12*a**3*b**6 + 36*a**2*b**7*x**2 + 36*a*b**8*x**4 + 12*
b**9*x**6) + x**4/(4*b**4)
```

3.322 $\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=77

$$-\frac{a^4}{6b^5\left(a+bx^2\right)^3}+\frac{a^3}{b^5\left(a+bx^2\right)^2}-\frac{3a^2}{b^5\left(a+bx^2\right)}-\frac{2a\log\left(a+bx^2\right)}{b^5}+\frac{x^2}{2b^4}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^4}{6b^5\left(a+bx^2\right)^3}+\frac{a^3}{b^5\left(a+bx^2\right)^2}-\frac{3a^2}{b^5\left(a+bx^2\right)}-\frac{2a\log\left(a+bx^2\right)}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x^2/(2*b^4) - a^4/(6*b^5*(a + b*x^2)^3) + a^3/(b^5*(a + b*x^2)^2) - (3*a^2)
/(b^5*(a + b*x^2)) - (2*a*Log[a + b*x^2])/b^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^9}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{2}b^4\,\mathrm{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\,\mathrm{Subst}\left(\int \left(\frac{1}{b^8}+\frac{a^4}{b^8(a+bx)^4}-\frac{4a^3}{b^8(a+bx)^3}+\frac{6a^2}{b^8(a+bx)^2}-\frac{4a}{b^8(a+bx)}\right)dx, x,$$
$$= \frac{x^2}{2b^4}-\frac{a^4}{6b^5\left(a+bx^2\right)^3}+\frac{a^3}{b^5\left(a+bx^2\right)^2}-\frac{3a^2}{b^5\left(a+bx^2\right)}-\frac{2a\log\left(a+bx^2\right)}{b^5}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.77

$$-\frac{\frac{a^2\left(13a^2+30abx^2+18b^2x^4\right)}{\left(a+bx^2\right)^3}+12a\log\left(a+bx^2\right)-3bx^2}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(-3*b*x^2 + (a^2*(13*a^2 + 30*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 1
2*a*Log[a + b*x^2])/b^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.78, size = 124, normalized size = 1.61

$$\frac{3\,b^4x^8+9\,ab^3x^6-9\,a^2b^2x^4-27\,a^3bx^2-13\,a^4-12\left(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{6\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 9*a*b^3*x^6 - 9*a^2*b^2*x^4 - 27*a^3*b*x^2 - 13*a^4 - 12*(
a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^8*x^6 + 3
*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

giac [A] time = 0.16, size = 73, normalized size = 0.95

$$\frac{x^2}{2\,b^4} - \frac{2\,a\log\left(\left|bx^2 + a\right|\right)}{b^5} + \frac{22\,ab^3x^6 + 48\,a^2b^2x^4 + 36\,a^3bx^2 + 9\,a^4}{6\left(bx^2 + a\right)^3 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^4 - 2*a*log(abs(b*x^2 + a))/b^5 + 1/6*(22*a*b^3*x^6 + 48*a^2*b^2*
x^4 + 36*a^3*b*x^2 + 9*a^4)/((b*x^2 + a)^3*b^5)
```

maple [A] time = 0.01, size = 74, normalized size = 0.96

$$-\frac{a^4}{6\left(b\,x^2 + a\right)^3 b^5} + \frac{a^3}{\left(b\,x^2 + a\right)^2 b^5} + \frac{x^2}{2b^4} - \frac{3a^2}{\left(b\,x^2 + a\right) b^5} - \frac{2a\ln\left(b\,x^2 + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/2*x^2/b^4-1/6*a^4/b^5/(b*x^2+a)^3+a^3/b^5/(b*x^2+a)^2-3*a^2/b^5/(b*x^2+a)
-2*a*ln(b*x^2+a)/b^5
```

maxima [A] time = 1.40, size = 88, normalized size = 1.14

$$-\frac{18\,a^2b^2x^4 + 30\,a^3bx^2 + 13\,a^4}{6\left(b^8x^6 + 3\,ab^7x^4 + 3\,a^2b^6x^2 + a^3b^5\right)} + \frac{x^2}{2\,b^4} - \frac{2\,a\log\left(bx^2 + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(18*a^2*b^2*x^4 + 30*a^3*b*x^2 + 13*a^4)/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^
2*b^6*x^2 + a^3*b^5) + 1/2*x^2/b^4 - 2*a*log(b*x^2 + a)/b^5
```

mupad [B] time = 4.51, size = 88, normalized size = 1.14

$$\frac{x^2}{2\,b^4} - \frac{\frac{13\,a^4}{6\,b} + 5\,a^3\,x^2 + 3\,a^2\,b\,x^4}{a^3\,b^4 + 3\,a^2\,b^5\,x^2 + 3\,a\,b^6\,x^4 + b^7\,x^6} - \frac{2\,a\,\ln\left(b\,x^2 + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] x^2/(2*b^4) - ((13*a^4)/(6*b) + 5*a^3*x^2 + 3*a^2*b*x^4)/(a^3*b^4 + b^7*x^6
 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2) - (2*a*log(a + b*x^2))/b^5
```

sympy [A] time = 0.59, size = 90, normalized size = 1.17

$$-\frac{2a\log\left(a+bx^2\right)}{b^5}+\frac{-13a^4-30a^3bx^2-18a^2b^2x^4}{6a^3b^5+18a^2b^6x^2+18ab^7x^4+6b^8x^6}+\frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -2*a*log(a + b*x**2)/b**5 + (-13*a**4 - 30*a**3*b*x**2 - 18*a**2*b**2*x**4)
/(6*a**3*b**5 + 18*a**2*b**6*x**2 + 18*a*b**7*x**4 + 6*b**8*x**6) + x**2/(2
*b**4)
```

3.323 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=71

$$\frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^3/(6*b^4*(a + b*x^2)^3) - (3*a^2)/(4*b^4*(a + b*x^2)^2) + (3*a)/(2*b^4*(a
 + b*x^2)) + Log[a + b*x^2]/(2*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^7}{\left(ab+b^2x^2\right)^4}\,dx \\
&= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^4}\,dx,x,x^2\right) \\
&= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(-\frac{a^3}{b^7(a+bx)^4}+\frac{3a^2}{b^7(a+bx)^3}-\frac{3a}{b^7(a+bx)^2}+\frac{1}{b^7(a+bx)}\right)dx,x,x^2\right) \\
&= \frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.70

$$
\frac{\frac{a\left(11a^2+27abx^2+18b^2x^4\right)}{\left(a+bx^2\right)^3}+6\log\left(a+bx^2\right)}{12b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((a*(11*a^2 + 27*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 6*Log[a + b*x^2])/(
12*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.73, size = 102, normalized size = 1.44

$$
\frac{18\,ab^2x^4+27\,a^2bx^2+11\,a^3+6\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{12\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3 + 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*a
^2*b*x^2 + a^3)*log(b*x^2 + a))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^
3*b^4)
```

giac [A] time = 0.17, size = 53, normalized size = 0.75

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^4}-\frac{11\,b^2x^6+15\,abx^4+6\,a^2x^2}{12\,\left(bx^2+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^4 - 1/12*(11*b^2*x^6 + 15*a*b*x^4 + 6*a^2*x^2)/((
b*x^2 + a)^3*b^3)
```

maple [A] time = 0.01, size = 64, normalized size = 0.90

$$\frac{a^3}{6\left(b\,x^2+a\right)^3b^4}-\frac{3a^2}{4\left(b\,x^2+a\right)^2b^4}+\frac{3a}{2\left(b\,x^2+a\right)b^4}+\frac{\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*a^3/b^4/(b*x^2+a)^3-3/4*a^2/b^4/(b*x^2+a)^2+3/2*a/b^4/(b*x^2+a)+1/2*ln(
b*x^2+a)/b^4
```

maxima [A] time = 1.31, size = 77, normalized size = 1.08

$$\frac{18\,ab^2x^4+27\,a^2bx^2+11\,a^3}{12\,\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3)/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*
b^5*x^2 + a^3*b^4) + 1/2*log(b*x^2 + a)/b^4
```

mupad [B] time = 4.33, size = 75, normalized size = 1.06

$$\frac{\frac{11\,a^3}{12\,b^4}+\frac{3\,a\,x^4}{2\,b^2}+\frac{9\,a^2\,x^2}{4\,b^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}+\frac{\ln\left(b\,x^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((11*a^3)/(12*b^4) + (3*a*x^4)/(2*b^2) + (9*a^2*x^2)/(4*b^3))/(a^3 + b^3*x^
6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) + log(a + b*x^2)/(2*b^4)
```

sympy [**A**] time = 0.47, size = 76, normalized size = 1.07

$$\frac{11a^3 + 27a^2bx^2 + 18ab^2x^4}{12a^3b^4 + 36a^2b^5x^2 + 36ab^6x^4 + 12b^7x^6} + \frac{\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x**2 + 18*a*b**2*x**4)/(12*a**3*b**4 + 36*a**2*b**5*x*
*2 + 36*a*b**6*x**4 + 12*b**7*x**6) + log(a + b*x**2)/(2*b**4)
```

3.324 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=19

$$\frac{x^6}{6a(a+bx^2)^3}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$\frac{x^6}{6a(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x^6/(6*a*(a + b*x^2)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^2} dx &= b^4 \int \frac{x^5}{(ab+b^2x^2)^4} dx \\ &= \frac{x^6}{6a(a+bx^2)^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.84

$$-\frac{a^2+3abx^2+3b^2x^4}{6b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] -1/6*(a^2 + 3*a*b*x^2 + 3*b^2*x^4)/(b^3*(a + b*x^2)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [B] time = 0.49, size = 58, normalized size = 3.05

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
 a^3*b^3)
```

giac [A] time = 0.19, size = 33, normalized size = 1.74

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\left(bx^2+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/((b*x^2 + a)^3*b^3)
```

maple [B] time = 0.01, size = 48, normalized size = 2.53

$$-\frac{a^{2}}{6\left(b\,x^{2}+a\right)^{3} b^{3}}+\frac{a}{2\left(b\,x^{2}+a\right)^{2} b^{3}}-\frac{1}{2\left(b\,x^{2}+a\right) b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/6*a^2/b^3/(b*x^2+a)^3+1/2*a/b^3/(b*x^2+a)^2-1/2/b^3/(b*x^2+a)
```

maxima [B] time = 1.38, size = 58, normalized size = 3.05

$$-\frac{3\,b^{2}x^{4}+3\,abx^{2}+a^{2}}{6\left(b^{6}x^{6}+3\,ab^{5}x^{4}+3\,a^{2}b^{4}x^{2}+a^{3}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
 a^3*b^3)
```

mupad [B] time = 4.29, size = 60, normalized size = 3.16

$$-\frac{a^{2}+3\,a\,b\,x^{2}+3\,b^{2}\,x^{4}}{6\,a^{3}\,b^{3}+18\,a^{2}\,b^{4}\,x^{2}+18\,a\,b^{5}\,x^{4}+6\,b^{6}\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] -(a^2 + 3*b^2*x^4 + 3*a*b*x^2)/(6*a^3*b^3 + 6*b^6*x^6 + 18*a*b^5*x^4 + 18*a
^2*b^4*x^2)
```

sympy [B] time = 0.40, size = 60, normalized size = 3.16

$$\frac{-a^{2}-3abx^{2}-3b^{2}x^{4}}{6a^{3}b^{3}+18a^{2}b^{4}x^{2}+18ab^{5}x^{4}+6b^{6}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*a**3*b**3 + 18*a**2*b**4*x**2 + 18*a*
b**5*x**4 + 6*b**6*x**6)
```

3.325 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=34

$$\frac{a}{6b^2(a+bx^2)^3}-\frac{1}{4b^2(a+bx^2)^2}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a}{6b^2(a+bx^2)^3}-\frac{1}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a/(6*b^2*(a + b*x^2)^3) - 1/(4*b^2*(a + b*x^2)^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^3}{\left(ab+b^2x^2\right)^4}\,dx\\
&= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^4}\,dx,x,x^2\right)\\
&= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(-\frac{a}{b^5(a+bx)^4}+\frac{1}{b^5(a+bx)^3}\right)dx,x,x^2\right)\\
&= \frac{a}{6b^2\left(a+bx^2\right)^3}-\frac{1}{4b^2\left(a+bx^2\right)^2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$
-\frac{a+3bx^2}{12b^2\left(a+bx^2\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/12*(a + 3*b*x^2)/(b^2*(a + b*x^2)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.54, size = 47, normalized size = 1.38

$$
-\frac{3\,bx^2+a}{12\left(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

giac [A] time = 0.16, size = 22, normalized size = 0.65

$$-\frac{3\,bx^2+a}{12\left(bx^2+a\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
[Out] -1/12*(3*b*x^2 + a)/((b*x^2 + a)^3*b^2)
```

maple [A] time = 0.01, size = 31, normalized size = 0.91

$$\frac{a}{6\left(b\,x^2+a\right)^3b^2}-\frac{1}{4\left(b\,x^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
[Out] 1/6*a/b^2/(b*x^2+a)^3-1/4/b^2/(b*x^2+a)^2
```

maxima [A] time = 1.35, size = 47, normalized size = 1.38

$$-\frac{3\,bx^2+a}{12\left(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

mupad [B] time = 4.23, size = 48, normalized size = 1.41

$$-\frac{\frac{a}{12\,b^2}+\frac{x^2}{4\,b}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
[Out] -(a/(12*b^2) + x^2/(4*b))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)
```

sympy [A] time = 0.37, size = 48, normalized size = 1.41

$$\frac{-a-3bx^2}{12a^3b^2+36a^2b^3x^2+36ab^4x^4+12b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-a - 3*b*x**2)/(12*a**3*b**2 + 36*a**2*b**3*x**2 + 36*a*b**4*x**4 + 12*b**
5*x**6)
```

3.326 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/(6*b*(a + b*x^2)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x}{\left(ab+b^2x^2\right)^4}\,dx\\ &= -\frac{1}{6b\left(a+bx^2\right)^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] -1/6*1/(b*(a + b*x^2)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [B] time = 0.80, size = 37, normalized size = 2.31

$$-\frac{1}{6\left(b^4x^6+3\,ab^3x^4+3\,a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{1}{6\left(bx^2+a\right)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
[Out] -1/6/((b*x^2 + a)^3*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{6\left(b\,x^2+a\right)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/6/b/(b*x^2+a)^3
```

maxima [B] time = 1.34, size = 37, normalized size = 2.31

$$-\frac{1}{6\left(b^4x^6+3\,ab^3x^4+3\,a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

mupad [B] time = 4.28, size = 39, normalized size = 2.44

$$-\frac{1}{6\,a^3\,b+18\,a^2\,b^2\,x^2+18\,a\,b^3\,x^4+6\,b^4\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] -1/(6*a^3*b + 6*b^4*x^6 + 18*a*b^3*x^4 + 18*a^2*b^2*x^2)
```

sympy [B] time = 0.33, size = 39, normalized size = 2.44

$$-\frac{1}{6a^3b+18a^2b^2x^2+18ab^3x^4+6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -1/(6*a**3*b + 18*a**2*b**2*x**2 + 18*a*b**3*x**4 + 6*b**4*x**6)
```

3.327 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)}{a^4}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{6a\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{1}{2a^3\left(a+bx^2\right)}+\frac{1}{4a^2\left(a+bx^2\right)^2}-\frac{\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)}{a^4}+\frac{1}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] 1/(6*a*(a + b*x^2)^3) + 1/(4*a^2*(a + b*x^2)^2) + 1/(2*a^3*(a + b*x^2)) + L
og[x]/a^4 - Log[a + b*x^2]/(2*a^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{2}b^4\,\mathrm{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\,\mathrm{Subst}\left(\int \left(\frac{1}{a^4b^4x} - \frac{1}{ab^3(a+bx)^4} - \frac{1}{a^2b^3(a+bx)^3} - \frac{1}{a^3b^3(a+bx)^2} - \frac{1}{a^4b^3(a\right.\right.$$
$$= \frac{1}{6a\left(a+bx^2\right)^3} + \frac{1}{4a^2\left(a+bx^2\right)^2} + \frac{1}{2a^3\left(a+bx^2\right)} + \frac{\log(x)}{a^4} - \frac{\log\left(a+bx^2\right)}{2a^4}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.77

$$\frac{\frac{a\left(11a^2+15abx^2+6b^2x^4\right)}{\left(a+bx^2\right)^3} - 6\log\left(a+bx^2\right) + 12\log(x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x^2 + 6*b^2*x^4))/(a + b*x^2)^3 + 12*Log[x] - 6*Log[a
+ b*x^2])/(12*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2), x]
```

fricas [B] time = 2.42, size = 134, normalized size = 1.91

$$\frac{6\,ab^2x^4 + 15\,a^2bx^2 + 11\,a^3 - 6\left(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3\right)\log\left(bx^2+a\right) + 12\left(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3\right)\log(x)}{12\left(a^4b^3x^6 + 3\,a^5b^2x^4 + 3\,a^6bx^2 + a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*b^2*x^4 + 15*a^2*b*x^2 + 11*a^3 - 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^
2*b*x^2 + a^3)*log(b*x^2 + a) + 12*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3)*log(x))/(a^4*b^3*x^6 + 3*a^5*b^2*x^4 + 3*a^6*b*x^2 + a^7)
```

giac [A] time = 0.15, size = 70, normalized size = 1.00

$$\frac{\log\left(x^{2}\right)}{2\,a^{4}}-\frac{\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{4}}+\frac{11\,b^{3}x^{6}+39\,ab^{2}x^{4}+48\,a^{2}bx^{2}+22\,a^{3}}{12\,\left(bx^{2}+a\right)^{3}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^4 - 1/2*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6 + 39*a*b^
2*x^4 + 48*a^2*b*x^2 + 22*a^3)/((b*x^2 + a)^3*a^4)
```

maple [A] time = 0.01, size = 63, normalized size = 0.90

$$\frac{1}{6\left(b\,x^{2}+a\right)^{3}a}+\frac{1}{4\left(b\,x^{2}+a\right)^{2}a^{2}}+\frac{1}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{\ln(x)}{a^{4}}-\frac{\ln\left(b\,x^{2}+a\right)}{2a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6/a/(b*x^2+a)^3+1/4/a^2/(b*x^2+a)^2+1/2/a^3/(b*x^2+a)+ln(x)/a^4-1/2*ln(b*
x^2+a)/a^4
```

maxima [A] time = 1.42, size = 82, normalized size = 1.17

$$\frac{6\,b^{2}x^{4}+15\,abx^{2}+11\,a^{2}}{12\left(a^{3}b^{3}x^{6}+3\,a^{4}b^{2}x^{4}+3\,a^{5}bx^{2}+a^{6}\right)}-\frac{\log\left(bx^{2}+a\right)}{2\,a^{4}}+\frac{\log\left(x^{2}\right)}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^2*x^4 + 15*a*b*x^2 + 11*a^2)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5
*b*x^2 + a^6) - 1/2*log(b*x^2 + a)/a^4 + 1/2*log(x^2)/a^4
```

mupad [B] time = 4.47, size = 78, normalized size = 1.11

$$\frac{\ln(x)}{a^{4}}+\frac{\frac{11}{12\,a}+\frac{5\,b\,x^{2}}{4\,a^{2}}+\frac{b^{2}\,x^{4}}{2\,a^{3}}}{a^{3}+3\,a^{2}\,b\,x^{2}+3\,a\,b^{2}\,x^{4}+b^{3}\,x^{6}}-\frac{\ln\left(b\,x^{2}+a\right)}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] log(x)/a^4 + (11/(12*a) + (5*b*x^2)/(4*a^2) + (b^2*x^4)/(2*a^3))/(a^3 + b^3
*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) - log(a + b*x^2)/(2*a^4)
```

sympy [A] time = 0.56, size = 80, normalized size = 1.14

$$\frac{11a^2 + 15abx^2 + 6b^2x^4}{12a^6 + 36a^5bx^2 + 36a^4b^2x^4 + 12a^3b^3x^6} + \frac{\log(x)}{a^4} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**2 + 15*a*b*x**2 + 6*b**2*x**4)/(12*a**6 + 36*a**5*b*x**2 + 36*a**4*b
**2*x**4 + 12*a**3*b**3*x**6) + log(x)/a**4 - log(a/b + x**2)/(2*a**4)
```

3.328 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=84

$$\frac{2b\log\left(a+bx^2\right)}{a^5}-\frac{4b\log(x)}{a^5}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{1}{2a^4x^2}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}+\frac{2b\log\left(a+bx^2\right)}{a^5}-\frac{4b\log(x)}{a^5}-\frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/(2*a^4*x^2) - b/(6*a^2*(a + b*x^2)^3) - b/(2*a^3*(a + b*x^2)^2) - (3*b)/
(2*a^4*(a + b*x^2)) - (4*b*Log[x])/a^5 + (2*b*Log[a + b*x^2])/a^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a^2+2abx^2+b^2x^4\right)^2} \, dx = b^4 \int \frac{1}{x^3 \left(ab+b^2x^2\right)^4} \, dx$$
$$= \frac{1}{2} b^4 \operatorname{Subst}\left(\int \frac{1}{x^2 \left(ab+b^2x\right)^4} \, dx, x, x^2\right)$$
$$= \frac{1}{2} b^4 \operatorname{Subst}\left(\int \left(\frac{1}{a^4b^4x^2} - \frac{4}{a^5b^3x} + \frac{1}{a^2b^2(a+bx)^4} + \frac{2}{a^3b^2(a+bx)^3} + \frac{3}{a^4b^2(a+bx}\right.\right.$$
$$= -\frac{1}{2a^4x^2} - \frac{b}{6a^2\left(a+bx^2\right)^3} - \frac{b}{2a^3\left(a+bx^2\right)^2} - \frac{3b}{2a^4\left(a+bx^2\right)} - \frac{4b\log(x)}{a^5} + \frac{2b\log}{}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 0.83

$$-\frac{\frac{a\left(3a^3+22a^2bx^2+30ab^2x^4+12b^3x^6\right)}{x^2\left(a+bx^2\right)^3} - 12b\log\left(a+bx^2\right) + 24b\log(x)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/6*((a*(3*a^3 + 22*a^2*b*x^2 + 30*a*b^2*x^4 + 12*b^3*x^6))/(x^2*(a + b*x^
2)^3) + 24*b*Log[x] - 12*b*Log[a + b*x^2])/a^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a^2+2abx^2+b^2x^4\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2), x]
```

fricas [B] time = 0.92, size = 163, normalized size = 1.94

$$-\frac{12\,ab^3x^6 + 30\,a^2b^2x^4 + 22\,a^3bx^2 + 3\,a^4 - 12\left(b^4x^8 + 3\,ab^3x^6 + 3\,a^2b^2x^4 + a^3bx^2\right)\log\left(bx^2+a\right) + 24\left(b^4x^8 + 3\,ab^3x^6 + 3\,a^2b^2x^4 + a^3bx^2\right)\log(x)}{6\left(a^5b^3x^8 + 3\,a^6b^2x^6 + 3\,a^7bx^4 + a^8x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*b^3*x^6 + 30*a^2*b^2*x^4 + 22*a^3*b*x^2 + 3*a^4 - 12*(b^4*x^8 +
3*a*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(b*x^2 + a) + 24*(b^4*x^8 + 3*a
*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(x))/(a^5*b^3*x^8 + 3*a^6*b^2*x^6
+ 3*a^7*b*x^4 + a^8*x^2)
```

giac [A] time = 0.16, size = 93, normalized size = 1.11

$$-\frac{2\,b\log\left(x^2\right)}{a^5}+\frac{2\,b\log\left(\left|bx^2+a\right|\right)}{a^5}+\frac{4\,bx^2-a}{2\,a^5x^2}-\frac{22\,b^4x^6+75\,ab^3x^4+87\,a^2b^2x^2+35\,a^3b}{6\left(bx^2+a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(x^2)/a^5 + 2*b*log(abs(b*x^2 + a))/a^5 + 1/2*(4*b*x^2 - a)/(a^5*x^
2) - 1/6*(22*b^4*x^6 + 75*a*b^3*x^4 + 87*a^2*b^2*x^2 + 35*a^3*b)/((b*x^2 +
a)^3*a^5)
```

maple [A] time = 0.02, size = 77, normalized size = 0.92

$$-\frac{b}{6\left(b\,x^2+a\right)^3a^2}-\frac{b}{2\left(b\,x^2+a\right)^2a^3}-\frac{3b}{2\left(b\,x^2+a\right)a^4}-\frac{4b\ln(x)}{a^5}+\frac{2b\ln\left(b\,x^2+a\right)}{a^5}-\frac{1}{2a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/2/a^4/x^2-1/6*b/a^2/(b*x^2+a)^3-1/2*b/a^3/(b*x^2+a)^2-3/2*b/a^4/(b*x^2+a
)-4*b*ln(x)/a^5+2*b*ln(b*x^2+a)/a^5
```

maxima [A] time = 1.40, size = 99, normalized size = 1.18

$$-\frac{12\,b^3x^6+30\,ab^2x^4+22\,a^2bx^2+3\,a^3}{6\left(a^4b^3x^8+3\,a^5b^2x^6+3\,a^6bx^4+a^7x^2\right)}+\frac{2\,b\log\left(bx^2+a\right)}{a^5}-\frac{2\,b\log\left(x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(12*b^3*x^6 + 30*a*b^2*x^4 + 22*a^2*b*x^2 + 3*a^3)/(a^4*b^3*x^8 + 3*a^
5*b^2*x^6 + 3*a^6*b*x^4 + a^7*x^2) + 2*b*log(b*x^2 + a)/a^5 - 2*b*log(x^2)/
a^5
```

mupad [B] time = 0.15, size = 97, normalized size = 1.15

$$\frac{2\,b\,\ln\left(b\,x^2+a\right)}{a^5}-\frac{\frac{1}{2\,a}+\frac{11\,b\,x^2}{3\,a^2}+\frac{5\,b^2\,x^4}{a^3}+\frac{2\,b^3\,x^6}{a^4}}{a^3\,x^2+3\,a^2\,b\,x^4+3\,a\,b^2\,x^6+b^3\,x^8}-\frac{4\,b\,\ln(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] (2*b*log(a + b*x^2))/a^5 - (1/(2*a) + (11*b*x^2)/(3*a^2) + (5*b^2*x^4)/a^3
+ (2*b^3*x^6)/a^4)/(a^3*x^2 + b^3*x^8 + 3*a^2*b*x^4 + 3*a*b^2*x^6) - (4*b*l
og(x))/a^5
```

sympy [A] time = 0.67, size = 102, normalized size = 1.21

$$\frac{-3a^3-22a^2bx^2-30ab^2x^4-12b^3x^6}{6a^7x^2+18a^6bx^4+18a^5b^2x^6+6a^4b^3x^8}-\frac{4b\log(x)}{a^5}+\frac{2b\log\left(\frac{a}{b}+x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-3*a**3 - 22*a**2*b*x**2 - 30*a*b**2*x**4 - 12*b**3*x**6)/(6*a**7*x**2 + 1
8*a**6*b*x**4 + 18*a**5*b**2*x**6 + 6*a**4*b**3*x**8) - 4*b*log(x)/a**5 + 2
*b*log(a/b + x**2)/a**5
```

3.329 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{5b^2\log\left(a+bx^2\right)}{a^6}+\frac{10b^2\log(x)}{a^6}+\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{2b}{a^5x^2}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}-\frac{1}{4a^4x^4}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}-\frac{5b^2\log\left(a+bx^2\right)}{a^6}+\frac{10b^2\log(x)}{a^6}+\frac{2b}{a^5x^2}-\frac{1}{4a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/(4*a^4*x^4) + (2*b)/(a^5*x^2) + b^2/(6*a^3*(a + b*x^2)^3) + (3*b^2)/(4*a
^4*(a + b*x^2)^2) + (3*b^2)/(a^5*(a + b*x^2)) + (10*b^2*Log[x])/a^6 - (5*b^
2*Log[a + b*x^2])/a^6
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^5\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{1}{x^3\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$

$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(\frac{1}{a^4b^4x^3} - \frac{4}{a^5b^3x^2} + \frac{10}{a^6b^2x} - \frac{1}{a^3b(a+bx)^4} - \frac{3}{a^4b(a+bx)^3} - \frac{}{a^5b(}\right.\right.$$

$$= -\frac{1}{4a^4x^4} + \frac{2b}{a^5x^2} + \frac{b^2}{6a^3\left(a+bx^2\right)^3} + \frac{3b^2}{4a^4\left(a+bx^2\right)^2} + \frac{3b^2}{a^5\left(a+bx^2\right)} + \frac{10b^2\log(x)}{a^6}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 0.84

$$\frac{\frac{a\left(-3a^4+15a^3bx^2+110a^2b^2x^4+150ab^3x^6+60b^4x^8\right)}{x^4\left(a+bx^2\right)^3} - 60b^2\log\left(a+bx^2\right) + 120b^2\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x^2 + 110*a^2*b^2*x^4 + 150*a*b^3*x^6 + 60*b^4*x^8))
/(x^4*(a + b*x^2)^3) + 120*b^2*Log[x] - 60*b^2*Log[a + b*x^2])/(12*a^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2), x]
```

fricas [A] time = 4.94, size = 178, normalized size = 1.76

$$\frac{60\,ab^4x^8 + 150\,a^2b^3x^6 + 110\,a^3b^2x^4 + 15\,a^4bx^2 - 3\,a^5 - 60\left(b^5x^{10} + 3\,ab^4x^8 + 3\,a^2b^3x^6 + a^3b^2x^4\right)\log\left(bx^2+a\right) + 120\left(b^5x^{10} + 3\,ab^4x^8 + 3\,a^2b^3x^6 + a^3b^2x^4\right)\log(x)}{12\left(a^6b^3x^{10} + 3\,a^7b^2x^8 + 3\,a^8bx^6 + a^9x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^8 + 150*a^2*b^3*x^6 + 110*a^3*b^2*x^4 + 15*a^4*b*x^2 - 3*a
^5 - 60*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(b*x^2 +
a) + 120*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(x))/(a^
6*b^3*x^10 + 3*a^7*b^2*x^8 + 3*a^8*b*x^6 + a^9*x^4)
```

giac [A] time = 0.16, size = 108, normalized size = 1.07

$$\frac{5\,b^2\log\left(x^2\right)}{a^6}-\frac{5\,b^2\log\left(\left|bx^2+a\right|\right)}{a^6}+\frac{110\,b^5x^6+366\,ab^4x^4+411\,a^2b^3x^2+157\,a^3b^2}{12\left(bx^2+a\right)^3a^6}-\frac{30\,b^2x^4-8\,abx^2+a^2}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5*b^2*log(x^2)/a^6 - 5*b^2*log(abs(b*x^2 + a))/a^6 + 1/12*(110*b^5*x^6 + 36
6*a*b^4*x^4 + 411*a^2*b^3*x^2 + 157*a^3*b^2)/((b*x^2 + a)^3*a^6) - 1/4*(30*
b^2*x^4 - 8*a*b*x^2 + a^2)/(a^6*x^4)
```

maple [A] time = 0.02, size = 96, normalized size = 0.95

$$\frac{b^2}{6\left(b\,x^2+a\right)^3a^3}+\frac{3b^2}{4\left(b\,x^2+a\right)^2a^4}+\frac{3b^2}{\left(b\,x^2+a\right)a^5}+\frac{10b^2\ln(x)}{a^6}-\frac{5b^2\ln\left(b\,x^2+a\right)}{a^6}+\frac{2b}{a^5x^2}-\frac{1}{4a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/4/a^4/x^4+2*b/a^5/x^2+1/6*b^2/a^3/(b*x^2+a)^3+3/4*b^2/a^4/(b*x^2+a)^2+3*
b^2/a^5/(b*x^2+a)+10*b^2*ln(x)/a^6-5*b^2*ln(b*x^2+a)/a^6
```

maxima [A] time = 1.42, size = 114, normalized size = 1.13

$$\frac{60\,b^4x^8+150\,ab^3x^6+110\,a^2b^2x^4+15\,a^3bx^2-3\,a^4}{12\left(a^5b^3x^{10}+3\,a^6b^2x^8+3\,a^7bx^6+a^8x^4\right)}-\frac{5\,b^2\log\left(bx^2+a\right)}{a^6}+\frac{5\,b^2\log\left(x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^8 + 150*a*b^3*x^6 + 110*a^2*b^2*x^4 + 15*a^3*b*x^2 - 3*a^4)/
(a^5*b^3*x^10 + 3*a^6*b^2*x^8 + 3*a^7*b*x^6 + a^8*x^4) - 5*b^2*log(b*x^2 +
a)/a^6 + 5*b^2*log(x^2)/a^6
```

mupad [B] time = 4.66, size = 111, normalized size = 1.10

$$\frac{\frac{5\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{55\,b^2\,x^4}{6\,a^3}+\frac{25\,b^3\,x^6}{2\,a^4}+\frac{5\,b^4\,x^8}{a^5}}{a^3\,x^4+3\,a^2\,b\,x^6+3\,a\,b^2\,x^8+b^3\,x^{10}}-\frac{5\,b^2\,\ln\left(b\,x^2+a\right)}{a^6}+\frac{10\,b^2\,\ln(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((5*b*x^2)/(4*a^2) - 1/(4*a) + (55*b^2*x^4)/(6*a^3) + (25*b^3*x^6)/(2*a^4)
+ (5*b^4*x^8)/a^5)/(a^3*x^4 + b^3*x^10 + 3*a^2*b*x^6 + 3*a*b^2*x^8) - (5*b^
2*log(a + b*x^2))/a^6 + (10*b^2*log(x))/a^6
```

sympy [A] time = 0.72, size = 116, normalized size = 1.15

$$\frac{-3a^4+15a^3bx^2+110a^2b^2x^4+150ab^3x^6+60b^4x^8}{12a^8x^4+36a^7bx^6+36a^6b^2x^8+12a^5b^3x^{10}}+\frac{10b^2\log\left(x\right)}{a^6}-\frac{5b^2\log\left(\frac{a}{b}+x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x**2 + 110*a**2*b**2*x**4 + 150*a*b**3*x**6 + 60*b**4*
x**8)/(12*a**8*x**4 + 36*a**7*b*x**6 + 36*a**6*b**2*x**8 + 12*a**5*b**3*x**
10) + 10*b**2*log(x)/a**6 - 5*b**2*log(a/b + x**2)/a**6
```

3.330 $\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=117

$$-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{13/2}}+\frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{231x^5}{80b^4}$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{231a^2x}{16b^6}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{13/2}}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{77ax^3}{16b^5}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{231x^5}{80b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (231*a^2*x)/(16*b^6) - (77*a*x^3)/(16*b^5) + (231*x^5)/(80*b^4) - x^11/(6*b
*(a + b*x^2)^3) - (11*x^9)/(24*b^2*(a + b*x^2)^2) - (33*x^7)/(16*b^3*(a + b
*x^2)) - (231*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^{12}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{1}{6}\left(11b^2\right)\int \frac{x^{10}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}+\frac{33}{8}\int \frac{x^8}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}+\frac{231\int \frac{x^6}{ab+b^2x^2}\,dx}{16b^2}$$

$$= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}+\frac{231\int \left(\frac{a^2}{b^4}-\frac{ax^2}{b^3}+\frac{x^4}{b^2}-\frac{a^3}{b^3(ab+\cdots}\right.}{16b^2}$$

$$= \frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}+\frac{231x^5}{80b^4}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\cdots$$

$$= \frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}+\frac{231x^5}{80b^4}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\cdots$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.85

$$\frac{3465a^5x+9240a^4bx^3+7623a^3b^2x^5+1584a^2b^3x^7-176ab^4x^9+48b^5x^{11}}{240b^6\left(a+bx^2\right)^3}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

[Out] (3465*a^5*x + 9240*a^4*b*x^3 + 7623*a^3*b^2*x^5 + 1584*a^2*b^3*x^7 - 176*a*
b^4*x^9 + 48*b^5*x^11)/(240*b^6*(a + b*x^2)^3) - (231*a^(5/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(16*b^(13/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.58, size = 322, normalized size = 2.75

$$\left[\frac{96\,b^5x^{11}-352\,ab^4x^9+3168\,a^2b^3x^7+15246\,a^3b^2x^5+18480\,a^4bx^3+6930\,a^5x+3465\left(a^2b^3x^6+3\,a^3b^2x^4+3\,a^4bx^2+a^5\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{480\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)},\ \frac{48\,b^5x^{11}-176\,ab^4x^9+1584\,a^2b^3x^7+7623\,a^3b^2x^5+9240\,a^4bx^3+3465\,a^5x-3465\left(a^2b^3x^6+3\,a^3b^2x^4+3\,a^4bx^2+a^5\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{240\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/480*(96*b^5*x^11 - 352*a*b^4*x^9 + 3168*a^2*b^3*x^7 + 15246*a^3*b^2*x^5
+ 18480*a^4*b*x^3 + 6930*a^5*x + 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*
b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b
^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6), 1/240*(48*b^5*x^11 - 176*a
*b^4*x^9 + 1584*a^2*b^3*x^7 + 7623*a^3*b^2*x^5 + 9240*a^4*b*x^3 + 3465*a^5*
x - 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan
(b*x*sqrt(a/b)/a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)]
```

giac [A] time = 0.15, size = 96, normalized size = 0.82

$$-\frac{231\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^6}+\frac{267\,a^3b^2x^5+472\,a^4bx^3+213\,a^5x}{48\left(bx^2+a\right)^3b^6}+\frac{3\,b^{16}x^5-20\,ab^{15}x^3+150\,a^2b^{14}x}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/48*(267*a^3*b^2*x^5 +
 472*a^4*b*x^3 + 213*a^5*x)/((b*x^2 + a)^3*b^6) + 1/15*(3*b^16*x^5 - 20*a*b
^15*x^3 + 150*a^2*b^14*x)/b^20
```

maple [A] time = 0.02, size = 108, normalized size = 0.92

$$\frac{89a^3x^5}{16\left(bx^2+a\right)^3b^4}+\frac{59a^4x^3}{6\left(bx^2+a\right)^3b^5}+\frac{x^5}{5b^4}+\frac{71a^5x}{16\left(bx^2+a\right)^3b^6}-\frac{4a\,x^3}{3b^5}-\frac{231a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;b^6}+\frac{10a^2x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/5*x^5/b^4-4/3*a*x^3/b^5+10*a^2*x/b^6+89/16/b^4*a^3/(b*x^2+a)^3*x^5+59/6/b
^5*a^4/(b*x^2+a)^3*x^3+71/16/b^6*a^5/(b*x^2+a)^3*x-231/16/b^6*a^3/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 116, normalized size = 0.99

$$\frac{267\,a^3b^2x^5+472\,a^4bx^3+213\,a^5x}{48\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)}-\frac{231\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^6}+\frac{3\,b^2x^5-20\,abx^3+150\,a^2x}{15\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(267*a^3*b^2*x^5 + 472*a^4*b*x^3 + 213*a^5*x)/(b^9*x^6 + 3*a*b^8*x^4 +
 3*a^2*b^7*x^2 + a^3*b^6) - 231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6
) + 1/15*(3*b^2*x^5 - 20*a*b*x^3 + 150*a^2*x)/b^6
```

mupad [B] time = 0.06, size = 109, normalized size = 0.93

$$\frac{\frac{71\,a^5\,x}{16}+\frac{59\,a^4\,b\,x^3}{6}+\frac{89\,a^3\,b^2\,x^5}{16}}{a^3\,b^6+3\,a^2\,b^7\,x^2+3\,a\,b^8\,x^4+b^9\,x^6}+\frac{x^5}{5\,b^4}-\frac{4\,a\,x^3}{3\,b^5}+\frac{10\,a^2\,x}{b^6}-\frac{231\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((71*a^5*x)/16 + (59*a^4*b*x^3)/6 + (89*a^3*b^2*x^5)/16)/(a^3*b^6 + b^9*x^6
 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2) + x^5/(5*b^4) - (4*a*x^3)/(3*b^5) + (10*a^2
*x)/b^6 - (231*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(16*b^(13/2))
```

sympy [A] time = 0.66, size = 172, normalized size = 1.47

$$\frac{10a^2x}{b^6}-\frac{4ax^3}{3b^5}+\frac{231\sqrt{-\frac{a^5}{b^{13}}}\,\log\left(x-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32}-\frac{231\sqrt{-\frac{a^5}{b^{13}}}\,\log\left(x+\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32}+\frac{213a^5x+472a^4bx^3+267a^3b^2x^5}{48a^3b^6+144a^2b^7x^2+144ab^8x^4+48b^9x^6}+\frac{x^5}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 10*a**2*x/b**6 - 4*a*x**3/(3*b**5) + 231*sqrt(-a**5/b**13)*log(x - b**6*sqr
t(-a**5/b**13)/a**2)/32 - 231*sqrt(-a**5/b**13)*log(x + b**6*sqrt(-a**5/b**
13)/a**2)/32 + (213*a**5*x + 472*a**4*b*x**3 + 267*a**3*b**2*x**5)/(48*a**3
*b**6 + 144*a**2*b**7*x**2 + 144*a*b**8*x**4 + 48*b**9*x**6) + x**5/(5*b**4
)
```

3.331 $\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=104

$$\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{11/2}}-\frac{105ax}{16b^5}-\frac{21x^5}{16b^3\left(a+bx^2\right)}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{x^9}{6b\left(a+bx^2\right)^3}+\frac{35x^3}{16b^4}$$

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{11/2}}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}-\frac{105ax}{16b^5}-\frac{x^9}{6b\left(a+bx^2\right)^3}+\frac{35x^3}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-105*a*x)/(16*b^5) + (35*x^3)/(16*b^4) - x^9/(6*b*(a + b*x^2)^3) - (3*x^7)
/(8*b^2*(a + b*x^2)^2) - (21*x^5)/(16*b^3*(a + b*x^2)) + (105*a^(3/2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^{10}}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}+\frac{1}{2}\left(3b^2\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^3}\,dx$$
$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}+\frac{21}{8}\int \frac{x^6}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105\int \frac{x^4}{ab+b^2x^2}\,dx}{16b^2}$$
$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2\left(ab+b^2x^2\right)}\right.}{16b^2}$$
$$= -\frac{105ax}{16b^5}+\frac{35x^3}{16b^4}-\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{\left(105a^2\right)\int}{16b}$$
$$= -\frac{105ax}{16b^5}+\frac{35x^3}{16b^4}-\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105a^{3/2}\tan}{16b^{1}}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 0.86

$$\frac{315a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\frac{\sqrt{b}\,x\left(-315a^4-840a^3bx^2-693a^2b^2x^4-144ab^3x^6+16b^4x^8\right)}{\left(a+bx^2\right)^3}}{48b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((Sqrt[b]*x*(-315*a^4 - 840*a^3*b*x^2 - 693*a^2*b^2*x^4 - 144*a*b^3*x^6 + 1
6*b^4*x^8))/(a + b*x^2)^3 + 315*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(48*b^
(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 1.18, size = 296, normalized size = 2.85

$$\left[\frac{32\,b^4x^9-288\,ab^3x^7-1386\,a^2b^2x^5-1680\,a^3bx^3-630\,a^4x+315\left(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{96\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)},\ \frac{16\,b^4x^9-144\,ab^3x^7-693\,a^2b^2x^5-840\,a^3bx^3-315\,a^4x+315\left(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{48\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(32*b^4*x^9 - 288*a*b^3*x^7 - 1386*a^2*b^2*x^5 - 1680*a^3*b*x^3 - 630
*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*sqrt(-a/b)*log
((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^
2*b^6*x^2 + a^3*b^5), 1/48*(16*b^4*x^9 - 144*a*b^3*x^7 - 693*a^2*b^2*x^5 -
840*a^3*b*x^3 - 315*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 +
a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*
x^2 + a^3*b^5)]
```

giac [A] time = 0.16, size = 84, normalized size = 0.81

$$\frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^5}-\frac{165\,a^2b^2x^5+280\,a^3bx^3+123\,a^4x}{48\left(bx^2+a\right)^3b^5}+\frac{b^8x^3-12\,ab^7x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/48*(165*a^2*b^2*x^5 +
280*a^3*b*x^3 + 123*a^4*x)/((b*x^2 + a)^3*b^5) + 1/3*(b^8*x^3 - 12*a*b^7*x)
/b^12
```

maple [A] time = 0.01, size = 97, normalized size = 0.93

$$-\frac{55a^2x^5}{16\left(b\,x^2+a\right)^3 b^3}-\frac{35a^3x^3}{6\left(b\,x^2+a\right)^3 b^4}-\frac{41a^4x}{16\left(b\,x^2+a\right)^3 b^5}+\frac{x^3}{3b^4}+\frac{105a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;b^5}-\frac{4ax}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/3*x^3/b^4-4*a*x/b^5-55/16/b^3*a^2/(b*x^2+a)^3*x^5-35/6/b^4*a^3/(b*x^2+a)^
3*x^3-41/16/b^5*a^4/(b*x^2+a)^3*x+105/16/b^5*a^2/(a*b)^(1/2)*arctan(1/(a*b)
^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 104, normalized size = 1.00

$$-\frac{165\,a^2b^2x^5+280\,a^3bx^3+123\,a^4x}{48\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}+\frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^5}+\frac{bx^3-12\,ax}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(165*a^2*b^2*x^5 + 280*a^3*b*x^3 + 123*a^4*x)/(b^8*x^6 + 3*a*b^7*x^4
+ 3*a^2*b^6*x^2 + a^3*b^5) + 105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^
5) + 1/3*(b*x^3 - 12*a*x)/b^5
```

mupad [B] time = 4.36, size = 99, normalized size = 0.95

$$\frac{x^3}{3\,b^4}-\frac{\frac{41\,a^4\,x}{16}+\frac{35\,a^3\,b\,x^3}{6}+\frac{55\,a^2\,b^2\,x^5}{16}}{a^3\,b^5+3\,a^2\,b^6\,x^2+3\,a\,b^7\,x^4+b^8\,x^6}+\frac{105\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{11/2}}-\frac{4\,a\,x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] x^3/(3*b^4) - ((41*a^4*x)/16 + (35*a^3*b*x^3)/6 + (55*a^2*b^2*x^5)/16)/(a^3
*b^5 + b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2) + (105*a^(3/2)*atan((b^(1/2)*
x)/a^(1/2)))/(16*b^(11/2)) - (4*a*x)/b^5
```

sympy [A] time = 0.63, size = 156, normalized size = 1.50

$$-\frac{4ax}{b^5}-\frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32}+\frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32}+\frac{-123a^4x-280a^3bx^3-165a^2b^2x^5}{48a^3b^5+144a^2b^6x^2+144ab^7x^4+48b^8x^6}+\frac{x^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -4*a*x/b**5 - 105*sqrt(-a**3/b**11)*log(x - b**5*sqrt(-a**3/b**11)/a)/32 +
105*sqrt(-a**3/b**11)*log(x + b**5*sqrt(-a**3/b**11)/a)/32 + (-123*a**4*x -
 280*a**3*b*x**3 - 165*a**2*b**2*x**5)/(48*a**3*b**5 + 144*a**2*b**6*x**2 +
 144*a*b**7*x**4 + 48*b**8*x**6) + x**3/(3*b**4)
```

3.332 $\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{x^7}{6b\left(a+bx^2\right)^3}+\frac{35x}{16b^4}$$

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 321, 205}

$$-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}-\frac{x^7}{6b\left(a+bx^2\right)^3}+\frac{35x}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (35*x)/(16*b^4) - x^7/(6*b*(a + b*x^2)^3) - (7*x^5)/(24*b^2*(a + b*x^2)^2)
- (35*x^3)/(48*b^3*(a + b*x^2)) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(16*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^8}{\left(ab+b^2x^2\right)^4}\,dx\\
&= -\frac{x^7}{6b\left(a+bx^2\right)^3}+\frac{1}{6}\left(7b^2\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^3}\,dx\\
&= -\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}+\frac{35}{24}\int \frac{x^4}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}+\frac{35\int \frac{x^2}{ab+b^2x^2}\,dx}{16b^2}\\
&= \frac{35x}{16b^4}-\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{(35a)\int \frac{1}{ab+b^2x^2}\,dx}{16b^3}\\
&= \frac{35x}{16b^4}-\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{35\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.83

$$\frac{105a^3x+280a^2bx^3+231ab^2x^5+48b^3x^7}{48b^4\left(a+bx^2\right)^3}-\frac{35\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (105*a^3*x + 280*a^2*b*x^3 + 231*a*b^2*x^5 + 48*b^3*x^7)/(48*b^4*(a + b*x^2
)^3) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.52, size = 268, normalized size = 2.88

$$\left[\frac{96\,b^3x^7+462\,ab^2x^5+560\,a^2bx^3+210\,a^3x+105\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{96\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)}, \frac{48\,b^3x^7+231\,ab^2x^5+280\,a^2bx^3+105\,a^3x-105\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{48\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(96*b^3*x^7 + 462*a*b^2*x^5 + 560*a^2*b*x^3 + 210*a^3*x + 105*(b^3*x^
6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/
b) - a)/(b*x^2 + a)))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4), 1/
48*(48*b^3*x^7 + 231*a*b^2*x^5 + 280*a^2*b*x^3 + 105*a^3*x - 105*(b^3*x^6 +
 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x
^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.16, size = 65, normalized size = 0.70

$$-\frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^4}+\frac{x}{b^4}+\frac{87\,ab^2x^5+136\,a^2bx^3+57\,a^3x}{48\,\left(bx^2+a\right)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x/b^4 + 1/48*(87*a*b^2*x^5
 + 136*a^2*b*x^3 + 57*a^3*x)/((b*x^2 + a)^3*b^4)
```

maple [A] time = 0.01, size = 83, normalized size = 0.89

$$\frac{29a\,x^5}{16\left(b\,x^2+a\right)^3b^2}+\frac{17a^2x^3}{6\left(b\,x^2+a\right)^3b^3}+\frac{19a^3x}{16\left(b\,x^2+a\right)^3b^4}-\frac{35a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,b^4}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] x/b^4+29/16/b^2*a/(b*x^2+a)^3*x^5+17/6/b^3*a^2/(b*x^2+a)^3*x^3+19/16/b^4*a^
3/(b*x^2+a)^3*x-35/16/b^4*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.85, size = 90, normalized size = 0.97

$$\frac{87\,ab^{2}x^{5}+136\,a^{2}bx^{3}+57\,a^{3}x}{48\,\left(b^{7}x^{6}+3\,ab^{6}x^{4}+3\,a^{2}b^{5}x^{2}+a^{3}b^{4}\right)}-\frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^{4}}+\frac{x}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(87*a*b^2*x^5 + 136*a^2*b*x^3 + 57*a^3*x)/(b^7*x^6 + 3*a*b^6*x^4 + 3*a
^2*b^5*x^2 + a^3*b^4) - 35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x/b
^4
```

mupad [B] time = 0.10, size = 86, normalized size = 0.92

$$\frac{x}{b^{4}}+\frac{\frac{19\,a^{3}\,x}{16}+\frac{17\,a^{2}\,b\,x^{3}}{6}+\frac{29\,a\,b^{2}\,x^{5}}{16}}{a^{3}\,b^{4}+3\,a^{2}\,b^{5}\,x^{2}+3\,a\,b^{6}\,x^{4}+b^{7}\,x^{6}}-\frac{35\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] x/b^4 + ((19*a^3*x)/16 + (17*a^2*b*x^3)/6 + (29*a*b^2*x^5)/16)/(a^3*b^4 + b
^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2) - (35*a^(1/2)*atan((b^(1/2)*x)/a^(1/2
)))/(16*b^(9/2))
```

sympy [A] time = 0.57, size = 131, normalized size = 1.41

$$\frac{35\sqrt{-\frac{a}{b^{9}}}\log\left(-b^{4}\sqrt{-\frac{a}{b^{9}}}+x\right)}{32}-\frac{35\sqrt{-\frac{a}{b^{9}}}\log\left(b^{4}\sqrt{-\frac{a}{b^{9}}}+x\right)}{32}+\frac{57a^{3}x+136a^{2}bx^{3}+87ab^{2}x^{5}}{48a^{3}b^{4}+144a^{2}b^{5}x^{2}+144ab^{6}x^{4}+48b^{7}x^{6}}+\frac{x}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-a/b**9)*log(-b**4*sqrt(-a/b**9) + x)/32 - 35*sqrt(-a/b**9)*log(b**
4*sqrt(-a/b**9) + x)/32 + (57*a**3*x + 136*a**2*b*x**3 + 87*a*b**2*x**5)/(4
8*a**3*b**4 + 144*a**2*b**5*x**2 + 144*a*b**6*x**4 + 48*b**7*x**6) + x/b**4
```

3.333 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}-\frac{5x}{16b^3\left(a+bx^2\right)}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{x^5}{6b\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 288, 205}

$$-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}-\frac{x^5}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x^5/(6*b*(a + b*x^2)^3) - (5*x^3)/(24*b^2*(a + b*x^2)^2) - (5*x)/(16*b^3*(
a + b*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^6}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= -\frac{x^5}{6b\left(a+bx^2\right)^3}+\frac{1}{6}\left(5b^2\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,dx$$
$$= -\frac{x^5}{6b\left(a+bx^2\right)^3}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}+\frac{5}{8}\int \frac{x^2}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= -\frac{x^5}{6b\left(a+bx^2\right)^3}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\int \frac{1}{ab+b^2x^2}\,dx}{16b^2}$$
$$= -\frac{x^5}{6b\left(a+bx^2\right)^3}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.80

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}-\frac{x\left(15a^2+40abx^2+33b^2x^4\right)}{48b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/48*(x*(15*a^2 + 40*a*b*x^2 + 33*b^2*x^4))/(b^3*(a + b*x^2)^3) + (5*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.85, size = 254, normalized size = 3.06

$$\left[-\frac{66\,ab^3x^5+80\,a^2b^2x^3+30\,a^3bx+15\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{96\left(ab^7x^6+3\,a^2b^6x^4+3\,a^3b^5x^2+a^4b^4\right)},\,-\frac{33\,ab^3x^5+40\,a^2b^2x^3+15\,a^3bx-15\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{48\left(ab^7x^6+3\,a^2b^6x^4+3\,a^3b^5x^2+a^4b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(66*a*b^3*x^5 + 80*a^2*b^2*x^3 + 30*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*
x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)))/(a*b^7*x^6 + 3*a^2*b^6*x^4 + 3*a^3*b^5*x^2 + a^4*b^4), -1/48*(33*a*
b^3*x^5 + 40*a^2*b^2*x^3 + 15*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b
*x^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^7*x^6 + 3*a^2*b^6*x^4 + 3
*a^3*b^5*x^2 + a^4*b^4)]
```

giac [A] time = 0.17, size = 56, normalized size = 0.67

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \sqrt{ab}\, b^3} - \frac{33 \, b^2 x^5 + 40 \, abx^3 + 15 \, a^2 x}{48 \left(bx^2 + a\right)^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/48*(33*b^2*x^5 + 40*a*b*x^3
+ 15*a^2*x)/((b*x^2 + a)^3*b^3)
```

maple [A] time = 0.01, size = 58, normalized size = 0.70

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\ b^3} + \frac{-\frac{11x^5}{16b} - \frac{5a\,x^3}{6b^2} - \frac{5a^2x}{16b^3}}{\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (-11/16/b*x^5-5/6*a/b^2*x^3-5/16*a^2/b^3*x)/(b*x^2+a)^3+5/16/b^3/(a*b)^(1/2
)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.07, size = 81, normalized size = 0.98

$$-\frac{33 \, b^2 x^5 + 40 \, abx^3 + 15 \, a^2 x}{48 \left(b^6 x^6 + 3 \, ab^5 x^4 + 3 \, a^2 b^4 x^2 + a^3 b^3\right)} + \frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \sqrt{ab}\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(33*b^2*x^5 + 40*a*b*x^3 + 15*a^2*x)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b
^4*x^2 + a^3*b^3) + 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
```

mupad [B] time = 4.39, size = 78, normalized size = 0.94

$$\frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,\sqrt{a}\,b^{7/2}}-\frac{\frac{11\,x^5}{16\,b}+\frac{5\,a\,x^3}{6\,b^2}+\frac{5\,a^2\,x}{16\,b^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (5*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(1/2)*b^(7/2)) - ((11*x^5)/(16*b) + (5*
a*x^3)/(6*b^2) + (5*a^2*x)/(16*b^3))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2
*x^4)
```

sympy [A] time = 0.49, size = 134, normalized size = 1.61

$$-\frac{5\sqrt{-\frac{1}{ab^7}}\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{ab^7}}\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32}+\frac{-15a^2x-40abx^3-33b^2x^5}{48a^3b^3+144a^2b^4x^2+144ab^5x^4+48b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a*b**7))*log(-a*b**3*sqrt(-1/(a*b**7)) + x)/32 + 5*sqrt(-1/(a*b
**7))*log(a*b**3*sqrt(-1/(a*b**7)) + x)/32 + (-15*a**2*x - 40*a*b*x**3 - 33
*b**2*x**5)/(48*a**3*b**3 + 144*a**2*b**4*x**2 + 144*a*b**5*x**4 + 48*b**6*
x**6)
```

3.334 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=84

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{x}{16ab^2\left(a+bx^2\right)}-\frac{x}{8b^2\left(a+bx^2\right)^2}-\frac{x^3}{6b\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{x}{16ab^2\left(a+bx^2\right)}-\frac{x}{8b^2\left(a+bx^2\right)^2}-\frac{x^3}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x^3/(6*b*(a + b*x^2)^3) - x/(8*b^2*(a + b*x^2)^2) + x/(16*a*b^2*(a + b*x^2
)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx &= b^4\int \frac{x^4}{(ab+b^2x^2)^4}\,dx\\ &= -\frac{x^3}{6b(a+bx^2)^3}+\frac{1}{2}b^2\int \frac{x^2}{(ab+b^2x^2)^3}\,dx\\ &= -\frac{x^3}{6b(a+bx^2)^3}-\frac{x}{8b^2(a+bx^2)^2}+\frac{1}{8}\int \frac{1}{(ab+b^2x^2)^2}\,dx\\ &= -\frac{x^3}{6b(a+bx^2)^3}-\frac{x}{8b^2(a+bx^2)^2}+\frac{x}{16ab^2(a+bx^2)}+\frac{\int \frac{1}{ab+b^2x^2}\,dx}{16ab}\\ &= -\frac{x^3}{6b(a+bx^2)^3}-\frac{x}{8b^2(a+bx^2)^2}+\frac{x}{16ab^2(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.82

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{-3a^2x-8abx^3+3b^2x^5}{48ab^2(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x - 8*a*b*x^3 + 3*b^2*x^5)/(48*a*b^2*(a + b*x^2)^3) + ArcTan[(Sqrt[
b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 0.85, size = 258, normalized size = 3.07

$$\left[\frac{6\,ab^{3}x^{5}-16\,a^{2}b^{2}x^{3}-6\,a^{3}bx-3\,\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{96\,\left(a^{2}b^{6}x^{6}+3\,a^{3}b^{5}x^{4}+3\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)},\ \frac{3\,ab^{3}x^{5}-8\,a^{2}b^{2}x^{3}-3\,a^{3}bx+3\,\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{48\,\left(a^{2}b^{6}x^{6}+3\,a^{3}b^{5}x^{4}+3\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(6*a*b^3*x^5 - 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
+ 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3*a^4*b^4*x^2 + a^5*b^3), 1/48*(3*a*b^3*
x^5 - 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3*a^4*
b^4*x^2 + a^5*b^3)]
```

giac [A] time = 0.16, size = 62, normalized size = 0.74

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,ab^{2}}+\frac{3\,b^{2}x^{5}-8\,abx^{3}-3\,a^{2}x}{48\,\left(bx^{2}+a\right)^{3}ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/48*(3*b^2*x^5 - 8*a*b*x^3
- 3*a^2*x)/((b*x^2 + a)^3*a*b^2)
```

maple [A] time = 0.01, size = 58, normalized size = 0.69

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;a\,b^{2}}+\frac{\frac{x^{5}}{16a}-\frac{x^{3}}{6b}-\frac{ax}{16b^{2}}}{\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (1/16/a*x^5-1/6/b*x^3-1/16*a/b^2*x)/(b*x^2+a)^3+1/16/a/b^2/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.93, size = 87, normalized size = 1.04

$$\frac{3\,b^{2}x^{5}-8\,abx^{3}-3\,a^{2}x}{48\left(ab^{5}x^{6}+3\,a^{2}b^{4}x^{4}+3\,a^{3}b^{3}x^{2}+a^{4}b^{2}\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*x^5 - 8*a*b*x^3 - 3*a^2*x)/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b
^3*x^2 + a^4*b^2) + 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

mupad [B] time = 4.35, size = 75, normalized size = 0.89

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{3/2}\,b^{5/2}}-\frac{\frac{x^3}{6\,b}-\frac{x^5}{16\,a}+\frac{a\,x}{16\,b^2}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(16*a^(3/2)*b^(5/2)) - (x^3/(6*b) - x^5/(16*a) +
(a*x)/(16*b^2))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)
```

sympy [B] time = 0.45, size = 143, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{-3a^2x-8abx^3+3b^2x^5}{48a^4b^2+144a^3b^3x^2+144a^2b^4x^4+48ab^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + sqrt(-1
/(a**3*b**5))*log(a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + (-3*a**2*x - 8*a
*b*x**3 + 3*b**2*x**5)/(48*a**4*b**2 + 144*a**3*b**3*x**2 + 144*a**2*b**4*x
**4 + 48*a*b**5*x**6)
```

3.335 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{x}{24ab\left(a+bx^2\right)^2}-\frac{x}{6b\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{x}{24ab\left(a+bx^2\right)^2}-\frac{x}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x/(6*b*(a + b*x^2)^3) + x/(24*a*b*(a + b*x^2)^2) + x/(16*a^2*b*(a + b*x^2)
) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^2}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{1}{6}b^2\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx$$
$$= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{x}{24ab\left(a+bx^2\right)^2}+\frac{b\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{8a}$$
$$= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{x}{24ab\left(a+bx^2\right)^2}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{\int \frac{1}{ab+b^2x^2}\,dx}{16a^2}$$
$$= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{x}{24ab\left(a+bx^2\right)^2}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{-3a^2x+8abx^3+3b^2x^5}{48a^2b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x + 8*a*b*x^3 + 3*b^2*x^5)/(48*a^2*b*(a + b*x^2)^3) + ArcTan[(Sqrt[
b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [A] time = 1.14, size = 258, normalized size = 3.04

$$\left[\frac{6\,ab^{3}x^{5}+16\,a^{2}b^{2}x^{3}-6\,a^{3}bx-3\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{96\left(a^{3}b^{5}x^{6}+3\,a^{4}b^{4}x^{4}+3\,a^{5}b^{3}x^{2}+a^{6}b^{2}\right)},\ \frac{3\,ab^{3}x^{5}+8\,a^{2}b^{2}x^{3}-3\,a^{3}bx+3\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{48\left(a^{3}b^{5}x^{6}+3\,a^{4}b^{4}x^{4}+3\,a^{5}b^{3}x^{2}+a^{6}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(6*a*b^3*x^5 + 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
+ 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3*a^5*b^3*x^2 + a^6*b^2), 1/48*(3*a*b^3*
x^5 + 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3*a^5*
b^3*x^2 + a^6*b^2)]
```

giac [A] time = 0.18, size = 62, normalized size = 0.73

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^{2}b}+\frac{3\,b^{2}x^{5}+8\,abx^{3}-3\,a^{2}x}{48\left(bx^{2}+a\right)^{3}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/48*(3*b^2*x^5 + 8*a*b*x^3
- 3*a^2*x)/((b*x^2 + a)^3*a^2*b)
```

maple [A] time = 0.01, size = 58, normalized size = 0.68

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;a^{2}b}+\frac{\frac{b\,x^{5}}{16a^{2}}+\frac{x^{3}}{6a}-\frac{x}{16b}}{\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (1/16/a^2*b*x^5+1/6/a*x^3-1/16/b*x)/(b*x^2+a)^3+1/16/a^2/b/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 87, normalized size = 1.02

$$\frac{3\,b^2x^5+8\,abx^3-3\,a^2x}{48\left(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*x^5 + 8*a*b*x^3 - 3*a^2*x)/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4
*b^2*x^2 + a^5*b) + 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

mupad [B] time = 4.31, size = 74, normalized size = 0.87

$$\frac{\frac{x^3}{6\,a}-\frac{x}{16\,b}+\frac{b\,x^5}{16\,a^2}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (x^3/(6*a) - x/(16*b) + (b*x^5)/(16*a^2))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*
a*b^2*x^4) + atan((b^(1/2)*x)/a^(1/2))/(16*a^(5/2)*b^(3/2))
```

sympy [B] time = 0.44, size = 139, normalized size = 1.64

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{-3a^2x+8abx^3+3b^2x^5}{48a^5b+144a^4b^2x^2+144a^3b^3x^4+48a^2b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + sqrt(-1/(a
**5*b**3))*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + (-3*a**2*x + 8*a*b*x**
3 + 3*b**2*x**5)/(48*a**5*b + 144*a**4*b**2*x**2 + 144*a**3*b**3*x**4 + 48*
a**2*b**4*x**6)
```

3.336 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=79

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{x}{6a\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 199, 205}

$$\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{x}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

```
[Out] x/(6*a*(a + b*x^2)^3) + (5*x)/(24*a^2*(a + b*x^2)^2) + (5*x)/(16*a^3*(a + b
*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{\left(5b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{6a}$$
$$= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{\left(5b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{8a^2}$$
$$= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{(5b)\int \frac{1}{ab+b^2x^2}\,dx}{16a^3}$$
$$= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.84

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{33a^2x+40abx^3+15b^2x^5}{48a^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
[Out] (33*a^2*x + 40*a*b*x^3 + 15*b^2*x^5)/(48*a^3*(a + b*x^2)^3) + (5*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2), x]
```

fricas [A] time = 0.80, size = 254, normalized size = 3.22

$$\left[\frac{30\,ab^{3}x^{5}+80\,a^{2}b^{2}x^{3}+66\,a^{3}bx-15\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{96\left(a^{4}b^{4}x^{6}+3\,a^{5}b^{3}x^{4}+3\,a^{6}b^{2}x^{2}+a^{7}b\right)},\ \frac{15\,ab^{3}x^{5}+40\,a^{2}b^{2}x^{3}+33\,a^{3}bx+15\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{48\left(a^{4}b^{4}x^{6}+3\,a^{5}b^{3}x^{4}+3\,a^{6}b^{2}x^{2}+a^{7}b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(30*a*b^3*x^5 + 80*a^2*b^2*x^3 + 66*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*x
^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
+ a)))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3*a^6*b^2*x^2 + a^7*b), 1/48*(15*a*b^
3*x^5 + 40*a^2*b^2*x^3 + 33*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x
^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3
*a^6*b^2*x^2 + a^7*b)]
```

giac [A] time = 0.15, size = 56, normalized size = 0.71

$$\frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^{3}}+\frac{15\,b^{2}x^{5}+40\,abx^{3}+33\,a^{2}x}{48\left(bx^{2}+a\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/48*(15*b^2*x^5 + 40*a*b*x^3
+ 33*a^2*x)/((b*x^2 + a)^3*a^3)
```

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x}{6\left(b\,x^{2}+a\right)^{3}a}+\frac{5x}{24\left(b\,x^{2}+a\right)^{2}a^{2}}+\frac{5x}{16\left(b\,x^{2}+a\right)a^{3}}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*x/a/(b*x^2+a)^3+5/24*x/a^2/(b*x^2+a)^2+5/16*x/a^3/(b*x^2+a)+5/16/a^3/(a
*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 80, normalized size = 1.01

$$\frac{15\,b^{2}x^{5}+40\,abx^{3}+33\,a^{2}x}{48\left(a^{3}b^{3}x^{6}+3\,a^{4}b^{2}x^{4}+3\,a^{5}bx^{2}+a^{6}\right)}+\frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(15*b^2*x^5 + 40*a*b*x^3 + 33*a^2*x)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*
a^5*b*x^2 + a^6) + 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.36, size = 77, normalized size = 0.97

$$\frac{\frac{11\,x}{16\,a}+\frac{5\,b\,x^3}{6\,a^2}+\frac{5\,b^2\,x^5}{16\,a^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}+\frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((11*x)/(16*a) + (5*b*x^3)/(6*a^2) + (5*b^2*x^5)/(16*a^3))/(a^3 + b^3*x^6 +
 3*a^2*b*x^2 + 3*a*b^2*x^4) + (5*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(7/2)*b^(
1/2))
```

sympy [A] time = 0.44, size = 129, normalized size = 1.63

$$-\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{33a^2x+40abx^3+15b^2x^5}{48a^6+144a^5bx^2+144a^4b^2x^4+48a^3b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a**7*b))*log(-a**4*sqrt(-1/(a**7*b)) + x)/32 + 5*sqrt(-1/(a**7*
b))*log(a**4*sqrt(-1/(a**7*b)) + x)/32 + (33*a**2*x + 40*a*b*x**3 + 15*b**2
*x**5)/(48*a**6 + 144*a**5*b*x**2 + 144*a**4*b**2*x**4 + 48*a**3*b**3*x**6)
```

3.337 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{35\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{35}{16a^4x}+\frac{35}{48a^3x\left(a+bx^2\right)}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{1}{6ax\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.167, Rules used = {28, 290, 325, 205}

$$\frac{35}{48a^3x\left(a+bx^2\right)}+\frac{7}{24a^2x\left(a+bx^2\right)^2}-\frac{35\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{35}{16a^4x}+\frac{1}{6ax\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x) + 1/(6*a*x*(a + b*x^2)^3) + 7/(24*a^2*x*(a + b*x^2)^2) + 35/
(48*a^3*x*(a + b*x^2)) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/
2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{\left(7b^3\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^3}\,dx}{6a}$$

$$= \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{7}{24a^2x\left(a+bx^2\right)^2} + \frac{\left(35b^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx}{24a^2}$$

$$= \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{7}{24a^2x\left(a+bx^2\right)^2} + \frac{35}{48a^3x\left(a+bx^2\right)} + \frac{(35b)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{16a^3}$$

$$= -\frac{35}{16a^4x} + \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{7}{24a^2x\left(a+bx^2\right)^2} + \frac{35}{48a^3x\left(a+bx^2\right)} - \frac{\left(35b^2\right)\int \frac{1}{ab+}}{16a^4}$$

$$= -\frac{35}{16a^4x} + \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{7}{24a^2x\left(a+bx^2\right)^2} + \frac{35}{48a^3x\left(a+bx^2\right)} - \frac{35\sqrt{b}\,\tan^{-1}}{16a^{9/2}}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.83

$$-\frac{35\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{9/2}} - \frac{48a^3+231a^2bx^2+280ab^2x^4+105b^3x^6}{48a^4x\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/48*(48*a^3 + 231*a^2*b*x^2 + 280*a*b^2*x^4 + 105*b^3*x^6)/(a^4*x*(a + b*
x^2)^3) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2), x]
```

fricas [A] time = 0.82, size = 268, normalized size = 2.82

$$\left[-\frac{210\,b^3x^6+560\,ab^2x^4+462\,a^2bx^2+96\,a^3-105\,\left(b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{96\,\left(a^4b^3x^7+3\,a^5b^2x^5+3\,a^6bx^3+a^7x\right)},-\frac{105\,b^3x^6+280\,ab^2x^4+231\,a^2bx^2+48\,a^3+105\,\left(b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{48\,\left(a^4b^3x^7+3\,a^5b^2x^5+3\,a^6bx^3+a^7x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(210*b^3*x^6 + 560*a*b^2*x^4 + 462*a^2*b*x^2 + 96*a^3 - 105*(b^3*x^7
 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b
/a) - a)/(b*x^2 + a)))/(a^4*b^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x),
 -1/48*(105*b^3*x^6 + 280*a*b^2*x^4 + 231*a^2*b*x^2 + 48*a^3 + 105*(b^3*x^7
 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b
^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x)]
```

giac [A] time = 0.16, size = 68, normalized size = 0.72

$$-\frac{35\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^4}-\frac{1}{a^4x}-\frac{57\,b^3x^5+136\,ab^2x^3+87\,a^2bx}{48\,\left(bx^2+a\right)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -35/16*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/(a^4*x) - 1/48*(57*b^3*x
^5 + 136*a*b^2*x^3 + 87*a^2*b*x)/((b*x^2 + a)^3*a^4)
```

maple [A] time = 0.01, size = 86, normalized size = 0.91

$$-\frac{19b^3x^5}{16\left(b\,x^2+a\right)^3a^4}-\frac{17b^2x^3}{6\left(b\,x^2+a\right)^3a^3}-\frac{29bx}{16\left(b\,x^2+a\right)^3a^2}-\frac{35b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;a^4}-\frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/a^4/x-19/16*b^3/a^4/(b*x^2+a)^3*x^5-17/6*b^2/a^3/(b*x^2+a)^3*x^3-29/16*b
/a^2/(b*x^2+a)^3*x-35/16*b/a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.03, size = 93, normalized size = 0.98

$$-\frac{105\,b^3x^6 + 280\,ab^2x^4 + 231\,a^2bx^2 + 48\,a^3}{48\left(a^4b^3x^7 + 3\,a^5b^2x^5 + 3\,a^6bx^3 + a^7x\right)} - \frac{35\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(105*b^3*x^6 + 280*a*b^2*x^4 + 231*a^2*b*x^2 + 48*a^3)/(a^4*b^3*x^7 +
 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x) - 35/16*b*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^4)
```

mupad [B] time = 4.44, size = 88, normalized size = 0.93

$$-\frac{\frac{1}{a} + \frac{77\,b\,x^2}{16\,a^2} + \frac{35\,b^2\,x^4}{6\,a^3} + \frac{35\,b^3\,x^6}{16\,a^4}}{a^3\,x + 3\,a^2\,b\,x^3 + 3\,a\,b^2\,x^5 + b^3\,x^7} - \frac{35\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] - (1/a + (77*b*x^2)/(16*a^2) + (35*b^2*x^4)/(6*a^3) + (35*b^3*x^6)/(16*a^4)
)/(a^3*x + b^3*x^7 + 3*a^2*b*x^3 + 3*a*b^2*x^5) - (35*b^(1/2)*atan((b^(1/2)
*x)/a^(1/2)))/(16*a^(9/2))
```

sympy [A] time = 0.58, size = 139, normalized size = 1.46

$$\frac{35\sqrt{-\frac{b}{a^9}}\,\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}}{b} + x\right)}{32} - \frac{35\sqrt{-\frac{b}{a^9}}\,\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}}{b} + x\right)}{32} + \frac{-48a^3 - 231a^2bx^2 - 280ab^2x^4 - 105b^3x^6}{48a^7x + 144a^6bx^3 + 144a^5b^2x^5 + 48a^4b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-b/a**9)*log(-a**5*sqrt(-b/a**9)/b + x)/32 - 35*sqrt(-b/a**9)*log(a
**5*sqrt(-b/a**9)/b + x)/32 + (-48*a**3 - 231*a**2*b*x**2 - 280*a*b**2*x**4
 - 105*b**3*x**6)/(48*a**7*x + 144*a**6*b*x**3 + 144*a**5*b**2*x**5 + 48*a*
*4*b**3*x**7)
```

3.338 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=106

$$\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{11/2}}+\frac{105b}{16a^5x}-\frac{35}{16a^4x^3}+\frac{21}{16a^3x^3\left(a+bx^2\right)}+\frac{3}{8a^2x^3\left(a+bx^2\right)^2}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{11/2}}+\frac{21}{16a^3x^3\left(a+bx^2\right)}+\frac{3}{8a^2x^3\left(a+bx^2\right)^2}+\frac{105b}{16a^5x}-\frac{35}{16a^4x^3}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x^3) + (105*b)/(16*a^5*x) + 1/(6*a*x^3*(a + b*x^2)^3) + 3/(8*a^
2*x^3*(a + b*x^2)^2) + 21/(16*a^3*x^3*(a + b*x^2)) + (105*b^(3/2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(16*a^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^4\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{\left(3b^3\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^3}\,dx}{2a}$$

$$= \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{\left(21b^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx}{8a^2}$$

$$= \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} + \frac{(105b)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{16a^3}$$

$$= -\frac{35}{16a^4x^3} + \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} - \frac{\left(105b^2\right)\int}{1}$$

$$= -\frac{35}{16a^4x^3} + \frac{105b}{16a^5x} + \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} + \cdots$$

$$= -\frac{35}{16a^4x^3} + \frac{105b}{16a^5x} + \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} + \cdots$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.86

$$\frac{\frac{\sqrt{a}\left(-16a^4+144a^3bx^2+693a^2b^2x^4+840ab^3x^6+315b^4x^8\right)}{x^3\left(a+bx^2\right)^3}+315b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{48a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((Sqrt[a]*(-16*a^4 + 144*a^3*b*x^2 + 693*a^2*b^2*x^4 + 840*a*b^3*x^6 + 315*
b^4*x^8))/(x^3*(a + b*x^2)^3) + 315*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(4
8*a^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2), x]
```

fricas [A] time = 0.90, size = 304, normalized size = 2.87

$$\left[\frac{630\,b^4x^8+1680\,ab^3x^6+1386\,a^2b^2x^4+288\,a^3bx^2-32\,a^4+315\,\left(b^4x^9+3\,ab^3x^7+3\,a^2b^2x^5+a^3bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{96\,\left(a^5b^3x^9+3\,a^6b^2x^7+3\,a^7bx^5+a^8x^3\right)},\ \frac{315\,b^4x^8+840\,ab^3x^6+693\,a^2b^2x^4+144\,a^3bx^2-16\,a^4+315\,\left(b^4x^9+3\,ab^3x^7+3\,a^2b^2x^5+a^3bx^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{48\,\left(a^5b^3x^9+3\,a^6b^2x^7+3\,a^7bx^5+a^8x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(630*b^4*x^8 + 1680*a*b^3*x^6 + 1386*a^2*b^2*x^4 + 288*a^3*b*x^2 - 32
*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5 + a^3*b*x^3)*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^3*x^9 + 3*a^6*b^2*x^
7 + 3*a^7*b*x^5 + a^8*x^3), 1/48*(315*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2*b^2
*x^4 + 144*a^3*b*x^2 - 16*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5
+ a^3*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 +
3*a^7*b*x^5 + a^8*x^3)]
```

giac [A] time = 0.17, size = 82, normalized size = 0.77

$$\frac{105\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^5}+\frac{315\,b^4x^8+840\,ab^3x^6+693\,a^2b^2x^4+144\,a^3bx^2-16\,a^4}{48\,\left(bx^3+ax\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 105/16*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/48*(315*b^4*x^8 + 840*
a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a^4)/((b*x^3 + a*x)^3*a^5)
```

maple [A] time = 0.02, size = 99, normalized size = 0.93

$$\frac{41b^4x^5}{16\left(b\,x^2+a\right)^3a^5}+\frac{35b^3x^3}{6\left(b\,x^2+a\right)^3a^4}+\frac{55b^2x}{16\left(b\,x^2+a\right)^3a^3}+\frac{105b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,a^5}+\frac{4b}{a^5x}-\frac{1}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/3/a^4/x^3+4*b/a^5/x+41/16*b^4/a^5/(b*x^2+a)^3*x^5+35/6*b^3/a^4/(b*x^2+a)
^3*x^3+55/16*b^2/a^3/(b*x^2+a)^3*x+105/16*b^2/a^5/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x)
```

maxima [A] time = 3.05, size = 108, normalized size = 1.02

$$\frac{315\,b^4x^8+840\,ab^3x^6+693\,a^2b^2x^4+144\,a^3bx^2-16\,a^4}{48\left(a^5b^3x^9+3\,a^6b^2x^7+3\,a^7bx^5+a^8x^3\right)}+\frac{105\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(315*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a^
4)/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 + 3*a^7*b*x^5 + a^8*x^3) + 105/16*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 4.45, size = 102, normalized size = 0.96

$$\frac{\frac{3\,b\,x^2}{a^2}-\frac{1}{3\,a}+\frac{231\,b^2\,x^4}{16\,a^3}+\frac{35\,b^3\,x^6}{2\,a^4}+\frac{105\,b^4\,x^8}{16\,a^5}}{a^3\,x^3+3\,a^2\,b\,x^5+3\,a\,b^2\,x^7+b^3\,x^9}+\frac{105\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((3*b*x^2)/a^2 - 1/(3*a) + (231*b^2*x^4)/(16*a^3) + (35*b^3*x^6)/(2*a^4) +
(105*b^4*x^8)/(16*a^5))/(a^3*x^3 + b^3*x^9 + 3*a^2*b*x^5 + 3*a*b^2*x^7) + (
105*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(11/2))
```

sympy [A] time = 0.63, size = 162, normalized size = 1.53

$$-\frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2}+x\right)}{32}+\frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2}+x\right)}{32}+\frac{-16a^4+144a^3bx^2+693a^2b^2x^4+840ab^3x^6+315b^4x^8}{48a^8x^3+144a^7bx^5+144a^6b^2x^7+48a^5b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -105*sqrt(-b**3/a**11)*log(-a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + 105*sqrt(
-b**3/a**11)*log(a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + (-16*a**4 + 144*a**3
*b*x**2 + 693*a**2*b**2*x**4 + 840*a*b**3*x**6 + 315*b**4*x**8)/(48*a**8*x*
*3 + 144*a**7*b*x**5 + 144*a**6*b**2*x**7 + 48*a**5*b**3*x**9)
```

3.339 $\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{13/2}}-\frac{231b^2}{16a^6x}+\frac{77b}{16a^5x^3}-\frac{231}{80a^4x^5}+\frac{33}{16a^3x^5\left(a+bx^2\right)}+\frac{11}{24a^2x^5\left(a+bx^2\right)^2}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{231b^2}{16a^6x}-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{13/2}}+\frac{77b}{16a^5x^3}+\frac{33}{16a^3x^5\left(a+bx^2\right)}+\frac{11}{24a^2x^5\left(a+bx^2\right)^2}-\frac{231}{80a^4x^5}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -231/(80*a^4*x^5) + (77*b)/(16*a^5*x^3) - (231*b^2)/(16*a^6*x) + 1/(6*a*x^5
*(a + b*x^2)^3) + 11/(24*a^2*x^5*(a + b*x^2)^2) + 33/(16*a^3*x^5*(a + b*x^2
)) - (231*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^6\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{\left(11b^3\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^3}\,dx}{6a}$$

$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{\left(33b^2\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^2}\,dx}{8a^2}$$

$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)} + \frac{(231b)\int \frac{1}{x^6\left(ab+b^2x^2\right)}$$

$$= -\frac{231}{80a^4x^5} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)} - \frac{\left(231b^2\right.}{}$$

$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)}$$

$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} - \frac{231b^2}{16a^6x} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{3}{16a^3x^5}$$

$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} - \frac{231b^2}{16a^6x} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{3}{16a^3x^5}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.85

$$-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{13/2}} - \frac{48a^5-176a^4bx^2+1584a^3b^2x^4+7623a^2b^3x^6+9240ab^4x^8+3465b^5x^{10}}{240a^6x^5\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/240*(48*a^5 - 176*a^4*b*x^2 + 1584*a^3*b^2*x^4 + 7623*a^2*b^3*x^6 + 9240
*a*b^4*x^8 + 3465*b^5*x^10)/(a^6*x^5*(a + b*x^2)^3) - (231*b^(5/2)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(a^2 + 2abx^2 + b^2x^4\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2), x]
```

fricas [A] time = 0.87, size = 330, normalized size = 2.77

$$\left[-\frac{6930\,b^5x^{10} + 18480\,ab^4x^8 + 15246\,a^2b^3x^6 + 3168\,a^3b^2x^4 - 352\,a^4bx^2 + 96\,a^5 - 3465\left(b^5x^{11} + 3\,ab^4x^9 + 3\,a^2b^3x^7 + a^3b^2x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right)}{480\left(a^6b^3x^{11} + 3\,a^7b^2x^9 + 3\,a^8bx^7 + a^9x^5\right)}, -\frac{3465\,b^5x^{10} + 9240\,ab^4x^8 + 7623\,a^2b^3x^6 + 1584\,a^3b^2x^4 - 176\,a^4bx^2 + 48\,a^5 + 3465\left(b^5x^{11} + 3\,ab^4x^9 + 3\,a^2b^3x^7 + a^3b^2x^5\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{240\left(a^6b^3x^{11} + 3\,a^7b^2x^9 + 3\,a^8bx^7 + a^9x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/480*(6930*b^5*x^10 + 18480*a*b^4*x^8 + 15246*a^2*b^3*x^6 + 3168*a^3*b^2
*x^4 - 352*a^4*b*x^2 + 96*a^5 - 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^
7 + a^3*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
)/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a^9*x^5), -1/240*(3465*b^5*
x^10 + 9240*a*b^4*x^8 + 7623*a^2*b^3*x^6 + 1584*a^3*b^2*x^4 - 176*a^4*b*x^2
 + 48*a^5 + 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^7 + a^3*b^2*x^5)*sqr
t(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a
^9*x^5)]
```

giac [A] time = 0.16, size = 93, normalized size = 0.78

$$-\frac{231\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^6} - \frac{213\,b^5x^5 + 472\,ab^4x^3 + 267\,a^2b^3x}{48\left(bx^2 + a\right)^3a^6} - \frac{150\,b^2x^4 - 20\,abx^2 + 3\,a^2}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/48*(213*b^5*x^5 + 472
*a*b^4*x^3 + 267*a^2*b^3*x)/((b*x^2 + a)^3*a^6) - 1/15*(150*b^2*x^4 - 20*a*
b*x^2 + 3*a^2)/(a^6*x^5)
```

maple [A] time = 0.02, size = 110, normalized size = 0.92

$$-\frac{71b^5x^5}{16\left(b\,x^2+a\right)^3 a^6}-\frac{59b^4x^3}{6\left(b\,x^2+a\right)^3 a^5}-\frac{89b^3x}{16\left(b\,x^2+a\right)^3 a^4}-\frac{231b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;a^6}-\frac{10b^2}{a^6x}+\frac{4b}{3a^5x^3}-\frac{1}{5a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/5/a^4/x^5-10*b^2/a^6/x+4/3*b/a^5/x^3-71/16/a^6*b^5/(b*x^2+a)^3*x^5-59/6/
a^5*b^4/(b*x^2+a)^3*x^3-89/16/a^4*b^3/(b*x^2+a)^3*x-231/16/a^6*b^3/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 119, normalized size = 1.00

$$-\frac{3465\,b^5x^{10}+9240\,ab^4x^8+7623\,a^2b^3x^6+1584\,a^3b^2x^4-176\,a^4bx^2+48\,a^5}{240\left(a^6b^3x^{11}+3\,a^7b^2x^9+3\,a^8bx^7+a^9x^5\right)}-\frac{231\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/240*(3465*b^5*x^10 + 9240*a*b^4*x^8 + 7623*a^2*b^3*x^6 + 1584*a^3*b^2*x^
4 - 176*a^4*b*x^2 + 48*a^5)/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a
^9*x^5) - 231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 4.46, size = 114, normalized size = 0.96

$$-\frac{\frac{1}{5\,a}-\frac{11\,b\,x^2}{15\,a^2}+\frac{33\,b^2\,x^4}{5\,a^3}+\frac{2541\,b^3\,x^6}{80\,a^4}+\frac{77\,b^4\,x^8}{2\,a^5}+\frac{231\,b^5\,x^{10}}{16\,a^6}}{a^3\,x^5+3\,a^2\,b\,x^7+3\,a\,b^2\,x^9+b^3\,x^{11}}-\frac{231\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] - (1/(5*a) - (11*b*x^2)/(15*a^2) + (33*b^2*x^4)/(5*a^3) + (2541*b^3*x^6)/(8
0*a^4) + (77*b^4*x^8)/(2*a^5) + (231*b^5*x^10)/(16*a^6))/(a^3*x^5 + b^3*x^1
1 + 3*a^2*b*x^7 + 3*a*b^2*x^9) - (231*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(1
6*a^(13/2))
```

sympy [A] time = 0.71, size = 173, normalized size = 1.45

$$\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}-\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}+\frac{-48a^5+176a^4bx^2-1584a^3b^2x^4-7623a^2b^3x^6-9240ab^4x^8-3465b^5x^{10}}{240a^9x^5+720a^8bx^7+720a^7b^2x^9+240a^6b^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 231*sqrt(-b**5/a**13)*log(-a**7*sqrt(-b**5/a**13)/b**3 + x)/32 - 231*sqrt(-
b**5/a**13)*log(a**7*sqrt(-b**5/a**13)/b**3 + x)/32 + (-48*a**5 + 176*a**4*
b*x**2 - 1584*a**3*b**2*x**4 - 7623*a**2*b**3*x**6 - 9240*a*b**4*x**8 - 346
5*b**5*x**10)/(240*a**9*x**5 + 720*a**8*b*x**7 + 720*a**7*b**2*x**9 + 240*a
**6*b**3*x**11)
```

3.340 $\int \frac{x^{15}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=133

$$\frac{a^7}{10b^8\left(a+bx^2\right)^5}-\frac{7a^6}{8b^8\left(a+bx^2\right)^4}+\frac{7a^5}{2b^8\left(a+bx^2\right)^3}-\frac{35a^4}{4b^8\left(a+bx^2\right)^2}+\frac{35a^3}{2b^8\left(a+bx^2\right)}+\frac{21a^2\log\left(a+bx^2\right)}{2b^8}-\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}$$

Rubi [A] time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^7}{10b^8\left(a+bx^2\right)^5}-\frac{7a^6}{8b^8\left(a+bx^2\right)^4}+\frac{7a^5}{2b^8\left(a+bx^2\right)^3}-\frac{35a^4}{4b^8\left(a+bx^2\right)^2}+\frac{35a^3}{2b^8\left(a+bx^2\right)}+\frac{21a^2\log\left(a+bx^2\right)}{2b^8}-\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-3*a*x^2)/b^7 + x^4/(4*b^6) + a^7/(10*b^8*(a + b*x^2)^5) - (7*a^6)/(8*b^8*
(a + b*x^2)^4) + (7*a^5)/(2*b^8*(a + b*x^2)^3) - (35*a^4)/(4*b^8*(a + b*x^2
)^2) + (35*a^3)/(2*b^8*(a + b*x^2)) + (21*a^2*Log[a + b*x^2])/(2*b^8)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{15}}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^7}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(-\frac{6a}{b^{13}}+\frac{x}{b^{12}}-\frac{a^7}{b^{13}(a+bx)^6}+\frac{7a^6}{b^{13}(a+bx)^5}-\frac{21a^5}{b^{13}(a+bx)^4}+\frac{35}{b^{13}(a\right.\right.$$
$$= -\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}+\frac{a^7}{10b^8\left(a+bx^2\right)^5}-\frac{7a^6}{8b^8\left(a+bx^2\right)^4}+\frac{7a^5}{2b^8\left(a+bx^2\right)^3}-\frac{35a^4}{4b^8\left(a+bx^2\right.}$$

Mathematica [A] time = 0.03, size = 114, normalized size = 0.86

$$\frac{459a^7+1875a^6bx^2+2700a^5b^2x^4+1300a^4b^3x^6-400a^3b^4x^8-500a^2b^5x^{10}+420a^2\left(a+bx^2\right)^5\log\left(a+bx^2\right)-70ab^6x^{12}+10b^7x^{14}}{40b^8\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (459*a^7 + 1875*a^6*b*x^2 + 2700*a^5*b^2*x^4 + 1300*a^4*b^3*x^6 - 400*a^3*b
^4*x^8 - 500*a^2*b^5*x^10 - 70*a*b^6*x^12 + 10*b^7*x^14 + 420*a^2*(a + b*x^
2)^5*Log[a + b*x^2])/(40*b^8*(a + b*x^2)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.88, size = 203, normalized size = 1.53

$$\frac{10\,b^7x^{14}-70\,ab^6x^{12}-500\,a^2b^5x^{10}-400\,a^3b^4x^8+1300\,a^4b^3x^6+2700\,a^5b^2x^4+1875\,a^6bx^2+459\,a^7+420\left(a^2b^5x^{10}+5\,a^3b^4x^8+10\,a^4b^3x^6+10\,a^5b^2x^4+5\,a^6bx^2+a^7\right)\log\left(bx^2+a\right)}{40\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(10*b^7*x^14 - 70*a*b^6*x^12 - 500*a^2*b^5*x^10 - 400*a^3*b^4*x^8 + 13
00*a^4*b^3*x^6 + 2700*a^5*b^2*x^4 + 1875*a^6*b*x^2 + 459*a^7 + 420*(a^2*b^5
*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7
)*log(b*x^2 + a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10
*x^4 + 5*a^4*b^9*x^2 + a^5*b^8)
```

giac [A] time = 0.16, size = 113, normalized size = 0.85

$$\frac{21\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^8}+\frac{b^6x^4-12\,ab^5x^2}{4\,b^{12}}-\frac{959\,a^2b^5x^{10}+4095\,a^3b^4x^8+7140\,a^4b^3x^6+6300\,a^5b^2x^4+2800\,a^6bx^2+500\,a^7}{40\left(bx^2+a\right)^5b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 21/2*a^2*log(abs(b*x^2 + a))/b^8 + 1/4*(b^6*x^4 - 12*a*b^5*x^2)/b^12 - 1/40
*(959*a^2*b^5*x^10 + 4095*a^3*b^4*x^8 + 7140*a^4*b^3*x^6 + 6300*a^5*b^2*x^4
 + 2800*a^6*b*x^2 + 500*a^7)/((b*x^2 + a)^5*b^8)
```

maple [A] time = 0.02, size = 120, normalized size = 0.90

$$\frac{a^7}{10\left(b\,x^2+a\right)^5b^8}-\frac{7a^6}{8\left(b\,x^2+a\right)^4b^8}+\frac{x^4}{4b^6}+\frac{7a^5}{2\left(b\,x^2+a\right)^3b^8}-\frac{35a^4}{4\left(b\,x^2+a\right)^2b^8}-\frac{3a\,x^2}{b^7}+\frac{35a^3}{2\left(b\,x^2+a\right)b^8}+\frac{21a^2\ln\left(b\,x^2+a\right)}{2b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -3*a*x^2/b^7+1/4*x^4/b^6+1/10*a^7/b^8/(b*x^2+a)^5-7/8*a^6/b^8/(b*x^2+a)^4+7
/2*a^5/b^8/(b*x^2+a)^3-35/4*a^4/b^8/(b*x^2+a)^2+35/2*a^3/b^8/(b*x^2+a)+21/2
*a^2*ln(b*x^2+a)/b^8
```

maxima [A] time = 1.42, size = 143, normalized size = 1.08

$$\frac{700\,a^3b^4x^8+2450\,a^4b^3x^6+3290\,a^5b^2x^4+1995\,a^6bx^2+459\,a^7}{40\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8\right)}+\frac{21\,a^2\log\left(bx^2+a\right)}{2\,b^8}+\frac{bx^4-12\,ax^2}{4\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(700*a^3*b^4*x^8 + 2450*a^4*b^3*x^6 + 3290*a^5*b^2*x^4 + 1995*a^6*b*x^
2 + 459*a^7)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4
+ 5*a^4*b^9*x^2 + a^5*b^8) + 21/2*a^2*log(b*x^2 + a)/b^8 + 1/4*(b*x^4 - 12*
a*x^2)/b^7
```

mupad [B] time = 0.13, size = 142, normalized size = 1.07

$$\frac{\frac{459\,a^7}{40\,b}+\frac{399\,a^6\,x^2}{8}+\frac{329\,a^5\,b\,x^4}{4}+\frac{245\,a^4\,b^2\,x^6}{4}+\frac{35\,a^3\,b^3\,x^8}{2}}{a^5\,b^7+5\,a^4\,b^8\,x^2+10\,a^3\,b^9\,x^4+10\,a^2\,b^{10}\,x^6+5\,a\,b^{11}\,x^8+b^{12}\,x^{10}}+\frac{x^4}{4\,b^6}-\frac{3\,a\,x^2}{b^7}+\frac{21\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((459*a^7)/(40*b) + (399*a^6*x^2)/8 + (329*a^5*b*x^4)/4 + (245*a^4*b^2*x^6)
/4 + (35*a^3*b^3*x^8)/2)/(a^5*b^7 + b^12*x^10 + 5*a*b^11*x^8 + 5*a^4*b^8*x^
2 + 10*a^3*b^9*x^4 + 10*a^2*b^10*x^6) + x^4/(4*b^6) - (3*a*x^2)/b^7 + (21*a
^2*log(a + b*x^2))/(2*b^8)
```

sympy [A] time = 0.99, size = 150, normalized size = 1.13

$$\frac{21a^2\log\left(a+bx^2\right)}{2b^8}-\frac{3ax^2}{b^7}+\frac{459a^7+1995a^6bx^2+3290a^5b^2x^4+2450a^4b^3x^6+700a^3b^4x^8}{40a^5b^8+200a^4b^9x^2+400a^3b^{10}x^4+400a^2b^{11}x^6+200ab^{12}x^8+40b^{13}x^{10}}+\frac{x^4}{4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*log(a + b*x**2)/(2*b**8) - 3*a*x**2/b**7 + (459*a**7 + 1995*a**6*b*
x**2 + 3290*a**5*b**2*x**4 + 2450*a**4*b**3*x**6 + 700*a**3*b**4*x**8)/(40*
a**5*b**8 + 200*a**4*b**9*x**2 + 400*a**3*b**10*x**4 + 400*a**2*b**11*x**6
+ 200*a*b**12*x**8 + 40*b**13*x**10) + x**4/(4*b**6)
```

3.341 $\int \frac{x^{13}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=118

$$-\frac{a^6}{10b^7\left(a+bx^2\right)^5}+\frac{3a^5}{4b^7\left(a+bx^2\right)^4}-\frac{5a^4}{2b^7\left(a+bx^2\right)^3}+\frac{5a^3}{b^7\left(a+bx^2\right)^2}-\frac{15a^2}{2b^7\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{b^7}+\frac{x^2}{2b^6}$$

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^6}{10b^7\left(a+bx^2\right)^5}+\frac{3a^5}{4b^7\left(a+bx^2\right)^4}-\frac{5a^4}{2b^7\left(a+bx^2\right)^3}+\frac{5a^3}{b^7\left(a+bx^2\right)^2}-\frac{15a^2}{2b^7\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{b^7}+\frac{x^2}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^2/(2*b^6) - a^6/(10*b^7*(a + b*x^2)^5) + (3*a^5)/(4*b^7*(a + b*x^2)^4) -
(5*a^4)/(2*b^7*(a + b*x^2)^3) + (5*a^3)/(b^7*(a + b*x^2)^2) - (15*a^2)/(2*b
^7*(a + b*x^2)) - (3*a*Log[a + b*x^2])/b^7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{x^{13}}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2} b^6 \,\text{Subst}\left(\int \frac{x^6}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2} b^6 \,\text{Subst}\left(\int \left(\frac{1}{b^{12}} + \frac{a^6}{b^{12}(a+bx)^6} - \frac{6a^5}{b^{12}(a+bx)^5} + \frac{15a^4}{b^{12}(a+bx)^4} - \frac{20a^3}{b^{12}(a+bx)^3} +\right.\right.$$
$$= \frac{x^2}{2b^6} - \frac{a^6}{10b^7\left(a+bx^2\right)^5} + \frac{3a^5}{4b^7\left(a+bx^2\right)^4} - \frac{5a^4}{2b^7\left(a+bx^2\right)^3} + \frac{5a^3}{b^7\left(a+bx^2\right)^2} - \frac{15}{2b^7\left(a\right.}$$

Mathematica [A] time = 0.03, size = 101, normalized size = 0.86

$$-\frac{87a^6+375a^5bx^2+600a^4b^2x^4+400a^3b^3x^6+50a^2b^4x^8-50ab^5x^{10}+60a\left(a+bx^2\right)^5\log\left(a+bx^2\right)-10b^6x^{12}}{20b^7\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20*(87*a^6 + 375*a^5*b*x^2 + 600*a^4*b^2*x^4 + 400*a^3*b^3*x^6 + 50*a^2*
b^4*x^8 - 50*a*b^5*x^10 - 10*b^6*x^12 + 60*a*(a + b*x^2)^5*Log[a + b*x^2])/
(b^7*(a + b*x^2)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.86, size = 190, normalized size = 1.61

$$\frac{10\,b^6x^{12}+50\,ab^5x^{10}-50\,a^2b^4x^8-400\,a^3b^3x^6-600\,a^4b^2x^4-375\,a^5bx^2-87\,a^6-60\left(ab^5x^{10}+5\,a^2b^4x^8+10\,a^3b^3x^6+10\,a^4b^2x^4+5\,a^5bx^2+a^6\right)\log\left(bx^2+a\right)}{20\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 50*a*b^5*x^10 - 50*a^2*b^4*x^8 - 400*a^3*b^3*x^6 - 600*
a^4*b^2*x^4 - 375*a^5*b*x^2 - 87*a^6 - 60*(a*b^5*x^10 + 5*a^2*b^4*x^8 + 10*
a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^12*x^1
0 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b
^7)
```

giac [A] time = 0.23, size = 95, normalized size = 0.81

$$\frac{x^2}{2\,b^6} - \frac{3\,a\log\left(\left|bx^2 + a\right|\right)}{b^7} + \frac{137\,ab^5x^{10} + 535\,a^2b^4x^8 + 870\,a^3b^3x^6 + 720\,a^4b^2x^4 + 300\,a^5bx^2 + 50\,a^6}{20\left(bx^2 + a\right)^5 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^6 - 3*a*log(abs(b*x^2 + a))/b^7 + 1/20*(137*a*b^5*x^10 + 535*a^2*
b^4*x^8 + 870*a^3*b^3*x^6 + 720*a^4*b^2*x^4 + 300*a^5*b*x^2 + 50*a^6)/((b*x
^2 + a)^5*b^7)
```

maple [A] time = 0.01, size = 109, normalized size = 0.92

$$-\frac{a^6}{10\left(b\,x^2+a\right)^5 b^7} + \frac{3a^5}{4\left(b\,x^2+a\right)^4 b^7} - \frac{5a^4}{2\left(b\,x^2+a\right)^3 b^7} + \frac{5a^3}{\left(b\,x^2+a\right)^2 b^7} + \frac{x^2}{2b^6} - \frac{15a^2}{2\left(b\,x^2+a\right) b^7} - \frac{3a\ln\left(b\,x^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/2*x^2/b^6-1/10*a^6/b^7/(b*x^2+a)^5+3/4*a^5/b^7/(b*x^2+a)^4-5/2*a^4/b^7/(b
*x^2+a)^3+5*a^3/b^7/(b*x^2+a)^2-15/2*a^2/b^7/(b*x^2+a)-3*a*ln(b*x^2+a)/b^7
```

maxima [A] time = 1.47, size = 132, normalized size = 1.12

$$-\frac{150\,a^2b^4x^8 + 500\,a^3b^3x^6 + 650\,a^4b^2x^4 + 385\,a^5bx^2 + 87\,a^6}{20\left(b^{12}x^{10} + 5\,ab^{11}x^8 + 10\,a^2b^{10}x^6 + 10\,a^3b^9x^4 + 5\,a^4b^8x^2 + a^5b^7\right)} + \frac{x^2}{2\,b^6} - \frac{3\,a\log\left(bx^2 + a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(150*a^2*b^4*x^8 + 500*a^3*b^3*x^6 + 650*a^4*b^2*x^4 + 385*a^5*b*x^2
+ 87*a^6)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*
a^4*b^8*x^2 + a^5*b^7) + 1/2*x^2/b^6 - 3*a*log(b*x^2 + a)/b^7
```

mupad [B] time = 4.60, size = 132, normalized size = 1.12

$$\frac{x^2}{2\,b^6}-\frac{\frac{87\,a^6}{20\,b}+\frac{77\,a^5\,x^2}{4}+\frac{65\,a^4\,b\,x^4}{2}+25\,a^3\,b^2\,x^6+\frac{15\,a^2\,b^3\,x^8}{2}}{a^5\,b^6+5\,a^4\,b^7\,x^2+10\,a^3\,b^8\,x^4+10\,a^2\,b^9\,x^6+5\,a\,b^{10}\,x^8+b^{11}\,x^{10}}-\frac{3\,a\,\ln\left(b\,x^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] x^2/(2*b^6) - ((87*a^6)/(20*b) + (77*a^5*x^2)/4 + (65*a^4*b*x^4)/2 + 25*a^3
*b^2*x^6 + (15*a^2*b^3*x^8)/2)/(a^5*b^6 + b^11*x^10 + 5*a*b^10*x^8 + 5*a^4*
b^7*x^2 + 10*a^3*b^8*x^4 + 10*a^2*b^9*x^6) - (3*a*log(a + b*x^2))/b^7
```

sympy [A] time = 0.97, size = 138, normalized size = 1.17

$$-\frac{3a\log\left(a+bx^2\right)}{b^7}+\frac{-87a^6-385a^5bx^2-650a^4b^2x^4-500a^3b^3x^6-150a^2b^4x^8}{20a^5b^7+100a^4b^8x^2+200a^3b^9x^4+200a^2b^{10}x^6+100ab^{11}x^8+20b^{12}x^{10}}+\frac{x^2}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*a*log(a + b*x**2)/b**7 + (-87*a**6 - 385*a**5*b*x**2 - 650*a**4*b**2*x**
4 - 500*a**3*b**3*x**6 - 150*a**2*b**4*x**8)/(20*a**5*b**7 + 100*a**4*b**8*
x**2 + 200*a**3*b**9*x**4 + 200*a**2*b**10*x**6 + 100*a*b**11*x**8 + 20*b**
12*x**10) + x**2/(2*b**6)
```

3.342 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=109

$$\frac{a^5}{10b^6\left(a+bx^2\right)^5}-\frac{5a^4}{8b^6\left(a+bx^2\right)^4}+\frac{5a^3}{3b^6\left(a+bx^2\right)^3}-\frac{5a^2}{2b^6\left(a+bx^2\right)^2}+\frac{5a}{2b^6\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^6}$$

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^5}{10b^6\left(a+bx^2\right)^5}-\frac{5a^4}{8b^6\left(a+bx^2\right)^4}+\frac{5a^3}{3b^6\left(a+bx^2\right)^3}-\frac{5a^2}{2b^6\left(a+bx^2\right)^2}+\frac{5a}{2b^6\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^5/(10*b^6*(a + b*x^2)^5) - (5*a^4)/(8*b^6*(a + b*x^2)^4) + (5*a^3)/(3*b^6
*(a + b*x^2)^3) - (5*a^2)/(2*b^6*(a + b*x^2)^2) + (5*a)/(2*b^6*(a + b*x^2))
 + Log[a + b*x^2]/(2*b^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{11}}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(-\frac{a^5}{b^{11}(a+bx)^6}+\frac{5a^4}{b^{11}(a+bx)^5}-\frac{10a^3}{b^{11}(a+bx)^4}+\frac{10a^2}{b^{11}(a+bx)^3}-\frac{}{b^{11}(}\right.\right.$$
$$= \frac{a^5}{10b^6\left(a+bx^2\right)^5}-\frac{5a^4}{8b^6\left(a+bx^2\right)^4}+\frac{5a^3}{3b^6\left(a+bx^2\right)^3}-\frac{5a^2}{2b^6\left(a+bx^2\right)^2}+\frac{5a}{2b^6\left(a+bx\right.}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.66

$$\frac{\frac{a\left(137a^4+625a^3bx^2+1100a^2b^2x^4+900ab^3x^6+300b^4x^8\right)}{\left(a+bx^2\right)^5}+60\log\left(a+bx^2\right)}{120b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((a*(137*a^4 + 625*a^3*b*x^2 + 1100*a^2*b^2*x^4 + 900*a*b^3*x^6 + 300*b^4*x
^8))/(a + b*x^2)^5 + 60*Log[a + b*x^2])/(120*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.77, size = 168, normalized size = 1.54

$$\frac{300\,ab^4x^8+900\,a^2b^3x^6+1100\,a^3b^2x^4+625\,a^4bx^2+137\,a^5+60\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\log\left(bx^2+a\right)}{120\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2 +
 137*a^5 + 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5
*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^
6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

giac [A] time = 0.16, size = 75, normalized size = 0.69

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^6}-\frac{137\,b^4x^{10}+385\,ab^3x^8+470\,a^2b^2x^6+270\,a^3bx^4+60\,a^4x^2}{120\,\left(bx^2+a\right)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^6 - 1/120*(137*b^4*x^10 + 385*a*b^3*x^8 + 470*a^2
*b^2*x^6 + 270*a^3*b*x^4 + 60*a^4*x^2)/((b*x^2 + a)^5*b^5)
```

maple [A] time = 0.01, size = 98, normalized size = 0.90

$$\frac{a^5}{10\left(b\,x^2+a\right)^5b^6}-\frac{5a^4}{8\left(b\,x^2+a\right)^4b^6}+\frac{5a^3}{3\left(b\,x^2+a\right)^3b^6}-\frac{5a^2}{2\left(b\,x^2+a\right)^2b^6}+\frac{5a}{2\left(b\,x^2+a\right)b^6}+\frac{\ln\left(b\,x^2+a\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*a^5/b^6/(b*x^2+a)^5-5/8*a^4/b^6/(b*x^2+a)^4+5/3*a^3/b^6/(b*x^2+a)^3-5/
2*a^2/b^6/(b*x^2+a)^2+5/2*a/b^6/(b*x^2+a)+1/2*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.42, size = 121, normalized size = 1.11

$$\frac{300\,ab^4x^8+900\,a^2b^3x^6+1100\,a^3b^2x^4+625\,a^4bx^2+137\,a^5}{120\,\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2 +
 137*a^5)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a
^4*b^7*x^2 + a^5*b^6) + 1/2*log(b*x^2 + a)/b^6
```

mupad [B] time = 4.37, size = 119, normalized size = 1.09

$$\frac{\frac{137\,a^5}{120\,b^6}+\frac{5\,a\,x^8}{2\,b^2}+\frac{15\,a^2\,x^6}{2\,b^3}+\frac{55\,a^3\,x^4}{6\,b^4}+\frac{125\,a^4\,x^2}{24\,b^5}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{\ln\left(b\,x^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((137*a^5)/(120*b^6) + (5*a*x^8)/(2*b^2) + (15*a^2*x^6)/(2*b^3) + (55*a^3*x
^4)/(6*b^4) + (125*a^4*x^2)/(24*b^5))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b
^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + log(a + b*x^2)/(2*b^6)
```

sympy [A] time = 0.81, size = 124, normalized size = 1.14

$$\frac{137a^5+625a^4bx^2+1100a^3b^2x^4+900a^2b^3x^6+300ab^4x^8}{120a^5b^6+600a^4b^7x^2+1200a^3b^8x^4+1200a^2b^9x^6+600ab^{10}x^8+120b^{11}x^{10}}+\frac{\log\left(a+bx^2\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**5 + 625*a**4*b*x**2 + 1100*a**3*b**2*x**4 + 900*a**2*b**3*x**6 + 30
0*a*b**4*x**8)/(120*a**5*b**6 + 600*a**4*b**7*x**2 + 1200*a**3*b**8*x**4 +
1200*a**2*b**9*x**6 + 600*a*b**10*x**8 + 120*b**11*x**10) + log(a + b*x**2)
/(2*b**6)
```

3.343 $\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx$

Optimal. Leaf size=19

$$\frac{x^{10}}{10a\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$\frac{x^{10}}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^10/(10*a*(a + b*x^2)^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx &= b^6 \int \frac{x^9}{\left(ab+b^2x^2\right)^6} \, dx\\ &= \frac{x^{10}}{10a\left(a+bx^2\right)^5}\end{aligned}$$

Mathematica [**B**] time = 0.02, size = 57, normalized size = 3.00

$$-\frac{a^4+5a^3bx^2+10a^2b^2x^4+10ab^3x^6+5b^4x^8}{10b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(a^4 + 5*a^3*b*x^2 + 10*a^2*b^2*x^4 + 10*a*b^3*x^6 + 5*b^4*x^8)/(b^5*
(a + b*x^2)^5)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [**B**] time = 0.85, size = 102, normalized size = 5.37

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^10
*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5
*b^5)
```

giac [**B**] time = 0.20, size = 55, normalized size = 2.89

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\left(bx^2+a\right)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/((b*x
^2 + a)^5*b^5)
```

maple [B] time = 0.01, size = 81, normalized size = 4.26

$$-\frac{a^4}{10\left(b\,x^2+a\right)^5 b^5}+\frac{a^3}{2\left(b\,x^2+a\right)^4 b^5}-\frac{a^2}{\left(b\,x^2+a\right)^3 b^5}+\frac{a}{\left(b\,x^2+a\right)^2 b^5}-\frac{1}{2\left(b\,x^2+a\right) b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -a^2/b^5/(b*x^2+a)^3+1/2*a^3/b^5/(b*x^2+a)^4-1/10*a^4/b^5/(b*x^2+a)^5+a/b^5
/(b*x^2+a)^2-1/2/b^5/(b*x^2+a)
```

maxima [B] time = 1.37, size = 102, normalized size = 5.37

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^10
*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5
*b^5)
```

mupad [B] time = 4.45, size = 104, normalized size = 5.47

$$-\frac{a^4+5\,a^3\,b\,x^2+10\,a^2\,b^2\,x^4+10\,a\,b^3\,x^6+5\,b^4\,x^8}{10\,a^5\,b^5+50\,a^4\,b^6\,x^2+100\,a^3\,b^7\,x^4+100\,a^2\,b^8\,x^6+50\,a\,b^9\,x^8+10\,b^{10}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a^4 + 5*b^4*x^8 + 5*a^3*b*x^2 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4)/(10*a^5*b^
5 + 10*b^10*x^10 + 50*a*b^9*x^8 + 50*a^4*b^6*x^2 + 100*a^3*b^7*x^4 + 100*a^
2*b^8*x^6)
```

sympy [B] time = 0.72, size = 107, normalized size = 5.63

$$\frac{-a^4-5a^3bx^2-10a^2b^2x^4-10ab^3x^6-5b^4x^8}{10a^5b^5+50a^4b^6x^2+100a^3b^7x^4+100a^2b^8x^6+50ab^9x^8+10b^{10}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a**4 - 5*a**3*b*x**2 - 10*a**2*b**2*x**4 - 10*a*b**3*x**6 - 5*b**4*x**8)/
(10*a**5*b**5 + 50*a**4*b**6*x**2 + 100*a**3*b**7*x**4 + 100*a**2*b**8*x**6
 + 50*a*b**9*x**8 + 10*b**10*x**10)
```

3.344 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=39

$$\frac{x^8}{40a^2\left(a+bx^2\right)^4}+\frac{x^8}{10a\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{x^8}{40a^2\left(a+bx^2\right)^4}+\frac{x^8}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^8/(10*a*(a + b*x^2)^5) + x^8/(40*a^2*(a + b*x^2)^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^7}{\left(ab+b^2x^2\right)^6}\,dx\\
&= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^6}\,dx,x,x^2\right)\\
&= \frac{x^8}{10a\left(a+bx^2\right)^5}+\frac{b^5\,\mathrm{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{10a}\\
&= \frac{x^8}{10a\left(a+bx^2\right)^5}+\frac{x^8}{40a^2\left(a+bx^2\right)^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.18

$$-\frac{a^3+5a^2bx^2+10ab^2x^4+10b^3x^6}{40b^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/40*(a^3 + 5*a^2*b*x^2 + 10*a*b^2*x^4 + 10*b^3*x^6)/(b^4*(a + b*x^2)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [B] time = 0.69, size = 91, normalized size = 2.33

$$-\frac{10\,b^{3}x^{6}+10\,ab^{2}x^{4}+5\,a^{2}bx^{2}+a^{3}}{40\left(b^{9}x^{10}+5\,ab^{8}x^{8}+10\,a^{2}b^{7}x^{6}+10\,a^{3}b^{6}x^{4}+5\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*x
^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

giac [A] time = 0.16, size = 44, normalized size = 1.13

$$-\frac{10\,b^{3}x^{6}+10\,ab^{2}x^{4}+5\,a^{2}bx^{2}+a^{3}}{40\left(bx^{2}+a\right)^{5}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/((b*x^2 + a)^5*b^4)
```

maple [A] time = 0.01, size = 65, normalized size = 1.67

$$\frac{a^{3}}{10\left(b\,x^{2}+a\right)^{5}b^{4}}-\frac{3a^{2}}{8\left(b\,x^{2}+a\right)^{4}b^{4}}+\frac{a}{2\left(b\,x^{2}+a\right)^{3}b^{4}}-\frac{1}{4\left(b\,x^{2}+a\right)^{2}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/2*a/b^4/(b*x^2+a)^3-3/8*a^2/b^4/(b*x^2+a)^4+1/10*a^3/b^4/(b*x^2+a)^5-1/4/
b^4/(b*x^2+a)^2
```

maxima [B] time = 1.39, size = 91, normalized size = 2.33

$$-\frac{10\,b^{3}x^{6}+10\,ab^{2}x^{4}+5\,a^{2}bx^{2}+a^{3}}{40\left(b^{9}x^{10}+5\,ab^{8}x^{8}+10\,a^{2}b^{7}x^{6}+10\,a^{3}b^{6}x^{4}+5\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*x
^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

mupad [B] time = 0.05, size = 93, normalized size = 2.38

$$-\frac{a^3+5\,a^2\,b\,x^2+10\,a\,b^2\,x^4+10\,b^3\,x^6}{40\,a^5\,b^4+200\,a^4\,b^5\,x^2+400\,a^3\,b^6\,x^4+400\,a^2\,b^7\,x^6+200\,a\,b^8\,x^8+40\,b^9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a^3 + 10*b^3*x^6 + 5*a^2*b*x^2 + 10*a*b^2*x^4)/(40*a^5*b^4 + 40*b^9*x^10
+ 200*a*b^8*x^8 + 200*a^4*b^5*x^2 + 400*a^3*b^6*x^4 + 400*a^2*b^7*x^6)
```

sympy [B] time = 0.67, size = 95, normalized size = 2.44

$$\frac{-a^3-5a^2bx^2-10ab^2x^4-10b^3x^6}{40a^5b^4+200a^4b^5x^2+400a^3b^6x^4+400a^2b^7x^6+200ab^8x^8+40b^9x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a**3 - 5*a**2*b*x**2 - 10*a*b**2*x**4 - 10*b**3*x**6)/(40*a**5*b**4 + 200
*a**4*b**5*x**2 + 400*a**3*b**6*x**4 + 400*a**2*b**7*x**6 + 200*a*b**8*x**8
 + 40*b**9*x**10)
```

3.345 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=53

$$-\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -a^2/(10*b^3*(a + b*x^2)^5) + a/(4*b^3*(a + b*x^2)^4) - 1/(6*b^3*(a + b*x^2
)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^5}{\left(ab+b^2x^2\right)^6}\,dx\\ &= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^6}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{a^2}{b^8(a+bx)^6}-\frac{2a}{b^8(a+bx)^5}+\frac{1}{b^8(a+bx)^4}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.66

$$-\frac{a^2+5abx^2+10b^2x^4}{60b^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/60*(a^2 + 5*a*b*x^2 + 10*b^2*x^4)/(b^3*(a + b*x^2)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.81, size = 80, normalized size = 1.51

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\left(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*x
^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

giac [A] time = 0.19, size = 33, normalized size = 0.62

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,\left(bx^2+a\right)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/((b*x^2 + a)^5*b^3)
```

maple [A] time = 0.01, size = 48, normalized size = 0.91

$$-\frac{a^2}{10\left(b\,x^2+a\right)^5b^3}+\frac{a}{4\left(b\,x^2+a\right)^4b^3}-\frac{1}{6\left(b\,x^2+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
[Out] -1/10*a^2/b^3/(b*x^2+a)^5+1/4*a/b^3/(b*x^2+a)^4-1/6/b^3/(b*x^2+a)^3
```

maxima [A] time = 1.35, size = 80, normalized size = 1.51

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\left(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*x
^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

mupad [B] time = 4.62, size = 81, normalized size = 1.53

$$-\frac{\frac{a^2}{60\,b^3}+\frac{x^4}{6\,b}+\frac{a\,x^2}{12\,b^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a^2/(60*b^3) + x^4/(6*b) + (a*x^2)/(12*b^2))/(a^5 + b^5*x^10 + 5*a^4*b*x^
2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

sympy [A] time = 0.62, size = 83, normalized size = 1.57

$$\frac{-a^2 - 5abx^2 - 10b^2x^4}{60a^5b^3 + 300a^4b^4x^2 + 600a^3b^5x^4 + 600a^2b^6x^6 + 300ab^7x^8 + 60b^8x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a**2 - 5*a*b*x**2 - 10*b**2*x**4)/(60*a**5*b**3 + 300*a**4*b**4*x**2 + 60
0*a**3*b**5*x**4 + 600*a**2*b**6*x**6 + 300*a*b**7*x**8 + 60*b**8*x**10)
```

3.346 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=34

$$\frac{a}{10b^2(a+bx^2)^5}-\frac{1}{8b^2(a+bx^2)^4}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a}{10b^2(a+bx^2)^5}-\frac{1}{8b^2(a+bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a/(10*b^2*(a + b*x^2)^5) - 1/(8*b^2*(a + b*x^2)^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^3}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(-\frac{a}{b^7(a+bx)^6}+\frac{1}{b^7(a+bx)^5}\right)dx, x, x^2\right)$$
$$= \frac{a}{10b^2\left(a+bx^2\right)^5}-\frac{1}{8b^2\left(a+bx^2\right)^4}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$-\frac{a+5bx^2}{40b^2\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
[Out] -1/40*(a + 5*b*x^2)/(b^2*(a + b*x^2)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
[Out] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [B] time = 0.76, size = 69, normalized size = 2.03

$$-\frac{5\,bx^2+a}{40\left(b^7x^{10}+5\,ab^6x^8+10\,a^2b^5x^6+10\,a^3b^4x^4+5\,a^4b^3x^2+a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*x
^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

giac [A] time = 0.16, size = 22, normalized size = 0.65

$$-\frac{5\,bx^2 + a}{40\,\left(bx^2 + a\right)^5 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(5*b*x^2 + a)/((b*x^2 + a)^5*b^2)
```

maple [A] time = 0.01, size = 31, normalized size = 0.91

$$\frac{a}{10\,\left(b\,x^2 + a\right)^5 b^2} - \frac{1}{8\,\left(b\,x^2 + a\right)^4 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*a/b^2/(b*x^2+a)^5-1/8/b^2/(b*x^2+a)^4
```

maxima [B] time = 1.35, size = 69, normalized size = 2.03

$$-\frac{5\,bx^2 + a}{40\,\left(b^7x^{10} + 5\,ab^6x^8 + 10\,a^2b^5x^6 + 10\,a^3b^4x^4 + 5\,a^4b^3x^2 + a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*x
^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

mupad [B] time = 4.48, size = 70, normalized size = 2.06

$$-\frac{\frac{a}{40\,b^2} + \frac{x^2}{8\,b}}{a^5 + 5\,a^4\,b\,x^2 + 10\,a^3\,b^2\,x^4 + 10\,a^2\,b^3\,x^6 + 5\,a\,b^4\,x^8 + b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a/(40*b^2) + x^2/(8*b))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*
a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

sympy [B] time = 0.58, size = 71, normalized size = 2.09

$$\frac{-a-5bx^2}{40a^5b^2+200a^4b^3x^2+400a^3b^4x^4+400a^2b^5x^6+200ab^6x^8+40b^7x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a - 5*b*x**2)/(40*a**5*b**2 + 200*a**4*b**3*x**2 + 400*a**3*b**4*x**4 + 4
00*a**2*b**5*x**6 + 200*a*b**6*x**8 + 40*b**7*x**10)
```

3.347 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/(10*b*(a + b*x^2)^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x}{\left(ab+b^2x^2\right)^6}\,dx\\ &= -\frac{1}{10b\left(a+bx^2\right)^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*1/(b*(a + b*x^2)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [B] time = 0.77, size = 59, normalized size = 3.69

$$-\frac{1}{10\left(b^6x^{10}+5\,ab^5x^8+10\,a^2b^4x^6+10\,a^3b^3x^4+5\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^2
*x^2 + a^5*b)
```

giac [A] time = 0.19, size = 14, normalized size = 0.88

$$-\frac{1}{10\left(bx^2+a\right)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/10/((b*x^2 + a)^5*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{10\left(b\,x^{2}+a\right)^{5} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/10/b/(b*x^2+a)^5
```

maxima [B] time = 1.31, size = 59, normalized size = 3.69

$$-\frac{1}{10\left(b^{6}x^{10}+5\,ab^{5}x^{8}+10\,a^{2}b^{4}x^{6}+10\,a^{3}b^{3}x^{4}+5\,a^{4}b^{2}x^{2}+a^{5}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^2
*x^2 + a^5*b)
```

mupad [B] time = 0.06, size = 61, normalized size = 3.81

$$-\frac{1}{10\,a^{5}\,b+50\,a^{4}\,b^{2}\,x^{2}+100\,a^{3}\,b^{3}\,x^{4}+100\,a^{2}\,b^{4}\,x^{6}+50\,a\,b^{5}\,x^{8}+10\,b^{6}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -1/(10*a^5*b + 10*b^6*x^10 + 50*a*b^5*x^8 + 50*a^4*b^2*x^2 + 100*a^3*b^3*x^
4 + 100*a^2*b^4*x^6)
```

sympy [B] time = 0.50, size = 63, normalized size = 3.94

$$-\frac{1}{10a^{5}b+50a^{4}b^{2}x^{2}+100a^{3}b^{3}x^{4}+100a^{2}b^{4}x^{6}+50ab^{5}x^{8}+10b^{6}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -1/(10*a**5*b + 50*a**4*b**2*x**2 + 100*a**3*b**3*x**4 + 100*a**2*b**4*x**6
 + 50*a*b**5*x**8 + 10*b**6*x**10)
```

3.348 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=102

$$-\frac{\log\left(a+bx^2\right)}{2a^6}+\frac{\log(x)}{a^6}+\frac{1}{2a^5\left(a+bx^2\right)}+\frac{1}{4a^4\left(a+bx^2\right)^2}+\frac{1}{6a^3\left(a+bx^2\right)^3}+\frac{1}{8a^2\left(a+bx^2\right)^4}+\frac{1}{10a\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{1}{2a^5\left(a+bx^2\right)}+\frac{1}{4a^4\left(a+bx^2\right)^2}+\frac{1}{6a^3\left(a+bx^2\right)^3}+\frac{1}{8a^2\left(a+bx^2\right)^4}-\frac{\log\left(a+bx^2\right)}{2a^6}+\frac{\log(x)}{a^6}+\frac{1}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] 1/(10*a*(a + b*x^2)^5) + 1/(8*a^2*(a + b*x^2)^4) + 1/(6*a^3*(a + b*x^2)^3)
+ 1/(4*a^4*(a + b*x^2)^2) + 1/(2*a^5*(a + b*x^2)) + Log[x]/a^6 - Log[a + b*
x^2]/(2*a^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{1}{x\left(ab+b^2x^2\right)^6}\,dx \\
&= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^6}\,dx, x, x^2\right) \\
&= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{1}{a^6b^6x}-\frac{1}{ab^5(a+bx)^6}-\frac{1}{a^2b^5(a+bx)^5}-\frac{1}{a^3b^5(a+bx)^4}-\frac{1}{a^4b^5(a\right.\right. \\
&= \frac{1}{10a\left(a+bx^2\right)^5}+\frac{1}{8a^2\left(a+bx^2\right)^4}+\frac{1}{6a^3\left(a+bx^2\right)^3}+\frac{1}{4a^4\left(a+bx^2\right)^2}+\frac{1}{2a^5\left(a+\right.}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.75

$$
\frac{\frac{a\left(137a^4+385a^3bx^2+470a^2b^2x^4+270ab^3x^6+60b^4x^8\right)}{\left(a+bx^2\right)^5}-60\log\left(a+bx^2\right)+120\log(x)}{120a^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(137*a^4 + 385*a^3*b*x^2 + 470*a^2*b^2*x^4 + 270*a*b^3*x^6 + 60*b^4*x^8
))/(a + b*x^2)^5 + 120*Log[x] - 60*Log[a + b*x^2])/(120*a^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3), x]
```

fricas [B] time = 0.79, size = 222, normalized size = 2.18

$$
\frac{60\,ab^4x^8+270\,a^2b^3x^6+470\,a^3b^2x^4+385\,a^4bx^2+137\,a^5-60\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\log\left(bx^2+a\right)+120\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\log(x)}{120\left(a^6b^5x^{10}+5\,a^7b^4x^8+10\,a^8b^3x^6+10\,a^9b^2x^4+5\,a^{10}bx^2+a^{11}\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")

[Out] 1/120*(60*a*b^4*x^8 + 270*a^2*b^3*x^6 + 470*a^3*b^2*x^4 + 385*a^4*b*x^2 + 137*a^5 - 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*log(b*x^2 + a) + 120*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*log(x))/(a^6*b^5*x^10 + 5*a^7*b^4*x^8 + 10*a^8*b^3*x^6 + 10*a^9*b^2*x^4 + 5*a^10*b*x^2 + a^11)

giac [A] time = 0.15, size = 92, normalized size = 0.90

$$\frac{\log\left(x^2\right)}{2\,a^6} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a^6} + \frac{137\,b^5x^{10} + 745\,ab^4x^8 + 1640\,a^2b^3x^6 + 1840\,a^3b^2x^4 + 1070\,a^4bx^2 + 274\,a^5}{120\left(bx^2 + a\right)^5a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")

[Out] 1/2*log(x^2)/a^6 - 1/2*log(abs(b*x^2 + a))/a^6 + 1/120*(137*b^5*x^10 + 745*a*b^4*x^8 + 1640*a^2*b^3*x^6 + 1840*a^3*b^2*x^4 + 1070*a^4*b*x^2 + 274*a^5)/((b*x^2 + a)^5*a^6)

maple [A] time = 0.02, size = 91, normalized size = 0.89

$$\frac{1}{10\left(b\,x^2 + a\right)^5 a} + \frac{1}{8\left(b\,x^2 + a\right)^4 a^2} + \frac{1}{6\left(b\,x^2 + a\right)^3 a^3} + \frac{1}{4\left(b\,x^2 + a\right)^2 a^4} + \frac{1}{2\left(b\,x^2 + a\right)a^5} + \frac{\ln(x)}{a^6} - \frac{\ln\left(b\,x^2 + a\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)

[Out] 1/10/a/(b*x^2+a)^5+1/8/a^2/(b*x^2+a)^4+1/6/a^3/(b*x^2+a)^3+1/4/a^4/(b*x^2+a)^2+1/2/a^5/(b*x^2+a)+ln(x)/a^6-1/2*ln(b*x^2+a)/a^6

maxima [A] time = 1.45, size = 126, normalized size = 1.24

$$\frac{60\,b^4x^8 + 270\,ab^3x^6 + 470\,a^2b^2x^4 + 385\,a^3bx^2 + 137\,a^4}{120\left(a^5b^5x^{10} + 5\,a^6b^4x^8 + 10\,a^7b^3x^6 + 10\,a^8b^2x^4 + 5\,a^9bx^2 + a^{10}\right)} - \frac{\log\left(bx^2 + a\right)}{2\,a^6} + \frac{\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")

[Out] 1/120*(60*b^4*x^8 + 270*a*b^3*x^6 + 470*a^2*b^2*x^4 + 385*a^3*b*x^2 + 137*a^4)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10) - 1/2*log(b*x^2 + a)/a^6 + 1/2*log(x^2)/a^6

mupad [B] time = 0.24, size = 122, normalized size = 1.20

$$\frac{\ln(x)}{a^6}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^6}+\frac{\frac{137}{120\,a}+\frac{77\,b\,x^2}{24\,a^2}+\frac{47\,b^2\,x^4}{12\,a^3}+\frac{9\,b^3\,x^6}{4\,a^4}+\frac{b^4\,x^8}{2\,a^5}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] log(x)/a^6 - log(a + b*x^2)/(2*a^6) + (137/(120*a) + (77*b*x^2)/(24*a^2) +
(47*b^2*x^4)/(12*a^3) + (9*b^3*x^6)/(4*a^4) + (b^4*x^8)/(2*a^5))/(a^5 + b^5
*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

sympy [A] time = 0.81, size = 128, normalized size = 1.25

$$\frac{137a^4+385a^3bx^2+470a^2b^2x^4+270ab^3x^6+60b^4x^8}{120a^{10}+600a^9bx^2+1200a^8b^2x^4+1200a^7b^3x^6+600a^6b^4x^8+120a^5b^5x^{10}}+\frac{\log(x)}{a^6}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**4 + 385*a**3*b*x**2 + 470*a**2*b**2*x**4 + 270*a*b**3*x**6 + 60*b**
4*x**8)/(120*a**10 + 600*a**9*b*x**2 + 1200*a**8*b**2*x**4 + 1200*a**7*b**3
*x**6 + 600*a**6*b**4*x**8 + 120*a**5*b**5*x**10) + log(x)/a**6 - log(a/b +
 x**2)/(2*a**6)
```

3.349 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=116

$$\frac{3b\log\left(a+bx^2\right)}{a^7}-\frac{6b\log(x)}{a^7}-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{1}{2a^6x^2}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{10a^2\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{10a^2\left(a+bx^2\right)^5}+\frac{3b\log\left(a+bx^2\right)}{a^7}-\frac{6b\log(x)}{a^7}-\frac{1}{2a^6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/(2*a^6*x^2) - b/(10*a^2*(a + b*x^2)^5) - b/(4*a^3*(a + b*x^2)^4) - b/(2*
a^4*(a + b*x^2)^3) - b/(a^5*(a + b*x^2)^2) - (5*b)/(2*a^6*(a + b*x^2)) - (6
*b*Log[x])/a^7 + (3*b*Log[a + b*x^2])/a^7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{1}{x^3\left(ab+b^2x^2\right)^6}\,dx \\
&= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^6}\,dx, x, x^2\right) \\
&= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \left(\frac{1}{a^6b^6x^2}-\frac{6}{a^7b^5x}+\frac{1}{a^2b^4(a+bx)^6}+\frac{2}{a^3b^4(a+bx)^5}+\frac{3}{a^4b^4(a+b\right.\right. \\
&= -\frac{1}{2a^6x^2}-\frac{b}{10a^2\left(a+bx^2\right)^5}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{a^5\left(a+bx^2\right)^2}-
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 92, normalized size = 0.79

$$
-\frac{\frac{a\left(10a^5+137a^4bx^2+385a^3b^2x^4+470a^2b^3x^6+270ab^4x^8+60b^5x^{10}\right)}{x^2\left(a+bx^2\right)^5}-60b\log\left(a+bx^2\right)+120b\log(x)}{20a^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/20*((a*(10*a^5 + 137*a^4*b*x^2 + 385*a^3*b^2*x^4 + 470*a^2*b^3*x^6 + 270
*a*b^4*x^8 + 60*b^5*x^10))/(x^2*(a + b*x^2)^5) + 120*b*Log[x] - 60*b*Log[a
+ b*x^2])/a^7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3), x]
```

fricas [B] time = 0.81, size = 251, normalized size = 2.16

$$
-\frac{60\,ab^5x^{10}+270\,a^2b^4x^8+470\,a^3b^3x^6+385\,a^4b^2x^4+137\,a^5bx^2+10\,a^6-60\left(b^6x^{12}+5\,ab^5x^{10}+10\,a^2b^4x^8+10\,a^3b^3x^6+5\,a^4b^2x^4+a^5bx^2\right)\log\left(bx^2+a\right)+120\left(b^6x^{12}+5\,ab^5x^{10}+10\,a^2b^4x^8+10\,a^3b^3x^6+5\,a^4b^2x^4+a^5bx^2\right)\log(x)}{20\left(a^7b^5x^{12}+5\,a^8b^4x^{10}+10\,a^9b^3x^8+10\,a^{10}b^2x^6+5\,a^{11}bx^4+a^{12}x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/20*(60*a*b^5*x^10 + 270*a^2*b^4*x^8 + 470*a^3*b^3*x^6 + 385*a^4*b^2*x^4
+ 137*a^5*b*x^2 + 10*a^6 - 60*(b^6*x^12 + 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 1
0*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2)*log(b*x^2 + a) + 120*(b^6*x^12 +
 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2
)*log(x))/(a^7*b^5*x^12 + 5*a^8*b^4*x^10 + 10*a^9*b^3*x^8 + 10*a^10*b^2*x^6
 + 5*a^11*b*x^4 + a^12*x^2)
```

giac [A] time = 0.16, size = 115, normalized size = 0.99

$$-\frac{3\,b\log\left(x^2\right)}{a^7}+\frac{3\,b\log\left(\left|bx^2+a\right|\right)}{a^7}+\frac{6\,bx^2-a}{2\,a^7x^2}-\frac{137\,b^6x^{10}+735\,ab^5x^8+1590\,a^2b^4x^6+1740\,a^3b^3x^4+970\,a^4b^2x^2+224\,a^5b}{20\left(bx^2+a\right)^5a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -3*b*log(x^2)/a^7 + 3*b*log(abs(b*x^2 + a))/a^7 + 1/2*(6*b*x^2 - a)/(a^7*x^
2) - 1/20*(137*b^6*x^10 + 735*a*b^5*x^8 + 1590*a^2*b^4*x^6 + 1740*a^3*b^3*x
^4 + 970*a^4*b^2*x^2 + 224*a^5*b)/((b*x^2 + a)^5*a^7)
```

maple [A] time = 0.02, size = 107, normalized size = 0.92

$$-\frac{b}{10\left(b\,x^2+a\right)^5a^2}-\frac{b}{4\left(b\,x^2+a\right)^4a^3}-\frac{b}{2\left(b\,x^2+a\right)^3a^4}-\frac{b}{\left(b\,x^2+a\right)^2a^5}-\frac{5b}{2\left(b\,x^2+a\right)a^6}-\frac{6b\ln(x)}{a^7}+\frac{3b\ln\left(b\,x^2+a\right)}{a^7}-\frac{1}{2a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/2/a^6/x^2-1/10*b/a^2/(b*x^2+a)^5-1/4*b/a^3/(b*x^2+a)^4-1/2*b/a^4/(b*x^2+
a)^3-b/a^5/(b*x^2+a)^2-5/2*b/a^6/(b*x^2+a)-6*b*ln(x)/a^7+3*b*ln(b*x^2+a)/a^
7
```

maxima [A] time = 1.58, size = 143, normalized size = 1.23

$$-\frac{60\,b^5x^{10}+270\,ab^4x^8+470\,a^2b^3x^6+385\,a^3b^2x^4+137\,a^4bx^2+10\,a^5}{20\left(a^6b^5x^{12}+5\,a^7b^4x^{10}+10\,a^8b^3x^8+10\,a^9b^2x^6+5\,a^{10}bx^4+a^{11}x^2\right)}+\frac{3\,b\log\left(bx^2+a\right)}{a^7}-\frac{3\,b\log\left(x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(60*b^5*x^10 + 270*a*b^4*x^8 + 470*a^2*b^3*x^6 + 385*a^3*b^2*x^4 + 13
7*a^4*b*x^2 + 10*a^5)/(a^6*b^5*x^12 + 5*a^7*b^4*x^10 + 10*a^8*b^3*x^8 + 10*
```

```
a^9*b^2*x^6 + 5*a^10*b*x^4 + a^11*x^2) + 3*b*log(b*x^2 + a)/a^7 - 3*b*log(x
^2)/a^7
```

mupad [B] time = 4.68, size = 141, normalized size = 1.22

$$\frac{3\,b\,\ln\left(b\,x^2+a\right)}{a^7}-\frac{\frac{1}{2\,a}+\frac{137\,b\,x^2}{20\,a^2}+\frac{77\,b^2\,x^4}{4\,a^3}+\frac{47\,b^3\,x^6}{2\,a^4}+\frac{27\,b^4\,x^8}{2\,a^5}+\frac{3\,b^5\,x^{10}}{a^6}}{a^5\,x^2+5\,a^4\,b\,x^4+10\,a^3\,b^2\,x^6+10\,a^2\,b^3\,x^8+5\,a\,b^4\,x^{10}+b^5\,x^{12}}-\frac{6\,b\,\ln(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] (3*b*log(a + b*x^2))/a^7 - (1/(2*a) + (137*b*x^2)/(20*a^2) + (77*b^2*x^4)/(
4*a^3) + (47*b^3*x^6)/(2*a^4) + (27*b^4*x^8)/(2*a^5) + (3*b^5*x^10)/a^6)/(a
^5*x^2 + b^5*x^12 + 5*a^4*b*x^4 + 5*a*b^4*x^10 + 10*a^3*b^2*x^6 + 10*a^2*b^
3*x^8) - (6*b*log(x))/a^7
```

sympy [A] time = 0.90, size = 150, normalized size = 1.29

$$\frac{-10a^5-137a^4bx^2-385a^3b^2x^4-470a^2b^3x^6-270ab^4x^8-60b^5x^{10}}{20a^{11}x^2+100a^{10}bx^4+200a^9b^2x^6+200a^8b^3x^8+100a^7b^4x^{10}+20a^6b^5x^{12}}-\frac{6b\log(x)}{a^7}+\frac{3b\log\left(\frac{a}{b}+x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-10*a**5 - 137*a**4*b*x**2 - 385*a**3*b**2*x**4 - 470*a**2*b**3*x**6 - 270
*a*b**4*x**8 - 60*b**5*x**10)/(20*a**11*x**2 + 100*a**10*b*x**4 + 200*a**9*
b**2*x**6 + 200*a**8*b**3*x**8 + 100*a**7*b**4*x**10 + 20*a**6*b**5*x**12)
- 6*b*log(x)/a**7 + 3*b*log(a/b + x**2)/a**7
```

3.350 $\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}+\frac{21b^2\log(x)}{a^8}+\frac{15b^2}{2a^7\left(a+bx^2\right)}+\frac{3b}{a^7x^2}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}-\frac{1}{4a^6x^4}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+$$

Rubi [A] time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{15b^2}{2a^7\left(a+bx^2\right)}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{10a^3\left(a+bx^2\right)^5}-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}+\frac{21b^2\log(x)}{a^8}+\frac{3b}{a^7x^2}-\frac{1}{4a^6x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/(4*a^6*x^4) + (3*b)/(a^7*x^2) + b^2/(10*a^3*(a + b*x^2)^5) + (3*b^2)/(8*
a^4*(a + b*x^2)^4) + b^2/(a^5*(a + b*x^2)^3) + (5*b^2)/(2*a^6*(a + b*x^2)^2
) + (15*b^2)/(2*a^7*(a + b*x^2)) + (21*b^2*Log[x])/a^8 - (21*b^2*Log[a + b*
x^2])/(2*a^8)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{1}{x^5\left(ab+b^2x^2\right)^6}\,dx \\
&= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{1}{x^3\left(ab+b^2x\right)^6}\,dx,x,x^2\right) \\
&= \frac{1}{2}b^6\,\text{Subst}\left(\int\left(\frac{1}{a^6b^6x^3}-\frac{6}{a^7b^5x^2}+\frac{21}{a^8b^4x}-\frac{1}{a^3b^3(a+bx)^6}-\frac{3}{a^4b^3(a+bx)^5}-\dots\right.\right. \\
&= -\frac{1}{4a^6x^4}+\frac{3b}{a^7x^2}+\frac{b^2}{10a^3\left(a+bx^2\right)^5}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{5b\dots}{2a^6\left(a+\dots\right.}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 107, normalized size = 0.76

$$
\frac{\frac{a\left(-10a^6+70a^5bx^2+959a^4b^2x^4+2695a^3b^3x^6+3290a^2b^4x^8+1890ab^5x^{10}+420b^6x^{12}\right)}{x^4\left(a+bx^2\right)^5}-420b^2\log\left(a+bx^2\right)+840b^2\log(x)}{40a^8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(-10*a^6 + 70*a^5*b*x^2 + 959*a^4*b^2*x^4 + 2695*a^3*b^3*x^6 + 3290*a^2
*b^4*x^8 + 1890*a*b^5*x^10 + 420*b^6*x^12))/(x^4*(a + b*x^2)^5) + 840*b^2*L
og[x] - 420*b^2*Log[a + b*x^2])/(40*a^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3), x]
```

fricas [B] time = 0.89, size = 266, normalized size = 1.90

$$
\frac{420\,ab^6x^{12}+1890\,a^2b^5x^{10}+3290\,a^3b^4x^8+2695\,a^4b^3x^6+959\,a^5b^2x^4+70\,a^6bx^2-10\,a^7-420\left(b^7x^{14}+5\,ab^6x^{12}+10\,a^2b^5x^{10}+10\,a^3b^4x^8+5\,a^4b^3x^6+a^5b^2x^4\right)\log\left(bx^2+a\right)+840\left(b^7x^{14}+5\,ab^6x^{12}+10\,a^2b^5x^{10}+10\,a^3b^4x^8+5\,a^4b^3x^6+a^5b^2x^4\right)\log(x)}{40\left(a^8b^5x^{14}+5\,a^9b^4x^{12}+10\,a^{10}b^3x^{10}+10\,a^{11}b^2x^8+5\,a^{12}bx^6+a^{13}x^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(420*a*b^6*x^12 + 1890*a^2*b^5*x^10 + 3290*a^3*b^4*x^8 + 2695*a^4*b^3*
x^6 + 959*a^5*b^2*x^4 + 70*a^6*b*x^2 - 10*a^7 - 420*(b^7*x^14 + 5*a*b^6*x^1
2 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 + 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(b*x
^2 + a) + 840*(b^7*x^14 + 5*a*b^6*x^12 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 +
 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(x))/(a^8*b^5*x^14 + 5*a^9*b^4*x^12 + 10*a
^10*b^3*x^10 + 10*a^11*b^2*x^8 + 5*a^12*b*x^6 + a^13*x^4)
```

giac [A] time = 0.16, size = 130, normalized size = 0.93

$$\frac{21\,b^{2}\log\left(x^{2}\right)}{2\,a^{8}}-\frac{21\,b^{2}\log\left(|bx^{2}+a|\right)}{2\,a^{8}}-\frac{63\,b^{2}x^{4}-12\,abx^{2}+a^{2}}{4\,a^{8}x^{4}}+\frac{959\,b^{7}x^{10}+5095\,ab^{6}x^{8}+10890\,a^{2}b^{5}x^{6}+11730\,a^{3}b^{4}x^{4}+6390\,a^{4}b^{3}x^{2}+1418\,a^{5}b^{2}}{40\left(bx^{2}+a\right)^{5}a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 21/2*b^2*log(x^2)/a^8 - 21/2*b^2*log(abs(b*x^2 + a))/a^8 - 1/4*(63*b^2*x^4
- 12*a*b*x^2 + a^2)/(a^8*x^4) + 1/40*(959*b^7*x^10 + 5095*a*b^6*x^8 + 10890
*a^2*b^5*x^6 + 11730*a^3*b^4*x^4 + 6390*a^4*b^3*x^2 + 1418*a^5*b^2)/((b*x^2
 + a)^5*a^8)
```

maple [A] time = 0.02, size = 129, normalized size = 0.92

$$\frac{b^{2}}{10\left(b\,x^{2}+a\right)^{5}a^{3}}+\frac{3b^{2}}{8\left(b\,x^{2}+a\right)^{4}a^{4}}+\frac{b^{2}}{\left(b\,x^{2}+a\right)^{3}a^{5}}+\frac{5b^{2}}{2\left(b\,x^{2}+a\right)^{2}a^{6}}+\frac{15b^{2}}{2\left(b\,x^{2}+a\right)a^{7}}+\frac{21b^{2}\ln(x)}{a^{8}}-\frac{21b^{2}\ln\left(b\,x^{2}+a\right)}{2a^{8}}+\frac{3b}{a^{7}x^{2}}-\frac{1}{4a^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/4/a^6/x^4+3*b/a^7/x^2+1/10*b^2/a^3/(b*x^2+a)^5+3/8*b^2/a^4/(b*x^2+a)^4+b
^2/a^5/(b*x^2+a)^3+5/2*b^2/a^6/(b*x^2+a)^2+15/2*b^2/a^7/(b*x^2+a)+21*b^2*ln
(x)/a^8-21/2*b^2*ln(b*x^2+a)/a^8
```

maxima [A] time = 1.45, size = 158, normalized size = 1.13

$$\frac{420\,b^{6}x^{12}+1890\,ab^{5}x^{10}+3290\,a^{2}b^{4}x^{8}+2695\,a^{3}b^{3}x^{6}+959\,a^{4}b^{2}x^{4}+70\,a^{5}bx^{2}-10\,a^{6}}{40\left(a^{7}b^{5}x^{14}+5\,a^{8}b^{4}x^{12}+10\,a^{9}b^{3}x^{10}+10\,a^{10}b^{2}x^{8}+5\,a^{11}bx^{6}+a^{12}x^{4}\right)}-\frac{21\,b^{2}\log\left(bx^{2}+a\right)}{2\,a^{8}}+\frac{21\,b^{2}\log\left(x^{2}\right)}{2\,a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(420*b^6*x^12 + 1890*a*b^5*x^10 + 3290*a^2*b^4*x^8 + 2695*a^3*b^3*x^6
+ 959*a^4*b^2*x^4 + 70*a^5*b*x^2 - 10*a^6)/(a^7*b^5*x^14 + 5*a^8*b^4*x^12 +
```

```
10*a^9*b^3*x^10 + 10*a^10*b^2*x^8 + 5*a^11*b*x^6 + a^12*x^4) - 21/2*b^2*lo
g(b*x^2 + a)/a^8 + 21/2*b^2*log(x^2)/a^8
```

mupad [B] time = 4.91, size = 155, normalized size = 1.11

$$\frac{\frac{7\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{959\,b^2\,x^4}{40\,a^3}+\frac{539\,b^3\,x^6}{8\,a^4}+\frac{329\,b^4\,x^8}{4\,a^5}+\frac{189\,b^5\,x^{10}}{4\,a^6}+\frac{21\,b^6\,x^{12}}{2\,a^7}}{a^5\,x^4+5\,a^4\,b\,x^6+10\,a^3\,b^2\,x^8+10\,a^2\,b^3\,x^{10}+5\,a\,b^4\,x^{12}+b^5\,x^{14}}-\frac{21\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^8}+\frac{21\,b^2\,\ln(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((7*b*x^2)/(4*a^2) - 1/(4*a) + (959*b^2*x^4)/(40*a^3) + (539*b^3*x^6)/(8*a^
4) + (329*b^4*x^8)/(4*a^5) + (189*b^5*x^10)/(4*a^6) + (21*b^6*x^12)/(2*a^7)
)/(a^5*x^4 + b^5*x^14 + 5*a^4*b*x^6 + 5*a*b^4*x^12 + 10*a^3*b^2*x^8 + 10*a^
2*b^3*x^10) - (21*b^2*log(a + b*x^2))/(2*a^8) + (21*b^2*log(x))/a^8
```

sympy [A] time = 0.96, size = 165, normalized size = 1.18

$$\frac{-10a^6+70a^5bx^2+959a^4b^2x^4+2695a^3b^3x^6+3290a^2b^4x^8+1890ab^5x^{10}+420b^6x^{12}}{40a^{12}x^4+200a^{11}bx^6+400a^{10}b^2x^8+400a^9b^3x^{10}+200a^8b^4x^{12}+40a^7b^5x^{14}}+\frac{21b^2\log(x)}{a^8}-\frac{21b^2\log\left(\frac{a}{b}+x^2\right)}{2a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-10*a**6 + 70*a**5*b*x**2 + 959*a**4*b**2*x**4 + 2695*a**3*b**3*x**6 + 329
0*a**2*b**4*x**8 + 1890*a*b**5*x**10 + 420*b**6*x**12)/(40*a**12*x**4 + 200
*a**11*b*x**6 + 400*a**10*b**2*x**8 + 400*a**9*b**3*x**10 + 200*a**8*b**4*x
**12 + 40*a**7*b**5*x**14) + 21*b**2*log(x)/a**8 - 21*b**2*log(a/b + x**2)/
(2*a**8)
```

3.351 $\int \frac{x^{16}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{9009a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{17/2}}+\frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}-\frac{1287x^7}{256b^5\left(a+bx^2\right)}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{3x^{13}}{16b^2\left(a+bx^2\right.}$$

Rubi [A] time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{9009a^2x}{256b^8}-\frac{9009a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{17/2}}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{1287x^7}{256b^5\left(a+bx^2\right)}-\frac{3003ax^3}{256b^7}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}+\frac{9009x^5}{1280b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (9009*a^2*x)/(256*b^8) - (3003*a*x^3)/(256*b^7) + (9009*x^5)/(1280*b^6) - x
^15/(10*b*(a + b*x^2)^5) - (3*x^13)/(16*b^2*(a + b*x^2)^4) - (13*x^11)/(32*
b^3*(a + b*x^2)^3) - (143*x^9)/(128*b^4*(a + b*x^2)^2) - (1287*x^7)/(256*b^
5*(a + b*x^2)) - (9009*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(17/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{16}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{16}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}+\frac{1}{2}\left(3b^4\right)\int \frac{x^{14}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}+\frac{1}{16}\left(39b^2\right)\int \frac{x^{12}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}+\frac{143}{32}\int \frac{x^{10}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}+\frac{1287}{\cdots}$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{1}{256b\cdots}$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{1}{256b\cdots}$$

$$= \frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}+\frac{9009x^5}{1280b^6}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+b\cdots\right.}$$

$$= \frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}+\frac{9009x^5}{1280b^6}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+b\cdots\right.}$$

Mathematica [A] time = 0.06, size = 122, normalized size = 0.79

$$\frac{\frac{\sqrt{b}\,x\left(45045a^7+210210a^6bx^2+384384a^5b^2x^4+338910a^4b^3x^6+137995a^3b^4x^8+16640a^2b^5x^{10}-1280ab^6x^{12}+256b^7x^{14}\right)}{\left(a+bx^2\right)^5}-45045a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1280b^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(45045*a^7 + 210210*a^6*b*x^2 + 384384*a^5*b^2*x^4 + 338910*a^4
*b^3*x^6 + 137995*a^3*b^4*x^8 + 16640*a^2*b^5*x^10 - 1280*a*b^6*x^12 + 256*
b^7*x^14))/(a + b*x^2)^5 - 45045*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(1280
*b^(17/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{16}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.82, size = 454, normalized size = 2.93

$$\left[\frac{512\,b^7x^{15}-2560\,ab^6x^{13}+33280\,a^2b^5x^{11}+275990\,a^3b^4x^9+677820\,a^4b^3x^7+768768\,a^5b^2x^5+420420\,a^6bx^3+90090\,a^7x+45045\left(a^2b^5x^{10}+5\,a^3b^4x^8+10\,a^4b^3x^6+10\,a^5b^2x^4+5\,a^6bx^2+a^7\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{2560\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8\right)},\ \frac{256\,b^7x^{15}-1280\,ab^6x^{13}+16640\,a^2b^5x^{11}+137995\,a^3b^4x^9+338910\,a^4b^3x^7+384384\,a^5b^2x^5+210210\,a^6bx^3+45045\,a^7x-45045\left(a^2b^5x^{10}+5\,a^3b^4x^8+10\,a^4b^3x^6+10\,a^5b^2x^4+5\,a^6bx^2+a^7\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{1280\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(512*b^7*x^15 - 2560*a*b^6*x^13 + 33280*a^2*b^5*x^11 + 275990*a^3*b
^4*x^9 + 677820*a^4*b^3*x^7 + 768768*a^5*b^2*x^5 + 420420*a^6*b*x^3 + 90090
*a^7*x + 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*
x^4 + 5*a^6*b*x^2 + a^7)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5
*a^4*b^9*x^2 + a^5*b^8), 1/1280*(256*b^7*x^15 - 1280*a*b^6*x^13 + 16640*a^2
*b^5*x^11 + 137995*a^3*b^4*x^9 + 338910*a^4*b^3*x^7 + 384384*a^5*b^2*x^5 +
210210*a^6*b*x^3 + 45045*a^7*x - 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a
^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7)*sqrt(a/b)*arctan(b*x*sqrt(
a/b)/a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5*
a^4*b^9*x^2 + a^5*b^8)]
```

giac [A] time = 0.17, size = 117, normalized size = 0.75

$$-\frac{9009\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^{8}}+\frac{26635\,a^{3}b^{4}x^{9}+94430\,a^{4}b^{3}x^{7}+128128\,a^{5}b^{2}x^{5}+78370\,a^{6}bx^{3}+18165\,a^{7}x}{1280\,\left(bx^{2}+a\right)^{5}b^{8}}+\frac{b^{24}x^{5}-10\,ab^{23}x^{3}+105\,a^{2}b^{22}x}{5\,b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -9009/256*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^8) + 1/1280*(26635*a^3*b^4
*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370*a^6*b*x^3 + 18165*a^7
*x)/((b*x^2 + a)^5*b^8) + 1/5*(b^24*x^5 - 10*a*b^23*x^3 + 105*a^2*b^22*x)/b
^30
```

maple [A] time = 0.02, size = 148, normalized size = 0.95

$$\frac{5327a^{3}x^{9}}{256\left(b\,x^{2}+a\right)^{5}b^{4}}+\frac{9443a^{4}x^{7}}{128\left(b\,x^{2}+a\right)^{5}b^{5}}+\frac{1001a^{5}x^{5}}{10\left(b\,x^{2}+a\right)^{5}b^{6}}+\frac{7837a^{6}x^{3}}{128\left(b\,x^{2}+a\right)^{5}b^{7}}+\frac{3633a^{7}x}{256\left(b\,x^{2}+a\right)^{5}b^{8}}+\frac{x^{5}}{5b^{6}}-\frac{2a\,x^{3}}{b^{7}}-\frac{9009a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,b^{8}}+\frac{21a^{2}x}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/5*x^5/b^6-2*a*x^3/b^7+21*a^2*x/b^8+5327/256/b^4*a^3/(b*x^2+a)^5*x^9+9443/
128/b^5*a^4/(b*x^2+a)^5*x^7+1001/10/b^6*a^5/(b*x^2+a)^5*x^5+7837/128/b^7*a^
6/(b*x^2+a)^5*x^3+3633/256/b^8*a^7/(b*x^2+a)^5*x-9009/256/b^8*a^3/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.89, size = 159, normalized size = 1.03

$$\frac{26635\,a^{3}b^{4}x^{9}+94430\,a^{4}b^{3}x^{7}+128128\,a^{5}b^{2}x^{5}+78370\,a^{6}bx^{3}+18165\,a^{7}x}{1280\left(b^{13}x^{10}+5\,ab^{12}x^{8}+10\,a^{2}b^{11}x^{6}+10\,a^{3}b^{10}x^{4}+5\,a^{4}b^{9}x^{2}+a^{5}b^{8}\right)}-\frac{9009\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^{8}}+\frac{b^{2}x^{5}-10\,abx^{3}+105\,a^{2}x}{5\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(26635*a^3*b^4*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370*
a^6*b*x^3 + 18165*a^7*x)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a
^3*b^10*x^4 + 5*a^4*b^9*x^2 + a^5*b^8) - 9009/256*a^3*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^8) + 1/5*(b^2*x^5 - 10*a*b*x^3 + 105*a^2*x)/b^8
```

mupad [B] time = 0.11, size = 153, normalized size = 0.99

$$\frac{\frac{3633\,a^{7}\,x}{256}+\frac{7837\,a^{6}\,b\,x^{3}}{128}+\frac{1001\,a^{5}\,b^{2}\,x^{5}}{10}+\frac{9443\,a^{4}\,b^{3}\,x^{7}}{128}+\frac{5327\,a^{3}\,b^{4}\,x^{9}}{256}}{a^{5}\,b^{8}+5\,a^{4}\,b^{9}\,x^{2}+10\,a^{3}\,b^{10}\,x^{4}+10\,a^{2}\,b^{11}\,x^{6}+5\,a\,b^{12}\,x^{8}+b^{13}\,x^{10}}+\frac{x^{5}}{5\,b^{6}}-\frac{2\,a\,x^{3}}{b^{7}}+\frac{21\,a^{2}\,x}{b^{8}}-\frac{9009\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((3633*a^7*x)/256 + (7837*a^6*b*x^3)/128 + (1001*a^5*b^2*x^5)/10 + (9443*a^
4*b^3*x^7)/128 + (5327*a^3*b^4*x^9)/256)/(a^5*b^8 + b^13*x^10 + 5*a*b^12*x^
8 + 5*a^4*b^9*x^2 + 10*a^3*b^10*x^4 + 10*a^2*b^11*x^6) + x^5/(5*b^6) - (2*a
*x^3)/b^7 + (21*a^2*x)/b^8 - (9009*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*
b^(17/2))
```

sympy [A] time = 1.09, size = 218, normalized size = 1.41

$$\frac{21a^2x}{b^8} - \frac{2ax^3}{b^7} + \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x - \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512} - \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x + \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512} + \frac{18165a^7x + 78370a^6bx^3 + 128128a^5b^2x^5 + 94430a^4b^3x^7 + 26635a^3b^4x^9}{1280a^5b^8 + 6400a^4b^9x^2 + 12800a^3b^{10}x^4 + 12800a^2b^{11}x^6 + 6400ab^{12}x^8 + 1280b^{13}x^{10}} + \frac{x^5}{5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**16/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*x/b**8 - 2*a*x**3/b**7 + 9009*sqrt(-a**5/b**17)*log(x - b**8*sqrt(-
a**5/b**17)/a**2)/512 - 9009*sqrt(-a**5/b**17)*log(x + b**8*sqrt(-a**5/b**1
7)/a**2)/512 + (18165*a**7*x + 78370*a**6*b*x**3 + 128128*a**5*b**2*x**5 +
94430*a**4*b**3*x**7 + 26635*a**3*b**4*x**9)/(1280*a**5*b**8 + 6400*a**4*b*
*9*x**2 + 12800*a**3*b**10*x**4 + 12800*a**2*b**11*x**6 + 6400*a*b**12*x**8
 + 1280*b**13*x**10) + x**5/(5*b**6)
```

3.352 $\int \frac{x^{14}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{3003a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{15/2}}-\frac{3003ax}{256b^7}-\frac{3003x^5}{1280b^5\left(a+bx^2\right)}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{}{10b\,(}$$

Rubi [A] time = 0.09, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{3003a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{15/2}}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}-\frac{3003x^5}{1280b^5\left(a+bx^2\right)}-\frac{3003ax}{256b^7}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}+\frac{1001x^3}{256b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-3003*a*x)/(256*b^7) + (1001*x^3)/(256*b^6) - x^13/(10*b*(a + b*x^2)^5) -
(13*x^11)/(80*b^2*(a + b*x^2)^4) - (143*x^9)/(480*b^3*(a + b*x^2)^3) - (429
*x^7)/(640*b^4*(a + b*x^2)^2) - (3003*x^5)/(1280*b^5*(a + b*x^2)) + (3003*a
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(15/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{14}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(13b^4\right)\int \frac{x^{12}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(143b^2\right)\int \frac{x^{10}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}+\frac{429}{160}\int \frac{x^8}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}+\frac{3003}{}$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}-\frac{}{1280}$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}-\frac{}{1280}$$

$$= -\frac{3003ax}{256b^7}+\frac{1001x^3}{256b^6}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{}{640}$$

$$= -\frac{3003ax}{256b^7}+\frac{1001x^3}{256b^6}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{}{640}$$

Mathematica [A] time = 0.06, size = 111, normalized size = 0.78

$$\frac{45045a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\frac{\sqrt{b}\,x\left(-45045a^6-210210a^5bx^2-384384a^4b^2x^4-338910a^3b^3x^6-137995a^2b^4x^8-16640ab^5x^{10}+1280b^6x^{12}\right)}{\left(a+bx^2\right)^5}}{3840b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(-45045*a^6 - 210210*a^5*b*x^2 - 384384*a^4*b^2*x^4 - 338910*a^
3*b^3*x^6 - 137995*a^2*b^4*x^8 - 16640*a*b^5*x^10 + 1280*b^6*x^12))/(a + b*
x^2)^5 + 45045*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*b^(15/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.90, size = 428, normalized size = 3.01

$$\left[\frac{2560\,b^6x^{13}-33280\,ab^5x^{11}-275990\,a^2b^4x^9-677820\,a^3b^3x^7-768768\,a^4b^2x^5-420420\,a^5bx^3-90090\,a^6x+45045\left(ab^5x^{10}+5\,a^2b^4x^8+10\,a^3b^3x^6+10\,a^4b^2x^4+5\,a^5bx^2+a^6\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{7680\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)},\ \frac{1280\,b^6x^{13}-16640\,ab^5x^{11}-137995\,a^2b^4x^9-338910\,a^3b^3x^7-384384\,a^4b^2x^5-210210\,a^5bx^3-45045\,a^6x+45045\left(ab^5x^{10}+5\,a^2b^4x^8+10\,a^3b^3x^6+10\,a^4b^2x^4+5\,a^5bx^2+a^6\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{3840\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(2560*b^6*x^13 - 33280*a*b^5*x^11 - 275990*a^2*b^4*x^9 - 677820*a^3
*b^3*x^7 - 768768*a^4*b^2*x^5 - 420420*a^5*b*x^3 - 90090*a^6*x + 45045*(a*b
^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 + a
^6)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^12*x^10
+ 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7
), 1/3840*(1280*b^6*x^13 - 16640*a*b^5*x^11 - 137995*a^2*b^4*x^9 - 338910*a
^3*b^3*x^7 - 384384*a^4*b^2*x^5 - 210210*a^5*b*x^3 - 45045*a^6*x + 45045*(a
*b^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 +
 a^6)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2
*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)]
```

giac [A] time = 0.16, size = 106, normalized size = 0.75

$$\frac{3003\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^7}-\frac{35595\,a^2b^4x^9+121310\,a^3b^3x^7+160384\,a^4b^2x^5+96290\,a^5bx^3+22005\,a^6x}{3840\left(bx^2+a\right)^5b^7}+\frac{b^{12}x^3-18\,ab^{11}x}{3\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3003/256*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) - 1/3840*(35595*a^2*b^4*
x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 96290*a^5*b*x^3 + 22005*a^6
*x)/((b*x^2 + a)^5*b^7) + 1/3*(b^12*x^3 - 18*a*b^11*x)/b^18
```

maple [A] time = 0.02, size = 137, normalized size = 0.96

$$-\frac{2373a^2x^9}{256\left(bx^2+a\right)^5b^3}-\frac{12131a^3x^7}{384\left(bx^2+a\right)^5b^4}-\frac{1253a^4x^5}{30\left(bx^2+a\right)^5b^5}-\frac{9629a^5x^3}{384\left(bx^2+a\right)^5b^6}-\frac{1467a^6x}{256\left(bx^2+a\right)^5b^7}+\frac{x^3}{3b^6}+\frac{3003a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,b^7}-\frac{6ax}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/3*x^3/b^6-6*a*x/b^7-2373/256/b^3*a^2/(b*x^2+a)^5*x^9-12131/384/b^4*a^3/(b
*x^2+a)^5*x^7-1253/30/b^5*a^4/(b*x^2+a)^5*x^5-9629/384/b^6*a^5/(b*x^2+a)^5*
x^3-1467/256/b^7*a^6/(b*x^2+a)^5*x+3003/256/b^7*a^2/(a*b)^(1/2)*arctan(1/(a
*b)^(1/2)*b*x)
```

maxima [A] time = 2.91, size = 148, normalized size = 1.04

$$-\frac{35595\,a^2b^4x^9+121310\,a^3b^3x^7+160384\,a^4b^2x^5+96290\,a^5bx^3+22005\,a^6x}{3840\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)}+\frac{3003\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^7}+\frac{bx^3-18\,ax}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/3840*(35595*a^2*b^4*x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 9629
0*a^5*b*x^3 + 22005*a^6*x)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10
*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7) + 3003/256*a^2*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^7) + 1/3*(b*x^3 - 18*a*x)/b^7
```

mupad [B] time = 4.52, size = 143, normalized size = 1.01

$$\frac{x^3}{3\,b^6}-\frac{\frac{1467\,a^6\,x}{256}+\frac{9629\,a^5\,b\,x^3}{384}+\frac{1253\,a^4\,b^2\,x^5}{30}+\frac{12131\,a^3\,b^3\,x^7}{384}+\frac{2373\,a^2\,b^4\,x^9}{256}}{a^5\,b^7+5\,a^4\,b^8\,x^2+10\,a^3\,b^9\,x^4+10\,a^2\,b^{10}\,x^6+5\,a\,b^{11}\,x^8+b^{12}\,x^{10}}+\frac{3003\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{15/2}}-\frac{6\,a\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] x^3/(3*b^6) - ((1467*a^6*x)/256 + (9629*a^5*b*x^3)/384 + (1253*a^4*b^2*x^5)
/30 + (12131*a^3*b^3*x^7)/384 + (2373*a^2*b^4*x^9)/256)/(a^5*b^7 + b^12*x^1
```

```
0 + 5*a*b^11*x^8 + 5*a^4*b^8*x^2 + 10*a^3*b^9*x^4 + 10*a^2*b^10*x^6) + (300
3*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*b^(15/2)) - (6*a*x)/b^7
```

sympy [A] time = 1.03, size = 204, normalized size = 1.44

$$-\frac{6ax}{b^7}-\frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x-\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512}+\frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x+\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512}+\frac{-22005a^6x-96290a^5bx^3-160384a^4b^2x^5-121310a^3b^3x^7-35595a^2b^4x^9}{3840a^5b^7+19200a^4b^8x^2+38400a^3b^9x^4+38400a^2b^{10}x^6+19200ab^{11}x^8+3840b^{12}x^{10}}+\frac{x^3}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -6*a*x/b**7 - 3003*sqrt(-a**3/b**15)*log(x - b**7*sqrt(-a**3/b**15)/a)/512
+ 3003*sqrt(-a**3/b**15)*log(x + b**7*sqrt(-a**3/b**15)/a)/512 + (-22005*a*
*6*x - 96290*a**5*b*x**3 - 160384*a**4*b**2*x**5 - 121310*a**3*b**3*x**7 -
35595*a**2*b**4*x**9)/(3840*a**5*b**7 + 19200*a**4*b**8*x**2 + 38400*a**3*b
**9*x**4 + 38400*a**2*b**10*x**6 + 19200*a*b**11*x**8 + 3840*b**12*x**10) +
 x**3/(3*b**6)
```

3.353 $\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=131

$$-\frac{693\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{13/2}}-\frac{231x^3}{256b^5\left(a+bx^2\right)}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}+$$

Rubi [A] time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 321, 205}

$$-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{231x^3}{256b^5\left(a+bx^2\right)}-\frac{693\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{13/2}}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}+\frac{693x}{256b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (693*x)/(256*b^6) - x^11/(10*b*(a + b*x^2)^5) - (11*x^9)/(80*b^2*(a + b*x^2
)^4) - (33*x^7)/(160*b^3*(a + b*x^2)^3) - (231*x^5)/(640*b^4*(a + b*x^2)^2)
 - (231*x^3)/(256*b^5*(a + b*x^2)) - (693*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(256*b^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{12}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(11b^4\right)\int \frac{x^{10}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(99b^2\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}+\frac{231}{160}\int \frac{x^6}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}+\frac{231}{\cdots}$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{\cdots}{256\cdots}$$

$$= \frac{693x}{256b^6}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^{\cdots}}$$

$$= \frac{693x}{256b^6}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^{\cdots}}$$

Mathematica [A] time = 0.05, size = 100, normalized size = 0.76

$$\frac{\frac{\sqrt{b}\,x\left(3465a^5+16170a^4bx^2+29568a^3b^2x^4+26070a^2b^3x^6+10615ab^4x^8+1280b^5x^{10}\right)}{\left(a+bx^2\right)^5}-3465\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1280b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(3465*a^5 + 16170*a^4*b*x^2 + 29568*a^3*b^2*x^4 + 26070*a^2*b^3
*x^6 + 10615*a*b^4*x^8 + 1280*b^5*x^10))/(a + b*x^2)^5 - 3465*Sqrt[a]*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(1280*b^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.79, size = 400, normalized size = 3.05

$$\left[\frac{2560\,b^5x^{11}+21230\,ab^4x^9+52140\,a^2b^3x^7+59136\,a^3b^2x^5+32340\,a^4bx^3+6930\,a^5x+3465\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{2560\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)},\ \frac{1280\,b^5x^{11}+10615\,ab^4x^9+26070\,a^2b^3x^7+29568\,a^3b^2x^5+16170\,a^4bx^3+3465\,a^5x-3465\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{1280\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(2560*b^5*x^11 + 21230*a*b^4*x^9 + 52140*a^2*b^3*x^7 + 59136*a^3*b^
2*x^5 + 32340*a^4*b*x^3 + 6930*a^5*x + 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^
2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b
*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6
 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6), 1/1280*(1280*b^5*x^11 + 10615
*a*b^4*x^9 + 26070*a^2*b^3*x^7 + 29568*a^3*b^2*x^5 + 16170*a^4*b*x^3 + 3465
*a^5*x - 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5
*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^11*x^10 + 5*a*b^10*
x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)]
```

giac [A] time = 0.17, size = 87, normalized size = 0.66

$$-\frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^6}+\frac{x}{b^6}+\frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\left(bx^2+a\right)^5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + x/b^6 + 1/1280*(4215*a*b
^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a^4*b*x^3 + 2185*a^5*
x)/((b*x^2 + a)^5*b^6)
```

maple [A] time = 0.02, size = 123, normalized size = 0.94

$$\frac{843a\,x^9}{256\left(b\,x^2+a\right)^5 b^2}+\frac{1327a^2x^7}{128\left(b\,x^2+a\right)^5 b^3}+\frac{131a^3x^5}{10\left(b\,x^2+a\right)^5 b^4}+\frac{977a^4x^3}{128\left(b\,x^2+a\right)^5 b^5}+\frac{437a^5x}{256\left(b\,x^2+a\right)^5 b^6}-\frac{693a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,b^6}+\frac{x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] x/b^6+843/256/b^2*a/(b*x^2+a)^5*x^9+1327/128/b^3*a^2/(b*x^2+a)^5*x^7+131/10
/b^4*a^3/(b*x^2+a)^5*x^5+977/128/b^5*a^4/(b*x^2+a)^5*x^3+437/256/b^6*a^5/(b
*x^2+a)^5*x-693/256/b^6*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.99, size = 134, normalized size = 1.02

$$\frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)}-\frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^6}+\frac{x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(4215*a*b^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a^4*b
*x^3 + 2185*a^5*x)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*
x^4 + 5*a^4*b^7*x^2 + a^5*b^6) - 693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + x/b^6
```

mupad [B] time = 0.16, size = 130, normalized size = 0.99

$$\frac{\frac{437\,a^5\,x}{256}+\frac{977\,a^4\,b\,x^3}{128}+\frac{131\,a^3\,b^2\,x^5}{10}+\frac{1327\,a^2\,b^3\,x^7}{128}+\frac{843\,a\,b^4\,x^9}{256}}{a^5\,b^6+5\,a^4\,b^7\,x^2+10\,a^3\,b^8\,x^4+10\,a^2\,b^9\,x^6+5\,a\,b^{10}\,x^8+b^{11}\,x^{10}}+\frac{x}{b^6}-\frac{693\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((437*a^5*x)/256 + (977*a^4*b*x^3)/128 + (843*a*b^4*x^9)/256 + (131*a^3*b^2
*x^5)/10 + (1327*a^2*b^3*x^7)/128)/(a^5*b^6 + b^11*x^10 + 5*a*b^10*x^8 + 5*
a^4*b^7*x^2 + 10*a^3*b^8*x^4 + 10*a^2*b^9*x^6) + x/b^6 - (693*a^(1/2)*atan(
(b^(1/2)*x)/a^(1/2)))/(256*b^(13/2))
```

sympy [A] time = 0.95, size = 178, normalized size = 1.36

$$\frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(-b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512}-\frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512}+\frac{2185a^5x+9770a^4bx^3+16768a^3b^2x^5+13270a^2b^3x^7+4215ab^4x^9}{1280a^5b^6+6400a^4b^7x^2+12800a^3b^8x^4+12800a^2b^9x^6+6400ab^{10}x^8+1280b^{11}x^{10}}+\frac{x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-a/b**13)*log(-b**6*sqrt(-a/b**13) + x)/512 - 693*sqrt(-a/b**13)*l
og(b**6*sqrt(-a/b**13) + x)/512 + (2185*a**5*x + 9770*a**4*b*x**3 + 16768*a
**3*b**2*x**5 + 13270*a**2*b**3*x**7 + 4215*a*b**4*x**9)/(1280*a**5*b**6 +
6400*a**4*b**7*x**2 + 12800*a**3*b**8*x**4 + 12800*a**2*b**9*x**6 + 6400*a*
b**10*x**8 + 1280*b**11*x**10) + x/b**6
```

3.354 $\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=121

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{a}\,b^{11/2}}-\frac{63x}{256b^5\left(a+bx^2\right)}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{x^9}{10b\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 288, 205}

$$-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{63x}{256b^5\left(a+bx^2\right)}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{a}\,b^{11/2}}-\frac{x^9}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^9/(10*b*(a + b*x^2)^5) - (9*x^7)/(80*b^2*(a + b*x^2)^4) - (21*x^5)/(160*
b^3*(a + b*x^2)^3) - (21*x^3)/(128*b^4*(a + b*x^2)^2) - (63*x)/(256*b^5*(a
+ b*x^2)) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*Sqrt[a]*b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{10}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(9b^4\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(63b^2\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}+\frac{21}{32}\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}+\frac{63\int}{}$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{}{256b^5}$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{}{256b^5}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.73

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{a}\,b^{11/2}}-\frac{x\left(315a^4+1470a^3bx^2+2688a^2b^2x^4+2370ab^3x^6+965b^4x^8\right)}{1280b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/1280*(x*(315*a^4 + 1470*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 2370*a*b^3*x^6 +
965*b^4*x^8))/(b^5*(a + b*x^2)^5) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*S
qrt[a]*b^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.86, size = 386, normalized size = 3.19

$$\left[-\frac{1930\,ab^5x^9 + 4740\,a^2b^4x^7 + 5376\,a^3b^3x^5 + 2940\,a^4b^2x^3 + 630\,a^5bx + 315\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4bx^2 + a^5\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2560\left(ab^{11}x^{10} + 5\,a^2b^{10}x^8 + 10\,a^3b^9x^6 + 10\,a^4b^8x^4 + 5\,a^5b^7x^2 + a^6b^6\right)}, -\frac{965\,ab^5x^9 + 2370\,a^2b^4x^7 + 2688\,a^3b^3x^5 + 1470\,a^4b^2x^3 + 315\,a^5bx - 315\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4bx^2 + a^5\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{1280\left(ab^{11}x^{10} + 5\,a^2b^{10}x^8 + 10\,a^3b^9x^6 + 10\,a^4b^8x^4 + 5\,a^5b^7x^2 + a^6b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(1930*a*b^5*x^9 + 4740*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 2940*a^4*b
^2*x^3 + 630*a^5*b*x + 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^
3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 + 10*a^4*b^8*x
^4 + 5*a^5*b^7*x^2 + a^6*b^6), -1/1280*(965*a*b^5*x^9 + 2370*a^2*b^4*x^7 +
2688*a^3*b^3*x^5 + 1470*a^4*b^2*x^3 + 315*a^5*b*x - 315*(b^5*x^10 + 5*a*b^4
*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 + 10*a^4*b
^8*x^4 + 5*a^5*b^7*x^2 + a^6*b^6)]
```

giac [A] time = 0.19, size = 78, normalized size = 0.64

$$\frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^5} - \frac{965\,b^4x^9 + 2370\,ab^3x^7 + 2688\,a^2b^2x^5 + 1470\,a^3bx^3 + 315\,a^4x}{1280\left(bx^2 + a\right)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/1280*(965*b^4*x^9 + 2370*a
*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3 + 315*a^4*x)/((b*x^2 + a)^5*b^
5)
```

maple [A] time = 0.01, size = 80, normalized size = 0.66

$$\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\;b^5} + \frac{-\frac{193x^9}{256b} - \frac{237a\,x^7}{128b^2} - \frac{21a^2x^5}{10b^3} - \frac{147a^3x^3}{128b^4} - \frac{63a^4x}{256b^5}}{\left(b\,x^2 + a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

[Out] (-193/256/b*x^9-237/128*a/b^2*x^7-21/10*a^2/b^3*x^5-147/128*a^3/b^4*x^3-63/256*a^4/b^5*x)/(b*x^2+a)^5+63/256/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)

maxima [A] time = 3.07, size = 125, normalized size = 1.03

$$-\frac{965\,b^4x^9+2370\,ab^3x^7+2688\,a^2b^2x^5+1470\,a^3bx^3+315\,a^4x}{1280\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)}+\frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(965*b^4*x^9 + 2370*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3 + 315*a^4*x)/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5) + 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5)
```

mupad [B] time = 4.52, size = 122, normalized size = 1.01

$$\frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,\sqrt{a}\,b^{11/2}}-\frac{\frac{193\,x^9}{256\,b}+\frac{237\,a\,x^7}{128\,b^2}+\frac{63\,a^4\,x}{256\,b^5}+\frac{21\,a^2\,x^5}{10\,b^3}+\frac{147\,a^3\,x^3}{128\,b^4}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (63*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(1/2)*b^(11/2)) - ((193*x^9)/(256*b) + (237*a*x^7)/(128*b^2) + (63*a^4*x)/(256*b^5) + (21*a^2*x^5)/(10*b^3) + (147*a^3*x^3)/(128*b^4))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

sympy [A] time = 0.79, size = 182, normalized size = 1.50

$$-\frac{63\sqrt{-\frac{1}{ab^{11}}}\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{ab^{11}}}\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}+\frac{-315a^4x-1470a^3bx^3-2688a^2b^2x^5-2370ab^3x^7-965b^4x^9}{1280a^5b^5+6400a^4b^6x^2+12800a^3b^7x^4+12800a^2b^8x^6+6400ab^9x^8+1280b^{10}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a*b**11))*log(-a*b**5*sqrt(-1/(a*b**11)) + x)/512 + 63*sqrt(-1/(a*b**11))*log(a*b**5*sqrt(-1/(a*b**11)) + x)/512 + (-315*a**4*x - 1470*a**3*b*x**3 - 2688*a**2*b**2*x**5 - 2370*a*b**3*x**7 - 965*b**4*x**9)/(1280*a**5*b**5 + 6400*a**4*b**6*x**2 + 12800*a**3*b**7*x**4 + 12800*a**2*b**8*x**6 + 6400*a*b**9*x**8 + 1280*b**10*x**10)
```

3.355 $\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=122

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}+\frac{7x}{256ab^4\left(a+bx^2\right)}-\frac{7x}{128b^4\left(a+bx^2\right)^2}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}+\frac{7x}{256ab^4\left(a+bx^2\right)}-\frac{7x}{128b^4\left(a+bx^2\right)^2}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^7/(10*b*(a + b*x^2)^5) - (7*x^5)/(80*b^2*(a + b*x^2)^4) - (7*x^3)/(96*b^
3*(a + b*x^2)^3) - (7*x)/(128*b^4*(a + b*x^2)^2) + (7*x)/(256*a*b^4*(a + b*
x^2)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(3/2)*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^8}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(7b^4\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^5}\,dx$$
$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}+\frac{1}{16}\left(7b^2\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}+\frac{7}{32}\int \frac{x^2}{\left(ab+b^2x^2\right)^3}\,dx$$
$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x}{128b^4\left(a+bx^2\right)^2}+\frac{7\int \frac{}{(ab}}{1}$$
$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x}{128b^4\left(a+bx^2\right)^2}+\frac{}{256ab}$$
$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x}{128b^4\left(a+bx^2\right)^2}+\frac{}{256ab}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.75

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}-\frac{x\left(105a^4+490a^3bx^2+896a^2b^2x^4+790ab^3x^6-105b^4x^8\right)}{3840ab^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/3840*(x*(105*a^4 + 490*a^3*b*x^2 + 896*a^2*b^2*x^4 + 790*a*b^3*x^6 - 105
*b^4*x^8))/(a*b^4*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(
3/2)*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.85, size = 390, normalized size = 3.20

$$\left[\frac{210\,ab^5x^9-1580\,a^2b^4x^7-1792\,a^3b^3x^5-980\,a^4b^2x^3-210\,a^5bx-105\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{7680\left(a^2b^{10}x^{10}+5\,a^3b^9x^8+10\,a^4b^8x^6+10\,a^5b^7x^4+5\,a^6b^6x^2+a^7b^5\right)},\ \frac{105\,ab^5x^9-790\,a^2b^4x^7-896\,a^3b^3x^5-490\,a^4b^2x^3-105\,a^5bx+105\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{3840\left(a^2b^{10}x^{10}+5\,a^3b^9x^8+10\,a^4b^8x^6+10\,a^5b^7x^4+5\,a^6b^6x^2+a^7b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(210*a*b^5*x^9 - 1580*a^2*b^4*x^7 - 1792*a^3*b^3*x^5 - 980*a^4*b^2*
x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b
^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^5*b^7*x^4
 + 5*a^6*b^6*x^2 + a^7*b^5), 1/3840*(105*a*b^5*x^9 - 790*a^2*b^4*x^7 - 896*
a^3*b^3*x^5 - 490*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4*x^8 +
 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^5*b^7*x^
4 + 5*a^6*b^6*x^2 + a^7*b^5)]
```

giac [A] time = 0.16, size = 84, normalized size = 0.69

$$\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,ab^4}+\frac{105\,b^4x^9-790\,ab^3x^7-896\,a^2b^2x^5-490\,a^3bx^3-105\,a^4x}{3840\left(bx^2+a\right)^5ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/3840*(105*b^4*x^9 - 790*a
*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a*b^
4)
```

maple [A] time = 0.01, size = 80, normalized size = 0.66

$$\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a\,b^{4}}+\frac{\frac{7x^{9}}{256a}-\frac{79x^{7}}{384b}-\frac{7a\,x^{5}}{30b^{2}}-\frac{49a^{2}x^{3}}{384b^{3}}-\frac{7a^{3}x}{256b^{4}}}{\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (7/256/a*x^9-79/384/b*x^7-7/30*a/b^2*x^5-49/384*a^2/b^3*x^3-7/256*a^3/b^4*x
)/(b*x^2+a)^5+7/256/a/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.03, size = 131, normalized size = 1.07

$$\frac{105\,b^{4}x^{9}-790\,ab^{3}x^{7}-896\,a^{2}b^{2}x^{5}-490\,a^{3}bx^{3}-105\,a^{4}x}{3840\left(ab^{9}x^{10}+5\,a^{2}b^{8}x^{8}+10\,a^{3}b^{7}x^{6}+10\,a^{4}b^{6}x^{4}+5\,a^{5}b^{5}x^{2}+a^{6}b^{4}\right)}+\frac{7\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,ab^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(105*b^4*x^9 - 790*a*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 105
*a^4*x)/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a
^5*b^5*x^2 + a^6*b^4) + 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

mupad [B] time = 4.42, size = 119, normalized size = 0.98

$$\frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{3/2}\,b^{9/2}}-\frac{\frac{79\,x^{7}}{384\,b}-\frac{7\,x^{9}}{256\,a}+\frac{7\,a\,x^{5}}{30\,b^{2}}+\frac{7\,a^{3}\,x}{256\,b^{4}}+\frac{49\,a^{2}\,x^{3}}{384\,b^{3}}}{a^{5}+5\,a^{4}\,b\,x^{2}+10\,a^{3}\,b^{2}\,x^{4}+10\,a^{2}\,b^{3}\,x^{6}+5\,a\,b^{4}\,x^{8}+b^{5}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (7*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(3/2)*b^(9/2)) - ((79*x^7)/(384*b) - (
7*x^9)/(256*a) + (7*a*x^5)/(30*b^2) + (7*a^3*x)/(256*b^4) + (49*a^2*x^3)/(3
84*b^3))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*
a^2*b^3*x^6)
```

sympy [A] time = 0.76, size = 194, normalized size = 1.59

$$-\frac{7\sqrt{-\frac{1}{a^{3}b^{9}}}\log\left(-a^{2}b^{4}\sqrt{-\frac{1}{a^{3}b^{9}}}+x\right)}{512}+\frac{7\sqrt{-\frac{1}{a^{3}b^{9}}}\log\left(a^{2}b^{4}\sqrt{-\frac{1}{a^{3}b^{9}}}+x\right)}{512}+\frac{-105a^{4}x-490a^{3}bx^{3}-896a^{2}b^{2}x^{5}-790ab^{3}x^{7}+105b^{4}x^{9}}{3840a^{6}b^{4}+19200a^{5}b^{5}x^{2}+38400a^{4}b^{6}x^{4}+38400a^{3}b^{7}x^{6}+19200a^{2}b^{8}x^{8}+3840ab^{9}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**3*b**9))*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + 7*sq
rt(-1/(a**3*b**9))*log(a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + (-105*a**4
*x - 490*a**3*b*x**3 - 896*a**2*b**2*x**5 - 790*a*b**3*x**7 + 105*b**4*x**9
)/(3840*a**6*b**4 + 19200*a**5*b**5*x**2 + 38400*a**4*b**6*x**4 + 38400*a**
3*b**7*x**6 + 19200*a**2*b**8*x**8 + 3840*a*b**9*x**10)
```

3.356 $\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=123

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}+\frac{3x}{256a^2b^3(a+bx^2)}+\frac{x}{128ab^3(a+bx^2)^2}-\frac{x}{32b^3(a+bx^2)^3}-\frac{x^3}{16b^2(a+bx^2)^4}-\frac{x^5}{10b(a+bx^2)^5}$$

Rubi [A] time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{3x}{256a^2b^3(a+bx^2)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}-\frac{x^3}{16b^2(a+bx^2)^4}+\frac{x}{128ab^3(a+bx^2)^2}-\frac{x}{32b^3(a+bx^2)^3}-\frac{x^5}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^5/(10*b*(a + b*x^2)^5) - x^3/(16*b^2*(a + b*x^2)^4) - x/(32*b^3*(a + b*x
^2)^3) + x/(128*a*b^3*(a + b*x^2)^2) + (3*x)/(256*a^2*b^3*(a + b*x^2)) + (3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^6}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}+\frac{1}{2}b^4\int \frac{x^4}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}+\frac{1}{16}\left(3b^2\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{1}{32}\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{x}{128ab^3\left(a+bx^2\right)^2}+\frac{3\int}{}$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{x}{128ab^3\left(a+bx^2\right)^2}+\frac{}{256}$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{x}{128ab^3\left(a+bx^2\right)^2}+\frac{}{256}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.74

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}+\frac{-15a^4x-70a^3bx^3-128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^2b^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-15*a^4*x - 70*a^3*b*x^3 - 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(1
280*a^2*b^3*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b
^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.86, size = 390, normalized size = 3.17

$$\left[\frac{30\,ab^5x^9+140\,a^2b^4x^7-256\,a^3b^3x^5-140\,a^4b^2x^3-30\,a^5bx-15\,\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2560\,\left(a^3b^9x^{10}+5\,a^4b^8x^8+10\,a^5b^7x^6+10\,a^6b^6x^4+5\,a^7b^5x^2+a^8b^4\right)},\ \frac{15\,ab^5x^9+70\,a^2b^4x^7-128\,a^3b^3x^5-70\,a^4b^2x^3-15\,a^5bx+15\,\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{1280\,\left(a^3b^9x^{10}+5\,a^4b^8x^8+10\,a^5b^7x^6+10\,a^6b^6x^4+5\,a^7b^5x^2+a^8b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 - 256*a^3*b^3*x^5 - 140*a^4*b^2*x^3
 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^
4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)))/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 + 5*a
^7*b^5*x^2 + a^8*b^4), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 - 128*a^3*b^3*
x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3
*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 + 5*a^7*b^
5*x^2 + a^8*b^4)]
```

giac [A] time = 0.16, size = 84, normalized size = 0.68

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^2b^3}+\frac{15\,b^4x^9+70\,ab^3x^7-128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\,\left(bx^2+a\right)^5a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/1280*(15*b^4*x^9 + 70*a
*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^2*b^
3)
```

maple [A] time = 0.01, size = 78, normalized size = 0.63

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^2b^3}+\frac{\frac{3b\,x^9}{256a^2}+\frac{7x^7}{128a}-\frac{x^5}{10b}-\frac{7a\,x^3}{128b^2}-\frac{3a^2x}{256b^3}}{\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (3/256/a^2*b*x^9+7/128/a*x^7-1/10/b*x^5-7/128*a/b^2*x^3-3/256*a^2/b^3*x)/(b
*x^2+a)^5+3/256/a^2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 133, normalized size = 1.08

$$\frac{15\,b^4x^9+70\,ab^3x^7-128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\left(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3\right)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(15*b^4*x^9 + 70*a*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4
*x)/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6
*b^4*x^2 + a^7*b^3) + 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3)
```

mupad [B] time = 4.50, size = 117, normalized size = 0.95

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{5/2}\,b^{7/2}}-\frac{\frac{x^5}{10\,b}-\frac{7\,x^7}{128\,a}+\frac{7\,a\,x^3}{128\,b^2}+\frac{3\,a^2\,x}{256\,b^3}-\frac{3\,b\,x^9}{256\,a^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(5/2)*b^(7/2)) - (x^5/(10*b) - (7*x^7)
/(128*a) + (7*a*x^3)/(128*b^2) + (3*a^2*x)/(256*b^3) - (3*b*x^9)/(256*a^2))
/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*
x^6)
```

sympy [A] time = 0.70, size = 196, normalized size = 1.59

$$-\frac{3\sqrt{-\frac{1}{a^5b^7}}\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{512}+\frac{3\sqrt{-\frac{1}{a^5b^7}}\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{512}+\frac{-15a^4x-70a^3bx^3-128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^7b^3+6400a^6b^4x^2+12800a^5b^5x^4+12800a^4b^6x^6+6400a^3b^7x^8+1280a^2b^8x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*b**7))*log(-a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + 3*sq
rt(-1/(a**5*b**7))*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + (-15*a**4*
x - 70*a**3*b*x**3 - 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)/(1
280*a**7*b**3 + 6400*a**6*b**4*x**2 + 12800*a**5*b**5*x**4 + 12800*a**4*b**
6*x**6 + 6400*a**3*b**7*x**8 + 1280*a**2*b**8*x**10)
```

3.357 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=124

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{3x}{256a^3b^2\left(a+bx^2\right)}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{x}{160ab^2\left(a+bx^2\right)^3}-\frac{3x}{80b^2\left(a+bx^2\right)^4}-\frac{x^3}{10b\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{3x}{256a^3b^2\left(a+bx^2\right)}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{x}{160ab^2\left(a+bx^2\right)^3}-\frac{3x}{80b^2\left(a+bx^2\right)^4}-\frac{x^3}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^3/(10*b*(a + b*x^2)^5) - (3*x)/(80*b^2*(a + b*x^2)^4) + x/(160*a*b^2*(a
+ b*x^2)^3) + x/(128*a^2*b^2*(a + b*x^2)^2) + (3*x)/(256*a^3*b^2*(a + b*x^2
)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^4}{\left(ab+b^2x^2\right)^6}\,dx\\
&= -\frac{x^3}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(3b^4\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^5}\,dx\\
&= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(3b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx\\
&= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{b\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{32a}\\
&= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\\
&= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\\
&= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+
\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.73

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{-15a^4x-70a^3bx^3+128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^3b^2\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

[Out] (-15*a^4*x - 70*a^3*b*x^3 + 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(1280*a^3*b^2*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)*b^(5/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]

fricas [A] time = 1.48, size = 390, normalized size = 3.15

$$\left[\frac{30\,ab^5x^9+140\,a^2b^4x^7+256\,a^3b^3x^5-140\,a^4b^2x^3-30\,a^5bx-15\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2560\left(a^4b^8x^{10}+5\,a^5b^7x^8+10\,a^6b^6x^6+10\,a^7b^5x^4+5\,a^8b^4x^2+a^9b^3\right)},\ \frac{15\,ab^5x^9+70\,a^2b^4x^7+128\,a^3b^3x^5-70\,a^4b^2x^3-15\,a^5bx+15\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{1280\left(a^4b^8x^{10}+5\,a^5b^7x^8+10\,a^6b^6x^6+10\,a^7b^5x^4+5\,a^8b^4x^2+a^9b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")

[Out] [1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 + 256*a^3*b^3*x^5 - 140*a^4*b^2*x^3 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 + 5*a^8*b^4*x^2 + a^9*b^3), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 + 128*a^3*b^3*x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 + 5*a^8*b^4*x^2 + a^9*b^3)]

giac [A] time = 0.16, size = 84, normalized size = 0.68

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^3b^2}+\frac{15\,b^4x^9+70\,ab^3x^7+128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\left(bx^2+a\right)^5a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")

[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2) + 1/1280*(15*b^4*x^9 + 70*a*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^3*b^2)

maple [A] time = 0.01, size = 78, normalized size = 0.63

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^3b^2}+\frac{\frac{3b^2x^9}{256a^3}+\frac{7b\,x^7}{128a^2}+\frac{x^5}{10a}-\frac{7x^3}{128b}-\frac{3ax}{256b^2}}{\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (3/256/a^3*b^2*x^9+7/128/a^2*b*x^7+1/10/a*x^5-7/128/b*x^3-3/256*a/b^2*x)/(b
*x^2+a)^5+3/256/a^3/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.04, size = 133, normalized size = 1.07

$$\frac{15\,b^4x^9+70\,ab^3x^7+128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\left(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2\right)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(15*b^4*x^9 + 70*a*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4
*x)/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7
*b^3*x^2 + a^8*b^2) + 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

mupad [B] time = 4.47, size = 116, normalized size = 0.94

$$\frac{\frac{x^5}{10\,a}-\frac{7\,x^3}{128\,b}+\frac{7\,b\,x^7}{128\,a^2}+\frac{3\,b^2\,x^9}{256\,a^3}-\frac{3\,a\,x}{256\,b^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{7/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (x^5/(10*a) - (7*x^3)/(128*b) + (7*b*x^7)/(128*a^2) + (3*b^2*x^9)/(256*a^3)
 - (3*a*x)/(256*b^2))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*
b^2*x^4 + 10*a^2*b^3*x^6) + (3*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(7/2)*b^(5
/2))
```

sympy [A] time = 0.64, size = 196, normalized size = 1.58

$$-\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512}+\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512}+\frac{-15a^4x-70a^3bx^3+128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^8b^2+6400a^7b^3x^2+12800a^6b^4x^4+12800a^5b^5x^6+6400a^4b^6x^8+1280a^3b^7x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**7*b**5))*log(-a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + 3*sq
rt(-1/(a**7*b**5))*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + (-15*a**4*
x - 70*a**3*b*x**3 + 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)/(1
280*a**8*b**2 + 6400*a**7*b**3*x**2 + 12800*a**6*b**4*x**4 + 12800*a**5*b**
5*x**6 + 6400*a**4*b**6*x**8 + 1280*a**3*b**7*x**10)
```

3.358 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=125

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{7x}{256a^4b\left(a+bx^2\right)}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{x}{80ab\left(a+bx^2\right)^4}-\frac{x}{10b\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{7x}{256a^4b\left(a+bx^2\right)}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{x}{80ab\left(a+bx^2\right)^4}-\frac{x}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x/(10*b*(a + b*x^2)^5) + x/(80*a*b*(a + b*x^2)^4) + (7*x)/(480*a^2*b*(a +
b*x^2)^3) + (7*x)/(384*a^3*b*(a + b*x^2)^2) + (7*x)/(256*a^4*b*(a + b*x^2))
 + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/2)*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^2}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{1}{10}b^4\int \frac{1}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{\left(7b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{80a}$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{\left(7b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{96a^2}$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\ldots$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\ldots$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\ldots$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.73

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{-105a^4x+790a^3bx^3+896a^2b^2x^5+490ab^3x^7+105b^4x^9}{3840a^4b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-105*a^4*x + 790*a^3*b*x^3 + 896*a^2*b^2*x^5 + 490*a*b^3*x^7 + 105*b^4*x^9
)/(3840*a^4*b*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/2)
*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [A] time = 0.87, size = 390, normalized size = 3.12

$$\left[\frac{210\,ab^5x^9+980\,a^2b^4x^7+1792\,a^3b^3x^5+1580\,a^4b^2x^3-210\,a^5bx-105\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{7680\left(a^5b^7x^{10}+5\,a^6b^6x^8+10\,a^7b^5x^6+10\,a^8b^4x^4+5\,a^9b^3x^2+a^{10}b^2\right)},\ \frac{105\,ab^5x^9+490\,a^2b^4x^7+896\,a^3b^3x^5+790\,a^4b^2x^3-105\,a^5bx+105\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{3840\left(a^5b^7x^{10}+5\,a^6b^6x^8+10\,a^7b^5x^6+10\,a^8b^4x^4+5\,a^9b^3x^2+a^{10}b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(210*a*b^5*x^9 + 980*a^2*b^4*x^7 + 1792*a^3*b^3*x^5 + 1580*a^4*b^2*
x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b
^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8*b^4*x^4
+ 5*a^9*b^3*x^2 + a^10*b^2), 1/3840*(105*a*b^5*x^9 + 490*a^2*b^4*x^7 + 896*
a^3*b^3*x^5 + 790*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4*x^8 +
 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8*b^4*x^4
 + 5*a^9*b^3*x^2 + a^10*b^2)]
```

giac [A] time = 0.17, size = 84, normalized size = 0.67

$$\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^4b}+\frac{105\,b^4x^9+490\,ab^3x^7+896\,a^2b^2x^5+790\,a^3bx^3-105\,a^4x}{3840\left(bx^2+a\right)^5a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b) + 1/3840*(105*b^4*x^9 + 490*a
*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a^4*
b)
```

maple [A] time = 0.01, size = 80, normalized size = 0.64

$$\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\ a^4b}+\frac{\frac{7b^3x^9}{256a^4}+\frac{49b^2x^7}{384a^3}+\frac{7b\,x^5}{30a^2}+\frac{79x^3}{384a}-\frac{7x}{256b}}{\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (7/256/a^4*b^3*x^9+49/384/a^3*b^2*x^7+7/30/a^2*b*x^5+79/384/a*x^3-7/256/b*x
)/(b*x^2+a)^5+7/256/a^4/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 131, normalized size = 1.05

$$\frac{105\,b^4x^9+490\,ab^3x^7+896\,a^2b^2x^5+790\,a^3bx^3-105\,a^4x}{3840\left(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b\right)}+\frac{7\ \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(105*b^4*x^9 + 490*a*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 105
*a^4*x)/(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5
*a^8*b^2*x^2 + a^9*b) + 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

mupad [B] time = 4.48, size = 118, normalized size = 0.94

$$\frac{\frac{79\,x^3}{384\,a}-\frac{7\,x}{256\,b}+\frac{7\,b\,x^5}{30\,a^2}+\frac{49\,b^2\,x^7}{384\,a^3}+\frac{7\,b^3\,x^9}{256\,a^4}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{9/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((79*x^3)/(384*a) - (7*x)/(256*b) + (7*b*x^5)/(30*a^2) + (49*b^2*x^7)/(384*
a^3) + (7*b^3*x^9)/(256*a^4))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 +
 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + (7*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(9
/2)*b^(3/2))
```

sympy [A] time = 0.62, size = 190, normalized size = 1.52

$$-\frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{512}+\frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{512}+\frac{-105a^4x+790a^3bx^3+896a^2b^2x^5+490ab^3x^7+105b^4x^9}{3840a^9b+19200a^8b^2x^2+38400a^7b^3x^4+38400a^6b^4x^6+19200a^5b^5x^8+3840a^4b^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**9*b**3))*log(-a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + 7*sqrt(
-1/(a**9*b**3))*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + (-105*a**4*x + 7
90*a**3*b*x**3 + 896*a**2*b**2*x**5 + 490*a*b**3*x**7 + 105*b**4*x**9)/(384
0*a**9*b + 19200*a**8*b**2*x**2 + 38400*a**7*b**3*x**4 + 38400*a**6*b**4*x*
*6 + 19200*a**5*b**5*x**8 + 3840*a**4*b**6*x**10)
```

3.359 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=113

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}+\frac{63x}{256a^5\left(a+bx^2\right)}+\frac{21x}{128a^4\left(a+bx^2\right)^2}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{x}{10a\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 199, 205}

$$\frac{63x}{256a^5\left(a+bx^2\right)}+\frac{21x}{128a^4\left(a+bx^2\right)^2}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}+\frac{x}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] x/(10*a*(a + b*x^2)^5) + (9*x)/(80*a^2*(a + b*x^2)^4) + (21*x)/(160*a^3*(a
+ b*x^2)^3) + (21*x)/(128*a^4*(a + b*x^2)^2) + (63*x)/(256*a^5*(a + b*x^2))
 + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(11/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{\left(9b^5\right)\int \frac{1}{\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{\left(63b^4\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{\left(21b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{32a^3}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{21x}{128a^4\left(a+bx^2\right)^2} + \frac{\left(63b^2\right.}{}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{21x}{128a^4\left(a+bx^2\right)^2} + \frac{}{256a^5}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{21x}{128a^4\left(a+bx^2\right)^2} + \frac{}{256a^5}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 0.79

$$\frac{\frac{\sqrt{a}\,x\left(965a^4+2370a^3bx^2+2688a^2b^2x^4+1470ab^3x^6+315b^4x^8\right)}{\left(a+bx^2\right)^5}+\frac{315\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}}{1280a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] ((Sqrt[a]*x*(965*a^4 + 2370*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 1470*a*b^3*x^6 +
 315*b^4*x^8))/(a + b*x^2)^5 + (315*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(
1280*a^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3), x]
```

fricas [A] time = 0.73, size = 386, normalized size = 3.42

$$\left[\frac{630\,ab^5x^9+2940\,a^2b^4x^7+5376\,a^3b^3x^5+4740\,a^4b^2x^3+1930\,a^5bx-315\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2560\left(a^6b^6x^{10}+5\,a^7b^5x^8+10\,a^8b^4x^6+10\,a^9b^3x^4+5\,a^{10}b^2x^2+a^{11}b\right)},\ \frac{315\,ab^5x^9+1470\,a^2b^4x^7+2688\,a^3b^3x^5+2370\,a^4b^2x^3+965\,a^5bx+315\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{1280\left(a^6b^6x^{10}+5\,a^7b^5x^8+10\,a^8b^4x^6+10\,a^9b^3x^4+5\,a^{10}b^2x^2+a^{11}b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(630*a*b^5*x^9 + 2940*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 4740*a^4*b^2
*x^3 + 1930*a^5*b*x - 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3
*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10*a^9*b^3*x^
4 + 5*a^10*b^2*x^2 + a^11*b), 1/1280*(315*a*b^5*x^9 + 1470*a^2*b^4*x^7 + 26
88*a^3*b^3*x^5 + 2370*a^4*b^2*x^3 + 965*a^5*b*x + 315*(b^5*x^10 + 5*a*b^4*x
^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10*a^9*b^3
*x^4 + 5*a^10*b^2*x^2 + a^11*b)]
```

giac [A] time = 0.17, size = 78, normalized size = 0.69

$$\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^5}+\frac{315\,b^4x^9+1470\,ab^3x^7+2688\,a^2b^2x^5+2370\,a^3bx^3+965\,a^4x}{1280\left(bx^2+a\right)^5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/1280*(315*b^4*x^9 + 1470*a
*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 + 965*a^4*x)/((b*x^2 + a)^5*a^
5)
```

maple [A] time = 0.01, size = 96, normalized size = 0.85

$$\frac{x}{10\left(bx^2+a\right)^5a}+\frac{9x}{80\left(bx^2+a\right)^4a^2}+\frac{21x}{160\left(bx^2+a\right)^3a^3}+\frac{21x}{128\left(bx^2+a\right)^2a^4}+\frac{63x}{256\left(bx^2+a\right)a^5}+\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*x/a/(b*x^2+a)^5+9/80*x/a^2/(b*x^2+a)^4+21/160*x/a^3/(b*x^2+a)^3+21/128
*x/a^4/(b*x^2+a)^2+63/256*x/a^5/(b*x^2+a)+63/256/a^5/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)
```

maxima [A] time = 3.04, size = 124, normalized size = 1.10

$$\frac{315\,b^4x^9 + 1470\,ab^3x^7 + 2688\,a^2b^2x^5 + 2370\,a^3bx^3 + 965\,a^4x}{1280\left(a^5b^5x^{10} + 5\,a^6b^4x^8 + 10\,a^7b^3x^6 + 10\,a^8b^2x^4 + 5\,a^9bx^2 + a^{10}\right)} + \frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(315*b^4*x^9 + 1470*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 +
965*a^4*x)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4
+ 5*a^9*b*x^2 + a^10) + 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 4.71, size = 121, normalized size = 1.07

$$\frac{\frac{193\,x}{256\,a} + \frac{237\,b\,x^3}{128\,a^2} + \frac{21\,b^2\,x^5}{10\,a^3} + \frac{147\,b^3\,x^7}{128\,a^4} + \frac{63\,b^4\,x^9}{256\,a^5}}{a^5 + 5\,a^4\,b\,x^2 + 10\,a^3\,b^2\,x^4 + 10\,a^2\,b^3\,x^6 + 5\,a\,b^4\,x^8 + b^5\,x^{10}} + \frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{11/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((193*x)/(256*a) + (237*b*x^3)/(128*a^2) + (21*b^2*x^5)/(10*a^3) + (147*b^3
*x^7)/(128*a^4) + (63*b^4*x^9)/(256*a^5))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5
*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + (63*atan((b^(1/2)*x)/a^(1/2
)))/(256*a^(11/2)*b^(1/2))
```

sympy [A] time = 0.68, size = 177, normalized size = 1.57

$$-\frac{63\sqrt{-\frac{1}{a^{11}b}}\,\log\left(-a^6\sqrt{-\frac{1}{a^{11}b}} + x\right)}{512} + \frac{63\sqrt{-\frac{1}{a^{11}b}}\,\log\left(a^6\sqrt{-\frac{1}{a^{11}b}} + x\right)}{512} + \frac{965a^4x + 2370a^3bx^3 + 2688a^2b^2x^5 + 1470ab^3x^7 + 315b^4x^9}{1280a^{10} + 6400a^9bx^2 + 12800a^8b^2x^4 + 12800a^7b^3x^6 + 6400a^6b^4x^8 + 1280a^5b^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a**11*b))*log(-a**6*sqrt(-1/(a**11*b)) + x)/512 + 63*sqrt(-1/(
a**11*b))*log(a**6*sqrt(-1/(a**11*b)) + x)/512 + (965*a**4*x + 2370*a**3*b*
```

```
x**3 + 2688*a**2*b**2*x**5 + 1470*a*b**3*x**7 + 315*b**4*x**9)/(1280*a**10
+ 6400*a**9*b*x**2 + 12800*a**8*b**2*x**4 + 12800*a**7*b**3*x**6 + 6400*a**
6*b**4*x**8 + 1280*a**5*b**5*x**10)
```

3.360 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=133

$$-\frac{693\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{693}{256a^6x}+\frac{231}{256a^5x\left(a+bx^2\right)}+\frac{231}{640a^4x\left(a+bx^2\right)^2}+\frac{33}{160a^3x\left(a+bx^2\right)^3}+\frac{11}{80a^2x\left(a+bx^2\right)^4}+\frac{}{10}$$

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{231}{256a^5x\left(a+bx^2\right)}+\frac{231}{640a^4x\left(a+bx^2\right)^2}+\frac{33}{160a^3x\left(a+bx^2\right)^3}+\frac{11}{80a^2x\left(a+bx^2\right)^4}-\frac{693\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{693}{256a^6x}+\frac{1}{10ax\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -693/(256*a^6*x) + 1/(10*a*x*(a + b*x^2)^5) + 11/(80*a^2*x*(a + b*x^2)^4) +
 33/(160*a^3*x*(a + b*x^2)^3) + 231/(640*a^4*x*(a + b*x^2)^2) + 231/(256*a^
5*x*(a + b*x^2)) - (693*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^2\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{\left(11b^5\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{\left(99b^4\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{\left(231b^3\right)\int \frac{1}{x^2\left(ab+b^2\right.}}{160a^3}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right)^2}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right)^2}$$

$$= -\frac{693}{256a^6x} + \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{}{640a^4}$$

$$= -\frac{693}{256a^6x} + \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{}{640a^4}$$

Mathematica [A] time = 0.06, size = 101, normalized size = 0.76

$$-\frac{693\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{13/2}} - \frac{1280a^5+10615a^4bx^2+26070a^3b^2x^4+29568a^2b^3x^6+16170ab^4x^8+3465b^5x^{10}}{1280a^6x\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/1280*(1280*a^5 + 10615*a^4*b*x^2 + 26070*a^3*b^2*x^4 + 29568*a^2*b^3*x^6
 + 16170*a*b^4*x^8 + 3465*b^5*x^10)/(a^6*x*(a + b*x^2)^5) - (693*Sqrt[b]*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3), x]
```

fricas [A] time = 3.13, size = 400, normalized size = 3.01

$$\left[-\frac{6930\,b^5x^{10}+32340\,ab^4x^8+59136\,a^2b^3x^6+52140\,a^3b^2x^4+21230\,a^4bx^2+2560\,a^5-3465\left(b^5x^{11}+5\,ab^4x^9+10\,a^2b^3x^7+10\,a^3b^2x^5+5\,a^4bx^3+a^5x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2560\left(a^6b^5x^{11}+5\,a^7b^4x^9+10\,a^8b^3x^7+10\,a^9b^2x^5+5\,a^{10}bx^3+a^{11}x\right)},\ -\frac{3465\,b^5x^{10}+16170\,ab^4x^8+29568\,a^2b^3x^6+26070\,a^3b^2x^4+10615\,a^4bx^2+1280\,a^5+3465\left(b^5x^{11}+5\,ab^4x^9+10\,a^2b^3x^7+10\,a^3b^2x^5+5\,a^4bx^3+a^5x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{1280\left(a^6b^5x^{11}+5\,a^7b^4x^9+10\,a^8b^3x^7+10\,a^9b^2x^5+5\,a^{10}bx^3+a^{11}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(6930*b^5*x^10 + 32340*a*b^4*x^8 + 59136*a^2*b^3*x^6 + 52140*a^3*b
^2*x^4 + 21230*a^4*b*x^2 + 2560*a^5 - 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2
*b^3*x^7 + 10*a^3*b^2*x^5 + 5*a^4*b*x^3 + a^5*x)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^8*b^
3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x), -1/1280*(3465*b^5*x^10 + 1
6170*a*b^4*x^8 + 29568*a^2*b^3*x^6 + 26070*a^3*b^2*x^4 + 10615*a^4*b*x^2 +
1280*a^5 + 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2*b^3*x^7 + 10*a^3*b^2*x^5 +
 5*a^4*b*x^3 + a^5*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^5*x^11 + 5*a^7*
b^4*x^9 + 10*a^8*b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x)]
```

giac [A] time = 0.16, size = 90, normalized size = 0.68

$$-\frac{693\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^6}-\frac{1}{a^6x}-\frac{2185\,b^5x^9+9770\,ab^4x^7+16768\,a^2b^3x^5+13270\,a^3b^2x^3+4215\,a^4bx}{1280\left(bx^2+a\right)^5a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -693/256*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/(a^6*x) - 1/1280*(2185
*b^5*x^9 + 9770*a*b^4*x^7 + 16768*a^2*b^3*x^5 + 13270*a^3*b^2*x^3 + 4215*a^
4*b*x)/((b*x^2 + a)^5*a^6)
```

maple [A] time = 0.02, size = 126, normalized size = 0.95

$$-\frac{437b^5x^9}{256\left(b\,x^2+a\right)^5a^6}-\frac{977b^4x^7}{128\left(b\,x^2+a\right)^5a^5}-\frac{131b^3x^5}{10\left(b\,x^2+a\right)^5a^4}-\frac{1327b^2x^3}{128\left(b\,x^2+a\right)^5a^3}-\frac{843bx}{256\left(b\,x^2+a\right)^5a^2}-\frac{693b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^6}-\frac{1}{a^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/a^6/x-437/256*b^5/a^6/(b*x^2+a)^5*x^9-977/128*b^4/a^5/(b*x^2+a)^5*x^7-13
1/10*b^3/a^4/(b*x^2+a)^5*x^5-1327/128*b^2/a^3/(b*x^2+a)^5*x^3-843/256*b/a^2
/(b*x^2+a)^5*x-693/256*b/a^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.12, size = 137, normalized size = 1.03

$$-\frac{3465\,b^5x^{10}+16170\,ab^4x^8+29568\,a^2b^3x^6+26070\,a^3b^2x^4+10615\,a^4bx^2+1280\,a^5}{1280\left(a^6b^5x^{11}+5\,a^7b^4x^9+10\,a^8b^3x^7+10\,a^9b^2x^5+5\,a^{10}bx^3+a^{11}x\right)}-\frac{693\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(3465*b^5*x^10 + 16170*a*b^4*x^8 + 29568*a^2*b^3*x^6 + 26070*a^3*b^
2*x^4 + 10615*a^4*b*x^2 + 1280*a^5)/(a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^8*
b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x) - 693/256*b*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 4.58, size = 132, normalized size = 0.99

$$-\frac{\frac{1}{a}+\frac{2123\,b\,x^2}{256\,a^2}+\frac{2607\,b^2\,x^4}{128\,a^3}+\frac{231\,b^3\,x^6}{10\,a^4}+\frac{1617\,b^4\,x^8}{128\,a^5}+\frac{693\,b^5\,x^{10}}{256\,a^6}}{a^5\,x+5\,a^4\,b\,x^3+10\,a^3\,b^2\,x^5+10\,a^2\,b^3\,x^7+5\,a\,b^4\,x^9+b^5\,x^{11}}-\frac{693\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] - (1/a + (2123*b*x^2)/(256*a^2) + (2607*b^2*x^4)/(128*a^3) + (231*b^3*x^6)/
(10*a^4) + (1617*b^4*x^8)/(128*a^5) + (693*b^5*x^10)/(256*a^6))/(a^5*x + b^
5*x^11 + 5*a^4*b*x^3 + 5*a*b^4*x^9 + 10*a^3*b^2*x^5 + 10*a^2*b^3*x^7) - (69
3*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(13/2))
```

sympy [A] time = 0.83, size = 187, normalized size = 1.41

$$\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}-\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}+\frac{-1280a^5-10615a^4bx^2-26070a^3b^2x^4-29568a^2b^3x^6-16170ab^4x^8-3465b^5x^{10}}{1280a^{11}x+6400a^{10}bx^3+12800a^9b^2x^5+12800a^8b^3x^7+6400a^7b^4x^9+1280a^6b^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-b/a**13)*log(-a**7*sqrt(-b/a**13)/b + x)/512 - 693*sqrt(-b/a**13)
*log(a**7*sqrt(-b/a**13)/b + x)/512 + (-1280*a**5 - 10615*a**4*b*x**2 - 260
70*a**3*b**2*x**4 - 29568*a**2*b**3*x**6 - 16170*a*b**4*x**8 - 3465*b**5*x*
*10)/(1280*a**11*x + 6400*a**10*b*x**3 + 12800*a**9*b**2*x**5 + 12800*a**8*
b**3*x**7 + 6400*a**7*b**4*x**9 + 1280*a**6*b**5*x**11)
```

3.361 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=144

$$\frac{3003b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{15/2}}+\frac{3003b}{256a^7x}-\frac{1001}{256a^6x^3}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{}{80a^2}$$

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{3003b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{15/2}}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{13}{80a^2x^3\left(a+bx^2\right)^4}+\frac{3003b}{256a^7x}-\frac{1001}{256a^6x^3}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1001/(256*a^6*x^3) + (3003*b)/(256*a^7*x) + 1/(10*a*x^3*(a + b*x^2)^5) + 1
3/(80*a^2*x^3*(a + b*x^2)^4) + 143/(480*a^3*x^3*(a + b*x^2)^3) + 429/(640*a
^4*x^3*(a + b*x^2)^2) + 3003/(1280*a^5*x^3*(a + b*x^2)) + (3003*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(15/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^4\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{\left(13b^5\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{\left(143b^4\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{\left(429b^3\right)\int \frac{1}{x^4\left(ab+\right.}}{160a^3}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{429}{640a^4x^3\left(a+bx\right.}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{429}{640a^4x^3\left(a+bx\right.}$$

$$= -\frac{1001}{256a^6x^3} + \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{}{640}$$

$$= -\frac{1001}{256a^6x^3} + \frac{3003b}{256a^7x} + \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+b\right.}$$

$$= -\frac{1001}{256a^6x^3} + \frac{3003b}{256a^7x} + \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+b\right.}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 0.78

$$\frac{\frac{\sqrt{a}\left(-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910a^3b^3x^6+384384a^2b^4x^8+210210ab^5x^{10}+45045b^6x^{12}\right)}{x^3\left(a+bx^2\right)^5}+45045b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3840a^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((Sqrt[a]*(-1280*a^6 + 16640*a^5*b*x^2 + 137995*a^4*b^2*x^4 + 338910*a^3*b^
3*x^6 + 384384*a^2*b^4*x^8 + 210210*a*b^5*x^10 + 45045*b^6*x^12))/(x^3*(a +
 b*x^2)^5) + 45045*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*a^(15/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3), x]
```

fricas [A] time = 0.89, size = 436, normalized size = 3.03

$$\left[\frac{90090\,b^6x^{12}+420420\,ab^5x^{10}+768768\,a^2b^4x^8+677820\,a^3b^3x^6+275990\,a^4b^2x^4+33280\,a^5bx^2-2560\,a^6+45045\left(b^6x^{13}+5\,ab^5x^{11}+10\,a^2b^4x^9+10\,a^3b^3x^7+5\,a^4b^2x^5+a^5bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{7680\left(a^7b^5x^{13}+5\,a^8b^4x^{11}+10\,a^9b^3x^9+10\,a^{10}b^2x^7+5\,a^{11}bx^5+a^{12}x^3\right)},\ \frac{45045\,b^6x^{12}+210210\,ab^5x^{10}+384384\,a^2b^4x^8+338910\,a^3b^3x^6+137995\,a^4b^2x^4+16640\,a^5bx^2-1280\,a^6+45045\left(b^6x^{13}+5\,ab^5x^{11}+10\,a^2b^4x^9+10\,a^3b^3x^7+5\,a^4b^2x^5+a^5bx^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{3840\left(a^7b^5x^{13}+5\,a^8b^4x^{11}+10\,a^9b^3x^9+10\,a^{10}b^2x^7+5\,a^{11}bx^5+a^{12}x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(90090*b^6*x^12 + 420420*a*b^5*x^10 + 768768*a^2*b^4*x^8 + 677820*a
^3*b^3*x^6 + 275990*a^4*b^2*x^4 + 33280*a^5*b*x^2 - 2560*a^6 + 45045*(b^6*x
^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*a^4*b^2*x^5 + a^5*
b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^5
*x^13 + 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 +
a^12*x^3), 1/3840*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^4*x^8
+ 338910*a^3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a^6 + 45
045*(b^6*x^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*a^4*b^2*
x^5 + a^5*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^7*b^5*x^13 + 5*a^8*b^4*x
^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x^3)]
```

giac [A] time = 0.16, size = 104, normalized size = 0.72

$$\frac{3003\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^7}+\frac{18\,bx^2-a}{3\,a^7x^3}+\frac{22005\,b^6x^9+96290\,ab^5x^7+160384\,a^2b^4x^5+121310\,a^3b^3x^3+35595\,a^4b^2x}{3840\left(bx^2+a\right)^5a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7) + 1/3*(18*b*x^2 - a)/(a^
7*x^3) + 1/3840*(22005*b^6*x^9 + 96290*a*b^5*x^7 + 160384*a^2*b^4*x^5 + 121
310*a^3*b^3*x^3 + 35595*a^4*b^2*x)/((b*x^2 + a)^5*a^7)
```

maple [A] time = 0.02, size = 139, normalized size = 0.97

$$\frac{1467b^6x^9}{256\left(b\,x^2+a\right)^5a^7}+\frac{9629b^5x^7}{384\left(b\,x^2+a\right)^5a^6}+\frac{1253b^4x^5}{30\left(b\,x^2+a\right)^5a^5}+\frac{12131b^3x^3}{384\left(b\,x^2+a\right)^5a^4}+\frac{2373b^2x}{256\left(b\,x^2+a\right)^5a^3}+\frac{3003b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^7}+\frac{6b}{a^7x}-\frac{1}{3a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/3/a^6/x^3+6*b/a^7/x+1467/256/a^7*b^6/(b*x^2+a)^5*x^9+9629/384/a^6*b^5/(b
*x^2+a)^5*x^7+1253/30/a^5*b^4/(b*x^2+a)^5*x^5+12131/384/a^4*b^3/(b*x^2+a)^5
*x^3+2373/256/a^3*b^2/(b*x^2+a)^5*x+3003/256/a^7*b^2/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)
```

maxima [A] time = 3.10, size = 152, normalized size = 1.06

$$\frac{45045\,b^6x^{12}+210210\,ab^5x^{10}+384384\,a^2b^4x^8+338910\,a^3b^3x^6+137995\,a^4b^2x^4+16640\,a^5bx^2-1280\,a^6}{3840\left(a^7b^5x^{13}+5\,a^8b^4x^{11}+10\,a^9b^3x^9+10\,a^{10}b^2x^7+5\,a^{11}bx^5+a^{12}x^3\right)}+\frac{3003\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^4*x^8 + 338910*a^
3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a^6)/(a^7*b^5*x^13
+ 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x
^3) + 3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

mupad [B] time = 4.62, size = 146, normalized size = 1.01

$$\frac{\frac{13\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{27599\,b^2\,x^4}{768\,a^3}+\frac{11297\,b^3\,x^6}{128\,a^4}+\frac{1001\,b^4\,x^8}{10\,a^5}+\frac{7007\,b^5\,x^{10}}{128\,a^6}+\frac{3003\,b^6\,x^{12}}{256\,a^7}}{a^5\,x^3+5\,a^4\,b\,x^5+10\,a^3\,b^2\,x^7+10\,a^2\,b^3\,x^9+5\,a\,b^4\,x^{11}+b^5\,x^{13}}+\frac{3003\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{15/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((13*b*x^2)/(3*a^2) - 1/(3*a) + (27599*b^2*x^4)/(768*a^3) + (11297*b^3*x^6)
/(128*a^4) + (1001*b^4*x^8)/(10*a^5) + (7007*b^5*x^10)/(128*a^6) + (3003*b^
```

```
6*x^12)/(256*a^7))/(a^5*x^3 + b^5*x^13 + 5*a^4*b*x^5 + 5*a*b^4*x^11 + 10*a^
3*b^2*x^7 + 10*a^2*b^3*x^9) + (3003*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(256
*a^(15/2))
```

sympy [A] time = 0.89, size = 209, normalized size = 1.45

$$-\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(-\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910a^3b^3x^6+384384a^2b^4x^8+210210ab^5x^{10}+45045b^6x^{12}}{3840a^{12}x^3+19200a^{11}bx^5+38400a^{10}b^2x^7+38400a^9b^3x^9+19200a^8b^4x^{11}+3840a^7b^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3003*sqrt(-b**3/a**15)*log(-a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + 3003*sq
rt(-b**3/a**15)*log(a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + (-1280*a**6 + 16
640*a**5*b*x**2 + 137995*a**4*b**2*x**4 + 338910*a**3*b**3*x**6 + 384384*a*
*2*b**4*x**8 + 210210*a*b**5*x**10 + 45045*b**6*x**12)/(3840*a**12*x**3 + 1
9200*a**11*b*x**5 + 38400*a**10*b**2*x**7 + 38400*a**9*b**3*x**9 + 19200*a*
*8*b**4*x**11 + 3840*a**7*b**5*x**13)
```

3.362 $\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{17/2}}-\frac{9009b^2}{256a^8x}+\frac{3003b}{256a^7x^3}-\frac{9009}{1280a^6x^5}+\frac{1287}{256a^5x^5\left(a+bx^2\right)}+\frac{143}{128a^4x^5\left(a+bx^2\right)^2}+\frac{13}{32a^3x^5\left(a+bx^2\right.}$$

Rubi [A] time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{9009b^2}{256a^8x}-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{17/2}}+\frac{3003b}{256a^7x^3}+\frac{1287}{256a^5x^5\left(a+bx^2\right)}+\frac{143}{128a^4x^5\left(a+bx^2\right)^2}+\frac{13}{32a^3x^5\left(a+bx^2\right)^3}+\frac{3}{16a^2x^5\left(a+bx^2\right)^4}-\frac{9009}{1280a^6x^5}+\frac{1}{10ax^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -9009/(1280*a^6*x^5) + (3003*b)/(256*a^7*x^3) - (9009*b^2)/(256*a^8*x) + 1/
(10*a*x^5*(a + b*x^2)^5) + 3/(16*a^2*x^5*(a + b*x^2)^4) + 13/(32*a^3*x^5*(a
 + b*x^2)^3) + 143/(128*a^4*x^5*(a + b*x^2)^2) + 1287/(256*a^5*x^5*(a + b*x
^2)) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(17/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^6\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{\left(3b^5\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^5}\,dx}{2a}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{\left(39b^4\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^4}\,dx}{16a^2}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{\left(143b^3\right)\int \frac{1}{x^6\left(ab+b^2\right.}}{32a^3}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{143}{128a^4x^5\left(a+bx^2\right)}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{143}{128a^4x^5\left(a+bx^2\right)}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{}{128}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+b\right.}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} - \frac{9009b^2}{256a^8x} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{}{32a}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} - \frac{9009b^2}{256a^8x} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{}{32a}$$

Mathematica [A] time = 0.06, size = 123, normalized size = 0.78

$$-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{17/2}} - \frac{256a^7-1280a^6bx^2+16640a^5b^2x^4+137995a^4b^3x^6+338910a^3b^4x^8+384384a^2b^5x^{10}+210210ab^6x^{12}+45045b^7x^{14}}{1280a^8x^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/1280*(256*a^7 - 1280*a^6*b*x^2 + 16640*a^5*b^2*x^4 + 137995*a^4*b^3*x^6
+ 338910*a^3*b^4*x^8 + 384384*a^2*b^5*x^10 + 210210*a*b^6*x^12 + 45045*b^7*
x^14)/(a^8*x^5*(a + b*x^2)^5) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(256*a^(17/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(a^2 + 2abx^2 + b^2x^4\right)^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3), x]
```

fricas [A] time = 0.83, size = 462, normalized size = 2.94

$$\left[-\frac{90090\, b^7x^{14} + 420420\, ab^6x^{12} + 768768\, a^2b^5x^{10} + 677820\, a^3b^4x^8 + 275990\, a^4b^3x^6 + 33280\, a^5b^2x^4 - 2560\, a^6bx^2 + 512\, a^7 - 45045 \left(b^7x^{15} + 5\, ab^6x^{13} + 10\, a^2b^5x^{11} + 10\, a^3b^4x^9 + 5\, a^4b^3x^7 + a^5b^2x^5\right)\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2 - 2\, ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right)}{2560 \left(a^8b^5x^{15} + 5\, a^9b^4x^{13} + 10\, a^{10}b^3x^{11} + 10\, a^{11}b^2x^9 + 5\, a^{12}bx^7 + a^{13}x^5\right)}, -\frac{45045\, b^7x^{14} + 210210\, ab^6x^{12} + 384384\, a^2b^5x^{10} + 338910\, a^3b^4x^8 + 137995\, a^4b^3x^6 + 16640\, a^5b^2x^4 - 1280\, a^6bx^2 + 256\, a^7 + 45045 \left(b^7x^{15} + 5\, ab^6x^{13} + 10\, a^2b^5x^{11} + 10\, a^3b^4x^9 + 5\, a^4b^3x^7 + a^5b^2x^5\right)\sqrt{\frac{b}{a}} \arctan\left(x\sqrt{\frac{b}{a}}\right)}{1280 \left(a^8b^5x^{15} + 5\, a^9b^4x^{13} + 10\, a^{10}b^3x^{11} + 10\, a^{11}b^2x^9 + 5\, a^{12}bx^7 + a^{13}x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(90090*b^7*x^14 + 420420*a*b^6*x^12 + 768768*a^2*b^5*x^10 + 677820
*a^3*b^4*x^8 + 275990*a^4*b^3*x^6 + 33280*a^5*b^2*x^4 - 2560*a^6*b*x^2 + 51
2*a^7 - 45045*(b^7*x^15 + 5*a*b^6*x^13 + 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 +
 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)
/(b*x^2 + a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*
b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5), -1/1280*(45045*b^7*x^14 + 210210*a*b^6*
x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a^4*b^3*x^6 + 1664
0*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7 + 45045*(b^7*x^15 + 5*a*b^6*x^13 +
 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 + 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(b/a)*
arctan(x*sqrt(b/a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10
*a^11*b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5)]
```

giac [A] time = 0.16, size = 115, normalized size = 0.73

$$-\frac{9009\, b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\, \sqrt{ab}\, a^8} - \frac{45045\, b^7x^{14} + 210210\, ab^6x^{12} + 384384\, a^2b^5x^{10} + 338910\, a^3b^4x^8 + 137995\, a^4b^3x^6 + 16640\, a^5b^2x^4 - 1280\, a^6bx^2 + 256\, a^7}{1280 \left(bx^3 + ax\right)^5 a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8) - 1/1280*(45045*b^7*x^1
4 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a
^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7)/((b*x^3 + a*x)^5
*a^8)
```

maple [A] time = 0.02, size = 150, normalized size = 0.96

$$-\frac{3633b^7x^9}{256\left(bx^2+a\right)^5a^8}-\frac{7837b^6x^7}{128\left(bx^2+a\right)^5a^7}-\frac{1001b^5x^5}{10\left(bx^2+a\right)^5a^6}-\frac{9443b^4x^3}{128\left(bx^2+a\right)^5a^5}-\frac{5327b^3x}{256\left(bx^2+a\right)^5a^4}-\frac{9009b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^8}-\frac{21b^2}{a^8x}+\frac{2b}{a^7x^3}-\frac{1}{5a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/5/a^6/x^5-21*b^2/a^8/x+2*b/a^7/x^3-3633/256*b^7/a^8/(b*x^2+a)^5*x^9-7837
/128*b^6/a^7/(b*x^2+a)^5*x^7-1001/10*b^5/a^6/(b*x^2+a)^5*x^5-9443/128*b^4/a
^5/(b*x^2+a)^5*x^3-5327/256*b^3/a^4/(b*x^2+a)^5*x-9009/256*b^3/a^8/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.11, size = 163, normalized size = 1.04

$$-\frac{45045\,b^7x^{14}+210210\,ab^6x^{12}+384384\,a^2b^5x^{10}+338910\,a^3b^4x^8+137995\,a^4b^3x^6+16640\,a^5b^2x^4-1280\,a^6bx^2+256\,a^7}{1280\left(a^8b^5x^{15}+5\,a^9b^4x^{13}+10\,a^{10}b^3x^{11}+10\,a^{11}b^2x^9+5\,a^{12}bx^7+a^{13}x^5\right)}-\frac{9009\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(45045*b^7*x^14 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*
a^3*b^4*x^8 + 137995*a^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256
*a^7)/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*b^2*x^9 +
 5*a^12*b*x^7 + a^13*x^5) - 9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a
^8)
```

mupad [B] time = 4.65, size = 158, normalized size = 1.01

$$-\frac{\frac{1}{5\,a}-\frac{b\,x^2}{a^2}+\frac{13\,b^2\,x^4}{a^3}+\frac{27599\,b^3\,x^6}{256\,a^4}+\frac{33891\,b^4\,x^8}{128\,a^5}+\frac{3003\,b^5\,x^{10}}{10\,a^6}+\frac{21021\,b^6\,x^{12}}{128\,a^7}+\frac{9009\,b^7\,x^{14}}{256\,a^8}}{a^5\,x^5+5\,a^4\,b\,x^7+10\,a^3\,b^2\,x^9+10\,a^2\,b^3\,x^{11}+5\,a\,b^4\,x^{13}+b^5\,x^{15}}-\frac{9009\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] - (1/(5*a) - (b*x^2)/a^2 + (13*b^2*x^4)/a^3 + (27599*b^3*x^6)/(256*a^4) + (
33891*b^4*x^8)/(128*a^5) + (3003*b^5*x^10)/(10*a^6) + (21021*b^6*x^12)/(128
```

```
*a^7) + (9009*b^7*x^14)/(256*a^8))/(a^5*x^5 + b^5*x^15 + 5*a^4*b*x^7 + 5*a*
b^4*x^13 + 10*a^3*b^2*x^9 + 10*a^2*b^3*x^11) - (9009*b^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/(256*a^(17/2))
```

sympy [A] time = 0.95, size = 221, normalized size = 1.41

$$\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(-\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}-\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}+\frac{-256a^7+1280a^6bx^2-16640a^5b^2x^4-137995a^4b^3x^6-338910a^3b^4x^8-384384a^2b^5x^{10}-210210ab^6x^{12}-45045b^7x^{14}}{1280a^{13}x^5+6400a^{12}bx^7+12800a^{11}b^2x^9+12800a^{10}b^3x^{11}+6400a^9b^4x^{13}+1280a^8b^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 9009*sqrt(-b**5/a**17)*log(-a**9*sqrt(-b**5/a**17)/b**3 + x)/512 - 9009*sqr
t(-b**5/a**17)*log(a**9*sqrt(-b**5/a**17)/b**3 + x)/512 + (-256*a**7 + 1280
*a**6*b*x**2 - 16640*a**5*b**2*x**4 - 137995*a**4*b**3*x**6 - 338910*a**3*b
**4*x**8 - 384384*a**2*b**5*x**10 - 210210*a*b**6*x**12 - 45045*b**7*x**14)
/(1280*a**13*x**5 + 6400*a**12*b*x**7 + 12800*a**11*b**2*x**9 + 12800*a**10
*b**3*x**11 + 6400*a**9*b**4*x**13 + 1280*a**8*b**5*x**15)
```

3.363 $\int \frac{1}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=19

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 199, 203}

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+2x^2+x^4}\,dx = \int \frac{1}{\left(1+x^2\right)^2}\,dx$$
$$= \frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x)$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{x^2+1} + \tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + x^4)^(-1),x]
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+2x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*x^2 + x^4)^(-1),x]
[Out] IntegrateAlgebraic[(1 + 2*x^2 + x^4)^(-1), x]
```

fricas [A] time = 1.98, size = 19, normalized size = 1.00

$$\frac{\left(x^2+1\right)\arctan(x) + x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

giac [A] time = 0.15, size = 15, normalized size = 0.79

$$\frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*x^2+1),x)
```

```
[Out] 1/2/(x^2+1)*x+1/2*arctan(x)
```

maxima [A] time = 3.06, size = 15, normalized size = 0.79

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.84

$$\frac{\mathrm{atan}(x)}{2}+\frac{x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + x^4 + 1),x)
```

```
[Out] atan(x)/2 + x/(2*(x^2 + 1))
```

sympy [A] time = 0.11, size = 12, normalized size = 0.63

$$\frac{x}{2x^2+2}+\frac{\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*x**2+1),x)
```

```
[Out] x/(2*x**2 + 2) + atan(x)/2
```

3.364 $\int \frac{x}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2\left(x^2+1\right)}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 261}

$$-\frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/(2*(1 + x^2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+2x^2+x^4}\,dx &= \int \frac{x}{\left(1+x^2\right)^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/2*1/(1 + x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1+2x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 + 2*x^2 + x^4),x]
```

```
[Out] IntegrateAlgebraic[x/(1 + 2*x^2 + x^4), x]
```

fricas [A] time = 0.85, size = 9, normalized size = 0.82

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2/(x^2 + 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.82

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+1),x)
```

```
[Out] -1/2/(x^2+1)
```

maxima [A] time = 1.35, size = 9, normalized size = 0.82

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] -1/2/(x^2 + 1)
```

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2 + x^4 + 1),x)
[Out] -1/(2*(x^2 + 1))
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+1),x)
[Out] -1/(2*x**2 + 2)
```

3.365 $\int \frac{x^2}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x)-\frac{x}{2\left(x^2+1\right)}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 288, 203}

$$\frac{1}{2}\tan^{-1}(x)-\frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x^2 + x^4),x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = \int \frac{x^2}{\left(1+x^2\right)^2}\,dx$$
$$= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x)$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x^2 + x^4),x]
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1+2x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 + 2*x^2 + x^4),x]
[Out] IntegrateAlgebraic[x^2/(1 + 2*x^2 + x^4), x]
```

fricas [A] time = 0.83, size = 21, normalized size = 1.11

$$\frac{\left(x^2+1\right)\arctan(x) - x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

giac [A] time = 0.16, size = 15, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{x}{2\left(x^2+1\right)}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+2*x^2+1),x)
[Out] -1/2/(x^2+1)*x+1/2*arctan(x)
```

maxima [A] time = 2.97, size = 15, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2 + x^4 + 1),x)
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

sympy [A] time = 0.11, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+2*x**2+1),x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

3.366 $\int \frac{x^3}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2(x^2+1)}+\frac{1}{2}\log(x^2+1)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{1}{2(x^2+1)}+\frac{1}{2}\log(x^2+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 2*x^2 + x^4),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{1+2x^2+x^4}\,dx = \int \frac{x^3}{\left(1+x^2\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(1+x)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, x^2\right)$$
$$= \frac{1}{2\left(1+x^2\right)}+\frac{1}{2}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.82

$$\frac{1}{2}\left(\frac{1}{x^2+1}+\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x^2 + x^4),x]
[Out] ((1 + x^2)^(-1) + Log[1 + x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1+2x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(1 + 2*x^2 + x^4),x]
[Out] IntegrateAlgebraic[x^3/(1 + 2*x^2 + x^4), x]
```

fricas [A] time = 0.86, size = 23, normalized size = 1.05

$$\frac{\left(x^2+1\right)\log\left(x^2+1\right)+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*log(x^2 + 1) + 1)/(x^2 + 1)
```

giac [A] time = 0.20, size = 18, normalized size = 0.82

$$\frac{1}{2\left(x^{2}+1\right)}+\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="giac")
[Out] 1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{\ln\left(x^{2}+1\right)}{2}+\frac{1}{2x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+2*x^2+1),x)
[Out] 1/2/(x^2+1)+1/2*ln(x^2+1)
```

maxima [A] time = 1.28, size = 18, normalized size = 0.82

$$\frac{1}{2\left(x^{2}+1\right)}+\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] 1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$\frac{\ln\left(x^{2}+1\right)}{2}+\frac{1}{2\,\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x^2 + x^4 + 1),x)
[Out] log(x^2 + 1)/2 + 1/(2*(x^2 + 1))
```

sympy [A] time = 0.09, size = 15, normalized size = 0.68

$$\frac{\log\left(x^{2}+1\right)}{2}+\frac{1}{2x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+2*x**2+1),x)
```

```
[Out] log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

3.367 $\int \frac{x}{81-18x^2+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2(9-x^2)}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 261}

$$\frac{1}{2(9-x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(81 - 18*x^2 + x^4),x]
```

```
[Out] 1/(2*(9 - x^2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{81-18x^2+x^4}\,dx &= \int \frac{x}{\left(-9+x^2\right)^2}\,dx\\ &= \frac{1}{2(9-x^2)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$-\frac{1}{2(x^2-9)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(81 - 18*x^2 + x^4),x]
```

```
[Out] -1/2*1/(-9 + x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{81-18x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(81 - 18*x^2 + x^4),x]
```

```
[Out] IntegrateAlgebraic[x/(81 - 18*x^2 + x^4), x]
```

fricas [A] time = 0.78, size = 9, normalized size = 0.69

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="fricas")
```

```
[Out] -1/2/(x^2 - 9)
```

giac [A] time = 0.16, size = 9, normalized size = 0.69

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 - 9)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-18*x^2+81),x)
```

```
[Out] -1/2/(x^2-9)
```

maxima [A] time = 1.33, size = 9, normalized size = 0.69

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="maxima")
[Out] -1/2/(x^2 - 9)
```

mupad [B] time = 0.05, size = 11, normalized size = 0.85

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 - 18*x^2 + 81),x)
[Out] -1/(2*(x^2 - 9))
```

sympy [A] time = 0.09, size = 8, normalized size = 0.62

$$-\frac{1}{2x^2-18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-18*x**2+81),x)
[Out] -1/(2*x**2 - 18)
```

3.368 $\int \frac{x^3}{16-8x^2+x^4} \, dx$

Optimal. Leaf size=24

$$\frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(16 - 8*x^2 + x^4),x]
```

```
[Out] 2/(4 - x^2) + Log[4 - x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{16-8x^2+x^4}\,dx &= \int \frac{x^3}{\left(-4+x^2\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(-4+x)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{4}{(-4+x)^2}+\frac{1}{-4+x}\right)dx,x,x^2\right)\\ &= \frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-4\right)-\frac{2}{x^2-4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(16 - 8*x^2 + x^4),x]
[Out] -2/(-4 + x^2) + Log[-4 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{16-8x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(16 - 8*x^2 + x^4),x]
[Out] IntegrateAlgebraic[x^3/(16 - 8*x^2 + x^4), x]
```

fricas [A] time = 1.15, size = 23, normalized size = 0.96

$$\frac{\left(x^2-4\right)\log\left(x^2-4\right)-4}{2\left(x^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="fricas")
[Out] 1/2*((x^2 - 4)*log(x^2 - 4) - 4)/(x^2 - 4)
```

giac [A] time = 0.15, size = 19, normalized size = 0.79

$$-\frac{2}{x^2-4}+\frac{1}{2}\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="giac")
[Out] -2/(x^2 - 4) + 1/2*log(abs(x^2 - 4))
```

maple [A] time = 0.00, size = 19, normalized size = 0.79

$$\frac{\ln\left(x^2-4\right)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4-8*x^2+16),x)
[Out] 1/2*ln(x^2-4)-2/(x^2-4)
```

maxima [A] time = 1.32, size = 18, normalized size = 0.75

$$-\frac{2}{x^2-4}+\frac{1}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="maxima")
[Out] -2/(x^2 - 4) + 1/2*log(x^2 - 4)
```

mupad [B] time = 4.23, size = 18, normalized size = 0.75

$$\frac{\ln\left(x^2-4\right)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 - 8*x^2 + 16),x)
[Out] log(x^2 - 4)/2 - 2/(x^2 - 4)
```

sympy [A] time = 0.10, size = 14, normalized size = 0.58

$$\frac{\log\left(x^2-4\right)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4-8*x**2+16),x)
```

```
[Out] log(x**2 - 4)/2 - 2/(x**2 - 4)
```

3.369 $\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^2\left(ab+b^2x\right)dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(abx^2+b^2x^3\right)dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}$$
$$= \frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(4ax^6+3bx^8\right)}{24\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(4*a*x^6 + 3*b*x^8))/(24*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 6.68, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(4ax^6+3bx^8\right)}{24\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(4*a*x^6 + 3*b*x^8))/(24*(a + b*x^2))
```

fricas [A] time = 0.76, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^8+\frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

[Out] 1/8*b*x^8 + 1/6*a*x^6

giac [A] time = 0.16, size = 29, normalized size = 0.37

$$\frac{1}{8}\,bx^8\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{6}\,ax^6\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/8*b*x^8*sgn(b*x^2 + a) + 1/6*a*x^6*sgn(b*x^2 + a)

maple [A] time = 0.01, size = 36, normalized size = 0.46

$$\frac{\left(3b\,x^2+4a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x^6}{24b\,x^2+24a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*((b*x^2+a)^2)^(1/2),x)

[Out] 1/24*x^6*(3*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)

maxima [A] time = 1.35, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^8+\frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/8*b*x^8 + 1/6*a*x^6

mupad [B] time = 4.45, size = 71, normalized size = 0.90

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\;\left(a^3-4\,a^2\,b\,x^2-5\,a\,b^2\,x^4+3\,b\,x^2\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)\right)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*((a + b*x^2)^2)^(1/2),x)

[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a^3 - 4*a^2*b*x^2 - 5*a*b^2*x^4 + 3*b*x
^2*(a^2 + b^2*x^4 + 2*a*b*x^2)))/(24*b^3)

sympy [A] time = 0.10, size = 12, normalized size = 0.15

$$\frac{ax^6}{6} + \frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**6/6 + b*x**8/8
```

3.370 $\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}-\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 609}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}-\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b^2) + (a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/2)/(6*b^2)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2} - \frac{a\,\text{Subst}\left(\int\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)}{2b}$$
$$= -\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2} + \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.58

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax^4+2bx^6\right)}{12\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x^4 + 2*b*x^6))/(12*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 6.19, size = 39, normalized size = 0.58

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax^4+2bx^6\right)}{12\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x^4 + 2*b*x^6))/(12*(a + b*x^2))
```

fricas [A] time = 1.07, size = 13, normalized size = 0.19

$$\frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

giac [A] time = 0.18, size = 23, normalized size = 0.34

$$\frac{1}{12}\left(2\,bx^{6}+3\,ax^{4}\right)\operatorname{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*b*x^6 + 3*a*x^4)*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.54

$$\frac{\left(2b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}\;x^{4}}{12b\,x^{2}+12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/12*x^4*(2*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.28, size = 13, normalized size = 0.19

$$\frac{1}{6}\,bx^{6}+\frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

mupad [B] time = 4.31, size = 59, normalized size = 0.88

$$\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}\;\left(8\,b^{2}\,\left(a^{2}+b^{2}\,x^{4}\right)-12\,a^{2}\,b^{2}+4\,a\,b^{3}\,x^{2}\right)}{48\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(8*b^2*(a^2 + b^2*x^4) - 12*a^2*b^2 + 4*
a*b^3*x^2))/(48*b^4)
```

sympy [A] time = 0.10, size = 12, normalized size = 0.18

$$\frac{ax^{4}}{4}+\frac{bx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

3.371 $\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 609}

$$\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\ &= \frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a+bx^2)^2}\,(2ax^2+bx^4)}{4(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(2*a*x^2 + b*x^4))/(4*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 5.68, size = 38, normalized size = 1.06

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(2ax^2+bx^4\right)}{4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(2*a*x^2 + b*x^4))/(4*(a + b*x^2))
```

fricas [A] time = 0.67, size = 13, normalized size = 0.36

$$\frac{1}{4}bx^4+\frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

giac [A] time = 0.16, size = 22, normalized size = 0.61

$$\frac{1}{4}\left(bx^4+2\,ax^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/4*(b*x^4 + 2*a*x^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 35, normalized size = 0.97

$$\frac{\left(b\,x^2+2a\right)\sqrt{\left(b\,x^2+a\right)^2}\,x^2}{4b\,x^2+4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^2)^(1/2),x)
```

[Out] 1/4*x^2*(b*x^2+2*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)

maxima [A] time = 1.32, size = 14, normalized size = 0.39

$$\frac{\left(bx^2 + a\right)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/4*(b*x^2 + a)^2/b

mupad [B] time = 4.35, size = 33, normalized size = 0.92

$$\left(\frac{a}{4\,b} + \frac{x^2}{4}\right)\,\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((a + b*x^2)^2)^(1/2),x)

[Out] (a/(4*b) + x^2/4)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)

sympy [A] time = 0.10, size = 12, normalized size = 0.33

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x**2+a)**2)**(1/2),x)

[Out] a*x**2/2 + b*x**4/4

3.372 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal. Leaf size=75

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{a\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{a\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

```
[Out] (b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (a*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x}+b^2x\right)dx}{ab+b^2x^2}$$
$$= \frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)} + \frac{a\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(2a\log(x)+bx^2\right)}{2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2 + 2*a*Log[x]))/(2*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 0.21, size = 197, normalized size = 2.63

$$\frac{1}{4}\sqrt{a^2+2abx^2+b^2x^4}+\frac{1}{4}a\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)-\frac{a\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{4b}-\frac{a\sqrt{b^2}\log\left(b\sqrt{a^2+2abx^2+b^2x^4}-ab-b\sqrt{b^2}\,x^2\right)}{4b}-\frac{1}{4}\sqrt{b^2}\,x^2$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

```
[Out] -1/4*(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/4 + (a*Log[-a - Sqrt
[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/4 - (a*(b + Sqrt[b^2])*Log[a
- Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*b) - (a*Sqrt[b^2]*Lo
g[-(a*b) - b*Sqrt[b^2]*x^2 + b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*b)
```

fricas [A] time = 0.63, size = 11, normalized size = 0.15

$$\frac{1}{2}\,bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

giac [A] time = 0.16, size = 30, normalized size = 0.40

$$\frac{1}{2}\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/2*b*x^2*sgn(b*x^2 + a) + 1/2*a*log(x^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 34, normalized size = 0.45

$$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(b\,x^2+2a\ln(x)\right)}{2b\,x^2+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x,x)
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(b*x^2+2*a*ln(x))/(b*x^2+a)
```

maxima [A] time = 1.39, size = 14, normalized size = 0.19

$$\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

mupad [B] time = 4.39, size = 109, normalized size = 1.45

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2}-\frac{\ln\left(a\,b+\frac{a^2}{x^2}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^2}\right)\,\sqrt{a^2}}{2}+\frac{a\,b\,\ln\left(a\,b+\sqrt{\left(b\,x^2+a\right)^2}\,\sqrt{b^2}+b^2\,x^2\right)}{2\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x,x)
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/2 - (log(a*b + a^2/x^2 + ((a^2)^(1/2)*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/x^2)*(a^2)^(1/2))/2 + (a*b*log(a*b + ((a +
b*x^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(1/2))
```

sympy [A] time = 0.12, size = 10, normalized size = 0.13

$$a\log(x) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x**2/2
```

3.373 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3} \, dx$

Optimal. Leaf size=75

$$\frac{b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^3}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^3}+\frac{b^2}{x}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)} + \frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\log(x)}{a+bx^2}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.52

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a-2bx^2\log(x)\right)}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

```
[Out] -1/2*(Sqrt[(a + b*x^2)^2]*(a - 2*b*x^2*Log[x]))/(x^2*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 0.51, size = 734, normalized size = 9.79

$$\frac{-\frac{1}{2}\left(b^2\right)^{3/2}x^4\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}x^2\right)-\frac{1}{2}\left(b^2\right)^{3/2}x^4\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)-\frac{1}{2}ab\sqrt{b^2}x^2\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}x^2\right)+\frac{1}{2}b^2x^2\sqrt{a^2+2abx^2+b^2x^4}\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}x^2\right)-\frac{1}{2}ab\sqrt{b^2}x^2\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)+\frac{1}{2}b^2x^2\sqrt{a^2+2abx^2+b^2x^4}\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)+a^2\sqrt{b^2}}{\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}x^2\right)\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)}+\frac{-ab\sqrt{a^2+2abx^2+b^2x^4}-ab^2x^2\tanh^{-1}\left(\frac{\sqrt{a^2+2abx^2+b^2x^4}-\sqrt{b^2}x^2}{a}\right)+b\sqrt{b^2}x^2\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{a^2+2abx^2+b^2x^4}-\sqrt{b^2}x^2}{a}\right)-b^3x^4\tanh^{-1}\left(\frac{\sqrt{a^2+2abx^2+b^2x^4}-\sqrt{b^2}x^2}{a}\right)+ab\sqrt{b^2}x^2}{\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}x^2\right)\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

```
[Out] (a*b*Sqrt[b^2]*x^2 - a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] - a*b^2*x^2*ArcTan
h[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a] - b^3*x^4*ArcTanh
[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a] + b*Sqrt[b^2]*x^2*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/a])/((-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (a^2*Sqrt[b^2]
- (a*b*Sqrt[b^2]*x^2*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]])/2 - ((b^2)^(3/2)*x^4*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]])/2 + (b^2*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-a - Sqrt[b^2]*x
^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/2 - (a*b*Sqrt[b^2]*x^2*Log[a - Sqrt[
b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/2 - ((b^2)^(3/2)*x^4*Log[a - S
qrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/2 + (b^2*x^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]]
```

```
)/2)/((-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])*(a - Sqrt[b^2]
*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]))
```

fricas [A] time = 1.97, size = 17, normalized size = 0.23

$$\frac{2\,bx^2\log(x) - a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b*x^2*log(x) - a)/x^2
```

giac [A] time = 0.23, size = 45, normalized size = 0.60

$$\frac{1}{2}\,b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right) - \frac{bx^2\mathrm{sgn}\left(bx^2+a\right) + a\mathrm{sgn}\left(bx^2+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
[Out] 1/2*b*log(x^2)*sgn(b*x^2 + a) - 1/2*(b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a
))/x^2
```

maple [A] time = 0.01, size = 38, normalized size = 0.51

$$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(2b\,x^2\ln(x) - a\right)}{2\left(b\,x^2+a\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^3,x)
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*b*ln(x)*x^2-a)/(b*x^2+a)/x^2
```

maxima [A] time = 1.40, size = 14, normalized size = 0.19

$$\frac{1}{2}\,b\log\left(x^2\right) - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] 1/2*b*log(x^2) - 1/2*a/x^2
```

mupad [B] time = 4.45, size = 112, normalized size = 1.49

$$\frac{\ln\left(a\,b+\sqrt{\left(b\,x^2+a\right)^2}\,\sqrt{b^2}+b^2\,x^2\right)\sqrt{b^2}}{2}-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^2}-\frac{a\,b\,\ln\left(a\,b+\frac{a^2}{x^2}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^2}\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^3,x)
```

```
[Out] (log(a*b + ((a + b*x^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2)*(b^2)^(1/2))/2 - (a
^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*x^2) - (a*b*log(a*b + a^2/x^2 + ((a^2)^(
1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/x^2))/(2*(a^2)^(1/2))
```

sympy [A] time = 0.15, size = 10, normalized size = 0.13

$$-\frac{a}{2x^2}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x)
```

3.374 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx$

Optimal. Leaf size=39

$$-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{ab+b^2x}{x^3}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$
$$= -\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.95

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a+2bx^2\right)}{4x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

```
[Out] -1/4*(Sqrt[(a + b*x^2)^2]*(a + 2*b*x^2))/(x^4*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 0.38, size = 118, normalized size = 3.03

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-ab-2b^2x^2\right)+\sqrt{b^2}\left(a^2+3abx^2+2b^2x^4\right)}{4x^4\left(ab+b^2x^2\right)-4\sqrt{b^2}\,x^4\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

```
[Out] ((-(a*b) - 2*b^2*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] + Sqrt[b^2]*(a^2 + 3*
a*b*x^2 + 2*b^2*x^4))/(4*x^4*(a*b + b^2*x^2) - 4*Sqrt[b^2]*x^4*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])
```

fricas [A] time = 0.91, size = 13, normalized size = 0.33

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

giac [A] time = 0.16, size = 30, normalized size = 0.77

$$-\frac{2\,bx^2\mathrm{sgn}\left(bx^2+a\right)+a\mathrm{sgn}\left(bx^2+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="giac")
[Out] -1/4*(2*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^4
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$-\frac{\left(2b\,x^2+a\right)\sqrt{\left(b\,x^2+a\right)^2}}{4\left(b\,x^2+a\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^5,x)
[Out] -1/4*(2*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.33

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

mupad [B] time = 4.21, size = 33, normalized size = 0.85

$$-\frac{\left(2\,b\,x^2+a\right)\sqrt{\left(b\,x^2+a\right)^2}}{4\,x^4\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^5,x)
[Out] -((a + 2*b*x^2)*((a + b*x^2)^2)^(1/2))/(4*x^4*(a + b*x^2))
```

sympy [A] time = 0.17, size = 14, normalized size = 0.36

$$\frac{-a-2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**5,x)
```

```
[Out] (-a - 2*b*x**2)/(4*x**4)
```

3.375 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1110}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^6) + (a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/2)/(12*a^2*x^6)
```

Rule 1110

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(4*a*d*(p + 1)*(2*p +
1)), x] - Simp[((d*x)^(m + 1)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^p)/(4*a*d*(
2*p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx=-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.54

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(2a+3bx^2\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^2)^2]*(2*a + 3*b*x^2))/(x^6*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 3.25, size = 751, normalized size = 10.43

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-2a^{15}b^3-55a^{14}b^4x^2-704a^{13}b^5x^4-5563a^{12}b^6x^6-30344a^{11}b^7x^8-121000a^{10}b^8x^{10}-364320a^9b^9x^{12}-843216a^8b^{10}x^{14}-1512192a^7b^{11}x^{16}-2100736a^6b^{12}x^{18}-2241536a^5b^{13}x^{20}-1803520a^4b^{14}x^{22}-1058816a^3b^{15}x^{24}-428032a^2b^{16}x^{26}-106496ab^{17}x^{28}-12288b^{18}x^{30}\right)+\sqrt{b^2}\left(2a^{16}b^2+57a^{15}b^3x^2+759a^{14}b^4x^4+6267a^{13}b^5x^6+35907a^{12}b^6x^8+151344a^{11}b^7x^{10}+485320a^{10}b^8x^{12}+1207536a^9b^9x^{14}+2355408a^8b^{10}x^{16}+3612928a^7b^{11}x^{18}+4342272a^6b^{12}x^{20}+4045056a^5b^{13}x^{22}+2862336a^4b^{14}x^{24}+1486848a^3b^{15}x^{26}+534528a^2b^{16}x^{28}+118784ab^{17}x^{30}+12288b^{18}x^{32}\right)}{3\sqrt{b^2}x^6\sqrt{a^2+2abx^2+b^2x^4}\left(-4a^{14}b^2-104a^{13}b^3x^2-1252a^{12}b^4x^4-9248a^{11}b^5x^6-46816a^{10}b^6x^8-171776a^9b^7x^{10}-470976a^8b^8x^{12}-979968a^7b^9x^{14}-1554432a^6b^{10}x^{16}-1869824a^5b^{11}x^{18}-1678336a^4b^{12}x^{20}-1089536a^3b^{13}x^{22}-483328a^2b^{14}x^{24}-131072ab^{15}x^{26}-16384b^{16}x^{28}\right)+3x^6\left(4a^{15}b^3+108a^{14}b^4x^2+1356a^{13}b^5x^4+10500a^{12}b^6x^6+56064a^{11}b^7x^8+218592a^{10}b^8x^{10}+642752a^9b^9x^{12}+1450944a^8b^{10}x^{14}+2534400a^7b^{11}x^{16}+3424256a^6b^{12}x^{18}+3548160a^5b^{13}x^{20}+2767872a^4b^{14}x^{22}+1572864a^3b^{15}x^{24}+614400a^2b^{16}x^{26}+147456ab^{17}x^{28}+16384b^{18}x^{30}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-2*a^15*b^3 - 55*a^14*b^4*x^2 - 704*a^13*
b^5*x^4 - 5563*a^12*b^6*x^6 - 30344*a^11*b^7*x^8 - 121000*a^10*b^8*x^10 - 3
64320*a^9*b^9*x^12 - 843216*a^8*b^10*x^14 - 1512192*a^7*b^11*x^16 - 2100736
*a^6*b^12*x^18 - 2241536*a^5*b^13*x^20 - 1803520*a^4*b^14*x^22 - 1058816*a^
3*b^15*x^24 - 428032*a^2*b^16*x^26 - 106496*a*b^17*x^28 - 12288*b^18*x^30)
+ Sqrt[b^2]*(2*a^16*b^2 + 57*a^15*b^3*x^2 + 759*a^14*b^4*x^4 + 6267*a^13*b^
5*x^6 + 35907*a^12*b^6*x^8 + 151344*a^11*b^7*x^10 + 485320*a^10*b^8*x^12 +
1207536*a^9*b^9*x^14 + 2355408*a^8*b^10*x^16 + 3612928*a^7*b^11*x^18 + 4342
272*a^6*b^12*x^20 + 4045056*a^5*b^13*x^22 + 2862336*a^4*b^14*x^24 + 1486848
*a^3*b^15*x^26 + 534528*a^2*b^16*x^28 + 118784*a*b^17*x^30 + 12288*b^18*x^3
2))/(3*Sqrt[b^2]*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-4*a^14*b^2 - 104*a^1
3*b^3*x^2 - 1252*a^12*b^4*x^4 - 9248*a^11*b^5*x^6 - 46816*a^10*b^6*x^8 - 17
1776*a^9*b^7*x^10 - 470976*a^8*b^8*x^12 - 979968*a^7*b^9*x^14 - 1554432*a^6
*b^10*x^16 - 1869824*a^5*b^11*x^18 - 1678336*a^4*b^12*x^20 - 1089536*a^3*b^
13*x^22 - 483328*a^2*b^14*x^24 - 131072*a*b^15*x^26 - 16384*b^16*x^28) + 3*
x^6*(4*a^15*b^3 + 108*a^14*b^4*x^2 + 1356*a^13*b^5*x^4 + 10500*a^12*b^6*x^6
 + 56064*a^11*b^7*x^8 + 218592*a^10*b^8*x^10 + 642752*a^9*b^9*x^12 + 145094
4*a^8*b^10*x^14 + 2534400*a^7*b^11*x^16 + 3424256*a^6*b^12*x^18 + 3548160*a
^5*b^13*x^20 + 2767872*a^4*b^14*x^22 + 1572864*a^3*b^15*x^24 + 614400*a^2*b
^16*x^26 + 147456*a*b^17*x^28 + 16384*b^18*x^30))
```

fricas [A] time = 0.87, size = 15, normalized size = 0.21

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

giac [A] time = 0.16, size = 31, normalized size = 0.43

$$-\frac{3\,bx^2\mathrm{sgn}\left(bx^2+a\right)+2\,a\mathrm{sgn}\left(bx^2+a\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2*sgn(b*x^2 + a) + 2*a*sgn(b*x^2 + a))/x^6
```

maple [A] time = 0.00, size = 36, normalized size = 0.50

$$-\frac{\left(3b\,x^2+2a\right)\sqrt{\left(b\,x^2+a\right)^2}}{12\left(b\,x^2+a\right)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^7,x)
```

```
[Out] -1/12*(3*b*x^2+2*a)*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)
```

maxima [A] time = 1.30, size = 15, normalized size = 0.21

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

mupad [B] time = 4.24, size = 35, normalized size = 0.49

$$-\frac{\left(3\,b\,x^2+2\,a\right)\,\sqrt{\left(b\,x^2+a\right)^2}}{12\,x^6\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^7,x)
```

```
[Out] -((2*a + 3*b*x^2)*((a + b*x^2)^2)^(1/2))/(12*x^6*(a + b*x^2))
```

sympy [A] time = 0.19, size = 15, normalized size = 0.21

$$\frac{-2a-3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**7,x)
```

```
[Out] (-2*a - 3*b*x**2)/(12*x**6)
```

3.376 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \frac{ab+b^2x}{x^5}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \left(\frac{ab}{x^5}+\frac{b^2}{x^4}\right)dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a+4bx^2\right)}{24x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] -1/24*(Sqrt[(a + b*x^2)^2]*(3*a + 4*b*x^2))/(x^8*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 0.58, size = 266, normalized size = 3.37

$$\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\left(-3a^4b-13a^3b^2x^2-21a^2b^3x^4-15ab^4x^6-4b^5x^8\right)+\sqrt{b^2}\,b^3\left(3a^5+16a^4bx^2+34a^3b^2x^4+36a^2b^3x^6+19ab^4x^8+4b^5x^{10}\right)}{3\sqrt{b^2}\,x^8\sqrt{a^2+2abx^2+b^2x^4}\left(-8a^3b^3-24a^2b^4x^2-24ab^5x^4-8b^6x^6\right)+3x^8\left(8a^4b^4+32a^3b^5x^2+48a^2b^6x^4+32ab^7x^6+8b^8x^8\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-3*a^4*b - 13*a^3*b^2*x^2 - 21*a^2*b^
3*x^4 - 15*a*b^4*x^6 - 4*b^5*x^8) + b^3*Sqrt[b^2]*(3*a^5 + 16*a^4*b*x^2 + 3
4*a^3*b^2*x^4 + 36*a^2*b^3*x^6 + 19*a*b^4*x^8 + 4*b^5*x^10))/(3*Sqrt[b^2]*x
^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-8*a^3*b^3 - 24*a^2*b^4*x^2 - 24*a*b^5*
x^4 - 8*b^6*x^6) + 3*x^8*(8*a^4*b^4 + 32*a^3*b^5*x^2 + 48*a^2*b^6*x^4 + 32*
a*b^7*x^6 + 8*b^8*x^8))
```

fricas [A] time = 0.82, size = 15, normalized size = 0.19

$$-\frac{4\,bx^2+3\,a}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="fricas")
[Out] -1/24*(4*b*x^2 + 3*a)/x^8
```

giac [A] time = 0.16, size = 31, normalized size = 0.39

$$-\frac{4\,bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a\mathrm{sgn}\left(bx^2+a\right)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="giac")
[Out] -1/24*(4*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^8
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(4b\,x^2+3a\right)\sqrt{\left(b\,x^2+a\right)^2}}{24\left(b\,x^2+a\right)x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^9,x)
[Out] -1/24*(4*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

maxima [A] time = 1.35, size = 15, normalized size = 0.19

$$-\frac{4\,bx^2+3\,a}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="maxima")
[Out] -1/24*(4*b*x^2 + 3*a)/x^8
```

mupad [B] time = 4.24, size = 35, normalized size = 0.44

$$-\frac{\left(4\,b\,x^2+3\,a\right)\,\sqrt{\left(b\,x^2+a\right)^2}}{24\,x^8\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^9,x)
[Out] -((3*a + 4*b*x^2)*((a + b*x^2)^2)^(1/2))/(24*x^8*(a + b*x^2))
```

sympy [A] time = 0.20, size = 15, normalized size = 0.19

$$\frac{-3a - 4bx^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**9,x)
[Out] (-3*a - 4*b*x**2)/(24*x**8)
```

3.377 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx,x,x^2\right)\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{ab+b^2x}{x^6}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{ab}{x^6}+\frac{b^2}{x^5}\right)dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\ &= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(4a+5bx^2\right)}{40x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^2)^2]*(4*a + 5*b*x^2))/(x^10*(a + b*x^2))
```

IntegrateAlgebraic [**B**] time = 0.64, size = 312, normalized size = 3.95

$$\frac{2b^4\sqrt{a^2+2abx^2+b^2x^4}\left(-4a^5b-21a^4b^2x^2-44a^3b^3x^4-46a^2b^4x^6-24ab^5x^8-5b^6x^{10}\right)+2\sqrt{b^2}\,b^4\left(4a^6+25a^5bx^2+65a^4b^2x^4+90a^3b^3x^6+70a^2b^4x^8+29ab^5x^{10}+5b^6x^{12}\right)}{5\sqrt{b^2}\,x^{10}\sqrt{a^2+2abx^2+b^2x^4}\left(-16a^4b^4-64a^3b^5x^2-96a^2b^6x^4-64ab^7x^6-16b^8x^8\right)+5x^{10}\left(16a^5b^5+80a^4b^6x^2+160a^3b^7x^4+160a^2b^8x^6+80ab^9x^8+16b^{10}x^{10}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] (2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-4*a^5*b - 21*a^4*b^2*x^2 - 44*a^3*
b^3*x^4 - 46*a^2*b^4*x^6 - 24*a*b^5*x^8 - 5*b^6*x^10) + 2*b^4*Sqrt[b^2]*(4*
a^6 + 25*a^5*b*x^2 + 65*a^4*b^2*x^4 + 90*a^3*b^3*x^6 + 70*a^2*b^4*x^8 + 29*
a*b^5*x^10 + 5*b^6*x^12))/(5*Sqrt[b^2]*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*(-16*a^4*b^4 - 64*a^3*b^5*x^2 - 96*a^2*b^6*x^4 - 64*a*b^7*x^6 - 16*b^8*x^8
) + 5*x^10*(16*a^5*b^5 + 80*a^4*b^6*x^2 + 160*a^3*b^7*x^4 + 160*a^2*b^8*x^6
 + 80*a*b^9*x^8 + 16*b^10*x^10))
```

fricas [A] time = 0.82, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+4\,a}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="fricas")
[Out] -1/40*(5*b*x^2 + 4*a)/x^10
```

giac [A] time = 0.16, size = 31, normalized size = 0.39

$$-\frac{5\,bx^2\mathrm{sgn}\left(bx^2+a\right)+4\,a\mathrm{sgn}\left(bx^2+a\right)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="giac")
[Out] -1/40*(5*b*x^2*sgn(b*x^2 + a) + 4*a*sgn(b*x^2 + a))/x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(5b\,x^2+4a\right)\sqrt{\left(b\,x^2+a\right)^2}}{40\left(b\,x^2+a\right)x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^11,x)
[Out] -1/40*(5*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

maxima [A] time = 1.31, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+4\,a}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="maxima")
[Out] -1/40*(5*b*x^2 + 4*a)/x^10
```

mupad [B] time = 4.21, size = 35, normalized size = 0.44

$$-\frac{\left(5\,b\,x^2+4\,a\right)\,\sqrt{\left(b\,x^2+a\right)^2}}{40\,x^{10}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^11,x)
```

```
[Out] -((4*a + 5*b*x^2)*((a + b*x^2)^2)^(1/2))/(40*x^10*(a + b*x^2))
```

sympy [A] time = 0.22, size = 15, normalized size = 0.19

$$\frac{-4a-5bx^2}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**11,x)
```

```
[Out] (-4*a - 5*b*x**2)/(40*x**10)
```

3.378 $\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(abx^4+b^2x^6\right)dx}{ab+b^2x^2}$$
$$= \frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)} + \frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(7ax^5+5bx^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(7*a*x^5 + 5*b*x^7))/(35*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 4.53, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(7ax^5+5bx^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(7*a*x^5 + 5*b*x^7))/(35*(a + b*x^2))
```

fricas [A] time = 0.77, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

giac [A] time = 0.17, size = 29, normalized size = 0.37

$$\frac{1}{7}\,bx^7\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{5}\,ax^5\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/7*b*x^7*sgn(b*x^2 + a) + 1/5*a*x^5*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(5b\,x^2+7a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x^5}{35b\,x^2+35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^2+a)^2)^(1/2),x)
[Out] 1/35*x^5*(5*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{\left(b\,x^2+a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a + b*x^2)^2)^(1/2),x)
[Out] int(x^4*((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.10, size = 12, normalized size = 0.15

$$\frac{ax^5}{5}+\frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**7/7
```

3.379 $\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(abx^2+b^2x^4\right)dx}{ab+b^2x^2}$$
$$= \frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Mathematica [**A**] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(5ax^3+3bx^5\right)}{15\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(5*a*x^3 + 3*b*x^5))/(15*(a + b*x^2))
```

IntegrateAlgebraic [**A**] time = 4.35, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(5ax^3+3bx^5\right)}{15\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(5*a*x^3 + 3*b*x^5))/(15*(a + b*x^2))
```

fricas [**A**] time = 1.37, size = 13, normalized size = 0.16

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

giac [A] time = 0.15, size = 29, normalized size = 0.37

$$\frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,ax^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/5*b*x^5*sgn(b*x^2 + a) + 1/3*a*x^3*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(3b\,x^2+5a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x^3}{15b\,x^2+15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^2+a)^2)^(1/2),x)
[Out] 1/15*x^3*(3*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.31, size = 13, normalized size = 0.16

$$\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{\left(b\,x^2+a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x^2)^2)^(1/2),x)
[Out] int(x^2*((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.15, size = 12, normalized size = 0.15

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

3.380 $\int \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=74

$$\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1088}

$$\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*x^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(2ab+2b^2x^2\right)dx}{2ab+2b^2x^2}\\ &= \frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax+bx^3\right)}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x + b*x^3))/(3*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 4.22, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax+bx^3\right)}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x + b*x^3))/(3*(a + b*x^2))
```

fricas [A] time = 0.76, size = 10, normalized size = 0.14

$$\frac{1}{3}bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*b*x^3 + a*x
```

giac [A] time = 0.18, size = 20, normalized size = 0.27

$$\frac{1}{3}\left(bx^3+3\,ax\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(b*x^3 + 3*a*x)*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 33, normalized size = 0.45

$$\frac{\left(b\,x^2+3a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x}{3b\,x^2+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*x*(b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.14

$$\frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*b*x^3 + a*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2),x)
[Out] int(((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.11

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2),x)
[Out] a*x + b*x**3/3
```

3.381 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal. Leaf size=72

$$\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (b*x*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^2}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2+\frac{ab}{x^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} + \frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.49

$$\frac{\left(bx^2-a\right)\sqrt{\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
[Out] ((-a + b*x^2)*Sqrt[(a + b*x^2)^2])/(x*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 8.06, size = 35, normalized size = 0.49

$$\frac{\left(bx^2-a\right)\sqrt{\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
[Out] ((-a + b*x^2)*Sqrt[(a + b*x^2)^2])/(x*(a + b*x^2))
```

fricas [A] time = 1.10, size = 13, normalized size = 0.18

$$\frac{bx^2-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] (b*x^2 - a)/x
```

giac [A] time = 0.16, size = 26, normalized size = 0.36

$$bx\operatorname{sgn}\left(bx^2+a\right)-\frac{a\operatorname{sgn}\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*x*sgn(b*x^2 + a) - a*sgn(b*x^2 + a)/x
```

maple [A] time = 0.00, size = 34, normalized size = 0.47

$$-\frac{\left(-b\,x^2+a\right)\sqrt{\left(b\,x^2+a\right)^2}}{\left(b\,x^2+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^2,x)
```

```
[Out] -(-b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/x
```

maxima [A] time = 1.34, size = 10, normalized size = 0.14

$$bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] b*x - a/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\left(b\,x^2+a\right)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^2,x)
```

```
[Out] int(((a + b*x^2)^2)^(1/2)/x^2, x)
```

sympy [A] time = 0.13, size = 5, normalized size = 0.07

$$-\frac{a}{x}+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a/x + b*x
```

3.382 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4} dx$

Optimal. Leaf size=77

$$-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^4}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^4}+\frac{b^2}{x^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.48

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a+3bx^2\right)}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
[Out] -1/3*(Sqrt[(a + b*x^2)^2]*(a + 3*b*x^2))/(x^3*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 13.39, size = 39, normalized size = 0.51

$$\frac{\left(-a-3bx^2\right)\sqrt{\left(a+bx^2\right)^2}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
[Out] ((-a - 3*b*x^2)*Sqrt[(a + b*x^2)^2])/(3*x^3*(a + b*x^2))
```

fricas [A] time = 0.59, size = 13, normalized size = 0.17

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/3*(3*b*x^2 + a)/x^3
```

giac [A] time = 0.17, size = 30, normalized size = 0.39

$$-\frac{3\,bx^2\mathrm{sgn}\left(bx^2+a\right)+a\mathrm{sgn}\left(bx^2+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^3
```

maple [A] time = 0.00, size = 34, normalized size = 0.44

$$-\frac{\left(3b\,x^2+a\right)\sqrt{\left(b\,x^2+a\right)^2}}{3\left(b\,x^2+a\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^4,x)
```

```
[Out] -1/3*(3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)
```

maxima [A] time = 1.38, size = 13, normalized size = 0.17

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

mupad [B] time = 4.24, size = 33, normalized size = 0.43

$$-\frac{\left(3\,b\,x^2+a\right)\,\sqrt{\left(b\,x^2+a\right)^2}}{3\,x^3\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^4,x)
```

```
[Out] -((a + 3*b*x^2)*((a + b*x^2)^2)^(1/2))/(3*x^3*(a + b*x^2))
```

sympy [A] time = 0.16, size = 14, normalized size = 0.18

$$\frac{-a-3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**4,x)
```

```
[Out] (-a - 3*b*x**2)/(3*x**3)
```

3.383 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^6}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^4}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a+5bx^2\right)}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
[Out] -1/15*(Sqrt[(a + b*x^2)^2]*(3*a + 5*b*x^2))/(x^5*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 16.71, size = 39, normalized size = 0.49

$$\frac{\left(-3a-5bx^2\right)\sqrt{\left(a+bx^2\right)^2}}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
[Out] ((-3*a - 5*b*x^2)*Sqrt[(a + b*x^2)^2])/(15*x^5*(a + b*x^2))
```

fricas [A] time = 0.87, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="fricas")
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

giac [A] time = 0.15, size = 31, normalized size = 0.39

$$-\frac{5\,bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a\mathrm{sgn}\left(bx^2+a\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(5b\,x^2+3a\right)\sqrt{\left(b\,x^2+a\right)^2}}{15\left(b\,x^2+a\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^6,x)
```

```
[Out] -1/15*(5*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)
```

maxima [A] time = 1.31, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

mupad [B] time = 4.21, size = 35, normalized size = 0.44

$$-\frac{\left(5\,b\,x^2+3\,a\right)\,\sqrt{\left(b\,x^2+a\right)^2}}{15\,x^5\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^6,x)
```

```
[Out] -((3*a + 5*b*x^2)*((a + b*x^2)^2)^(1/2))/(15*x^5*(a + b*x^2))
```

sympy [A] time = 0.18, size = 15, normalized size = 0.19

$$\frac{-3a-5bx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**6,x)
```

```
[Out] (-3*a - 5*b*x**2)/(15*x**5)
```

3.384 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^8}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^8}+\frac{b^2}{x^6}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(5a+7bx^2\right)}{35x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
[Out] -1/35*(Sqrt[(a + b*x^2)^2]*(5*a + 7*b*x^2))/(x^7*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 19.61, size = 39, normalized size = 0.49

$$\frac{\left(-5a-7bx^2\right)\sqrt{\left(a+bx^2\right)^2}}{35x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
[Out] ((-5*a - 7*b*x^2)*Sqrt[(a + b*x^2)^2])/(35*x^7*(a + b*x^2))
```

fricas [A] time = 1.40, size = 15, normalized size = 0.19

$$-\frac{7\,bx^2+5\,a}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="fricas")
[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

giac [A] time = 0.16, size = 31, normalized size = 0.39

$$-\frac{7 \, b x^{2} \mathrm{sgn}\left(b x^{2} + a\right) + 5 \, a \mathrm{sgn}\left(b x^{2} + a\right)}{35 \, x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="giac")
[Out] -1/35*(7*b*x^2*sgn(b*x^2 + a) + 5*a*sgn(b*x^2 + a))/x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(7 b \,x^{2}+5 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{35 \left(b \,x^{2}+a \right) x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^8,x)
[Out] -1/35*(7*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)
```

maxima [A] time = 1.27, size = 15, normalized size = 0.19

$$-\frac{7 \, b x^{2} + 5 \, a}{35 \, x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="maxima")
[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

mupad [B] time = 4.18, size = 35, normalized size = 0.44

$$-\frac{\left(7\,b\,x^2+5\,a\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{35\,x^7\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^8,x)
[Out] -((5*a + 7*b*x^2)*((a + b*x^2)^2)^(1/2))/(35*x^7*(a + b*x^2))
```

sympy [A] time = 0.20, size = 15, normalized size = 0.19

$$\frac{-5a-7bx^2}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**8,x)
```

```
[Out] (-5*a - 7*b*x**2)/(35*x**7)
```

3.385 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^{10}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^{10}}+\frac{b^2}{x^8}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(7a+9bx^2\right)}{63x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
[Out] -1/63*(Sqrt[(a + b*x^2)^2]*(7*a + 9*b*x^2))/(x^9*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 21.36, size = 39, normalized size = 0.49

$$\frac{\left(-7a-9bx^2\right)\sqrt{\left(a+bx^2\right)^2}}{63x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
[Out] ((-7*a - 9*b*x^2)*Sqrt[(a + b*x^2)^2])/(63*x^9*(a + b*x^2))
```

fricas [A] time = 0.85, size = 15, normalized size = 0.19

$$-\frac{9\,bx^2+7\,a}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="fricas")
[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

giac [A] time = 0.16, size = 31, normalized size = 0.39

$$-\frac{9\,bx^2\mathrm{sgn}\left(bx^2+a\right)+7\,a\mathrm{sgn}\left(bx^2+a\right)}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="giac")
[Out] -1/63*(9*b*x^2*sgn(b*x^2 + a) + 7*a*sgn(b*x^2 + a))/x^9
```

maple [A] time = 0.01, size = 36, normalized size = 0.46

$$-\frac{\left(9b\,x^2+7a\right)\sqrt{\left(b\,x^2+a\right)^2}}{63\left(b\,x^2+a\right)x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^10,x)
[Out] -1/63*(9*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

maxima [A] time = 1.29, size = 15, normalized size = 0.19

$$-\frac{9\,bx^2+7\,a}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="maxima")
[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

mupad [B] time = 4.20, size = 35, normalized size = 0.44

$$-\frac{\left(9\,b\,x^2+7\,a\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{63\,x^9\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^10,x)
[Out] -((7*a + 9*b*x^2)*((a + b*x^2)^2)^(1/2))/(63*x^9*(a + b*x^2))
```

sympy [A] time = 0.22, size = 15, normalized size = 0.19

$$\frac{-7a-9bx^2}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**10,x)
```

```
[Out] (-7*a - 9*b*x**2)/(63*x**9)
```

3.386 $\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}$$

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a^2*b*x^12*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a*b^2*x^14*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(16*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^{3/2}dx=\frac{1}{2}\text{Subst}\left(\int x^4\left(a^2+2abx+b^2x^2\right)^{3/2}dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^4\left(ab+b^2x\right)^3dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(a^3b^3x^4+3a^2b^4x^5+3ab^5x^6+b^6x^7\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$=\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2ab}}{14\left(a+bx\right.}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{\left(a+bx^2\right)^2}\left(56a^3+140a^2bx^2+120ab^2x^4+35b^3x^6\right)}{560\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(56*a^3 + 140*a^2*b*x^2 + 120*a*b^2*x^4 + 35*b^3*
x^6))/(560*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 12.31, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{\left(a+bx^2\right)^2}\left(56a^3+140a^2bx^2+120ab^2x^4+35b^3x^6\right)}{560\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(56*a^3 + 140*a^2*b*x^2 + 120*a*b^2*x^4 + 35*b^3*
x^6))/(560*(a + b*x^2))
```

fricas [A] time = 0.80, size = 35, normalized size = 0.21

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{16}\,b^{3}x^{16}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{3}{14}\,ab^{2}x^{14}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{1}{4}\,a^{2}bx^{12}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{1}{10}\,a^{3}x^{10}\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16*sgn(b*x^2 + a) + 3/14*a*b^2*x^14*sgn(b*x^2 + a) + 1/4*a^2*b*x
^12*sgn(b*x^2 + a) + 1/10*a^3*x^10*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(35b^{3}x^{6}+120a\,b^{2}x^{4}+140a^{2}b\,x^{2}+56a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x^{10}}{560\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/560*x^10*(35*b^3*x^6+120*a*b^2*x^4+140*a^2*b*x^2+56*a^3)*((b*x^2+a)^2)^(3
/2)/(b*x^2+a)^3
```

maxima [A] time = 1.31, size = 35, normalized size = 0.21

$$\frac{1}{16}\,b^{3}x^{16}+\frac{3}{14}\,ab^{2}x^{14}+\frac{1}{4}\,a^{2}bx^{12}+\frac{1}{10}\,a^{3}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral(x**9*((a + b*x**2)**2)**(3/2), x)
```

3.387 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a*b^2*x^12*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(14*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{2}\operatorname{Subst}\left(\int x^3\left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int x^3\left(ab+b^2x\right)^3 dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(a^3b^3x^3+3a^2b^4x^4+3ab^5x^5+b^6x^6\right) dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)} + \frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)} + \frac{ab^2x^{12}\sqrt{a^2+2ab}}{4\left(a+bx\right.}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{\left(a+bx^2\right)^2}\left(35a^3+84a^2bx^2+70ab^2x^4+20b^3x^6\right)}{280\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(35*a^3 + 84*a^2*b*x^2 + 70*a*b^2*x^4 + 20*b^3*x^6
))/(280*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 10.51, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{\left(a+bx^2\right)^2}\left(35a^3+84a^2bx^2+70ab^2x^4+20b^3x^6\right)}{280\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(35*a^3 + 84*a^2*b*x^2 + 70*a*b^2*x^4 + 20*b^3*x^6
))/(280*(a + b*x^2))
```

fricas [A] time = 0.79, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14} + \frac{1}{4}\,ab^2x^{12} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{14}\,b^3x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,ab^2x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{10}\,a^2bx^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{8}\,a^3x^8\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sgn(b*x^2 + a) + 1/4*a*b^2*x^12*sgn(b*x^2 + a) + 3/10*a^2*b*x
^10*sgn(b*x^2 + a) + 1/8*a^3*x^8*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(20b^3x^6+70a\,b^2x^4+84a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^8}{280\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/280*x^8*(20*b^3*x^6+70*a*b^2*x^4+84*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

maxima [A] time = 1.32, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(3/2), x)
```

3.388 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=106

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^3}-\frac{a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{5b^3}+\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^3}$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1111, 645}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^4}{5b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^3}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^2*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^3) - (a*(a + b*x^2
)^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*b^3) + ((a + b*x^2)^5*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(12*b^3)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x^2 \left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(\frac{a^2(ab+b^2x)^3}{b^2} - \frac{2a(ab+b^2x)^4}{b^3} + \frac{(ab+b^2x)^5}{b^4}\right) dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8b^3} - \frac{a\left(a+bx^2\right)^4\sqrt{a^2+2abx^2+b^2x^4}}{5b^3} + \frac{(a+$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.58

$$\frac{x^6\sqrt{\left(a+bx^2\right)^2}\left(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6\right)}{120\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3*x^6
))/(120*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 9.45, size = 61, normalized size = 0.58

$$\frac{x^6\sqrt{\left(a+bx^2\right)^2}\left(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6\right)}{120\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3*x^6
))/(120*(a + b*x^2))
```

fricas [A] time = 0.81, size = 35, normalized size = 0.33

$$\frac{1}{12}b^3x^{12} + \frac{3}{10}ab^2x^{10} + \frac{3}{8}a^2bx^8 + \frac{1}{6}a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

giac [A] time = 0.22, size = 67, normalized size = 0.63

$$\frac{1}{12}\,b^3x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{10}\,ab^2x^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{8}\,a^2bx^8\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{6}\,a^3x^6\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12*sgn(b*x^2 + a) + 3/10*a*b^2*x^10*sgn(b*x^2 + a) + 3/8*a^2*b*x
^8*sgn(b*x^2 + a) + 1/6*a^3*x^6*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.55

$$\frac{\left(10b^3x^6+36a\,b^2x^4+45a^2b\,x^2+20a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^6}{120\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

maxima [A] time = 1.34, size = 35, normalized size = 0.33

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(\left(a + bx^2\right)^2 \right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**2)**2)**(3/2), x)
```

3.389 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 609}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b^2) + (a^2 + 2*a*b*x
^2 + b^2*x^4)^(5/2)/(10*b^2)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x\left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2} - \frac{a\,\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)}{2b}$$
$$= -\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2} + \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.91

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6\right)}{40\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*x^6)
)/(40*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 8.59, size = 61, normalized size = 0.91

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6\right)}{40\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*x^6)
)/(40*(a + b*x^2))
```

fricas [A] time = 0.90, size = 35, normalized size = 0.52

$$\frac{1}{10}\,b^3x^{10} + \frac{3}{8}\,ab^2x^8 + \frac{1}{2}\,a^2bx^6 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

giac [A] time = 0.15, size = 45, normalized size = 0.67

$$\frac{1}{40} \left(4 b^3 x^{10} + 15 a b^2 x^8 + 20 a^2 b x^6 + 10 a^3 x^4\right) \operatorname{sgn}\left(b x^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.87

$$\frac{\left(4b^3x^6 + 15a\,b^2x^4 + 20a^2b\,x^2 + 10a^3\right)\left(\left(b\,x^2 + a\right)^2\right)^{\frac{3}{2}} x^4}{40\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3
```

maxima [A] time = 1.33, size = 35, normalized size = 0.52

$$\frac{1}{10} b^3 x^{10} + \frac{3}{8} a b^2 x^8 + \frac{1}{2} a^2 b x^6 + \frac{1}{4} a^3 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

mupad [B] time = 4.29, size = 46, normalized size = 0.69

$$\frac{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}\,\left(-a^2 + 3\,a\,b\,x^2 + 4\,b^2\,x^4\right)}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)*(4*b^2*x^4 - a^2 + 3*a*b*x^2))/(40*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**3*((a + b*x**2)**2)**(3/2), x)
```

3.390 $\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,x^2\right)\\&=\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^(3/2))/(8*b)
```

IntegrateAlgebraic [A] time = 8.06, size = 60, normalized size = 1.67

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(4a^3x^2+6a^2bx^4+4ab^2x^6+b^3x^8\right)}{8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(4*a^3*x^2 + 6*a^2*b*x^4 + 4*a*b^2*x^6 + b^3*x^8))/(8*
(a + b*x^2))
```

fricas [A] time = 0.79, size = 35, normalized size = 0.97

$$\frac{1}{8}\,b^3x^8+\frac{1}{2}\,ab^2x^6+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

giac [A] time = 0.15, size = 44, normalized size = 1.22

$$\frac{1}{8}\left(2\left(bx^4+2\,ax^2\right)a^2+\left(bx^4+2\,ax^2\right)^2b\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^4 + 2*a*x^2)*a^2 + (b*x^4 + 2*a*x^2)^2*b)*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 57, normalized size = 1.58

$$\frac{\left(b^3x^6+4a\,b^2x^4+6a^2b\,x^2+4a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^2}{8\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*x^2*(b^3*x^6+4*a*b^2*x^4+6*a^2*b*x^2+4*a^3)*((b*x^2+a)^2)^(3/2)/(b*x^2+
a)^3
```

maxima [A] time = 1.36, size = 35, normalized size = 0.97

$$\frac{1}{8}\,b^3x^8+\frac{1}{2}\,ab^2x^6+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

mupad [B] time = 4.25, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^2+a\,b\right)\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] ((a*b + b^2*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2))/(8*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(3/2), x)
```

3.391 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x} \, dx$

Optimal. Leaf size=163

$$\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^2+b}}{a+bx^2}$$

Rubi [A] time = 0.05, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*x^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^6*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*
Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{b^3x^6\sqrt{a^2+2abx^2+}}{6\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(12a^3\log(x)+bx^2\left(18a^2+9abx^2+2b^2x^4\right)\right)}{12\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*Log[x
]))/(12*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.38, size = 256, normalized size = 1.57

$$\frac{1}{24}\sqrt{a^2+2abx^2+b^2x^4}\left(11a^2+7abx^2+2b^2x^4\right)+\frac{1}{24}\left(-18a^2\sqrt{b^2}\,x^2-9ab\sqrt{b^2}\,x^4-2\left(b^2\right)^{3/2}x^6\right)+\frac{1}{4}a^3\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)-\frac{a^3\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{4b}-\frac{a^3\sqrt{b^2}\log\left(b\sqrt{a^2+2abx^2+b^2x^4}-ab-b\sqrt{b^2}\,x^2\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(11*a^2 + 7*a*b*x^2 + 2*b^2*x^4))/24 + (-1
8*a^2*Sqrt[b^2]*x^2 - 9*a*b*Sqrt[b^2]*x^4 - 2*(b^2)^(3/2)*x^6)/24 + (a^3*Lo
g[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/4 - (a^3*(b + Sqrt
[b^2])*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*b) - (a
^3*Sqrt[b^2]*Log[-(a*b) - b*Sqrt[b^2]*x^2 + b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]])/(4*b)
```

fricas [A] time = 0.64, size = 33, normalized size = 0.20

$$\frac{1}{6}\,b^3x^6+\frac{3}{4}\,ab^2x^4+\frac{3}{2}\,a^2bx^2+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

giac [A] time = 0.19, size = 68, normalized size = 0.42

$$\frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x^2 + a) + 3/4*a*b^2*x^4*sgn(b*x^2 + a) + 3/2*a^2*b*x^2*s
gn(b*x^2 + a) + 1/2*a^3*log(x^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 57, normalized size = 0.35

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^6+9a\,b^2x^4+18a^2b\,x^2+12a^3\ln(x)\right)}{12\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+9*a*b^2*x^4+18*a^2*b*x^2+12*a^3*ln(x))/
(b*x^2+a)^3
```

maxima [A] time = 1.33, size = 33, normalized size = 0.20

$$\frac{1}{6}\,b^3x^6+\frac{3}{4}\,ab^2x^4+\frac{3}{2}\,a^2bx^2+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x, x)
```

3.392 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=164

$$\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

Rubi [A] time = 0.05, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (3*a*b^2*x^2*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (b^3*x^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^3}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^2}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)} + \frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)} + \frac{b^3x^4\sqrt{a^2+2abx^2+b^2x}}{4\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-2a^3+12a^2bx^2\log(x)+6ab^2x^4+b^3x^6\right)}{4x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^3 + 6*a*b^2*x^4 + b^3*x^6 + 12*a^2*b*x^2*Log[x])
)/(4*x^2*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.66, size = 320, normalized size = 1.95

$$-\frac{3}{4}a^2\sqrt{b^2}\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}x^2\right)-\frac{3}{4}a^2\sqrt{b^2}\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)+\frac{3}{2}a^2b\tanh^{-1}\left(\frac{\sqrt{b^2}x^2}{a}-\frac{\sqrt{a^2+2abx^2+b^2x^4}}{a}\right)+\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-8a^3b-21a^2b^2x^2+24ab^3x^4+4b^4x^6\right)+\sqrt{b^2}\left(8a^4+29a^3bx^2-3a^2b^2x^4-28ab^3x^6-4b^4x^8\right)}{16x^2\left(ab+b^2x^2\right)-16\sqrt{b^2}x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-8*a^3*b - 21*a^2*b^2*x^2 + 24*a*b^3*x^4
+ 4*b^4*x^6) + Sqrt[b^2]*(8*a^4 + 29*a^3*b*x^2 - 3*a^2*b^2*x^4 - 28*a*b^3*x
^6 - 4*b^4*x^8))/(16*x^2*(a*b + b^2*x^2) - 16*Sqrt[b^2]*x^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) + (3*a^2*b*ArcTanh[(Sqrt[b^2]*x^2)/a - Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]/a])/2 - (3*a^2*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]])/4 - (3*a^2*Sqrt[b^2]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]])/4
```

fricas [A] time = 0.81, size = 38, normalized size = 0.23

$$\frac{b^3x^6 + 6\,ab^2x^4 + 12\,a^2bx^2\log(x) - 2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

giac [A] time = 0.16, size = 87, normalized size = 0.53

$$\frac{1}{4}\,b^3x^4\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{2}\,ab^2x^2\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{2}\,a^2b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right) - \frac{3\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right) + a^3\mathrm{sgn}\left(bx^2+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sgn(b*x^2 + a) + 3/2*a*b^2*x^2*sgn(b*x^2 + a) + 3/2*a^2*b*log(x
^2)*sgn(b*x^2 + a) - 1/2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/
x^2
```

maple [A] time = 0.01, size = 59, normalized size = 0.36

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(b^3x^6 + 6a\,b^2x^4 + 12a^2b\,x^2\ln(x) - 2a^3\right)}{4\left(b\,x^2+a\right)^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x)
```

```
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(b^3*x^6+6*a*b^2*x^4+12*a^2*b*ln(x)*x^2-2*a^3)/(b*x
^2+a)^3/x^2
```

maxima [A] time = 1.29, size = 34, normalized size = 0.21

$$\frac{1}{4}\,b^3x^4 + \frac{3}{2}\,ab^2x^2 + 3\,a^2b\log(x) - \frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^3,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**3,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**3, x)
```

3.393 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=164

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}$$

Rubi [A] time = 0.05, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*x^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\frac{\left(ab+b^2x^2\right)^3}{x^5}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\frac{\left(ab+b^2x\right)^3}{x^3}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^3+6a^2bx^2-12ab^2x^4\log(x)-2b^3x^6\right)}{4x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] -1/4*(Sqrt[(a + b*x^2)^2]*(a^3 + 6*a^2*b*x^2 - 2*b^3*x^6 - 12*a*b^2*x^4*Log
[x]))/(x^4*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.55, size = 1170, normalized size = 7.13

$$-\frac{3b^2\log\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)a^5}{4\left(-\sqrt{b^2}x^2-a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2}-\frac{3b\sqrt{b^2}\log\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)a^5}{4\left(-\sqrt{b^2}x^2-a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2}+\frac{3b^2\left(\sqrt{b^2x^4+2abx^2+a^2}-\sqrt{b^2}x^2\right)^2\log\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)a^3}{2\left(-\sqrt{b^2}x^2-a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2}+\frac{3b\sqrt{b^2}\left(\sqrt{b^2x^4+2abx^2+a^2}-\sqrt{b^2}x^2\right)^2\log\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)a^3}{2\left(-\sqrt{b^2}x^2-a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2}+\frac{3}{4}b^2\log\left(-\sqrt{b^2}x^2-a+\sqrt{b^2x^4+2abx^2+a^2}\right)a-\frac{3}{4}b\sqrt{b^2}\log\left(-\sqrt{b^2}x^2-a+\sqrt{b^2x^4+2abx^2+a^2}\right)a-\frac{3b^2\left(\sqrt{b^2x^4+2abx^2+a^2}-\sqrt{b^2}x^2\right)^4\log\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)a}{4\left(-\sqrt{b^2}x^2-a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2}-\frac{3b\sqrt{b^2}\left(\sqrt{b^2x^4+2abx^2+a^2}-\sqrt{b^2}x^2\right)^4\log\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)a}{4\left(-\sqrt{b^2}x^2-a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2\left(-\sqrt{b^2}x^2+a+\sqrt{b^2x^4+2abx^2+a^2}\right)^2}+\frac{\sqrt{b^2x^4+2abx^2+a^2}\left(2b^4x^6+ab^3x^4-6a^2b^2x^2-a^3b\right)+\sqrt{b^2}\left(-2b^4x^8-3ab^3x^6+5a^2b^2x^4+7a^3bx^2+a^4\right)}{4x^4\left(b^2x^2+ab\right)-4\sqrt{b^2}x^4\sqrt{b^2x^4+2abx^2+a^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^3*b) - 6*a^2*b^2*x^2 + a*b^3*x^4 + 2*
b^4*x^6) + Sqrt[b^2]*(a^4 + 7*a^3*b*x^2 + 5*a^2*b^2*x^4 - 3*a*b^3*x^6 - 2*b
^4*x^8))/(4*x^4*(a*b + b^2*x^2) - 4*Sqrt[b^2]*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) + (3*a*b^2*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
])/4 - (3*a*b*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]])/4 - (3*a^5*b^2*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*(a - Sqrt
```

```
[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2) - (3*a^5*b*Sqrt[b^2]*Log[a
- Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt[b^2]*x^2
+ Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])^2) + (3*a^3*b^2*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])^2*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(2*(-
a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*(a - Sqrt[b^2]*x^2 +
 Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2) + (3*a^3*b*Sqrt[b^2]*(-(Sqrt[b^2]*x^2)
 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]])/(2*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])^2*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2) - (3*a*b^2*(
-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4*Log[a - Sqrt[b^2]*x^2
 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])^2*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
^2) - (3*a*b*Sqrt[b^2]*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
^4*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt[
b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*(a - Sqrt[b^2]*x^2 + Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])^2)
```

fricas [A] time = 1.48, size = 39, normalized size = 0.24

$$\frac{2\,b^3x^6+12\,ab^2x^4\log(x)-6\,a^2bx^2-a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

giac [A] time = 0.17, size = 87, normalized size = 0.53

$$\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}\,ab^2\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{9\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+6\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^2 + a) + 3/2*a*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(9*a*b
^2*x^4*sgn(b*x^2 + a) + 6*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^
4
```

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^6+12a\,b^2x^4\ln(x)-6a^2b\,x^2-a^3\right)}{4\left(b\,x^2+a\right)^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x)
```

```
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+12*a*b^2*ln(x)*x^4-6*a^2*b*x^2-a^3)/(b*x
^2+a)^3/x^4
```

maxima [A] time = 1.31, size = 34, normalized size = 0.21

$$\frac{1}{2}b^3x^2 + 3ab^2\log(x) - \frac{3a^2b}{2x^2} - \frac{a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2 + 3*a*b^2*log(x) - 3/2*a^2*b/x^2 - 1/4*a^3/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + 2abx^2 + b^2x^4\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**5, x)
```

3.394 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^7} dx$

Optimal. Leaf size=163

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}$$

Rubi [A] time = 0.05, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^7}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^4}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.39

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a\left(2a^2+9abx^2+18b^2x^4\right)-12b^3x^6\log(x)\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^2)^2]*(a*(2*a^2 + 9*a*b*x^2 + 18*b^2*x^4) - 12*b^3*x^6
*Log[x]))/(x^6*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 4.94, size = 944, normalized size = 5.79

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-2*a^17*b^3 - 61*a^16*b^4*x^2 - 878*a^15*
b^5*x^4 - 7925*a^14*b^6*x^6 - 50266*a^13*b^7*x^8 - 237848*a^12*b^8*x^10 - 8
69648*a^11*b^9*x^12 - 2509936*a^10*b^10*x^14 - 5788640*a^9*b^11*x^16 - 1072
6144*a^8*b^12*x^18 - 15961088*a^7*b^13*x^20 - 18952960*a^6*b^14*x^22 - 1772
4928*a^5*b^15*x^24 - 12769280*a^4*b^16*x^26 - 6836224*a^3*b^17*x^28 - 25600
00*a^2*b^18*x^30 - 598016*a*b^19*x^32 - 65536*b^20*x^34) + Sqrt[b^2]*(2*a^1
```

```
8*b^2 + 63*a^17*b^3*x^2 + 939*a^16*b^4*x^4 + 8803*a^15*b^5*x^6 + 58191*a^14
*b^6*x^8 + 288114*a^13*b^7*x^10 + 1107496*a^12*b^8*x^12 + 3379584*a^11*b^9*
x^14 + 8298576*a^10*b^10*x^16 + 16514784*a^9*b^11*x^18 + 26687232*a^8*b^12*
x^20 + 34914048*a^7*b^13*x^22 + 36677888*a^6*b^14*x^24 + 30494208*a^5*b^15*
x^26 + 19605504*a^4*b^16*x^28 + 9396224*a^3*b^17*x^30 + 3158016*a^2*b^18*x^
32 + 663552*a*b^19*x^34 + 65536*b^20*x^36))/(3*Sqrt[b^2]*x^6*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]*(-4*a^14*b^2 - 104*a^13*b^3*x^2 - 1252*a^12*b^4*x^4 - 924
8*a^11*b^5*x^6 - 46816*a^10*b^6*x^8 - 171776*a^9*b^7*x^10 - 470976*a^8*b^8*
x^12 - 979968*a^7*b^9*x^14 - 1554432*a^6*b^10*x^16 - 1869824*a^5*b^11*x^18
- 1678336*a^4*b^12*x^20 - 1089536*a^3*b^13*x^22 - 483328*a^2*b^14*x^24 - 13
1072*a*b^15*x^26 - 16384*b^16*x^28) + 3*x^6*(4*a^15*b^3 + 108*a^14*b^4*x^2
+ 1356*a^13*b^5*x^4 + 10500*a^12*b^6*x^6 + 56064*a^11*b^7*x^8 + 218592*a^10
*b^8*x^10 + 642752*a^9*b^9*x^12 + 1450944*a^8*b^10*x^14 + 2534400*a^7*b^11*
x^16 + 3424256*a^6*b^12*x^18 + 3548160*a^5*b^13*x^20 + 2767872*a^4*b^14*x^2
2 + 1572864*a^3*b^15*x^24 + 614400*a^2*b^16*x^26 + 147456*a*b^17*x^28 + 163
84*b^18*x^30)) + (b^3*ArcTanh[(Sqrt[b^2]*x^2)/a - Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]/a])/2 - ((b^2)^(3/2)*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]])/4 - ((b^2)^(3/2)*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]])/4
```

fricas [A] time = 0.63, size = 39, normalized size = 0.24

$$\frac{12\, b^{3} x^{6} \log\left(x\right) - 18\, a b^{2} x^{4} - 9\, a^{2} b x^{2} - 2\, a^{3}}{12\, x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

giac [A] time = 0.19, size = 87, normalized size = 0.53

$$\frac{1}{2}\, b^{3} \log\left(x^{2}\right) \mathrm{sgn}\left(b x^{2} + a\right) - \frac{11\, b^{3} x^{6} \mathrm{sgn}\left(b x^{2} + a\right) + 18\, a b^{2} x^{4} \mathrm{sgn}\left(b x^{2} + a\right) + 9\, a^{2} b x^{2} \mathrm{sgn}\left(b x^{2} + a\right) + 2\, a^{3} \mathrm{sgn}\left(b x^{2} + a\right)}{12\, x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(x^2)*sgn(b*x^2 + a) - 1/12*(11*b^3*x^6*sgn(b*x^2 + a) + 18*a*b^
2*x^4*sgn(b*x^2 + a) + 9*a^2*b*x^2*sgn(b*x^2 + a) + 2*a^3*sgn(b*x^2 + a))/x
^6
```

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(12b^{3}x^{6}\ln(x)-18a\,b^{2}x^{4}-9a^{2}b\,x^{2}-2a^{3}\right)}{12\left(b\,x^{2}+a\right)^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x)
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(12*b^3*ln(x)*x^6-18*a*b^2*x^4-9*a^2*b*x^2-2*a^3)/
(b*x^2+a)^3/x^6
```

maxima [A] time = 1.26, size = 33, normalized size = 0.20

$$b^{3}\log(x)-\frac{3\,ab^{2}}{2\,x^{2}}-\frac{3\,a^{2}b}{4\,x^{4}}-\frac{a^{3}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="maxima")
[Out] b^3*log(x) - 3/2*a*b^2/x^2 - 3/4*a^2*b/x^4 - 1/6*a^3/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{3/2}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^7,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**7,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**7, x)
```

3.395 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$-\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] -((a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*a*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^5}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.44

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^3+4a^2bx^2+6ab^2x^4+4b^3x^6\right)}{8x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/8*(Sqrt[(a + b*x^2)^2]*(a^3 + 4*a^2*b*x^2 + 6*a*b^2*x^4 + 4*b^3*x^6))/(x
^8*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.19, size = 306, normalized size = 7.46

$$\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\left(-a^6b-7a^5b^2x^2-21a^4b^3x^4-35a^3b^4x^6-34a^2b^5x^8-18ab^6x^{10}-4b^7x^{12}\right)+\sqrt{b^2}\,b^3\left(a^7+8a^6bx^2+28a^5b^2x^4+56a^4b^3x^6+69a^3b^4x^8+52a^2b^5x^{10}+22ab^6x^{12}+4b^7x^{14}\right)}{\sqrt{b^2}\,x^8\sqrt{a^2+2abx^2+b^2x^4}\left(-8a^3b^3-24a^2b^4x^2-24ab^5x^4-8b^6x^6\right)+x^8\left(8a^4b^4+32a^3b^5x^2+48a^2b^6x^4+32ab^7x^6+8b^8x^8\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^6*b) - 7*a^5*b^2*x^2 - 21*a^4*b^3
*x^4 - 35*a^3*b^4*x^6 - 34*a^2*b^5*x^8 - 18*a*b^6*x^10 - 4*b^7*x^12) + b^3*
Sqrt[b^2]*(a^7 + 8*a^6*b*x^2 + 28*a^5*b^2*x^4 + 56*a^4*b^3*x^6 + 69*a^3*b^4
*x^8 + 52*a^2*b^5*x^10 + 22*a*b^6*x^12 + 4*b^7*x^14))/(Sqrt[b^2]*x^8*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]*(-8*a^3*b^3 - 24*a^2*b^4*x^2 - 24*a*b^5*x^4 - 8*b
^6*x^6) + x^8*(8*a^4*b^4 + 32*a^3*b^5*x^2 + 48*a^2*b^6*x^4 + 32*a*b^7*x^6 +
 8*b^8*x^8))
```

fricas [A] time = 0.76, size = 35, normalized size = 0.85

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

giac [B] time = 0.17, size = 68, normalized size = 1.66

$$-\frac{4\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+6\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+4\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+a\right)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6*sgn(b*x^2 + a) + 6*a*b^2*x^4*sgn(b*x^2 + a) + 4*a^2*b*x^2*s
gn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^8
```

maple [A] time = 0.00, size = 56, normalized size = 1.37

$$-\frac{\left(4b^3x^6+6a\,b^2x^4+4a^2b\,x^2+a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{8\left(b\,x^2+a\right)^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x)
```

```
[Out] -1/8*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^8/(b*x^2
+a)^3
```

maxima [A] time = 1.36, size = 35, normalized size = 0.85

$$-\frac{b^3}{2\,x^2}-\frac{3\,ab^2}{4\,x^4}-\frac{a^2b}{2\,x^6}-\frac{a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/2*b^3/x^2 - 3/4*a*b^2/x^4 - 1/2*a^2*b/x^6 - 1/8*a^3/x^8
```

mupad [B] time = 4.24, size = 151, normalized size = 3.68

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^2\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\left(b\,x^2+a\right)}-\frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^6\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^9,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (b^3*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*x^2*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(2*x^6*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**9, x)
```

3.396 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1110}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*a*x^10) + (a^2 + 2*a*b*
x^2 + b^2*x^4)^(5/2)/(40*a^2*x^10)
```

Rule 1110

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(4*a*d*(p + 1)*(2*p +
1)), x] - Simp[((d*x)^(m + 1)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^p)/(4*a*d*(
2*p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}\,dx=-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.85

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(4a^3+15a^2bx^2+20ab^2x^4+10b^3x^6\right)}{40x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^2)^2]*(4*a^3 + 15*a^2*b*x^2 + 20*a*b^2*x^4 + 10*b^3*x^
6))/(x^10*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 0.95, size = 356, normalized size = 4.94

$$\frac{2b^4\sqrt{a^2+2abx^2+b^2x^4}\left(-4a^7b-31a^6b^2x^2-104a^5b^3x^4-196a^4b^4x^6-224a^3b^5x^8-155a^2b^6x^{10}-60ab^7x^{12}-10b^8x^{14}\right)+2\sqrt{b^2}\,b^4\left(4a^8+35a^7bx^2+135a^6b^2x^4+300a^5b^3x^6+420a^4b^4x^8+379a^3b^5x^{10}+215a^2b^6x^{12}+70ab^7x^{14}+10b^8x^{16}\right)}{5\sqrt{b^2}\,x^{10}\sqrt{a^2+2abx^2+b^2x^4}\left(-16a^4b^4-64a^3b^5x^2-96a^2b^6x^4-64ab^7x^6-16b^8x^8\right)+5x^{10}\left(16a^5b^5+80a^4b^6x^2+160a^3b^7x^4+160a^2b^8x^6+80ab^9x^8+16b^{10}x^{10}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] (2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-4*a^7*b - 31*a^6*b^2*x^2 - 104*a^5
*b^3*x^4 - 196*a^4*b^4*x^6 - 224*a^3*b^5*x^8 - 155*a^2*b^6*x^10 - 60*a*b^7*
x^12 - 10*b^8*x^14) + 2*b^4*Sqrt[b^2]*(4*a^8 + 35*a^7*b*x^2 + 135*a^6*b^2*x
^4 + 300*a^5*b^3*x^6 + 420*a^4*b^4*x^8 + 379*a^3*b^5*x^10 + 215*a^2*b^6*x^1
2 + 70*a*b^7*x^14 + 10*b^8*x^16))/(5*Sqrt[b^2]*x^10*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]*(-16*a^4*b^4 - 64*a^3*b^5*x^2 - 96*a^2*b^6*x^4 - 64*a*b^7*x^6 - 16
*b^8*x^8) + 5*x^10*(16*a^5*b^5 + 80*a^4*b^6*x^2 + 160*a^3*b^7*x^4 + 160*a^2
*b^8*x^6 + 80*a*b^9*x^8 + 16*b^10*x^10))
```

fricas [A] time = 0.85, size = 37, normalized size = 0.51

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

giac [A] time = 0.21, size = 69, normalized size = 0.96

$$-\frac{10\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+20\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+15\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+4\,a^3\mathrm{sgn}\left(bx^2+a\right)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6*sgn(b*x^2 + a) + 20*a*b^2*x^4*sgn(b*x^2 + a) + 15*a^2*b*x
^2*sgn(b*x^2 + a) + 4*a^3*sgn(b*x^2 + a))/x^10
```

maple [A] time = 0.01, size = 58, normalized size = 0.81

$$-\frac{\left(10b^{3}x^{6}+20a\,b^{2}x^{4}+15a^{2}b\,x^{2}+4a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{40\left(b\,x^{2}+a\right)^{3}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x)
```

```
[Out] -1/40*(10*b^3*x^6+20*a*b^2*x^4+15*a^2*b*x^2+4*a^3)*((b*x^2+a)^2)^(3/2)/x^10
/(b*x^2+a)^3
```

maxima [A] time = 1.43, size = 35, normalized size = 0.49

$$-\frac{b^{3}}{4\,x^{4}}-\frac{ab^{2}}{2\,x^{6}}-\frac{3\,a^{2}b}{8\,x^{8}}-\frac{a^{3}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/4*b^3/x^4 - 1/2*a*b^2/x^6 - 3/8*a^2*b/x^8 - 1/10*a^3/x^10
```

mupad [B] time = 4.20, size = 151, normalized size = 2.10

$$-\frac{a^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{10\,x^{10}\,\left(b\,x^{2}+a\right)}-\frac{b^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{4\,x^{4}\,\left(b\,x^{2}+a\right)}-\frac{a\,b^{2}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,x^{6}\,\left(b\,x^{2}+a\right)}-\frac{3\,a^{2}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{8\,x^{8}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^11,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(2*x^6*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(8*x^8*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**11, x)
```

3.397 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}$$

Rubi [A] time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(6*x^6*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{13}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx,x,x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^7}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \left(\frac{a^3b^3}{x^7}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^5}+\frac{b^6}{x^4}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(10a^3+36a^2bx^2+45ab^2x^4+20b^3x^6\right)}{120x^{12}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

```
[Out] -1/120*(Sqrt[(a + b*x^2)^2]*(10*a^3 + 36*a^2*b*x^2 + 45*a*b^2*x^4 + 20*b^3*
x^6))/(x^12*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.05, size = 400, normalized size = 2.40

$$\frac{4b^5\sqrt{a^2+2abx^2+b^2x^4}\left(-10a^8b-86a^7b^2x^2-325a^6b^3x^4-705a^5b^4x^6-960a^4b^5x^8-840a^3b^6x^{10}-461a^2b^7x^{12}-145ab^8x^{14}-20b^9x^{16}\right)+4\sqrt{b^2}\,b^5\left(10a^9+96a^8bx^2+411a^7b^2x^4+1030a^6b^3x^6+1665a^5b^4x^8+1800a^4b^5x^{10}+1301a^3b^6x^{12}+606a^2b^7x^{14}+165ab^8x^{16}+20b^9x^{18}\right)}{15\sqrt{b^2}\,x^{12}\sqrt{a^2+2abx^2+b^2x^4}\left(-32a^5b^5-160a^4b^6x^2-320a^3b^7x^4-320a^2b^8x^6-160ab^9x^8-32b^{10}x^{10}\right)+15x^{12}\left(32a^6b^6+192a^5b^7x^2+480a^4b^8x^4+640a^3b^9x^6+480a^2b^{10}x^8+192ab^{11}x^{10}+32b^{12}x^{12}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

```
[Out] (4*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-10*a^8*b - 86*a^7*b^2*x^2 - 325*a^
6*b^3*x^4 - 705*a^5*b^4*x^6 - 960*a^4*b^5*x^8 - 840*a^3*b^6*x^10 - 461*a^2*
b^7*x^12 - 145*a*b^8*x^14 - 20*b^9*x^16) + 4*b^5*Sqrt[b^2]*(10*a^9 + 96*a^8
```

```
*b*x^2 + 411*a^7*b^2*x^4 + 1030*a^6*b^3*x^6 + 1665*a^5*b^4*x^8 + 1800*a^4*b
^5*x^10 + 1301*a^3*b^6*x^12 + 606*a^2*b^7*x^14 + 165*a*b^8*x^16 + 20*b^9*x^
18))/(15*Sqrt[b^2]*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-32*a^5*b^5 - 160*
a^4*b^6*x^2 - 320*a^3*b^7*x^4 - 320*a^2*b^8*x^6 - 160*a*b^9*x^8 - 32*b^10*x
^10) + 15*x^12*(32*a^6*b^6 + 192*a^5*b^7*x^2 + 480*a^4*b^8*x^4 + 640*a^3*b^
9*x^6 + 480*a^2*b^10*x^8 + 192*a*b^11*x^10 + 32*b^12*x^12))
```

fricas [A] time = 0.97, size = 37, normalized size = 0.22

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

giac [A] time = 0.16, size = 69, normalized size = 0.41

$$-\frac{20\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+45\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+36\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+10\,a^3\mathrm{sgn}\left(bx^2+a\right)}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(20*b^3*x^6*sgn(b*x^2 + a) + 45*a*b^2*x^4*sgn(b*x^2 + a) + 36*a^2*b*
x^2*sgn(b*x^2 + a) + 10*a^3*sgn(b*x^2 + a))/x^12
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(20b^3x^6+45a\,b^2x^4+36a^2b\,x^2+10a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{120\left(b\,x^2+a\right)^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x)
```

```
[Out] -1/120*(20*b^3*x^6+45*a*b^2*x^4+36*a^2*b*x^2+10*a^3)*((b*x^2+a)^2)^(3/2)/x^
12/(b*x^2+a)^3
```

maxima [A] time = 1.31, size = 35, normalized size = 0.21

$$-\frac{b^3}{6\,x^6}-\frac{3\,ab^2}{8\,x^8}-\frac{3\,a^2b}{10\,x^{10}}-\frac{a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -1/6*b^3/x^6 - 3/8*a*b^2/x^8 - 3/10*a^2*b/x^10 - 1/12*a^3/x^12
```

mupad [B] time = 4.21, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^13,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(10*x^10*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**13, x)
```

3.398 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}$$

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(8*x^8*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{15}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^8}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^7}+\frac{3ab^5}{x^6}+\frac{b^6}{x^5}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$
-\frac{\sqrt{\left(a+bx^2\right)^2}\left(20a^3+70a^2bx^2+84ab^2x^4+35b^3x^6\right)}{280x^{14}\left(a+bx^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] -1/280*(Sqrt[(a + b*x^2)^2]*(20*a^3 + 70*a^2*b*x^2 + 84*a*b^2*x^4 + 35*b^3*
x^6))/(x^14*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.17, size = 444, normalized size = 2.66

$$
\frac{8b^6\sqrt{a^2+2abx^2+b^2x^4}\left(-20a^9b-190a^8b^2x^2-804a^7b^3x^4-1989a^6b^4x^6-3170a^5b^5x^8-3375a^4b^6x^{10}-2400a^3b^7x^{12}-1099a^2b^8x^{14}-294ab^9x^{16}-35b^{10}x^{18}\right)+8\sqrt{b^2}\,b^6\left(20a^{10}+210a^9bx^2+994a^8b^2x^4+2793a^7b^3x^6+5159a^6b^4x^8+6545a^5b^5x^{10}+5775a^4b^6x^{12}+3499a^3b^7x^{14}+1393a^2b^8x^{16}+329ab^9x^{18}+35b^{10}x^{20}\right)}{35\sqrt{b^2}\,x^{14}\sqrt{a^2+2abx^2+b^2x^4}\left(-64a^6b^6-384a^5b^7x^2-960a^4b^8x^4-1280a^3b^9x^6-960a^2b^{10}x^8-384ab^{11}x^{10}-64b^{12}x^{12}\right)+35x^{14}\left(64a^7b^7+448a^6b^8x^2+1344a^5b^9x^4+2240a^4b^{10}x^6+2240a^3b^{11}x^8+1344a^2b^{12}x^{10}+448ab^{13}x^{12}+64b^{14}x^{14}\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] (8*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-20*a^9*b - 190*a^8*b^2*x^2 - 804*a
^7*b^3*x^4 - 1989*a^6*b^4*x^6 - 3170*a^5*b^5*x^8 - 3375*a^4*b^6*x^10 - 2400
*a^3*b^7*x^12 - 1099*a^2*b^8*x^14 - 294*a*b^9*x^16 - 35*b^10*x^18) + 8*b^6*
```

```
Sqrt[b^2]*(20*a^10 + 210*a^9*b*x^2 + 994*a^8*b^2*x^4 + 2793*a^7*b^3*x^6 + 5
159*a^6*b^4*x^8 + 6545*a^5*b^5*x^10 + 5775*a^4*b^6*x^12 + 3499*a^3*b^7*x^14
 + 1393*a^2*b^8*x^16 + 329*a*b^9*x^18 + 35*b^10*x^20))/(35*Sqrt[b^2]*x^14*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-64*a^6*b^6 - 384*a^5*b^7*x^2 - 960*a^4*b^8
*x^4 - 1280*a^3*b^9*x^6 - 960*a^2*b^10*x^8 - 384*a*b^11*x^10 - 64*b^12*x^12
) + 35*x^14*(64*a^7*b^7 + 448*a^6*b^8*x^2 + 1344*a^5*b^9*x^4 + 2240*a^4*b^1
0*x^6 + 2240*a^3*b^11*x^8 + 1344*a^2*b^12*x^10 + 448*a*b^13*x^12 + 64*b^14*
x^14))
```

fricas [A] time = 1.08, size = 37, normalized size = 0.22

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

giac [A] time = 0.16, size = 69, normalized size = 0.41

$$-\frac{35\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+84\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+70\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+20\,a^3\mathrm{sgn}\left(bx^2+a\right)}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/280*(35*b^3*x^6*sgn(b*x^2 + a) + 84*a*b^2*x^4*sgn(b*x^2 + a) + 70*a^2*b*
x^2*sgn(b*x^2 + a) + 20*a^3*sgn(b*x^2 + a))/x^14
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(35b^3x^6+84a\,b^2x^4+70a^2b\,x^2+20a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{280\left(b\,x^2+a\right)^3x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x)
```

```
[Out] -1/280*(35*b^3*x^6+84*a*b^2*x^4+70*a^2*b*x^2+20*a^3)*((b*x^2+a)^2)^(3/2)/x^
14/(b*x^2+a)^3
```

maxima [A] time = 1.13, size = 35, normalized size = 0.21

$$-\frac{b^3}{8\,x^8}-\frac{3\,ab^2}{10\,x^{10}}-\frac{a^2b}{4\,x^{12}}-\frac{a^3}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/8*b^3/x^8 - 3/10*a*b^2/x^10 - 1/4*a^2*b/x^12 - 1/14*a^3/x^14
```

mupad [B] time = 4.21, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}-\frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{12}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^15,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(4*x^12*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**15, x)
```

3.399 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}$$

Rubi [A] time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(10*x^10*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{17}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^9}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \left(\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^7}+\frac{b^6}{x^6}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^3+120a^2bx^2+140ab^2x^4+56b^3x^6\right)}{560x^{16}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] -1/560*(Sqrt[(a + b*x^2)^2]*(35*a^3 + 120*a^2*b*x^2 + 140*a*b^2*x^4 + 56*b^
3*x^6))/(x^16*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.28, size = 488, normalized size = 2.92

$$\frac{8b^7\sqrt{a^2+2abx^2+b^2x^4}\left(-35a^{10}b-365a^9b^2x^2-1715a^8b^3x^4-4781a^7b^4x^6-8757a^6b^5x^8-11011a^5b^6x^{10}-9625a^4b^7x^{12}-5775a^3b^8x^{14}-2276a^2b^9x^{16}-532ab^{10}x^{18}-56b^{11}x^{20}\right)+8\sqrt{b^2}\,b^7\left(35a^{11}+400a^{10}bx^2+2080a^9b^2x^4+6496a^8b^3x^6+13538a^7b^4x^8+19768a^6b^5x^{10}+20636a^5b^6x^{12}+15400a^4b^7x^{14}+8051a^3b^8x^{16}+2808a^2b^9x^{18}+588ab^{10}x^{20}+56b^{11}x^{22}\right)}{35\sqrt{b^2}\,x^{16}\sqrt{a^2+2abx^2+b^2x^4}\left(-128a^7b^7-896a^6b^8x^2-2688a^5b^9x^4-4480a^4b^{10}x^6-4480a^3b^{11}x^8-2688a^2b^{12}x^{10}-896ab^{13}x^{12}-128b^{14}x^{14}\right)+35x^{16}\left(128a^8b^8+1024a^7b^9x^2+3584a^6b^{10}x^4+7168a^5b^{11}x^6+8960a^4b^{12}x^8+7168a^3b^{13}x^{10}+3584a^2b^{14}x^{12}+1024ab^{15}x^{14}+128b^{16}x^{16}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] (8*b^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-35*a^10*b - 365*a^9*b^2*x^2 - 1715
*a^8*b^3*x^4 - 4781*a^7*b^4*x^6 - 8757*a^6*b^5*x^8 - 11011*a^5*b^6*x^10 - 9
625*a^4*b^7*x^12 - 5775*a^3*b^8*x^14 - 2276*a^2*b^9*x^16 - 532*a*b^10*x^18
```

```
- 56*b^11*x^20) + 8*b^7*Sqrt[b^2]*(35*a^11 + 400*a^10*b*x^2 + 2080*a^9*b^2*
x^4 + 6496*a^8*b^3*x^6 + 13538*a^7*b^4*x^8 + 19768*a^6*b^5*x^10 + 20636*a^5
*b^6*x^12 + 15400*a^4*b^7*x^14 + 8051*a^3*b^8*x^16 + 2808*a^2*b^9*x^18 + 58
8*a*b^10*x^20 + 56*b^11*x^22))/(35*Sqrt[b^2]*x^16*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]*(-128*a^7*b^7 - 896*a^6*b^8*x^2 - 2688*a^5*b^9*x^4 - 4480*a^4*b^10*x
^6 - 4480*a^3*b^11*x^8 - 2688*a^2*b^12*x^10 - 896*a*b^13*x^12 - 128*b^14*x^
14) + 35*x^16*(128*a^8*b^8 + 1024*a^7*b^9*x^2 + 3584*a^6*b^10*x^4 + 7168*a^
5*b^11*x^6 + 8960*a^4*b^12*x^8 + 7168*a^3*b^13*x^10 + 3584*a^2*b^14*x^12 +
1024*a*b^15*x^14 + 128*b^16*x^16))
```

fricas [A] time = 0.83, size = 37, normalized size = 0.22

$$-\frac{56\,b^{3}x^{6}+140\,ab^{2}x^{4}+120\,a^{2}bx^{2}+35\,a^{3}}{560\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/560*(56*b^3*x^6 + 140*a*b^2*x^4 + 120*a^2*b*x^2 + 35*a^3)/x^16
```

giac [A] time = 0.18, size = 69, normalized size = 0.41

$$-\frac{56\,b^{3}x^{6}\mathrm{sgn}\left(bx^{2}+a\right)+140\,ab^{2}x^{4}\mathrm{sgn}\left(bx^{2}+a\right)+120\,a^{2}bx^{2}\mathrm{sgn}\left(bx^{2}+a\right)+35\,a^{3}\mathrm{sgn}\left(bx^{2}+a\right)}{560\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/560*(56*b^3*x^6*sgn(b*x^2 + a) + 140*a*b^2*x^4*sgn(b*x^2 + a) + 120*a^2*
b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^16
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(56b^{3}x^{6}+140a\,b^{2}x^{4}+120a^{2}b\,x^{2}+35a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{560\left(b\,x^{2}+a\right)^{3}x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x)
```

```
[Out] -1/560*(56*b^3*x^6+140*a*b^2*x^4+120*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)/
x^16/(b*x^2+a)^3
```

maxima [A] time = 1.32, size = 35, normalized size = 0.21

$$-\frac{b^3}{10\,x^{10}}-\frac{ab^2}{4\,x^{12}}-\frac{3\,a^2b}{14\,x^{14}}-\frac{a^3}{16\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/10*b^3/x^10 - 1/4*a*b^2/x^12 - 3/14*a^2*b/x^14 - 1/16*a^3/x^16
```

mupad [B] time = 4.23, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}-\frac{a\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{12}\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^17,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(4*x^12*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(14*x^14*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**17, x)
```

3.400 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (b^3*x^15*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(15*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^8 \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3x^8+3a^2b^4x^{10}+3ab^5x^{12}+b^6x^{14}\right) dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)} + \frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)} + \frac{3ab^2x^{13}\sqrt{a^2+2a}}{13\left(a+b\right.}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^9\sqrt{\left(a+bx^2\right)^2}\left(715a^3+1755a^2bx^2+1485ab^2x^4+429b^3x^6\right)}{6435\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^2)^2]*(715*a^3 + 1755*a^2*b*x^2 + 1485*a*b^2*x^4 + 429*b
^3*x^6))/(6435*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 7.30, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(715a^3x^9+1755a^2bx^{11}+1485ab^2x^{13}+429b^3x^{15}\right)}{6435\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(715*a^3*x^9 + 1755*a^2*b*x^11 + 1485*a*b^2*x^13 + 429
*b^3*x^15))/(6435*(a + b*x^2))
```

fricas [A] time = 0.80, size = 35, normalized size = 0.21

$$\frac{1}{15}b^3x^{15} + \frac{3}{13}ab^2x^{13} + \frac{3}{11}a^2bx^{11} + \frac{1}{9}a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{15}\,b^3x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{13}\,ab^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{11}\,a^2bx^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{9}\,a^3x^9\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*b^3*x^15*sgn(b*x^2 + a) + 3/13*a*b^2*x^13*sgn(b*x^2 + a) + 3/11*a^2*b*
x^11*sgn(b*x^2 + a) + 1/9*a^3*x^9*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(429b^3x^6+1485a\,b^2x^4+1755a^2b\,x^2+715a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^9}{6435\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/6435*x^9*(429*b^3*x^6+1485*a*b^2*x^4+1755*a^2*b*x^2+715*a^3)*((b*x^2+a)^2
)^(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.30, size = 35, normalized size = 0.21

$$\frac{1}{15}\,b^3x^{15}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**2)**2)**(3/2), x)
```

3.401 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a^2*b*x^9*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a*b^2*x^11*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(13*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^6 \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3x^6+3a^2b^4x^8+3ab^5x^{10}+b^6x^{12}\right) dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)} + \frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{3ab^2x^{11}\sqrt{a^2+2abx}}{11\left(a+bx^2\right.}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{\left(a+bx^2\right)^2}\left(429a^3+1001a^2bx^2+819ab^2x^4+231b^3x^6\right)}{3003\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(429*a^3 + 1001*a^2*b*x^2 + 819*a*b^2*x^4 + 231*b^
3*x^6))/(3003*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 6.46, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(429a^3x^7+1001a^2bx^9+819ab^2x^{11}+231b^3x^{13}\right)}{3003\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(429*a^3*x^7 + 1001*a^2*b*x^9 + 819*a*b^2*x^11 + 231*b
^3*x^13))/(3003*(a + b*x^2))
```

fricas [A] time = 0.81, size = 35, normalized size = 0.21

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{13}\,b^3x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{11}\,ab^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^2bx^9\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{7}\,a^3x^7\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sgn(b*x^2 + a) + 3/11*a*b^2*x^11*sgn(b*x^2 + a) + 1/3*a^2*b*x
^9*sgn(b*x^2 + a) + 1/7*a^3*x^7*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(231b^3x^6+819a\,b^2x^4+1001a^2b\,x^2+429a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^7}{3003\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/3003*x^7*(231*b^3*x^6+819*a*b^2*x^4+1001*a^2*b*x^2+429*a^3)*((b*x^2+a)^2)
^(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.27, size = 35, normalized size = 0.21

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(\left(a + bx^2 \right)^2 \right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(3/2), x)
```

3.402 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a*b^2*x^9*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(11*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^4\left(ab+b^2x^2\right)^3\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int\left(a^3b^3x^4+3a^2b^4x^6+3ab^5x^8+b^6x^{10}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ab^2x^9\sqrt{a^2+2abx^2}}{3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{\left(a+bx^2\right)^2}\left(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6\right)}{1155\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3
*x^6))/(1155*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 5.88, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(231a^3x^5+495a^2bx^7+385ab^2x^9+105b^3x^{11}\right)}{1155\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(231*a^3*x^5 + 495*a^2*b*x^7 + 385*a*b^2*x^9 + 105*b^3
*x^11))/(1155*(a + b*x^2))
```

fricas [A] time = 0.83, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

giac [A] time = 0.18, size = 67, normalized size = 0.40

$$\frac{1}{11}\,b^3x^{11}\text{sgn}\left(bx^2+a\right)+\frac{1}{3}\,ab^2x^9\text{sgn}\left(bx^2+a\right)+\frac{3}{7}\,a^2bx^7\text{sgn}\left(bx^2+a\right)+\frac{1}{5}\,a^3x^5\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*sgn(b*x^2 + a) + 1/3*a*b^2*x^9*sgn(b*x^2 + a) + 3/7*a^2*b*x^7
*sgn(b*x^2 + a) + 1/5*a^3*x^5*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(105b^3x^6+385a\,b^2x^4+495a^2b\,x^2+231a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^5}{1155\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*((b*x^2+a)^2)^
(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.39, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\left(a + b x^2 \right)^2 \right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral(x**4*((a + b*x**2)**2)**(3/2), x)
```

3.403 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a*b^2*x^7*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(9*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^2 \left(ab+b^2x^2\right)^3 dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3x^2+3a^2b^4x^4+3ab^5x^6+b^6x^8\right) dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)} + \frac{3ab^2x^7\sqrt{a^2+2abx}}{7\left(a+bx^2\right.}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2} \left(105a^3x^3+189a^2bx^5+135ab^2x^7+35b^3x^9\right)}{315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*b^3*
x^9))/(315*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 5.53, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2} \left(105a^3x^3+189a^2bx^5+135ab^2x^7+35b^3x^9\right)}{315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*b^3*
x^9))/(315*(a + b*x^2))
```

fricas [A] time = 0.53, size = 35, normalized size = 0.21

$$\frac{1}{9}b^3x^9 + \frac{3}{7}ab^2x^7 + \frac{3}{5}a^2bx^5 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{9}\,b^3x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{7}\,ab^2x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{5}\,a^2bx^5\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^3x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sgn(b*x^2 + a) + 3/7*a*b^2*x^7*sgn(b*x^2 + a) + 3/5*a^2*b*x^5*s
gn(b*x^2 + a) + 1/3*a^3*x^3*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^3}{315\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*((b*x^2+a)^2)^(3
/2)/(b*x^2+a)^3
```

maxima [A] time = 1.35, size = 35, normalized size = 0.21

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**(3/2), x)
```

3.404 $\int \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$\frac{3ab^2x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2+2abx^2+b^2\right.}{\left(a+bx^2\right)^3}$$

Rubi [A] time = 0.03, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1088, 194}

$$\frac{b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{3ab^2x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*x^3*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x^5*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(3/2))/(7*(a + b*x^2)^3)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2} \int \left(2ab+2b^2x^2\right)^3 dx}{\left(2ab+2b^2x^2\right)^3}$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2} \int \left(8a^3b^3+24a^2b^4x^2+24ab^5x^4+8b^6x^6\right) dx}{\left(2ab+2b^2x^2\right)^3}$$
$$= \frac{a^3x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{3ab^2x^5\left(a^2+2abx\right.}{5\left(a+bx\right.}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^3x+35a^2bx^3+21ab^2x^5+5b^3x^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^7))/
(35*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 5.30, size = 59, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^3x+35a^2bx^3+21ab^2x^5+5b^3x^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^7))/
(35*(a + b*x^2))
```

fricas [A] time = 0.79, size = 31, normalized size = 0.19

$$\frac{1}{7}b^3x^7 + \frac{3}{5}ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

giac [A] time = 0.18, size = 63, normalized size = 0.40

$$\frac{1}{7}\,b^3x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{5}\,ab^2x^5\mathrm{sgn}\left(bx^2+a\right)+a^2bx^3\mathrm{sgn}\left(bx^2+a\right)+a^3x\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sgn(b*x^2 + a) + 3/5*a*b^2*x^5*sgn(b*x^2 + a) + a^2*b*x^3*sgn(b
*x^2 + a) + a^3*x*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 56, normalized size = 0.35

$$\frac{\left(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x}{35\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)/(b*
x^2+a)^3
```

maxima [A] time = 1.36, size = 31, normalized size = 0.19

$$\frac{1}{7}\,b^3x^7+\frac{3}{5}\,ab^2x^5+a^2bx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2), x)
```

3.405 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^2} dx$

Optimal. Leaf size=158

$$\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (3*a^2*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (a*b^2*x^3*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(a + b*x^2) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a +
b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^2}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(3a^2b^4+\frac{a^3b^3}{x^2}+3ab^5x^2+b^6x^4\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b}}{a+bx^2}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6\right)}{5x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(5*x*
(a + b*x^2))
```

IntegrateAlgebraic [A] time = 9.71, size = 60, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6\right)}{5x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(5*x*
(a + b*x^2))
```

fricas [A] time = 0.61, size = 36, normalized size = 0.23

$$\frac{b^3x^6+5\,ab^2x^4+15\,a^2bx^2-5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

giac [A] time = 0.16, size = 64, normalized size = 0.41

$$\frac{1}{5}\,b^3x^5\mathrm{sgn}\left(bx^2+a\right)+ab^2x^3\mathrm{sgn}\left(bx^2+a\right)+3\,a^2bx\mathrm{sgn}\left(bx^2+a\right)-\frac{a^3\mathrm{sgn}\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sgn(b*x^2 + a) + a*b^2*x^3*sgn(b*x^2 + a) + 3*a^2*b*x*sgn(b*x^2
 + a) - a^3*sgn(b*x^2 + a)/x
```

maple [A] time = 0.01, size = 58, normalized size = 0.37

$$-\frac{\left(-b^3x^6-5a\,b^2x^4-15a^2b\,x^2+5a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{5\left(b\,x^2+a\right)^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2)/x/(b*x^2
+a)^3
```

maxima [A] time = 1.34, size = 32, normalized size = 0.20

$$\frac{1}{5}\,b^3x^5+ab^2x^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^2,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**2,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**2, x)
```

3.406 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=161

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (3*a*b^2*x*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(a + b*x^2) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(
a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^4}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(3ab^5+\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^2}+b^6x^2\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} + \frac{3ab^2x\sqrt{a^2+2abx^2+b^2}}{a+bx^2}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^3+9a^2bx^2-9ab^2x^4-b^3x^6\right)}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -1/3*(Sqrt[(a + b*x^2)^2]*(a^3 + 9*a^2*b*x^2 - 9*a*b^2*x^4 - b^3*x^6))/(x^3
*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 14.48, size = 60, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-a^3-9a^2bx^2+9ab^2x^4+b^3x^6\right)}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-a^3 - 9*a^2*b*x^2 + 9*a*b^2*x^4 + b^3*x^6))/(3*x^3*(
a + b*x^2))
```

fricas [A] time = 0.74, size = 36, normalized size = 0.22

$$\frac{b^3x^6+9\,ab^2x^4-9\,a^2bx^2-a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

giac [A] time = 0.16, size = 67, normalized size = 0.42

$$\frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^2+a\right)+3\,ab^2x\mathrm{sgn}\left(bx^2+a\right)-\frac{9\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x^2 + a) + 3*a*b^2*x*sgn(b*x^2 + a) - 1/3*(9*a^2*b*x^2*sg
n(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^3
```

maple [A] time = 0.01, size = 56, normalized size = 0.35

$$-\frac{\left(-b^3x^6-9a\,b^2x^4+9a^2b\,x^2+a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{3\left(b\,x^2+a\right)^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x)
```

```
[Out] -1/3*(-b^3*x^6-9*a*b^2*x^4+9*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^3/(b*x^2+
a)^3
```

maxima [A] time = 1.29, size = 33, normalized size = 0.20

$$\frac{1}{3}\,b^3x^3+3\,ab^2x-\frac{3\,a^2b}{x}-\frac{a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 3*a^2*b/x - 1/3*a^3/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**4, x)
```

3.407 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^6} dx$

Optimal. Leaf size=158

$$-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(x*(a + b*x^2)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a +
 b*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^6}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^6+\frac{a^3b^3}{x^6}+\frac{3a^2b^4}{x^4}+\frac{3ab^5}{x^2}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^3+5a^2bx^2+15ab^2x^4-5b^3x^6\right)}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] -1/5*(Sqrt[(a + b*x^2)^2]*(a^3 + 5*a^2*b*x^2 + 15*a*b^2*x^4 - 5*b^3*x^6))/(
x^5*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 17.50, size = 61, normalized size = 0.39

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-a^3-5a^2bx^2-15ab^2x^4+5b^3x^6\right)}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-a^3 - 5*a^2*b*x^2 - 15*a*b^2*x^4 + 5*b^3*x^6))/(5*x^
5*(a + b*x^2))
```

fricas [A] time = 0.81, size = 37, normalized size = 0.23

$$\frac{5\,b^3x^6-15\,ab^2x^4-5\,a^2bx^2-a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

giac [A] time = 0.19, size = 66, normalized size = 0.42

$$b^3x\text{sgn}\left(bx^2+a\right)-\frac{15\,ab^2x^4\text{sgn}\left(bx^2+a\right)+5\,a^2bx^2\text{sgn}\left(bx^2+a\right)+a^3\text{sgn}\left(bx^2+a\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] b^3*x*sgn(b*x^2 + a) - 1/5*(15*a*b^2*x^4*sgn(b*x^2 + a) + 5*a^2*b*x^2*sgn(b
*x^2 + a) + a^3*sgn(b*x^2 + a))/x^5
```

maple [A] time = 0.01, size = 56, normalized size = 0.35

$$-\frac{\left(-5b^3x^6+15a\,b^2x^4+5a^2b\,x^2+a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{5\left(b\,x^2+a\right)^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x)
```

```
[Out] -1/5*(-5*b^3*x^6+15*a*b^2*x^4+5*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^5/(b*x
^2+a)^3
```

maxima [A] time = 1.32, size = 32, normalized size = 0.20

$$b^3x-\frac{3\,ab^2}{x}-\frac{a^2b}{x^3}-\frac{a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] b^3*x - 3*a*b^2/x - a^2*b/x^3 - 1/5*a^3/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**6, x)
```

3.408 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=163

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x
*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^8}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^4}+\frac{b^6}{x^2}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(5a^3+21a^2bx^2+35ab^2x^4+35b^3x^6\right)}{35x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/35*(Sqrt[(a + b*x^2)^2]*(5*a^3 + 21*a^2*b*x^2 + 35*a*b^2*x^4 + 35*b^3*x^
6))/(x^7*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 17.90, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-5a^3-21a^2bx^2-35ab^2x^4-35b^3x^6\right)}{35x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-5*a^3 - 21*a^2*b*x^2 - 35*a*b^2*x^4 - 35*b^3*x^6))/(
35*x^7*(a + b*x^2))
```

fricas [A] time = 0.77, size = 37, normalized size = 0.23

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

giac [A] time = 0.22, size = 69, normalized size = 0.42

$$-\frac{35\,b^{3}x^{6}\mathrm{sgn}\left(bx^{2}+a\right)+35\,ab^{2}x^{4}\mathrm{sgn}\left(bx^{2}+a\right)+21\,a^{2}bx^{2}\mathrm{sgn}\left(bx^{2}+a\right)+5\,a^{3}\mathrm{sgn}\left(bx^{2}+a\right)}{35\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(35*b^3*x^6*sgn(b*x^2 + a) + 35*a*b^2*x^4*sgn(b*x^2 + a) + 21*a^2*b*x
^2*sgn(b*x^2 + a) + 5*a^3*sgn(b*x^2 + a))/x^7
```

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$-\frac{\left(35b^{3}x^{6}+35a\,b^{2}x^{4}+21a^{2}b\,x^{2}+5a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x)
```

```
[Out] -1/35*(35*b^3*x^6+35*a*b^2*x^4+21*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2)/x^7/
(b*x^2+a)^3
```

maxima [A] time = 1.27, size = 35, normalized size = 0.21

$$-\frac{b^{3}}{x}-\frac{ab^{2}}{x^{3}}-\frac{3\,a^{2}b}{5\,x^{5}}-\frac{a^{3}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] -b^3/x - a*b^2/x^3 - 3/5*a^2*b/x^5 - 1/7*a^3/x^7
```

mupad [B] time = 4.25, size = 151, normalized size = 0.93

$$-\frac{a^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{7\,x^{7}\left(b\,x^{2}+a\right)}-\frac{b^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{x\left(b\,x^{2}+a\right)}-\frac{a\,b^{2}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{x^{3}\left(b\,x^{2}+a\right)}-\frac{3\,a^{2}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{5\,x^{5}\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^8,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (b^3*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(x*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(x^3*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(5*x^5*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**8, x)
```

3.409 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(3*x^3*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{10}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{10}}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^6}+\frac{b^6}{x^4}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^3+135a^2bx^2+189ab^2x^4+105b^3x^6\right)}{315x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -1/315*(Sqrt[(a + b*x^2)^2]*(35*a^3 + 135*a^2*b*x^2 + 189*a*b^2*x^4 + 105*b
^3*x^6))/(x^9*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 18.94, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-35a^3-135a^2bx^2-189ab^2x^4-105b^3x^6\right)}{315x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-35*a^3 - 135*a^2*b*x^2 - 189*a*b^2*x^4 - 105*b^3*x^6
))/(315*x^9*(a + b*x^2))
```

fricas [A] time = 1.01, size = 37, normalized size = 0.22

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

giac [A] time = 0.19, size = 69, normalized size = 0.41

$$-\frac{105\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+189\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+135\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+35\,a^3\mathrm{sgn}\left(bx^2+a\right)}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(105*b^3*x^6*sgn(b*x^2 + a) + 189*a*b^2*x^4*sgn(b*x^2 + a) + 135*a^2
*b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^9
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(105b^3x^6+189a\,b^2x^4+135a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{315\left(b\,x^2+a\right)^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x)
```

```
[Out] -1/315*(105*b^3*x^6+189*a*b^2*x^4+135*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)
/x^9/(b*x^2+a)^3
```

maxima [A] time = 1.34, size = 35, normalized size = 0.21

$$-\frac{b^3}{3\,x^3}-\frac{3\,ab^2}{5\,x^5}-\frac{3\,a^2b}{7\,x^7}-\frac{a^3}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/3*b^3/x^3 - 3/5*a*b^2/x^5 - 3/7*a^2*b/x^7 - 1/9*a^3/x^9
```

mupad [B] time = 4.26, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^3\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^10,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (b^3*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(7*x^7*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**10, x)
```

3.410 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{12}} \, dx$

Optimal. Leaf size=167

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(5*x^5*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{12}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{12}}+\frac{3a^2b^4}{x^{10}}+\frac{3ab^5}{x^8}+\frac{b^6}{x^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(105a^3+385a^2bx^2+495ab^2x^4+231b^3x^6\right)}{1155x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -1/1155*(Sqrt[(a + b*x^2)^2]*(105*a^3 + 385*a^2*b*x^2 + 495*a*b^2*x^4 + 231
*b^3*x^6))/(x^11*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 19.62, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-105a^3-385a^2bx^2-495ab^2x^4-231b^3x^6\right)}{1155x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-105*a^3 - 385*a^2*b*x^2 - 495*a*b^2*x^4 - 231*b^3*x^
6))/(1155*x^11*(a + b*x^2))
```

fricas [A] time = 1.28, size = 37, normalized size = 0.22

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="fricas")

[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11

giac [A] time = 0.16, size = 69, normalized size = 0.41

$$-\frac{231\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+495\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+385\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+105\,a^3\mathrm{sgn}\left(bx^2+a\right)}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="giac")

[Out] -1/1155*(231*b^3*x^6*sgn(b*x^2 + a) + 495*a*b^2*x^4*sgn(b*x^2 + a) + 385*a^2*b*x^2*sgn(b*x^2 + a) + 105*a^3*sgn(b*x^2 + a))/x^11

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(231b^3x^6+495a\,b^2x^4+385a^2b\,x^2+105a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{1155\left(b\,x^2+a\right)^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x)

[Out] -1/1155*(231*b^3*x^6+495*a*b^2*x^4+385*a^2*b*x^2+105*a^3)*((b*x^2+a)^2)^(3/2)/x^11/(b*x^2+a)^3

maxima [A] time = 1.30, size = 35, normalized size = 0.21

$$-\frac{b^3}{5\,x^5}-\frac{3\,ab^2}{7\,x^7}-\frac{a^2b}{3\,x^9}-\frac{a^3}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="maxima")

[Out] -1/5*b^3/x^5 - 3/7*a*b^2/x^7 - 1/3*a^2*b/x^9 - 1/11*a^3/x^11

mupad [B] time = 4.55, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\left(b\,x^2+a\right)}-\frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^9\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^12,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(3*x^9*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**12, x)
```

3.411 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(7*x^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{14}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{14}}+\frac{3a^2b^4}{x^{12}}+\frac{3ab^5}{x^{10}}+\frac{b^6}{x^8}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(231a^3+819a^2bx^2+1001ab^2x^4+429b^3x^6\right)}{3003x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -1/3003*(Sqrt[(a + b*x^2)^2]*(231*a^3 + 819*a^2*b*x^2 + 1001*a*b^2*x^4 + 42
9*b^3*x^6))/(x^13*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 20.50, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-231a^3-819a^2bx^2-1001ab^2x^4-429b^3x^6\right)}{3003x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-231*a^3 - 819*a^2*b*x^2 - 1001*a*b^2*x^4 - 429*b^3*x
^6))/(3003*x^13*(a + b*x^2))
```

fricas [A] time = 0.77, size = 37, normalized size = 0.22

$$-\frac{429\,b^3x^6+1001\,ab^2x^4+819\,a^2bx^2+231\,a^3}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/3003*(429*b^3*x^6 + 1001*a*b^2*x^4 + 819*a^2*b*x^2 + 231*a^3)/x^13
```

giac [A] time = 0.16, size = 69, normalized size = 0.41

$$-\frac{429\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+1001\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+819\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+231\,a^3\mathrm{sgn}\left(bx^2+a\right)}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/3003*(429*b^3*x^6*sgn(b*x^2 + a) + 1001*a*b^2*x^4*sgn(b*x^2 + a) + 819*a
^2*b*x^2*sgn(b*x^2 + a) + 231*a^3*sgn(b*x^2 + a))/x^13
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(429b^3x^6+1001a\,b^2x^4+819a^2b\,x^2+231a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{3003\left(b\,x^2+a\right)^3\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x)
```

```
[Out] -1/3003*(429*b^3*x^6+1001*a*b^2*x^4+819*a^2*b*x^2+231*a^3)*((b*x^2+a)^2)^(3
/2)/x^13/(b*x^2+a)^3
```

maxima [A] time = 1.39, size = 35, normalized size = 0.21

$$-\frac{b^3}{7\,x^7}-\frac{ab^2}{3\,x^9}-\frac{3\,a^2b}{11\,x^{11}}-\frac{a^3}{13\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/7*b^3/x^7 - 1/3*a*b^2/x^9 - 3/11*a^2*b/x^11 - 1/13*a^3/x^13
```

mupad [B] time = 4.63, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}-\frac{a\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^9\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^14,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(3*x^9*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(11*x^11*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**14, x)
```

3.412 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(9*x^9*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{16}}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{16}}+\frac{3a^2b^4}{x^{14}}+\frac{3ab^5}{x^{12}}+\frac{b^6}{x^{10}}\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(429a^3+1485a^2bx^2+1755ab^2x^4+715b^3x^6\right)}{6435x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

```
[Out] -1/6435*(Sqrt[(a + b*x^2)^2]*(429*a^3 + 1485*a^2*b*x^2 + 1755*a*b^2*x^4 + 7
15*b^3*x^6))/(x^15*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 21.97, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-429a^3-1485a^2bx^2-1755ab^2x^4-715b^3x^6\right)}{6435x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-429*a^3 - 1485*a^2*b*x^2 - 1755*a*b^2*x^4 - 715*b^3*
x^6))/(6435*x^15*(a + b*x^2))
```

fricas [A] time = 1.20, size = 37, normalized size = 0.22

$$-\frac{715\,b^3x^6+1755\,ab^2x^4+1485\,a^2bx^2+429\,a^3}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/6435*(715*b^3*x^6 + 1755*a*b^2*x^4 + 1485*a^2*b*x^2 + 429*a^3)/x^15
```

giac [A] time = 0.21, size = 69, normalized size = 0.41

$$-\frac{715\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+1755\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+1485\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+429\,a^3\mathrm{sgn}\left(bx^2+a\right)}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/6435*(715*b^3*x^6*sgn(b*x^2 + a) + 1755*a*b^2*x^4*sgn(b*x^2 + a) + 1485*
a^2*b*x^2*sgn(b*x^2 + a) + 429*a^3*sgn(b*x^2 + a))/x^15
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(715b^3x^6+1755a\,b^2x^4+1485a^2b\,x^2+429a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{6435\left(b\,x^2+a\right)^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x)
```

```
[Out] -1/6435*(715*b^3*x^6+1755*a*b^2*x^4+1485*a^2*b*x^2+429*a^3)*((b*x^2+a)^2)^(
3/2)/x^15/(b*x^2+a)^3
```

maxima [A] time = 1.29, size = 35, normalized size = 0.21

$$-\frac{b^3}{9\,x^9}-\frac{3\,ab^2}{11\,x^{11}}-\frac{3\,a^2b}{13\,x^{13}}-\frac{a^3}{15\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/9*b^3/x^9 - 3/11*a*b^2/x^11 - 3/13*a^2*b/x^13 - 1/15*a^3/x^15
```

mupad [B] time = 4.30, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{15\,x^{15}\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^16,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(15*x^15*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(13*x^13*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**16, x)
```

3.413 $\int x^{13} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24\left(a+bx^2\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x}}{14\left(a+bx^2\right)}$$

Rubi [A] time = 0.16, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24\left(a+bx^2\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*(a + b*x^2)) + (5*a^3*b^2*x^18*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(22*(a + b*x^2)) + (b^5*x^24*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*(a
 + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^6\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^6\left(ab+b^2x\right)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(a^5b^5x^6+5a^4b^6x^7+10a^3b^7x^8+10a^2b^8x^9+5\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{a^2+}}{9\left(a+\right.}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{\left(a+bx^2\right)^2}\left(792a^5+3465a^4bx^2+6160a^3b^2x^4+5544a^2b^3x^6+2520ab^4x^8+462b^5x^{10}\right)}{11088\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^14*Sqrt[(a + b*x^2)^2]*(792*a^5 + 3465*a^4*b*x^2 + 6160*a^3*b^2*x^4 + 55
44*a^2*b^3*x^6 + 2520*a*b^4*x^8 + 462*b^5*x^10))/(11088*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 30.12, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(792a^5x^{14}+3465a^4bx^{16}+6160a^3b^2x^{18}+5544a^2b^3x^{20}+2520ab^4x^{22}+462b^5x^{24}\right)}{11088\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(792*a^5*x^14 + 3465*a^4*b*x^16 + 6160*a^3*b^2*x^18 +
5544*a^2*b^3*x^20 + 2520*a*b^4*x^22 + 462*b^5*x^24))/(11088*(a + b*x^2))
```

fricas [A] time = 0.77, size = 57, normalized size = 0.22

$$\frac{1}{24}\,b^{5}x^{24}+\frac{5}{22}\,ab^{4}x^{22}+\frac{1}{2}\,a^{2}b^{3}x^{20}+\frac{5}{9}\,a^{3}b^{2}x^{18}+\frac{5}{16}\,a^{4}bx^{16}+\frac{1}{14}\,a^{5}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{24}\,b^{5}x^{24}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{5}{22}\,ab^{4}x^{22}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{1}{2}\,a^{2}b^{3}x^{20}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{5}{9}\,a^{3}b^{2}x^{18}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{5}{16}\,a^{4}bx^{16}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{1}{14}\,a^{5}x^{14}\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sgn(b*x^2 + a) + 5/22*a*b^4*x^22*sgn(b*x^2 + a) + 1/2*a^2*b^3
*x^20*sgn(b*x^2 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^2 + a) + 5/16*a^4*b*x^16*sg
n(b*x^2 + a) + 1/14*a^5*x^14*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(462b^{5}x^{10}+2520a\,b^{4}x^{8}+5544a^{2}b^{3}x^{6}+6160a^{3}b^{2}x^{4}+3465a^{4}b\,x^{2}+792a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}x^{14}}{11088\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/11088*x^14*(462*b^5*x^10+2520*a*b^4*x^8+5544*a^2*b^3*x^6+6160*a^3*b^2*x^4
+3465*a^4*b*x^2+792*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.32, size = 57, normalized size = 0.22

$$\frac{1}{24}\,b^{5}x^{24}+\frac{5}{22}\,ab^{4}x^{22}+\frac{1}{2}\,a^{2}b^{3}x^{20}+\frac{5}{9}\,a^{3}b^{2}x^{18}+\frac{5}{16}\,a^{4}bx^{16}+\frac{1}{14}\,a^{5}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{13} \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int(x^13*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13} \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x**13*((a + b*x**2)**2)**(5/2), x)
```

3.414 $\int x^{11} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x}}{12\left(a+bx^2\right)}$$

Rubi [A] time = 0.16, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*(a + b*x^2)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (5*a^2*b^3*x^18*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(4*(a + b*x^2)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*(a
+ b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^5\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^5\left(ab+b^2x\right)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(a^5b^5x^5+5a^4b^6x^6+10a^3b^7x^7+10a^2b^8x^8+5\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\left(a+bx^2\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+}}{8\left(a+\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{12}\sqrt{\left(a+bx^2\right)^2}\left(462a^5+1980a^4bx^2+3465a^3b^2x^4+3080a^2b^3x^6+1386ab^4x^8+252b^5x^{10}\right)}{5544\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^12*Sqrt[(a + b*x^2)^2]*(462*a^5 + 1980*a^4*b*x^2 + 3465*a^3*b^2*x^4 + 30
80*a^2*b^3*x^6 + 1386*a*b^4*x^8 + 252*b^5*x^10))/(5544*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 22.75, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(462a^5x^{12}+1980a^4bx^{14}+3465a^3b^2x^{16}+3080a^2b^3x^{18}+1386ab^4x^{20}+252b^5x^{22}\right)}{5544\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(462*a^5*x^12 + 1980*a^4*b*x^14 + 3465*a^3*b^2*x^16 +
3080*a^2*b^3*x^18 + 1386*a*b^4*x^20 + 252*b^5*x^22))/(5544*(a + b*x^2))
```

fricas [A] time = 0.74, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

giac [A] time = 0.15, size = 105, normalized size = 0.41

$$\frac{1}{22}\,b^5x^{22}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^2b^3x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^3b^2x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{12}\,a^5x^{12}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22*sgn(b*x^2 + a) + 1/4*a*b^4*x^20*sgn(b*x^2 + a) + 5/9*a^2*b^3*
x^18*sgn(b*x^2 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^2 + a) + 5/14*a^4*b*x^14*sgn
(b*x^2 + a) + 1/12*a^5*x^12*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(252b^5x^{10}+1386a\,b^4x^8+3080a^2b^3x^6+3465a^3b^2x^4+1980a^4b\,x^2+462a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^{12}}{5544\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/5544*x^12*(252*b^5*x^10+1386*a*b^4*x^8+3080*a^2*b^3*x^6+3465*a^3*b^2*x^4+
1980*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.37, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{11} \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^11*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11} \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**2)**2)**(5/2), x)
```

3.415 $\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=201

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^9}{20b^5}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{8b^5}+\frac{a^4\sqrt{a^2}}{}$$

Rubi [A] time = 0.13, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1111, 645}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^9}{20b^5}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{8b^5}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{7b^5}+\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^4*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^5) - (2*a^3*(a +
b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^5) + (3*a^2*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^5) - (2*a*(a + b*x^2)^8*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(9*b^5) + ((a + b*x^2)^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(20*b^5)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{2}\text{Subst}\left(\int x^4 \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(\frac{a^4(ab+b^2x)^5}{b^4} - \frac{4a^3(ab+b^2x)^6}{b^5} + \frac{6a^2(ab+b^2x)^7}{b^6} - \frac{4a(ab+}{b^7}\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{a^4\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^5} - \frac{2a^3\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^5} + \frac{3}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.41

$$\frac{x^{10}\sqrt{\left(a+bx^2\right)^2}\left(252a^5+1050a^4bx^2+1800a^3b^2x^4+1575a^2b^3x^6+700ab^4x^8+126b^5x^{10}\right)}{2520\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(252*a^5 + 1050*a^4*b*x^2 + 1800*a^3*b^2*x^4 + 15
75*a^2*b^3*x^6 + 700*a*b^4*x^8 + 126*b^5*x^10))/(2520*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 17.02, size = 83, normalized size = 0.41

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(252a^5x^{10}+1050a^4bx^{12}+1800a^3b^2x^{14}+1575a^2b^3x^{16}+700ab^4x^{18}+126b^5x^{20}\right)}{2520\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(252*a^5*x^10 + 1050*a^4*b*x^12 + 1800*a^3*b^2*x^14 +
1575*a^2*b^3*x^16 + 700*a*b^4*x^18 + 126*b^5*x^20))/(2520*(a + b*x^2))
```

fricas [A] time = 1.02, size = 57, normalized size = 0.28

$$\frac{1}{20}b^5x^{20} + \frac{5}{18}ab^4x^{18} + \frac{5}{8}a^2b^3x^{16} + \frac{5}{7}a^3b^2x^{14} + \frac{5}{12}a^4bx^{12} + \frac{1}{10}a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

giac [A] time = 0.16, size = 105, normalized size = 0.52

$$\frac{1}{20}\,b^5x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{18}\,ab^4x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^2b^3x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,a^3b^2x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{12}\,a^4bx^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{10}\,a^5x^{10}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sgn(b*x^2 + a) + 5/18*a*b^4*x^18*sgn(b*x^2 + a) + 5/8*a^2*b^3
*x^16*sgn(b*x^2 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^2 + a) + 5/12*a^4*b*x^12*sg
n(b*x^2 + a) + 1/10*a^5*x^10*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.40

$$\frac{\left(126b^5x^{10}+700a\,b^4x^8+1575a^2b^3x^6+1800a^3b^2x^4+1050a^4b\,x^2+252a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^{10}}{2520\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/2520*x^10*(126*b^5*x^10+700*a*b^4*x^8+1575*a^2*b^3*x^6+1800*a^3*b^2*x^4+1
050*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.43, size = 57, normalized size = 0.28

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^9\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \left(\left(a + bx^2 \right)^2 \right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**9*((a + b*x**2)**2)**(5/2), x)
```

3.416 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^8}{18b^4}-\frac{3a\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^7}{16b^4}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^6}{14b^4}-\frac{a^3\sqrt{a^2}}{}$$

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^8}{18b^4}-\frac{3a\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^7}{16b^4}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^6}{14b^4}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^5}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^4) + (3*a^2*(a +
 b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*b^4) - (3*a*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*b^4) + ((a + b*x^2)^8*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(18*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{2}\text{Subst}\left(\int x^3 \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^3 \left(ab+b^2x\right)^5 dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(-\frac{a^3\left(ab+b^2x\right)^5}{b^3}+\frac{3a^2\left(ab+b^2x\right)^6}{b^4}-\frac{3a\left(ab+b^2x\right)^7}{b^5}+\frac{(ab+b}{b^6}\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^4}+\frac{3a^2\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{14b^4}-$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.52

$$\frac{x^8\sqrt{\left(a+bx^2\right)^2}\left(126a^5+504a^4bx^2+840a^3b^2x^4+720a^2b^3x^6+315ab^4x^8+56b^5x^{10}\right)}{1008\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(126*a^5 + 504*a^4*b*x^2 + 840*a^3*b^2*x^4 + 720*a
^2*b^3*x^6 + 315*a*b^4*x^8 + 56*b^5*x^10))/(1008*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 14.64, size = 83, normalized size = 0.52

$$\frac{x^8\sqrt{\left(a+bx^2\right)^2}\left(126a^5+504a^4bx^2+840a^3b^2x^4+720a^2b^3x^6+315ab^4x^8+56b^5x^{10}\right)}{1008\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(126*a^5 + 504*a^4*b*x^2 + 840*a^3*b^2*x^4 + 720*a
^2*b^3*x^6 + 315*a*b^4*x^8 + 56*b^5*x^10))/(1008*(a + b*x^2))
```

fricas [A] time = 1.11, size = 57, normalized size = 0.36

$$\frac{1}{18}b^5x^{18}+\frac{5}{16}ab^4x^{16}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{6}a^3b^2x^{12}+\frac{1}{2}a^4bx^{10}+\frac{1}{8}a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

giac [A] time = 0.16, size = 105, normalized size = 0.66

$$\frac{1}{18}\,b^5x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{16}\,ab^4x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,a^2b^3x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{6}\,a^3b^2x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^4bx^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{8}\,a^5x^8\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/18*b^5*x^18*sgn(b*x^2 + a) + 5/16*a*b^4*x^16*sgn(b*x^2 + a) + 5/7*a^2*b^3
*x^14*sgn(b*x^2 + a) + 5/6*a^3*b^2*x^12*sgn(b*x^2 + a) + 1/2*a^4*b*x^10*sgn
(b*x^2 + a) + 1/8*a^5*x^8*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.50

$$\frac{\left(56b^5x^{10}+315a\,b^4x^8+720a^2b^3x^6+840a^3b^2x^4+504a^4b\,x^2+126a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\,x^8}{1008\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/1008*x^8*(56*b^5*x^10+315*a*b^4*x^8+720*a^2*b^3*x^6+840*a^3*b^2*x^4+504*a
^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.40, size = 57, normalized size = 0.36

$$\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(5/2), x)
```

3.417 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^3}$$

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^3) - (a*(a + b*x^
2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^3) + ((a + b*x^2)^7*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(16*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{2}\operatorname{Subst}\left(\int x^2 \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int x^2 \left(ab+b^2x\right)^5 dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{a^2\left(ab+b^2x\right)^5}{b^2}-\frac{2a\left(ab+b^2x\right)^6}{b^3}+\frac{\left(ab+b^2x\right)^7}{b^4}\right) dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{a^2\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^3}-\frac{a\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^3}+\frac{(a+}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.70

$$\frac{x^6\sqrt{\left(a+bx^2\right)^2}\left(56a^5+210a^4bx^2+336a^3b^2x^4+280a^2b^3x^6+120ab^4x^8+21b^5x^{10}\right)}{336\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(56*a^5 + 210*a^4*b*x^2 + 336*a^3*b^2*x^4 + 280*a^
2*b^3*x^6 + 120*a*b^4*x^8 + 21*b^5*x^10))/(336*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 13.03, size = 83, normalized size = 0.70

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(56a^5x^6+210a^4bx^8+336a^3b^2x^{10}+280a^2b^3x^{12}+120ab^4x^{14}+21b^5x^{16}\right)}{336\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(56*a^5*x^6 + 210*a^4*b*x^8 + 336*a^3*b^2*x^10 + 280*a
^2*b^3*x^12 + 120*a*b^4*x^14 + 21*b^5*x^16))/(336*(a + b*x^2))
```

fricas [A] time = 0.52, size = 56, normalized size = 0.47

$$\frac{1}{16}b^5x^{16}+\frac{5}{14}ab^4x^{14}+\frac{5}{6}a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}a^4bx^8+\frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

giac [A] time = 0.19, size = 104, normalized size = 0.87

$$\frac{1}{16}\,b^5x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{14}\,ab^4x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{6}\,a^2b^3x^{12}\mathrm{sgn}\left(bx^2+a\right)+a^3b^2x^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^4bx^8\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{6}\,a^5x^6\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sgn(b*x^2 + a) + 5/14*a*b^4*x^14*sgn(b*x^2 + a) + 5/6*a^2*b^3
*x^12*sgn(b*x^2 + a) + a^3*b^2*x^10*sgn(b*x^2 + a) + 5/8*a^4*b*x^8*sgn(b*x^
2 + a) + 1/6*a^5*x^6*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.67

$$\frac{\left(21b^5x^{10}+120a\,b^4x^8+280a^2b^3x^6+336a^3b^2x^4+210a^4b\,x^2+56a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^6}{336\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/336*x^6*(21*b^5*x^10+120*a*b^4*x^8+280*a^2*b^3*x^6+336*a^3*b^2*x^4+210*a^
4*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 56, normalized size = 0.47

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x**2)**2)**(5/2), x)
```

3.418 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 609}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b^2) + (a^2 + 2*a*b*
x^2 + b^2*x^4)^(7/2)/(14*b^2)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int x\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$
$$=\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\,\text{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)}{2b}$$
$$=-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.24

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(21a^5+70a^4bx^2+105a^3b^2x^4+84a^2b^3x^6+35ab^4x^8+6b^5x^{10}\right)}{84\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(21*a^5 + 70*a^4*b*x^2 + 105*a^3*b^2*x^4 + 84*a^2*
b^3*x^6 + 35*a*b^4*x^8 + 6*b^5*x^10))/(84*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 12.21, size = 83, normalized size = 1.24

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(21a^5+70a^4bx^2+105a^3b^2x^4+84a^2b^3x^6+35ab^4x^8+6b^5x^{10}\right)}{84\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(21*a^5 + 70*a^4*b*x^2 + 105*a^3*b^2*x^4 + 84*a^2*
b^3*x^6 + 35*a*b^4*x^8 + 6*b^5*x^10))/(84*(a + b*x^2))
```

fricas [A] time = 1.12, size = 56, normalized size = 0.84

$$\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

giac [A] time = 0.20, size = 67, normalized size = 1.00

$$\frac{1}{84}\left(6\,b^{5}x^{14}+35\,ab^{4}x^{12}+84\,a^{2}b^{3}x^{10}+105\,a^{3}b^{2}x^{8}+70\,a^{4}bx^{6}+21\,a^{5}x^{4}\right)\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/84*(6*b^5*x^14 + 35*a*b^4*x^12 + 84*a^2*b^3*x^10 + 105*a^3*b^2*x^8 + 70*a
^4*b*x^6 + 21*a^5*x^4)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 1.19

$$\frac{\left(6b^{5}x^{10}+35a\,b^{4}x^{8}+84a^{2}b^{3}x^{6}+105a^{3}b^{2}x^{4}+70a^{4}b\,x^{2}+21a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}x^{4}}{84\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/84*x^4*(6*b^5*x^10+35*a*b^4*x^8+84*a^2*b^3*x^6+105*a^3*b^2*x^4+70*a^4*b*x
^2+21*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.25, size = 56, normalized size = 0.84

$$\frac{1}{14}\,b^{5}x^{14}+\frac{5}{12}\,ab^{4}x^{12}+a^{2}b^{3}x^{10}+\frac{5}{4}\,a^{3}b^{2}x^{8}+\frac{5}{6}\,a^{4}bx^{6}+\frac{1}{4}\,a^{5}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3}\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x**3*((a + b*x**2)**2)**(5/2), x)
```

3.419 $\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)\\&=\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]

[Out] ((a + b*x^2)*((a + b*x^2)^2)^(5/2))/(12*b)

IntegrateAlgebraic [B] time = 10.48, size = 82, normalized size = 2.28

$$\frac{x^2\sqrt{(a+bx^2)^2}\,\left(6a^5+15a^4bx^2+20a^3b^2x^4+15a^2b^3x^6+6ab^4x^8+b^5x^{10}\right)}{12\,(a+bx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]

[Out] (x^2*Sqrt[(a + b*x^2)^2]*(6*a^5 + 15*a^4*b*x^2 + 20*a^3*b^2*x^4 + 15*a^2*b^3*x^6 + 6*a*b^4*x^8 + b^5*x^10))/(12*(a + b*x^2))

fricas [A] time = 0.80, size = 57, normalized size = 1.58

$$\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a^4*b*x^4 + 1/2*a^5*x^2

giac [B] time = 0.16, size = 66, normalized size = 1.83

$$\frac{1}{12}\left(3\left(bx^4+2\,ax^2\right)a^4+3\left(bx^4+2\,ax^2\right)^2a^2b+\left(bx^4+2\,ax^2\right)^3b^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")

[Out] 1/12*(3*(b*x^4 + 2*a*x^2)*a^4 + 3*(b*x^4 + 2*a*x^2)^2*a^2*b + (b*x^4 + 2*a*x^2)^3*b^2)*sgn(b*x^2 + a)

maple [B] time = 0.01, size = 79, normalized size = 2.19

$$\frac{\left(b^5x^{10}+6a\,b^4x^8+15a^2b^3x^6+20a^3b^2x^4+15a^4b\,x^2+6a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\,x^2}{12\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/12*x^2*(b^5*x^10+6*a*b^4*x^8+15*a^2*b^3*x^6+20*a^3*b^2*x^4+15*a^4*b*x^2+6
*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.31, size = 57, normalized size = 1.58

$$\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a^
4*b*x^4 + 1/2*a^5*x^2
```

mupad [B] time = 4.40, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^2+a\,b\right)\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] ((a*b + b^2*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2))/(12*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(5/2), x)
```

3.420 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x} \, dx$

Optimal. Leaf size=251

$$\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^5\log(x)\sqrt{a^2+2abx^2+}}{a+bx^2}$$

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a^5\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (5*a^4*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^3*b^2*
x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^2*b^3*x^6*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(10*(a + b*x^2)) + (a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x
^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3+b^1\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2ab}}{3\left(a+bx^2\right.}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(120a^5\log(x)+bx^2\left(300a^4+300a^3bx^2+200a^2b^2x^4+75ab^3x^6+12b^4x^8\right)\right)}{120\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(300*a^4 + 300*a^3*b*x^2 + 200*a^2*b^2*x^4 + 75
*a*b^3*x^6 + 12*b^4*x^8) + 120*a^5*Log[x]))/(120*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.56, size = 314, normalized size = 1.25

$$\frac{1}{4}a^5\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)-\frac{a^5\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{4b}-\frac{a^5\sqrt{b^2}\log\left(b\sqrt{a^2+2abx^2+b^2x^4}-ab-b\sqrt{b^2}\,x^2\right)}{4b}+\frac{1}{240}\sqrt{a^2+2abx^2+b^2x^4}\left(137a^4+163a^3bx^2+137a^2b^2x^4+63ab^3x^6+12b^4x^8\right)+\frac{1}{240}\left(-300a^4\sqrt{b^2}\,x^2-300a^3b\sqrt{b^2}\,x^4-200a^2\left(b^2\right)^{3/2}x^6-75ab^3\sqrt{b^2}\,x^8-12b^4\sqrt{b^2}\,x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(137*a^4 + 163*a^3*b*x^2 + 137*a^2*b^2*x^4
 + 63*a*b^3*x^6 + 12*b^4*x^8))/240 + (-300*a^4*Sqrt[b^2]*x^2 - 300*a^3*b*Sq
rt[b^2]*x^4 - 200*a^2*(b^2)^(3/2)*x^6 - 75*a*b^3*Sqrt[b^2]*x^8 - 12*b^4*Sqr
t[b^2]*x^10)/240 + (a^5*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]])/4 - (a^5*(b + Sqrt[b^2])*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x
```

```
^2 + b^2*x^4]])/(4*b) - (a^5*Sqrt[b^2]*Log[-(a*b) - b*Sqrt[b^2]*x^2 + b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*b)
```

fricas [A] time = 0.86, size = 55, normalized size = 0.22

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

giac [A] time = 0.19, size = 106, normalized size = 0.42

$$\frac{1}{10}\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^5\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x^2 + a) + 5/8*a*b^4*x^8*sgn(b*x^2 + a) + 5/3*a^2*b^3*x
^6*sgn(b*x^2 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^2 + a) + 5/2*a^4*b*x^2*sgn(b*x^
2 + a) + 1/2*a^5*log(x^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 79, normalized size = 0.31

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\left(12b^5x^{10}+75a\,b^4x^8+200a^2b^3x^6+300a^3b^2x^4+300a^4b\,x^2+120a^5\ln(x)\right)}{120\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+75*a*b^4*x^8+200*a^2*b^3*x^6+300*a^3
*b^2*x^4+300*a^4*b*x^2+120*a^5*ln(x))/(b*x^2+a)^5
```

maxima [A] time = 1.36, size = 55, normalized size = 0.22

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x, x)
```

3.421 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+$$

Rubi [A] time = 0.07, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{5a^4b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^3*b^2*x^2
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(6*(a + b*x^2)) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a
 + b*x^2)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^3}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^2}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b^{10}x^3\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2-}}{2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-12a^5+120a^4bx^2\log(x)+120a^3b^2x^4+60a^2b^3x^6+20ab^4x^8+3b^5x^{10}\right)}{24x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-12*a^5 + 120*a^3*b^2*x^4 + 60*a^2*b^3*x^6 + 20*a*b^4
*x^8 + 3*b^5*x^10 + 120*a^4*b*x^2*Log[x]))/(24*x^2*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.95, size = 364, normalized size = 1.46

$$-\frac{5}{4}a^4\sqrt{b^2}\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)-\frac{5}{4}a^4\sqrt{b^2}\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)+\frac{5}{2}a^4b\tanh^{-1}\left(\frac{\sqrt{b^2}\,x^2}{a}-\frac{\sqrt{a^2+2abx^2+b^2x^4}}{a}\right)+\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-192a^5b-395a^4b^2x^2+1920a^3b^3x^4+960a^2b^4x^6+320ab^5x^8+48b^6x^{10}\right)+\sqrt{b^2}\left(192a^6+587a^5bx^2-1525a^4b^2x^4-2880a^3b^3x^6-1280a^2b^4x^8-368ab^5x^{10}-48b^6x^{12}\right)}{384x^2\left(ab+b^2x^2\right)-384\sqrt{b^2}\,x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-192*a^5*b - 395*a^4*b^2*x^2 + 1920*a^3*b
^3*x^4 + 960*a^2*b^4*x^6 + 320*a*b^5*x^8 + 48*b^6*x^10) + Sqrt[b^2]*(192*a^
6 + 587*a^5*b*x^2 - 1525*a^4*b^2*x^4 - 2880*a^3*b^3*x^6 - 1280*a^2*b^4*x^8
- 368*a*b^5*x^10 - 48*b^6*x^12))/(384*x^2*(a*b + b^2*x^2) - 384*Sqrt[b^2]*x
^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*a^4*b*ArcTanh[(Sqrt[b^2]*x^2)/a -
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/a])/2 - (5*a^4*Sqrt[b^2]*Log[-a - Sqrt[b^2]
```

```
*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/4 - (5*a^4*Sqrt[b^2]*Log[a - Sqrt[
b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/4
```

fricas [A] time = 0.85, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{10} + 20\,ab^4x^8 + 60\,a^2b^3x^6 + 120\,a^3b^2x^4 + 120\,a^4bx^2\log(x) - 12\,a^5}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a^
4*b*x^2*log(x) - 12*a^5)/x^2
```

giac [A] time = 0.16, size = 125, normalized size = 0.50

$$\frac{1}{8}\,b^5x^8\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{2}\,a^2b^3x^4\mathrm{sgn}\left(bx^2 + a\right) + 5\,a^3b^2x^2\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{2}\,a^4b\log\left(x^2\right)\mathrm{sgn}\left(bx^2 + a\right) - \frac{5\,a^4bx^2\mathrm{sgn}\left(bx^2 + a\right) + a^5\mathrm{sgn}\left(bx^2 + a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sgn(b*x^2 + a) + 5/6*a*b^4*x^6*sgn(b*x^2 + a) + 5/2*a^2*b^3*x^4
*sgn(b*x^2 + a) + 5*a^3*b^2*x^2*sgn(b*x^2 + a) + 5/2*a^4*b*log(x^2)*sgn(b*x
^2 + a) - 1/2*(5*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^2
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^2 + a\right)^2\right)^{\frac{5}{2}}\left(3b^5x^{10} + 20a\,b^4x^8 + 60a^2b^3x^6 + 120a^3b^2x^4 + 120a^4b\,x^2\ln(x) - 12a^5\right)}{24\left(b\,x^2 + a\right)^5 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x)
```

```
[Out] 1/24*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+20*a*b^4*x^8+60*a^2*b^3*x^6+120*a^3*b^
2*x^4+120*a^4*b*ln(x)*x^2-12*a^5)/(b*x^2+a)^5/x^2
```

maxima [A] time = 1.34, size = 56, normalized size = 0.22

$$\frac{1}{8}\,b^5x^8 + \frac{5}{6}\,ab^4x^6 + \frac{5}{2}\,a^2b^3x^4 + 5\,a^3b^2x^2 + 5\,a^4b\log(x) - \frac{a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5*a^4*b*log
(x) - 1/2*a^5/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**3, x)
```

3.422 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-$$

Rubi [A] time = 0.07, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}+\frac{10a^3b^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^2*b^3*x^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(4*(a + b*x^2)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a +
 b*x^2)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^5}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^3}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x^2\right) d\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)} + \frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2}}{a+bx^2}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-3a^5-30a^4bx^2+120a^3b^2x^4\log(x)+60a^2b^3x^6+15ab^4x^8+2b^5x^{10}\right)}{12x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 30*a^4*b*x^2 + 60*a^2*b^3*x^6 + 15*a*b^4*x^8
 + 2*b^5*x^10 + 120*a^3*b^2*x^4*Log[x]))/(12*x^4*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 1.04, size = 366, normalized size = 1.46

$$-\frac{5}{2}a^3b\sqrt{b^2}\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}x^2\right)-\frac{5}{2}a^3b\sqrt{b^2}\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2\right)+5a^3b^2\tanh^{-1}\left(\frac{\sqrt{b^2}x^2}{a}-\frac{\sqrt{a^2+2abx^2+b^2x^4}}{a}\right)+\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-6a^5b-60a^4b^2x^2+53a^3b^3x^4+120a^2b^4x^6+30ab^5x^8+4b^6x^{10}\right)+\sqrt{b^2}\left(6a^6+66a^5bx^2+7a^4b^2x^4-173a^3b^3x^6-150a^2b^4x^8-34ab^5x^{10}-4b^6x^{12}\right)}{24x^4\left(ab+b^2x^2\right)-24\sqrt{b^2}x^4\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-6*a^5*b - 60*a^4*b^2*x^2 + 53*a^3*b^3*x^
4 + 120*a^2*b^4*x^6 + 30*a*b^5*x^8 + 4*b^6*x^10) + Sqrt[b^2]*(6*a^6 + 66*a^
5*b*x^2 + 7*a^4*b^2*x^4 - 173*a^3*b^3*x^6 - 150*a^2*b^4*x^8 - 34*a*b^5*x^10
 - 4*b^6*x^12))/(24*x^4*(a*b + b^2*x^2) - 24*Sqrt[b^2]*x^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + 5*a^3*b^2*ArcTanh[(Sqrt[b^2]*x^2)/a - Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]/a] - (5*a^3*b*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2
```

```
*a*b*x^2 + b^2*x^4]])/2 - (5*a^3*b*Sqrt[b^2]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]])/2
```

fricas [A] time = 0.83, size = 61, normalized size = 0.24

$$\frac{2\,b^5x^{10}+15\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4\log(x)-30\,a^4bx^2-3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x) -
 30*a^4*b*x^2 - 3*a^5)/x^4
```

giac [A] time = 0.16, size = 127, normalized size = 0.51

$$\frac{1}{6}\,b^5x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{4}\,ab^4x^4\mathrm{sgn}\left(bx^2+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^2+a\right)+5\,a^3b^2\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{30\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+10\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+a^5\mathrm{sgn}\left(bx^2+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x^2 + a) + 5/4*a*b^4*x^4*sgn(b*x^2 + a) + 5*a^2*b^3*x^2*s
gn(b*x^2 + a) + 5*a^3*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(30*a^3*b^2*x^4*sgn
(b*x^2 + a) + 10*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^4
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\left(2b^5x^{10}+15a\,b^4x^8+60a^2b^3x^6+120a^3b^2x^4\ln(x)-30a^4b\,x^2-3a^5\right)}{12\left(b\,x^2+a\right)^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(5/2)*(2*b^5*x^10+15*a*b^4*x^8+60*a^2*b^3*x^6+120*a^3*b^
2*ln(x)*x^4-30*a^4*b*x^2-3*a^5)/(b*x^2+a)^5/x^4
```

maxima [A] time = 1.34, size = 56, normalized size = 0.22

$$\frac{1}{6}\,b^5x^6+\frac{5}{4}\,ab^4x^4+5\,a^2b^3x^2+10\,a^3b^2\log(x)-\frac{5\,a^4b}{2\,x^2}-\frac{a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 10*a^3*b^2*log(x) - 5/2*a^4*b
/x^2 - 1/4*a^5/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**5, x)
```

3.423 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b}}{6x^6\left(a+bx^2\right)}$$

Rubi [A] time = 0.07, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{b^5x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(2*(a + b*x^2)) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a
 + b*x^2)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^7}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^7}\,dx}{b^4\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^4}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right)dx,x\right.}{2b^4\left(ab+b^2x^2\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$
\frac{\sqrt{\left(a+bx^2\right)^2}\left(-2a^5-15a^4bx^2-60a^3b^2x^4+120a^2b^3x^6\log(x)+30ab^4x^8+3b^5x^{10}\right)}{12x^6\left(a+bx^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^5 - 15*a^4*b*x^2 - 60*a^3*b^2*x^4 + 30*a*b^4*x^8
 + 3*b^5*x^10 + 120*a^2*b^3*x^6*Log[x]))/(12*x^6*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 1.80, size = 364, normalized size = 1.46

$$
-\frac{5}{2}a^2\left(b^2\right)^{3/2}\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)-\frac{5}{2}a^2\left(b^2\right)^{3/2}\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)+5a^2b^3\tanh^{-1}\left(\frac{\sqrt{b^2}\,x^2}{a}-\frac{\sqrt{a^2+2abx^2+b^2x^4}}{a}\right)+\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-8a^5b-60a^4b^2x^2-240a^3b^3x^4-391a^2b^4x^6+120ab^5x^8+12b^6x^{10}\right)+\sqrt{b^2}\left(8a^6+68a^5bx^2+300a^4b^2x^4+631a^3b^3x^6+271a^2b^4x^8-132ab^5x^{10}-12b^6x^{12}\right)}{48x^6\left(ab+b^2x^2\right)-48\sqrt{b^2}\,x^6\sqrt{a^2+2abx^2+b^2x^4}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-8*a^5*b - 60*a^4*b^2*x^2 - 240*a^3*b^3*x
^4 - 391*a^2*b^4*x^6 + 120*a*b^5*x^8 + 12*b^6*x^10) + Sqrt[b^2]*(8*a^6 + 68
*a^5*b*x^2 + 300*a^4*b^2*x^4 + 631*a^3*b^3*x^6 + 271*a^2*b^4*x^8 - 132*a*b^
5*x^10 - 12*b^6*x^12))/(48*x^6*(a*b + b^2*x^2) - 48*Sqrt[b^2]*x^6*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) + 5*a^2*b^3*ArcTanh[(Sqrt[b^2]*x^2)/a - Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]/a] - (5*a^2*(b^2)^(3/2)*Log[-a - Sqrt[b^2]*x^2 + Sqrt[
```

```
a^2 + 2*a*b*x^2 + b^2*x^4]])/2 - (5*a^2*(b^2)^(3/2)*Log[a - Sqrt[b^2]*x^2 +
 Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/2
```

fricas [A] time = 0.96, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{10} + 30\,ab^4x^8 + 120\,a^2b^3x^6\log(x) - 60\,a^3b^2x^4 - 15\,a^4bx^2 - 2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4 -
 15*a^4*b*x^2 - 2*a^5)/x^6
```

giac [A] time = 0.16, size = 128, normalized size = 0.51

$$\frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx^2+a\right) + \frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx^2+a\right) + 5\,a^2b^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right) - \frac{110\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right) + 60\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right) + 15\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right) + 2\,a^5\mathrm{sgn}\left(bx^2+a\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sgn(b*x^2 + a) + 5/2*a*b^4*x^2*sgn(b*x^2 + a) + 5*a^2*b^3*log(x
^2)*sgn(b*x^2 + a) - 1/12*(110*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^4*
sgn(b*x^2 + a) + 15*a^4*b*x^2*sgn(b*x^2 + a) + 2*a^5*sgn(b*x^2 + a))/x^6
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\left(3b^5x^{10} + 30a\,b^4x^8 + 120a^2b^3x^6\ln(x) - 60a^3b^2x^4 - 15a^4b\,x^2 - 2a^5\right)}{12\left(b\,x^2+a\right)^5 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+30*a*b^4*x^8+120*a^2*b^3*ln(x)*x^6-60*
a^3*b^2*x^4-15*a^4*b*x^2-2*a^5)/(b*x^2+a)^5/x^6
```

maxima [A] time = 1.38, size = 56, normalized size = 0.22

$$\frac{1}{4}\,b^5x^4 + \frac{5}{2}\,ab^4x^2 + 10\,a^2b^3\log(x) - \frac{5\,a^3b^2}{x^2} - \frac{5\,a^4b}{4\,x^4} - \frac{a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 10*a^2*b^3*log(x) - 5*a^3*b^2/x^2 - 5/4*a^4*b
/x^4 - 1/6*a^5/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**7, x)
```

3.424 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

Rubi [A] time = 0.07, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}+\frac{b^5x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(x^2*(a + b*x^2)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*
(a + b*x^2)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^9}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^9}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^5}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right)dx,x,x\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a^5+20a^4bx^2+60a^3b^2x^4+120a^2b^3x^6-120ab^4x^8\log(x)-12b^5x^{10}\right)}{24x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -1/24*(Sqrt[(a + b*x^2)^2]*(3*a^5 + 20*a^4*b*x^2 + 60*a^3*b^2*x^4 + 120*a^2
*b^3*x^6 - 12*b^5*x^10 - 120*a*b^4*x^8*Log[x]))/(x^8*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 2.75, size = 2027, normalized size = 8.11

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] (-(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(3*a^8*b + 29*a^7*b^2*x^2 + 129*a^6*
b^3*x^4 + 363*a^5*b^4*x^6 + 554*a^4*b^5*x^8 + 390*a^3*b^6*x^10 + 66*a^2*b^7
*x^12 - 42*a*b^8*x^14 - 12*b^9*x^16)) - b^3*Sqrt[b^2]*(-3*a^9 - 32*a^8*b*x^
2 - 158*a^7*b^2*x^4 - 492*a^6*b^3*x^6 - 917*a^5*b^4*x^8 - 944*a^4*b^5*x^10
- 456*a^3*b^6*x^12 - 24*a^2*b^7*x^14 + 54*a*b^8*x^16 + 12*b^9*x^18))/(3*Sqr
t[b^2]*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-8*a^3*b^3 - 24*a^2*b^4*x^2 - 2
```

```
4*a*b^5*x^4 - 8*b^6*x^6) + 3*x^8*(8*a^4*b^4 + 32*a^3*b^5*x^2 + 48*a^2*b^6*x
^4 + 32*a*b^7*x^6 + 8*b^8*x^8)) + (5*a*b^4*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]])/4 - (5*a*b^3*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^2 +
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/4 - (5*a^9*b^4*Log[a - Sqrt[b^2]*x^2 + Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4) -
 (5*a^9*b^3*Sqrt[b^2]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4*(a - Sqrt[
b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4) + (5*a^7*b^4*(-(Sqrt[b^2]*x^
2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]])/((-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4) + (5*a^7*b^3
*Sqrt[b^2]*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*Log[a - S
qrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/((-a - Sqrt[b^2]*x^2 + Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])^4) - (15*a^5*b^4*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])^4*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(2*(-a -
Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])^4) - (15*a^5*b^3*Sqrt[b^2]*(-(Sqrt[b^2]*x^2)
+ Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]])/(2*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4) + (5*a^3*b^4*
(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^6*Log[a - Sqrt[b^2]*x^
2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/((-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^
4) + (5*a^3*b^3*Sqrt[b^2]*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])^6*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/((-a - Sqrt
[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])^4) - (5*a*b^4*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])^8*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4*(a - Sqrt[b^
2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4) - (5*a*b^3*Sqrt[b^2]*(-(Sqrt[b
^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^8*Log[a - Sqrt[b^2]*x^2 + Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4)
```

fricas [A] time = 0.83, size = 61, normalized size = 0.24

$$\frac{12\,b^5x^{10}+120\,ab^4x^8\log(x)-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^4
```

```
- 20*a^4*b*x^2 - 3*a^5)/x^8
```

giac [A] time = 0.19, size = 126, normalized size = 0.50

$$\frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,ab^4\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{125\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+120\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+60\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+20\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a^5\mathrm{sgn}\left(bx^2+a\right)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x^2 + a) + 5/2*a*b^4*log(x^2)*sgn(b*x^2 + a) - 1/24*(125*
a*b^4*x^8*sgn(b*x^2 + a) + 120*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^4*
sgn(b*x^2 + a) + 20*a^4*b*x^2*sgn(b*x^2 + a) + 3*a^5*sgn(b*x^2 + a))/x^8
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\left(12b^5x^{10}+120a\,b^4x^8\ln(x)-120a^2b^3x^6-60a^3b^2x^4-20a^4b\,x^2-3a^5\right)}{24\left(b\,x^2+a\right)^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x)
```

```
[Out] 1/24*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+120*a*b^4*ln(x)*x^8-120*a^2*b^3*x^6-6
0*a^3*b^2*x^4-20*a^4*b*x^2-3*a^5)/(b*x^2+a)^5/x^8
```

maxima [A] time = 1.36, size = 56, normalized size = 0.22

$$\frac{1}{2}\,b^5x^2+5\,ab^4\log(x)-\frac{5\,a^2b^3}{x^2}-\frac{5\,a^3b^2}{2\,x^4}-\frac{5\,a^4b}{6\,x^6}-\frac{a^5}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*log(x) - 5*a^2*b^3/x^2 - 5/2*a^3*b^2/x^4 - 5/6*a^4*b/
x^6 - 1/8*a^5/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**9, x)
```

3.425 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=251

$$\frac{b^5\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-5a$$

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^5\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(3*x^6*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(2*x^2*(a + b*x^2)) + (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*
x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{11}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{11}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^6}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right)dx,x,\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x}}{3x^6\left(a+bx^2\right)}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a\left(12a^4+75a^3bx^2+200a^2b^2x^4+300ab^3x^6+300b^4x^8\right)-120b^5x^{10}\log(x)\right)}{120x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -1/120*(Sqrt[(a + b*x^2)^2]*(a*(12*a^4 + 75*a^3*b*x^2 + 200*a^2*b^2*x^4 + 3
00*a*b^3*x^6 + 300*b^4*x^8) - 120*b^5*x^10*Log[x]))/(x^10*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 3.60, size = 2386, normalized size = 9.51

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] (2*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-12*a^8*b - 123*a^7*b^2*x^2 - 572
*a^6*b^3*x^4 - 1598*a^5*b^4*x^6 - 3012*a^4*b^5*x^8 - 3875*a^3*b^6*x^10 - 32
00*a^2*b^7*x^12 - 1500*a*b^8*x^14 - 300*b^9*x^16) + 2*a*b^4*Sqrt[b^2]*(12*a
^9 + 135*a^8*b*x^2 + 695*a^7*b^2*x^4 + 2170*a^6*b^3*x^6 + 4610*a^5*b^4*x^8
+ 6887*a^4*b^5*x^10 + 7075*a^3*b^6*x^12 + 4700*a^2*b^7*x^14 + 1800*a*b^8*x^
16 + 300*b^9*x^18))/(15*Sqrt[b^2]*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-16
```

```
*a^4*b^4 - 64*a^3*b^5*x^2 - 96*a^2*b^6*x^4 - 64*a*b^7*x^6 - 16*b^8*x^8) + 1
5*x^10*(16*a^5*b^5 + 80*a^4*b^6*x^2 + 160*a^3*b^7*x^4 + 160*a^2*b^8*x^6 + 8
0*a*b^9*x^8 + 16*b^10*x^10)) + (b^5*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]])/4 - (b^4*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]])/4 + (a^10*b^5*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])^5*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5) + (a^10*b^4*S
qrt[b^2]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a -
 Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5*(a - Sqrt[b^2]*x^2 + Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])^5) - (5*a^8*b^5*(-(Sqrt[b^2]*x^2) + Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])^2*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5*(a -
Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5) - (5*a^8*b^4*Sqrt[b^2]*
(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*Log[a - Sqrt[b^2]*x^
2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])^5*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)^5) + (5*a^6*b^5*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4*Lo
g[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(2*(-a - Sqrt[b^2]*
x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])^5) + (5*a^6*b^4*Sqrt[b^2]*(-(Sqrt[b^2]*x^2) + Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])^4*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]])/(2*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5*(a - S
qrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5) - (5*a^4*b^5*(-(Sqrt[b^2
]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^6*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]])/(2*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])^5*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5) - (5*a
^4*b^4*Sqrt[b^2]*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^6*Log
[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(2*(-a - Sqrt[b^2]*x
^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])^5) + (5*a^2*b^5*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])^8*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4
*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5*(a - Sqrt[b^2]*x^
2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5) + (5*a^2*b^4*Sqrt[b^2]*(-(Sqrt[b^2]
*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^8*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])^5*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5) - (b^5*
(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^10*Log[a - Sqrt[b^2]*x
^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt[b^2]*x^2 + Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])^5*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])^5) - (b^4*Sqrt[b^2]*(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
^10*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(-a - Sqrt
[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^5*(a - Sqrt[b^2]*x^2 + Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])^5)
```

fricas [A] time = 0.60, size = 61, normalized size = 0.24

$$\frac{120 \, b^{5} x^{10} \log\left(x\right) - 300 \, a b^{4} x^{8} - 300 \, a^{2} b^{3} x^{6} - 200 \, a^{3} b^{2} x^{4} - 75 \, a^{4} b x^{2} - 12 \, a^{5}}{120 \, x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2*
x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

giac [A] time = 0.16, size = 125, normalized size = 0.50

$$\frac{1}{2} \, b^{5} \log\left(x^{2}\right) \mathrm{sgn}\left(b x^{2} + a\right) - \frac{137 \, b^{5} x^{10} \mathrm{sgn}\left(b x^{2} + a\right) + 300 \, a b^{4} x^{8} \mathrm{sgn}\left(b x^{2} + a\right) + 300 \, a^{2} b^{3} x^{6} \mathrm{sgn}\left(b x^{2} + a\right) + 200 \, a^{3} b^{2} x^{4} \mathrm{sgn}\left(b x^{2} + a\right) + 75 \, a^{4} b x^{2} \mathrm{sgn}\left(b x^{2} + a\right) + 12 \, a^{5} \mathrm{sgn}\left(b x^{2} + a\right)}{120 \, x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(x^2)*sgn(b*x^2 + a) - 1/120*(137*b^5*x^10*sgn(b*x^2 + a) + 300*
a*b^4*x^8*sgn(b*x^2 + a) + 300*a^2*b^3*x^6*sgn(b*x^2 + a) + 200*a^3*b^2*x^4
*sgn(b*x^2 + a) + 75*a^4*b*x^2*sgn(b*x^2 + a) + 12*a^5*sgn(b*x^2 + a))/x^10
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}} \left(120 b^{5} x^{10} \ln\left(x\right)-300 a\,b^{4} x^{8}-300 a^{2} b^{3} x^{6}-200 a^{3} b^{2} x^{4}-75 a^{4} b\,x^{2}-12 a^{5}\right)}{120 \left(b\,x^{2}+a\right)^{5} x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(120*b^5*ln(x)*x^10-300*a*b^4*x^8-300*a^2*b^3*x^6
-200*a^3*b^2*x^4-75*a^4*b*x^2-12*a^5)/(b*x^2+a)^5/x^10
```

maxima [A] time = 1.32, size = 55, normalized size = 0.22

$$b^{5} \log\left(x\right) - \frac{5 \, a b^{4}}{2 \, x^{2}} - \frac{5 \, a^{2} b^{3}}{2 \, x^{4}} - \frac{5 \, a^{3} b^{2}}{3 \, x^{6}} - \frac{5 \, a^{4} b}{8 \, x^{8}} - \frac{a^{5}}{10 \, x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] b^5*log(x) - 5/2*a*b^4/x^2 - 5/2*a^2*b^3/x^4 - 5/3*a^3*b^2/x^6 - 5/8*a^4*b/
x^8 - 1/10*a^5/x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^11,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**11, x)
```

3.426 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*a*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^7}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.98

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^5+6a^4bx^2+15a^3b^2x^4+20a^2b^3x^6+15ab^4x^8+6b^5x^{10}\right)}{12x^{12}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^2)^2]*(a^5 + 6*a^4*b*x^2 + 15*a^3*b^2*x^4 + 20*a^2*b^3
*x^6 + 15*a*b^4*x^8 + 6*b^5*x^10))/(x^12*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 2.63, size = 442, normalized size = 10.78

$$\frac{8b^5\sqrt{a^2+2abx^2+b^2x^4}\left(-a^{10}b-11a^9b^2x^2-55a^8b^3x^4-165a^7b^4x^6-330a^6b^5x^8-462a^5b^6x^{10}-461a^4b^7x^{12}-325a^3b^8x^{14}-155a^2b^9x^{16}-45ab^{10}x^{18}-6b^{11}x^{20}\right)+8\sqrt{b^2}b^5\left(a^{11}+12a^{10}bx^2+66a^9b^2x^4+220a^8b^3x^6+495a^7b^4x^8+792a^6b^5x^{10}+923a^5b^6x^{12}+786a^4b^7x^{14}+480a^3b^8x^{16}+200a^2b^9x^{18}+51ab^{10}x^{20}+6b^{11}x^{22}\right)}{3\sqrt{b^2}x^{12}\sqrt{a^2+2abx^2+b^2x^4}\left(-32a^5b^5-160a^4b^6x^2-320a^3b^7x^4-320a^2b^8x^6-160ab^9x^8-32b^{10}x^{10}\right)+3x^{12}\left(32a^6b^6+192a^5b^7x^2+480a^4b^8x^4+640a^3b^9x^6+480a^2b^{10}x^8+192ab^{11}x^{10}+32b^{12}x^{12}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] (8*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^10*b) - 11*a^9*b^2*x^2 - 55*a^8
*b^3*x^4 - 165*a^7*b^4*x^6 - 330*a^6*b^5*x^8 - 462*a^5*b^6*x^10 - 461*a^4*b
^7*x^12 - 325*a^3*b^8*x^14 - 155*a^2*b^9*x^16 - 45*a*b^10*x^18 - 6*b^11*x^2
0) + 8*b^5*Sqrt[b^2]*(a^11 + 12*a^10*b*x^2 + 66*a^9*b^2*x^4 + 220*a^8*b^3*x
^6 + 495*a^7*b^4*x^8 + 792*a^6*b^5*x^10 + 923*a^5*b^6*x^12 + 786*a^4*b^7*x^
14 + 480*a^3*b^8*x^16 + 200*a^2*b^9*x^18 + 51*a*b^10*x^20 + 6*b^11*x^22))/(
3*Sqrt[b^2]*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-32*a^5*b^5 - 160*a^4*b^6
*x^2 - 320*a^3*b^7*x^4 - 320*a^2*b^8*x^6 - 160*a*b^9*x^8 - 32*b^10*x^10) +
3*x^12*(32*a^6*b^6 + 192*a^5*b^7*x^2 + 480*a^4*b^8*x^4 + 640*a^3*b^9*x^6 +
480*a^2*b^10*x^8 + 192*a*b^11*x^10 + 32*b^12*x^12))
```

fricas [B] time = 0.87, size = 57, normalized size = 1.39

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

giac [B] time = 0.16, size = 106, normalized size = 2.59

$$-\frac{6\,b^5x^{10}\operatorname{sgn}\left(bx^2+a\right)+15\,ab^4x^8\operatorname{sgn}\left(bx^2+a\right)+20\,a^2b^3x^6\operatorname{sgn}\left(bx^2+a\right)+15\,a^3b^2x^4\operatorname{sgn}\left(bx^2+a\right)+6\,a^4bx^2\operatorname{sgn}\left(bx^2+a\right)+a^5\operatorname{sgn}\left(bx^2+a\right)}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*x^10*sgn(b*x^2 + a) + 15*a*b^4*x^8*sgn(b*x^2 + a) + 20*a^2*b^3
*x^6*sgn(b*x^2 + a) + 15*a^3*b^2*x^4*sgn(b*x^2 + a) + 6*a^4*b*x^2*sgn(b*x^2
 + a) + a^5*sgn(b*x^2 + a))/x^12
```

maple [B] time = 0.01, size = 78, normalized size = 1.90

$$-\frac{\left(6b^5x^{10}+15a\,b^4x^8+20a^2b^3x^6+15a^3b^2x^4+6a^4b\,x^2+a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{12\left(b\,x^2+a\right)^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x)
```

```
[Out] -1/12*(6*b^5*x^10+15*a*b^4*x^8+20*a^2*b^3*x^6+15*a^3*b^2*x^4+6*a^4*b*x^2+a^
5)*((b*x^2+a)^2)^(5/2)/x^12/(b*x^2+a)^5
```

maxima [B] time = 1.40, size = 57, normalized size = 1.39

$$-\frac{b^5}{2\,x^2}-\frac{5\,ab^4}{4\,x^4}-\frac{5\,a^2b^3}{3\,x^6}-\frac{5\,a^3b^2}{4\,x^8}-\frac{a^4b}{2\,x^{10}}-\frac{a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

[Out] -1/2*b^5/x^2 - 5/4*a*b^4/x^4 - 5/3*a^2*b^3/x^6 - 5/4*a^3*b^2/x^8 - 1/2*a^4*b/x^10 - 1/12*a^5/x^12

mupad [B] time = 4.18, size = 231, normalized size = 5.63

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^2\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,\left(b\,x^2+a\right)}-\frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^{10}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^6\,\left(b\,x^2+a\right)}-\frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^8\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^13,x)

[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (b^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*x^2*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*x^10*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(3*x^6*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^8*(a + b*x^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**13,x)

[Out] Integral(((a + b*x**2)**2)**(5/2)/x**13, x)

3.427 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{15}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{84a^2x^{14}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12ax^{14}}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1110}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{84a^2x^{14}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*a*x^14) + (a^2 + 2*a*b
*x^2 + b^2*x^4)^(7/2)/(84*a^2*x^14)
```

Rule 1110

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(4*a*d*(p + 1)*(2*p +
1)), x] - Simp[((d*x)^(m + 1)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^p)/(4*a*d*(
2*p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{15}}\,dx=-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12ax^{14}}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{84a^2x^{14}}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.15

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(6a^5+35a^4bx^2+84a^3b^2x^4+105a^2b^3x^6+70ab^4x^8+21b^5x^{10}\right)}{84x^{14}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -1/84*(Sqrt[(a + b*x^2)^2]*(6*a^5 + 35*a^4*b*x^2 + 84*a^3*b^2*x^4 + 105*a^2
*b^3*x^6 + 70*a*b^4*x^8 + 21*b^5*x^10))/(x^14*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.34, size = 488, normalized size = 6.78

$$\frac{16b^6\sqrt{a^2+2abx^2+b^2x^4}\left(-6a^{11}b-71a^{10}b^2x^2-384a^9b^3x^4-1254a^8b^4x^6-2750a^7b^5x^8-4257a^6b^6x^{10}-4752a^5b^7x^{12}-3829a^4b^8x^{14}-2184a^3b^9x^{16}-840a^2b^{10}x^{18}-196ab^{11}x^{20}-21b^{12}x^{22}\right)+16\sqrt{b^2}\,b^6\left(6a^{12}+77a^{11}bx^2+455a^{10}b^2x^4+1638a^9b^3x^6+4004a^8b^4x^8+7007a^7b^5x^{10}+9009a^6b^6x^{12}+8581a^5b^7x^{14}+6013a^4b^8x^{16}+3024a^3b^9x^{18}+1036a^2b^{10}x^{20}+217ab^{11}x^{22}+21b^{12}x^{24}\right)}{21\sqrt{b^2}\,x^{14}\sqrt{a^2+2abx^2+b^2x^4}\left(-64a^6b^6-384a^5b^7x^2-960a^4b^8x^4-1280a^3b^9x^6-960a^2b^{10}x^8-384ab^{11}x^{10}-64b^{12}x^{12}\right)+21x^{14}\left(64a^7b^7+448a^6b^8x^2+1344a^5b^9x^4+2240a^4b^{10}x^6+2240a^3b^{11}x^8+1344a^2b^{12}x^{10}+448ab^{13}x^{12}+64b^{14}x^{14}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] (16*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-6*a^11*b - 71*a^10*b^2*x^2 - 384*
a^9*b^3*x^4 - 1254*a^8*b^4*x^6 - 2750*a^7*b^5*x^8 - 4257*a^6*b^6*x^10 - 475
2*a^5*b^7*x^12 - 3829*a^4*b^8*x^14 - 2184*a^3*b^9*x^16 - 840*a^2*b^10*x^18
- 196*a*b^11*x^20 - 21*b^12*x^22) + 16*b^6*Sqrt[b^2]*(6*a^12 + 77*a^11*b*x^
2 + 455*a^10*b^2*x^4 + 1638*a^9*b^3*x^6 + 4004*a^8*b^4*x^8 + 7007*a^7*b^5*x
^10 + 9009*a^6*b^6*x^12 + 8581*a^5*b^7*x^14 + 6013*a^4*b^8*x^16 + 3024*a^3*
b^9*x^18 + 1036*a^2*b^10*x^20 + 217*a*b^11*x^22 + 21*b^12*x^24))/(21*Sqrt[b
^2]*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-64*a^6*b^6 - 384*a^5*b^7*x^2 - 9
60*a^4*b^8*x^4 - 1280*a^3*b^9*x^6 - 960*a^2*b^10*x^8 - 384*a*b^11*x^10 - 64
*b^12*x^12) + 21*x^14*(64*a^7*b^7 + 448*a^6*b^8*x^2 + 1344*a^5*b^9*x^4 + 22
40*a^4*b^10*x^6 + 2240*a^3*b^11*x^8 + 1344*a^2*b^12*x^10 + 448*a*b^13*x^12
+ 64*b^14*x^14))
```

fricas [A] time = 1.26, size = 59, normalized size = 0.82

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

giac [A] time = 0.16, size = 107, normalized size = 1.49

$$-\frac{21\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+70\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+105\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+84\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+35\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+6\,a^5\mathrm{sgn}\left(bx^2+a\right)}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(21*b^5*x^10*sgn(b*x^2 + a) + 70*a*b^4*x^8*sgn(b*x^2 + a) + 105*a^2*b
^3*x^6*sgn(b*x^2 + a) + 84*a^3*b^2*x^4*sgn(b*x^2 + a) + 35*a^4*b*x^2*sgn(b*
x^2 + a) + 6*a^5*sgn(b*x^2 + a))/x^14
```

maple [A] time = 0.01, size = 80, normalized size = 1.11

$$-\frac{\left(21b^5x^{10}+70a\,b^4x^8+105a^2b^3x^6+84a^3b^2x^4+35a^4b\,x^2+6a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{84\left(b\,x^2+a\right)^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x)
```

```
[Out] -1/84*(21*b^5*x^10+70*a*b^4*x^8+105*a^2*b^3*x^6+84*a^3*b^2*x^4+35*a^4*b*x^2
+6*a^5)*((b*x^2+a)^2)^(5/2)/x^14/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.79

$$-\frac{b^5}{4\,x^4}-\frac{5\,ab^4}{6\,x^6}-\frac{5\,a^2b^3}{4\,x^8}-\frac{a^3b^2}{x^{10}}-\frac{5\,a^4b}{12\,x^{12}}-\frac{a^5}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/4*b^5/x^4 - 5/6*a*b^4/x^6 - 5/4*a^2*b^3/x^8 - a^3*b^2/x^10 - 5/12*a^4*b/
x^12 - 1/14*a^5/x^14
```

mupad [B] time = 4.22, size = 231, normalized size = 3.21

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^8\left(b\,x^2+a\right)}-\frac{a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^{10}\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^15,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)
^(1/2))/(4*x^8*(a + b*x^2)) - (a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(
x^10*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**15, x)
```

3.428 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{17}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{16ax^{16}}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{56a^2x^{14}}-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{336a^3x^{12}}$$

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1111, 646, 45, 37}

$$-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{336a^3x^{12}}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{56a^2x^{14}}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*a*x^16) + (b*(a + b*x^
2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(56*a^2*x^14) - (b^2*(a + b*x^2)^5*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(336*a^3*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{17}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx,x,x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^9}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx,x,\right.}{8ab^3\left(ab+b^2x^2\right)}$$

$$= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} + \frac{b\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}} + \frac{\sqrt{a^2+2a}}{}$$

$$= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} + \frac{b\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}} - \frac{b^2\left(a+b\right.}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.65

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(21a^5+120a^4bx^2+280a^3b^2x^4+336a^2b^3x^6+210ab^4x^8+56b^5x^{10}\right)}{336x^{16}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -1/336*(Sqrt[(a + b*x^2)^2]*(21*a^5 + 120*a^4*b*x^2 + 280*a^3*b^2*x^4 + 336
*a^2*b^3*x^6 + 210*a*b^4*x^8 + 56*b^5*x^10))/(x^16*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.44, size = 532, normalized size = 4.16

$$\frac{8b^7\sqrt{a^2+2abx^2+b^2x^4}\left(-21a^{12}b-267a^{11}b^2x^2-1561a^{10}b^3x^4-5551a^9b^4x^6-13377a^8b^5x^8-23023a^7b^6x^{10}-29029a^6b^7x^{12}-27027a^5b^8x^{14}-18446a^4b^9x^{16}-9002a^3b^{10}x^{18}-2982a^2b^{11}x^{20}-602ab^{12}x^{22}-56b^{13}x^{24}\right)+8\sqrt{b^2}\,b^7\left(21a^{13}+288a^{12}bx^2+1828a^{11}b^2x^4+7112a^{10}b^3x^6+18928a^9b^4x^8+36400a^8b^5x^{10}+52052a^7b^6x^{12}+56056a^6b^7x^{14}+45473a^5b^8x^{16}+27448a^4b^9x^{18}+11984a^3b^{10}x^{20}+3584a^2b^{11}x^{22}+658ab^{12}x^{24}+56b^{13}x^{26}\right)}{21\sqrt{b^2}\,x^{16}\sqrt{a^2+2abx^2+b^2x^4}\left(-128a^7b^7-896a^6b^8x^2-2688a^5b^9x^4-4480a^4b^{10}x^6-4480a^3b^{11}x^8-2688a^2b^{12}x^{10}-896ab^{13}x^{12}-128b^{14}x^{14}\right)+21x^{16}\left(128a^8b^8+1024a^7b^9x^2+3584a^6b^{10}x^4+7168a^5b^{11}x^6+8960a^4b^{12}x^8+7168a^3b^{13}x^{10}+3584a^2b^{14}x^{12}+1024ab^{15}x^{14}+128b^{16}x^{16}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] (8*b^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-21*a^12*b - 267*a^11*b^2*x^2 - 156
1*a^10*b^3*x^4 - 5551*a^9*b^4*x^6 - 13377*a^8*b^5*x^8 - 23023*a^7*b^6*x^10
- 29029*a^6*b^7*x^12 - 27027*a^5*b^8*x^14 - 18446*a^4*b^9*x^16 - 9002*a^3*b
^10*x^18 - 2982*a^2*b^11*x^20 - 602*a*b^12*x^22 - 56*b^13*x^24) + 8*b^7*Sqr
t[b^2]*(21*a^13 + 288*a^12*b*x^2 + 1828*a^11*b^2*x^4 + 7112*a^10*b^3*x^6 +
18928*a^9*b^4*x^8 + 36400*a^8*b^5*x^10 + 52052*a^7*b^6*x^12 + 56056*a^6*b^7
*x^14 + 45473*a^5*b^8*x^16 + 27448*a^4*b^9*x^18 + 11984*a^3*b^10*x^20 + 358
4*a^2*b^11*x^22 + 658*a*b^12*x^24 + 56*b^13*x^26))/(21*Sqrt[b^2]*x^16*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]*(-128*a^7*b^7 - 896*a^6*b^8*x^2 - 2688*a^5*b^9*x
^4 - 4480*a^4*b^10*x^6 - 4480*a^3*b^11*x^8 - 2688*a^2*b^12*x^10 - 896*a*b^1
3*x^12 - 128*b^14*x^14) + 21*x^16*(128*a^8*b^8 + 1024*a^7*b^9*x^2 + 3584*a^
6*b^10*x^4 + 7168*a^5*b^11*x^6 + 8960*a^4*b^12*x^8 + 7168*a^3*b^13*x^10 + 3
584*a^2*b^14*x^12 + 1024*a*b^15*x^14 + 128*b^16*x^16))
```

fricas [A] time = 0.81, size = 59, normalized size = 0.46

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

giac [A] time = 0.21, size = 107, normalized size = 0.84

$$-\frac{56\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+210\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+336\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+280\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+120\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+21\,a^5\mathrm{sgn}\left(bx^2+a\right)}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(56*b^5*x^10*sgn(b*x^2 + a) + 210*a*b^4*x^8*sgn(b*x^2 + a) + 336*a^2
*b^3*x^6*sgn(b*x^2 + a) + 280*a^3*b^2*x^4*sgn(b*x^2 + a) + 120*a^4*b*x^2*sg
n(b*x^2 + a) + 21*a^5*sgn(b*x^2 + a))/x^16
```

maple [A] time = 0.01, size = 80, normalized size = 0.62

$$-\frac{\left(56b^{5}x^{10}+210a\,b^{4}x^{8}+336a^{2}b^{3}x^{6}+280a^{3}b^{2}x^{4}+120a^{4}b\,x^{2}+21a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{336\left(b\,x^{2}+a\right)^{5}x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x)
```

```
[Out] -1/336*(56*b^5*x^10+210*a*b^4*x^8+336*a^2*b^3*x^6+280*a^3*b^2*x^4+120*a^4*b
*x^2+21*a^5)*((b*x^2+a)^2)^(5/2)/x^16/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.45

$$-\frac{b^{5}}{6\,x^{6}}-\frac{5\,ab^{4}}{8\,x^{8}}-\frac{a^{2}b^{3}}{x^{10}}-\frac{5\,a^{3}b^{2}}{6\,x^{12}}-\frac{5\,a^{4}b}{14\,x^{14}}-\frac{a^{5}}{16\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/6*b^5/x^6 - 5/8*a*b^4/x^8 - a^2*b^3/x^10 - 5/6*a^3*b^2/x^12 - 5/14*a^4*b
/x^14 - 1/16*a^5/x^16
```

mupad [B] time = 4.24, size = 231, normalized size = 1.80

$$-\frac{a^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{16\,x^{16}\left(b\,x^{2}+a\right)}-\frac{b^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{6\,x^{6}\left(b\,x^{2}+a\right)}-\frac{5\,a\,b^{4}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{8\,x^{8}\left(b\,x^{2}+a\right)}-\frac{5\,a^{4}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{14\,x^{14}\left(b\,x^{2}+a\right)}-\frac{a^{2}\,b^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{x^{10}\left(b\,x^{2}+a\right)}-\frac{5\,a^{3}\,b^{2}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{6\,x^{12}\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^17,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(x^10*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6
*x^12*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**17, x)
```

3.429 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{19}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^{10}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2}}{16x}$$

Rubi [A] time = 0.16, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^{10}\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(6*x^12*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(2*x^10*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a +
 b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{19}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx,x,x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{10}}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(\frac{a^5b^5}{x^{10}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^7}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^5}\right)dx,x,\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x}}{7x^{14}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(56a^5+315a^4bx^2+720a^3b^2x^4+840a^2b^3x^6+504ab^4x^8+126b^5x^{10}\right)}{1008x^{18}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] -1/1008*(Sqrt[(a + b*x^2)^2]*(56*a^5 + 315*a^4*b*x^2 + 720*a^3*b^2*x^4 + 84
0*a^2*b^3*x^6 + 504*a*b^4*x^8 + 126*b^5*x^10))/(x^18*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.59, size = 576, normalized size = 2.26

$$\frac{16b^8\sqrt{a^2+2abx^2+b^2x^4}\left(-56a^{13}b-763a^{12}b^2x^2-4808a^{11}b^3x^4-18556a^{10}b^4x^6-48944a^9b^5x^8-93184a^8b^6x^{10}-131768a^7b^7x^{12}-140140a^6b^8x^{14}-112112a^5b^9x^{16}-66639a^4b^{10}x^{18}-28608a^3b^{11}x^{20}-8400a^2b^{12}x^{22}-1512ab^{13}x^{24}-126b^{14}x^{26}\right)+16\sqrt{b^2}\,b^8\left(56a^{14}+819a^{13}bx^2+5571a^{12}b^2x^4+23364a^{11}b^3x^6+67500a^{10}b^4x^8+142128a^9b^5x^{10}+224952a^8b^6x^{12}+271908a^7b^7x^{14}+252252a^6b^8x^{16}+178751a^5b^9x^{18}+95247a^4b^{10}x^{20}+37008a^3b^{11}x^{22}+9912a^2b^{12}x^{24}+1638ab^{13}x^{26}+126b^{14}x^{28}\right)}{63\sqrt{b^2}\,x^{18}\sqrt{a^2+2abx^2+b^2x^4}\left(-256a^8b^8-2048a^7b^9x^2-7168a^6b^{10}x^4-14336a^5b^{11}x^6-17920a^4b^{12}x^8-14336a^3b^{13}x^{10}-7168a^2b^{14}x^{12}-2048ab^{15}x^{14}-256b^{16}x^{16}\right)+63x^{18}\left(256a^9b^9+2304a^8b^{10}x^2+9216a^7b^{11}x^4+21504a^6b^{12}x^6+32256a^5b^{13}x^8+32256a^4b^{14}x^{10}+21504a^3b^{15}x^{12}+9216a^2b^{16}x^{14}+2304ab^{17}x^{16}+256b^{18}x^{18}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] (16*b^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-56*a^13*b - 763*a^12*b^2*x^2 - 48
08*a^11*b^3*x^4 - 18556*a^10*b^4*x^6 - 48944*a^9*b^5*x^8 - 93184*a^8*b^6*x^
```

```
10 - 131768*a^7*b^7*x^12 - 140140*a^6*b^8*x^14 - 112112*a^5*b^9*x^16 - 6663
9*a^4*b^10*x^18 - 28608*a^3*b^11*x^20 - 8400*a^2*b^12*x^22 - 1512*a*b^13*x^
24 - 126*b^14*x^26) + 16*b^8*Sqrt[b^2]*(56*a^14 + 819*a^13*b*x^2 + 5571*a^1
2*b^2*x^4 + 23364*a^11*b^3*x^6 + 67500*a^10*b^4*x^8 + 142128*a^9*b^5*x^10 +
 224952*a^8*b^6*x^12 + 271908*a^7*b^7*x^14 + 252252*a^6*b^8*x^16 + 178751*a
^5*b^9*x^18 + 95247*a^4*b^10*x^20 + 37008*a^3*b^11*x^22 + 9912*a^2*b^12*x^2
4 + 1638*a*b^13*x^26 + 126*b^14*x^28))/(63*Sqrt[b^2]*x^18*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4]*(-256*a^8*b^8 - 2048*a^7*b^9*x^2 - 7168*a^6*b^10*x^4 - 14336
*a^5*b^11*x^6 - 17920*a^4*b^12*x^8 - 14336*a^3*b^13*x^10 - 7168*a^2*b^14*x^
12 - 2048*a*b^15*x^14 - 256*b^16*x^16) + 63*x^18*(256*a^9*b^9 + 2304*a^8*b^
10*x^2 + 9216*a^7*b^11*x^4 + 21504*a^6*b^12*x^6 + 32256*a^5*b^13*x^8 + 3225
6*a^4*b^14*x^10 + 21504*a^3*b^15*x^12 + 9216*a^2*b^16*x^14 + 2304*a*b^17*x^
16 + 256*b^18*x^18))
```

fricas [A] time = 0.82, size = 59, normalized size = 0.23

$$-\frac{126\,b^{5}x^{10}+504\,ab^{4}x^{8}+840\,a^{2}b^{3}x^{6}+720\,a^{3}b^{2}x^{4}+315\,a^{4}bx^{2}+56\,a^{5}}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

giac [A] time = 0.17, size = 107, normalized size = 0.42

$$-\frac{126\,b^{5}x^{10}\mathrm{sgn}\left(bx^{2}+a\right)+504\,ab^{4}x^{8}\mathrm{sgn}\left(bx^{2}+a\right)+840\,a^{2}b^{3}x^{6}\mathrm{sgn}\left(bx^{2}+a\right)+720\,a^{3}b^{2}x^{4}\mathrm{sgn}\left(bx^{2}+a\right)+315\,a^{4}bx^{2}\mathrm{sgn}\left(bx^{2}+a\right)+56\,a^{5}\mathrm{sgn}\left(bx^{2}+a\right)}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10*sgn(b*x^2 + a) + 504*a*b^4*x^8*sgn(b*x^2 + a) + 840*a
^2*b^3*x^6*sgn(b*x^2 + a) + 720*a^3*b^2*x^4*sgn(b*x^2 + a) + 315*a^4*b*x^2*
sgn(b*x^2 + a) + 56*a^5*sgn(b*x^2 + a))/x^18
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(126b^{5}x^{10}+504a\,b^{4}x^{8}+840a^{2}b^{3}x^{6}+720a^{3}b^{2}x^{4}+315a^{4}b\,x^{2}+56a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{1008\left(b\,x^{2}+a\right)^{5}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x)
```

```
[Out] -1/1008*(126*b^5*x^10+504*a*b^4*x^8+840*a^2*b^3*x^6+720*a^3*b^2*x^4+315*a^4
*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/x^18/(b*x^2+a)^5
```

maxima [A] time = 1.33, size = 57, normalized size = 0.22

$$-\frac{b^5}{8\,x^8}-\frac{ab^4}{2\,x^{10}}-\frac{5\,a^2b^3}{6\,x^{12}}-\frac{5\,a^3b^2}{7\,x^{14}}-\frac{5\,a^4b}{16\,x^{16}}-\frac{a^5}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] -1/8*b^5/x^8 - 1/2*a*b^4/x^10 - 5/6*a^2*b^3/x^12 - 5/7*a^3*b^2/x^14 - 5/16*
a^4*b/x^16 - 1/18*a^5/x^18
```

mupad [B] time = 4.27, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{18\,x^{18}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^{10}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^{12}\,\left(b\,x^2+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^{14}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^19,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(18*x^18*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(2*x^10*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^
(1/2))/(6*x^12*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))
/(7*x^14*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{19}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**19, x)
```

3.430 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{21}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}$$

Rubi [A] time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(20*x^20*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(12*x^12*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10
*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{21}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{11}}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\operatorname{Subst}\left(\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^{10}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^7}+\frac{b^{10}}{x^6}\right) dx, x,\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x}}{8x^{16}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(126a^5+700a^4bx^2+1575a^3b^2x^4+1800a^2b^3x^6+1050ab^4x^8+252b^5x^{10}\right)}{2520x^{20}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

```
[Out] -1/2520*(Sqrt[(a + b*x^2)^2]*(126*a^5 + 700*a^4*b*x^2 + 1575*a^3*b^2*x^4 +
1800*a^2*b^3*x^6 + 1050*a*b^4*x^8 + 252*b^5*x^10))/(x^20*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.67, size = 620, normalized size = 2.43

$$\frac{64b^9\sqrt{a^2+2abx^2+b^2x^4}\left(-126a^{14}b-1834a^{13}b^2x^2-12411a^{12}b^3x^4-51759a^{11}b^4x^6-148626a^{10}b^5x^8-310878a^9b^6x^{10}-488502a^8b^7x^{12}-585858a^7b^8x^{14}-538902a^6b^9x^{16}-378378a^5b^{10}x^{18}-199627a^4b^{11}x^{20}-76743a^3b^{12}x^{22}-20322a^2b^{13}x^{24}-3318ab^{14}x^{26}-252b^{15}x^{28}\right)+64\sqrt{b^2}\,b^9\left(126a^{15}+1960a^{14}bx^2+14245a^{13}b^2x^4+64170a^{12}b^3x^6+200385a^{11}b^4x^8+459504a^{10}b^5x^{10}+799380a^9b^6x^{12}+1074360a^8b^7x^{14}+1124760a^7b^8x^{16}+917280a^6b^9x^{18}+578005a^5b^{10}x^{20}+276370a^4b^{11}x^{22}+97065a^3b^{12}x^{24}+23640a^2b^{13}x^{26}+3570ab^{14}x^{28}+252b^{15}x^{30}\right)}{315\sqrt{b^2}\,x^{20}\sqrt{a^2+2abx^2+b^2x^4}\left(-512a^9b^9-4608a^8b^{10}x^2-18432a^7b^{11}x^4-43008a^6b^{12}x^6-64512a^5b^{13}x^8-64512a^4b^{14}x^{10}-43008a^3b^{15}x^{12}-18432a^2b^{16}x^{14}-4608ab^{17}x^{16}-512b^{18}x^{18}\right)+315x^{20}\left(512a^{10}b^{10}+5120a^9b^{11}x^2+23040a^8b^{12}x^4+61440a^7b^{13}x^6+107520a^6b^{14}x^8+129024a^5b^{15}x^{10}+107520a^4b^{16}x^{12}+61440a^3b^{17}x^{14}+23040a^2b^{18}x^{16}+5120ab^{19}x^{18}+512b^{20}x^{20}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

```
[Out] (64*b^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-126*a^14*b - 1834*a^13*b^2*x^2 -
12411*a^12*b^3*x^4 - 51759*a^11*b^4*x^6 - 148626*a^10*b^5*x^8 - 310878*a^9*
b^6*x^10 - 488502*a^8*b^7*x^12 - 585858*a^7*b^8*x^14 - 538902*a^6*b^9*x^16
```

```
- 378378*a^5*b^10*x^18 - 199627*a^4*b^11*x^20 - 76743*a^3*b^12*x^22 - 20322
*a^2*b^13*x^24 - 3318*a*b^14*x^26 - 252*b^15*x^28) + 64*b^9*Sqrt[b^2]*(126*
a^15 + 1960*a^14*b*x^2 + 14245*a^13*b^2*x^4 + 64170*a^12*b^3*x^6 + 200385*a
^11*b^4*x^8 + 459504*a^10*b^5*x^10 + 799380*a^9*b^6*x^12 + 1074360*a^8*b^7*
x^14 + 1124760*a^7*b^8*x^16 + 917280*a^6*b^9*x^18 + 578005*a^5*b^10*x^20 +
276370*a^4*b^11*x^22 + 97065*a^3*b^12*x^24 + 23640*a^2*b^13*x^26 + 3570*a*b
^14*x^28 + 252*b^15*x^30))/(315*Sqrt[b^2]*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]*(-512*a^9*b^9 - 4608*a^8*b^10*x^2 - 18432*a^7*b^11*x^4 - 43008*a^6*b^12
*x^6 - 64512*a^5*b^13*x^8 - 64512*a^4*b^14*x^10 - 43008*a^3*b^15*x^12 - 184
32*a^2*b^16*x^14 - 4608*a*b^17*x^16 - 512*b^18*x^18) + 315*x^20*(512*a^10*b
^10 + 5120*a^9*b^11*x^2 + 23040*a^8*b^12*x^4 + 61440*a^7*b^13*x^6 + 107520*
a^6*b^14*x^8 + 129024*a^5*b^15*x^10 + 107520*a^4*b^16*x^12 + 61440*a^3*b^17
*x^14 + 23040*a^2*b^18*x^16 + 5120*a*b^19*x^18 + 512*b^20*x^20))
```

fricas [A] time = 0.73, size = 59, normalized size = 0.23

$$-\frac{252 \, b^{5} x^{10} + 1050 \, a b^{4} x^{8} + 1800 \, a^{2} b^{3} x^{6} + 1575 \, a^{3} b^{2} x^{4} + 700 \, a^{4} b x^{2} + 126 \, a^{5}}{2520 \, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

giac [A] time = 0.16, size = 107, normalized size = 0.42

$$-\frac{252 \, b^{5} x^{10} \mathrm{sgn}\left(b x^{2} + a\right) + 1050 \, a b^{4} x^{8} \mathrm{sgn}\left(b x^{2} + a\right) + 1800 \, a^{2} b^{3} x^{6} \mathrm{sgn}\left(b x^{2} + a\right) + 1575 \, a^{3} b^{2} x^{4} \mathrm{sgn}\left(b x^{2} + a\right) + 700 \, a^{4} b x^{2} \mathrm{sgn}\left(b x^{2} + a\right) + 126 \, a^{5} \mathrm{sgn}\left(b x^{2} + a\right)}{2520 \, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/2520*(252*b^5*x^10*sgn(b*x^2 + a) + 1050*a*b^4*x^8*sgn(b*x^2 + a) + 1800
*a^2*b^3*x^6*sgn(b*x^2 + a) + 1575*a^3*b^2*x^4*sgn(b*x^2 + a) + 700*a^4*b*x
^2*sgn(b*x^2 + a) + 126*a^5*sgn(b*x^2 + a))/x^20
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(252 b^{5} x^{10}+1050 a \,b^{4} x^{8}+1800 a^{2} b^{3} x^{6}+1575 a^{3} b^{2} x^{4}+700 a^{4} b \,x^{2}+126 a^{5}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}{2520 \left(b \,x^{2}+a \right)^{5} x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x)
```

```
[Out] -1/2520*(252*b^5*x^10+1050*a*b^4*x^8+1800*a^2*b^3*x^6+1575*a^3*b^2*x^4+700*
a^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/x^20/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 57, normalized size = 0.22

$$-\frac{b^5}{10\,x^{10}}-\frac{5\,ab^4}{12\,x^{12}}-\frac{5\,a^2b^3}{7\,x^{14}}-\frac{5\,a^3b^2}{8\,x^{16}}-\frac{5\,a^4b}{18\,x^{18}}-\frac{a^5}{20\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="maxima")
```

```
[Out] -1/10*b^5/x^10 - 5/12*a*b^4/x^12 - 5/7*a^2*b^3/x^14 - 5/8*a^3*b^2/x^16 - 5/
18*a^4*b/x^18 - 1/20*a^5/x^20
```

mupad [B] time = 4.22, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{20\,x^{20}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{18\,x^{18}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^{14}\,\left(b\,x^2+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^{16}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^21,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(20*x^20*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(18*x^18*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(7*x^14*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(8*x^16*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{21}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**21,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**21, x)
```

3.431 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2}}{4x}$$

Rubi [A] time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^20*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(14*x^14*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a
 + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{23}}\,dx=\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx,x,x^2\right)$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\;\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{12}}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\;\mathrm{Subst}\left(\int \left(\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^{10}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^7}\right)dx,x,\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$=-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(252a^5+1386a^4bx^2+3080a^3b^2x^4+3465a^2b^3x^6+1980ab^4x^8+462b^5x^{10}\right)}{5544x^{22}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

```
[Out] -1/5544*(Sqrt[(a + b*x^2)^2]*(252*a^5 + 1386*a^4*b*x^2 + 3080*a^3*b^2*x^4 +
 3465*a^2*b^3*x^6 + 1980*a*b^4*x^8 + 462*b^5*x^10))/(x^22*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.78, size = 664, normalized size = 2.60

$$\frac{128b^{10}\sqrt{a^2+2abx^2+b^2x^4}\left(-252a^{15}b-3906a^{14}b^2x^2-28280a^{13}b^3x^4-126875a^{12}b^4x^6-394470a^{11}b^5x^8-900351a^{10}b^6x^{10}-1558512a^9b^7x^{12}-2083500a^8b^8x^{14}-2168880a^7b^9x^{16}-1758120a^6b^{10}x^{18}-1100736a^5b^{11}x^{20}-522731a^4b^{12}x^{22}-182270a^3b^{13}x^{24}-44055a^2b^{14}x^{26}-6600ab^{15}x^{28}-462b^{16}x^{30}\right)+128\sqrt{b^2}\,b^{10}\left(252a^{16}+4158a^{15}bx^2+32186a^{14}b^2x^4+155155a^{13}b^3x^6+521345a^{12}b^4x^8+1294821a^{11}b^5x^{10}+2458863a^{10}b^6x^{12}+3642012a^9b^7x^{14}+4252380a^8b^8x^{16}+3927000a^7b^9x^{18}+2858856a^6b^{10}x^{20}+1623467a^5b^{11}x^{22}+705001a^4b^{12}x^{24}+226325a^3b^{13}x^{26}+50655a^2b^{14}x^{28}+7062ab^{15}x^{30}+462b^{16}x^{32}\right)}{693\sqrt{b^2}\,x^{22}\sqrt{a^2+2abx^2+b^2x^4}\left(-1024a^{10}b^{10}-10240a^9b^{11}x^2-46080a^8b^{12}x^4-122880a^7b^{13}x^6-215040a^6b^{14}x^8-258048a^5b^{15}x^{10}-215040a^4b^{16}x^{12}-122880a^3b^{17}x^{14}-46080a^2b^{18}x^{16}-10240ab^{19}x^{18}-1024b^{20}x^{20}\right)+693x^{22}\left(1024a^{11}b^{11}+11264a^{10}b^{12}x^2+56320a^9b^{13}x^4+168960a^8b^{14}x^6+337920a^7b^{15}x^8+473088a^6b^{16}x^{10}+473088a^5b^{17}x^{12}+337920a^4b^{18}x^{14}+168960a^3b^{19}x^{16}+56320a^2b^{20}x^{18}+11264ab^{21}x^{20}+1024b^{22}x^{22}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

```
[Out] (128*b^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-252*a^15*b - 3906*a^14*b^2*x^2
- 28280*a^13*b^3*x^4 - 126875*a^12*b^4*x^6 - 394470*a^11*b^5*x^8 - 900351*a
```

```
^10*b^6*x^10 - 1558512*a^9*b^7*x^12 - 2083500*a^8*b^8*x^14 - 2168880*a^7*b^
9*x^16 - 1758120*a^6*b^10*x^18 - 1100736*a^5*b^11*x^20 - 522731*a^4*b^12*x^
22 - 182270*a^3*b^13*x^24 - 44055*a^2*b^14*x^26 - 6600*a*b^15*x^28 - 462*b^
16*x^30) + 128*b^10*Sqrt[b^2]*(252*a^16 + 4158*a^15*b*x^2 + 32186*a^14*b^2*
x^4 + 155155*a^13*b^3*x^6 + 521345*a^12*b^4*x^8 + 1294821*a^11*b^5*x^10 + 2
458863*a^10*b^6*x^12 + 3642012*a^9*b^7*x^14 + 4252380*a^8*b^8*x^16 + 392700
0*a^7*b^9*x^18 + 2858856*a^6*b^10*x^20 + 1623467*a^5*b^11*x^22 + 705001*a^4
*b^12*x^24 + 226325*a^3*b^13*x^26 + 50655*a^2*b^14*x^28 + 7062*a*b^15*x^30
+ 462*b^16*x^32))/(693*Sqrt[b^2]*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-102
4*a^10*b^10 - 10240*a^9*b^11*x^2 - 46080*a^8*b^12*x^4 - 122880*a^7*b^13*x^6
 - 215040*a^6*b^14*x^8 - 258048*a^5*b^15*x^10 - 215040*a^4*b^16*x^12 - 1228
80*a^3*b^17*x^14 - 46080*a^2*b^18*x^16 - 10240*a*b^19*x^18 - 1024*b^20*x^20
) + 693*x^22*(1024*a^11*b^11 + 11264*a^10*b^12*x^2 + 56320*a^9*b^13*x^4 + 1
68960*a^8*b^14*x^6 + 337920*a^7*b^15*x^8 + 473088*a^6*b^16*x^10 + 473088*a^
5*b^17*x^12 + 337920*a^4*b^18*x^14 + 168960*a^3*b^19*x^16 + 56320*a^2*b^20*
x^18 + 11264*a*b^21*x^20 + 1024*b^22*x^22))
```

fricas [A] time = 0.96, size = 59, normalized size = 0.23

$$-\frac{462\,b^5x^{10}+1980\,ab^4x^8+3465\,a^2b^3x^6+3080\,a^3b^2x^4+1386\,a^4bx^2+252\,a^5}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/5544*(462*b^5*x^10 + 1980*a*b^4*x^8 + 3465*a^2*b^3*x^6 + 3080*a^3*b^2*x^
4 + 1386*a^4*b*x^2 + 252*a^5)/x^22
```

giac [A] time = 0.19, size = 107, normalized size = 0.42

$$-\frac{462\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+1980\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+3465\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+3080\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+1386\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+252\,a^5\mathrm{sgn}\left(bx^2+a\right)}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/5544*(462*b^5*x^10*sgn(b*x^2 + a) + 1980*a*b^4*x^8*sgn(b*x^2 + a) + 3465
*a^2*b^3*x^6*sgn(b*x^2 + a) + 3080*a^3*b^2*x^4*sgn(b*x^2 + a) + 1386*a^4*b*
x^2*sgn(b*x^2 + a) + 252*a^5*sgn(b*x^2 + a))/x^22
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(462b^5x^{10}+1980a\,b^4x^8+3465a^2b^3x^6+3080a^3b^2x^4+1386a^4b\,x^2+252a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{5544\left(b\,x^2+a\right)^5x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x)
```

```
[Out] -1/5544*(462*b^5*x^10+1980*a*b^4*x^8+3465*a^2*b^3*x^6+3080*a^3*b^2*x^4+1386
*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/x^22/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 57, normalized size = 0.22

$$-\frac{b^5}{12\,x^{12}}-\frac{5\,ab^4}{14\,x^{14}}-\frac{5\,a^2b^3}{8\,x^{16}}-\frac{5\,a^3b^2}{9\,x^{18}}-\frac{a^4b}{4\,x^{20}}-\frac{a^5}{22\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] -1/12*b^5/x^12 - 5/14*a*b^4/x^14 - 5/8*a^2*b^3/x^16 - 5/9*a^3*b^2/x^18 - 1/
4*a^4*b/x^20 - 1/22*a^5/x^22
```

mupad [B] time = 4.23, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{22\,x^{22}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}-\frac{a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{20}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^{16}\,\left(b\,x^2+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^{18}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^23,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(22*x^22*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(4*x^20*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(8*x^16*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(9*x^18*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{23}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**23,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**23, x)
```

3.432 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{25}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a}}{22}$$

Rubi [A] time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*x^24*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(2*x^20*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(16*x^16*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(
a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{25}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{13}}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(\frac{a^5b^5}{x^{13}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^{11}}+\frac{10a^2b^8}{x^{10}}+\frac{5ab^9}{x^9}+\frac{b^{10}}{x^8}\right) dx, x, \right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)} - \frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(462a^5+2520a^4bx^2+5544a^3b^2x^4+6160a^2b^3x^6+3465ab^4x^8+792b^5x^{10}\right)}{11088x^{24}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -1/11088*(Sqrt[(a + b*x^2)^2]*(462*a^5 + 2520*a^4*b*x^2 + 5544*a^3*b^2*x^4
+ 6160*a^2*b^3*x^6 + 3465*a*b^4*x^8 + 792*b^5*x^10))/(x^24*(a + b*x^2))
```

IntegrateAlgebraic [B] time = 1.90, size = 708, normalized size = 2.78

$$\frac{128b^{11}\sqrt{a^2+2abx^2+b^2x^4}\left(-462a^{16}b-7602a^{15}b^2x^2-58674a^{14}b^3x^4-281974a^{13}b^4x^6-944405a^{12}b^5x^8-2337511a^{11}b^6x^{10}-4422891a^{10}b^7x^{12}-6526113a^9b^8x^{14}-7589208a^8b^9x^{16}-6978840a^7b^{10}x^{18}-5057976a^6b^{11}x^{20}-2858856a^5b^{12}x^{22}-1235389a^4b^{13}x^{24}-394559a^3b^{14}x^{26}-87835a^2b^{15}x^{28}-12177ab^{16}x^{30}-792b^{17}x^{32}\right)+128\sqrt{b^2}\,b^{11}\left(462a^{17}+8064a^{16}bx^2+66276a^{15}b^2x^4+340648a^{14}b^3x^6+1226379a^{13}b^4x^8+3281916a^{12}b^5x^{10}+6760402a^{11}b^6x^{12}+10949004a^{10}b^7x^{14}+14115321a^9b^8x^{16}+14568048a^8b^9x^{18}+12036816a^7b^{10}x^{20}+7916832a^6b^{11}x^{22}+4094245a^5b^{12}x^{24}+1629948a^4b^{13}x^{26}+482394a^3b^{14}x^{28}+100012a^2b^{15}x^{30}+12969ab^{16}x^{32}+792b^{17}x^{34}\right)}{693\sqrt{b^2}\,x^{24}\sqrt{a^2+2abx^2+b^2x^4}\left(-2048a^{11}b^{11}-22528a^{10}b^{12}x^2-112640a^9b^{13}x^4-337920a^8b^{14}x^6-675840a^7b^{15}x^8-946176a^6b^{16}x^{10}-946176a^5b^{17}x^{12}-675840a^4b^{18}x^{14}-337920a^3b^{19}x^{16}-112640a^2b^{20}x^{18}-22528ab^{21}x^{20}-2048b^{22}x^{22}\right)+693x^{24}\left(2048a^{12}b^{12}+24576a^{11}b^{13}x^2+135168a^{10}b^{14}x^4+450560a^9b^{15}x^6+1013760a^8b^{16}x^8+1622016a^7b^{17}x^{10}+1892352a^6b^{18}x^{12}+1622016a^5b^{19}x^{14}+1013760a^4b^{20}x^{16}+450560a^3b^{21}x^{18}+135168a^2b^{22}x^{20}+24576ab^{23}x^{22}+2048b^{24}x^{24}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] (128*b^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-462*a^16*b - 7602*a^15*b^2*x^2
- 58674*a^14*b^3*x^4 - 281974*a^13*b^4*x^6 - 944405*a^12*b^5*x^8 - 2337511*
```

a^11*b^6*x^10 - 4422891*a^10*b^7*x^12 - 6526113*a^9*b^8*x^14 - 7589208*a^8*
b^9*x^16 - 6978840*a^7*b^10*x^18 - 5057976*a^6*b^11*x^20 - 2858856*a^5*b^12
*x^22 - 1235389*a^4*b^13*x^24 - 394559*a^3*b^14*x^26 - 87835*a^2*b^15*x^28
- 12177*a*b^16*x^30 - 792*b^17*x^32) + 128*b^11*Sqrt[b^2]*(462*a^17 + 8064*
a^16*b*x^2 + 66276*a^15*b^2*x^4 + 340648*a^14*b^3*x^6 + 1226379*a^13*b^4*x^
8 + 3281916*a^12*b^5*x^10 + 6760402*a^11*b^6*x^12 + 10949004*a^10*b^7*x^14
+ 14115321*a^9*b^8*x^16 + 14568048*a^8*b^9*x^18 + 12036816*a^7*b^10*x^20 +
7916832*a^6*b^11*x^22 + 4094245*a^5*b^12*x^24 + 1629948*a^4*b^13*x^26 + 482
394*a^3*b^14*x^28 + 100012*a^2*b^15*x^30 + 12969*a*b^16*x^32 + 792*b^17*x^3
4))/(693*Sqrt[b^2]*x^24*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-2048*a^11*b^11 -
22528*a^10*b^12*x^2 - 112640*a^9*b^13*x^4 - 337920*a^8*b^14*x^6 - 675840*a^
7*b^15*x^8 - 946176*a^6*b^16*x^10 - 946176*a^5*b^17*x^12 - 675840*a^4*b^18*
x^14 - 337920*a^3*b^19*x^16 - 112640*a^2*b^20*x^18 - 22528*a*b^21*x^20 - 20
48*b^22*x^22) + 693*x^24*(2048*a^12*b^12 + 24576*a^11*b^13*x^2 + 135168*a^1
0*b^14*x^4 + 450560*a^9*b^15*x^6 + 1013760*a^8*b^16*x^8 + 1622016*a^7*b^17*
x^10 + 1892352*a^6*b^18*x^12 + 1622016*a^5*b^19*x^14 + 1013760*a^4*b^20*x^1
6 + 450560*a^3*b^21*x^18 + 135168*a^2*b^22*x^20 + 24576*a*b^23*x^22 + 2048*
b^24*x^24))

fricas [A] time = 0.76, size = 59, normalized size = 0.23

$$-\frac{792\,b^5x^{10}+3465\,ab^4x^8+6160\,a^2b^3x^6+5544\,a^3b^2x^4+2520\,a^4bx^2+462\,a^5}{11088\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="fricas")

[Out] -1/11088*(792*b^5*x^10 + 3465*a*b^4*x^8 + 6160*a^2*b^3*x^6 + 5544*a^3*b^2*x
^4 + 2520*a^4*b*x^2 + 462*a^5)/x^24

giac [A] time = 0.17, size = 107, normalized size = 0.42

$$-\frac{792\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+3465\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+6160\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+5544\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+2520\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+462\,a^5\mathrm{sgn}\left(bx^2+a\right)}{11088\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="giac")

[Out] -1/11088*(792*b^5*x^10*sgn(b*x^2 + a) + 3465*a*b^4*x^8*sgn(b*x^2 + a) + 616
0*a^2*b^3*x^6*sgn(b*x^2 + a) + 5544*a^3*b^2*x^4*sgn(b*x^2 + a) + 2520*a^4*b
*x^2*sgn(b*x^2 + a) + 462*a^5*sgn(b*x^2 + a))/x^24

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(792b^{5}x^{10}+3465a\,b^{4}x^{8}+6160a^{2}b^{3}x^{6}+5544a^{3}b^{2}x^{4}+2520a^{4}b\,x^{2}+462a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{11088\left(b\,x^{2}+a\right)^{5}x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x)
```

```
[Out] -1/11088*(792*b^5*x^10+3465*a*b^4*x^8+6160*a^2*b^3*x^6+5544*a^3*b^2*x^4+252
0*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/x^24/(b*x^2+a)^5
```

maxima [A] time = 1.38, size = 57, normalized size = 0.22

$$-\frac{b^{5}}{14\,x^{14}}-\frac{5\,ab^{4}}{16\,x^{16}}-\frac{5\,a^{2}b^{3}}{9\,x^{18}}-\frac{a^{3}b^{2}}{2\,x^{20}}-\frac{5\,a^{4}b}{22\,x^{22}}-\frac{a^{5}}{24\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

```
[Out] -1/14*b^5/x^14 - 5/16*a*b^4/x^16 - 5/9*a^2*b^3/x^18 - 1/2*a^3*b^2/x^20 - 5/
22*a^4*b/x^22 - 1/24*a^5/x^24
```

mupad [B] time = 4.22, size = 231, normalized size = 0.91

$$-\frac{a^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{24\,x^{24}\left(b\,x^{2}+a\right)}-\frac{b^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{14\,x^{14}\left(b\,x^{2}+a\right)}-\frac{5\,a\,b^{4}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{16\,x^{16}\left(b\,x^{2}+a\right)}-\frac{5\,a^{4}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{22\,x^{22}\left(b\,x^{2}+a\right)}-\frac{5\,a^{2}\,b^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{9\,x^{18}\left(b\,x^{2}+a\right)}-\frac{a^{3}\,b^{2}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,x^{20}\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^25,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(24*x^24*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(22*x^22*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(9*x^18*(a + b*x^2)) - (a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/
2))/(2*x^20*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**25,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**25, x)
```

3.433 $\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23\left(a+bx^2\right)}+\frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{a^5x^{13}\sqrt{a^2+2abx^2+}}{13\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23\left(a+bx^2\right)}+\frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{a^4bx^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a^4*b*x^15*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^3*b^2*x^17*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(19*(a + b*x^2)) + (5*a*b^4*x^21*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(21*(a + b*x^2)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23
*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{12}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5x^{12}+5a^4b^6x^{14}+10a^3b^7x^{16}+10a^2b^8x^{18}+5ab^9x^{\cdots}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{a^4bx^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2\cdots}}{17\left(a+b\cdots\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{\left(a+bx^2\right)^2}\left(156009a^5+676039a^4bx^2+1193010a^3b^2x^4+1067430a^2b^3x^6+482885ab^4x^8+88179b^5x^{10}\right)}{2028117\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^2)^2]*(156009*a^5 + 676039*a^4*b*x^2 + 1193010*a^3*b^2*
x^4 + 1067430*a^2*b^3*x^6 + 482885*a*b^4*x^8 + 88179*b^5*x^10))/(2028117*(a
 + b*x^2))
```

IntegrateAlgebraic [A] time = 19.39, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(156009a^5x^{13}+676039a^4bx^{15}+1193010a^3b^2x^{17}+1067430a^2b^3x^{19}+482885ab^4x^{21}+88179b^5x^{23}\right)}{2028117\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(156009*a^5*x^13 + 676039*a^4*b*x^15 + 1193010*a^3*b^2
*x^17 + 1067430*a^2*b^3*x^19 + 482885*a*b^4*x^21 + 88179*b^5*x^23))/(202811
7*(a + b*x^2))
```

fricas [A] time = 1.07, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{5}{21}\,ab^4x^{21}+\frac{10}{19}\,a^2b^3x^{19}+\frac{10}{17}\,a^3b^2x^{17}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17 +
 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{23}\,b^5x^{23}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{21}\,ab^4x^{21}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{19}\,a^2b^3x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^4bx^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{13}\,a^5x^{13}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sgn(b*x^2 + a) + 5/21*a*b^4*x^21*sgn(b*x^2 + a) + 10/19*a^2*b
^3*x^19*sgn(b*x^2 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^2 + a) + 1/3*a^4*b*x^15
*sgn(b*x^2 + a) + 1/13*a^5*x^13*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(88179b^5x^{10}+482885a\,b^4x^8+1067430a^2b^3x^6+1193010a^3b^2x^4+676039a^4b\,x^2+156009a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^{13}}{2028117\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/2028117*x^13*(88179*b^5*x^10+482885*a*b^4*x^8+1067430*a^2*b^3*x^6+1193010
*a^3*b^2*x^4+676039*a^4*b*x^2+156009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.28, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{5}{21}\,ab^4x^{21}+\frac{10}{19}\,a^2b^3x^{19}+\frac{10}{17}\,a^3b^2x^{17}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17 +
 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{12}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int(x^12*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x**12*((a + b*x**2)**2)**(5/2), x)
```

3.434 $\int x^{10} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{a^5x^{11}\sqrt{a^2+2abx^2+\cdots}}{11\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^3*b^2*x^15*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(19*(a + b*x^2)) + (b^5*x^21*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2
1*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^{10}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \left(a^5b^5x^{10}+5a^4b^6x^{12}+10a^3b^7x^{14}+10a^2b^8x^{16}+5ab^9x\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)} + \frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)} + \frac{2a^3b^2x^{15}\sqrt{a^2+2}}{3\left(a+b\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{\left(a+bx^2\right)^2}\left(88179a^5+373065a^4bx^2+646646a^3b^2x^4+570570a^2b^3x^6+255255ab^4x^8+46189b^5x^{10}\right)}{969969\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^2)^2]*(88179*a^5 + 373065*a^4*b*x^2 + 646646*a^3*b^2*x^
4 + 570570*a^2*b^3*x^6 + 255255*a*b^4*x^8 + 46189*b^5*x^10))/(969969*(a + b
*x^2))
```

IntegrateAlgebraic [A] time = 13.15, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(88179a^5x^{11}+373065a^4bx^{13}+646646a^3b^2x^{15}+570570a^2b^3x^{17}+255255ab^4x^{19}+46189b^5x^{21}\right)}{969969\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(88179*a^5*x^11 + 373065*a^4*b*x^13 + 646646*a^3*b^2*x
^15 + 570570*a^2*b^3*x^17 + 255255*a*b^4*x^19 + 46189*b^5*x^21))/(969969*(a
 + b*x^2))
```

fricas [A] time = 0.97, size = 57, normalized size = 0.22

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 + 5
/13*a^4*b*x^13 + 1/11*a^5*x^11
```

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{21}b^5x^{21}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{19}ab^4x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{17}a^2b^3x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}a^3b^2x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{13}a^4bx^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{11}a^5x^{11}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/21*b^5*x^21*sgn(b*x^2 + a) + 5/19*a*b^4*x^19*sgn(b*x^2 + a) + 10/17*a^2*b
^3*x^17*sgn(b*x^2 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^2 + a) + 5/13*a^4*b*x^13*
sgn(b*x^2 + a) + 1/11*a^5*x^11*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(46189b^5x^{10}+255255a\,b^4x^8+570570a^2b^3x^6+646646a^3b^2x^4+373065a^4b\,x^2+88179a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^{11}}{969969\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/969969*x^11*(46189*b^5*x^10+255255*a*b^4*x^8+570570*a^2*b^3*x^6+646646*a^
3*b^2*x^4+373065*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.22

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 + 5
/13*a^4*b*x^13 + 1/11*a^5*x^11
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{10}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^10*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{10}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**10*((a + b*x**2)**2)**(5/2), x)
```

3.435 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^5x^9\sqrt{a^2+2abx^2+b^2}}{9\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^3*b^2*x^13*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^2*b^3*x^15*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(17*(a + b*x^2)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*
(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^8 \left(ab+b^2x^2\right)^5 dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^5b^5x^8+5a^4b^6x^{10}+10a^3b^7x^{12}+10a^2b^8x^{14}+5ab^9x^{16}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)} + \frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)} + \frac{10a^3b^2x^{13}\sqrt{a^2+2a}}{13\left(a+bx\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^9\sqrt{\left(a+bx^2\right)^2}\left(46189a^5+188955a^4bx^2+319770a^3b^2x^4+277134a^2b^3x^6+122265ab^4x^8+21879b^5x^{10}\right)}{415701\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^2)^2]*(46189*a^5 + 188955*a^4*b*x^2 + 319770*a^3*b^2*x^4
 + 277134*a^2*b^3*x^6 + 122265*a*b^4*x^8 + 21879*b^5*x^10))/(415701*(a + b*
x^2))
```

IntegrateAlgebraic [A] time = 10.43, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(46189a^5x^9+188955a^4bx^{11}+319770a^3b^2x^{13}+277134a^2b^3x^{15}+122265ab^4x^{17}+21879b^5x^{19}\right)}{415701\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(46189*a^5*x^9 + 188955*a^4*b*x^11 + 319770*a^3*b^2*x^
13 + 277134*a^2*b^3*x^15 + 122265*a*b^4*x^17 + 21879*b^5*x^19))/(415701*(a
+ b*x^2))
```

fricas [A] time = 0.47, size = 57, normalized size = 0.22

$$\frac{1}{19}b^5x^{19}+\frac{5}{17}ab^4x^{17}+\frac{2}{3}a^2b^3x^{15}+\frac{10}{13}a^3b^2x^{13}+\frac{5}{11}a^4bx^{11}+\frac{1}{9}a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{19}\,b^5x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\,a^2b^3x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{9}\,a^5x^9\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19*sgn(b*x^2 + a) + 5/17*a*b^4*x^17*sgn(b*x^2 + a) + 2/3*a^2*b^3
*x^15*sgn(b*x^2 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^2 + a) + 5/11*a^4*b*x^11*
sgn(b*x^2 + a) + 1/9*a^5*x^9*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(21879b^5x^{10}+122265a\,b^4x^8+277134a^2b^3x^6+319770a^3b^2x^4+188955a^4b\,x^2+46189a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^9}{415701\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/415701*x^9*(21879*b^5*x^10+122265*a*b^4*x^8+277134*a^2*b^3*x^6+319770*a^3
*b^2*x^4+188955*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.33, size = 57, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**8*((a + b*x**2)**2)**(5/2), x)
```

3.436 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{a^5x^7\sqrt{a^2+2abx^2+b^2}}{7\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (5*a^4*b*x^9*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a*b^4*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(3*(a + b*x^2)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a
+ b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^6\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5x^6+5a^4b^6x^8+10a^3b^7x^{10}+10a^2b^8x^{12}+5ab^9x^{14}+\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2a}}{11\left(a+bx^2\right.}$$

Mathematica [A]   time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{\left(a+bx^2\right)^2}\left(21879a^5+85085a^4bx^2+139230a^3b^2x^4+117810a^2b^3x^6+51051ab^4x^8+9009b^5x^{10}\right)}{153153\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(21879*a^5 + 85085*a^4*b*x^2 + 139230*a^3*b^2*x^4
+ 117810*a^2*b^3*x^6 + 51051*a*b^4*x^8 + 9009*b^5*x^10))/(153153*(a + b*x^2
))
```

IntegrateAlgebraic [A]   time = 8.62, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(21879a^5x^7+85085a^4bx^9+139230a^3b^2x^{11}+117810a^2b^3x^{13}+51051ab^4x^{15}+9009b^5x^{17}\right)}{153153\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(21879*a^5*x^7 + 85085*a^4*b*x^9 + 139230*a^3*b^2*x^11
 + 117810*a^2*b^3*x^13 + 51051*a*b^4*x^15 + 9009*b^5*x^17))/(153153*(a + b*
x^2))
```

fricas [A]   time = 0.94, size = 57, normalized size = 0.22

$$\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

giac [A] time = 0.17, size = 105, normalized size = 0.41

$$\frac{1}{17}b^5x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}ab^4x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{13}a^2b^3x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{11}a^3b^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}a^4bx^9\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{7}a^5x^7\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sgn(b*x^2 + a) + 1/3*a*b^4*x^15*sgn(b*x^2 + a) + 10/13*a^2*b^
3*x^13*sgn(b*x^2 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^2 + a) + 5/9*a^4*b*x^9*s
gn(b*x^2 + a) + 1/7*a^5*x^7*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(9009b^5x^{10}+51051a\,b^4x^8+117810a^2b^3x^6+139230a^3b^2x^4+85085a^4b\,x^2+21879a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^7}{153153\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/153153*x^7*(9009*b^5*x^10+51051*a*b^4*x^8+117810*a^2*b^3*x^6+139230*a^3*b
^2*x^4+85085*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.29, size = 57, normalized size = 0.22

$$\frac{1}{17}b^5x^{17}+\frac{1}{3}ab^4x^{15}+\frac{10}{13}a^2b^3x^{13}+\frac{10}{11}a^3b^2x^{11}+\frac{5}{9}a^4bx^9+\frac{1}{7}a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{6}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(5/2), x)
```

3.437 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^5x^5\sqrt{a^2+2abx^2+b}}{5\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^3*b^2*x^9*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a*b^4*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(13*(a + b*x^2)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*(a
 + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^4\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int\left(a^5b^5x^4+5a^4b^6x^6+10a^3b^7x^8+10a^2b^8x^{10}+5ab^9x^{12}+\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{10a^3b^2x^9\sqrt{a^2+2ab}}{9\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{\left(a+bx^2\right)^2}\left(9009a^5+32175a^4bx^2+50050a^3b^2x^4+40950a^2b^3x^6+17325ab^4x^8+3003b^5x^{10}\right)}{45045\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(9009*a^5 + 32175*a^4*b*x^2 + 50050*a^3*b^2*x^4 +
40950*a^2*b^3*x^6 + 17325*a*b^4*x^8 + 3003*b^5*x^10))/(45045*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 7.54, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(9009a^5x^5+32175a^4bx^7+50050a^3b^2x^9+40950a^2b^3x^{11}+17325ab^4x^{13}+3003b^5x^{15}\right)}{45045\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(9009*a^5*x^5 + 32175*a^4*b*x^7 + 50050*a^3*b^2*x^9 +
40950*a^2*b^3*x^11 + 17325*a*b^4*x^13 + 3003*b^5*x^15))/(45045*(a + b*x^2))
```

fricas [A] time = 0.97, size = 57, normalized size = 0.22

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5/7*a^4*b*x^7 + 1/5*a^5*x^5

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{15}b^5x^{15}\operatorname{sgn}\left(bx^2+a\right)+\frac{5}{13}ab^4x^{13}\operatorname{sgn}\left(bx^2+a\right)+\frac{10}{11}a^2b^3x^{11}\operatorname{sgn}\left(bx^2+a\right)+\frac{10}{9}a^3b^2x^9\operatorname{sgn}\left(bx^2+a\right)+\frac{5}{7}a^4bx^7\operatorname{sgn}\left(bx^2+a\right)+\frac{1}{5}a^5x^5\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")

[Out] 1/15*b^5*x^15*sgn(b*x^2 + a) + 5/13*a*b^4*x^13*sgn(b*x^2 + a) + 10/11*a^2*b^3*x^11*sgn(b*x^2 + a) + 10/9*a^3*b^2*x^9*sgn(b*x^2 + a) + 5/7*a^4*b*x^7*sgn(b*x^2 + a) + 1/5*a^5*x^5*sgn(b*x^2 + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(3003b^5x^{10}+17325a\,b^4x^8+40950a^2b^3x^6+50050a^3b^2x^4+32175a^4b\,x^2+9009a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^5}{45045\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)

[Out] 1/45045*x^5*(3003*b^5*x^10+17325*a*b^4*x^8+40950*a^2*b^3*x^6+50050*a^3*b^2*x^4+32175*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5

maxima [A] time = 1.33, size = 57, normalized size = 0.22

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5/7*a^4*b*x^7 + 1/5*a^5*x^5

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)

```
[Out] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**(5/2), x)
```

3.438 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=252

$$\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^5x^3\sqrt{a^2+2abx^2+b^2}}{3\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^7*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^2*b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(9*(a + b*x^2)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
11*(a + b*x^2)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2
))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^2\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int\left(a^5b^5x^2+5a^4b^6x^4+10a^3b^7x^6+10a^2b^8x^8+5ab^9x^{10}+\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^7\sqrt{a^2+2abx}}{7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^3\sqrt{\left(a+bx^2\right)^2}\left(3003a^5+9009a^4bx^2+12870a^3b^2x^4+10010a^2b^3x^6+4095ab^4x^8+693b^5x^{10}\right)}{9009\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^2)^2]*(3003*a^5 + 9009*a^4*b*x^2 + 12870*a^3*b^2*x^4 + 1
0010*a^2*b^3*x^6 + 4095*a*b^4*x^8 + 693*b^5*x^10))/(9009*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 6.75, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(3003a^5x^3+9009a^4bx^5+12870a^3b^2x^7+10010a^2b^3x^9+4095ab^4x^{11}+693b^5x^{13}\right)}{9009\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(3003*a^5*x^3 + 9009*a^4*b*x^5 + 12870*a^3*b^2*x^7 + 1
0010*a^2*b^3*x^9 + 4095*a*b^4*x^11 + 693*b^5*x^13))/(9009*(a + b*x^2))
```

fricas [A] time = 0.99, size = 56, normalized size = 0.22

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

giac [A] time = 0.18, size = 104, normalized size = 0.41

$$\frac{1}{13}\,b^5x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{9}\,a^2b^3x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}\,a^3b^2x^7\mathrm{sgn}\left(bx^2+a\right)+a^4bx^5\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^5x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13*sgn(b*x^2 + a) + 5/11*a*b^4*x^11*sgn(b*x^2 + a) + 10/9*a^2*b^
3*x^9*sgn(b*x^2 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^2 + a) + a^4*b*x^5*sgn(b*x^
2 + a) + 1/3*a^5*x^3*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{\left(693b^5x^{10}+4095a\,b^4x^8+10010a^2b^3x^6+12870a^3b^2x^4+9009a^4b\,x^2+3003a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^3}{9009\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/9009*x^3*(693*b^5*x^10+4095*a*b^4*x^8+10010*a^2*b^3*x^6+12870*a^3*b^2*x^4
+9009*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.36, size = 56, normalized size = 0.22

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**(5/2), x)
```

3.439 $\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=248

$$\frac{b^5x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{11\left(a+bx^2\right)^5}+\frac{5ab^4x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{9\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{7\left(a+bx^2\right)^5}+\frac{a^5x\left(a^2+2abx\right.}{\left(a+bx\right.}$$

Rubi [A] time = 0.05, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1088, 194}

$$\frac{b^5x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{11\left(a+bx^2\right)^5}+\frac{5ab^4x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{9\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{7\left(a+bx^2\right)^5}+\frac{2a^3b^2x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}+\frac{5a^4bx^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{3\left(a+bx^2\right)^5}+\frac{a^5x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (5*a^4*b*x^3*(a^2
 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(3*(a + b*x^2)^5) + (2*a^3*b^2*x^5*(a^2 + 2*
a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (10*a^2*b^3*x^7*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(5/2))/(7*(a + b*x^2)^5) + (5*a*b^4*x^9*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(5/2))/(9*(a + b*x^2)^5) + (b^5*x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
/(11*(a + b*x^2)^5)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2} \int \left(2ab+2b^2x^2\right)^5 dx}{\left(2ab+2b^2x^2\right)^5}$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2} \int \left(32a^5b^5+160a^4b^6x^2+320a^3b^7x^4+320a^2b^8x^6+160ab\right.}{\left(2ab+2b^2x^2\right)^5}$$
$$= \frac{a^5x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5} + \frac{5a^4bx^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{3\left(a+bx^2\right)^5} + \frac{2a^3b^2x^5\left(a^2+2ab\right.}{\left(a+bx^2\right.}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(693a^5x+1155a^4bx^3+1386a^3b^2x^5+990a^2b^3x^7+385ab^4x^9+63b^5x^{11}\right)}{693\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(693*a^5*x + 1155*a^4*b*x^3 + 1386*a^3*b^2*x^5 + 990*a
^2*b^3*x^7 + 385*a*b^4*x^9 + 63*b^5*x^11))/(693*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 6.35, size = 81, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(693a^5x+1155a^4bx^3+1386a^3b^2x^5+990a^2b^3x^7+385ab^4x^9+63b^5x^{11}\right)}{693\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(693*a^5*x + 1155*a^4*b*x^3 + 1386*a^3*b^2*x^5 + 990*a
^2*b^3*x^7 + 385*a*b^4*x^9 + 63*b^5*x^11))/(693*(a + b*x^2))
```

fricas [A] time = 0.81, size = 54, normalized size = 0.22

$$\frac{1}{11}b^5x^{11} + \frac{5}{9}ab^4x^9 + \frac{10}{7}a^2b^3x^7 + 2\,a^3b^2x^5 + \frac{5}{3}a^4bx^3 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

giac [A] time = 0.16, size = 102, normalized size = 0.41

$$\frac{1}{11}\,b^{5}x^{11}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{5}{9}\,ab^{4}x^{9}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{10}{7}\,a^{2}b^{3}x^{7}\mathrm{sgn}\left(bx^{2}+a\right)+2\,a^{3}b^{2}x^{5}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{5}{3}\,a^{4}bx^{3}\mathrm{sgn}\left(bx^{2}+a\right)+a^{5}x\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sgn(b*x^2 + a) + 5/9*a*b^4*x^9*sgn(b*x^2 + a) + 10/7*a^2*b^3*
x^7*sgn(b*x^2 + a) + 2*a^3*b^2*x^5*sgn(b*x^2 + a) + 5/3*a^4*b*x^3*sgn(b*x^2
 + a) + a^5*x*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 78, normalized size = 0.31

$$\frac{\left(63b^{5}x^{10}+385a\,b^{4}x^{8}+990a^{2}b^{3}x^{6}+1386a^{3}b^{2}x^{4}+1155a^{4}b\,x^{2}+693a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}x}{693\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/693*x*(63*b^5*x^10+385*a*b^4*x^8+990*a^2*b^3*x^6+1386*a^3*b^2*x^4+1155*a^
4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.32, size = 54, normalized size = 0.22

$$\frac{1}{11}\,b^{5}x^{11}+\frac{5}{9}\,ab^{4}x^{9}+\frac{10}{7}\,a^{2}b^{3}x^{7}+2\,a^{3}b^{2}x^{5}+\frac{5}{3}\,a^{4}bx^{3}+a^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2), x)
```

3.440 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+$$

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (5*a^4*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(3*(a + b*x^2)) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(a + b*x^2) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a +
b*x^2)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^2}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(5a^4b^6+\frac{a^5b^5}{x^2}+10a^3b^7x^2+10a^2b^8x^4+5ab^9x^6+b^{10}x^8\right) d}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+}}{3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-63a^5+315a^4bx^2+210a^3b^2x^4+126a^2b^3x^6+45ab^4x^8+7b^5x^{10}\right)}{63x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-63*a^5 + 315*a^4*b*x^2 + 210*a^3*b^2*x^4 + 126*a^2*b
^3*x^6 + 45*a*b^4*x^8 + 7*b^5*x^10))/(63*x*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 10.72, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-63a^5+315a^4bx^2+210a^3b^2x^4+126a^2b^3x^6+45ab^4x^8+7b^5x^{10}\right)}{63x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-63*a^5 + 315*a^4*b*x^2 + 210*a^3*b^2*x^4 + 126*a^2*b
^3*x^6 + 45*a*b^4*x^8 + 7*b^5*x^10))/(63*x*(a + b*x^2))
```

fricas [A] time = 0.85, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a
^4*b*x^2 - 63*a^5)/x
```

giac [A] time = 0.16, size = 103, normalized size = 0.42

$$\frac{1}{9}\,b^5x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,ab^4x^7\mathrm{sgn}\left(bx^2+a\right)+2\,a^2b^3x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{3}\,a^3b^2x^3\mathrm{sgn}\left(bx^2+a\right)+5\,a^4bx\mathrm{sgn}\left(bx^2+a\right)-\frac{a^5\mathrm{sgn}\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sgn(b*x^2 + a) + 5/7*a*b^4*x^7*sgn(b*x^2 + a) + 2*a^2*b^3*x^5*s
gn(b*x^2 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^2 + a) + 5*a^4*b*x*sgn(b*x^2 + a)
- a^5*sgn(b*x^2 + a)/x
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-7b^5x^{10}-45a\,b^4x^8-126a^2b^3x^6-210a^3b^2x^4-315a^4b\,x^2+63a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{63\left(b\,x^2+a\right)^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x)
```

```
[Out] -1/63*(-7*b^5*x^10-45*a*b^4*x^8-126*a^2*b^3*x^6-210*a^3*b^2*x^4-315*a^4*b*x
^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x/(b*x^2+a)^5
```

maxima [A] time = 1.32, size = 55, normalized size = 0.22

$$\frac{1}{9}\,b^5x^9+\frac{5}{7}\,ab^4x^7+2\,a^2b^3x^5+\frac{10}{3}\,a^3b^2x^3+5\,a^4bx-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**2, x)
```

3.441 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=246

$$\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-$$

Rubi [A] time = 0.06, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^3*b^2*x*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^2*b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(3*(a + b*x^2)) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*
x^2) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^4}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(10a^3b^7+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^2}+10a^2b^8x^2+5ab^9x^4+b^{10}x^6\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} + \frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2}}{a+bx^2}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-7a^5-105a^4bx^2+210a^3b^2x^4+70a^2b^3x^6+21ab^4x^8+3b^5x^{10}\right)}{21x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-7*a^5 - 105*a^4*b*x^2 + 210*a^3*b^2*x^4 + 70*a^2*b^3
*x^6 + 21*a*b^4*x^8 + 3*b^5*x^10))/(21*x^3*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 15.89, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-7a^5-105a^4bx^2+210a^3b^2x^4+70a^2b^3x^6+21ab^4x^8+3b^5x^{10}\right)}{21x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-7*a^5 - 105*a^4*b*x^2 + 210*a^3*b^2*x^4 + 70*a^2*b^3
*x^6 + 21*a*b^4*x^8 + 3*b^5*x^10))/(21*x^3*(a + b*x^2))
```

fricas [A] time = 0.65, size = 59, normalized size = 0.24

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^
4*b*x^2 - 7*a^5)/x^3
```

giac [A] time = 0.16, size = 104, normalized size = 0.42

$$\frac{1}{7}\,b^5x^7\mathrm{sgn}\left(bx^2+a\right)+ab^4x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx^2+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx^2+a\right)-\frac{15\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+a^5\mathrm{sgn}\left(bx^2+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sgn(b*x^2 + a) + a*b^4*x^5*sgn(b*x^2 + a) + 10/3*a^2*b^3*x^3*sg
n(b*x^2 + a) + 10*a^3*b^2*x*sgn(b*x^2 + a) - 1/3*(15*a^4*b*x^2*sgn(b*x^2 +
a) + a^5*sgn(b*x^2 + a))/x^3
```

maple [A] time = 0.01, size = 80, normalized size = 0.33

$$-\frac{\left(-3b^5x^{10}-21a\,b^4x^8-70a^2b^3x^6-210a^3b^2x^4+105a^4b\,x^2+7a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{21\left(b\,x^2+a\right)^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x)
```

```
[Out] -1/21*(-3*b^5*x^10-21*a*b^4*x^8-70*a^2*b^3*x^6-210*a^3*b^2*x^4+105*a^4*b*x^
2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^3/(b*x^2+a)^5
```

maxima [A] time = 1.33, size = 54, normalized size = 0.22

$$\frac{1}{7}\,b^5x^7+ab^4x^5+\frac{10}{3}\,a^2b^3x^3+10\,a^3b^2x-\frac{5\,a^4b}{x}-\frac{a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 5*a^4*b/x - 1/3
*a^5/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**4, x)
```

3.442 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6} \, dx$

Optimal. Leaf size=249

$$\frac{b^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}+\frac{5ab^4x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-$$

Rubi [A] time = 0.06, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5ab^4x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{b^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a
 + b*x^2)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^6}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(10a^2b^8+\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^2}+5ab^9x^2+b^{10}x^4\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{x\left(a+bx^2\right)}$$

Mathematica [**A**] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-3a^5-25a^4bx^2-150a^3b^2x^4+150a^2b^3x^6+25ab^4x^8+3b^5x^{10}\right)}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 25*a^4*b*x^2 - 150*a^3*b^2*x^4 + 150*a^2*b^3
*x^6 + 25*a*b^4*x^8 + 3*b^5*x^10))/(15*x^5*(a + b*x^2))
```

IntegrateAlgebraic [**A**] time = 18.62, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-3a^5-25a^4bx^2-150a^3b^2x^4+150a^2b^3x^6+25ab^4x^8+3b^5x^{10}\right)}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 25*a^4*b*x^2 - 150*a^3*b^2*x^4 + 150*a^2*b^3
*x^6 + 25*a*b^4*x^8 + 3*b^5*x^10))/(15*x^5*(a + b*x^2))
```

fricas [**A**] time = 0.87, size = 59, normalized size = 0.24

$$\frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^
4*b*x^2 - 3*a^5)/x^5
```

giac [A] time = 0.17, size = 106, normalized size = 0.43

$$\frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx^2+a\right)+10\,a^2b^3x\mathrm{sgn}\left(bx^2+a\right)-\frac{150\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+25\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a^5\mathrm{sgn}\left(bx^2+a\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sgn(b*x^2 + a) + 5/3*a*b^4*x^3*sgn(b*x^2 + a) + 10*a^2*b^3*x*sg
n(b*x^2 + a) - 1/15*(150*a^3*b^2*x^4*sgn(b*x^2 + a) + 25*a^4*b*x^2*sgn(b*x^
2 + a) + 3*a^5*sgn(b*x^2 + a))/x^5
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-3b^5x^{10}-25a\,b^4x^8-150a^2b^3x^6+150a^3b^2x^4+25a^4b\,x^2+3a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{15\left(b\,x^2+a\right)^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x)
```

```
[Out] -1/15*(-3*b^5*x^10-25*a*b^4*x^8-150*a^2*b^3*x^6+150*a^3*b^2*x^4+25*a^4*b*x^
2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^5/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 55, normalized size = 0.22

$$\frac{1}{5}\,b^5x^5+\frac{5}{3}\,ab^4x^3+10\,a^2b^3x-\frac{10\,a^3b^2}{x}-\frac{5\,a^4b}{3\,x^3}-\frac{a^5}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 10*a^3*b^2/x - 5/3*a^4*b/x^3 -
 1/5*a^5/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**6, x)
```

3.443 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{5ab^4x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-a^4$$

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{5ab^4x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a^4*b*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(x*(a + b*x^2)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a +
b*x^2) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^8}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(5ab^9+\frac{a^5b^5}{x^8}+\frac{5a^4b^6}{x^6}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^2}+b^{10}x^2\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Mathematica [**A**] time = 0.02, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a^5+21a^4bx^2+70a^3b^2x^4+210a^2b^3x^6-105ab^4x^8-7b^5x^{10}\right)}{21x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -1/21*(Sqrt[(a + b*x^2)^2]*(3*a^5 + 21*a^4*b*x^2 + 70*a^3*b^2*x^4 + 210*a^2
*b^3*x^6 - 105*a*b^4*x^8 - 7*b^5*x^10))/(x^7*(a + b*x^2))
```

IntegrateAlgebraic [**A**] time = 18.69, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-3a^5-21a^4bx^2-70a^3b^2x^4-210a^2b^3x^6+105ab^4x^8+7b^5x^{10}\right)}{21x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 21*a^4*b*x^2 - 70*a^3*b^2*x^4 - 210*a^2*b^3*
x^6 + 105*a*b^4*x^8 + 7*b^5*x^10))/(21*x^7*(a + b*x^2))
```

fricas [**A**] time = 0.85, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{10}+105\,ab^4x^8-210\,a^2b^3x^6-70\,a^3b^2x^4-21\,a^4bx^2-3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^
4*b*x^2 - 3*a^5)/x^7
```

giac [A] time = 0.16, size = 106, normalized size = 0.43

$$\frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx^2+a\right)+5\,ab^4x\mathrm{sgn}\left(bx^2+a\right)-\frac{210\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+70\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+21\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a^5\mathrm{sgn}\left(bx^2+a\right)}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x^2 + a) + 5*a*b^4*x*sgn(b*x^2 + a) - 1/21*(210*a^2*b^3*x
^6*sgn(b*x^2 + a) + 70*a^3*b^2*x^4*sgn(b*x^2 + a) + 21*a^4*b*x^2*sgn(b*x^2
+ a) + 3*a^5*sgn(b*x^2 + a))/x^7
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-7b^5x^{10}-105a\,b^4x^8+210a^2b^3x^6+70a^3b^2x^4+21a^4b\,x^2+3a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{21\left(b\,x^2+a\right)^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x)
```

```
[Out] -1/21*(-7*b^5*x^10-105*a*b^4*x^8+210*a^2*b^3*x^6+70*a^3*b^2*x^4+21*a^4*b*x^
2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^7/(b*x^2+a)^5
```

maxima [A] time = 1.29, size = 55, normalized size = 0.22

$$\frac{1}{3}\,b^5x^3+5\,ab^4x-\frac{10\,a^2b^3}{x}-\frac{10\,a^3b^2}{3\,x^3}-\frac{a^4b}{x^5}-\frac{a^5}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 10*a^2*b^3/x - 10/3*a^3*b^2/x^3 - a^4*b/x^5 - 1/7
*a^5/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^8,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**8, x)
```

3.444 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=246

$$\frac{b^5x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5a^4b}{}$$

Rubi [A] time = 0.06, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{b^5x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x
*(a + b*x^2)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{10}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^{10}+\frac{a^5b^5}{x^{10}}+\frac{5a^4b^6}{x^8}+\frac{10a^3b^7}{x^6}+\frac{10a^2b^8}{x^4}+\frac{5ab^9}{x^2}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(7a^5+45a^4bx^2+126a^3b^2x^4+210a^2b^3x^6+315ab^4x^8-63b^5x^{10}\right)}{63x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -1/63*(Sqrt[(a + b*x^2)^2]*(7*a^5 + 45*a^4*b*x^2 + 126*a^3*b^2*x^4 + 210*a^
2*b^3*x^6 + 315*a*b^4*x^8 - 63*b^5*x^10))/(x^9*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 18.69, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-7a^5-45a^4bx^2-126a^3b^2x^4-210a^2b^3x^6-315ab^4x^8+63b^5x^{10}\right)}{63x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-7*a^5 - 45*a^4*b*x^2 - 126*a^3*b^2*x^4 - 210*a^2*b^3
*x^6 - 315*a*b^4*x^8 + 63*b^5*x^10))/(63*x^9*(a + b*x^2))
```

fricas [A] time = 0.82, size = 59, normalized size = 0.24

$$\frac{63\,b^5x^{10}-315\,ab^4x^8-210\,a^2b^3x^6-126\,a^3b^2x^4-45\,a^4bx^2-7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*
a^4*b*x^2 - 7*a^5)/x^9
```

giac [A] time = 0.17, size = 105, normalized size = 0.43

$$b^5 x\,\mathrm{sgn}\left(bx^2+a\right) - \frac{315\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right) + 210\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right) + 126\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right) + 45\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right) + 7\,a^5\mathrm{sgn}\left(bx^2+a\right)}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x^2 + a) - 1/63*(315*a*b^4*x^8*sgn(b*x^2 + a) + 210*a^2*b^3*x^6
*sgn(b*x^2 + a) + 126*a^3*b^2*x^4*sgn(b*x^2 + a) + 45*a^4*b*x^2*sgn(b*x^2 +
 a) + 7*a^5*sgn(b*x^2 + a))/x^9
```

maple [A] time = 0.01, size = 80, normalized size = 0.33

$$-\frac{\left(-63b^5x^{10}+315a\,b^4x^8+210a^2b^3x^6+126a^3b^2x^4+45a^4b\,x^2+7a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{63\left(b\,x^2+a\right)^5 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x)
```

```
[Out] -1/63*(-63*b^5*x^10+315*a*b^4*x^8+210*a^2*b^3*x^6+126*a^3*b^2*x^4+45*a^4*b*
x^2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^9/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 54, normalized size = 0.22

$$b^5x - \frac{5\,ab^4}{x} - \frac{10\,a^2b^3}{3\,x^3} - \frac{2\,a^3b^2}{x^5} - \frac{5\,a^4b}{7\,x^7} - \frac{a^5}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] b^5*x - 5*a*b^4/x - 10/3*a^2*b^3/x^3 - 2*a^3*b^2/x^5 - 5/7*a^4*b/x^7 - 1/9*
a^5/x^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**10, x)
```

3.445 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=251

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{}}{}$$

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(x^5*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/
(3*x^3*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{12}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^{10}}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^6}+\frac{5ab^9}{x^4}+\frac{b^{10}}{x^2}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{7x^7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(63a^5+385a^4bx^2+990a^3b^2x^4+1386a^2b^3x^6+1155ab^4x^8+693b^5x^{10}\right)}{693x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -1/693*(Sqrt[(a + b*x^2)^2]*(63*a^5 + 385*a^4*b*x^2 + 990*a^3*b^2*x^4 + 138
6*a^2*b^3*x^6 + 1155*a*b^4*x^8 + 693*b^5*x^10))/(x^11*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 18.96, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-63a^5-385a^4bx^2-990a^3b^2x^4-1386a^2b^3x^6-1155ab^4x^8-693b^5x^{10}\right)}{693x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-63*a^5 - 385*a^4*b*x^2 - 990*a^3*b^2*x^4 - 1386*a^2*
b^3*x^6 - 1155*a*b^4*x^8 - 693*b^5*x^10))/(693*x^11*(a + b*x^2))
```

fricas [A] time = 0.94, size = 59, normalized size = 0.24

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="fricas")

[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4 + 385*a^4*b*x^2 + 63*a^5)/x^11

giac [A] time = 0.16, size = 107, normalized size = 0.43

$$-\frac{693\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+1155\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+1386\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+990\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+385\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+63\,a^5\mathrm{sgn}\left(bx^2+a\right)}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="giac")

[Out] -1/693*(693*b^5*x^10*sgn(b*x^2 + a) + 1155*a*b^4*x^8*sgn(b*x^2 + a) + 1386*a^2*b^3*x^6*sgn(b*x^2 + a) + 990*a^3*b^2*x^4*sgn(b*x^2 + a) + 385*a^4*b*x^2*sgn(b*x^2 + a) + 63*a^5*sgn(b*x^2 + a))/x^11

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(693b^5x^{10}+1155a\,b^4x^8+1386a^2b^3x^6+990a^3b^2x^4+385a^4b\,x^2+63a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{693\left(b\,x^2+a\right)^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x)

[Out] -1/693*(693*b^5*x^10+1155*a*b^4*x^8+1386*a^2*b^3*x^6+990*a^3*b^2*x^4+385*a^4*b*x^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x^11/(b*x^2+a)^5

maxima [A] time = 1.33, size = 57, normalized size = 0.23

$$-\frac{b^5}{x}-\frac{5\,ab^4}{3\,x^3}-\frac{2\,a^2b^3}{x^5}-\frac{10\,a^3b^2}{7\,x^7}-\frac{5\,a^4b}{9\,x^9}-\frac{a^5}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="maxima")

[Out] -b^5/x - 5/3*a*b^4/x^3 - 2*a^2*b^3/x^5 - 10/7*a^3*b^2/x^7 - 5/9*a^4*b/x^9 - 1/11*a^5/x^11

mupad [B] time = 4.22, size = 231, normalized size = 0.92

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^3\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\left(b\,x^2+a\right)}-\frac{2\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^5\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^12,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(x*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^4 +
2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(9*x^9*(a + b*x^2)) - (2*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
)/(x^5*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7
*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**12, x)
```

3.446 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=253

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{}}{1}$$

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(x^5*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*
x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{14}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{14}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^{10}}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^4}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{9x^9\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(693a^5+4095a^4bx^2+10010a^3b^2x^4+12870a^2b^3x^6+9009ab^4x^8+3003b^5x^{10}\right)}{9009x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

```
[Out] -1/9009*(Sqrt[(a + b*x^2)^2]*(693*a^5 + 4095*a^4*b*x^2 + 10010*a^3*b^2*x^4
+ 12870*a^2*b^3*x^6 + 9009*a*b^4*x^8 + 3003*b^5*x^10))/(x^13*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 19.15, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-693a^5-4095a^4bx^2-10010a^3b^2x^4-12870a^2b^3x^6-9009ab^4x^8-3003b^5x^{10}\right)}{9009x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-693*a^5 - 4095*a^4*b*x^2 - 10010*a^3*b^2*x^4 - 12870
*a^2*b^3*x^6 - 9009*a*b^4*x^8 - 3003*b^5*x^10))/(9009*x^13*(a + b*x^2))
```

fricas [A] time = 1.06, size = 59, normalized size = 0.23

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="fricas")

[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13

giac [A] time = 0.18, size = 107, normalized size = 0.42

$$-\frac{3003\,b^{5}x^{10}\mathrm{sgn}\left(bx^{2}+a\right)+9009\,ab^{4}x^{8}\mathrm{sgn}\left(bx^{2}+a\right)+12870\,a^{2}b^{3}x^{6}\mathrm{sgn}\left(bx^{2}+a\right)+10010\,a^{3}b^{2}x^{4}\mathrm{sgn}\left(bx^{2}+a\right)+4095\,a^{4}bx^{2}\mathrm{sgn}\left(bx^{2}+a\right)+693\,a^{5}\mathrm{sgn}\left(bx^{2}+a\right)}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="giac")

[Out] -1/9009*(3003*b^5*x^10*sgn(b*x^2 + a) + 9009*a*b^4*x^8*sgn(b*x^2 + a) + 12870*a^2*b^3*x^6*sgn(b*x^2 + a) + 10010*a^3*b^2*x^4*sgn(b*x^2 + a) + 4095*a^4*b*x^2*sgn(b*x^2 + a) + 693*a^5*sgn(b*x^2 + a))/x^13

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(3003b^{5}x^{10}+9009a\,b^{4}x^{8}+12870a^{2}b^{3}x^{6}+10010a^{3}b^{2}x^{4}+4095a^{4}b\,x^{2}+693a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{9009\left(b\,x^{2}+a\right)^{5}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x)

[Out] -1/9009*(3003*b^5*x^10+9009*a*b^4*x^8+12870*a^2*b^3*x^6+10010*a^3*b^2*x^4+4095*a^4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/x^13/(b*x^2+a)^5

maxima [A] time = 1.34, size = 57, normalized size = 0.23

$$-\frac{b^{5}}{3\,x^{3}}-\frac{ab^{4}}{x^{5}}-\frac{10\,a^{2}b^{3}}{7\,x^{7}}-\frac{10\,a^{3}b^{2}}{9\,x^{9}}-\frac{5\,a^{4}b}{11\,x^{11}}-\frac{a^{5}}{13\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="maxima")

[Out] -1/3*b^5/x^3 - a*b^4/x^5 - 10/7*a^2*b^3/x^7 - 10/9*a^3*b^2/x^9 - 5/11*a^4*b/x^11 - 1/13*a^5/x^13

mupad [B] time = 4.35, size = 231, normalized size = 0.91

$$-\frac{a^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{13\,x^{13}\left(b\,x^{2}+a\right)}-\frac{b^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{3\,x^{3}\left(b\,x^{2}+a\right)}-\frac{a\,b^{4}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{x^{5}\left(b\,x^{2}+a\right)}-\frac{5\,a^{4}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{11\,x^{11}\left(b\,x^{2}+a\right)}-\frac{10\,a^{2}\,b^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{7\,x^{7}\left(b\,x^{2}+a\right)}-\frac{10\,a^{3}\,b^{2}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{9\,x^{9}\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^14,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(x^5*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(11*x^11*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1
/2))/(7*x^7*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(
9*x^9*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**14, x)
```

3.447 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b}{}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(7*x^7*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(
a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{16}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{16}}+\frac{5a^4b^6}{x^{14}}+\frac{10a^3b^7}{x^{12}}+\frac{10a^2b^8}{x^{10}}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^6}\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{11x^{11}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3003a^5+17325a^4bx^2+40950a^3b^2x^4+50050a^2b^3x^6+32175ab^4x^8+9009b^5x^{10}\right)}{45045x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -1/45045*(Sqrt[(a + b*x^2)^2]*(3003*a^5 + 17325*a^4*b*x^2 + 40950*a^3*b^2*x
^4 + 50050*a^2*b^3*x^6 + 32175*a*b^4*x^8 + 9009*b^5*x^10))/(x^15*(a + b*x^2
))
```

IntegrateAlgebraic [A] time = 19.78, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-3003a^5-17325a^4bx^2-40950a^3b^2x^4-50050a^2b^3x^6-32175ab^4x^8-9009b^5x^{10}\right)}{45045x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3003*a^5 - 17325*a^4*b*x^2 - 40950*a^3*b^2*x^4 - 500
50*a^2*b^3*x^6 - 32175*a*b^4*x^8 - 9009*b^5*x^10))/(45045*x^15*(a + b*x^2))
```

fricas [A] time = 0.72, size = 59, normalized size = 0.23

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

giac [A] time = 0.16, size = 107, normalized size = 0.42

$$-\frac{9009\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+32175\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+50050\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+40950\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+17325\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+3003\,a^5\mathrm{sgn}\left(bx^2+a\right)}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(9009*b^5*x^10*sgn(b*x^2 + a) + 32175*a*b^4*x^8*sgn(b*x^2 + a) + 5
0050*a^2*b^3*x^6*sgn(b*x^2 + a) + 40950*a^3*b^2*x^4*sgn(b*x^2 + a) + 17325*
a^4*b*x^2*sgn(b*x^2 + a) + 3003*a^5*sgn(b*x^2 + a))/x^15
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(9009b^5x^{10}+32175a\,b^4x^8+50050a^2b^3x^6+40950a^3b^2x^4+17325a^4b\,x^2+3003a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{45045\left(b\,x^2+a\right)^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x)
```

```
[Out] -1/45045*(9009*b^5*x^10+32175*a*b^4*x^8+50050*a^2*b^3*x^6+40950*a^3*b^2*x^4
+17325*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/x^15/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.22

$$-\frac{b^5}{5\,x^5}-\frac{5\,ab^4}{7\,x^7}-\frac{10\,a^2b^3}{9\,x^9}-\frac{10\,a^3b^2}{11\,x^{11}}-\frac{5\,a^4b}{13\,x^{13}}-\frac{a^5}{15\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/5*b^5/x^5 - 5/7*a*b^4/x^7 - 10/9*a^2*b^3/x^9 - 10/11*a^3*b^2/x^11 - 5/13
*a^4*b/x^13 - 1/15*a^5/x^15
```

mupad [B] time = 4.21, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{15\,x^{15}\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\left(b\,x^2+a\right)}-\frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^16,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(15*x^15*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(9*x^9*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(11*x^11*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**16, x)
```

3.448 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{}}{}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(9*x^9*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a
 + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{18}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{18}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{18}}+\frac{5a^4b^6}{x^{16}}+\frac{10a^3b^7}{x^{14}}+\frac{10a^2b^8}{x^{12}}+\frac{5ab^9}{x^{10}}+\frac{b^{10}}{x^8}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(9009a^5+51051a^4bx^2+117810a^3b^2x^4+139230a^2b^3x^6+85085ab^4x^8+21879b^5x^{10}\right)}{153153x^{17}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] -1/153153*(Sqrt[(a + b*x^2)^2]*(9009*a^5 + 51051*a^4*b*x^2 + 117810*a^3*b^2
*x^4 + 139230*a^2*b^3*x^6 + 85085*a*b^4*x^8 + 21879*b^5*x^10))/(x^17*(a + b
*x^2))
```

IntegrateAlgebraic [A] time = 21.83, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-9009a^5-51051a^4bx^2-117810a^3b^2x^4-139230a^2b^3x^6-85085ab^4x^8-21879b^5x^{10}\right)}{153153x^{17}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-9009*a^5 - 51051*a^4*b*x^2 - 117810*a^3*b^2*x^4 - 13
9230*a^2*b^3*x^6 - 85085*a*b^4*x^8 - 21879*b^5*x^10))/(153153*x^17*(a + b*x
^2))
```

fricas [A] time = 0.77, size = 59, normalized size = 0.23

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

giac [A] time = 0.17, size = 107, normalized size = 0.42

$$-\frac{21879\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+85085\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+139230\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+117810\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+51051\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+9009\,a^5\mathrm{sgn}\left(bx^2+a\right)}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/153153*(21879*b^5*x^10*sgn(b*x^2 + a) + 85085*a*b^4*x^8*sgn(b*x^2 + a) +
 139230*a^2*b^3*x^6*sgn(b*x^2 + a) + 117810*a^3*b^2*x^4*sgn(b*x^2 + a) + 51
051*a^4*b*x^2*sgn(b*x^2 + a) + 9009*a^5*sgn(b*x^2 + a))/x^17
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(21879b^5x^{10}+85085a\,b^4x^8+139230a^2b^3x^6+117810a^3b^2x^4+51051a^4b\,x^2+9009a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{153153\left(b\,x^2+a\right)^5x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x)
```

```
[Out] -1/153153*(21879*b^5*x^10+85085*a*b^4*x^8+139230*a^2*b^3*x^6+117810*a^3*b^2
*x^4+51051*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/x^17/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.22

$$-\frac{b^5}{7\,x^7}-\frac{5\,ab^4}{9\,x^9}-\frac{10\,a^2b^3}{11\,x^{11}}-\frac{10\,a^3b^2}{13\,x^{13}}-\frac{a^4b}{3\,x^{15}}-\frac{a^5}{17\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/7*b^5/x^7 - 5/9*a*b^4/x^9 - 10/11*a^2*b^3/x^11 - 10/13*a^3*b^2/x^13 - 1/
3*a^4*b/x^15 - 1/17*a^5/x^17
```

mupad [B] time = 4.31, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^18,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(17*x^17*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(3*x^15*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(11*x^11*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
)/(13*x^13*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{18}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**18, x)
```

3.449 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{20}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{5a^4b}{}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(11*x^11*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9
*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{20}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{20}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{20}}+\frac{5a^4b^6}{x^{18}}+\frac{10a^3b^7}{x^{16}}+\frac{10a^2b^8}{x^{14}}+\frac{5ab^9}{x^{12}}+\frac{b^{10}}{x^{10}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)} - \frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(21879a^5+122265a^4bx^2+277134a^3b^2x^4+319770a^2b^3x^6+188955ab^4x^8+46189b^5x^{10}\right)}{415701x^{19}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -1/415701*(Sqrt[(a + b*x^2)^2]*(21879*a^5 + 122265*a^4*b*x^2 + 277134*a^3*b
^2*x^4 + 319770*a^2*b^3*x^6 + 188955*a*b^4*x^8 + 46189*b^5*x^10))/(x^19*(a
+ b*x^2))
```

IntegrateAlgebraic [A] time = 22.46, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-21879a^5-122265a^4bx^2-277134a^3b^2x^4-319770a^2b^3x^6-188955ab^4x^8-46189b^5x^{10}\right)}{415701x^{19}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-21879*a^5 - 122265*a^4*b*x^2 - 277134*a^3*b^2*x^4 -
319770*a^2*b^3*x^6 - 188955*a*b^4*x^8 - 46189*b^5*x^10))/(415701*x^19*(a +
b*x^2))
```

fricas [A] time = 0.74, size = 59, normalized size = 0.23

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

giac [A] time = 0.20, size = 107, normalized size = 0.42

$$-\frac{46189\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+188955\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+319770\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+277134\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+122265\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+21879\,a^5\mathrm{sgn}\left(bx^2+a\right)}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/415701*(46189*b^5*x^10*sgn(b*x^2 + a) + 188955*a*b^4*x^8*sgn(b*x^2 + a)
+ 319770*a^2*b^3*x^6*sgn(b*x^2 + a) + 277134*a^3*b^2*x^4*sgn(b*x^2 + a) + 1
22265*a^4*b*x^2*sgn(b*x^2 + a) + 21879*a^5*sgn(b*x^2 + a))/x^19
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(46189b^5x^{10}+188955a\,b^4x^8+319770a^2b^3x^6+277134a^3b^2x^4+122265a^4b\,x^2+21879a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{415701\left(b\,x^2+a\right)^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x)
```

```
[Out] -1/415701*(46189*b^5*x^10+188955*a*b^4*x^8+319770*a^2*b^3*x^6+277134*a^3*b^
2*x^4+122265*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/x^19/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 57, normalized size = 0.22

$$-\frac{b^5}{9\,x^9}-\frac{5\,ab^4}{11\,x^{11}}-\frac{10\,a^2b^3}{13\,x^{13}}-\frac{2\,a^3b^2}{3\,x^{15}}-\frac{5\,a^4b}{17\,x^{17}}-\frac{a^5}{19\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/9*b^5/x^9 - 5/11*a*b^4/x^11 - 10/13*a^2*b^3/x^13 - 2/3*a^3*b^2/x^15 - 5/
17*a^4*b/x^17 - 1/19*a^5/x^19
```

mupad [B] time = 4.27, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{19\,x^{19}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{2\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^20,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(19*x^19*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(17*x^17*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(13*x^13*(a + b*x^2)) - (2*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(3*x^15*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{20}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**20, x)
```

3.450 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-5a^4b$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(13*x^13*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^
11*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{22}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{22}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{22}}+\frac{5a^4b^6}{x^{20}}+\frac{10a^3b^7}{x^{18}}+\frac{10a^2b^8}{x^{16}}+\frac{5ab^9}{x^{14}}+\frac{b^{10}}{x^{12}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{17x^{17}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(46189a^5+255255a^4bx^2+570570a^3b^2x^4+646646a^2b^3x^6+373065ab^4x^8+88179b^5x^{10}\right)}{969969x^{21}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -1/969969*(Sqrt[(a + b*x^2)^2]*(46189*a^5 + 255255*a^4*b*x^2 + 570570*a^3*b
^2*x^4 + 646646*a^2*b^3*x^6 + 373065*a*b^4*x^8 + 88179*b^5*x^10))/(x^21*(a
+ b*x^2))
```

IntegrateAlgebraic [A] time = 24.64, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-46189a^5-255255a^4bx^2-570570a^3b^2x^4-646646a^2b^3x^6-373065ab^4x^8-88179b^5x^{10}\right)}{969969x^{21}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-46189*a^5 - 255255*a^4*b*x^2 - 570570*a^3*b^2*x^4 -
646646*a^2*b^3*x^6 - 373065*a*b^4*x^8 - 88179*b^5*x^10))/(969969*x^21*(a +
b*x^2))
```

fricas [A] time = 1.51, size = 59, normalized size = 0.23

$$-\frac{88179\,b^5x^{10}+373065\,ab^4x^8+646646\,a^2b^3x^6+570570\,a^3b^2x^4+255255\,a^4bx^2+46189\,a^5}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/969969*(88179*b^5*x^10 + 373065*a*b^4*x^8 + 646646*a^2*b^3*x^6 + 570570*
a^3*b^2*x^4 + 255255*a^4*b*x^2 + 46189*a^5)/x^21
```

giac [A] time = 0.16, size = 107, normalized size = 0.42

$$-\frac{88179\, b^5 x^{10} \mathrm{sgn}\left(bx^2+a\right) + 373065\, ab^4 x^8 \mathrm{sgn}\left(bx^2+a\right) + 646646\, a^2 b^3 x^6 \mathrm{sgn}\left(bx^2+a\right) + 570570\, a^3 b^2 x^4 \mathrm{sgn}\left(bx^2+a\right) + 255255\, a^4 b x^2 \mathrm{sgn}\left(bx^2+a\right) + 46189\, a^5 \mathrm{sgn}\left(bx^2+a\right)}{969969\, x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/969969*(88179*b^5*x^10*sgn(b*x^2 + a) + 373065*a*b^4*x^8*sgn(b*x^2 + a)
+ 646646*a^2*b^3*x^6*sgn(b*x^2 + a) + 570570*a^3*b^2*x^4*sgn(b*x^2 + a) + 2
55255*a^4*b*x^2*sgn(b*x^2 + a) + 46189*a^5*sgn(b*x^2 + a))/x^21
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(88179 b^5 x^{10}+373065 a\, b^4 x^8+646646 a^2 b^3 x^6+570570 a^3 b^2 x^4+255255 a^4 b\, x^2+46189 a^5\right)\left(\left(b\, x^2+a\right)^2\right)^{\frac{5}{2}}}{969969 \left(b\, x^2+a\right)^5 x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x)
```

```
[Out] -1/969969*(88179*b^5*x^10+373065*a*b^4*x^8+646646*a^2*b^3*x^6+570570*a^3*b^
2*x^4+255255*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/x^21/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 57, normalized size = 0.22

$$-\frac{b^5}{11\, x^{11}}-\frac{5\, ab^4}{13\, x^{13}}-\frac{2\, a^2 b^3}{3\, x^{15}}-\frac{10\, a^3 b^2}{17\, x^{17}}-\frac{5\, a^4 b}{19\, x^{19}}-\frac{a^5}{21\, x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] -1/11*b^5/x^11 - 5/13*a*b^4/x^13 - 2/3*a^2*b^3/x^15 - 10/17*a^3*b^2/x^17 -
5/19*a^4*b/x^19 - 1/21*a^5/x^21
```

mupad [B] time = 4.34, size = 231, normalized size = 0.91

$$-\frac{a^5 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{21\,x^{21}\left(b\,x^2+a\right)}-\frac{b^5 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\left(b\,x^2+a\right)}-\frac{5\,a\,b^4 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\left(b\,x^2+a\right)}-\frac{5\,a^4\,b \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{19\,x^{19}\left(b\,x^2+a\right)}-\frac{2\,a^2\,b^3 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^22,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(21*x^21*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(19*x^19*(a + b*x^2)) - (2*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(3*x^15*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^
(1/2))/(17*x^17*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{22}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**22,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**22, x)
```

3.451 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{24}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{\ }}{2}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*x^23*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(3*x^15*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^1
3*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{24}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{24}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{24}}+\frac{5a^4b^6}{x^{22}}+\frac{10a^3b^7}{x^{20}}+\frac{10a^2b^8}{x^{18}}+\frac{5ab^9}{x^{16}}+\frac{b^{10}}{x^{14}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{19x^{19}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(88179a^5+482885a^4bx^2+1067430a^3b^2x^4+1193010a^2b^3x^6+676039ab^4x^8+156009b^5x^{10}\right)}{2028117x^{23}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -1/2028117*(Sqrt[(a + b*x^2)^2]*(88179*a^5 + 482885*a^4*b*x^2 + 1067430*a^3
*b^2*x^4 + 1193010*a^2*b^3*x^6 + 676039*a*b^4*x^8 + 156009*b^5*x^10))/(x^23
*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 28.04, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-88179a^5-482885a^4bx^2-1067430a^3b^2x^4-1193010a^2b^3x^6-676039ab^4x^8-156009b^5x^{10}\right)}{2028117x^{23}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-88179*a^5 - 482885*a^4*b*x^2 - 1067430*a^3*b^2*x^4 -
 1193010*a^2*b^3*x^6 - 676039*a*b^4*x^8 - 156009*b^5*x^10))/(2028117*x^23*(
a + b*x^2))
```

fricas [A] time = 1.34, size = 59, normalized size = 0.23

$$-\frac{156009\,b^5x^{10}+676039\,ab^4x^8+1193010\,a^2b^3x^6+1067430\,a^3b^2x^4+482885\,a^4bx^2+88179\,a^5}{2028117\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/2028117*(156009*b^5*x^10 + 676039*a*b^4*x^8 + 1193010*a^2*b^3*x^6 + 1067
430*a^3*b^2*x^4 + 482885*a^4*b*x^2 + 88179*a^5)/x^23
```

giac [A] time = 0.16, size = 107, normalized size = 0.42

$$-\frac{156009\, b^5 x^{10} \mathrm{sgn}\left(bx^2+a\right) + 676039\, ab^4 x^8 \mathrm{sgn}\left(bx^2+a\right) + 1193010\, a^2 b^3 x^6 \mathrm{sgn}\left(bx^2+a\right) + 1067430\, a^3 b^2 x^4 \mathrm{sgn}\left(bx^2+a\right) + 482885\, a^4 b x^2 \mathrm{sgn}\left(bx^2+a\right) + 88179\, a^5 \mathrm{sgn}\left(bx^2+a\right)}{2028117\, x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/2028117*(156009*b^5*x^10*sgn(b*x^2 + a) + 676039*a*b^4*x^8*sgn(b*x^2 + a
) + 1193010*a^2*b^3*x^6*sgn(b*x^2 + a) + 1067430*a^3*b^2*x^4*sgn(b*x^2 + a)
 + 482885*a^4*b*x^2*sgn(b*x^2 + a) + 88179*a^5*sgn(b*x^2 + a))/x^23
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(156009 b^5 x^{10} + 676039 a\, b^4 x^8 + 1193010 a^2 b^3 x^6 + 1067430 a^3 b^2 x^4 + 482885 a^4 b\, x^2 + 88179 a^5\right) \left(\left(b\, x^2+a\right)^2\right)^{\frac{5}{2}}}{2028117 \left(b\, x^2+a\right)^5 x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x)
```

```
[Out] -1/2028117*(156009*b^5*x^10+676039*a*b^4*x^8+1193010*a^2*b^3*x^6+1067430*a^
3*b^2*x^4+482885*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/2)/x^23/(b*x^2+a)^5
```

maxima [A] time = 1.38, size = 57, normalized size = 0.22

$$-\frac{b^5}{13\, x^{13}} - \frac{ab^4}{3\, x^{15}} - \frac{10\, a^2 b^3}{17\, x^{17}} - \frac{10\, a^3 b^2}{19\, x^{19}} - \frac{5\, a^4 b}{21\, x^{21}} - \frac{a^5}{23\, x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/13*b^5/x^13 - 1/3*a*b^4/x^15 - 10/17*a^2*b^3/x^17 - 10/19*a^3*b^2/x^19 -
 5/21*a^4*b/x^21 - 1/23*a^5/x^23
```

mupad [B] time = 4.31, size = 231, normalized size = 0.91

$$-\frac{a^5 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{23\,x^{23}\,\left(b\,x^2+a\right)} - \frac{b^5 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)} - \frac{a\,b^4 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)} - \frac{5\,a^4\,b \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{21\,x^{21}\,\left(b\,x^2+a\right)} - \frac{10\,a^2\,b^3 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)} - \frac{10\,a^3\,b^2 \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{19\,x^{19}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^24,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(23*x^23*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(3*x^15*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(21*x^21*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^
2)^(1/2))/(17*x^17*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(19*x^19*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{24}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**24,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**24, x)
```

3.452 $\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=127

$$\frac{x^4(a+bx^2)}{4b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax^2(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2(a+bx^2)\log(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{x^4(a+bx^2)}{4b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax^2(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2(a+bx^2)\log(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(a*x^2*(a + b*x^2))/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^4*(a + b*
x^2))/(4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^2*(a + b*x^2)*Log[a + b*x^
2])/(2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)\\
&= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \frac{x^2}{ab+b^2x}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{ax^2\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^4\left(a+bx^2\right)}{4b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.43

$$
\frac{\left(a+bx^2\right)\left(2a^2\log\left(a+bx^2\right)+bx^2\left(bx^2-2a\right)\right)}{4b^3\sqrt{\left(a+bx^2\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*x^2*(-2*a + b*x^2) + 2*a^2*Log[a + b*x^2]))/(4*b^3*Sqrt[(a
+ b*x^2)^2])
```

IntegrateAlgebraic [A] time = 0.29, size = 182, normalized size = 1.43

$$
-\frac{a^2\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)}{4b^4}-\frac{a^2\left(\sqrt{b^2}-b\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{4b^4}+\frac{\left(bx^2-3a\right)\sqrt{a^2+2abx^2+b^2x^4}}{8b^3}+\frac{2ax^2-bx^4}{8b\sqrt{b^2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*x^2 - b*x^4)/(8*b*Sqrt[b^2]) + ((-3*a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(8*b^3) - (a^2*(b + Sqrt[b^2])*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]])/(4*b^4) - (a^2*(-b + Sqrt[b^2])*Log[a - Sqrt[b^2]*
x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*b^4)
```

fricas [A] time = 0.71, size = 33, normalized size = 0.26

$$
\frac{b^2x^4-2\,abx^2+2\,a^2\log\left(bx^2+a\right)}{4\,b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

giac [A] time = 0.18, size = 59, normalized size = 0.46

$$\frac{a^2 \log\left(\left|bx^2 + a\right|\right) \operatorname{sgn}\left(bx^2 + a\right)}{2\,b^3} + \frac{bx^4 \operatorname{sgn}\left(bx^2 + a\right) - 2\,ax^2 \operatorname{sgn}\left(bx^2 + a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b^3 + 1/4*(b*x^4*sgn(b*x^2 + a)
- 2*a*x^2*sgn(b*x^2 + a))/b^2
```

maple [A] time = 0.01, size = 52, normalized size = 0.41

$$\frac{\left(b\,x^2 + a\right)\left(b^2 x^4 - 2ab\,x^2 + 2a^2 \ln\left(b\,x^2 + a\right)\right)}{4\sqrt{\left(b\,x^2 + a\right)^2}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)*(b^2*x^4-2*a*b*x^2+2*a^2*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/b^3
```

maxima [A] time = 1.39, size = 34, normalized size = 0.27

$$\frac{a^2 \log\left(bx^2 + a\right)}{2\,b^3} + \frac{bx^4 - 2\,ax^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{\left(b\,x^2 + a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^5/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.20, size = 32, normalized size = 0.25

$$\frac{a^2 \log \left(a + bx^2\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

3.453 $\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}} dx$

Optimal. Leaf size=75

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1111, 640, 608, 31}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(2*b^2) - (a*(a + b*x^2)*Log[a + b*x^2])/(2
*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)}{2b}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{\left(a\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left(bx^2-a\log\left(a+bx^2\right)\right)}{2b^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*(b*x^2 - a*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 0.22, size = 156, normalized size = 2.08

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{4b^2} + \frac{a\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)}{4b^3} + \frac{a\left(\sqrt{b^2}-b\right)\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{4b^3} - \frac{x^2}{4\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] -1/4*x^2/Sqrt[b^2] + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(4*b^2) + (a*(b + Sqrt
[b^2])*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*b^3) +
 (a*(-b + Sqrt[b^2])*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]])/(4*b^3)
```

fricas [A] time = 0.67, size = 22, normalized size = 0.29

$$\frac{bx^2 - a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

giac [A] time = 0.17, size = 33, normalized size = 0.44

$$\frac{1}{2}\left(\frac{x^2}{b} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{b^2}\right)\operatorname{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(x^2/b - a*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 41, normalized size = 0.55

$$-\frac{\left(b\,x^2 + a\right)\left(-b\,x^2 + a\ln\left(b\,x^2 + a\right)\right)}{2\sqrt{\left(b\,x^2 + a\right)^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(-b*x^2+a*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/b^2
```

maxima [A] time = 1.31, size = 23, normalized size = 0.31

$$\frac{x^2}{2\,b} - \frac{a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

mupad [B] time = 4.52, size = 64, normalized size = 0.85

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^2}-\frac{a\,b\,\ln\left(a\,b+\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{b^2}+b^2\,x^2\right)}{2\,{\left(b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*b^2) - (a*b*log(a*b + ((a + b*x^2)^2)^
(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2))
```

sympy [A] time = 0.18, size = 20, normalized size = 0.27

$$-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

3.454 $\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1107, 608, 31}

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)$$
$$= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.80

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 0.25, size = 149, normalized size = 3.39

$$-\frac{\log\left(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}\,x^2\right)}{4\sqrt{b^2}}-\frac{\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{4\sqrt{b^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b^2}\,x^2}{a}-\frac{\sqrt{a^2+2abx^2+b^2x^4}}{a}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[b^2]*x^2)/a - Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/a]/b - Log
[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]]/(4*Sqrt[b^2]) - Log[
a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]]/(4*Sqrt[b^2])
```

fricas [A] time = 1.63, size = 13, normalized size = 0.30

$$\frac{\log(bx^2+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

giac [A] time = 0.15, size = 22, normalized size = 0.50

$$\frac{\log\left(\left|bx^2+a\right|\right)\operatorname{sgn}\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.73

$$\frac{\left(b\,x^2+a\right)\ln\left(b\,x^2+a\right)}{2\sqrt{\left(b\,x^2+a\right)^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^2+a)^2)^(1/2),x)
[Out] 1/2*(b*x^2+a)*ln(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.30

$$\frac{\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 + a)/b
```

mupad [B] time = 4.42, size = 33, normalized size = 0.75

$$\frac{\ln\left(b^2\,x^2+a\,b\right)\,\mathrm{sign}\left(2\,b^2\,x^2+2\,a\,b\right)}{2\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2)^(1/2),x)
[Out] (log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(1/2))
```

sympy [A] time = 0.15, size = 10, normalized size = 0.23

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] log(a + b*x**2)/(2*b)
```

3.455 $\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=80

$$\frac{\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1112, 266, 36, 29, 31}

$$\frac{\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log
[a + b*x^2])/(2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{1}{x\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(a+bx^2\right)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.52

$$\frac{\left(a+bx^2\right)\left(2\log(x)-\log\left(a+bx^2\right)\right)}{2a\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(2*Log[x] - Log[a + b*x^2]))/(2*a*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 0.19, size = 94, normalized size = 1.18

$$\frac{\log\left(-a\sqrt{a^2+2abx^2+b^2x^4}+a^2+a\sqrt{b^2}\,x^2\right)}{2a}-\frac{\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/2*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]]/a + Log[a^2 +
 a*Sqrt[b^2]*x^2 - a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]]/(2*a)
```

fricas [A] time = 0.89, size = 18, normalized size = 0.22

$$-\frac{\log\left(bx^2+a\right)-2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

giac [A] time = 0.15, size = 33, normalized size = 0.41

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a}-\frac{\log\left(\left|bx^2+a\right|\right)}{a}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(log(x^2)/a - log(abs(b*x^2 + a))/a)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\left(b\,x^2+a\right)\left(2\ln(x)-\ln\left(b\,x^2+a\right)\right)}{2\sqrt{\left(b\,x^2+a\right)^2}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^2+a)^2)^(1/2),x)
[Out] 1/2*(b*x^2+a)*(2*ln(x)-ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/a
```

maxima [A] time = 1.28, size = 23, normalized size = 0.29

$$-\frac{\log\left(bx^2+a\right)}{2\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + a)/a + 1/2*log(x^2)/a
```

mupad [B] time = 4.45, size = 40, normalized size = 0.50

$$-\frac{\ln\left(\sqrt{\left(b\,x^2+a\right)^2}\,\sqrt{a^2}\,+a^2+a\,b\,x^2\right)+\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] -(log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a*b*x^2) + log(1/x^2))/(2*(
a^2)^(1/2))
```

sympy [A] time = 0.26, size = 15, normalized size = 0.19

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

3.456 $\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=125

$$\frac{-a-bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\log(x)\left(a+bx^2\right)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$-\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\log(x)\left(a+bx^2\right)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(a + b*x^2)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*(a + b*x^2)*Log
[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2)*Log[a + b*x^2])
/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{x^3\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b\left(a+bx^2\right)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.43

$$-\frac{\left(a+bx^2\right)\left(-bx^2\log\left(a+bx^2\right)+a+2bx^2\log(x)\right)}{2a^2x^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/2*((a + b*x^2)*(a + 2*b*x^2*Log[x] - b*x^2*Log[a + b*x^2]))/(a^2*x^2*Sqr
t[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 0.60, size = 380, normalized size = 3.04

$$\frac{\left(\sqrt{a^2+2abx^2+b^2x^4}-\sqrt{b^2}\,x^2\right)^2\left(\frac{b\log\left(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}\,x^2\right)}{2a^2}-\frac{b\log\left(a^3+a^2\sqrt{b^2}\,x^2-a^2\sqrt{a^2+2abx^2+b^2x^4}\right)}{2a^2}\right)}{-2\sqrt{b^2}\,x^2\sqrt{a^2+2abx^2+b^2x^4}+a^2+2abx^2+2b^2x^4}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a^2b+4ab^2x^2+4b^3x^4\right)+\sqrt{b^2}\left(-a^3-5a^2bx^2-8ab^2x^4-4b^3x^6\right)}{a\sqrt{b^2}\sqrt{a^2+2abx^2+b^2x^4}\left(2a^2x^2+8abx^4+8b^2x^6\right)+a\left(-2a^3bx^2-10a^2b^2x^4-16ab^3x^6-8b^4x^8\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(a^2*b + 4*a*b^2*x^2 + 4*b^3*x^4) + Sqrt[b
^2]*(-a^3 - 5*a^2*b*x^2 - 8*a*b^2*x^4 - 4*b^3*x^6))/(a*Sqrt[b^2]*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]*(2*a^2*x^2 + 8*a*b*x^4 + 8*b^2*x^6) + a*(-2*a^3*b*x^2
 - 10*a^2*b^2*x^4 - 16*a*b^3*x^6 - 8*b^4*x^8)) + ((-(Sqrt[b^2]*x^2) + Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])^2*((b*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4]])/(2*a^2) - (b*Log[a^3 + a^2*Sqrt[b^2]*x^2 - a^2*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]])/(2*a^2)))/(a^2 + 2*a*b*x^2 + 2*b^2*x^4 - 2*Sqrt[b^2]
*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

fricas [A] time = 0.58, size = 33, normalized size = 0.26

$$\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log(x)-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

giac [A] time = 0.16, size = 52, normalized size = 0.42

$$-\frac{1}{2}\left(\frac{b\log\left(x^2\right)}{a^2}-\frac{b\log\left(\left|bx^2+a\right|\right)}{a^2}-\frac{bx^2-a}{a^2x^2}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(x^2)/a^2 - b*log(abs(b*x^2 + a))/a^2 - (b*x^2 - a)/(a^2*x^2))*s
gn(b*x^2 + a)
```

maple [A] time = 0.01, size = 51, normalized size = 0.41

$$-\frac{\left(b\,x^2+a\right)\left(2b\,x^2\ln(x)-b\,x^2\ln\left(b\,x^2+a\right)+a\right)}{2\sqrt{\left(b\,x^2+a\right)^2}\;a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(2*b*x^2*ln(x)-b*ln(b*x^2+a)*x^2+a)/((b*x^2+a)^2)^(1/2)/x^2/
a^2
```

maxima [A] time = 1.29, size = 33, normalized size = 0.26

$$\frac{b\log\left(bx^2+a\right)}{2\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

mupad [B] time = 4.45, size = 75, normalized size = 0.60

$$\frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^2}{\sqrt{a^2}\,\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\right)}{2\,{\left(a^2\right)}^{3/2}}-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,a^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] (a*b*atanh((a^2 + a*b*x^2)/((a^2)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)))
)/(2*(a^2)^(3/2)) - (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*a^2*x^2)
```

sympy [A] time = 0.32, size = 31, normalized size = 0.25

$$-\frac{1}{2ax^2}-\frac{b\log{(x)}}{a^2}+\frac{b\log{\left(\frac{a}{b}+x^2\right)}}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

3.457 $\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=129

$$-\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 302, 205}

$$\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((a*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (x^3*(a + b*x^
2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/2)*(a + b*x^2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{x^4}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2\left(ab+b^2x^2\right)}\right)dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a^2\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{b^2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.51

$$\frac{\left(a+bx^2\right)\left(3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x\left(bx^2-3a\right)\right)}{3b^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 4.97, size = 63, normalized size = 0.49

$$\frac{\left(a+bx^2\right)\left(\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{bx^3-3ax}{3b^2}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*((-3*a*x + b*x^3)/(3*b^2) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/b^(5/2)))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 3.01, size = 99, normalized size = 0.77

$$\left[\frac{2\,bx^{3}+3\,a\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-6\,ax}{6\,b^{2}},\,\frac{bx^{3}+3\,a\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,ax}{3\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*a*
x)/b^2]
```

giac [A] time = 0.16, size = 64, normalized size = 0.50

$$\frac{a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{sgn}\left(bx^{2}+a\right)}{\sqrt{ab}\,b^{2}}+\frac{b^{2}x^{3}\operatorname{sgn}\left(bx^{2}+a\right)-3\,abx\operatorname{sgn}\left(bx^{2}+a\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3*sgn
(b*x^2 + a) - 3*a*b*x*sgn(b*x^2 + a))/b^3
```

maple [A] time = 0.01, size = 63, normalized size = 0.49

$$\frac{\left(b\,x^{2}+a\right)\left(\sqrt{ab}\,b\,x^{3}+3a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-3\sqrt{ab}\,ax\right)}{3\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\sqrt{ab}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((a*b)^(1/2)*x^3*b-3*(a*b)^(1/2)*x*a+3*a^2*arctan(1/(a*b)^(1/
2)*b*x))/((b*x^2+a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

maxima [A] time = 2.93, size = 37, normalized size = 0.29

$$\frac{a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{bx^{3}-3\,ax}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*x^3 - 3*a*x)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^4/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.21, size = 80, normalized size = 0.62

$$-\frac{ax}{b^2} - \frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x - \frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x + \frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 + sqrt(-a**
3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

3.458 $\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 321, 205}

$$\frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(a + b*x^2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{x^2}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{x\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{x\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left(\sqrt{b}\,x-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*Sq
rt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 4.36, size = 52, normalized size = 0.58

$$\frac{\left(a+bx^2\right)\left(\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/Sqrt[(a
 + b*x^2)^2]
```

fricas [A] time = 0.79, size = 82, normalized size = 0.92

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

giac [A] time = 0.23, size = 42, normalized size = 0.47

$$-\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right) \operatorname{sgn}\left(bx^2 + a\right)}{\sqrt{ab}\, b} + \frac{x \operatorname{sgn}\left(bx^2 + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b) + x*sgn(b*x^2 + a)/b
```

maple [A] time = 0.01, size = 48, normalized size = 0.54

$$\frac{\left(b\,x^2 + a\right)\left(-a \arctan\left(\frac{bx}{\sqrt{ab}}\right) + \sqrt{ab}\; x\right)}{\sqrt{\left(b\,x^2 + a\right)^2}\;\sqrt{ab}\; b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] (b*x^2+a)*(x*(a*b)^(1/2)-a*arctan(1/(a*b)^(1/2)*b*x))/((b*x^2+a)^2)^(1/2)/b
/(a*b)^(1/2)
```

maxima [A] time = 2.91, size = 26, normalized size = 0.29

$$-\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\left(b\,x^2 + a\right)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^2/((a + b*x^2)^2)^(1/2), x)
```

sympy [A]   time = 0.19, size = 56, normalized size = 0.63

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

3.459 $\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1088, 205}

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(2ab+2b^2x^2)\int \frac{1}{2ab+2b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.83

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[(a + b*x^2)
^2])
```

IntegrateAlgebraic [A] time = 4.17, size = 44, normalized size = 0.83

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[(a + b*x^2)
^2])
```

fricas [A] time = 1.27, size = 67, normalized size = 1.26

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

giac [A] time = 0.18, size = 23, normalized size = 0.43

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/sqrt(a*b)
```

maple [A] time = 0.00, size = 34, normalized size = 0.64

$$\frac{\left(b\,x^{2}+a\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.03, size = 15, normalized size = 0.28

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\left(b\,x^{2}+a\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.18, size = 53, normalized size = 1.00

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

3.460 $\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=92

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 325, 205}

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((a + b*x^2)/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - (Sqrt[b]*(a + b*x^2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.61

$$-\frac{\left(a+bx^2\right)\left(\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{a}\right)}{a^{3/2}x\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(a^(3/2)*
x*Sqrt[(a + b*x^2)^2]))
```

IntegrateAlgebraic [A] time = 8.78, size = 55, normalized size = 0.60

$$\frac{\left(a+bx^2\right)\left(-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/
Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 1.56, size = 82, normalized size = 0.89

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

giac [A] time = 0.15, size = 37, normalized size = 0.40

$$-\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{1}{ax}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^2+a\right)\left(bx\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\right)}{\sqrt{\left(b\,x^2+a\right)^2}\,\sqrt{ab}\,\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -(b*x^2+a)*(b*arctan(1/(a*b)^(1/2)*b*x)*x+(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/
a/x/(a*b)^(1/2)
```

maxima [A] time = 2.94, size = 29, normalized size = 0.32

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*((a + b*x^2)^2)^(1/2)), x)
```

sympy [A] time = 0.23, size = 65, normalized size = 0.71

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

3.461 $\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=133

$$\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{-a-bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.04, antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 325, 205}

$$\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(a + b*x^2)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2))/(a
^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\right.}{a^{5/2}\sqrt{a^2+2abx^2+b^2}}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 0.53

$$-\frac{\left(a+bx^2\right)\left(\sqrt{a}\left(a-3bx^2\right)-3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{3a^{5/2}x^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/3*((a + b*x^2)*(Sqrt[a]*(a - 3*b*x^2) - 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)
/Sqrt[a]]))/(a^(5/2)*x^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 17.68, size = 66, normalized size = 0.50

$$\frac{\left(a+bx^2\right)\left(\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3bx^2-a}{3a^2x^3}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*((-a + 3*b*x^2)/(3*a^2*x^3) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/a^(5/2)))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 1.20, size = 106, normalized size = 0.80

$$\left[\frac{3\,bx^{3}\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+6\,bx^{2}-2\,a}{6\,a^{2}x^{3}},\ \frac{3\,bx^{3}\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^{2}-a}{3\,a^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

giac [A] time = 0.16, size = 50, normalized size = 0.38

$$\frac{1}{3}\left(\frac{3\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{3\,bx^{2}-a}{a^{2}x^{3}}\right)\operatorname{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/(a^2*x^3))
*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 69, normalized size = 0.52

$$\frac{\left(b\,x^{2}+a\right)\left(3b^{2}x^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3\sqrt{ab}\;b\,x^{2}-\sqrt{ab}\;a\right)}{3\sqrt{\left(b\,x^{2}+a\right)^{2}}\;\sqrt{ab}\;a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*(3*b^2*arctan(1/(a*b)^(1/2)*b*x)*x^3+3*b*x^2*(a*b)^(1/2)-a*(a
*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a^2/x^3/(a*b)^(1/2)
```

maxima [A] time = 3.01, size = 40, normalized size = 0.30

$$\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{3\,bx^{2}-a}{3\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/(x^4*((a + b*x^2)^2)^(1/2)), x)
```

sympy [A] time = 0.28, size = 87, normalized size = 0.65

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

3.462 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} \, dx$

Optimal. Leaf size=158

$$-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*a^2)/(2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^3/(4*b^4*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^2*(a + b*x^2))/(2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (3*a*(a + b*x^2)*Log[a + b*x^2])/(2*b^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right) \\
&= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right)dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= -\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+\right.}{2b^3\sqrt{a^2+2ab}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.51

$$
\frac{-5a^3-4a^2bx^2+4ab^2x^4-6a\left(a+bx^2\right)^2\log\left(a+bx^2\right)+2b^3x^6}{4b^4\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*a^3 - 4*a^2*b*x^2 + 4*a*b^2*x^4 + 2*b^3*x^6 - 6*a*(a + b*x^2)^2*Log[a +
 b*x^2])/(4*b^4*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 1.26, size = 1386, normalized size = 8.77

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((8*a^4*Sqrt[b^2]*x^2)/b^4 + (16*a^3*Sqrt[b^2]*x^4)/b^3 + (4*a^2*(b^2)^(3/2
)*x^6)/b^4 - (10*a*Sqrt[b^2]*x^8)/b - 4*Sqrt[b^2]*x^10 - (2*a^4*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/b^4 - (6*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^3
```

```
 - (10*a^2*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2 + (6*a*x^6*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/b + 4*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] - (12*a^3*x^
4*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a])/b^2 - (2
4*a^2*x^6*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a])/
b - 12*a*x^8*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a
] + (12*a^2*Sqrt[b^2]*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(-(Sqrt[b
^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a])/b^3 + (12*a*(b^2)^(3/2)*x^6
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/a])/b^4)/((-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])^2*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2) + ((-2*
a^5)/(b^3*Sqrt[b^2]) - (8*a^4*x^2)/(b^2)^(3/2) - (8*a^3*x^4)/(b*Sqrt[b^2])
+ (8*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^3 + (6*a^3*x^4*Log[-a - Sqr
t[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(b*Sqrt[b^2]) + (12*a^2*x^6*
Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] + (6*a
*b*x^8*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2]
 - (6*a^2*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-a - Sqrt[b^2]*x^2 + Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]])/b^2 - (6*a*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b + (6*a^3*x^4
*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(b*Sqrt[b^2]) +
(12*a^2*x^6*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[
b^2] + (6*a*b*x^8*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])
/Sqrt[b^2] - (6*a^2*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x
^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b^2 - (6*a*x^6*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b)/((-
a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2*(a - Sqrt[b^2]*x^2 +
 Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^2)
```

fricas [A] time = 1.49, size = 91, normalized size = 0.58

$$\frac{2\,b^3x^6+4\,ab^2x^4-4\,a^2bx^2-5\,a^3-6\left(ab^2x^4+2\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*b
*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

giac [A] time = 0.25, size = 83, normalized size = 0.53

$$\frac{x^2}{2\,b^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{3\,a\log\left(\left|bx^2+a\right|\right)}{2\,b^4\mathrm{sgn}\left(bx^2+a\right)}-\frac{6\,a^2bx^2+5\,a^3}{4\left(bx^2+a\right)^2b^4\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/(b^3*sgn(b*x^2 + a)) - 3/2*a*log(abs(b*x^2 + a))/(b^4*sgn(b*x^2 + a
)) - 1/4*(6*a^2*b*x^2 + 5*a^3)/((b*x^2 + a)^2*b^4*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 103, normalized size = 0.65

$$-\frac{\left(-2b^{3}x^{6}+6a\,b^{2}x^{4}\ln\left(b\,x^{2}+a\right)-4a\,b^{2}x^{4}+12a^{2}b\,x^{2}\ln\left(b\,x^{2}+a\right)+4a^{2}b\,x^{2}+6a^{3}\ln\left(b\,x^{2}+a\right)+5a^{3}\right)\left(b\,x^{2}+a\right)}{4\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(-2*b^3*x^6+6*ln(b*x^2+a)*x^4*a*b^2-4*a*b^2*x^4+12*ln(b*x^2+a)*x^2*a^2
*b+4*a^2*b*x^2+6*ln(b*x^2+a)*a^3+5*a^3)*(b*x^2+a)/b^4/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.36, size = 66, normalized size = 0.42

$$-\frac{6\,a^{2}bx^{2}+5\,a^{3}}{4\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)}+\frac{x^{2}}{2\,b^{3}}-\frac{3\,a\log\left(bx^{2}+a\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3
- 3/2*a*log(b*x^2 + a)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7}}{\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**7/((a + b*x**2)**2)**(3/2), x)
```

3.463 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] a/(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^2/(4*b^3*(a + b*x^2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+\right.}{2b^3\sqrt{a^2+2abx^2+}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.54

$$\frac{a\left(3a+4bx^2\right)+2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4b^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a*(3*a + 4*b*x^2) + 2*(a + b*x^2)^2*Log[a + b*x^2])/(4*b^3*(a + b*x^2)*Sqr
t[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 1.17, size = 1590, normalized size = 14.07

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((2*a^4*Sqrt[b^2])/b^4 + (6*a^3*Sqrt[b^2]*x^2)/b^3 + (6*a^2*(b^2)^(3/2)*x^4
)/b^4 - (6*a^2*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2 - (2*a^2*(b^2)^(3/2
)*x^4*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b^4 - (4*a
```

```
*Sqrt[b^2]*x^6*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b
 - 2*Sqrt[b^2]*x^8*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
] + (2*a*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]])/b + 2*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[
-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] - (2*a^2*(b^2)^(3/2)*
x^4*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b^4 - (4*a*Sq
rt[b^2]*x^6*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b - 2
*Sqrt[b^2]*x^8*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] + (
2*a*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]])/b + 2*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sq
rt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/((-a - Sqrt[b^2]*x^2 + Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])^2*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])^2) + ((-6*a^3*x^2)/(b*Sqrt[b^2]) - (12*a^2*x^4)/Sqrt[b^2] - (8*a*
b*x^6)/Sqrt[b^2] + (2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^3 + (4*a^2*x^2
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2 + (8*a*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/b + (2*a^2*x^4*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]])/b + 4*a*x^6*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] +
 2*b*x^8*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] - (2*a*x^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4]])/Sqrt[b^2] - (2*b*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a
 - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] - (2*a^2*x^4
*Log[-(a*b^3) - b^3*Sqrt[b^2]*x^2 + b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b
 - 4*a*x^6*Log[-(a*b^3) - b^3*Sqrt[b^2]*x^2 + b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]] - 2*b*x^8*Log[-(a*b^3) - b^3*Sqrt[b^2]*x^2 + b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]] + (2*a*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-(a*b^3) - b^
3*Sqrt[b^2]*x^2 + b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] + (2*b*x^
6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-(a*b^3) - b^3*Sqrt[b^2]*x^2 + b^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2])/((-a - Sqrt[b^2]*x^2 + Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])^2*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])^2)
```

fricas [A] time = 1.03, size = 69, normalized size = 0.61

$$\frac{4\,abx^2+3\,a^2+2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^5
*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.23, size = 64, normalized size = 0.57

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{4\,ax^2+\frac{3\,a^2}{b}}{4\left(bx^2+a\right)^2b^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/(b^3*sgn(b*x^2 + a)) + 1/4*(4*a*x^2 + 3*a^2/b)/((b*
x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 81, normalized size = 0.72

$$\frac{\left(2b^2x^4\ln\left(b\,x^2+a\right)+4ab\,x^2\ln\left(b\,x^2+a\right)+4ab\,x^2+2a^2\ln\left(b\,x^2+a\right)+3a^2\right)\left(b\,x^2+a\right)}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(2*ln(b*x^2+a)*x^4*b^2+4*ln(b*x^2+a)*x^2*a*b+4*a*b*x^2+2*a^2*ln(b*x^2+a
)+3*a^2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.37, size = 55, normalized size = 0.49

$$\frac{4\,abx^2+3\,a^2}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*log(b*x^2 +
 a)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**2)**(3/2), x)
```

3.464 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^4}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.68, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 607}

$$\frac{a}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a/(4*b^2*(a + b*x^2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)$$
$$= -\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)}{2b}$$
$$= -\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.95

$$\frac{-a-2bx^2}{4b^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-a - 2*b*x^2)/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 0.57, size = 157, normalized size = 3.83

$$\frac{-a^3b+\sqrt{b^2}\left(-a^2+abx^2-2b^2x^4\right)\sqrt{a^2+2abx^2+b^2x^4}+ab^3x^4+2b^4x^6}{2x^4\left(-2ab^5-2b^6x^2\right)\sqrt{a^2+2abx^2+b^2x^4}+2\sqrt{b^2}\,x^4\left(2a^2b^4+4ab^5x^2+2b^6x^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(a^3*b) + a*b^3*x^4 + 2*b^4*x^6 + Sqrt[b^2]*(-a^2 + a*b*x^2 - 2*b^2*x^4)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^4*(-2*a*b^5 - 2*b^6*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4] + 2*Sqrt[b^2]*x^4*(2*a^2*b^4 + 4*a*b^5*x^2 + 2*b^6*x^4)
)
```

fricas [A] time = 0.72, size = 36, normalized size = 0.88

$$-\frac{2\,bx^2+a}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.23, size = 32, normalized size = 0.78

$$-\frac{2\,bx^2+a}{4\left(bx^2+a\right)^2 b^2 \mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] -1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$-\frac{\left(b\,x^2+a\right)\left(2b\,x^2+a\right)}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
[Out] -1/4*(b*x^2+a)*(2*b*x^2+a)/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.38, size = 36, normalized size = 0.88

$$-\frac{2\,bx^2+a}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

mupad [B] time = 4.24, size = 42, normalized size = 1.02

$$-\frac{\left(2\,b\,x^2+a\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,b^2\,\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] -((a + 2*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*b^2*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2)**(3/2), x)
```

3.465 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 607}

$$-\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{4b\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] -1/4*(a + b*x^2)/(b*((a + b*x^2)^2)^(3/2))
```

IntegrateAlgebraic [B] time = 0.52, size = 137, normalized size = 3.61

$$\frac{\sqrt{b^2}\left(a-bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}+a^2b+b^3x^4}{2b\sqrt{b^2}\,x^4\left(2a^2b^2+4ab^3x^2+2b^4x^4\right)+2bx^4\left(-2ab^3-2b^4x^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (a^2*b + b^3*x^4 + Sqrt[b^2]*(a - b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
2*b*x^4*(-2*a*b^3 - 2*b^4*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] + 2*b*Sqrt[b
^2]*x^4*(2*a^2*b^2 + 4*a*b^3*x^2 + 2*b^4*x^4))
```

fricas [A] time = 0.87, size = 26, normalized size = 0.68

$$-\frac{1}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

giac [A] time = 0.20, size = 24, normalized size = 0.63

$$-\frac{1}{4\left(bx^2+a\right)^2 b\operatorname{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] -1/4/((b*x^2 + a)^2*b*sgn(b*x^2 + a))
```

maple [A] time = 0.00, size = 24, normalized size = 0.63

$$-\frac{b\,x^{2}+a}{4\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
[Out] -1/4*(b*x^2+a)/b/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.32, size = 26, normalized size = 0.68

$$-\frac{1}{4\left(b^{3}x^{4}+2\,ab^{2}x^{2}+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

mupad [B] time = 4.34, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{4\,b\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] -(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(4*b*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral(x/((a + b*x**2)**2)**(3/2), x)
```

3.466 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - ((a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log}{a^3\sqrt{a^2+2abx^2}}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.50

$$\frac{a\left(3a+2bx^2\right)+4\log(x)\left(a+bx^2\right)^2-2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^2) + 4*(a + b*x^2)^2*Log[x] - 2*(a + b*x^2)^2*Log[a + b*x^2
])/(4*a^3*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 2.25, size = 755, normalized size = 5.14

$$\frac{\left(\sqrt{a^2+2abx^2+b^2x^4}-\sqrt{b^2}\,x^2\right)^4\tanh^{-1}\left(\frac{\sqrt{b^2}\,x^2}{a}-\frac{\sqrt{a^2+2abx^2+b^2x^4}}{a}\right)}{a^3\left(a^4+4a^3bx^2+12a^2b^2x^4-8ab\sqrt{b^2}\,x^4\sqrt{a^2+2abx^2+b^2x^4}-4a^2\sqrt{b^2}\,x^2\sqrt{a^2+2abx^2+b^2x^4}-8\left(b^2\right)^{3/2}x^6\sqrt{a^2+2abx^2+b^2x^4}+16ab^3x^6+8b^4x^8\right)}+\frac{\sqrt{b^2}\left(-a^{11}-3a^9b^2x^4-50a^8b^3x^6-368a^7b^4x^8-1568a^6b^5x^{10}-4256a^5b^6x^{12}-7616a^4b^7x^{14}-8960a^3b^8x^{16}-6656a^2b^9x^{18}-2816ab^{10}x^{20}-512b^{11}x^{22}\right)+\sqrt{a^2+2abx^2+b^2x^4}\left(-a^{10}b+a^9b^2x^2+2a^8b^3x^4+48a^7b^4x^6+320a^6b^5x^8+1248a^5b^6x^{10}+3008a^4b^7x^{12}+4608a^3b^8x^{14}+4352a^2b^9x^{16}+2304ab^{10}x^{18}+512b^{11}x^{20}\right)}{2a^2b\sqrt{b^2}\,x^4\sqrt{a^2+2abx^2+b^2x^4}\left(-2a^9b-34a^8b^2x^2-256a^7b^3x^4-1120a^6b^4x^6-3136a^5b^5x^8-5824a^4b^6x^{10}-7168a^3b^7x^{12}-5632a^2b^8x^{14}-2560ab^9x^{16}-512b^{10}x^{18}\right)+2a^2bx^4\left(2a^{10}b^2+36a^9b^3x^2+290a^8b^4x^4+1376a^7b^5x^6+4256a^6b^6x^8+8960a^5b^7x^{10}+12992a^4b^8x^{12}+12800a^3b^9x^{14}+8192a^2b^{10}x^{16}+3072ab^{11}x^{18}+512b^{12}x^{20}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^10*b) + a^9*b^2*x^2 + 2*a^8*b^3*x^4 +
 48*a^7*b^4*x^6 + 320*a^6*b^5*x^8 + 1248*a^5*b^6*x^10 + 3008*a^4*b^7*x^12 +
 4608*a^3*b^8*x^14 + 4352*a^2*b^9*x^16 + 2304*a*b^10*x^18 + 512*b^11*x^20)
+ Sqrt[b^2]*(-a^11 - 3*a^9*b^2*x^4 - 50*a^8*b^3*x^6 - 368*a^7*b^4*x^8 - 156
8*a^6*b^5*x^10 - 4256*a^5*b^6*x^12 - 7616*a^4*b^7*x^14 - 8960*a^3*b^8*x^16
```

```
- 6656*a^2*b^9*x^18 - 2816*a*b^10*x^20 - 512*b^11*x^22))/(2*a^2*b*Sqrt[b^2]
*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-2*a^9*b - 34*a^8*b^2*x^2 - 256*a^7*b
^3*x^4 - 1120*a^6*b^4*x^6 - 3136*a^5*b^5*x^8 - 5824*a^4*b^6*x^10 - 7168*a^3
*b^7*x^12 - 5632*a^2*b^8*x^14 - 2560*a*b^9*x^16 - 512*b^10*x^18) + 2*a^2*b*
x^4*(2*a^10*b^2 + 36*a^9*b^3*x^2 + 290*a^8*b^4*x^4 + 1376*a^7*b^5*x^6 + 425
6*a^6*b^6*x^8 + 8960*a^5*b^7*x^10 + 12992*a^4*b^8*x^12 + 12800*a^3*b^9*x^14
 + 8192*a^2*b^10*x^16 + 3072*a*b^11*x^18 + 512*b^12*x^20)) + ((-(Sqrt[b^2]*
x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4*ArcTanh[(Sqrt[b^2]*x^2)/a - Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]/a])/(a^3*(a^4 + 4*a^3*b*x^2 + 12*a^2*b^2*x^4 + 1
6*a*b^3*x^6 + 8*b^4*x^8 - 4*a^2*Sqrt[b^2]*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4] - 8*a*b*Sqrt[b^2]*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] - 8*(b^2)^(3/2)*x^
6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]))
```

fricas [A] time = 2.92, size = 90, normalized size = 0.61

$$\frac{2\,abx^2+3\,a^2-2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+2\,abx^2+a^2\right)\log(x)}{4\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*(
b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)
```

giac [A] time = 0.27, size = 79, normalized size = 0.54

$$-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{\log\left(|x|\right)}{a^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{2\,abx^2+3\,a^2}{4\left(bx^2+a\right)^2a^3\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(b*x^2 + a))/(a^3*sgn(b*x^2 + a)) + log(abs(x))/(a^3*sgn(b*x^2
+ a)) + 1/4*(2*a*b*x^2 + 3*a^2)/((b*x^2 + a)^2*a^3*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 107, normalized size = 0.73

$$\frac{\left(4b^2x^4\ln(x)-2b^2x^4\ln\left(b\,x^2+a\right)+8ab\,x^2\ln(x)-4ab\,x^2\ln\left(b\,x^2+a\right)+2ab\,x^2+4a^2\ln(x)-2a^2\ln\left(b\,x^2+a\right)+3a^2\right)\left(b\,x^2+a\right)}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(4*ln(x)*x^4*b^2-2*b^2*x^4*ln(b*x^2+a)+8*ln(x)*x^2*a*b-4*a*b*x^2*ln(b*x
^2+a)+2*a*b*x^2+4*a^2*ln(x)-2*a^2*ln(b*x^2+a)+3*a^2)*(b*x^2+a)/a^3/((b*x^2+
a)^2)^(3/2)
```

maxima [A] time = 1.44, size = 57, normalized size = 0.39

$$\frac{2\,bx^2+3\,a}{4\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^3}+\frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 + a)/
a^3 + log(x)/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**2)**2)**(3/2)), x)
```

3.467 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b\log(x)\left(a+bx^2\right)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.10, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b\log(x)\left(a+bx^2\right)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -(b/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - b/(4*a^2*(a + b*x^2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) - (a + b*x^2)/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (3*b*(a + b*x^2)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3*b*(a + b*x^2)*Log[a + b*x^2])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^3\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x^2}-\frac{3}{a^4b^2x}+\frac{1}{a^2b(a+bx)^3}+\frac{2}{a^3b(a+bx)^2}+\frac{3}{a^4b(a+bx)}\right) dx}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a+b}{2a^3x^2\sqrt{a^2+2}}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.51

$$\frac{-a\left(2a^2+9abx^2+6b^2x^4\right)-12bx^2\log(x)\left(a+bx^2\right)^2+6bx^2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)) - 12*b*x^2*(a + b*x^2)^2*Log[x] + 6*b
*x^2*(a + b*x^2)^2*Log[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^
2])
```

IntegrateAlgebraic [B] time = 3.30, size = 796, normalized size = 4.21

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(a^16*b) + 2*a^15*b^2*x^2 + 61*a^14*b^3*x^4 + 864*a^13*b^4*x^6 + 7540*a^1
2*b^5*x^8 + 45344*a^11*b^6*x^10 + 199056*a^10*b^7*x^12 + 658944*a^9*b^8*x^1
4 + 1674816*a^8*b^9*x^16 + 3294720*a^7*b^10*x^18 + 5015296*a^6*b^11*x^20 +
5857280*a^5*b^12*x^22 + 5151744*a^4*b^13*x^24 + 3301376*a^3*b^14*x^26 + 145
4080*a^2*b^15*x^28 + 393216*a*b^16*x^30 + 49152*b^17*x^32 + Sqrt[b^2]*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]*(-a^15 - a^14*b*x^2 - 60*a^13*b^2*x^4 - 804*a^12
```

```
*b^3*x^6 - 6736*a^11*b^4*x^8 - 38608*a^10*b^5*x^10 - 160448*a^9*b^6*x^12 -
498496*a^8*b^7*x^14 - 1176320*a^7*b^8*x^16 - 2118400*a^6*b^9*x^18 - 2896896
*a^5*b^10*x^20 - 2960384*a^4*b^11*x^22 - 2191360*a^3*b^12*x^24 - 1110016*a^
2*b^13*x^26 - 344064*a*b^14*x^28 - 49152*b^15*x^30))/(2*a^3*x^4*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]*(-2*a^14*b^2 - 54*a^13*b^3*x^2 - 676*a^12*b^4*x^4 - 52
00*a^11*b^5*x^6 - 27456*a^10*b^6*x^8 - 105248*a^9*b^7*x^10 - 302016*a^8*b^8
*x^12 - 658944*a^7*b^9*x^14 - 1098240*a^6*b^10*x^16 - 1391104*a^5*b^11*x^18
 - 1317888*a^4*b^12*x^20 - 905216*a^3*b^13*x^22 - 425984*a^2*b^14*x^24 - 12
2880*a*b^15*x^26 - 16384*b^16*x^28) + 2*a^3*Sqrt[b^2]*x^4*(2*a^15*b + 56*a^
14*b^2*x^2 + 730*a^13*b^3*x^4 + 5876*a^12*b^4*x^6 + 32656*a^11*b^5*x^8 + 13
2704*a^10*b^6*x^10 + 407264*a^9*b^7*x^12 + 960960*a^8*b^8*x^14 + 1757184*a^
7*b^9*x^16 + 2489344*a^6*b^10*x^18 + 2708992*a^5*b^11*x^20 + 2223104*a^4*b^
12*x^22 + 1331200*a^3*b^13*x^24 + 548864*a^2*b^14*x^26 + 139264*a*b^15*x^28
 + 16384*b^16*x^30)) - (3*b*ArcTanh[(Sqrt[b^2]*x^2)/a - Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]/a])/a^4
```

fricas [A] time = 1.06, size = 119, normalized size = 0.63

$$-\frac{6\,ab^2x^4+9\,a^2bx^2+2\,a^3-6\left(b^3x^6+2\,ab^2x^4+a^2bx^2\right)\log\left(bx^2+a\right)+12\left(b^3x^6+2\,ab^2x^4+a^2bx^2\right)\log(x)}{4\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

giac [A] time = 0.24, size = 96, normalized size = 0.51

$$\frac{3\,b\log\left(\left|bx^2+a\right|\right)}{2\,a^4\mathrm{sgn}\left(bx^2+a\right)}-\frac{3\,b\log\left(|x|\right)}{a^4\mathrm{sgn}\left(bx^2+a\right)}-\frac{6\,ab^2x^4+9\,a^2bx^2+2\,a^3}{4\left(bx^2+a\right)^2a^4x^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/2*b*log(abs(b*x^2 + a))/(a^4*sgn(b*x^2 + a)) - 3*b*log(abs(x))/(a^4*sgn(b
*x^2 + a)) - 1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/((b*x^2 + a)^2*a^4*x^2
*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 133, normalized size = 0.70

$$-\frac{\left(12b^3x^6\ln(x)-6b^3x^6\ln\left(b\,x^2+a\right)+24a\,b^2x^4\ln(x)-12a\,b^2x^4\ln\left(b\,x^2+a\right)+6a\,b^2x^4+12a^2b\,x^2\ln(x)-6a^2b\,x^2\ln\left(b\,x^2+a\right)+9a^2b\,x^2+2a^3\right)\left(b\,x^2+a\right)}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(12*b^3*x^6*ln(x)-6*ln(b*x^2+a)*x^6*b^3+24*a*b^2*x^4*ln(x)-12*a*b^2*x^
4*ln(b*x^2+a)+6*a*b^2*x^4+12*a^2*b*x^2*ln(x)-6*a^2*b*x^2*ln(b*x^2+a)+9*a^2*
b*x^2+2*a^3)*(b*x^2+a)/a^4/x^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.37, size = 75, normalized size = 0.40

$$-\frac{6\,b^2x^4+9\,abx^2+2\,a^2}{4\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)}+\frac{3\,b\log\left(bx^2+a\right)}{2\,a^4}-\frac{3\,b\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
+ 3/2*b*log(b*x^2 + a)/a^4 - 3*b*log(x)/a^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

3.468 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 288, 205}

$$-\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*x)/(8*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(4*b*(a + b*x^2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*Sqrt[a]*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3\left(ab+b^2x^2\right)\right)}{8b^2\sqrt{a^2+2ab}}$$

$$= -\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3\left(a+bx^2\right)\tan}{8\sqrt{a}\,b^{5/2}\sqrt{a^2+2}}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{a}\,\sqrt{b}\,x\left(3a+5bx^2\right)}{8\sqrt{a}\,b^{5/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a + 5*b*x^2)) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*Sqrt[a]*b^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 7.06, size = 76, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left(\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}+\frac{-3ax-5bx^3}{8b^2\left(a+bx^2\right)^2}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*((-3*a*x - 5*b*x^3)/(8*b^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2))))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 1.33, size = 188, normalized size = 1.47

$$\left[-\frac{10\,ab^{2}x^{3}+6\,a^{2}bx+3\,\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{16\,\left(ab^{5}x^{4}+2\,a^{2}b^{4}x^{2}+a^{3}b^{3}\right)},-\frac{5\,ab^{2}x^{3}+3\,a^{2}bx-3\,\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\,\left(ab^{5}x^{4}+2\,a^{2}b^{4}x^{2}+a^{3}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^2
+ a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 97, normalized size = 0.76

$$-\frac{\left(-3b^{2}x^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-6ab\,x^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+5\sqrt{ab}\,b\,x^{3}-3a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3\sqrt{ab}\,ax\right)\left(b\,x^{2}+a\right)}{8\sqrt{ab}\,\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(-3*arctan(1/(a*b)^(1/2)*b*x)*x^4*b^2+5*(a*b)^(1/2)*b*x^3-6*arctan(1/(
a*b)^(1/2)*b*x)*x^2*a*b+3*(a*b)^(1/2)*a*x-3*a^2*arctan(1/(a*b)^(1/2)*b*x))*
(b*x^2+a)/(a*b)^(1/2)/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.03, size = 59, normalized size = 0.46

$$-\frac{5\,bx^{3}+3\,ax}{8\,\left(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2}\right)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 3/8*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(3/2), x)
```

3.469 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=129

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] x/(8*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(4*b*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)
*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(ab+b^2x^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(ab+b^2x^2\right)\int \frac{1}{ab+\dots}}{8ab\sqrt{a^2+2abx^2+\dots}}\\
&= \frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-\dots}}{8a^{3/2}b^{3/2}\sqrt{a^2+2a\dots}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.63

$$\frac{\sqrt{a}\,\sqrt{b}\,x\left(bx^2-a\right)+\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-a + b*x^2) + (a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(3/2)*b^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 5.91, size = 77, normalized size = 0.60

$$\frac{\left(a+bx^2\right)\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}-\frac{x\left(a-bx^2\right)}{8ab\left(a+bx^2\right)^2}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(-1/8*(x*(a - b*x^2))/(a*b*(a + b*x^2)^2) + ArcTan[(Sqrt[b]*x)
/Sqrt[a]]/(8*a^(3/2)*b^(3/2))))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 1.21, size = 190, normalized size = 1.47

$$\left[\frac{2\,ab^2x^3-2\,a^2bx-\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)},\ \frac{ab^2x^3-a^2bx+\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*log
((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 +
a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 97, normalized size = 0.75

$$\frac{\left(b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)+2ab\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\;b\,x^3+a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)-\sqrt{ab}\;ax\right)\left(b\,x^2+a\right)}{8\sqrt{ab}\;\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(b^2*x^4*arctan(1/(a*b)^(1/2)*b*x)+(a*b)^(1/2)*b*x^3+2*a*b*x^2*arctan(1
/(a*b)^(1/2)*b*x)-(a*b)^(1/2)*a*x+a^2*arctan(1/(a*b)^(1/2)*b*x))*(b*x^2+a)/
(a*b)^(1/2)/b/a/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 2.93, size = 62, normalized size = 0.48

$$\frac{bx^3 - ax}{8\left(ab^3x^4 + 2\,a^2b^2x^2 + a^3b\right)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 1/8*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(3/2), x)
```

3.470 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{3\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1088, 199, 205}

$$\frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{3\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] (x*(a + b*x^2))/(4*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*x*(a + b*x^2)^
2)/(8*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*(a + b*x^2)^3*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(2ab+2b^2x^2\right)^3 \int \frac{1}{\left(2ab+2b^2x^2\right)^3}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

$$= \frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{\left(3\left(2ab+2b^2x^2\right)^3\right)\int \frac{1}{\left(2ab+2b^2x^2\right)^2}\,dx}{8ab\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

$$= \frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{\left(3\left(2ab+2b^2x^2\right)^3\right)\int \frac{}{2}}{32a^2b^2\left(a^2+2abx^2+\right.}$$

$$= \frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{3\left(a+bx^2\right)^3\tan^{-1}\left(\right.}{8a^{5/2}\sqrt{b}\left(a^2+2abx^2+\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.61

$$\frac{\sqrt{a}\sqrt{b}\,x\left(5a+3bx^2\right)+3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(5*a + 3*b*x^2) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*Sqrt[b]*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 6.01, size = 76, normalized size = 0.56

$$\frac{\left(a+bx^2\right)\left(\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{5ax+3bx^3}{8a^2\left(a+bx^2\right)^2}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] ((a + b*x^2)*((5*a*x + 3*b*x^3)/(8*a^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b])))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 1.16, size = 188, normalized size = 1.39

$$\left[\frac{6\,ab^2x^3 + 10\,a^2bx - 3\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{16\left(a^3b^3x^4 + 2\,a^4b^2x^2 + a^5b\right)}, \frac{3\,ab^2x^3 + 5\,a^2bx + 3\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\left(a^3b^3x^4 + 2\,a^4b^2x^2 + a^5b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2
 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 97, normalized size = 0.72

$$\frac{\left(3b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right) + 6ab\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right) + 3\sqrt{ab}\;b\,x^3 + 3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right) + 5\sqrt{ab}\;ax\right)\left(b\,x^2 + a\right)}{8\sqrt{ab}\;\left(\left(b\,x^2 + a\right)^2\right)^{\frac{3}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(3*b^2*x^4*arctan(1/(a*b)^(1/2)*b*x)+3*(a*b)^(1/2)*b*x^3+6*a*b*x^2*arct
an(1/(a*b)^(1/2)*b*x)+5*(a*b)^(1/2)*a*x+3*a^2*arctan(1/(a*b)^(1/2)*b*x))*(b
*x^2+a)/(a*b)^(1/2)/a^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 2.99, size = 58, normalized size = 0.43

$$\frac{3\,bx^3 + 5\,ax}{8\left(a^2b^2x^4 + 2\,a^3bx^2 + a^4\right)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3/8*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/2), x)
```

3.471 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=169

$$\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)}-\frac{15\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15\left(a+bx^2\right.}{8a^3x\sqrt{a^2+2abx^2}}$$

Rubi [A] time = 0.07, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$-\frac{15\left(a+bx^2\right)}{8a^3x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)}-\frac{15\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 5/(8*a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x*(a + b*x^2)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) - (15*(a + b*x^2))/(8*a^3*x*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (15*Sqrt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15\left(ab+b^2\right.\right.}{8a^2\sqrt{a^2}}$$

$$= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{15\left(a\right.}{8a^3x\sqrt{a^2+2}}$$

$$= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{15\left(a\right.}{8a^3x\sqrt{a^2+2}}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 0.55

$$\frac{-\sqrt{a}\left(8a^2+25abx^2+15b^2x^4\right)-15\sqrt{b}\,x\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)) - 15*Sqrt[b]*x*(a + b*x^2)^2*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 11.67, size = 89, normalized size = 0.53

$$\frac{\left(a+bx^2\right)\left(\frac{-8a^2-25abx^2-15b^2x^4}{8a^3x\left(a+bx^2\right)^2}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]

[Out] ((a + b*x^2)*((-8*a^2 - 25*a*b*x^2 - 15*b^2*x^4)/(8*a^3*x*(a + b*x^2)^2) - (15*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))))/Sqrt[(a + b*x^2)^2]

fricas [A] time = 1.30, size = 202, normalized size = 1.20

$$\left[-\frac{30\,b^2x^4+50\,abx^2-15\left(b^2x^5+2\,abx^3+a^2x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+16\,a^2}{16\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)},-\frac{15\,b^2x^4+25\,abx^2+15\left(b^2x^5+2\,abx^3+a^2x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+8\,a^2}{8\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")

[Out] [-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.02, size = 119, normalized size = 0.70

$$-\frac{\left(15b^3x^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)+30a\,b^2x^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)+15\sqrt{ab}\;b^2x^4+15a^2bx\arctan\left(\frac{bx}{\sqrt{ab}}\right)+25\sqrt{ab}\;ab\,x^2+8\sqrt{ab}\;a^2\right)\left(b\,x^2+a\right)}{8\sqrt{ab}\;\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(15*arctan(1/(a*b)^(1/2)*b*x)*x^5*b^3+15*(a*b)^(1/2)*x^4*b^2+30*arctan
(1/(a*b)^(1/2)*b*x)*x^3*a*b^2+25*(a*b)^(1/2)*x^2*a*b+15*arctan(1/(a*b)^(1/2
)*b*x)*x*a^2*b+8*(a*b)^(1/2)*a^2)*(b*x^2+a)/(a*b)^(1/2)/x/a^3/((b*x^2+a)^2)
^(3/2)
```

maxima [A] time = 2.98, size = 71, normalized size = 0.42

$$-\frac{15\,b^{2}x^{4}+25\,abx^{2}+8\,a^{2}}{8\left(a^{3}b^{2}x^{5}+2\,a^{4}bx^{3}+a^{5}x\right)}-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
- 15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(\left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

3.472 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^{3/2}} dx$

Optimal. Leaf size=209

$$\frac{1}{4ax^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b(a+}{8a^4x\sqrt{a^2+2ab}$$

Rubi [A] time = 0.08, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$\frac{35b(a+bx^2)}{8a^4x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35(a+bx^2)}{24a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 7/(8*a^2*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x^3*(a + b*x^2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*(a + b*x^2))/(24*a^3*x^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) + (35*b*(a + b*x^2))/(8*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (35*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(35\left(ab+\right.\right.}{8a^2\sqrt{a}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35}{24a^3x^3\sqrt{a^2}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35}{24a^3x^3\sqrt{a^2}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35}{24a^3x^3\sqrt{a^2}}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 0.50

$$\frac{\sqrt{a}\left(-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6\right)+105b^{3/2}x^3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{24a^{9/2}x^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6) + 105*b^(3/2
)*x^3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(24*a^(9/2)*x^3*(a + b*x^2
)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 19.09, size = 100, normalized size = 0.48

$$\frac{\left(a+bx^2\right)\left(\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^4x^3\left(a+bx^2\right)^2}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] ((a + b*x^2)*((-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6)/(24*a^4
*x^3*(a + b*x^2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)))
)/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 2.22, size = 238, normalized size = 1.14

$$\left[\frac{210\,b^3x^6+350\,ab^2x^4+112\,a^2bx^2-16\,a^3+105\,\left(b^3x^7+2\,ab^2x^5+a^2bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48\,\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)},\ \frac{105\,b^3x^6+175\,ab^2x^4+56\,a^2bx^2-8\,a^3+105\,\left(b^3x^7+2\,ab^2x^5+a^2bx^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{24\,\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
+ 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 175*
a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 139, normalized size = 0.67

$$\frac{\left(105 b^4 x^7 \arctan\left(\frac{bx}{\sqrt{ab}}\right)+210 a\, b^3 x^5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)+105 \sqrt{ab}\, b^3 x^6+105 a^2 b^2 x^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)+175 \sqrt{ab}\, a\, b^2 x^4+56 \sqrt{ab}\, a^2 b\, x^2-8 \sqrt{ab}\, a^3\right)\left(b\,x^2+a\right)}{24 \sqrt{ab}\, \left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}} a^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/24*(105*arctan(1/(a*b)^(1/2)*b*x)*x^7*b^4+105*(a*b)^(1/2)*x^6*b^3+210*arc
tan(1/(a*b)^(1/2)*b*x)*x^5*a*b^3+175*(a*b)^(1/2)*x^4*a*b^2+105*arctan(1/(a*
b)^(1/2)*b*x)*x^3*a^2*b^2+56*(a*b)^(1/2)*x^2*a^2*b-8*(a*b)^(1/2)*a^3)*(b*x^
2+a)/(a*b)^(1/2)/x^3/a^4/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.03, size = 86, normalized size = 0.41

$$\frac{105\, b^3 x^6+175\, a b^2 x^4+56\, a^2 b x^2-8\, a^3}{24\left(a^4 b^2 x^7+2\, a^5 b x^5+a^6 x^3\right)}+\frac{35\, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\, \sqrt{ab}\, a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*(105*b^3*x^6 + 175*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3)/(a^4*b^2*x^7 + 2*
a^5*b*x^5 + a^6*x^3) + 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**2)**2)**(3/2)), x)
```

3.473 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=238

$$-\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.19, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^4}{6b^6\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5a^3}{2b^6\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^6\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-5*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^5/(8*b^6*(a + b*x^2)^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a^4)/(6*b^6*(a + b*x^2)^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (5*a^3)/(2*b^6*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) + (x^2*(a + b*x^2))/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a*
(a + b*x^2)*Log[a + b*x^2])/(2*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)$$
$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^{10}}-\frac{a^5}{b^{10}(a+bx)^5}+\frac{5a^4}{b^{10}(a+bx)^4}-\frac{10a^3}{b^{10}(a+bx)^3}+\frac{10a^2}{b^{10}(a+bx)^2}-\frac{}{b^{10}}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{6b^6\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.04, size = 103, normalized size = 0.43

$$\frac{-77a^5-248a^4bx^2-252a^3b^2x^4-48a^2b^3x^6+48ab^4x^8-60a\left(a+bx^2\right)^4\log\left(a+bx^2\right)+12b^5x^{10}}{24b^6\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^2 - 252*a^3*b^2*x^4 - 48*a^2*b^3*x^6 + 48*a*b^4*x^8
+ 12*b^5*x^10 - 60*a*(a + b*x^2)^4*Log[a + b*x^2])/(24*b^6*(a + b*x^2)^3*Sq
rt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 2.50, size = 2541, normalized size = 10.68

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((256*a^8*x^2)/(3*b^4*Sqrt[b^2]) + (1264*a^7*x^4)/(3*b^3*Sqrt[b^2]) + (1312
*a^6*x^6)/(b^2)^(3/2) + (7904*a^5*x^8)/(3*b*Sqrt[b^2]) + (9344*a^4*x^10)/(3
```

*Sqrt[b^2]) + (1792*a^3*b*x^12)/Sqrt[b^2] + 128*a^2*Sqrt[b^2]*x^14 - (288*a
*b^3*x^16)/Sqrt[b^2] - (64*b^4*x^18)/Sqrt[b^2] - (16*a^8*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/b^6 - (208*a^7*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*b^5)
 - (352*a^6*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^4 - (960*a^5*x^6*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/b^3 - (5024*a^4*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(3*b^2) - (1440*a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b - 352*a^2*x^
12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] + 224*a*b*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4] + 64*b^2*x^16*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] - (320*a^5*x^8*ArcTanh
[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a])/b^2 - (1280*a^4*x
^10*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a])/b - 19
20*a^3*x^12*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a]
 - 1280*a^2*b*x^14*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/a] - 320*a*b^2*x^16*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/a] + (320*a^4*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(-(Sqr
t[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a])/(b*Sqrt[b^2]) + (960*a^3
*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/a])/Sqrt[b^2] + (960*a^2*b*x^12*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]*ArcTanh[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a
])/Sqrt[b^2] + 320*a*Sqrt[b^2]*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh
[(-(Sqrt[b^2]*x^2) + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/a])/((-a - Sqrt[b^2]*
x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])^4) + ((-16*a^9)/(b^5*Sqrt[b^2]) - (256*a^8*x^2)/(3*b^4*
Sqrt[b^2]) - (1264*a^7*x^4)/(3*b^3*Sqrt[b^2]) - (1312*a^6*x^6)/(b^2)^(3/2)
 - (2256*a^5*x^8)/(b*Sqrt[b^2]) - (1920*a^4*x^10)/Sqrt[b^2] - (640*a^3*b*x^1
2)/Sqrt[b^2] + (256*a^7*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*b^5) + (336
*a^6*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^4 + (976*a^5*x^6*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/b^3 + (1280*a^4*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2
 + (640*a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b + (160*a^5*x^8*Log[-a -
 Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(b*Sqrt[b^2]) + (640*a^4
*x^10*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2]
+ (960*a^3*b*x^12*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]]
)/Sqrt[b^2] + 640*a^2*Sqrt[b^2]*x^14*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]] + (160*a*b^3*x^16*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] - (160*a^4*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b^2 - (480*a
^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]])/b - 480*a^2*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Lo
g[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] - 160*a*b*x^14*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]] + (160*a^5*x^8*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]])/(b*Sqrt[b^2]) + (640*a^4*x^10*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] + (960*a^3*b*x^12*Log[a - Sqrt[b^2]*x^2 + S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] + 640*a^2*Sqrt[b^2]*x^14*Log[a -
 Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] + (160*a*b^3*x^16*Log[a -
 Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] - (160*a^4*x^8

```
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]])/b^2 - (480*a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a -
 Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b - 480*a^2*x^12*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]] - 160*a*b*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2
 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/((-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4
)
```

fricas [A] time = 0.63, size = 157, normalized size = 0.66

$$\frac{12\,b^{5}x^{10}+48\,ab^{4}x^{8}-48\,a^{2}b^{3}x^{6}-252\,a^{3}b^{2}x^{4}-248\,a^{4}bx^{2}-77\,a^{5}-60\left(ab^{4}x^{8}+4\,a^{2}b^{3}x^{6}+6\,a^{3}b^{2}x^{4}+4\,a^{4}bx^{2}+a^{5}\right)\log\left(bx^{2}+a\right)}{24\left(b^{10}x^{8}+4\,ab^{9}x^{6}+6\,a^{2}b^{8}x^{4}+4\,a^{3}b^{7}x^{2}+a^{4}b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 48*a*b^4*x^8 - 48*a^2*b^3*x^6 - 252*a^3*b^2*x^4 - 248*a
^4*b*x^2 - 77*a^5 - 60*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b
*x^2 + a^5)*log(b*x^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3
*b^7*x^2 + a^4*b^6)
```

giac [A] time = 0.25, size = 105, normalized size = 0.44

$$\frac{x^{2}}{2\,b^{5}\mathrm{sgn}\left(bx^{2}+a\right)}-\frac{5\,a\log\left(\left|bx^{2}+a\right|\right)}{2\,b^{6}\mathrm{sgn}\left(bx^{2}+a\right)}-\frac{120\,a^{2}b^{3}x^{6}+300\,a^{3}b^{2}x^{4}+260\,a^{4}bx^{2}+77\,a^{5}}{24\left(bx^{2}+a\right)^{4}b^{6}\mathrm{sgn}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/(b^5*sgn(b*x^2 + a)) - 5/2*a*log(abs(b*x^2 + a))/(b^6*sgn(b*x^2 + a
)) - 1/24*(120*a^2*b^3*x^6 + 300*a^3*b^2*x^4 + 260*a^4*b*x^2 + 77*a^5)/((b*
x^2 + a)^4*b^6*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 163, normalized size = 0.68

$$-\frac{\left(-12b^{5}x^{10}+60a\,b^{4}x^{8}\ln\left(b\,x^{2}+a\right)-48a\,b^{4}x^{8}+240a^{2}b^{3}x^{6}\ln\left(b\,x^{2}+a\right)+48a^{2}b^{3}x^{6}+360a^{3}b^{2}x^{4}\ln\left(b\,x^{2}+a\right)+252a^{3}b^{2}x^{4}+240a^{4}b\,x^{2}\ln\left(b\,x^{2}+a\right)+248a^{4}b\,x^{2}+60a^{5}\ln\left(b\,x^{2}+a\right)+77a^{5}\right)\left(b\,x^{2}+a\right)}{24\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(-12*b^5*x^10+60*ln(b*x^2+a)*x^8*a*b^4-48*a*b^4*x^8+240*ln(b*x^2+a)*x
^6*a^2*b^3+48*a^2*b^3*x^6+360*ln(b*x^2+a)*x^4*a^3*b^2+252*a^3*b^2*x^4+240*l
```

```
n(b*x^2+a)*x^2*a^4*b+248*a^4*b*x^2+60*ln(b*x^2+a)*a^5+77*a^5)*(b*x^2+a)/b^6
/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.38, size = 110, normalized size = 0.46

$$-\frac{120\,a^2b^3x^6+300\,a^3b^2x^4+260\,a^4bx^2+77\,a^5}{24\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)}+\frac{x^2}{2\,b^5}-\frac{5\,a\log\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(120*a^2*b^3*x^6 + 300*a^3*b^2*x^4 + 260*a^4*b*x^2 + 77*a^5)/(b^10*x^
8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6) + 1/2*x^2/b^5 -
5/2*a*log(b*x^2 + a)/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11/((a + b*x**2)**2)**(5/2), x)
```

3.474 $\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{3a^2}{2b^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2ab}}$$

Rubi [A] time = 0.16, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a^3}{3b^5\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a^2}{2b^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a)/(b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^4/(8*b^5*(a + b*x^2)^3*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*a^3)/(3*b^5*(a + b*x^2)^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) - (3*a^2)/(2*b^5*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)$$
$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\frac{1}{b^9(a+bx)}\right)\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{3b^5\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.42

$$\frac{a\left(25a^3+88a^2bx^2+108ab^2x^4+48b^3x^6\right)+12\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24b^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a*(25*a^3 + 88*a^2*b*x^2 + 108*a*b^2*x^4 + 48*b^3*x^6) + 12*(a + b*x^2)^4*
Log[a + b*x^2])/(24*b^5*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 2.28, size = 3027, normalized size = 15.44

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((16*a^8)/(b^4*Sqrt[b^2]) + (224*a^7*x^2)/(3*b^3*Sqrt[b^2]) + (944*a^6*x^4)
/(3*(b^2)^(3/2)) + (800*a^5*x^6)/(b*Sqrt[b^2]) + (1200*a^4*x^8)/Sqrt[b^2] +
 (960*a^3*b*x^10)/Sqrt[b^2] + 320*a^2*Sqrt[b^2]*x^12 - (224*a^6*x^2*Sqrt[a^
```

2 + 2*a*b*x^2 + b^2*x^4])/(3*b^4) - (240*a^5*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/b^3 - (560*a^4*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2 - (640*a^3*x
^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b - 320*a^2*x^10*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4] - (32*a^4*x^8*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]])/Sqrt[b^2] - (128*a^3*b*x^10*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]])/Sqrt[b^2] - 192*a^2*Sqrt[b^2]*x^12*Log[-a - Sqrt[b^2]*x
^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] - (128*a*b^3*x^14*Log[-a - Sqrt[b^2]*
x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] - (32*b^4*x^16*Log[-a - S
qrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] + (32*a^3*x^8*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]])/b + 96*a^2*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-a - Sqrt
[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] + 96*a*b*x^12*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] +
 32*b^2*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-a - Sqrt[b^2]*x^2 + Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]] - (32*a^4*x^8*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] - (128*a^3*b*x^10*Log[a - Sqrt[b^2]*x^2
+ Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] - 192*a^2*Sqrt[b^2]*x^12*Log[
a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] - (128*a*b^3*x^14*Log[
a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] - (32*b^4*x
^16*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] + (
32*a^3*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]])/b + 96*a^2*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*L
og[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] + 96*a*b*x^12*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]] + 32*b^2*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^
2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/((-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^
4) + ((-224*a^7*x^2)/(3*b^3*Sqrt[b^2]) - (944*a^6*x^4)/(3*(b^2)^(3/2)) - (8
00*a^5*x^6)/(b*Sqrt[b^2]) - (4000*a^4*x^8)/(3*Sqrt[b^2]) - (4288*a^3*b*x^10
)/(3*Sqrt[b^2]) - 896*a^2*Sqrt[b^2]*x^12 - (256*a*b^3*x^14)/Sqrt[b^2] + (16
*a^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^5 + (176*a^6*x^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(3*b^4) + (256*a^5*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^3
+ (544*a^4*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2 + (2368*a^3*x^8*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(3*b) + 640*a^2*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4] + 256*a*b*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] + (32*a^4*x^8*Log[a - Sq
rt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b + 128*a^3*x^10*Log[a - Sq
rt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] + 192*a^2*b*x^12*Log[a - Sqr
t[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] + 128*a*b^2*x^14*Log[a - Sqrt
[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] + 32*b^3*x^16*Log[a - Sqrt[b^2
]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] - (32*a^3*x^8*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[
b^2] - (96*a^2*b*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2
 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] - 96*a*Sqrt[b^2]*x^12*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]] - (32*b^3*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[a - Sqrt[b^2]*x^

```
2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] - (32*a^4*x^8*Log[-(a*b^5)
- b^5*Sqrt[b^2]*x^2 + b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/b - 128*a^3*x^1
0*Log[-(a*b^5) - b^5*Sqrt[b^2]*x^2 + b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] -
 192*a^2*b*x^12*Log[-(a*b^5) - b^5*Sqrt[b^2]*x^2 + b^5*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]] - 128*a*b^2*x^14*Log[-(a*b^5) - b^5*Sqrt[b^2]*x^2 + b^5*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]] - 32*b^3*x^16*Log[-(a*b^5) - b^5*Sqrt[b^2]*x^2 +
 b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] + (32*a^3*x^8*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]*Log[-(a*b^5) - b^5*Sqrt[b^2]*x^2 + b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]])/Sqrt[b^2] + (96*a^2*b*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-(a*b
^5) - b^5*Sqrt[b^2]*x^2 + b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] +
 96*a*Sqrt[b^2]*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[-(a*b^5) - b^5*Sqr
t[b^2]*x^2 + b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]] + (32*b^3*x^14*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]*Log[-(a*b^5) - b^5*Sqrt[b^2]*x^2 + b^5*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]])/Sqrt[b^2])/((-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])^4*(a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])^4)
```

fricas [A] time = 2.77, size = 135, normalized size = 0.69

$$\frac{48\,ab^{3}x^{6}+108\,a^{2}b^{2}x^{4}+88\,a^{3}bx^{2}+25\,a^{4}+12\left(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4}\right)\log\left(bx^{2}+a\right)}{24\left(b^{9}x^{8}+4\,ab^{8}x^{6}+6\,a^{2}b^{7}x^{4}+4\,a^{3}b^{6}x^{2}+a^{4}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4 + 12*(b^4*x^8
+ 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^9*x^8
 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

giac [A] time = 0.21, size = 84, normalized size = 0.43

$$\frac{\log\left(\left|bx^{2}+a\right|\right)}{2\,b^{5}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{48\,ab^{2}x^{6}+108\,a^{2}bx^{4}+88\,a^{3}x^{2}+\frac{25\,a^{4}}{b}}{24\left(bx^{2}+a\right)^{4}b^{4}\mathrm{sgn}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/(b^5*sgn(b*x^2 + a)) + 1/24*(48*a*b^2*x^6 + 108*a^2
*b*x^4 + 88*a^3*x^2 + 25*a^4/b)/((b*x^2 + a)^4*b^4*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 141, normalized size = 0.72

$$\frac{\left(12b^{4}x^{8}\ln\left(b\,x^{2}+a\right)+48a\,b^{3}x^{6}\ln\left(b\,x^{2}+a\right)+48a\,b^{3}x^{6}+72a^{2}b^{2}x^{4}\ln\left(b\,x^{2}+a\right)+108a^{2}b^{2}x^{4}+48a^{3}b\,x^{2}\ln\left(b\,x^{2}+a\right)+88a^{3}b\,x^{2}+12a^{4}\ln\left(b\,x^{2}+a\right)+25a^{4}\right)\left(b\,x^{2}+a\right)}{24\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24*(12*ln(b*x^2+a)*x^8*b^4+48*ln(b*x^2+a)*x^6*a*b^3+48*a*b^3*x^6+72*ln(b*
x^2+a)*x^4*a^2*b^2+108*a^2*b^2*x^4+48*ln(b*x^2+a)*x^2*a^3*b+88*a^3*b*x^2+12
*ln(b*x^2+a)*a^4+25*a^4)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.46, size = 99, normalized size = 0.51

$$\frac{48\,ab^3x^6+108\,a^2b^2x^4+88\,a^3bx^2+25\,a^4}{24\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4)/(b^9*x^8 + 4*
a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5) + 1/2*log(b*x^2 + a)/b
^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**9/((a + b*x**2)**2)**(5/2), x)
```

3.475 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^8}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$\frac{x^8}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] x^8/(8*a*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^5}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{x^8}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.49

$$\frac{-a^3-4a^2bx^2-6ab^2x^4-4b^3x^6}{8b^4\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a^3 - 4*a^2*b*x^2 - 6*a*b^2*x^4 - 4*b^3*x^6)/(8*b^4*(a + b*x^2)^3*Sqrt[(a
 + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 0.73, size = 274, normalized size = 6.68

$$\frac{\sqrt{b^2}\left(-a^7+a^3b^4x^8+4a^2b^5x^{10}+6ab^6x^{12}+4b^7x^{14}\right)+\sqrt{a^2+2abx^2+b^2x^4}\left(-a^6b+a^5b^2x^2-a^4b^3x^4+a^3b^4x^6-2a^2b^5x^8-2ab^6x^{10}-4b^7x^{12}\right)}{b^5\sqrt{b^2}\,x^8\sqrt{a^2+2abx^2+b^2x^4}\left(-8a^3b^3-24a^2b^4x^2-24ab^5x^4-8b^6x^6\right)+b^5x^8\left(8a^4b^4+32a^3b^5x^2+48a^2b^6x^4+32ab^7x^6+8b^8x^8\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^6*b) + a^5*b^2*x^2 - a^4*b^3*x^4 + a^
3*b^4*x^6 - 2*a^2*b^5*x^8 - 2*a*b^6*x^10 - 4*b^7*x^12) + Sqrt[b^2]*(-a^7 +
a^3*b^4*x^8 + 4*a^2*b^5*x^10 + 6*a*b^6*x^12 + 4*b^7*x^14))/(b^5*Sqrt[b^2]*x
^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-8*a^3*b^3 - 24*a^2*b^4*x^2 - 24*a*b^5*
x^4 - 8*b^6*x^6) + b^5*x^8*(8*a^4*b^4 + 32*a^3*b^5*x^2 + 48*a^2*b^6*x^4 + 3
2*a*b^7*x^6 + 8*b^8*x^8))
```

fricas [B] time = 1.14, size = 80, normalized size = 1.95

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a*b^7*x^6 +
 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

giac [A] time = 0.22, size = 54, normalized size = 1.32

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\left(bx^2+a\right)^4b^4\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/((b*x^2 + a)^4*b^4*sgn(b
*x^2 + a))
```

maple [A] time = 0.01, size = 54, normalized size = 1.32

$$-\frac{\left(b\,x^2+a\right)\left(4b^3x^6+6a\,b^2x^4+4a^2b\,x^2+a^3\right)}{8\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/8*(b*x^2+a)*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)/b^4/((b*x^2+a)^2)^(5
/2)
```

maxima [B] time = 1.40, size = 80, normalized size = 1.95

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a*b^7*x^6 +
 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

mupad [B] time = 4.29, size = 144, normalized size = 3.51

$$\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,b^4\left(b\,x^2+a\right)^5}-\frac{a^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^4\left(b\,x^2+a\right)^4}-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^4\left(b\,x^2+a\right)^2}+\frac{3\,a\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,b^4\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*b^4*(a + b*x^2)^5) - (a^2*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^4*(a + b*x^2)^4) - (a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2)/(2*b^4*(a + b*x^2)^2) + (3*a*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))
/(4*b^4*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**7/((a + b*x**2)**2)**(5/2), x)
```

3.476 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1109}

$$\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] x^6/(24*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + x^6/(8*a*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 1109

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(2*(d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(d*(m + 3)*(2*a + b*
x^2)), x] - Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*d*(m + 3)
*(p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.68

$$\frac{-a^2-4abx^2-6b^2x^4}{24b^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] (-a^2 - 4*a*b*x^2 - 6*b^2*x^4)/(24*b^3*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 0.69, size = 256, normalized size = 3.46

$$\frac{\sqrt{b^2}\left(3a^6+a^2b^4x^8+4ab^5x^{10}+6b^6x^{12}\right)+\sqrt{a^2+2abx^2+b^2x^4}\left(3a^5b-3a^4b^2x^2+3a^3b^3x^4-3a^2b^4x^6+2ab^5x^8-6b^6x^{10}\right)}{3b^4\sqrt{b^2}\,x^8\sqrt{a^2+2abx^2+b^2x^4}\left(-8a^3b^3-24a^2b^4x^2-24ab^5x^4-8b^6x^6\right)+3b^4x^8\left(8a^4b^4+32a^3b^5x^2+48a^2b^6x^4+32ab^7x^6+8b^8x^8\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] (Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(3*a^5*b - 3*a^4*b^2*x^2 + 3*a^3*b^3*x^4 -
 3*a^2*b^4*x^6 + 2*a*b^5*x^8 - 6*b^6*x^10) + Sqrt[b^2]*(3*a^6 + a^2*b^4*x^8
 + 4*a*b^5*x^10 + 6*b^6*x^12))/(3*b^4*Sqrt[b^2]*x^8*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]*(-8*a^3*b^3 - 24*a^2*b^4*x^2 - 24*a*b^5*x^4 - 8*b^6*x^6) + 3*b^4*x
^8*(8*a^4*b^4 + 32*a^3*b^5*x^2 + 48*a^2*b^6*x^4 + 32*a*b^7*x^6 + 8*b^8*x^8)
)
```

fricas [A] time = 2.67, size = 69, normalized size = 0.93

$$-\frac{6\,b^2x^4+4\,abx^2+a^2}{24\left(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4
+ 4*a^3*b^4*x^2 + a^4*b^3)
```

giac [A] time = 0.21, size = 43, normalized size = 0.58

$$-\frac{6\,b^2x^4+4\,abx^2+a^2}{24\left(bx^2+a\right)^4b^3\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/((b*x^2 + a)^4*b^3*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 43, normalized size = 0.58

$$-\frac{\left(b\,x^{2}+a\right)\left(6b^{2}x^{4}+4ab\,x^{2}+a^{2}\right)}{24\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(b*x^2+a)*(6*b^2*x^4+4*a*b*x^2+a^2)/b^3/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.79, size = 69, normalized size = 0.93

$$-\frac{6\,b^{2}x^{4}+4\,abx^{2}+a^{2}}{24\left(b^{7}x^{8}+4\,ab^{6}x^{6}+6\,a^{2}b^{5}x^{4}+4\,a^{3}b^{4}x^{2}+a^{4}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4
+ 4*a^3*b^4*x^2 + a^4*b^3)
```

mupad [B] time = 4.23, size = 53, normalized size = 0.72

$$-\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}\,\left(a^{2}+4\,a\,b\,x^{2}+6\,b^{2}\,x^{4}\right)}{24\,b^{3}\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a^2 + 6*b^2*x^4 + 4*a*b*x^2))/(24*b^3*
(a + b*x^2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{5}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**2)**(5/2), x)
```

3.477 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=69

$$\frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 607}

$$\frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/(6*b^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + a/(8*b^2*(a + b*x^2)*(a^2 +
2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)$$
$$= -\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)}{2b}$$
$$= -\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.57

$$\frac{-a-4bx^2}{24b^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a - 4*b*x^2)/(24*b^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 0.69, size = 224, normalized size = 3.25

$$\frac{-3a^5b+\sqrt{b^2}\sqrt{a^2+2abx^2+b^2x^4}\left(-3a^4+3a^3bx^2-3a^2b^2x^4+3ab^3x^6-4b^4x^8\right)+ab^5x^8+4b^6x^{10}}{3x^8\sqrt{a^2+2abx^2+b^2x^4}\left(-8a^3b^7-24a^2b^8x^2-24ab^9x^4-8b^{10}x^6\right)+3\sqrt{b^2}\,x^8\left(8a^4b^6+32a^3b^7x^2+48a^2b^8x^4+32ab^9x^6+8b^{10}x^8\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-3*a^5*b + a*b^5*x^8 + 4*b^6*x^10 + Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]*(-3*a^4 + 3*a^3*b*x^2 - 3*a^2*b^2*x^4 + 3*a*b^3*x^6 - 4*b^4*x^8))/(3*x^
8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-8*a^3*b^7 - 24*a^2*b^8*x^2 - 24*a*b^9*x
^4 - 8*b^10*x^6) + 3*Sqrt[b^2]*x^8*(8*a^4*b^6 + 32*a^3*b^7*x^2 + 48*a^2*b^8
*x^4 + 32*a*b^9*x^6 + 8*b^10*x^8))
```

fricas [A] time = 1.21, size = 58, normalized size = 0.84

$$-\frac{4\,bx^2+a}{24\left(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x^2
+ a^4*b^2)
```

giac [A] time = 0.21, size = 32, normalized size = 0.46

$$-\frac{4\,bx^2+a}{24\,\left(bx^2+a\right)^4 b^2 \mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(4*b*x^2 + a)/((b*x^2 + a)^4*b^2*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 32, normalized size = 0.46

$$-\frac{\left(b\,x^2+a\right)\left(4b\,x^2+a\right)}{24\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(b*x^2+a)*(4*b*x^2+a)/b^2/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.36, size = 58, normalized size = 0.84

$$-\frac{4\,bx^2+a}{24\left(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x^2
+ a^4*b^2)
```

mupad [B] time = 4.26, size = 42, normalized size = 0.61

$$-\frac{\left(4\,b\,x^2+a\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{24\,b^2\,\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] -((a + 4*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(24*b^2*(a + b*x^2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2)**(5/2), x)
```

3.478 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 607}

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/(8*b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^2\right)\\
&= -\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{8b\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/8*(a + b*x^2)/(b*((a + b*x^2)^2)^(5/2))
```

IntegrateAlgebraic [B] time = 0.75, size = 200, normalized size = 5.26

$$\frac{a^4b+\sqrt{b^2}\sqrt{a^2+2abx^2+b^2x^4}\left(a^3-a^2bx^2+ab^2x^4-b^3x^6\right)+b^5x^8}{bx^8\sqrt{a^2+2abx^2+b^2x^4}\left(-8a^3b^5-24a^2b^6x^2-24ab^7x^4-8b^8x^6\right)+b\sqrt{b^2}\,x^8\left(8a^4b^4+32a^3b^5x^2+48a^2b^6x^4+32ab^7x^6+8b^8x^8\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^4*b + b^5*x^8 + Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(a^3 - a^2*b*x
^2 + a*b^2*x^4 - b^3*x^6))/(b*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-8*a^3*b
^5 - 24*a^2*b^6*x^2 - 24*a*b^7*x^4 - 8*b^8*x^6) + b*Sqrt[b^2]*x^8*(8*a^4*b^
4 + 32*a^3*b^5*x^2 + 48*a^2*b^6*x^4 + 32*a*b^7*x^6 + 8*b^8*x^8))
```

fricas [A] time = 4.60, size = 48, normalized size = 1.26

$$-\frac{1}{8\left(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)
```

giac [A] time = 0.21, size = 24, normalized size = 0.63

$$-\frac{1}{8\left(bx^2+a\right)^4b\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/8/((b*x^2 + a)^4*b*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 24, normalized size = 0.63

$$-\frac{b\,x^{2}+a}{8\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
[Out] -1/8*(b*x^2+a)/b/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.32, size = 48, normalized size = 1.26

$$-\frac{1}{8\left(b^{5}x^{8}+4\,ab^{4}x^{6}+6\,a^{2}b^{3}x^{4}+4\,a^{3}b^{2}x^{2}+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)
```

mupad [B] time = 4.27, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{8\,b\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] -(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(8*b*(a + b*x^2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x/((a + b*x**2)**2)**(5/2), x)
```

3.479 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log}{2a^5\sqrt{a^2+2ab}}$$

Rubi [A] time = 0.12, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$\frac{1}{4a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^5\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*(a + b*x^2)^3*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + 1/(6*a^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + 1/(4*a^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2
)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[a + b*x^
2])/(2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int\left(\frac{1}{a^5b^5x}-\frac{1}{ab^4(a+bx)^5}-\frac{1}{a^2b^4(a+bx)^4}-\frac{1}{a^3b^4(a+bx)^3}-\frac{1}{a^4b^4(a+bx)}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{6a^2\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 0.43

$$\frac{a\left(25a^3+52a^2bx^2+42ab^2x^4+12b^3x^6\right)+24\log(x)\left(a+bx^2\right)^4-12\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24a^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (a*(25*a^3 + 52*a^2*b*x^2 + 42*a*b^2*x^4 + 12*b^3*x^6) + 24*(a + b*x^2)^4*L
og[x] - 12*(a + b*x^2)^4*Log[a + b*x^2])/(24*a^5*(a + b*x^2)^3*Sqrt[(a + b*
x^2)^2])
```

IntegrateAlgebraic [B] time = 112.39, size = 3893, normalized size = 17.46

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-16*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(3*a^87 - 3*a^86*b*x^2 + 3*a
^85*b^2*x^4 - 3*a^84*b^3*x^6 + 28*a^83*b^4*x^8 + 4074*a^82*b^5*x^10 + 32839
2*a^81*b^6*x^12 + 17416344*a^80*b^7*x^14 + 684119040*a^79*b^8*x^16 + 212262
12480*a^78*b^9*x^18 + 541798692864*a^77*b^10*x^20 + 11700119129088*a^76*b^1
```

1*x^22 + 218177660073984*a^75*b^12*x^24 + 3568299926191104*a^74*b^13*x^26 +
51815594461802496*a^73*b^14*x^28 + 674671446755364864*a^72*b^15*x^30 + 794
1063256075862016*a^71*b^16*x^32 + 85067029477553700864*a^70*b^17*x^34 + 834
123879538833555456*a^69*b^18*x^36 + 7523474448566234578944*a^68*b^19*x^38 +
62685534036478928879616*a^67*b^20*x^40 + 484263949539613361700864*a^66*b^2
1*x^42 + 3479930411745645615906816*a^65*b^22*x^44 + 233278351340550948299732
504*a^64*b^23*x^46 + 146249615315137820985655296*a^63*b^24*x^48 + 859432059
122570824614150144*a^62*b^25*x^50 + 4743516432354559922458853376*a^61*b^26*
x^52 + 24634603647181064055495327744*a^60*b^27*x^54 + 120573287346692375306
185998336*a^59*b^28*x^56 + 556993306425737520204547620864*a^58*b^29*x^58 +
2431710423525963544007685439488*a^57*b^30*x^60 + 10044990811674606081019087
945728*a^56*b^31*x^62 + 39302890124942958048947321438208*a^55*b^32*x^64 + 1
45797964967106670900820916043776*a^54*b^33*x^66 + 5132188081386281544165184
96518144*a^53*b^34*x^68 + 1715578603609902451791538956533760*a^52*b^35*x^70
 + 5449694039783497785024773904924672*a^51*b^36*x^72 + 16460753314968192305
329000761262080*a^50*b^37*x^74 + 47301435355834570726904893777379328*a^49*b
^38*x^76 + 129374002716573907248459868192899072*a^48*b^39*x^78 + 3369306366
46549696381527413823111168*a^47*b^40*x^80 + 8357962833549846310713359905346
02752*a^46*b^41*x^82 + 1975361005020021175749282788611719168*a^45*b^42*x^84
 + 4449110127870895658771635322027507712*a^44*b^43*x^86 + 95510374042602168
47866175737338789888*a^43*b^44*x^88 + 1954450841640742220634082905114751795
2*a^42*b^45*x^90 + 38125736076483998462868401157222432768*a^41*b^46*x^92 +
70897347674846732883050381793665482752*a^40*b^47*x^94 + 1256711278815298466
97796348885994569728*a^39*b^48*x^96 + 2123179359483532098876933394756502814
72*a^38*b^49*x^98 + 341830976243852079520987249876704165888*a^37*b^50*x^100
 + 524340224188047425154680645061540052992*a^36*b^51*x^102 + 76607328158394
7528164240486507938316288*a^35*b^52*x^104 + 1065700613371013117842709614376
603615232*a^34*b^53*x^106 + 1411020168689744925287295159943475232768*a^33*b
^54*x^108 + 1777302527925165461341561537146824687616*a^32*b^55*x^110 + 2128
564412652177688937195991485965664256*a^31*b^56*x^112 + 24223954063903428741
20214563859260768256*a^30*b^57*x^114 + 261780782973537008796252510661123453
7472*a^29*b^58*x^116 + 2684295777372515995379929097395626835968*a^28*b^59*x
^118 + 2609450207845959606905876323113652715520*a^27*b^60*x^120 + 240257650
8782798093149416471150968438784*a^26*b^61*x^122 + 2092916507617038227414227
202520498831360*a^25*b^62*x^124 + 1722894322550200785772462871077208457216*
a^24*b^63*x^126 + 1338524199162955156171842117301576925184*a^23*b^64*x^128
+ 979986658223088202764220685370425081856*a^22*b^65*x^130 + 675048463780981
537495401786462871486464*a^21*b^66*x^132 + 43670174576430247080581921242995
0713856*a^20*b^67*x^134 + 264784901454740686099176120664272666624*a^19*b^68
*x^136 + 150133777416926053796721610962266750976*a^18*b^69*x^138 + 79403252
661685074394167343394234302464*a^17*b^70*x^140 + 39059779966378067682904413
566775853056*a^16*b^71*x^142 + 17813285973748092394987873783737483264*a^15*
b^72*x^144 + 7503690680834688849004036334253244416*a^14*b^73*x^146 + 290722
6777830056762419313398234742784*a^13*b^74*x^148 + 1030915112359687157926188
398124466176*a^12*b^75*x^150 + 332671302995605405712650389373845504*a^11*b^

76*x^152 + 97031595508821255286739673941016576*a^10*b^77*x^154 + 2537425706
0868870577491562299654144*a^9*b^78*x^156 + 5890738842117298554322640664002
56*a^8*b^79*x^158 + 1199289224076687944555612256337920*a^7*b^80*x^160 + 210
809734722622322011205101682688*a^6*b^81*x^162 + 313463645139987794070197816
52480*a^5*b^82*x^164 + 3833971628290179974483895386112*a^4*b^83*x^166 + 370
352006987302418412887605248*a^3*b^84*x^168 + 26492400411034983734511140864*
a^2*b^85*x^170 + 1247611445842297308296773632*a*b^86*x^172 + 29014219670751
100192948224*b^87*x^174) - 16*(3*a^88*b - 25*a^84*b^5*x^8 - 4102*a^83*b^6*x
^10 - 332466*a^82*b^7*x^12 - 17744736*a^81*b^8*x^14 - 701535384*a^80*b^9*x^
16 - 21910331520*a^79*b^10*x^18 - 563024905344*a^78*b^11*x^20 - 12241917821
952*a^77*b^12*x^22 - 229877779203072*a^76*b^13*x^24 - 3786477586265088*a^75
*b^14*x^26 - 55383894387993600*a^74*b^15*x^28 - 726487041217167360*a^73*b^1
6*x^30 - 8615734702831226880*a^72*b^17*x^32 - 93008092733629562880*a^71*b^1
8*x^34 - 919190909016387256320*a^70*b^19*x^36 - 8357598328105068134400*a^69
*b^20*x^38 - 70209008485045163458560*a^68*b^21*x^40 - 546949483576092290580
480*a^67*b^22*x^42 - 3964194361285258977607680*a^66*b^23*x^44 - 26807765545
800740445880320*a^65*b^24*x^46 - 169577450449192915815628800*a^64*b^25*x^48
 - 1005681674437708645599805440*a^63*b^26*x^50 - 560294849147713074707300035
20*a^62*b^27*x^52 - 29378120079535623977954181120*a^61*b^28*x^54 - 14520789
0993873439361681326080*a^60*b^29*x^56 - 677566593772429895510733619200*a^59
*b^30*x^58 - 2988703729951701064212233060352*a^58*b^31*x^60 - 1247670123520
0569625026773385216*a^57*b^32*x^62 - 49347880936617564129966409383936*a^56*
b^33*x^64 - 185100855092049628949768237481984*a^55*b^34*x^66 - 659016773105
734825317339412561920*a^54*b^35*x^68 - 2228797411748530606208057453051904*a
^53*b^36*x^70 - 7165272643393400236816312861458432*a^52*b^37*x^72 - 2191044
7354751690090353774666186752*a^51*b^38*x^74 - 63762188670802763032233894538
641408*a^50*b^39*x^76 - 176675438072408477975364761970278400*a^49*b^40*x^78
 - 466304639363123603629987282016010240*a^48*b^41*x^80 - 117272692000153432
7452863404357713920*a^47*b^42*x^82 - 2811157288375005806820618779146321920*
a^46*b^43*x^84 - 6424471132890916834520918110639226880*a^45*b^44*x^86 - 140
00147532131112506637811059366297600*a^44*b^45*x^88 - 2909554582066763905420
7004788486307840*a^43*b^46*x^90 - 57670244492891420669209230208369950720*a^
42*b^47*x^92 - 109023083751330731345918782950887915520*a^41*b^48*x^94 - 196
568475556376579580846730679660052480*a^40*b^49*x^96 - 337989063829883056585
489688361644851200*a^39*b^50*x^98 - 554148912192205289408680589352354447360
*a^38*b^51*x^100 - 866171200431899504675667894938244218880*a^37*b^52*x^102
- 1290413505771994953318921131569478369280*a^36*b^53*x^104 - 18317738949549
60646006950100884541931520*a^35*b^54*x^106 - 247672078206075804313000477432
0078848000*a^34*b^55*x^108 - 3188322696614910386628856697090299920384*a^33*
b^56*x^110 - 3905866940577343150278757528632790351872*a^32*b^57*x^112 - 455
0959819042520563057410555345226432512*a^31*b^58*x^114 - 5040203236125712962
082739670470495305728*a^30*b^59*x^116 - 53021036071078860833424542040068613
73440*a^29*b^60*x^118 - 5293745985218475602285805420509279551488*a^28*b^61*
x^120 - 5012026716628757700055292794264621154304*a^27*b^62*x^122 - 44954930
16399836320563643673671467270144*a^26*b^63*x^124 - 381581083016723901318669

0073597707288576*a^25*b^64*x^126 - 3061418521713155941944304988378785382400
*a^24*b^65*x^128 - 2318510857386043358936062802672002007040*a^23*b^66*x^130
 - 1655035122004069740259622471833296568320*a^22*b^67*x^132 - 1111750209545
284008301220998892822200320*a^21*b^68*x^134 - 701486647219043156904995333309
4223380480*a^20*b^69*x^136 - 414918678871666739895897731626539417600*a^19*b
^70*x^138 - 229537030078611128190888954356501053440*a^18*b^71*x^140 - 11846
3032628063142077071756961010155520*a^17*b^72*x^142 - 568730659401261600778 9
2287350513336320*a^16*b^73*x^144 - 253169766545827812439919101179907276 80*a
^15*b^74*x^146 - 10410917458664745611423349732487987200*a^14*b^75*x^148 - 3
938141890189743920345501796359208960*a^13*b^76*x^150 - 1363586415355292563 6
38838787498311680*a^12*b^77*x^152 - 429702898504426660999390063314862080*a^
11*b^78*x^154 - 122405852569690125864231236240670720*a^10*b^79*x^156 - 3126
4995902986169132923826366054400*a^9*b^80*x^158 - 709002806619398649998787 63
22738176*a^8*b^81*x^160 - 1410098958799310266566817358020608*a^7*b^82*x^162
 - 242156099236621101418224883335168*a^6*b^83*x^164 - 3518033614228895938 15
03677038592*a^5*b^84*x^166 - 420432363527748239289678299136 0*a^4*b^85*x^168
 - 3968444073983374021473987461 12*a^3*b^86*x^170 - 277400118568772810428079
14496*a^2*b^87*x^172 - 1276625665513048408489721856*a*b^88*x^174 - 29014219
670751100192948224*b^89*x^176))/(3*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(1
28*a^84*b^4*x^8 + 21120*a^83*b^5*x^10 + 1721472*a^82*b^6*x^12 + 92406656*a^
81*b^7*x^14 + 3674439936*a^80*b^8*x^16 + 115431837696*a^79*b^9*x^18 + 29837
81007360*a^78*b^10*x^20 + 65264862633984*a^77*b^11*x^22 + 1232951257055232*
a^76*b^12*x^24 + 20432990676713472*a^75*b^13*x^26 + 300716130445885440*a^74
*b^14*x^28 + 3969238746518323200*a^73*b^15*x^30 + 47370458487200808960*a^72
*b^16*x^32 + 514638307833890734080*a^71*b^17*x^34 + 5118983772087653498880*
a^70*b^18*x^36 + 46847576245279590973440*a^69*b^19*x^38 + 39614792984536390
0416000*a^68*b^20*x^40 + 3106721959632715288412160*a^67*b^21*x^42 + 2266902
3436505501851975680*a^66*b^22*x^44 + 154345386528573397590343680*a^65*b^23*
x^46 + 983080641935232659210895360*a^64*b^24*x^48 + 5870874650091141471613 7
47200*a^63*b^25*x^50 + 32939251658045736840163491840*a^62*b^26*x^52 + 17394
4179132957033591283384320*a^61*b^27*x^54 + 865962850098397774705978245120*a
^60*b^28*x^56 + 4070229396534870045831701987328*a^59*b^29*x^58 + 1808600721
9228693734214252625920*a^58*b^30*x^60 + 76065544328296704454991881961472*a^
57*b^31*x^62 + 303124377178837558835291835334656*a^56*b^32*x^64 + 114567402
0859022985294059076059136*a^55*b^33*x^66 + 411041679570111998141630978300 31
36*a^54*b^34*x^68 + 14009851676518597644792572002959360*a^53*b^35*x^70 + 45
394822957658124638063332524294144*a^52*b^36*x^72 + 139917789751193392435587
350802726912*a^51*b^37*x^74 + 410459873858094378326686815750193152*a^50*b^3
8*x^76 + 1146588188790904910371747976404008960*a^49*b^39*x^78 + 30511466309
23992568193710004423884800*a^48*b^40*x^80 + 773732433686962858387764248485 8
224640*a^47*b^41*x^82 + 18703305019991137633578654068606238720*a^46*b^42*x^
84 + 43107103684238284774282301309630545920*a^45*b^43*x^86 + 94746125967065
558286369665929351004160*a^44*b^44*x^88 + 198615000002424286648098587431403
520000*a^43*b^45*x^90 + 397130692504847361152873125569091338240*a^42*b^46*x
^92 + 757418302509245017132523963169657323520*a^41*b^47*x^94 + 137786447865

3181794400277391407075819520*a^40*b^48*x^96 + 2390618413362513124890580419918786723840*a^39*b^49*x^98 + 3955385015068042115189991389350644940800*a^38*b^50*x^100 + 6239651737215173635140320005662399528960*a^37*b^51*x^102 + 9382518809131986833287646369303054254080*a^36*b^52*x^104 + 13444227011136623928559678656620034785280*a^35*b^53*x^106 + 18350748809159245654215695088179610648576*a^34*b^54*x^108 + 23850139665038281564949273182080325386240*a^33*b^55*x^110 + 29501011442370924678193433541945719783424*a^32*b^56*x^112 + 34709779639129526516799866639524772708352*a^31*b^57*x^114 + 38820803891602154416762573083734716710912*a^30*b^58*x^116 + 41244886350924311041670535470940728328192*a^29*b^59*x^118 + 41593718455288022589388471161144339333120*a^28*b^60*x^120 + 39779337771714841625416239365458407456768*a^27*b^61*x^122 + 36044348394238624400918223255712630308864*a^26*b^62*x^124 + 30909917019058539460410388414124412370944*a^25*b^63*x^126 + 25056451871195099922047049138792730460160*a^24*b^64*x^128 + 19174574965327876626062519403677535436800*a^23*b^65*x^130 + 13831731297442951886921500151546055229440*a^22*b^66*x^132 + 93899062088695291297384823712174519 09120*a^21*b^67*x^134 + 59881004386659559986066004649813685043 20*a^20*b^68*x^136 + 35799592142852217151790468774867081625 60*a^19*b^69*x^138 + 2001897592698205886195342610222022656000*a^18*b^70*x^140 + 1044415366922352700482947391151763619840*a^17*b^71*x^142 + 506902354822510510083032763037195960320*a^16*b^72*x^144 + 2281294295258272742490135295754974003 20*a^15*b^73*x^146 + 94849133060508440059978288214562570240*a^14*b^74*x^148 + 36277068986647020067202576304360652800*a^13*b^75*x^150 + 12701061310210824422967616769422786560*a^12*b^76*x^152 + 4047255710028362687341349707846778880*a^11*b^77*x^154 + 1165860312209840554524188084357038080*a^10*b^78*x^156 + 301141170995507566848412970190372864*a^9*b^79*x^158 + 69062315585083645783933993101557760*a^8*b^80*x^160 + 138911064423229418036830116627 08736*a^7*b^81*x^162 + 2412615694461848378203398972899328*a^6*b^82*x^164 + 354493762970299331082628280352768*a^5*b^83*x^166 + 428479674960078587561443936 17408*a^4*b^84*x^168 + 4090618117313628445869793607680*a^3*b^85*x^170 + 289213741678046966723307896832*a^2*b^86*x^172 + 134625979272285104895279759 36*a*b^87*x^174 + 309485009821345068724781056*b^88*x^176) + 3*a^4*b*Sqrt[b^2]*(-128*a^85*b^3*x^8 - 21248*a^84*b^4*x^10 - 1742592*a^83*b^5*x^12 - 94128128*a^82*b^6*x^14 - 3766846592*a^81*b^7*x^16 - 119106277632*a^80*b^8*x^18 - 3099212845056*a^79*b^9*x^20 - 68248643641344*a^78*b^10*x^22 - 1298216119689216*a^77*b^11*x^24 - 21665941933768704*a^76*b^12*x^26 - 321149121122598912*a^75*b^13*x^28 - 4269954876964208640*a^74*b^14*x^30 - 51339697233719132160*a^73*b^15*x^32 - 562008766321091543040*a^72*b^16*x^34 - 5633622079921544232960*a^71*b^17*x^36 - 51966560017367244472320*a^70*b^18*x^38 - 442995506090643491389440*a^69*b^19*x^40 - 3502869889478079188828160*a^68*b^20*x^42 - 25775745396138217140387840*a^67*b^21*x^44 - 177014409965078899442319360*a^66*b^22*x^46 - 1137426028463806056801239040*a^65*b^23*x^48 - 6853955292026374130824642560*a^64*b^24*x^50 - 38810126308136878311777239040*a^63*b^25*x^52 - 206883430791002770431446876160*a^62*b^26*x^54 - 1039907029231354808297261629440*a^61*b^27*x^56 - 4936192246633267820537680232448*a^60*b^28*x^58 - 22156236615763563780045954613248*a^59*b^29*x^60 - 9415155154752539818920 6134

587392*a^58*b^30*x^62 - 379189921507134263290283717296128*a^57*b^31*x^64 - 1448798398037860544129350911393792*a^56*b^32*x^66 - 5256090816560142966710368859062272*a^55*b^33*x^68 - 18120268472219717626208881785962496*a^54*b^34*x^70 - 59404674634176722282855904527253504*a^53*b^35*x^72 - 185312612708851517073650683327021056*a^52*b^36*x^74 - 550377663609287770762274166552920064*a^51*b^37*x^76 - 1557048062648999288698434792154202112*a^50*b^38*x^78 - 4197734819714897478565457980827893760*a^49*b^39*x^80 - 10788470967793621152071352489282109440*a^48*b^40*x^82 - 26440629356860766217456296553464463360*a^47*b^41*x^84 - 61810408704229422407860955378236784640*a^46*b^42*x^86 - 137853229651303843060651967238981550080*a^45*b^43*x^88 - 293361125969489844934468253360754524160*a^44*b^44*x^90 - 595745692507271647800971713000494858240*a^43*b^45*x^92 - 1154548995014092378285397088738748661760*a^42*b^46*x^94 - 2135282781162426811532801354576733143040*a^41*b^47*x^96 - 3768482892015694919290857811325862543360*a^40*b^48*x^98 - 6346003428430555240080571809269431664640*a^39*b^49*x^100 - 10195036752283215750330311395013044469760*a^38*b^50*x^102 - 15622170546347160468427966374965453783040*a^37*b^51*x^104 - 22826745820268610761847325025923089039360*a^36*b^52*x^106 - 31794975820295869582775373744799645433856*a^35*b^53*x^108 - 42200888474197527219164968270259936034816*a^34*b^54*x^110 - 53351151107409206243142706724026045169664*a^33*b^55*x^112 - 64210791081500451194993300181470492491776*a^32*b^56*x^114 - 73530583530731680933562439723259489419264*a^31*b^57*x^116 - 80065690242526465458433108554675445039104*a^30*b^58*x^118 - 82838604806212333631059006632085067661312*a^29*b^59*x^120 - 81373056227002864214804710526602746789888*a^28*b^60*x^122 - 75823686165953466026334462621171037765632*a^27*b^61*x^124 - 66954265413297163861328611669837042679808*a^26*b^62*x^126 - 55966368890253639382457437552917142831104*a^25*b^63*x^128 - 44231026836522976548109568542470265896960*a^24*b^64*x^130 - 33006306262770828512984019555223590666240*a^23*b^65*x^132 - 23221637506312481016659982522763507138560*a^22*b^66*x^134 - 15378006647535485128345082836198820413440*a^21*b^67*x^136 - 9568059652951177713785647342468076666880*a^20*b^68*x^138 - 5581856806983427601374389487708730818560*a^19*b^69*x^140 - 3046312959620558586678290001373786275840*a^18*b^70*x^142 - 1551317721744863210565980154188959580160*a^17*b^71*x^144 - 735031784348337784332046292612693360640*a^16*b^72*x^146 - 322978562586335714308991817790059970560*a^15*b^73*x^148 - 131126202047155460127180864518923223040*a^14*b^74*x^150 - 48978130296857844490170193073783439360*a^13*b^75*x^152 - 16748317020239187110308966477269565440*a^12*b^76*x^154 - 5213116022238203241865537792203816960*a^11*b^77*x^156 - 1467001483205348121372601054547410944*a^10*b^78*x^158 - 370203486580591212632346963291930624*a^9*b^79*x^160 - 82953422027406587587617004764266496*a^8*b^80*x^162 - 16303722136784790181886410635608064*a^7*b^81*x^164 - 2767109457432147709286027253252096*a^6*b^82*x^166 - 397341730466307189838772673970176*a^5*b^83*x^168 - 46938585613321487202014187225088*a^4*b^84*x^170 - 4379831858991675412593101504512*a^3*b^85*x^172 - 302676339605275477212835872768*a^2*b^86*x^174 - 13772082937049855558252756992*a*b^87*x^176 - 309485009821345068724781056*b^88*x^178)) + ArcTanh[(Sqrt[b^2]*x^2)/a - Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/a]/a^5

fricas [A] time = 1.10, size = 178, normalized size = 0.80

$$\frac{12\,ab^{3}x^{6}+42\,a^{2}b^{2}x^{4}+52\,a^{3}bx^{2}+25\,a^{4}-12\left(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4}\right)\log\left(bx^{2}+a\right)+24\left(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4}\right)\log(x)}{24\left(a^{5}b^{4}x^{8}+4\,a^{6}b^{3}x^{6}+6\,a^{7}b^{2}x^{4}+4\,a^{8}bx^{2}+a^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*a*b^3*x^6 + 42*a^2*b^2*x^4 + 52*a^3*b*x^2 + 25*a^4 - 12*(b^4*x^8 +
 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a) + 24*(b^4*
x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4*x^8
 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9)
```

giac [A] time = 0.27, size = 101, normalized size = 0.45

$$-\frac{\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{5}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{\log\left(\left|x\right|\right)}{a^{5}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{12\,ab^{3}x^{6}+42\,a^{2}b^{2}x^{4}+52\,a^{3}bx^{2}+25\,a^{4}}{24\left(bx^{2}+a\right)^{4}a^{5}\mathrm{sgn}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(b*x^2 + a))/(a^5*sgn(b*x^2 + a)) + log(abs(x))/(a^5*sgn(b*x^2
+ a)) + 1/24*(12*a*b^3*x^6 + 42*a^2*b^2*x^4 + 52*a^3*b*x^2 + 25*a^4)/((b*x^
2 + a)^4*a^5*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 193, normalized size = 0.87

$$\frac{\left(24b^{4}x^{8}\ln(x)-12b^{4}x^{8}\ln\left(bx^{2}+a\right)+96a\,b^{3}x^{6}\ln(x)-48a\,b^{3}x^{6}\ln\left(bx^{2}+a\right)+12a\,b^{3}x^{6}+144a^{2}b^{2}x^{4}\ln(x)-72a^{2}b^{2}x^{4}\ln\left(bx^{2}+a\right)+42a^{2}b^{2}x^{4}+96a^{3}b\,x^{2}\ln(x)-48a^{3}b\,x^{2}\ln\left(bx^{2}+a\right)+52a^{3}b\,x^{2}+24a^{4}\ln(x)-12a^{4}\ln\left(bx^{2}+a\right)+25a^{4}\right)\left(bx^{2}+a\right)}{24\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{5}{2}}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24*(24*ln(x)*x^8*b^4-12*b^4*x^8*ln(b*x^2+a)+96*ln(x)*x^6*a*b^3-48*a*b^3*x
^6*ln(b*x^2+a)+12*a*b^3*x^6+144*ln(x)*x^4*a^2*b^2-72*a^2*b^2*x^4*ln(b*x^2+a
)+42*a^2*b^2*x^4+96*ln(x)*x^2*a^3*b-48*a^3*b*x^2*ln(b*x^2+a)+52*a^3*b*x^2+2
4*a^4*ln(x)-12*a^4*ln(b*x^2+a)+25*a^4)*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.44, size = 101, normalized size = 0.45

$$\frac{12\,b^{3}x^{6}+42\,ab^{2}x^{4}+52\,a^{2}bx^{2}+25\,a^{3}}{24\left(a^{4}b^{4}x^{8}+4\,a^{5}b^{3}x^{6}+6\,a^{6}b^{2}x^{4}+4\,a^{7}bx^{2}+a^{8}\right)}-\frac{\log\left(bx^{2}+a\right)}{2\,a^{5}}+\frac{\log(x)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(12*b^3*x^6 + 42*a*b^2*x^4 + 52*a^2*b*x^2 + 25*a^3)/(a^4*b^4*x^8 + 4*a
^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8) - 1/2*log(b*x^2 + a)/a^5 +
log(x)/a^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**2)**2)**(5/2)), x)
```

3.480 $\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5b\log(x)\left(a+bx^2\right)}{a^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.14, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$-\frac{3b}{4a^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^5x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5b\log(x)\left(a+bx^2\right)}{a^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^6\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*b)/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(8*a^2*(a + b*x^2)^3*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(3*a^3*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (3*b)/(4*a^4*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a
 + b*x^2)/(2*a^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*b*(a + b*x^2)*Lo
g[x])/(a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*b*(a + b*x^2)*Log[a + b*x^
2])/(2*a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^3\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x^2}-\frac{5}{a^6b^4x}+\frac{1}{a^2b^3(a+bx)^5}+\frac{2}{a^3b^3(a+bx)^4}+\frac{3}{a^4b^3(a+bx)}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{3a^3\left(a+bx\right.}$$

Mathematica [A] time = 0.05, size = 119, normalized size = 0.45

$$\frac{-a\left(12a^4+125a^3bx^2+260a^2b^2x^4+210ab^3x^6+60b^4x^8\right)-120bx^2\log(x)\left(a+bx^2\right)^4+60bx^2\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24a^6x^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^2 + 260*a^2*b^2*x^4 + 210*a*b^3*x^6 + 60*b^4*x^8
)) - 120*b*x^2*(a + b*x^2)^4*Log[x] + 60*b*x^2*(a + b*x^2)^4*Log[a + b*x^2]
)/(24*a^6*x^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [B] time = 81.20, size = 2844, normalized size = 10.65

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-3*a^64*b + 12*a^61*b^4*x^6 + 1493*a^60*b^5*x^8 + 91118*a^59*b^6*x^10 + 36
36810*a^58*b^7*x^12 + 106785760*a^57*b^8*x^14 + 2460076984*a^56*b^9*x^16 +
46310582976*a^55*b^10*x^18 + 732580660416*a^54*b^11*x^20 + 9938734502400*a^
53*b^12*x^22 + 117445946618880*a^52*b^13*x^24 + 1223637092620800*a^51*b^14*
x^26 + 11350399493322240*a^50*b^15*x^28 + 94488598194831360*a^49*b^16*x^30
```

+ 710617423734220800*a^48*b^17*x^32 + 4855121549399654400*a^47*b^18*x^34 + 30277909491232112640*a^46*b^19*x^36 + 173049266469218549760*a^45*b^20*x^38 + 909584595569746575360*a^44*b^21*x^40 + 4410135451820644761600*a^43*b^22*x^42 + 19775527127642947584000*a^42*b^23*x^44 + 82196556795830977167360*a^41*b^24*x^46 + 317307653352763348746240*a^40*b^25*x^48 + 1139583932697244587786240*a^39*b^26*x^50 + 3813174028256128637337600*a^38*b^27*x^52 + 11902704133849368389222400*a^37*b^28*x^54 + 34696391435737807567454208*a^36*b^29*x^56 + 94533796001415733929050112*a^35*b^30*x^58 + 240916941658790456262131712*a^34*b^31*x^60 + 574607755442020115378339840*a^33*b^32*x^62 + 1283164402851665258416701440*a^32*b^33*x^64 + 2683621494959139076750442496*a^31*b^34*x^66 + 5257145964851977442918662144*a^30*b^35*x^68 + 9646533665292061818700693504*a^29*b^36*x^70 + 16577868267430599772549939200*a^28*b^37*x^72 + 26675109984442157398669393920*a^27*b^38*x^74 + 40172301073084658684862136320*a^26*b^39*x^76 + 56591282297530387719562199040*a^25*b^40*x^78 + 74519077727500043969227653120*a^24*b^41*x^80 + 91643682630143098677205401600*a^23*b^42*x^82 + 105147917610427657204059340800*a^22*b^43*x^84 + 112415234614316153992359444480*a^21*b^44*x^86 + 111827437082254903764917944320*a^20*b^45*x^88 + 103335972403480938671190835520*a^19*b^46*x^90 + 88524263900650858976850739200*a^18*b^47*x^92 + 70151845667324737433370624000*a^17*b^48*x^94 + 51292238631349321490937937920*a^16*b^49*x^96 + 34499019413506196684176097280*a^15*b^50*x^98 + 21271946840740957336664801280*a^14*b^51*x^100 + 11976015405374637268480819200*a^13*b^52*x^102 + 6127537471799578147474636800*a^12*b^53*x^104 + 2833532513821046118461472768*a^11*b^54*x^106 + 1176474844646974332812132352*a^10*b^55*x^108 + 435119222530177299017367552*a^9*b^56*x^110 + 141970050175924377684541440*a^8*b^57*x^112 + 40374133580173208809635840*a^7*b^58*x^114 + 9854453508288460730925056*a^6*b^59*x^116 + 2022872405081186719760384*a^5*b^60*x^118 + 339632949089043788857344*a^4*b^61*x^120 + 44787397574274108620800*a^3*b^62*x^122 + 4350116952069709496320*a^2*b^63*x^124 + 276701161105643274240*a*b^64*x^126 + 8646911284551352320*b^65*x^128 + Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-3*a^63 + 3*a^62*b*x^2 - 3*a^61*b^2*x^4 - 9*a^60*b^3*x^6 - 1484*a^59*b^4*x^8 - 89634*a^58*b^5*x^10 - 3547176*a^57*b^6*x^12 - 103238584*a^56*b^7*x^14 - 2356838400*a^55*b^8*x^16 - 43953744576*a^54*b^9*x^18 - 688626915840*a^53*b^10*x^20 - 9250107586560*a^52*b^11*x^22 - 108195839032320*a^51*b^12*x^24 - 1115441253588480*a^50*b^13*x^26 - 10234958239733760*a^49*b^14*x^28 - 84253639955097600*a^48*b^15*x^30 - 626363783779123200*a^47*b^16*x^32 - 4228757765620531200*a^46*b^17*x^34 - 26049151725611581440*a^45*b^18*x^36 - 147000114743606968320*a^44*b^19*x^38 - 762584480826139607040*a^43*b^20*x^40 - 3647550970994505154560*a^42*b^21*x^42 - 16127976156648442429440*a^41*b^22*x^44 - 66068580639182534737920*a^40*b^23*x^46 - 251239072713580814008320*a^39*b^24*x^48 - 888344859983663773777920*a^38*b^25*x^50 - 2924829168272464863559680*a^37*b^26*x^52 - 8977874965576903525662720*a^36*b^27*x^54 - 25718516470160904041791488*a^35*b^28*x^56 - 68815279531254829887258624*a^34*b^29*x^58 - 172101662127535626374873088*a^33*b^30*x^60 - 402506093314484489003466752*a^32*b^31*x^62 - 880658309537180769413234688*a^31*b^32*x^64 - 1802963185421958307337207808*a^30*b^33*x^66 - 3454182779430019135

581454336*a^29*b^34*x^68 - 6192350885862042683119239168*a^28*b^35*x^70 - 10
38551738156855708943070003 2*a^27*b^36*x^72 - 16289592602873600309238693888*
a^26*b^37*x^74 - 23882708470211058375623442432*a^25*b^38*x^76 - 32708573827
319329343938756608*a^24*b^39*x^78 - 41810503900180714625288896512*a^23*b^40
*x^80 - 49833178729962384051916505088*a^22*b^41*x^82 - 55314738880465273152
142835712*a^21*b^42*x^84 - 57100495733850880840216608768*a^20*b^43*x^86 - 5
4726941348404022924701335552*a^19*b^44*x^88 - 48606655891944070942417747968
*a^18*b^45*x^90 - 39917608008706788034432991232*a^17*b^46*x^92 - 3023423765
8617949398937632768*a^16*b^47*x^94 - 21058000972731372092000305152*a^15*b^4
8*x^96 - 13441018440774824592175792128*a^14*b^49*x^98 - 783092839996613274 4
489009152*a^13*b^50*x^100 - 4145087005408504523991810048*a^12*b^51*x^102 -
1982450466391073623482826752*a^11*b^52*x^104 - 851082047429972494978646016*
a^10*b^53*x^106 - 325392797217001837833486336*a^9*b^54*x^108 - 109726425313
175461183881216*a^8*b^55*x^110 - 32243624862748916500660224*a^7*b^56*x^112
- 8130508717424292308975616*a^6*b^57*x^114 - 1723944790864168421949440*a^5*
b^58*x^116 - 298927614217018297810944*a^4*b^59*x^118 - 40705334872025491046
400*a^3*b^60*x^120 - 4082062702248617574400*a^2*b^61*x^122 - 26805424982109
1921920*a*b^62*x^124 - 8646911284551352320*b^63*x^126))/(3*a^5*x^8*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]*(-8*a^60*b^4 - 936*a^59*b^5*x^2 - 53832*a^58*b^6*x^
4 - 2028600*a^57*b^7*x^6 - 56333328*a^56*b^8*x^8 - 1229251968*a^55*b^9*x^10
 - 21948838912*a^54*b^10*x^12 - 329736109824*a^53*b^11*x^14 - 4253142643200
*a^52*b^12*x^16 - 47832699273216*a^51*b^13*x^18 - 474726976948224*a^50*b^14
*x^20 - 4198198392760320*a^49*b^15*x^22 - 33343446890557440*a^48*b^16*x^24
- 239403280048128000*a^47*b^17*x^26 - 1562459053231964160*a^46*b^18*x^28 -
9312555821128089600*a^45*b^19*x^30 - 50890640751160197120*a^44*b^20*x^32 -
255857896219504803840*a^43*b^21*x^34 - 1186944307588300800000*a^42*b^22*x^3
6 - 5093751076470545448960*a^41*b^23*x^38 - 20266419321282499706880*a^40*b^
24*x^40 - 74898737599223200481280*a^39*b^25*x^42 - 257538426184575127388160
*a^38*b^26*x^44 - 825067881224032223232000*a^37*b^27*x^46 - 246567411203255
2349859840*a^36*b^28*x^48 - 6880414330081729610514432*a^35*b^29*x^50 - 1794
2536074222169724289024*a^34*b^30*x^52 - 43755107777920626660474 88*a^33*b^3
1*x^54 - 99831297608843635615334400*a^32*b^32*x^56 - 213181721876762624526 3
85152*a^31*b^33*x^58 - 426155156775629047172431872*a^30*b^34*x^60 - 7975308
383645569998158561 28*a^29*b^35*x^62 - 1397189047727722213310201856*a^28*b^3
6*x^64 - 2290841170528074923297996800*a^27*b^37*x^66 - 35140508159487586048
45154304*a^26*b^38*x^68 - 5040462441364212140879118336*a^25*b^39*x^70 - 675
6005325714055365069373440*a^24*b^40*x^72 - 8454788378368570414312980480*a^2
3*b^41*x^74 - 9868868050118054737084416000*a^22*b^42*x^76 - 107314581937258
56494093598720*a^21*b^43*x^78 - 10855754855129860210792857600*a^20*b^44*x^8
0 - 10198877550514546149257379840*a^19*b^45*x^82 - 888183164052636027467071
4880*a^18*b^46*x^84 - 7153973538484081655808000000*a^17*b^47*x^86 - 5315869
468116817954964766720*a^16*b^48*x^88 - 3633267961926811266066677760*a^15*b^
49*x^90 - 2276282601414251560210268160*a^14*b^50*x^92 - 1302044770810745855
693291520*a^13*b^51*x^94 - 676809132086273815609344000*a^12*b^52*x^96 - 317
945254586734678093332480*a^11*b^53*x^98 - 134101230649370624636485632*a^10*

```
b^54*x^100 - 5038147038193984911861 3504*a^9*b^55*x^102 - 166979169084149600
```

fricas [A] time = 0.67, size = 207, normalized size = 0.78

$$-\frac{60\,ab^4x^8+210\,a^2b^3x^6+260\,a^3b^2x^4+125\,a^4bx^2+12\,a^5-60\left(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2\right)\log\left(bx^2+a\right)+120\left(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2\right)\log(x)}{24\left(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8b^2x^6+4\,a^9bx^4+a^{10}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(60*a*b^4*x^8 + 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 + 125*a^4*b*x^2 + 1
2*a^5 - 60*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b*
x^2)*log(b*x^2 + a) + 120*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b
^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 +
 4*a^9*b*x^4 + a^10*x^2)
```

giac [A] time = 0.22, size = 118, normalized size = 0.44

$$\frac{5\,b\log\left(\left|bx^2+a\right|\right)}{2\,a^6\operatorname{sgn}\left(bx^2+a\right)}-\frac{5\,b\log\left(|x|\right)}{a^6\operatorname{sgn}\left(bx^2+a\right)}-\frac{60\,ab^4x^8+210\,a^2b^3x^6+260\,a^3b^2x^4+125\,a^4bx^2+12\,a^5}{24\left(bx^2+a\right)^4a^6x^2\operatorname{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/2*b*log(abs(b*x^2 + a))/(a^6*sgn(b*x^2 + a)) - 5*b*log(abs(x))/(a^6*sgn(b
*x^2 + a)) - 1/24*(60*a*b^4*x^8 + 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 + 125*a
^4*b*x^2 + 12*a^5)/((b*x^2 + a)^4*a^6*x^2*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 219, normalized size = 0.82

$$-\frac{\left(120b^5x^{10}\ln(x)-60b^5x^{10}\ln\left(bx^2+a\right)+480a\,b^4x^8\ln(x)-240a\,b^4x^8\ln\left(bx^2+a\right)+60a\,b^4x^8+720a^2b^3x^6\ln(x)-360a^2b^3x^6\ln\left(bx^2+a\right)+210a^2b^3x^6+480a^3b^2x^4\ln(x)-240a^3b^2x^4\ln\left(bx^2+a\right)+260a^3b^2x^4+120a^4bx^2\ln(x)-60a^4bx^2\ln\left(bx^2+a\right)+125a^4bx^2+12a^5\right)\left(bx^2+a\right)}{24\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(120*b^5*x^10*ln(x)-60*ln(b*x^2+a)*x^10*b^5+480*a*b^4*x^8*ln(x)-240*a
*b^4*x^8*ln(b*x^2+a)+60*a*b^4*x^8+720*a^2*b^3*x^6*ln(x)-360*a^2*b^3*x^6*ln(
b*x^2+a)+210*a^2*b^3*x^6+480*a^3*b^2*x^4*ln(x)-240*a^3*b^2*x^4*ln(b*x^2+a)+
260*a^3*b^2*x^4+120*a^4*b*x^2*ln(x)-60*a^4*b*x^2*ln(b*x^2+a)+125*a^4*b*x^2+
12*a^5)*(b*x^2+a)/x^2/a^6/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.42, size = 119, normalized size = 0.45

$$-\frac{60\,b^4x^8+210\,ab^3x^6+260\,a^2b^2x^4+125\,a^3bx^2+12\,a^4}{24\left(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2\right)}+\frac{5\,b\log\left(bx^2+a\right)}{2\,a^6}-\frac{5\,b\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(60*b^4*x^8 + 210*a*b^3*x^6 + 260*a^2*b^2*x^4 + 125*a^3*b*x^2 + 12*a^
4)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 + 4*a^8*b*x^4 + a^9*x^2) +
 5/2*b*log(b*x^2 + a)/a^6 - 5*b*log(x)/a^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(5/2)), x)
```

3.481 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}} dx$

Optimal. Leaf size=211

$$-\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x}{64b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.08, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$-\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x}{64b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{3/2}b^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^5/(8*b*(a + b*x^2)^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*x^3)/(48*b^2*(a + b*x^2)^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) - (5*x)/(64*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(3/2)*b^(7/2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b^2\left(ab+b^2x^2\right)\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5}{}$$

$$= -\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{64}$$

$$= \frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{48b^2\left(a+bx^2\right.}$$

$$= \frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{48b^2\left(a+bx^2\right.}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 0.50

$$\frac{\sqrt{a}\sqrt{b}\,x\left(-15a^3-55a^2bx^2-73ab^2x^4+15b^3x^6\right)+15\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{3/2}b^{7/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 - 55*a^2*b*x^2 - 73*a*b^2*x^4 + 15*b^3*x^6) + 1
5*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(3/2)*b^(7/2)*(a + b*x^
2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 10.58, size = 101, normalized size = 0.48

$$\frac{\left(a+bx^2\right)\left(\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{3/2}b^{7/2}}+\frac{-15a^3x-55a^2bx^3-73ab^2x^5+15b^3x^7}{384ab^3\left(a+bx^2\right)^4}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*((-15*a^3*x - 55*a^2*b*x^3 - 73*a*b^2*x^5 + 15*b^3*x^7)/(384*a
*b^3*(a + b*x^2)^4) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(3/2)*b^(7/2))
))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 3.01, size = 324, normalized size = 1.54

$$\left[\frac{30\,ab^4x^7-146\,a^2b^3x^5-110\,a^3b^2x^3-30\,a^4bx-15\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{768\left(a^2b^8x^8+4\,a^3b^7x^6+6\,a^4b^6x^4+4\,a^5b^5x^2+a^6b^4\right)},\ \frac{15\,ab^4x^7-73\,a^2b^3x^5-55\,a^3b^2x^3-15\,a^4bx+15\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{384\left(a^2b^8x^8+4\,a^3b^7x^6+6\,a^4b^6x^4+4\,a^5b^5x^2+a^6b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 - 146*a^2*b^3*x^5 - 110*a^3*b^2*x^3 - 30*a^4*b*x - 15*
(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^8*x^8 + 4*a^3*b^7*x^6 + 6
*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4), 1/384*(15*a*b^4*x^7 - 73*a^2*b^3*x
^5 - 55*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^
4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^8*x^8 + 4*a^
3*b^7*x^6 + 6*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 172, normalized size = 0.82

$$\frac{\left(15b^4x^8 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 60a\,b^3x^6 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 15\sqrt{ab}\;b^3x^7 + 90a^2b^2x^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 73\sqrt{ab}\;a\,b^2x^5 + 60a^3b\,x^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 55\sqrt{ab}\;a^2b\,x^3 + 15a^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 15\sqrt{ab}\;a^3x\right)\left(b\,x^2+a\right)}{384\sqrt{ab}\;\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}} a\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(15*arctan(1/(a*b)^(1/2)*b*x)*x^8*b^4+15*(a*b)^(1/2)*x^7*b^3+60*arcta
n(1/(a*b)^(1/2)*b*x)*x^6*a*b^3-73*(a*b)^(1/2)*x^5*a*b^2+90*arctan(1/(a*b)^(
1/2)*b*x)*x^4*a^2*b^2-55*(a*b)^(1/2)*x^3*a^2*b+60*arctan(1/(a*b)^(1/2)*b*x)
*x^2*a^3*b-15*(a*b)^(1/2)*x*a^3+15*arctan(1/(a*b)^(1/2)*b*x)*a^4)*(b*x^2+a)
/(a*b)^(1/2)/b^3/a/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.03, size = 109, normalized size = 0.52

$$\frac{15\,b^3x^7 - 73\,ab^2x^5 - 55\,a^2bx^3 - 15\,a^3x}{384\left(ab^7x^8 + 4\,a^2b^6x^6 + 6\,a^3b^5x^4 + 4\,a^4b^4x^2 + a^5b^3\right)} + \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^3*x^7 - 73*a*b^2*x^5 - 55*a^2*b*x^3 - 15*a^3*x)/(a*b^7*x^8 + 4*
a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3) + 5/128*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**6/((a + b*x**2)**2)**(5/2), x)
```

3.482 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=212

$$\frac{x}{64ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\quad}{8b\left(a\right.}$$

Rubi [A] time = 0.09, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$-\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{64ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (3*x)/(128*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(8*b*(a + b*x^2)^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(16*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) + x/(64*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3b^2\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \ldots$$

$$= -\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \ldots$$

$$= \frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{16b^2\left(a+b\ldots\right.}$$

$$= \frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{16b^2\left(a+b\ldots\right.}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 0.50

$$\frac{\sqrt{a}\sqrt{b}\,x\left(-3a^3-11a^2bx^2+11ab^2x^4+3b^3x^6\right)+3\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-3*a^3 - 11*a^2*b*x^2 + 11*a*b^2*x^4 + 3*b^3*x^6) + 3*(
a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*(a + b*x^2)^
3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 14.15, size = 101, normalized size = 0.48

$$\frac{\left(a+bx^2\right)\left(\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}}+\frac{x\left(-3a^3-11a^2bx^2+11ab^2x^4+3b^3x^6\right)}{128a^2b^2\left(a+bx^2\right)^4}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*((x*(-3*a^3 - 11*a^2*b*x^2 + 11*a*b^2*x^4 + 3*b^3*x^6))/(128*a
^2*b^2*(a + b*x^2)^4) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2
))))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 1.66, size = 324, normalized size = 1.53

$$\left[\frac{6\,ab^4x^7+22\,a^2b^3x^5-22\,a^3b^2x^3-6\,a^4bx-3\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{256\left(a^3b^7x^8+4\,a^4b^6x^6+6\,a^5b^5x^4+4\,a^6b^4x^2+a^7b^3\right)},\ \frac{3\,ab^4x^7+11\,a^2b^3x^5-11\,a^3b^2x^3-3\,a^4bx+3\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{128\left(a^3b^7x^8+4\,a^4b^6x^6+6\,a^5b^5x^4+4\,a^6b^4x^2+a^7b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/256*(6*a*b^4*x^7 + 22*a^2*b^3*x^5 - 22*a^3*b^2*x^3 - 6*a^4*b*x - 3*(b^4*
x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log((b*x^
2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^7*x^8 + 4*a^4*b^6*x^6 + 6*a^5*
b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3), 1/128*(3*a*b^4*x^7 + 11*a^2*b^3*x^5 - 1
1*a^3*b^2*x^3 - 3*a^4*b*x + 3*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^
3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^7*x^8 + 4*a^4*b^6*x^
6 + 6*a^5*b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 172, normalized size = 0.81

$$\frac{\left(3b^4x^8\arctan\left(\frac{bx}{\sqrt{ab}}\right)+12a\,b^3x^6\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3\sqrt{ab}\,b^3x^7+18a^2b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)+11\sqrt{ab}\,a\,b^2x^5+12a^3b\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)-11\sqrt{ab}\,a^2b\,x^3+3a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)-3\sqrt{ab}\,a^3x\right)\left(b\,x^2+a\right)}{128\sqrt{ab}\,\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/128*(3*b^4*x^8*arctan(1/(a*b)^(1/2)*b*x)+3*(a*b)^(1/2)*b^3*x^7+12*a*b^3*x
^6*arctan(1/(a*b)^(1/2)*b*x)+11*(a*b)^(1/2)*a*b^2*x^5+18*a^2*b^2*x^4*arctan
(1/(a*b)^(1/2)*b*x)-11*(a*b)^(1/2)*a^2*b*x^3+12*a^3*b*x^2*arctan(1/(a*b)^(1
/2)*b*x)-3*(a*b)^(1/2)*a^3*x+3*a^4*arctan(1/(a*b)^(1/2)*b*x))*(b*x^2+a)/(a*
b)^(1/2)/b^2/a^2/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 2.99, size = 111, normalized size = 0.52

$$\frac{3\,b^3x^7+11\,ab^2x^5-11\,a^2bx^3-3\,a^3x}{128\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/128*(3*b^3*x^7 + 11*a*b^2*x^5 - 11*a^2*b*x^3 - 3*a^3*x)/(a^2*b^6*x^8 + 4*
a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) + 3/128*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(5/2), x)
```

3.483 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}} dx$

Optimal. Leaf size=213

$$\frac{5x}{192a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.08, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$\frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{192a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(8*b*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(48*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]) + (5*x)/(192*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(5}{}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{19}$$

$$= \frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{48ab\left(a+bx^2\right.}$$

$$= \frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{48ab\left(a+bx^2\right.}$$

Mathematica [A] time = 0.03, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\sqrt{b}\,x\left(-15a^3+73a^2bx^2+55ab^2x^4+15b^3x^6\right)+15\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{7/2}b^{3/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 + 73*a^2*b*x^2 + 55*a*b^2*x^4 + 15*b^3*x^6) + 1
5*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(7/2)*b^(3/2)*(a + b*x^
2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 17.35, size = 101, normalized size = 0.47

$$\frac{\left(a+bx^2\right)\left(\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}+\frac{-15a^3x+73a^2bx^3+55ab^2x^5+15b^3x^7}{384a^3b\left(a+bx^2\right)^4}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*((-15*a^3*x + 73*a^2*b*x^3 + 55*a*b^2*x^5 + 15*b^3*x^7)/(384*a
^3*b*(a + b*x^2)^4) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2))
))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 1.27, size = 324, normalized size = 1.52

$$\left[\frac{30\,ab^4x^7+110\,a^2b^3x^5+146\,a^3b^2x^3-30\,a^4bx-15\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{768\left(a^4b^6x^8+4\,a^5b^5x^6+6\,a^6b^4x^4+4\,a^7b^3x^2+a^8b^2\right)},\frac{15\,ab^4x^7+55\,a^2b^3x^5+73\,a^3b^2x^3-15\,a^4bx+15\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{384\left(a^4b^6x^8+4\,a^5b^5x^6+6\,a^6b^4x^4+4\,a^7b^3x^2+a^8b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 + 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 - 30*a^4*b*x - 15*
(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^6*x^8 + 4*a^5*b^5*x^6 + 6
*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 + 55*a^2*b^3*x
^5 + 73*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^
4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^6*x^8 + 4*a^
5*b^5*x^6 + 6*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 172, normalized size = 0.81

$$\frac{\left(15b^4x^8\arctan\left(\frac{bx}{\sqrt{ab}}\right)+60a\,b^3x^6\arctan\left(\frac{bx}{\sqrt{ab}}\right)+15\sqrt{ab}\,b^3x^7+90a^2b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)+55\sqrt{ab}\,a\,b^2x^5+60a^3b\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)+73\sqrt{ab}\,a^2b\,x^3+15a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)-15\sqrt{ab}\,a^3x\right)\left(b\,x^2+a\right)}{384\sqrt{ab}\,\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(15*b^4*x^8*arctan(1/(a*b)^(1/2)*b*x)+15*(a*b)^(1/2)*b^3*x^7+60*a*b^3
*x^6*arctan(1/(a*b)^(1/2)*b*x)+55*(a*b)^(1/2)*a*b^2*x^5+90*a^2*b^2*x^4*arct
an(1/(a*b)^(1/2)*b*x)+73*(a*b)^(1/2)*a^2*b*x^3+60*a^3*b*x^2*arctan(1/(a*b)^
(1/2)*b*x)-15*(a*b)^(1/2)*a^3*x+15*a^4*arctan(1/(a*b)^(1/2)*b*x))*(b*x^2+a)
/(a*b)^(1/2)/b/a^3/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 2.90, size = 109, normalized size = 0.51

$$\frac{15\,b^3x^7+55\,ab^2x^5+73\,a^2bx^3-15\,a^3x}{384\left(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b\right)}+\frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^3*x^7 + 55*a*b^2*x^5 + 73*a^2*b*x^3 - 15*a^3*x)/(a^3*b^5*x^8 +
4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) + 5/128*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^3*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(5/2), x)
```

3.484 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35\left(a+bx^2\right)^5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right.}{128a^4\left(a^2+2abx^2+\right.}$$

Rubi [A] time = 0.07, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1088, 199, 205}

$$\frac{35x\left(a+bx^2\right)^4}{128a^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35\left(a+bx^2\right)^5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] (x*(a + b*x^2))/(8*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (7*x*(a + b*x^2)^
2)/(48*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^3)/(192*a
^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^4)/(128*a^4*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*(a + b*x^2)^5*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(128*a^(9/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(2ab+2b^2x^2\right)^5 \int \frac{1}{\left(2ab+2b^2x^2\right)^5}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{\left(7\left(2ab+2b^2x^2\right)^5\right)\int \frac{1}{\left(2ab+2b^2x^2\right)^4}\,dx}{16ab\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{\left(35\left(2ab+2b^2x^2\right)^5\right)}{192a^2b^2\left(a^2+2a\right.}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x\left(a+bx^2\right.}{192a^3\left(a^2+2abx^2+\right.}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x\left(a+bx^2\right.}{192a^3\left(a^2+2abx^2+\right.}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x\left(a+bx^2\right.}{192a^3\left(a^2+2abx^2+\right.}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\sqrt{b}\,x\left(279a^3+511a^2bx^2+385ab^2x^4+105b^3x^6\right)+105\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{9/2}\sqrt{b}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(279*a^3 + 511*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3*x^6)
+ 105*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(9/2)*Sqrt[b]*(a +
b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 14.07, size = 98, normalized size = 0.46

$$\frac{\left(a+bx^2\right)\left(\frac{35\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}+\frac{x\left(279a^3+511a^2bx^2+385ab^2x^4+105b^3x^6\right)}{384a^4\left(a+bx^2\right)^4}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] ((a + b*x^2)*((x*(279*a^3 + 511*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3*x^6))/(
384*a^4*(a + b*x^2)^4) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(9/2)*Sqrt
[b])))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 2.22, size = 320, normalized size = 1.50

$$\left[\frac{210\,ab^4x^7+770\,a^2b^3x^5+1022\,a^3b^2x^3+558\,a^4bx-105\,\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{768\,\left(a^5b^5x^8+4\,a^6b^4x^6+6\,a^7b^3x^4+4\,a^8b^2x^2+a^9b\right)},\ \frac{105\,ab^4x^7+385\,a^2b^3x^5+511\,a^3b^2x^3+279\,a^4bx+105\,\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{384\,\left(a^5b^5x^8+4\,a^6b^4x^6+6\,a^7b^3x^4+4\,a^8b^2x^2+a^9b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(210*a*b^4*x^7 + 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 + 558*a^4*b*x -
105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6
 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b), 1/384*(105*a*b^4*x^7 + 385*a^2*b
^3*x^5 + 511*a^3*b^2*x^3 + 279*a^4*b*x + 105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2
*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^5*x^8
 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 169, normalized size = 0.79

$$\frac{\left(105b^4x^8\arctan\left(\frac{bx}{\sqrt{ab}}\right)+420a\,b^3x^6\arctan\left(\frac{bx}{\sqrt{ab}}\right)+105\sqrt{ab}\;b^3x^7+630a^2b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)+385\sqrt{ab}\;a\,b^2x^5+420a^3b\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)+511\sqrt{ab}\;a^2b\,x^3+105a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)+279\sqrt{ab}\;a^3x\right)\left(b\,x^2+a\right)}{384\sqrt{ab}\;\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(105*b^4*x^8*arctan(1/(a*b)^(1/2)*b*x)+105*(a*b)^(1/2)*b^3*x^7+420*a*
b^3*x^6*arctan(1/(a*b)^(1/2)*b*x)+385*(a*b)^(1/2)*a*b^2*x^5+630*a^2*b^2*x^4
*arctan(1/(a*b)^(1/2)*b*x)+511*(a*b)^(1/2)*a^2*b*x^3+420*a^3*b*x^2*arctan(1
/(a*b)^(1/2)*b*x)+279*(a*b)^(1/2)*a^3*x+105*a^4*arctan(1/(a*b)^(1/2)*b*x))*
(b*x^2+a)/(a*b)^(1/2)/a^4/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.05, size = 102, normalized size = 0.48

$$\frac{105\,b^3x^7+385\,ab^2x^5+511\,a^2bx^3+279\,a^3x}{384\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}+\frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(105*b^3*x^7 + 385*a*b^2*x^5 + 511*a^2*b*x^3 + 279*a^3*x)/(a^4*b^4*x^
8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8) + 35/128*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} + 2abx^{2} + b^{2}x^{4}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/2), x)
```

3.485 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$\frac{3}{16a^2x\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ax\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}-\frac{315\sqrt{b}\,\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\ldots}{128a \ldots}$$

Rubi [A] time = 0.11, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$-\frac{315\left(a+bx^2\right)}{128a^5x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21}{64a^3x\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}+\frac{3}{16a^2x\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ax\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}-\frac{315\sqrt{b}\,\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 105/(128*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 3/(16*a^2*x*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + 21/(64*a^3*x*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (315*(a + b*x^2))/(128*a^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (315*Sq
rt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3}{16a^2x\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3}{16a^2x\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{16a^2x\left(\right.}$$

$$= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{16a^2x\left(\right.}$$

$$= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{16a^2x\left(\right.}$$

Mathematica [A] time = 0.04, size = 115, normalized size = 0.46

$$\frac{-\sqrt{a}\left(128a^4+837a^3bx^2+1533a^2b^2x^4+1155ab^3x^6+315b^4x^8\right)-315\sqrt{b}\,x\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}x\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(Sqrt[a]*(128*a^4 + 837*a^3*b*x^2 + 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 + 3
15*b^4*x^8)) - 315*Sqrt[b]*x*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(12
8*a^(11/2)*x*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 16.44, size = 111, normalized size = 0.44

$$\frac{\left(a+bx^2\right)\left(\frac{-128a^4-837a^3bx^2-1533a^2b^2x^4-1155ab^3x^6-315b^4x^8}{128a^5x\left(a+bx^2\right)^4}-\frac{315\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] ((a + b*x^2)*((-128*a^4 - 837*a^3*b*x^2 - 1533*a^2*b^2*x^4 - 1155*a*b^3*x^6
 - 315*b^4*x^8)/(128*a^5*x*(a + b*x^2)^4) - (315*Sqrt[b]*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(128*a^(11/2))))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 3.02, size = 334, normalized size = 1.33

$$\left[-\frac{630\,b^4x^8+2310\,ab^3x^6+3066\,a^2b^2x^4+1674\,a^3bx^2+256\,a^4-315\left(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{256\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)},\,-\frac{315\,b^4x^8+1155\,ab^3x^6+1533\,a^2b^2x^4+837\,a^3bx^2+128\,a^4+315\left(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{128\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/256*(630*b^4*x^8 + 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 + 1674*a^3*b*x^2 +
 256*a^4 - 315*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x
)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^4*x^9
+ 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x^8
 + 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 + 837*a^3*b*x^2 + 128*a^4 + 315*(b^4*x
^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*sqrt(b/a)*arctan(x*
sqrt(b/a)))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^
9*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 191, normalized size = 0.76

$$-\frac{\left(315b^5x^9\arctan\left(\frac{bx}{\sqrt{ab}}\right)+1260a\,b^4x^7\arctan\left(\frac{bx}{\sqrt{ab}}\right)+315\sqrt{ab}\,b^4x^8+1890a^2b^3x^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)+1155\sqrt{ab}\,a\,b^3x^6+1260a^3b^2x^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)+1533\sqrt{ab}\,a^2b^2x^4+315a^4bx\arctan\left(\frac{bx}{\sqrt{ab}}\right)+837\sqrt{ab}\,a^3b\,x^2+128\sqrt{ab}\,a^4\right)\left(b\,x^2+a\right)}{128\sqrt{ab}\,\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/128*(315*arctan(1/(a*b)^(1/2)*b*x)*x^9*b^5+315*(a*b)^(1/2)*x^8*b^4+1260*
arctan(1/(a*b)^(1/2)*b*x)*x^7*a*b^4+1155*(a*b)^(1/2)*x^6*a*b^3+1890*arctan(
1/(a*b)^(1/2)*b*x)*x^5*a^2*b^3+1533*(a*b)^(1/2)*x^4*a^2*b^2+1260*arctan(1/(
a*b)^(1/2)*b*x)*x^3*a^3*b^2+837*(a*b)^(1/2)*x^2*a^3*b+315*arctan(1/(a*b)^(1
/2)*b*x)*x*a^4*b+128*(a*b)^(1/2)*a^4)*(b*x^2+a)/(a*b)^(1/2)/x/a^5/((b*x^2+a
)^2)^(5/2)
```

maxima [A] time = 3.04, size = 115, normalized size = 0.46

$$-\frac{315\,b^4x^8+1155\,ab^3x^6+1533\,a^2b^2x^4+837\,a^3bx^2+128\,a^4}{128\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}-\frac{315\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/128*(315*b^4*x^8 + 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 + 837*a^3*b*x^2 + 1
28*a^4)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
 - 315/128*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(5/2)), x)
```

3.486 $\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=291

$$\frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}+$$

Rubi [A] time = 0.12, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$\frac{1155b\left(a+bx^2\right)}{128a^6x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{385\left(a+bx^2\right)}{128a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{33}{64a^3x^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 231/(128*a^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x^3*(a + b*x^2)^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 11/(48*a^2*x^3*(a + b*x^2)^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) + 33/(64*a^3*x^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (385*(a + b*x^2))/(128*a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (1155*b*(a + b*x^2))/(128*a^6*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(13/2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^4}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48a^2x^3}$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48a^2x^3}$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48a^2x^3}$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48a^2x^3}$$

Mathematica [A] time = 0.05, size = 127, normalized size = 0.44

$$\frac{\sqrt{a}\left(-128a^5+1408a^4bx^2+9207a^3b^2x^4+16863a^2b^3x^6+12705ab^4x^8+3465b^5x^{10}\right)+3465b^{3/2}x^3\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{13/2}x^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (Sqrt[a]*(-128*a^5 + 1408*a^4*b*x^2 + 9207*a^3*b^2*x^4 + 16863*a^2*b^3*x^6
+ 12705*a*b^4*x^8 + 3465*b^5*x^10) + 3465*b^(3/2)*x^3*(a + b*x^2)^4*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(384*a^(13/2)*x^3*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 24.28, size = 122, normalized size = 0.42

$$\frac{\left(a+bx^2\right)\left(\frac{1155b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{13/2}}+\frac{-128a^5+1408a^4bx^2+9207a^3b^2x^4+16863a^2b^3x^6+12705ab^4x^8+3465b^5x^{10}}{384a^6x^3\left(a+bx^2\right)^4}\right)}{\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] ((a + b*x^2)*((-128*a^5 + 1408*a^4*b*x^2 + 9207*a^3*b^2*x^4 + 16863*a^2*b^3
*x^6 + 12705*a*b^4*x^8 + 3465*b^5*x^10)/(384*a^6*x^3*(a + b*x^2)^4) + (1155
*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(13/2))))/Sqrt[(a + b*x^2)^2]
```

fricas [A] time = 0.96, size = 370, normalized size = 1.27

$$\left[\frac{6930\,b^5x^{10}+25410\,ab^4x^8+33726\,a^2b^3x^6+18414\,a^3b^2x^4+2816\,a^4bx^2-256\,a^5+3465\left(b^5x^{11}+4\,ab^4x^9+6\,a^2b^3x^7+4\,a^3b^2x^5+a^4bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{768\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)},\ \frac{3465\,b^5x^{10}+12705\,ab^4x^8+16863\,a^2b^3x^6+9207\,a^3b^2x^4+1408\,a^4bx^2-128\,a^5+3465\left(b^5x^{11}+4\,ab^4x^9+6\,a^2b^3x^7+4\,a^3b^2x^5+a^4bx^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{384\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(6930*b^5*x^10 + 25410*a*b^4*x^8 + 33726*a^2*b^3*x^6 + 18414*a^3*b^2
*x^4 + 2816*a^4*b*x^2 - 256*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*
x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x
^5 + a^10*x^3), 1/384*(3465*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3*x^6
+ 9207*a^3*b^2*x^4 + 1408*a^4*b*x^2 - 128*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^
9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)
))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 211, normalized size = 0.73

$$\frac{\left(3465b^6x^{11}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+13860a\,b^5x^9\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3465\sqrt{ab}\,b^5x^{10}+20790a^2b^4x^7\arctan\left(\frac{bx}{\sqrt{ab}}\right)+12705\sqrt{ab}\,a\,b^4x^8+13860a^3b^3x^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)+16863\sqrt{ab}\,a^2b^3x^6+3465a^4b^2x^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)+9207\sqrt{ab}\,a^3b^2x^4+1408\sqrt{ab}\,a^4b\,x^2-128\sqrt{ab}\,a^5\right)\left(b\,x^2+a\right)}{384\sqrt{ab}\,\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(3465*arctan(1/(a*b)^(1/2)*b*x)*x^11*b^6+3465*(a*b)^(1/2)*x^10*b^5+13
860*arctan(1/(a*b)^(1/2)*b*x)*x^9*a*b^5+12705*(a*b)^(1/2)*x^8*a*b^4+20790*a
rctan(1/(a*b)^(1/2)*b*x)*x^7*a^2*b^4+16863*(a*b)^(1/2)*x^6*a^2*b^3+13860*ar
ctan(1/(a*b)^(1/2)*b*x)*x^5*a^3*b^3+9207*(a*b)^(1/2)*x^4*a^3*b^2+3465*arcta
n(1/(a*b)^(1/2)*b*x)*x^3*a^4*b^2+1408*(a*b)^(1/2)*x^2*a^4*b-128*(a*b)^(1/2)
*a^5)*(b*x^2+a)/(a*b)^(1/2)/x^3/a^6/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.13, size = 130, normalized size = 0.45

$$\frac{3465\,b^5x^{10}+12705\,ab^4x^8+16863\,a^2b^3x^6+9207\,a^3b^2x^4+1408\,a^4bx^2-128\,a^5}{384\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)}+\frac{1155\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(3465*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3*x^6 + 9207*a^3*b^2*x
^4 + 1408*a^4*b*x^2 - 128*a^5)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^
7 + 4*a^9*b*x^5 + a^10*x^3) + 1155/128*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\left(a + b x^2 \right)^2 \right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(1/(x**4*((a + b*x**2)**2)**(5/2)), x)
```

3.487 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(7/2))/(7*d) + (4*a*b*(d*x)^(11/2))/(11*d^3) + (2*b^2*(d*x)^(1
5/2))/(15*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)\,dx &= \int \left(a^2(dx)^{5/2}+\frac{2ab(dx)^{9/2}}{d^2}+\frac{b^2(dx)^{13/2}}{d^4}\right)\,dx\\ &= \frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.65

$$\frac{2x(dx)^{5/2}\left(165a^2+210abx^2+77b^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*(d*x)^(5/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

IntegrateAlgebraic [A] time = 0.03, size = 44, normalized size = 0.86

$$\frac{2(dx)^{7/2}\left(165a^2d^4+210abd^4x^2+77b^2d^4x^4\right)}{1155d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (2*(d*x)^(7/2)*(165*a^2*d^4 + 210*a*b*d^4*x^2 + 77*b^2*d^4*x^4))/(1155*d^5)
```

fricas [A] time = 1.01, size = 40, normalized size = 0.78

$$\frac{2}{1155}\left(77\,b^2d^2x^7+210\,abd^2x^5+165\,a^2d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 2/1155*(77*b^2*d^2*x^7 + 210*a*b*d^2*x^5 + 165*a^2*d^2*x^3)*sqrt(d*x)
```

giac [A] time = 0.17, size = 48, normalized size = 0.94

$$\frac{2}{15}\sqrt{dx}\,b^2d^2x^7+\frac{4}{11}\sqrt{dx}\,abd^2x^5+\frac{2}{7}\sqrt{dx}\,a^2d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 2/15*sqrt(d*x)*b^2*d^2*x^7 + 4/11*sqrt(d*x)*a*b*d^2*x^5 + 2/7*sqrt(d*x)*a^2
*d^2*x^3
```

maple [A] time = 0.01, size = 30, normalized size = 0.59

$$\frac{2\left(77b^2x^4+210ab\,x^2+165a^2\right)(dx)^{\frac{5}{2}}\,x}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] 2/1155*x*(77*b^2*x^4+210*a*b*x^2+165*a^2)*(d*x)^(5/2)
```

maxima [A] time = 1.30, size = 41, normalized size = 0.80

$$\frac{2\left(77\,(dx)^{\frac{15}{2}}\,b^2+210\,(dx)^{\frac{11}{2}}\,abd^2+165\,(dx)^{\frac{7}{2}}\,a^2d^4\right)}{1155\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 2/1155*(77*(d*x)^(15/2)*b^2 + 210*(d*x)^(11/2)*a*b*d^2 + 165*(d*x)^(7/2)*a^
2*d^4)/d^5
```

mupad [B] time = 0.07, size = 40, normalized size = 0.78

$$\frac{\frac{2\,b^2\,(d\,x)^{15/2}}{15}+\frac{2\,a^2\,d^4\,(d\,x)^{7/2}}{7}+\frac{4\,a\,b\,d^2\,(d\,x)^{11/2}}{11}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] ((2*b^2*(d*x)^(15/2))/15 + (2*a^2*d^4*(d*x)^(7/2))/7 + (4*a*b*d^2*(d*x)^(11
/2))/11)/d^5
```

sympy [A] time = 2.67, size = 49, normalized size = 0.96

$$\frac{2a^2d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{4abd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+\frac{2b^2d^{\frac{5}{2}}x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*d**(5/2)*x**(7/2)/7 + 4*a*b*d**(5/2)*x**(11/2)/11 + 2*b**2*d**(5/2)*
x**(15/2)/15
```

3.488 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(5/2))/(5*d) + (4*a*b*(d*x)^(9/2))/(9*d^3) + (2*b^2*(d*x)^(13/
2))/(13*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)\,dx&=\int\left(a^2(dx)^{3/2}+\frac{2ab(dx)^{7/2}}{d^2}+\frac{b^2(dx)^{11/2}}{d^4}\right)dx\\&=\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.65

$$\frac{2}{585}x(dx)^{3/2}\left(117a^2+130abx^2+45b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*(d*x)^(3/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

IntegrateAlgebraic [A] time = 0.03, size = 49, normalized size = 0.96

$$\frac{2\left(117a^2d^4(dx)^{5/2}+130abd^2(dx)^{9/2}+45b^2(dx)^{13/2}\right)}{585d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*(117*a^2*d^4*(d*x)^(5/2) + 130*a*b*d^2*(d*x)^(9/2) + 45*b^2*(d*x)^(13/2)
))/(585*d^5)
```

fricas [A] time = 1.69, size = 34, normalized size = 0.67

$$\frac{2}{585}\left(45\,b^2dx^6+130\,abdx^4+117\,a^2dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d*x^6 + 130*a*b*d*x^4 + 117*a^2*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.15, size = 42, normalized size = 0.82

$$\frac{2}{585}\left(45\,\sqrt{dx}\,b^2x^6+130\,\sqrt{dx}\,abx^4+117\,\sqrt{dx}\,a^2x^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 2/585*(45*sqrt(d*x)*b^2*x^6 + 130*sqrt(d*x)*a*b*x^4 + 117*sqrt(d*x)*a^2*x^2
)*d
```

maple [A] time = 0.01, size = 30, normalized size = 0.59

$$\frac{2\left(45b^2x^4+130ab\,x^2+117a^2\right)(dx)^{\frac{3}{2}}\,x}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/585*x*(45*b^2*x^4+130*a*b*x^2+117*a^2)*(d*x)^(3/2)
```

maxima [A] time = 1.34, size = 41, normalized size = 0.80

$$\frac{2\left(45\,(dx)^{\frac{13}{2}}\,b^2+130\,(dx)^{\frac{9}{2}}\,abd^2+117\,(dx)^{\frac{5}{2}}\,a^2d^4\right)}{585\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 2/585*(45*(d*x)^(13/2)*b^2 + 130*(d*x)^(9/2)*a*b*d^2 + 117*(d*x)^(5/2)*a^2*
d^4)/d^5
```

mupad [B] time = 4.22, size = 41, normalized size = 0.80

$$\frac{90\,b^2\,(d\,x)^{13/2}+234\,a^2\,d^4\,(d\,x)^{5/2}+260\,a\,b\,d^2\,(d\,x)^{9/2}}{585\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (90*b^2*(d*x)^(13/2) + 234*a^2*d^4*(d*x)^(5/2) + 260*a*b*d^2*(d*x)^(9/2))/(
585*d^5)
```

sympy [A] time = 1.24, size = 49, normalized size = 0.96

$$\frac{2a^2d^{\frac{3}{2}}x^{\frac{5}{2}}}{5}+\frac{4abd^{\frac{3}{2}}x^{\frac{9}{2}}}{9}+\frac{2b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*d**(3/2)*x**(5/2)/5 + 4*a*b*d**(3/2)*x**(9/2)/9 + 2*b**2*d**(3/2)*x*
*(13/2)/13
```

3.489 $\int \sqrt{dx} \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(3/2))/(3*d) + (4*a*b*(d*x)^(7/2))/(7*d^3) + (2*b^2*(d*x)^(11/
2))/(11*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right) dx &= \int \left(a^2\sqrt{dx}+\frac{2ab(dx)^{5/2}}{d^2}+\frac{b^2(dx)^{9/2}}{d^4}\right) dx\\ &= \frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.65

$$\frac{2}{231}x\sqrt{dx}\left(77a^2+66abx^2+21b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*Sqrt[d*x]*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

IntegrateAlgebraic [A] time = 0.03, size = 44, normalized size = 0.86

$$\frac{2(dx)^{3/2}\left(77a^2d^4+66abd^4x^2+21b^2d^4x^4\right)}{231d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (2*(d*x)^(3/2)*(77*a^2*d^4 + 66*a*b*d^4*x^2 + 21*b^2*d^4*x^4))/(231*d^5)
```

fricas [A] time = 1.25, size = 29, normalized size = 0.57

$$\frac{2}{231}\left(21\,b^2x^5+66\,abx^3+77\,a^2x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="fricas")
[Out] 2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(d*x)
```

giac [A] time = 0.15, size = 37, normalized size = 0.73

$$\frac{2}{11}\sqrt{dx}\,b^2x^5+\frac{4}{7}\sqrt{dx}\,abx^3+\frac{2}{3}\sqrt{dx}\,a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="giac")
[Out] 2/11*sqrt(d*x)*b^2*x^5 + 4/7*sqrt(d*x)*a*b*x^3 + 2/3*sqrt(d*x)*a^2*x
```

maple [A] time = 0.01, size = 30, normalized size = 0.59

$$\frac{2\left(21b^2x^4+66ab\,x^2+77a^2\right)\sqrt{dx}\;x}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x)
[Out] 2/231*x*(21*b^2*x^4+66*a*b*x^2+77*a^2)*(d*x)^(1/2)
```

maxima [A] time = 1.36, size = 41, normalized size = 0.80

$$\frac{2\left(21\,(dx)^{\frac{11}{2}}\,b^2+66\,(dx)^{\frac{7}{2}}\,abd^2+77\,(dx)^{\frac{3}{2}}\,a^2d^4\right)}{231\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*(d*x)^(11/2)*b^2 + 66*(d*x)^(7/2)*a*b*d^2 + 77*(d*x)^(3/2)*a^2*d^
4)/d^5
```

mupad [B] time = 0.05, size = 41, normalized size = 0.80

$$\frac{42\,b^2\,{\left(d\,x\right)}^{11/2}+154\,a^2\,d^4\,{\left(d\,x\right)}^{3/2}+132\,a\,b\,d^2\,{\left(d\,x\right)}^{7/2}}{231\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (42*b^2*(d*x)^(11/2) + 154*a^2*d^4*(d*x)^(3/2) + 132*a*b*d^2*(d*x)^(7/2))/(
231*d^5)
```

sympy [A] time = 0.48, size = 49, normalized size = 0.96

$$\frac{2a^2\sqrt{d}\,x^{\frac{3}{2}}}{3}+\frac{4ab\sqrt{d}\,x^{\frac{7}{2}}}{7}+\frac{2b^2\sqrt{d}\,x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)*(d*x)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(d)*x**(3/2)/3 + 4*a*b*sqrt(d)*x**(7/2)/7 + 2*b**2*sqrt(d)*x**(1
1/2)/11
```

3.490 $\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx$

Optimal. Leaf size=49

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

```
[Out] (2*a^2*Sqrt[d*x])/d + (4*a*b*(d*x)^(5/2))/(5*d^3) + (2*b^2*(d*x)^(9/2))/(9*
d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx &= \int \left(\frac{a^2}{\sqrt{dx}}+\frac{2ab(dx)^{3/2}}{d^2}+\frac{b^2(dx)^{7/2}}{d^4}\right)dx\\ &= \frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{2\left(45a^2x+18abx^3+5b^2x^5\right)}{45\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

```
[Out] (2*(45*a^2*x + 18*a*b*x^3 + 5*b^2*x^5))/(45*Sqrt[d*x])
```

IntegrateAlgebraic [A] time = 0.04, size = 49, normalized size = 1.00

$$\frac{2\left(45a^2d^4\sqrt{dx}+18abd^2(dx)^{5/2}+5b^2(dx)^{9/2}\right)}{45d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

```
[Out] (2*(45*a^2*d^4*Sqrt[d*x] + 18*a*b*d^2*(d*x)^(5/2) + 5*b^2*(d*x)^(9/2)))/(45
*d^5)
```

fricas [A] time = 1.36, size = 31, normalized size = 0.63

$$\frac{2\left(5\,b^2x^4+18\,abx^2+45\,a^2\right)\sqrt{dx}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(d*x)/d
```

giac [A] time = 0.15, size = 41, normalized size = 0.84

$$\frac{2\left(5\,\sqrt{dx}\,b^2x^4+18\,\sqrt{dx}\,abx^2+45\,\sqrt{dx}\,a^2\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(5*sqrt(d*x)*b^2*x^4 + 18*sqrt(d*x)*a*b*x^2 + 45*sqrt(d*x)*a^2)/d
```

maple [A] time = 0.01, size = 30, normalized size = 0.61

$$\frac{2\left(5b^2x^4+18ab\,x^2+45a^2\right)x}{45\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x)
```

```
[Out] 2/45*(5*b^2*x^4+18*a*b*x^2+45*a^2)*x/(d*x)^(1/2)
```

maxima [A] time = 1.27, size = 41, normalized size = 0.84

$$\frac{2\left(45\sqrt{dx}\,a^2+\frac{5\,(dx)^{\frac{9}{2}}b^2}{d^4}+\frac{18\,(dx)^{\frac{5}{2}}ab}{d^2}\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(45*sqrt(d*x)*a^2 + 5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)/d
```

mupad [B] time = 0.05, size = 41, normalized size = 0.84

$$\frac{10\,b^2\,(d\,x)^{9/2}+90\,a^2\,d^4\,\sqrt{d\,x}\,+36\,a\,b\,d^2\,(d\,x)^{5/2}}{45\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(1/2),x)
```

```
[Out] (10*b^2*(d*x)^(9/2) + 90*a^2*d^4*(d*x)^(1/2) + 36*a*b*d^2*(d*x)^(5/2))/(45*
d^5)
```

sympy [A] time = 0.63, size = 48, normalized size = 0.98

$$\frac{2a^2\sqrt{x}}{\sqrt{d}}+\frac{4abx^{\frac{5}{2}}}{5\sqrt{d}}+\frac{2b^2x^{\frac{9}{2}}}{9\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x)/sqrt(d) + 4*a*b*x**(5/2)/(5*sqrt(d)) + 2*b**2*x**(9/2)/(9*sq
rt(d))
```

3.491 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(d*Sqrt[d*x]) + (4*a*b*(d*x)^(3/2))/(3*d^3) + (2*b^2*(d*x)^(7/2))/
(7*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{3/2}}+\frac{2ab\sqrt{dx}}{d^2}+\frac{b^2(dx)^{5/2}}{d^4}\right)dx\\ &= -\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{2x\left(-21a^2+14abx^2+3b^2x^4\right)}{21(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-21*a^2 + 14*a*b*x^2 + 3*b^2*x^4))/(21*(d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.90

$$\frac{2\left(-21a^2d^4+14abd^4x^2+3b^2d^4x^4\right)}{21d^5\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
[Out] (2*(-21*a^2*d^4 + 14*a*b*d^4*x^2 + 3*b^2*d^4*x^4))/(21*d^5*Sqrt[d*x])
```

fricas [A] time = 1.92, size = 34, normalized size = 0.69

$$\frac{2\left(3\,b^2x^4+14\,abx^2-21\,a^2\right)\sqrt{dx}}{21\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="fricas")
[Out] 2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.22, size = 51, normalized size = 1.04

$$-\frac{2\left(\frac{21\,a^2}{\sqrt{dx}}-\frac{3\,\sqrt{dx}\,b^2d^{27}x^3+14\,\sqrt{dx}\,abd^{27}x}{d^{28}}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="giac")
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*sqrt(d*x)*b^2*d^27*x^3 + 14*sqrt(d*x)*a*b*d^27
*x)/d^28)/d
```

maple [A] time = 0.01, size = 30, normalized size = 0.61

$$-\frac{2\left(-3b^2x^4-14ab\,x^2+21a^2\right)x}{21\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x)
[Out] -2/21*(-3*b^2*x^4-14*a*b*x^2+21*a^2)*x/(d*x)^(3/2)
```

maxima [A] time = 1.36, size = 44, normalized size = 0.90

$$-\frac{2\left(\frac{21\,a^2}{\sqrt{dx}}-\frac{3\,(dx)^{\frac{7}{2}}b^2+14\,(dx)^{\frac{3}{2}}abd^2}{d^4}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*(d*x)^(7/2)*b^2 + 14*(d*x)^(3/2)*a*b*d^2)/d^4)
/d
```

mupad [B] time = 0.05, size = 31, normalized size = 0.63

$$\frac{-42\,a^2+28\,a\,b\,x^2+6\,b^2\,x^4}{21\,d\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(3/2),x)
```

```
[Out] (6*b^2*x^4 - 42*a^2 + 28*a*b*x^2)/(21*d*(d*x)^(1/2))
```

sympy [A] time = 0.66, size = 48, normalized size = 0.98

$$-\frac{2a^2}{d^{\frac{3}{2}}\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3d^{\frac{3}{2}}}+\frac{2b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(3/2),x)
```

```
[Out] -2*a**2/(d**(3/2)*sqrt(x)) + 4*a*b*x**(3/2)/(3*d**(3/2)) + 2*b**2*x**(7/2)/
(7*d**(3/2))
```

3.492 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*d*(d*x)^(3/2)) + (4*a*b*Sqrt[d*x])/d^3 + (2*b^2*(d*x)^(5/2))/(5
*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{5/2}}+\frac{2ab}{d^2\sqrt{dx}}+\frac{b^2(dx)^{3/2}}{d^4}\right)dx\\ &= -\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{x\left(-10a^2+60abx^2+6b^2x^4\right)}{15(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

[Out] (x*(-10*a^2 + 60*a*b*x^2 + 6*b^2*x^4))/(15*(d*x)^(5/2))

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.90

$$\frac{2\left(-5a^2d^4+30abd^4x^2+3b^2d^4x^4\right)}{15d^5(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]

[Out] (2*(-5*a^2*d^4 + 30*a*b*d^4*x^2 + 3*b^2*d^4*x^4))/(15*d^5*(d*x)^(3/2))

fricas [A] time = 0.98, size = 34, normalized size = 0.69

$$\frac{2\left(3\,b^2x^4+30\,abx^2-5\,a^2\right)\sqrt{dx}}{15\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="fricas")

[Out] 2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)*sqrt(d*x)/(d^3*x^2)

giac [A] time = 0.15, size = 53, normalized size = 1.08

$$-\frac{2\left(\frac{5\,a^2d}{\sqrt{dx}\,x}-\frac{3\left(\sqrt{dx}\,b^2d^{10}x^2+10\,\sqrt{dx}\,abd^{10}\right)}{d^{10}}\right)}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="giac")

[Out] -2/15*(5*a^2*d/(sqrt(d*x)*x) - 3*(sqrt(d*x)*b^2*d^10*x^2 + 10*sqrt(d*x)*a*b
*d^10)/d^10)/d^3

maple [A] time = 0.01, size = 30, normalized size = 0.61

$$-\frac{2\left(-3b^2x^4-30ab\,x^2+5a^2\right)x}{15\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x)

[Out] -2/15*(-3*b^2*x^4-30*a*b*x^2+5*a^2)*x/(d*x)^(5/2)

maxima [A] time = 1.40, size = 43, normalized size = 0.88

$$-\frac{2\left(\frac{5\,a^2}{(dx)^{\frac{3}{2}}}-\frac{3\left((dx)^{\frac{5}{2}}b^2+10\,\sqrt{dx}\,abd^2\right)}{d^4}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*a^2/(d*x)^(3/2) - 3*((d*x)^(5/2)*b^2 + 10*sqrt(d*x)*a*b*d^2)/d^4)/
d
```

mupad [B] time = 4.23, size = 34, normalized size = 0.69

$$\frac{-10\,a^2+60\,a\,b\,x^2+6\,b^2\,x^4}{15\,d^2\,x\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(5/2),x)
```

```
[Out] (6*b^2*x^4 - 10*a^2 + 60*a*b*x^2)/(15*d^2*x*(d*x)^(1/2))
```

sympy [A] time = 0.91, size = 48, normalized size = 0.98

$$-\frac{2a^2}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{4ab\sqrt{x}}{d^{\frac{5}{2}}}+\frac{2b^2x^{\frac{5}{2}}}{5d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(5/2),x)
```

```
[Out] -2*a**2/(3*d**(5/2)*x**(3/2)) + 4*a*b*sqrt(x)/d**(5/2) + 2*b**2*x**(5/2)/(5
*d**(5/2))
```

3.493 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^2)/(5*d*(d*x)^(5/2)) - (4*a*b)/(d^3*Sqrt[d*x]) + (2*b^2*(d*x)^(3/2))/
(3*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{7/2}}+\frac{2ab}{d^2(dx)^{3/2}}+\frac{b^2\sqrt{dx}}{d^4}\right)dx\\ &= -\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{2\sqrt{dx}\left(-3a^2-30abx^2+5b^2x^4\right)}{15d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*d^4*x^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 44, normalized size = 0.90

$$\frac{2\left(-3a^2d^4-30abd^4x^2+5b^2d^4x^4\right)}{15d^5(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
[Out] (2*(-3*a^2*d^4 - 30*a*b*d^4*x^2 + 5*b^2*d^4*x^4))/(15*d^5*(d*x)^(5/2))
```

fricas [A] time = 1.02, size = 34, normalized size = 0.69

$$\frac{2\left(5\,b^2x^4-30\,abx^2-3\,a^2\right)\sqrt{dx}}{15\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="fricas")
[Out] 2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.15, size = 48, normalized size = 0.98

$$\frac{2\left(5\,\sqrt{dx}\,b^2x-\frac{3\left(10\,abd^3x^2+a^2d^3\right)}{\sqrt{dx}\,d^2x^2}\right)}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="giac")
[Out] 2/15*(5*sqrt(d*x)*b^2*x - 3*(10*a*b*d^3*x^2 + a^2*d^3)/(sqrt(d*x)*d^2*x^2))
/d^4
```

maple [A] time = 0.01, size = 30, normalized size = 0.61

$$-\frac{2\left(-5b^2x^4+30ab\,x^2+3a^2\right)x}{15\,(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x)
[Out] -2/15*(-5*b^2*x^4+30*a*b*x^2+3*a^2)*x/(d*x)^(7/2)
```

maxima [A] time = 1.30, size = 47, normalized size = 0.96

$$\frac{2\left(\frac{5\,(dx)^{\frac{3}{2}}b^2}{d^4}-\frac{3\left(10\,abd^2x^2+a^2d^2\right)}{(dx)^{\frac{5}{2}}d^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*(d*x)^(3/2)*b^2/d^4 - 3*(10*a*b*d^2*x^2 + a^2*d^2)/((d*x)^(5/2)*d^2
))/d
```

mupad [B] time = 0.05, size = 34, normalized size = 0.69

$$-\frac{6\,a^2+60\,a\,b\,x^2-10\,b^2\,x^4}{15\,d^3\,x^2\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(7/2),x)
```

```
[Out] -(6*a^2 - 10*b^2*x^4 + 60*a*b*x^2)/(15*d^3*x^2*(d*x)^(1/2))
```

sympy [A] time = 1.97, size = 48, normalized size = 0.98

$$-\frac{2a^2}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{4ab}{d^{\frac{7}{2}}\sqrt{x}}+\frac{2b^2x^{\frac{3}{2}}}{3d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(7/2),x)
```

```
[Out] -2*a**2/(5*d**(7/2)*x**(5/2)) - 4*a*b/(d**(7/2)*sqrt(x)) + 2*b**2*x**(3/2)/
(3*d**(7/2))
```

3.494 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{2a^4(dx)^{7/2}}{7d}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (4*a^2*b^2*(d
*x)^(15/2))/(5*d^5) + (8*a*b^3*(d*x)^(19/2))/(19*d^7) + (2*b^4*(d*x)^(23/2)
)/(23*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int (dx)^{5/2}\left(ab+b^2x^2\right)^4\,dx}{b^4}\\
&= \frac{\int \left(a^4b^4(dx)^{5/2}+\frac{4a^3b^5(dx)^{9/2}}{d^2}+\frac{6a^2b^6(dx)^{13/2}}{d^4}+\frac{4ab^7(dx)^{17/2}}{d^6}+\frac{b^8(dx)^{21/2}}{d^8}\right)dx}{b^4}\\
&= \frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.60

$$\frac{2x(dx)^{5/2}\left(24035a^4+61180a^3bx^2+67298a^2b^2x^4+35420ab^3x^6+7315b^4x^8\right)}{168245}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(24035*a^4 + 61180*a^3*b*x^2 + 67298*a^2*b^2*x^4 + 35420*a
*b^3*x^6 + 7315*b^4*x^8))/168245
```

IntegrateAlgebraic [A] time = 0.05, size = 85, normalized size = 0.93

$$\frac{2\left(24035a^4d^8(dx)^{7/2}+61180a^3bd^6(dx)^{11/2}+67298a^2b^2d^4(dx)^{15/2}+35420ab^3d^2(dx)^{19/2}+7315b^4(dx)^{23/2}\right)}{168245d^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*(24035*a^4*d^8*(d*x)^(7/2) + 61180*a^3*b*d^6*(d*x)^(11/2) + 67298*a^2*b^
2*d^4*(d*x)^(15/2) + 35420*a*b^3*d^2*(d*x)^(19/2) + 7315*b^4*(d*x)^(23/2)))
/(168245*d^9)
```

fricas [A] time = 1.49, size = 68, normalized size = 0.75

$$\frac{2}{168245}\left(7315\,b^4d^2x^{11}+35420\,ab^3d^2x^9+67298\,a^2b^2d^2x^7+61180\,a^3bd^2x^5+24035\,a^4d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*b^4*d^2*x^11 + 35420*a*b^3*d^2*x^9 + 67298*a^2*b^2*d^2*x^7 +
 61180*a^3*b*d^2*x^5 + 24035*a^4*d^2*x^3)*sqrt(d*x)
```

giac [A] time = 0.15, size = 86, normalized size = 0.95

$$\frac{2}{23}\sqrt{dx}\,b^4d^2x^{11}+\frac{8}{19}\sqrt{dx}\,ab^3d^2x^9+\frac{4}{5}\sqrt{dx}\,a^2b^2d^2x^7+\frac{8}{11}\sqrt{dx}\,a^3bd^2x^5+\frac{2}{7}\sqrt{dx}\,a^4d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/23*sqrt(d*x)*b^4*d^2*x^11 + 8/19*sqrt(d*x)*a*b^3*d^2*x^9 + 4/5*sqrt(d*x)*
a^2*b^2*d^2*x^7 + 8/11*sqrt(d*x)*a^3*b*d^2*x^5 + 2/7*sqrt(d*x)*a^4*d^2*x^3
```

maple [A] time = 0.01, size = 52, normalized size = 0.57

$$\frac{2\left(7315 b^{4} x^{8}+35420 a\, b^{3} x^{6}+67298 a^{2} b^{2} x^{4}+61180 a^{3} b\, x^{2}+24035 a^{4}\right) (d x)^{\frac{5}{2}} x}{168245}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/168245*x*(7315*b^4*x^8+35420*a*b^3*x^6+67298*a^2*b^2*x^4+61180*a^3*b*x^2+
24035*a^4)*(d*x)^(5/2)
```

maxima [A] time = 1.35, size = 73, normalized size = 0.80

$$\frac{2\left(7315\,(dx)^{\frac{23}{2}} b^{4}+35420\,(dx)^{\frac{19}{2}} a b^{3} d^{2}+67298\,(dx)^{\frac{15}{2}} a^{2} b^{2} d^{4}+61180\,(dx)^{\frac{11}{2}} a^{3} b d^{6}+24035\,(dx)^{\frac{7}{2}} a^{4} d^{8}\right)}{168245\, d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/168245*(7315*(d*x)^(23/2)*b^4 + 35420*(d*x)^(19/2)*a*b^3*d^2 + 67298*(d*x
)^(15/2)*a^2*b^2*d^4 + 61180*(d*x)^(11/2)*a^3*b*d^6 + 24035*(d*x)^(7/2)*a^4
*d^8)/d^9
```

mupad [B] time = 4.20, size = 71, normalized size = 0.78

$$\frac{2\, a^{4}\, (d\, x)^{7/2}}{7\, d}+\frac{2\, b^{4}\, (d\, x)^{23/2}}{23\, d^{9}}+\frac{4\, a^{2}\, b^{2}\, (d\, x)^{15/2}}{5\, d^{5}}+\frac{8\, a^{3}\, b\, (d\, x)^{11/2}}{11\, d^{3}}+\frac{8\, a\, b^{3}\, (d\, x)^{19/2}}{19\, d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (2*b^4*(d*x)^(23/2))/(23*d^9) + (4*a^2*b^2*(d*x
)^(15/2))/(5*d^5) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (8*a*b^3*(d*x)^(19/2)
)/(19*d^7)
```

sympy [A] time = 5.68, size = 90, normalized size = 0.99

$$\frac{2a^{4}d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{8a^{3}bd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+\frac{4a^{2}b^{2}d^{\frac{5}{2}}x^{\frac{15}{2}}}{5}+\frac{8ab^{3}d^{\frac{5}{2}}x^{\frac{19}{2}}}{19}+\frac{2b^{4}d^{\frac{5}{2}}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*d**(5/2)*x**(7/2)/7 + 8*a**3*b*d**(5/2)*x**(11/2)/11 + 4*a**2*b**2*d
**(5/2)*x**(15/2)/5 + 8*a*b**3*d**(5/2)*x**(19/2)/19 + 2*b**4*d**(5/2)*x**(
23/2)/23
```

3.495 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{2a^4(dx)^{5/2}}{5d}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (12*a^2*b^2*(d*
x)^(13/2))/(13*d^5) + (8*a*b^3*(d*x)^(17/2))/(17*d^7) + (2*b^4*(d*x)^(21/2)
)/(21*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2 \, dx &= \frac{\int (dx)^{3/2}\left(ab+b^2x^2\right)^4 \, dx}{b^4}\\
&= \frac{\int \left(a^4b^4(dx)^{3/2}+\frac{4a^3b^5(dx)^{7/2}}{d^2}+\frac{6a^2b^6(dx)^{11/2}}{d^4}+\frac{4ab^7(dx)^{15/2}}{d^6}+\frac{b^8(dx)^{19/2}}{d^8}\right) dx}{b^4}\\
&= \frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.60

$$\frac{2x(dx)^{3/2}\left(13923a^4+30940a^3bx^2+32130a^2b^2x^4+16380ab^3x^6+3315b^4x^8\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(13923*a^4 + 30940*a^3*b*x^2 + 32130*a^2*b^2*x^4 + 16380*a
*b^3*x^6 + 3315*b^4*x^8))/69615
```

IntegrateAlgebraic [A] time = 0.05, size = 85, normalized size = 0.93

$$\frac{2\left(13923a^4d^8(dx)^{5/2}+30940a^3bd^6(dx)^{9/2}+32130a^2b^2d^4(dx)^{13/2}+16380ab^3d^2(dx)^{17/2}+3315b^4(dx)^{21/2}\right)}{69615d^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*(13923*a^4*d^8*(d*x)^(5/2) + 30940*a^3*b*d^6*(d*x)^(9/2) + 32130*a^2*b^2
*d^4*(d*x)^(13/2) + 16380*a*b^3*d^2*(d*x)^(17/2) + 3315*b^4*(d*x)^(21/2)))/
(69615*d^9)
```

fricas [A] time = 2.23, size = 58, normalized size = 0.64

$$\frac{2}{69615}\left(3315\,b^4dx^{10}+16380\,ab^3dx^8+32130\,a^2b^2dx^6+30940\,a^3bdx^4+13923\,a^4dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^4*d*x^10 + 16380*a*b^3*d*x^8 + 32130*a^2*b^2*d*x^6 + 30940*
a^3*b*d*x^4 + 13923*a^4*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.17, size = 74, normalized size = 0.81

$$\frac{2}{69615}\left(3315\,\sqrt{dx}\,b^4x^{10}+16380\,\sqrt{dx}\,ab^3x^8+32130\,\sqrt{dx}\,a^2b^2x^6+30940\,\sqrt{dx}\,a^3bx^4+13923\,\sqrt{dx}\,a^4x^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/69615*(3315*sqrt(d*x)*b^4*x^10 + 16380*sqrt(d*x)*a*b^3*x^8 + 32130*sqrt(d
*x)*a^2*b^2*x^6 + 30940*sqrt(d*x)*a^3*b*x^4 + 13923*sqrt(d*x)*a^4*x^2)*d
```

maple [A] time = 0.01, size = 52, normalized size = 0.57

$$\frac{2\left(3315b^4x^8+16380a\,b^3x^6+32130a^2b^2x^4+30940a^3b\,x^2+13923a^4\right)(dx)^{\frac{3}{2}}\,x}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/69615*x*(3315*b^4*x^8+16380*a*b^3*x^6+32130*a^2*b^2*x^4+30940*a^3*b*x^2+1
3923*a^4)*(d*x)^(3/2)
```

maxima [A] time = 1.35, size = 73, normalized size = 0.80

$$\frac{2\left(3315\,(dx)^{\frac{21}{2}}\,b^4+16380\,(dx)^{\frac{17}{2}}\,ab^3d^2+32130\,(dx)^{\frac{13}{2}}\,a^2b^2d^4+30940\,(dx)^{\frac{9}{2}}\,a^3bd^6+13923\,(dx)^{\frac{5}{2}}\,a^4d^8\right)}{69615\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/69615*(3315*(d*x)^(21/2)*b^4 + 16380*(d*x)^(17/2)*a*b^3*d^2 + 32130*(d*x)
^(13/2)*a^2*b^2*d^4 + 30940*(d*x)^(9/2)*a^3*b*d^6 + 13923*(d*x)^(5/2)*a^4*d
^8)/d^9
```

mupad [B] time = 0.03, size = 71, normalized size = 0.78

$$\frac{2\,a^4\,(d\,x)^{5/2}}{5\,d}+\frac{2\,b^4\,(d\,x)^{21/2}}{21\,d^9}+\frac{12\,a^2\,b^2\,(d\,x)^{13/2}}{13\,d^5}+\frac{8\,a^3\,b\,(d\,x)^{9/2}}{9\,d^3}+\frac{8\,a\,b^3\,(d\,x)^{17/2}}{17\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (2*b^4*(d*x)^(21/2))/(21*d^9) + (12*a^2*b^2*(d*
x)^(13/2))/(13*d^5) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (8*a*b^3*(d*x)^(17/2)
)/(17*d^7)
```

sympy [A] time = 2.69, size = 90, normalized size = 0.99

$$\frac{2a^4d^{\frac{3}{2}}x^{\frac{5}{2}}}{5}+\frac{8a^3bd^{\frac{3}{2}}x^{\frac{9}{2}}}{9}+\frac{12a^2b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}+\frac{8ab^3d^{\frac{3}{2}}x^{\frac{17}{2}}}{17}+\frac{2b^4d^{\frac{3}{2}}x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*d**(3/2)*x**(5/2)/5 + 8*a**3*b*d**(3/2)*x**(9/2)/9 + 12*a**2*b**2*d*
*(3/2)*x**(13/2)/13 + 8*a*b**3*d**(3/2)*x**(17/2)/17 + 2*b**4*d**(3/2)*x**(
21/2)/21
```

3.496 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{2a^4(dx)^{3/2}}{3d}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (12*a^2*b^2*(d*
x)^(11/2))/(11*d^5) + (8*a*b^3*(d*x)^(15/2))/(15*d^7) + (2*b^4*(d*x)^(19/2)
)/(19*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int \sqrt{dx}\left(ab+b^2x^2\right)^4\,dx}{b^4}\\ &= \frac{\int \left(a^4b^4\sqrt{dx}+\frac{4a^3b^5(dx)^{5/2}}{d^2}+\frac{6a^2b^6(dx)^{9/2}}{d^4}+\frac{4ab^7(dx)^{13/2}}{d^6}+\frac{b^8(dx)^{17/2}}{d^8}\right)dx}{b^4}\\ &= \frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.60

$$\frac{2x\sqrt{dx}\left(7315a^4+12540a^3bx^2+11970a^2b^2x^4+5852ab^3x^6+1155b^4x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*Sqrt[d*x]*(7315*a^4 + 12540*a^3*b*x^2 + 11970*a^2*b^2*x^4 + 5852*a*b^3
*x^6 + 1155*b^4*x^8))/21945
```

IntegrateAlgebraic [A] time = 0.05, size = 85, normalized size = 0.93

$$\frac{2\left(7315a^4d^8(dx)^{3/2}+12540a^3bd^6(dx)^{7/2}+11970a^2b^2d^4(dx)^{11/2}+5852ab^3d^2(dx)^{15/2}+1155b^4(dx)^{19/2}\right)}{21945d^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*(7315*a^4*d^8*(d*x)^(3/2) + 12540*a^3*b*d^6*(d*x)^(7/2) + 11970*a^2*b^2*
d^4*(d*x)^(11/2) + 5852*a*b^3*d^2*(d*x)^(15/2) + 1155*b^4*(d*x)^(19/2)))/(2
1945*d^9)
```

fricas [A] time = 1.80, size = 51, normalized size = 0.56

$$\frac{2}{21945}\left(1155\,b^4x^9+5852\,ab^3x^7+11970\,a^2b^2x^5+12540\,a^3bx^3+7315\,a^4x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^4*x^9 + 5852*a*b^3*x^7 + 11970*a^2*b^2*x^5 + 12540*a^3*b*x^
3 + 7315*a^4*x)*sqrt(d*x)
```

giac [A] time = 0.16, size = 69, normalized size = 0.76

$$\frac{2}{19}\sqrt{dx}\,b^4x^9+\frac{8}{15}\sqrt{dx}\,ab^3x^7+\frac{12}{11}\sqrt{dx}\,a^2b^2x^5+\frac{8}{7}\sqrt{dx}\,a^3bx^3+\frac{2}{3}\sqrt{dx}\,a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*sqrt(d*x)*b^4*x^9 + 8/15*sqrt(d*x)*a*b^3*x^7 + 12/11*sqrt(d*x)*a^2*b^2
*x^5 + 8/7*sqrt(d*x)*a^3*b*x^3 + 2/3*sqrt(d*x)*a^4*x
```

maple [A] time = 0.01, size = 52, normalized size = 0.57

$$\frac{2\left(1155b^{4}x^{8}+5852a\,b^{3}x^{6}+11970a^{2}b^{2}x^{4}+12540a^{3}b\,x^{2}+7315a^{4}\right)\sqrt{dx}\,x}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x)
```

```
[Out] 2/21945*x*(1155*b^4*x^8+5852*a*b^3*x^6+11970*a^2*b^2*x^4+12540*a^3*b*x^2+73
15*a^4)*(d*x)^(1/2)
```

maxima [A] time = 1.33, size = 73, normalized size = 0.80

$$\frac{2\left(1155\,(dx)^{\frac{19}{2}}\,b^{4}+5852\,(dx)^{\frac{15}{2}}\,ab^{3}d^{2}+11970\,(dx)^{\frac{11}{2}}\,a^{2}b^{2}d^{4}+12540\,(dx)^{\frac{7}{2}}\,a^{3}bd^{6}+7315\,(dx)^{\frac{3}{2}}\,a^{4}d^{8}\right)}{21945\,d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21945*(1155*(d*x)^(19/2)*b^4 + 5852*(d*x)^(15/2)*a*b^3*d^2 + 11970*(d*x)^
(11/2)*a^2*b^2*d^4 + 12540*(d*x)^(7/2)*a^3*b*d^6 + 7315*(d*x)^(3/2)*a^4*d^8
)/d^9
```

mupad [B] time = 0.03, size = 71, normalized size = 0.78

$$\frac{2\,a^{4}\,(d\,x)^{3/2}}{3\,d}+\frac{2\,b^{4}\,(d\,x)^{19/2}}{19\,d^{9}}+\frac{12\,a^{2}\,b^{2}\,(d\,x)^{11/2}}{11\,d^{5}}+\frac{8\,a^{3}\,b\,(d\,x)^{7/2}}{7\,d^{3}}+\frac{8\,a\,b^{3}\,(d\,x)^{15/2}}{15\,d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (2*b^4*(d*x)^(19/2))/(19*d^9) + (12*a^2*b^2*(d*
x)^(11/2))/(11*d^5) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (8*a*b^3*(d*x)^(15/2)
)/(15*d^7)
```

sympy [A] time = 1.27, size = 90, normalized size = 0.99

$$\frac{2a^{4}\sqrt{d}\,x^{\frac{3}{2}}}{3}+\frac{8a^{3}b\sqrt{d}\,x^{\frac{7}{2}}}{7}+\frac{12a^{2}b^{2}\sqrt{d}\,x^{\frac{11}{2}}}{11}+\frac{8ab^{3}\sqrt{d}\,x^{\frac{15}{2}}}{15}+\frac{2b^{4}\sqrt{d}\,x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2*(d*x)**(1/2),x)
```

```
[Out] 2*a**4*sqrt(d)*x**(3/2)/3 + 8*a**3*b*sqrt(d)*x**(7/2)/7 + 12*a**2*b**2*sqrt
(d)*x**(11/2)/11 + 8*a*b**3*sqrt(d)*x**(15/2)/15 + 2*b**4*sqrt(d)*x**(19/2)
/19
```

3.497 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{\sqrt{dx}}\,dx$

Optimal. Leaf size=89

$$\frac{2a^4\sqrt{dx}}{d}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{2a^4\sqrt{dx}}{d}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*a^4*Sqrt[d*x])/d + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (4*a^2*b^2*(d*x)^(9/2
))/(3*d^5) + (8*a*b^3*(d*x)^(13/2))/(13*d^7) + (2*b^4*(d*x)^(17/2))/(17*d^9
)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{\sqrt{dx}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{\sqrt{dx}} + \frac{4a^3b^5(dx)^{3/2}}{d^2} + \frac{6a^2b^6(dx)^{7/2}}{d^4} + \frac{4ab^7(dx)^{11/2}}{d^6} + \frac{b^8(dx)^{15/2}}{d^8}\right)dx}{b^4}$$
$$= \frac{2a^4\sqrt{dx}}{d} + \frac{8a^3b(dx)^{5/2}}{5d^3} + \frac{4a^2b^2(dx)^{9/2}}{3d^5} + \frac{8ab^3(dx)^{13/2}}{13d^7} + \frac{2b^4(dx)^{17/2}}{17d^9}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.62

$$\frac{2(3315a^4x+2652a^3bx^3+2210a^2b^2x^5+1020ab^3x^7+195b^4x^9)}{3315\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*(3315*a^4*x + 2652*a^3*b*x^3 + 2210*a^2*b^2*x^5 + 1020*a*b^3*x^7 + 195*b
^4*x^9))/(3315*Sqrt[d*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 85, normalized size = 0.96

$$\frac{2\left(3315a^4d^8\sqrt{dx}+2652a^3bd^6(dx)^{5/2}+2210a^2b^2d^4(dx)^{9/2}+1020ab^3d^2(dx)^{13/2}+195b^4(dx)^{17/2}\right)}{3315d^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*(3315*a^4*d^8*Sqrt[d*x] + 2652*a^3*b*d^6*(d*x)^(5/2) + 2210*a^2*b^2*d^4*
(d*x)^(9/2) + 1020*a*b^3*d^2*(d*x)^(13/2) + 195*b^4*(d*x)^(17/2)))/(3315*d^
9)
```

fricas [A] time = 0.88, size = 53, normalized size = 0.60

$$\frac{2\left(195\,b^4x^8+1020\,ab^3x^6+2210\,a^2b^2x^4+2652\,a^3bx^2+3315\,a^4\right)\sqrt{dx}}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="fricas")
```

[Out] 2/3315*(195*b^4*x^8 + 1020*a*b^3*x^6 + 2210*a^2*b^2*x^4 + 2652*a^3*b*x^2 + 3315*a^4)*sqrt(d*x)/d

giac [A] time = 0.26, size = 73, normalized size = 0.82

$$\frac{2\left(195\sqrt{dx}\,b^4x^8 + 1020\sqrt{dx}\,ab^3x^6 + 2210\sqrt{dx}\,a^2b^2x^4 + 2652\sqrt{dx}\,a^3bx^2 + 3315\sqrt{dx}\,a^4\right)}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="giac")

[Out] 2/3315*(195*sqrt(d*x)*b^4*x^8 + 1020*sqrt(d*x)*a*b^3*x^6 + 2210*sqrt(d*x)*a^2*b^2*x^4 + 2652*sqrt(d*x)*a^3*b*x^2 + 3315*sqrt(d*x)*a^4)/d

maple [A] time = 0.01, size = 52, normalized size = 0.58

$$\frac{2\left(195b^4x^8 + 1020a\,b^3x^6 + 2210a^2b^2x^4 + 2652a^3b\,x^2 + 3315a^4\right)x}{3315\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x)

[Out] 2/3315*(195*b^4*x^8+1020*a*b^3*x^6+2210*a^2*b^2*x^4+2652*a^3*b*x^2+3315*a^4)*x/(d*x)^(1/2)

maxima [A] time = 1.36, size = 90, normalized size = 1.01

$$\frac{2\left(9945\sqrt{dx}\,a^4 + \frac{585\,(dx)^{\frac{17}{2}}\,b^4}{d^8} + \frac{3060\,(dx)^{\frac{13}{2}}\,ab^3}{d^6} + \frac{4420\,(dx)^{\frac{9}{2}}\,a^2b^2}{d^4} + 442\left(\frac{5\,(dx)^{\frac{9}{2}}\,b^2}{d^4} + \frac{18\,(dx)^{\frac{5}{2}}\,ab}{d^2}\right)a^2\right)}{9945\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="maxima")

[Out] 2/9945*(9945*sqrt(d*x)*a^4 + 585*(d*x)^(17/2)*b^4/d^8 + 3060*(d*x)^(13/2)*a*b^3/d^6 + 4420*(d*x)^(9/2)*a^2*b^2/d^4 + 442*(5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)*a^2)/d

mupad [B] time = 0.03, size = 71, normalized size = 0.80

$$\frac{2\,a^4\,\sqrt{d\,x}}{d} + \frac{2\,b^4\,{\left(d\,x\right)}^{17/2}}{17\,d^9} + \frac{4\,a^2\,b^2\,{\left(d\,x\right)}^{9/2}}{3\,d^5} + \frac{8\,a^3\,b\,{\left(d\,x\right)}^{5/2}}{5\,d^3} + \frac{8\,a\,b^3\,{\left(d\,x\right)}^{13/2}}{13\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(1/2),x)
```

```
[Out] (2*a^4*(d*x)^(1/2))/d + (2*b^4*(d*x)^(17/2))/(17*d^9) + (4*a^2*b^2*(d*x)^(9
/2))/(3*d^5) + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (8*a*b^3*(d*x)^(13/2))/(13*d
^7)
```

sympy [**A**] time = 1.35, size = 88, normalized size = 0.99

$$\frac{2a^4\sqrt{x}}{\sqrt{d}} + \frac{8a^3bx^{\frac{5}{2}}}{5\sqrt{d}} + \frac{4a^2b^2x^{\frac{9}{2}}}{3\sqrt{d}} + \frac{8ab^3x^{\frac{13}{2}}}{13\sqrt{d}} + \frac{2b^4x^{\frac{17}{2}}}{17\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] 2*a**4*sqrt(x)/sqrt(d) + 8*a**3*b*x**(5/2)/(5*sqrt(d)) + 4*a**2*b**2*x**(9/
2)/(3*sqrt(d)) + 8*a*b**3*x**(13/2)/(13*sqrt(d)) + 2*b**4*x**(17/2)/(17*sqr
t(d))
```

3.498 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2a^4}{d\sqrt{dx}}+\frac{8a^3b(dx)^{3/2}}{3d^3}+\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8a^3b(dx)^{3/2}}{3d^3}-\frac{2a^4}{d\sqrt{dx}}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(d*Sqrt[d*x]) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (12*a^2*b^2*(d*x)^
(7/2))/(7*d^5) + (8*a*b^3*(d*x)^(11/2))/(11*d^7) + (2*b^4*(d*x)^(15/2))/(15
*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{3/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{(dx)^{3/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{3/2}} + \frac{4a^3b^5\sqrt{dx}}{d^2} + \frac{6a^2b^6(dx)^{5/2}}{d^4} + \frac{4ab^7(dx)^{9/2}}{d^6} + \frac{b^8(dx)^{13/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{d\sqrt{dx}} + \frac{8a^3b(dx)^{3/2}}{3d^3} + \frac{12a^2b^2(dx)^{7/2}}{7d^5} + \frac{8ab^3(dx)^{11/2}}{11d^7} + \frac{2b^4(dx)^{15/2}}{15d^9}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.62

$$\frac{2x\left(-1155a^4+1540a^3bx^2+990a^2b^2x^4+420ab^3x^6+77b^4x^8\right)}{1155(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-1155*a^4 + 1540*a^3*b*x^2 + 990*a^2*b^2*x^4 + 420*a*b^3*x^6 + 77*b^4
*x^8))/(1155*(d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 72, normalized size = 0.81

$$\frac{2\left(-1155a^4d^8+1540a^3bd^8x^2+990a^2b^2d^8x^4+420ab^3d^8x^6+77b^4d^8x^8\right)}{1155d^9\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (2*(-1155*a^4*d^8 + 1540*a^3*b*d^8*x^2 + 990*a^2*b^2*d^8*x^4 + 420*a*b^3*d^
8*x^6 + 77*b^4*d^8*x^8))/(1155*d^9*Sqrt[d*x])
```

fricas [A] time = 1.19, size = 56, normalized size = 0.63

$$\frac{2\left(77\,b^4x^8+420\,ab^3x^6+990\,a^2b^2x^4+1540\,a^3bx^2-1155\,a^4\right)\sqrt{dx}}{1155\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="fricas")
```

[Out] 2/1155*(77*b^4*x^8 + 420*a*b^3*x^6 + 990*a^2*b^2*x^4 + 1540*a^3*b*x^2 - 1155*a^4)*sqrt(d*x)/(d^2*x)

giac [A] time = 0.18, size = 89, normalized size = 1.00

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}}-\frac{77\,\sqrt{dx}\,b^4d^{119}x^7+420\,\sqrt{dx}\,ab^3d^{119}x^5+990\,\sqrt{dx}\,a^2b^2d^{119}x^3+1540\,\sqrt{dx}\,a^3bd^{119}x}{d^{120}}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="giac")

[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*sqrt(d*x)*b^4*d^119*x^7 + 420*sqrt(d*x)*a*b^3*d^119*x^5 + 990*sqrt(d*x)*a^2*b^2*d^119*x^3 + 1540*sqrt(d*x)*a^3*b*d^119*x)/d^120)/d

maple [A] time = 0.01, size = 52, normalized size = 0.58

$$-\frac{2\left(-77b^4x^8-420a\,b^3x^6-990a^2b^2x^4-1540a^3b\,x^2+1155a^4\right)x}{1155\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x)

[Out] -2/1155*(-77*b^4*x^8-420*a*b^3*x^6-990*a^2*b^2*x^4-1540*a^3*b*x^2+1155*a^4)*x/(d*x)^(3/2)

maxima [A] time = 1.21, size = 76, normalized size = 0.85

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}}-\frac{77\,(dx)^{\frac{15}{2}}\,b^4+420\,(dx)^{\frac{11}{2}}\,ab^3d^2+990\,(dx)^{\frac{7}{2}}\,a^2b^2d^4+1540\,(dx)^{\frac{3}{2}}\,a^3bd^6}{d^8}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="maxima")

[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*(d*x)^(15/2)*b^4 + 420*(d*x)^(11/2)*a*b^3*d^2 + 990*(d*x)^(7/2)*a^2*b^2*d^4 + 1540*(d*x)^(3/2)*a^3*b*d^6)/d^8)/d

mupad [B] time = 0.03, size = 71, normalized size = 0.80

$$\frac{2\,b^4\,(d\,x)^{15/2}}{15\,d^9}-\frac{2\,a^4}{d\,\sqrt{d\,x}}+\frac{12\,a^2\,b^2\,(d\,x)^{7/2}}{7\,d^5}+\frac{8\,a^3\,b\,(d\,x)^{3/2}}{3\,d^3}+\frac{8\,a\,b^3\,(d\,x)^{11/2}}{11\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(3/2),x)
```

```
[Out] (2*b^4*(d*x)^(15/2))/(15*d^9) - (2*a^4)/(d*(d*x)^(1/2)) + (12*a^2*b^2*(d*x)
^(7/2))/(7*d^5) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (8*a*b^3*(d*x)^(11/2))/(1
1*d^7)
```

sympy [A] time = 1.38, size = 88, normalized size = 0.99

$$-\frac{2a^4}{d^{\frac{3}{2}}\sqrt{x}}+\frac{8a^3bx^{\frac{3}{2}}}{3d^{\frac{3}{2}}}+\frac{12a^2b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}+\frac{8ab^3x^{\frac{11}{2}}}{11d^{\frac{3}{2}}}+\frac{2b^4x^{\frac{15}{2}}}{15d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(3/2),x)
```

```
[Out] -2*a**4/(d**(3/2)*sqrt(x)) + 8*a**3*b*x**(3/2)/(3*d**(3/2)) + 12*a**2*b**2*
x**(7/2)/(7*d**(3/2)) + 8*a*b**3*x**(11/2)/(11*d**(3/2)) + 2*b**4*x**(15/2)
/(15*d**(3/2))
```

3.499 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8a^3b\sqrt{dx}}{d^3}-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*d*(d*x)^(3/2)) + (8*a^3*b*Sqrt[d*x])/d^3 + (12*a^2*b^2*(d*x)^(5
/2))/(5*d^5) + (8*a*b^3*(d*x)^(9/2))/(9*d^7) + (2*b^4*(d*x)^(13/2))/(13*d^9
)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{5/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{(dx)^{5/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{5/2}}+\frac{4a^3b^5}{d^2\sqrt{dx}}+\frac{6a^2b^6(dx)^{3/2}}{d^4}+\frac{4ab^7(dx)^{7/2}}{d^6}+\frac{b^8(dx)^{11/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.62

$$\frac{x\left(-390a^4+4680a^3bx^2+1404a^2b^2x^4+520ab^3x^6+90b^4x^8\right)}{585(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (x*(-390*a^4 + 4680*a^3*b*x^2 + 1404*a^2*b^2*x^4 + 520*a*b^3*x^6 + 90*b^4*x
^8))/(585*(d*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 72, normalized size = 0.81

$$\frac{2\left(-195a^4d^8+2340a^3bd^8x^2+702a^2b^2d^8x^4+260ab^3d^8x^6+45b^4d^8x^8\right)}{585d^9(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (2*(-195*a^4*d^8 + 2340*a^3*b*d^8*x^2 + 702*a^2*b^2*d^8*x^4 + 260*a*b^3*d^8
*x^6 + 45*b^4*d^8*x^8))/(585*d^9*(d*x)^(3/2))
```

fricas [A] time = 0.85, size = 56, normalized size = 0.63

$$\frac{2\left(45\,b^4x^8+260\,ab^3x^6+702\,a^2b^2x^4+2340\,a^3bx^2-195\,a^4\right)\sqrt{dx}}{585\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="fricas")
```

[Out] 2/585*(45*b^4*x^8 + 260*a*b^3*x^6 + 702*a^2*b^2*x^4 + 2340*a^3*b*x^2 - 195*a^4)*sqrt(d*x)/(d^3*x^2)

giac [A] time = 0.16, size = 92, normalized size = 1.03

$$-\frac{2\left(\frac{195\,a^4d}{\sqrt{dx}\,x}-\frac{45\,\sqrt{dx}\,b^4d^{78}x^6+260\,\sqrt{dx}\,ab^3d^{78}x^4+702\,\sqrt{dx}\,a^2b^2d^{78}x^2+2340\,\sqrt{dx}\,a^3bd^{78}}{d^{78}}\right)}{585\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="giac")

[Out] -2/585*(195*a^4*d/(sqrt(d*x)*x) - (45*sqrt(d*x)*b^4*d^78*x^6 + 260*sqrt(d*x)*a*b^3*d^78*x^4 + 702*sqrt(d*x)*a^2*b^2*d^78*x^2 + 2340*sqrt(d*x)*a^3*b*d^78)/d^78)/d^3

maple [A] time = 0.01, size = 52, normalized size = 0.58

$$-\frac{2\left(-45b^4x^8-260a\,b^3x^6-702a^2b^2x^4-2340a^3b\,x^2+195a^4\right)x}{585\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x)

[Out] -2/585*(-45*b^4*x^8-260*a*b^3*x^6-702*a^2*b^2*x^4-2340*a^3*b*x^2+195*a^4)*x/(d*x)^(5/2)

maxima [A] time = 1.35, size = 76, normalized size = 0.85

$$-\frac{2\left(\frac{195\,a^4}{(dx)^{\frac{3}{2}}}-\frac{45\,(dx)^{\frac{13}{2}}\,b^4+260\,(dx)^{\frac{9}{2}}\,ab^3d^2+702\,(dx)^{\frac{5}{2}}\,a^2b^2d^4+2340\,\sqrt{dx}\,a^3bd^6}{d^8}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="maxima")

[Out] -2/585*(195*a^4/(d*x)^(3/2) - (45*(d*x)^(13/2)*b^4 + 260*(d*x)^(9/2)*a*b^3*d^2 + 702*(d*x)^(5/2)*a^2*b^2*d^4 + 2340*sqrt(d*x)*a^3*b*d^6)/d^8)/d

mupad [B] time = 0.03, size = 71, normalized size = 0.80

$$\frac{2\,b^4\,{\left(d\,x\right)}^{13/2}}{13\,d^9}-\frac{2\,a^4}{3\,d\,{\left(d\,x\right)}^{3/2}}+\frac{12\,a^2\,b^2\,{\left(d\,x\right)}^{5/2}}{5\,d^5}+\frac{8\,a^3\,b\,\sqrt{d\,x}}{d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{9/2}}{9\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(5/2),x)
```

```
[Out] (2*b^4*(d*x)^(13/2))/(13*d^9) - (2*a^4)/(3*d*(d*x)^(3/2)) + (12*a^2*b^2*(d*
x)^(5/2))/(5*d^5) + (8*a^3*b*(d*x)^(1/2))/d^3 + (8*a*b^3*(d*x)^(9/2))/(9*d^
7)
```

sympy [A] time = 1.74, size = 88, normalized size = 0.99

$$-\frac{2a^4}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{8a^3b\sqrt{x}}{d^{\frac{5}{2}}}+\frac{12a^2b^2x^{\frac{5}{2}}}{5d^{\frac{5}{2}}}+\frac{8ab^3x^{\frac{9}{2}}}{9d^{\frac{5}{2}}}+\frac{2b^4x^{\frac{13}{2}}}{13d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(5/2),x)
```

```
[Out] -2*a**4/(3*d**(5/2)*x**(3/2)) + 8*a**3*b*sqrt(x)/d**(5/2) + 12*a**2*b**2*x*
*(5/2)/(5*d**(5/2)) + 8*a*b**3*x**(9/2)/(9*d**(5/2)) + 2*b**4*x**(13/2)/(13
*d**(5/2))
```

3.500 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{4a^2b^2(dx)^{3/2}}{d^5}-\frac{8a^3b}{d^3\sqrt{dx}}-\frac{2a^4}{5d(dx)^{5/2}}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (-2*a^4)/(5*d*(d*x)^(5/2)) - (8*a^3*b)/(d^3*Sqrt[d*x]) + (4*a^2*b^2*(d*x)^(
3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7) + (2*b^4*(d*x)^(11/2))/(11*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{7/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{(dx)^{7/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{7/2}} + \frac{4a^3b^5}{d^2(dx)^{3/2}} + \frac{6a^2b^6\sqrt{dx}}{d^4} + \frac{4ab^7(dx)^{5/2}}{d^6} + \frac{b^8(dx)^{9/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{5d(dx)^{5/2}} - \frac{8a^3b}{d^3\sqrt{dx}} + \frac{4a^2b^2(dx)^{3/2}}{d^5} + \frac{8ab^3(dx)^{7/2}}{7d^7} + \frac{2b^4(dx)^{11/2}}{11d^9}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.69

$$\frac{2\sqrt{dx}\left(-77a^4-1540a^3bx^2+770a^2b^2x^4+220ab^3x^6+35b^4x^8\right)}{385d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-77*a^4 - 1540*a^3*b*x^2 + 770*a^2*b^2*x^4 + 220*a*b^3*x^6 +
35*b^4*x^8))/(385*d^4*x^3)
```

IntegrateAlgebraic [A] time = 0.06, size = 72, normalized size = 0.83

$$\frac{2\left(-77a^4d^8-1540a^3bd^8x^2+770a^2b^2d^8x^4+220ab^3d^8x^6+35b^4d^8x^8\right)}{385d^9(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (2*(-77*a^4*d^8 - 1540*a^3*b*d^8*x^2 + 770*a^2*b^2*d^8*x^4 + 220*a*b^3*d^8*
x^6 + 35*b^4*d^8*x^8))/(385*d^9*(d*x)^(5/2))
```

fricas [A] time = 1.81, size = 56, normalized size = 0.64

$$\frac{2\left(35\,b^4x^8+220\,ab^3x^6+770\,a^2b^2x^4-1540\,a^3bx^2-77\,a^4\right)\sqrt{dx}}{385\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/385*(35*b^4*x^8 + 220*a*b^3*x^6 + 770*a^2*b^2*x^4 - 1540*a^3*b*x^2 - 77*a
^4)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.18, size = 95, normalized size = 1.09

$$-\frac{2\left(\frac{77\left(20\,a^{3}bd^{3}x^{2}+a^{4}d^{3}\right)}{\sqrt{dx}\,d^{2}x^{2}}-\frac{5\left(7\,\sqrt{dx}\,b^{4}d^{55}x^{5}+44\,\sqrt{dx}\,ab^{3}d^{55}x^{3}+154\,\sqrt{dx}\,a^{2}b^{2}d^{55}x\right)}{d^{55}}\right)}{385\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/385*(77*(20*a^3*b*d^3*x^2 + a^4*d^3)/(sqrt(d*x)*d^2*x^2) - 5*(7*sqrt(d*x
)*b^4*d^55*x^5 + 44*sqrt(d*x)*a*b^3*d^55*x^3 + 154*sqrt(d*x)*a^2*b^2*d^55*x
)/d^55)/d^4
```

maple [A] time = 0.01, size = 52, normalized size = 0.60

$$-\frac{2\left(-35b^{4}x^{8}-220a\,b^{3}x^{6}-770a^{2}b^{2}x^{4}+1540a^{3}b\,x^{2}+77a^{4}\right)x}{385\left(dx\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x)
```

```
[Out] -2/385*(-35*b^4*x^8-220*a*b^3*x^6-770*a^2*b^2*x^4+1540*a^3*b*x^2+77*a^4)*x/
(d*x)^(7/2)
```

maxima [A] time = 1.44, size = 82, normalized size = 0.94

$$-\frac{2\left(\frac{77\left(20\,a^{3}bd^{2}x^{2}+a^{4}d^{2}\right)}{\left(dx\right)^{\frac{5}{2}}d^{2}}-\frac{5\left(7\left(dx\right)^{\frac{11}{2}}b^{4}+44\left(dx\right)^{\frac{7}{2}}ab^{3}d^{2}+154\left(dx\right)^{\frac{3}{2}}a^{2}b^{2}d^{4}\right)}{d^{8}}\right)}{385\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/385*(77*(20*a^3*b*d^2*x^2 + a^4*d^2)/((d*x)^(5/2)*d^2) - 5*(7*(d*x)^(11/
2)*b^4 + 44*(d*x)^(7/2)*a*b^3*d^2 + 154*(d*x)^(3/2)*a^2*b^2*d^4)/d^8)/d
```

mupad [B] time = 0.06, size = 75, normalized size = 0.86

$$\frac{2\,b^4\,{\left(d\,x\right)}^{11/2}}{11\,d^9}-\frac{\frac{2\,a^4\,d^2}{5}+8\,b\,a^3\,d^2\,x^2}{d^3\,{\left(d\,x\right)}^{5/2}}+\frac{4\,a^2\,b^2\,{\left(d\,x\right)}^{3/2}}{d^5}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{7/2}}{7\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(7/2),x)
```

```
[Out] (2*b^4*(d*x)^(11/2))/(11*d^9) - ((2*a^4*d^2)/5 + 8*a^3*b*d^2*x^2)/(d^3*(d*x
)^(5/2)) + (4*a^2*b^2*(d*x)^(3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7)
```

sympy [A] time = 2.47, size = 87, normalized size = 1.00

$$-\frac{2a^4}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{8a^3b}{d^{\frac{7}{2}}\sqrt{x}}+\frac{4a^2b^2x^{\frac{3}{2}}}{d^{\frac{7}{2}}}+\frac{8ab^3x^{\frac{7}{2}}}{7d^{\frac{7}{2}}}+\frac{2b^4x^{\frac{11}{2}}}{11d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(7/2),x)
```

```
[Out] -2*a**4/(5*d**(7/2)*x**(5/2)) - 8*a**3*b/(d**(7/2)*sqrt(x)) + 4*a**2*b**2*x
**(3/2)/d**(7/2) + 8*a*b**3*x**(7/2)/(7*d**(7/2)) + 2*b**4*x**(11/2)/(11*d*
*(7/2))
```

3.501 $\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=129

$$\frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^6(dx)^{7/2}}{7d}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (2*a^4*b^2*(
d*x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)^(
23/2))/(23*d^9) + (4*a*b^5*(d*x)^(27/2))/(9*d^11) + (2*b^6*(d*x)^(31/2))/(3
1*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{\int (dx)^{5/2}\left(ab+b^2x^2\right)^6\,dx}{b^6}$$

$$=\frac{\int\left(a^6b^6(dx)^{5/2}+\frac{6a^5b^7(dx)^{9/2}}{d^2}+\frac{15a^4b^8(dx)^{13/2}}{d^4}+\frac{20a^3b^9(dx)^{17/2}}{d^6}+\frac{15a^2b^{10}(dx)^{21/2}}{d^8}+6\right.}{b^6}$$

$$=\frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx}{23d^9}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.60

$$\frac{2x(dx)^{5/2}\left(1341153a^6+5120766a^5bx^2+9388071a^4b^2x^4+9882180a^3b^3x^6+6122655a^2b^4x^8+2086238ab^5x^{10}+302841b^6x^{12}\right)}{9388071}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(1341153*a^6 + 5120766*a^5*b*x^2 + 9388071*a^4*b^2*x^4 + 9
882180*a^3*b^3*x^6 + 6122655*a^2*b^4*x^8 + 2086238*a*b^5*x^10 + 302841*b^6*
x^12))/9388071
```

IntegrateAlgebraic [A] time = 0.07, size = 121, normalized size = 0.94

$$\frac{2\left(1341153a^6d^{12}(dx)^{7/2}+5120766a^5bd^{10}(dx)^{11/2}+9388071a^4b^2d^8(dx)^{15/2}+9882180a^3b^3d^6(dx)^{19/2}+6122655a^2b^4d^4(dx)^{23/2}+2086238ab^5d^2(dx)^{27/2}+302841b^6(dx)^{31/2}\right)}{9388071d^{13}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*(1341153*a^6*d^12*(d*x)^(7/2) + 5120766*a^5*b*d^10*(d*x)^(11/2) + 938807
1*a^4*b^2*d^8*(d*x)^(15/2) + 9882180*a^3*b^3*d^6*(d*x)^(19/2) + 6122655*a^2
*b^4*d^4*(d*x)^(23/2) + 2086238*a*b^5*d^2*(d*x)^(27/2) + 302841*b^6*(d*x)^(
31/2)))/(9388071*d^13)
```

fricas [A] time = 1.71, size = 96, normalized size = 0.74

$$\frac{2}{9388071}\left(302841\,b^6d^2x^{15}+2086238\,ab^5d^2x^{13}+6122655\,a^2b^4d^2x^{11}+9882180\,a^3b^3d^2x^9+9388071\,a^4b^2d^2x^7+5120766\,a^5bd^2x^5+1341153\,a^6d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/9388071*(302841*b^6*d^2*x^15 + 2086238*a*b^5*d^2*x^13 + 6122655*a^2*b^4*d
^2*x^11 + 9882180*a^3*b^3*d^2*x^9 + 9388071*a^4*b^2*d^2*x^7 + 5120766*a^5*b
*d^2*x^5 + 1341153*a^6*d^2*x^3)*sqrt(d*x)
```

giac [A] time = 0.16, size = 124, normalized size = 0.96

$$\frac{2}{31}\sqrt{dx}\,b^6d^2x^{15}+\frac{4}{9}\sqrt{dx}\,ab^5d^2x^{13}+\frac{30}{23}\sqrt{dx}\,a^2b^4d^2x^{11}+\frac{40}{19}\sqrt{dx}\,a^3b^3d^2x^9+2\sqrt{dx}\,a^4b^2d^2x^7+\frac{12}{11}\sqrt{dx}\,a^5bd^2x^5+\frac{2}{7}\sqrt{dx}\,a^6d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/31*sqrt(d*x)*b^6*d^2*x^15 + 4/9*sqrt(d*x)*a*b^5*d^2*x^13 + 30/23*sqrt(d*x
)*a^2*b^4*d^2*x^11 + 40/19*sqrt(d*x)*a^3*b^3*d^2*x^9 + 2*sqrt(d*x)*a^4*b^2*
d^2*x^7 + 12/11*sqrt(d*x)*a^5*b*d^2*x^5 + 2/7*sqrt(d*x)*a^6*d^2*x^3
```

maple [A] time = 0.01, size = 74, normalized size = 0.57

$$\frac{2\left(302841b^{6}x^{12}+2086238a\,b^{5}x^{10}+6122655a^{2}b^{4}x^{8}+9882180a^{3}b^{3}x^{6}+9388071a^{4}b^{2}x^{4}+5120766a^{5}b\,x^{2}+1341153a^{6}\right)(dx)^{\frac{5}{2}}x}{9388071}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/9388071*x*(302841*b^6*x^12+2086238*a*b^5*x^10+6122655*a^2*b^4*x^8+9882180
*a^3*b^3*x^6+9388071*a^4*b^2*x^4+5120766*a^5*b*x^2+1341153*a^6)*(d*x)^(5/2)
```

maxima [A] time = 1.40, size = 105, normalized size = 0.81

$$\frac{2\left(302841\,(dx)^{\frac{31}{2}}b^{6}+2086238\,(dx)^{\frac{27}{2}}ab^{5}d^{2}+6122655\,(dx)^{\frac{23}{2}}a^{2}b^{4}d^{4}+9882180\,(dx)^{\frac{19}{2}}a^{3}b^{3}d^{6}+9388071\,(dx)^{\frac{15}{2}}a^{4}b^{2}d^{8}+5120766\,(dx)^{\frac{11}{2}}a^{5}bd^{10}+1341153\,(dx)^{\frac{7}{2}}a^{6}d^{12}\right)}{9388071\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/9388071*(302841*(d*x)^(31/2)*b^6 + 2086238*(d*x)^(27/2)*a*b^5*d^2 + 61226
55*(d*x)^(23/2)*a^2*b^4*d^4 + 9882180*(d*x)^(19/2)*a^3*b^3*d^6 + 9388071*(d
*x)^(15/2)*a^4*b^2*d^8 + 5120766*(d*x)^(11/2)*a^5*b*d^10 + 1341153*(d*x)^(7
/2)*a^6*d^12)/d^13
```

mupad [B] time = 0.04, size = 103, normalized size = 0.80

$$\frac{2\,a^{6}\,(d\,x)^{7/2}}{7\,d}+\frac{2\,b^{6}\,(d\,x)^{31/2}}{31\,d^{13}}+\frac{2\,a^{4}\,b^{2}\,(d\,x)^{15/2}}{d^{5}}+\frac{40\,a^{3}\,b^{3}\,(d\,x)^{19/2}}{19\,d^{7}}+\frac{30\,a^{2}\,b^{4}\,(d\,x)^{23/2}}{23\,d^{9}}+\frac{12\,a^{5}\,b\,(d\,x)^{11/2}}{11\,d^{3}}+\frac{4\,a\,b^{5}\,(d\,x)^{27/2}}{9\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (2*b^6*(d*x)^(31/2))/(31*d^13) + (2*a^4*b^2*(d*
x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)^(23
/2))/(23*d^9) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (4*a*b^5*(d*x)^(27/2))/(
9*d^11)
```

sympy [A] time = 10.80, size = 129, normalized size = 1.00

$$\frac{2a^{6}d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{12a^{5}bd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+2a^{4}b^{2}d^{\frac{5}{2}}x^{\frac{15}{2}}+\frac{40a^{3}b^{3}d^{\frac{5}{2}}x^{\frac{19}{2}}}{19}+\frac{30a^{2}b^{4}d^{\frac{5}{2}}x^{\frac{23}{2}}}{23}+\frac{4ab^{5}d^{\frac{5}{2}}x^{\frac{27}{2}}}{9}+\frac{2b^{6}d^{\frac{5}{2}}x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*d**(5/2)*x**(7/2)/7 + 12*a**5*b*d**(5/2)*x**(11/2)/11 + 2*a**4*b**2*
d**(5/2)*x**(15/2) + 40*a**3*b**3*d**(5/2)*x**(19/2)/19 + 30*a**2*b**4*d**(
5/2)*x**(23/2)/23 + 4*a*b**5*d**(5/2)*x**(27/2)/9 + 2*b**6*d**(5/2)*x**(31/
2)/31
```

3.502 $\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=131

$$\frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{2a^6(dx)^{5/2}}{5d}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (30*a^4*b^2*(d*
x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(d*x
)^(21/2))/(7*d^9) + (12*a*b^5*(d*x)^(25/2))/(25*d^11) + (2*b^6*(d*x)^(29/2)
)/(29*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int (dx)^{3/2}\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6(dx)^{3/2}+\frac{6a^5b^7(dx)^{7/2}}{d^2}+\frac{15a^4b^8(dx)^{11/2}}{d^4}+\frac{20a^3b^9(dx)^{15/2}}{d^6}+\frac{15a^2b^{10}(dx)^{19/2}}{d^8}+\frac{6}{}\right.}{b^6}\\ &= \frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{}}{7d^9}\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.59

$$\frac{2x(dx)^{3/2}\left(672945a^6+2243150a^5bx^2+3882375a^4b^2x^4+3958500a^3b^3x^6+2403375a^2b^4x^8+807534ab^5x^{10}+116025b^6x^{12}\right)}{3364725}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(672945*a^6 + 2243150*a^5*b*x^2 + 3882375*a^4*b^2*x^4 + 39
58500*a^3*b^3*x^6 + 2403375*a^2*b^4*x^8 + 807534*a*b^5*x^10 + 116025*b^6*x^
12))/3364725
```

IntegrateAlgebraic [A] time = 0.06, size = 121, normalized size = 0.92

$$\frac{2\left(672945a^6d^{12}(dx)^{5/2}+2243150a^5bd^{10}(dx)^{9/2}+3882375a^4b^2d^8(dx)^{13/2}+3958500a^3b^3d^6(dx)^{17/2}+2403375a^2b^4d^4(dx)^{21/2}+807534ab^5d^2(dx)^{25/2}+116025b^6(dx)^{29/2}\right)}{3364725d^{13}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*(672945*a^6*d^12*(d*x)^(5/2) + 2243150*a^5*b*d^10*(d*x)^(9/2) + 3882375*
a^4*b^2*d^8*(d*x)^(13/2) + 3958500*a^3*b^3*d^6*(d*x)^(17/2) + 2403375*a^2*b
^4*d^4*(d*x)^(21/2) + 807534*a*b^5*d^2*(d*x)^(25/2) + 116025*b^6*(d*x)^(29/
2)))/(3364725*d^13)
```

fricas [A] time = 0.84, size = 82, normalized size = 0.63

$$\frac{2}{3364725}\left(116025\,b^6dx^{14}+807534\,ab^5dx^{12}+2403375\,a^2b^4dx^{10}+3958500\,a^3b^3dx^8+3882375\,a^4b^2dx^6+2243150\,a^5bdx^4+672945\,a^6dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*b^6*d*x^14 + 807534*a*b^5*d*x^12 + 2403375*a^2*b^4*d*x^10
 + 3958500*a^3*b^3*d*x^8 + 3882375*a^4*b^2*d*x^6 + 2243150*a^5*b*d*x^4 + 67
2945*a^6*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.19, size = 106, normalized size = 0.81

$$\frac{2}{3364725}\left(116025\,\sqrt{dx}\,b^6x^{14}+807534\,\sqrt{dx}\,ab^5x^{12}+2403375\,\sqrt{dx}\,a^2b^4x^{10}+3958500\,\sqrt{dx}\,a^3b^3x^8+3882375\,\sqrt{dx}\,a^4b^2x^6+2243150\,\sqrt{dx}\,a^5bx^4+672945\,\sqrt{dx}\,a^6x^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/3364725*(116025*sqrt(d*x)*b^6*x^14 + 807534*sqrt(d*x)*a*b^5*x^12 + 240337
5*sqrt(d*x)*a^2*b^4*x^10 + 3958500*sqrt(d*x)*a^3*b^3*x^8 + 3882375*sqrt(d*x
)*a^4*b^2*x^6 + 2243150*sqrt(d*x)*a^5*b*x^4 + 672945*sqrt(d*x)*a^6*x^2)*d
```

maple [A] time = 0.01, size = 74, normalized size = 0.56

$$\frac{2\left(116025b^6x^{12}+807534a\,b^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b^2x^4+2243150a^5b\,x^2+672945a^6\right)(dx)^{\frac{3}{2}}x}{3364725}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/3364725*x*(116025*b^6*x^12+807534*a*b^5*x^10+2403375*a^2*b^4*x^8+3958500*
a^3*b^3*x^6+3882375*a^4*b^2*x^4+2243150*a^5*b*x^2+672945*a^6)*(d*x)^(3/2)
```

maxima [A] time = 1.33, size = 105, normalized size = 0.80

$$\frac{2\left(116025\,(dx)^{\frac{29}{2}}b^6+807534\,(dx)^{\frac{25}{2}}ab^5d^2+2403375\,(dx)^{\frac{21}{2}}a^2b^4d^4+3958500\,(dx)^{\frac{17}{2}}a^3b^3d^6+3882375\,(dx)^{\frac{13}{2}}a^4b^2d^8+2243150\,(dx)^{\frac{9}{2}}a^5bd^{10}+672945\,(dx)^{\frac{5}{2}}a^6d^{12}\right)}{3364725\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/3364725*(116025*(d*x)^(29/2)*b^6 + 807534*(d*x)^(25/2)*a*b^5*d^2 + 240337
5*(d*x)^(21/2)*a^2*b^4*d^4 + 3958500*(d*x)^(17/2)*a^3*b^3*d^6 + 3882375*(d*
x)^(13/2)*a^4*b^2*d^8 + 2243150*(d*x)^(9/2)*a^5*b*d^10 + 672945*(d*x)^(5/2)
*a^6*d^12)/d^13
```

mupad [B] time = 0.04, size = 103, normalized size = 0.79

$$\frac{2\,a^6\,{\left(d\,x\right)}^{5/2}}{5\,d}+\frac{2\,b^6\,{\left(d\,x\right)}^{29/2}}{29\,d^{13}}+\frac{30\,a^4\,b^2\,{\left(d\,x\right)}^{13/2}}{13\,d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{17/2}}{17\,d^7}+\frac{10\,a^2\,b^4\,{\left(d\,x\right)}^{21/2}}{7\,d^9}+\frac{4\,a^5\,b\,{\left(d\,x\right)}^{9/2}}{3\,d^3}+\frac{12\,a\,b^5\,{\left(d\,x\right)}^{25/2}}{25\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (2*b^6*(d*x)^(29/2))/(29*d^13) + (30*a^4*b^2*(d
*x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(d*
x)^(21/2))/(7*d^9) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (12*a*b^5*(d*x)^(25/2)
)/(25*d^11)
```

sympy [A] time = 5.34, size = 131, normalized size = 1.00

$$\frac{2a^6d^{\frac{3}{2}}x^{\frac{5}{2}}}{5}+\frac{4a^5bd^{\frac{3}{2}}x^{\frac{9}{2}}}{3}+\frac{30a^4b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}+\frac{40a^3b^3d^{\frac{3}{2}}x^{\frac{17}{2}}}{17}+\frac{10a^2b^4d^{\frac{3}{2}}x^{\frac{21}{2}}}{7}+\frac{12ab^5d^{\frac{3}{2}}x^{\frac{25}{2}}}{25}+\frac{2b^6d^{\frac{3}{2}}x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*d**(3/2)*x**(5/2)/5 + 4*a**5*b*d**(3/2)*x**(9/2)/3 + 30*a**4*b**2*d*
*(3/2)*x**(13/2)/13 + 40*a**3*b**3*d**(3/2)*x**(17/2)/17 + 10*a**2*b**4*d**
(3/2)*x**(21/2)/7 + 12*a*b**5*d**(3/2)*x**(25/2)/25 + 2*b**6*d**(3/2)*x**(2
9/2)/29
```

3.503 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=131

$$\frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{2a^6(dx)^{3/2}}{3d}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (30*a^4*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*x)
^(19/2))/(19*d^9) + (12*a*b^5*(d*x)^(23/2))/(23*d^11) + (2*b^6*(d*x)^(27/2)
)/(27*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int \sqrt{dx}\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6\sqrt{dx}+\frac{6a^5b^7(dx)^{5/2}}{d^2}+\frac{15a^4b^8(dx)^{9/2}}{d^4}+\frac{20a^3b^9(dx)^{13/2}}{d^6}+\frac{15a^2b^{10}(dx)^{17/2}}{d^8}+\frac{6ab^{11}(}{d}\right.}{b^6}\\ &= \frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19}}{19d^9}\end{aligned}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.59

$$\frac{2x\sqrt{dx}\left(302841a^6+778734a^5bx^2+1238895a^4b^2x^4+1211364a^3b^3x^6+717255a^2b^4x^8+237006ab^5x^{10}+33649b^6x^{12}\right)}{908523}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*Sqrt[d*x]*(302841*a^6 + 778734*a^5*b*x^2 + 1238895*a^4*b^2*x^4 + 12113
64*a^3*b^3*x^6 + 717255*a^2*b^4*x^8 + 237006*a*b^5*x^10 + 33649*b^6*x^12))/
908523
```

IntegrateAlgebraic [A] time = 0.06, size = 121, normalized size = 0.92

$$\frac{2\left(302841a^6d^{12}(dx)^{3/2}+778734a^5bd^{10}(dx)^{7/2}+1238895a^4b^2d^8(dx)^{11/2}+1211364a^3b^3d^6(dx)^{15/2}+717255a^2b^4d^4(dx)^{19/2}+237006ab^5d^2(dx)^{23/2}+33649b^6(dx)^{27/2}\right)}{908523d^{13}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*(302841*a^6*d^12*(d*x)^(3/2) + 778734*a^5*b*d^10*(d*x)^(7/2) + 1238895*a
^4*b^2*d^8*(d*x)^(11/2) + 1211364*a^3*b^3*d^6*(d*x)^(15/2) + 717255*a^2*b^4
*d^4*(d*x)^(19/2) + 237006*a*b^5*d^2*(d*x)^(23/2) + 33649*b^6*(d*x)^(27/2))
)/(908523*d^13)
```

fricas [A] time = 2.07, size = 73, normalized size = 0.56

$$\frac{2}{908523}\left(33649\,b^6x^{13}+237006\,ab^5x^{11}+717255\,a^2b^4x^9+1211364\,a^3b^3x^7+1238895\,a^4b^2x^5+778734\,a^5bx^3+302841\,a^6x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^6*x^13 + 237006*a*b^5*x^11 + 717255*a^2*b^4*x^9 + 1211364
*a^3*b^3*x^7 + 1238895*a^4*b^2*x^5 + 778734*a^5*b*x^3 + 302841*a^6*x)*sqrt(
d*x)
```

giac [A] time = 0.18, size = 101, normalized size = 0.77

$$\frac{2}{27}\sqrt{dx}\,b^6x^{13}+\frac{12}{23}\sqrt{dx}\,ab^5x^{11}+\frac{30}{19}\sqrt{dx}\,a^2b^4x^9+\frac{8}{3}\sqrt{dx}\,a^3b^3x^7+\frac{30}{11}\sqrt{dx}\,a^4b^2x^5+\frac{12}{7}\sqrt{dx}\,a^5bx^3+\frac{2}{3}\sqrt{dx}\,a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(d*x)*b^6*x^13 + 12/23*sqrt(d*x)*a*b^5*x^11 + 30/19*sqrt(d*x)*a^2*
b^4*x^9 + 8/3*sqrt(d*x)*a^3*b^3*x^7 + 30/11*sqrt(d*x)*a^4*b^2*x^5 + 12/7*sq
rt(d*x)*a^5*b*x^3 + 2/3*sqrt(d*x)*a^6*x
```

maple [A] time = 0.01, size = 74, normalized size = 0.56

$$\frac{2\left(33649b^{6}x^{12}+237006a\,b^{5}x^{10}+717255a^{2}b^{4}x^{8}+1211364a^{3}b^{3}x^{6}+1238895a^{4}b^{2}x^{4}+778734a^{5}b\,x^{2}+302841a^{6}\right)\sqrt{dx}\;x}{908523}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x)
```

```
[Out] 2/908523*x*(33649*b^6*x^12+237006*a*b^5*x^10+717255*a^2*b^4*x^8+1211364*a^3
*b^3*x^6+1238895*a^4*b^2*x^4+778734*a^5*b*x^2+302841*a^6)*(d*x)^(1/2)
```

maxima [A] time = 1.39, size = 105, normalized size = 0.80

$$\frac{2\left(33649\,\left(dx\right)^{\frac{27}{2}}b^{6}+237006\,\left(dx\right)^{\frac{23}{2}}ab^{5}d^{2}+717255\,\left(dx\right)^{\frac{19}{2}}a^{2}b^{4}d^{4}+1211364\,\left(dx\right)^{\frac{15}{2}}a^{3}b^{3}d^{6}+1238895\,\left(dx\right)^{\frac{11}{2}}a^{4}b^{2}d^{8}+778734\,\left(dx\right)^{\frac{7}{2}}a^{5}bd^{10}+302841\,\left(dx\right)^{\frac{3}{2}}a^{6}d^{12}\right)}{908523\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/908523*(33649*(d*x)^(27/2)*b^6 + 237006*(d*x)^(23/2)*a*b^5*d^2 + 717255*(
d*x)^(19/2)*a^2*b^4*d^4 + 1211364*(d*x)^(15/2)*a^3*b^3*d^6 + 1238895*(d*x)^
(11/2)*a^4*b^2*d^8 + 778734*(d*x)^(7/2)*a^5*b*d^10 + 302841*(d*x)^(3/2)*a^6
*d^12)/d^13
```

mupad [B] time = 0.04, size = 103, normalized size = 0.79

$$\frac{2\,a^6\,{\left(d\,x\right)}^{3/2}}{3\,d}+\frac{2\,b^6\,{\left(d\,x\right)}^{27/2}}{27\,d^{13}}+\frac{30\,a^4\,b^2\,{\left(d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,a^3\,b^3\,{\left(d\,x\right)}^{15/2}}{3\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{19/2}}{19\,d^9}+\frac{12\,a^5\,b\,{\left(d\,x\right)}^{7/2}}{7\,d^3}+\frac{12\,a\,b^5\,{\left(d\,x\right)}^{23/2}}{23\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (2*b^6*(d*x)^(27/2))/(27*d^13) + (30*a^4*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*x)
^(19/2))/(19*d^9) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (12*a*b^5*(d*x)^(23/2)
)/(23*d^11)
```

sympy [A] time = 3.00, size = 131, normalized size = 1.00

$$\frac{2a^{6}\sqrt{d}\,x^{\frac{3}{2}}}{3}+\frac{12a^{5}b\sqrt{d}\,x^{\frac{7}{2}}}{7}+\frac{30a^{4}b^{2}\sqrt{d}\,x^{\frac{11}{2}}}{11}+\frac{8a^{3}b^{3}\sqrt{d}\,x^{\frac{15}{2}}}{3}+\frac{30a^{2}b^{4}\sqrt{d}\,x^{\frac{19}{2}}}{19}+\frac{12ab^{5}\sqrt{d}\,x^{\frac{23}{2}}}{23}+\frac{2b^{6}\sqrt{d}\,x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3*(d*x)**(1/2),x)
```

```
[Out] 2*a**6*sqrt(d)*x**(3/2)/3 + 12*a**5*b*sqrt(d)*x**(7/2)/7 + 30*a**4*b**2*sqr
t(d)*x**(11/2)/11 + 8*a**3*b**3*sqrt(d)*x**(15/2)/3 + 30*a**2*b**4*sqrt(d)*
x**(19/2)/19 + 12*a*b**5*sqrt(d)*x**(23/2)/23 + 2*b**6*sqrt(d)*x**(27/2)/27
```

3.504 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{\sqrt{dx}}\,dx$

Optimal. Leaf size=129

$$\frac{2a^6\sqrt{dx}}{d}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{2a^6\sqrt{dx}}{d}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*a^6*Sqrt[d*x])/d + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (10*a^4*b^2*(d*x)^(9
/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(17/2
))/(17*d^9) + (4*a*b^5*(d*x)^(21/2))/(7*d^11) + (2*b^6*(d*x)^(25/2))/(25*d^
13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{\sqrt{dx}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{\sqrt{dx}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{\sqrt{dx}} + \frac{6a^5b^7(dx)^{3/2}}{d^2} + \frac{15a^4b^8(dx)^{7/2}}{d^4} + \frac{20a^3b^9(dx)^{11/2}}{d^6} + \frac{15a^2b^{10}(dx)^{15/2}}{d^8} + \frac{6ab^{11}(dx)^{19/2}}{d^{10}} + \frac{b^{12}(}{}\right.}{b^6}$$
$$= \frac{2a^6\sqrt{dx}}{d} + \frac{12a^5b(dx)^{5/2}}{5d^3} + \frac{10a^4b^2(dx)^{9/2}}{3d^5} + \frac{40a^3b^3(dx)^{13/2}}{13d^7} + \frac{30a^2b^4(dx)^{17/2}}{17d^9} + \frac{4ab}{}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.60

$$\frac{2\left(116025a^6x+139230a^5bx^3+193375a^4b^2x^5+178500a^3b^3x^7+102375a^2b^4x^9+33150ab^5x^{11}+4641b^6x^{13}\right)}{116025\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*(116025*a^6*x + 139230*a^5*b*x^3 + 193375*a^4*b^2*x^5 + 178500*a^3*b^3*x
^7 + 102375*a^2*b^4*x^9 + 33150*a*b^5*x^11 + 4641*b^6*x^13))/(116025*Sqrt[d
*x])
```

IntegrateAlgebraic [A] time = 0.06, size = 121, normalized size = 0.94

$$\frac{2\left(116025a^6d^{12}\sqrt{dx}+139230a^5bd^{10}(dx)^{5/2}+193375a^4b^2d^8(dx)^{9/2}+178500a^3b^3d^6(dx)^{13/2}+102375a^2b^4d^4(dx)^{17/2}+33150ab^5d^2(dx)^{21/2}+4641b^6(dx)^{25/2}\right)}{116025d^{13}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*(116025*a^6*d^12*Sqrt[d*x] + 139230*a^5*b*d^10*(d*x)^(5/2) + 193375*a^4*
b^2*d^8*(d*x)^(9/2) + 178500*a^3*b^3*d^6*(d*x)^(13/2) + 102375*a^2*b^4*d^4*
(d*x)^(17/2) + 33150*a*b^5*d^2*(d*x)^(21/2) + 4641*b^6*(d*x)^(25/2)))/(1160
25*d^13)
```

fricas [A] time = 0.95, size = 75, normalized size = 0.58

$$\frac{2\left(4641\,b^6x^{12}+33150\,ab^5x^{10}+102375\,a^2b^4x^8+178500\,a^3b^3x^6+193375\,a^4b^2x^4+139230\,a^5bx^2+116025\,a^6\right)\sqrt{dx}}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*b^6*x^12 + 33150*a*b^5*x^10 + 102375*a^2*b^4*x^8 + 178500*a^
3*b^3*x^6 + 193375*a^4*b^2*x^4 + 139230*a^5*b*x^2 + 116025*a^6)*sqrt(d*x)/d
```

giac [A] time = 0.18, size = 105, normalized size = 0.81

$$\frac{2\left(4641\sqrt{dx}\,b^6x^{12}+33150\sqrt{dx}\,ab^5x^{10}+102375\sqrt{dx}\,a^2b^4x^8+178500\sqrt{dx}\,a^3b^3x^6+193375\sqrt{dx}\,a^4b^2x^4+139230\sqrt{dx}\,a^5bx^2+116025\sqrt{dx}\,a^6\right)}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/116025*(4641*sqrt(d*x)*b^6*x^12 + 33150*sqrt(d*x)*a*b^5*x^10 + 102375*sqr
t(d*x)*a^2*b^4*x^8 + 178500*sqrt(d*x)*a^3*b^3*x^6 + 193375*sqrt(d*x)*a^4*b^
2*x^4 + 139230*sqrt(d*x)*a^5*b*x^2 + 116025*sqrt(d*x)*a^6)/d
```

maple [A] time = 0.01, size = 74, normalized size = 0.57

$$\frac{2\left(4641b^6x^{12}+33150a\,b^5x^{10}+102375a^2b^4x^8+178500a^3b^3x^6+193375a^4b^2x^4+139230a^5b\,x^2+116025a^6\right)x}{116025\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x)
```

```
[Out] 2/116025*(4641*b^6*x^12+33150*a*b^5*x^10+102375*a^2*b^4*x^8+178500*a^3*b^3*
x^6+193375*a^4*b^2*x^4+139230*a^5*b*x^2+116025*a^6)*x/(d*x)^(1/2)
```

maxima [A] time = 1.36, size = 155, normalized size = 1.20

$$\frac{2\left(116025\sqrt{dx}\,a^6+\frac{4641\,(dx)^{\frac{25}{2}}b^6}{d^{12}}+\frac{33150\,(dx)^{\frac{21}{2}}ab^5}{d^{10}}+\frac{81900\,(dx)^{\frac{17}{2}}a^2b^4}{d^8}+\frac{71400\,(dx)^{\frac{13}{2}}a^3b^3}{d^6}+7735\left(\frac{5\,(dx)^{\frac{9}{2}}b^2}{d^4}+\frac{18\,(dx)^{\frac{5}{2}}ab}{d^2}\right)a^4+175\left(\frac{117\,(dx)^{\frac{17}{2}}b^4}{d^8}+\frac{612\,(dx)^{\frac{13}{2}}ab^3}{d^6}+\frac{884\,(dx)^{\frac{9}{2}}a^2b^2}{d^4}\right)a^2\right)}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/116025*(116025*sqrt(d*x)*a^6 + 4641*(d*x)^(25/2)*b^6/d^12 + 33150*(d*x)^(
21/2)*a*b^5/d^10 + 81900*(d*x)^(17/2)*a^2*b^4/d^8 + 71400*(d*x)^(13/2)*a^3*
b^3/d^6 + 7735*(5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)*a^4 + 175*(
117*(d*x)^(17/2)*b^4/d^8 + 612*(d*x)^(13/2)*a*b^3/d^6 + 884*(d*x)^(9/2)*a^2
*b^2/d^4)*a^2)/d
```

mupad [B] time = 0.04, size = 103, normalized size = 0.80

$$\frac{2\,a^6\,\sqrt{d\,x}}{d}+\frac{2\,b^6\,{\left(d\,x\right)}^{25/2}}{25\,d^{13}}+\frac{10\,a^4\,b^2\,{\left(d\,x\right)}^{9/2}}{3\,d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{13/2}}{13\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{17/2}}{17\,d^9}+\frac{12\,a^5\,b\,{\left(d\,x\right)}^{5/2}}{5\,d^3}+\frac{4\,a\,b^5\,{\left(d\,x\right)}^{21/2}}{7\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(1/2),x)
```

```
[Out] (2*a^6*(d*x)^(1/2))/d + (2*b^6*(d*x)^(25/2))/(25*d^13) + (10*a^4*b^2*(d*x)^
(9/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(17
/2))/(17*d^9) + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (4*a*b^5*(d*x)^(21/2))/(7*
d^11)
```

sympy [A] time = 2.94, size = 129, normalized size = 1.00

$$\frac{2a^6\sqrt{x}}{\sqrt{d}} + \frac{12a^5bx^{\frac{5}{2}}}{5\sqrt{d}} + \frac{10a^4b^2x^{\frac{9}{2}}}{3\sqrt{d}} + \frac{40a^3b^3x^{\frac{13}{2}}}{13\sqrt{d}} + \frac{30a^2b^4x^{\frac{17}{2}}}{17\sqrt{d}} + \frac{4ab^5x^{\frac{21}{2}}}{7\sqrt{d}} + \frac{2b^6x^{\frac{25}{2}}}{25\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] 2*a**6*sqrt(x)/sqrt(d) + 12*a**5*b*x**(5/2)/(5*sqrt(d)) + 10*a**4*b**2*x**(
9/2)/(3*sqrt(d)) + 40*a**3*b**3*x**(13/2)/(13*sqrt(d)) + 30*a**2*b**4*x**(1
7/2)/(17*sqrt(d)) + 4*a*b**5*x**(21/2)/(7*sqrt(d)) + 2*b**6*x**(25/2)/(25*s
qrt(d))
```

3.505 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2a^6}{d\sqrt{dx}}+\frac{4a^5b(dx)^{3/2}}{d^3}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

Rubi [A] time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{4a^5b(dx)^{3/2}}{d^3}-\frac{2a^6}{d\sqrt{dx}}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (-2*a^6)/(d*Sqrt[d*x]) + (4*a^5*b*(d*x)^(3/2))/d^3 + (30*a^4*b^2*(d*x)^(7/2
))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^(15/2))/
d^9 + (12*a*b^5*(d*x)^(19/2))/(19*d^11) + (2*b^6*(d*x)^(23/2))/(23*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{3/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{(dx)^{3/2}}\,dx}{b^6}$$

$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{3/2}} + \frac{6a^5b^7\sqrt{dx}}{d^2} + \frac{15a^4b^8(dx)^{5/2}}{d^4} + \frac{20a^3b^9(dx)^{9/2}}{d^6} + \frac{15a^2b^{10}(dx)^{13/2}}{d^8} + \frac{6ab^{11}(dx)^{17/2}}{d^{10}} + \frac{b^{12}(d}{d}\right.}{b^6}$$

$$= -\frac{2a^6}{d\sqrt{dx}} + \frac{4a^5b(dx)^{3/2}}{d^3} + \frac{30a^4b^2(dx)^{7/2}}{7d^5} + \frac{40a^3b^3(dx)^{11/2}}{11d^7} + \frac{2a^2b^4(dx)^{15/2}}{d^9} + \frac{12ab^5(}{19}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.62

$$\frac{2x\left(-33649a^6+67298a^5bx^2+72105a^4b^2x^4+61180a^3b^3x^6+33649a^2b^4x^8+10626ab^5x^{10}+1463b^6x^{12}\right)}{33649(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-33649*a^6 + 67298*a^5*b*x^2 + 72105*a^4*b^2*x^4 + 61180*a^3*b^3*x^6
+ 33649*a^2*b^4*x^8 + 10626*a*b^5*x^10 + 1463*b^6*x^12))/(33649*(d*x)^(3/2)
)
```

IntegrateAlgebraic [A] time = 0.06, size = 100, normalized size = 0.80

$$\frac{2\left(-33649a^6d^{12}+67298a^5bd^{12}x^2+72105a^4b^2d^{12}x^4+61180a^3b^3d^{12}x^6+33649a^2b^4d^{12}x^8+10626ab^5d^{12}x^{10}+1463b^6d^{12}x^{12}\right)}{33649d^{13}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (2*(-33649*a^6*d^12 + 67298*a^5*b*d^12*x^2 + 72105*a^4*b^2*d^12*x^4 + 61180
*a^3*b^3*d^12*x^6 + 33649*a^2*b^4*d^12*x^8 + 10626*a*b^5*d^12*x^10 + 1463*b
^6*d^12*x^12))/(33649*d^13*Sqrt[d*x])
```

fricas [A] time = 0.99, size = 78, normalized size = 0.62

$$\frac{2\left(1463\,b^6x^{12}+10626\,ab^5x^{10}+33649\,a^2b^4x^8+61180\,a^3b^3x^6+72105\,a^4b^2x^4+67298\,a^5bx^2-33649\,a^6\right)\sqrt{dx}}{33649\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/33649*(1463*b^6*x^12 + 10626*a*b^5*x^10 + 33649*a^2*b^4*x^8 + 61180*a^3*b
^3*x^6 + 72105*a^4*b^2*x^4 + 67298*a^5*b*x^2 - 33649*a^6)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.20, size = 127, normalized size = 1.02

$$-\frac{2\left(\frac{33649\,a^{6}}{\sqrt{dx}}-\frac{1463\,\sqrt{dx}\,b^{6}d^{275}x^{11}+10626\,\sqrt{dx}\,ab^{5}d^{275}x^{9}+33649\,\sqrt{dx}\,a^{2}b^{4}d^{275}x^{7}+61180\,\sqrt{dx}\,a^{3}b^{3}d^{275}x^{5}+72105\,\sqrt{dx}\,a^{4}b^{2}d^{275}x^{3}+67298\,\sqrt{dx}\,a^{5}bd^{275}x}{d^{276}}\right)}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*sqrt(d*x)*b^6*d^275*x^11 + 10626*sqrt
(d*x)*a*b^5*d^275*x^9 + 33649*sqrt(d*x)*a^2*b^4*d^275*x^7 + 61180*sqrt(d*x)
*a^3*b^3*d^275*x^5 + 72105*sqrt(d*x)*a^4*b^2*d^275*x^3 + 67298*sqrt(d*x)*a^
5*b*d^275*x)/d^276)/d
```

maple [A] time = 0.01, size = 74, normalized size = 0.59

$$-\frac{2\left(-1463b^{6}x^{12}-10626a\,b^{5}x^{10}-33649a^{2}b^{4}x^{8}-61180a^{3}b^{3}x^{6}-72105a^{4}b^{2}x^{4}-67298a^{5}b\,x^{2}+33649a^{6}\right)x}{33649\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x)
```

```
[Out] -2/33649*(-1463*b^6*x^12-10626*a*b^5*x^10-33649*a^2*b^4*x^8-61180*a^3*b^3*x
^6-72105*a^4*b^2*x^4-67298*a^5*b*x^2+33649*a^6)*x/(d*x)^(3/2)
```

maxima [A] time = 1.39, size = 108, normalized size = 0.86

$$-\frac{2\left(\frac{33649\,a^{6}}{\sqrt{dx}}-\frac{1463\,(dx)^{\frac{23}{2}}b^{6}+10626\,(dx)^{\frac{19}{2}}ab^{5}d^{2}+33649\,(dx)^{\frac{15}{2}}a^{2}b^{4}d^{4}+61180\,(dx)^{\frac{11}{2}}a^{3}b^{3}d^{6}+72105\,(dx)^{\frac{7}{2}}a^{4}b^{2}d^{8}+67298\,(dx)^{\frac{3}{2}}a^{5}bd^{10}}{d^{12}}\right)}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*(d*x)^(23/2)*b^6 + 10626*(d*x)^(19/2)
*a*b^5*d^2 + 33649*(d*x)^(15/2)*a^2*b^4*d^4 + 61180*(d*x)^(11/2)*a^3*b^3*d^
6 + 72105*(d*x)^(7/2)*a^4*b^2*d^8 + 67298*(d*x)^(3/2)*a^5*b*d^10)/d^12)/d
```

mupad [B] time = 0.04, size = 103, normalized size = 0.82

$$\frac{2\,b^{6}\,(d\,x)^{23/2}}{23\,d^{13}}-\frac{2\,a^{6}}{d\,\sqrt{d\,x}}+\frac{30\,a^{4}\,b^{2}\,(d\,x)^{7/2}}{7\,d^{5}}+\frac{40\,a^{3}\,b^{3}\,(d\,x)^{11/2}}{11\,d^{7}}+\frac{2\,a^{2}\,b^{4}\,(d\,x)^{15/2}}{d^{9}}+\frac{4\,a^{5}\,b\,(d\,x)^{3/2}}{d^{3}}+\frac{12\,a\,b^{5}\,(d\,x)^{19/2}}{19\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(3/2),x)
```

```
[Out] (2*b^6*(d*x)^(23/2))/(23*d^13) - (2*a^6)/(d*(d*x)^(1/2)) + (30*a^4*b^2*(d*x
)^(7/2))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^(1
5/2))/d^9 + (4*a^5*b*(d*x)^(3/2))/d^3 + (12*a*b^5*(d*x)^(19/2))/(19*d^11)
```

sympy [A] time = 3.01, size = 126, normalized size = 1.01

$$-\frac{2a^6}{d^{\frac{3}{2}}\sqrt{x}}+\frac{4a^5bx^{\frac{3}{2}}}{d^{\frac{3}{2}}}+\frac{30a^4b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}+\frac{40a^3b^3x^{\frac{11}{2}}}{11d^{\frac{3}{2}}}+\frac{2a^2b^4x^{\frac{15}{2}}}{d^{\frac{3}{2}}}+\frac{12ab^5x^{\frac{19}{2}}}{19d^{\frac{3}{2}}}+\frac{2b^6x^{\frac{23}{2}}}{23d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(3/2),x)
```

```
[Out] -2*a**6/(d**(3/2)*sqrt(x)) + 4*a**5*b*x**(3/2)/d**(3/2) + 30*a**4*b**2*x**(
7/2)/(7*d**(3/2)) + 40*a**3*b**3*x**(11/2)/(11*d**(3/2)) + 2*a**2*b**4*x**(
15/2)/d**(3/2) + 12*a*b**5*x**(19/2)/(19*d**(3/2)) + 2*b**6*x**(23/2)/(23*d
**(3/2))
```

3.506 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12a^5b\sqrt{dx}}{d^3}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{12a^5b\sqrt{dx}}{d^3}-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (-2*a^6)/(3*d*(d*x)^(3/2)) + (12*a^5*b*Sqrt[d*x])/d^3 + (6*a^4*b^2*(d*x)^(5
/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/2))/(13
*d^9) + (12*a*b^5*(d*x)^(17/2))/(17*d^11) + (2*b^6*(d*x)^(21/2))/(21*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{5/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{(dx)^{5/2}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{5/2}} + \frac{6a^5b^7}{d^2\sqrt{dx}} + \frac{15a^4b^8(dx)^{3/2}}{d^4} + \frac{20a^3b^9(dx)^{7/2}}{d^6} + \frac{15a^2b^{10}(dx)^{11/2}}{d^8} + \frac{6ab^{11}(dx)^{15/2}}{d^{10}} + \frac{b^{12}(dx)^{19}}{d^{12}}\right.}{b^6}$$
$$= -\frac{2a^6}{3d(dx)^{3/2}} + \frac{12a^5b\sqrt{dx}}{d^3} + \frac{6a^4b^2(dx)^{5/2}}{d^5} + \frac{40a^3b^3(dx)^{9/2}}{9d^7} + \frac{30a^2b^4(dx)^{13/2}}{13d^9} + \frac{12ab}{1}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.61

$$\frac{2x\left(-4641a^6+83538a^5bx^2+41769a^4b^2x^4+30940a^3b^3x^6+16065a^2b^4x^8+4914ab^5x^{10}+663b^6x^{12}\right)}{13923(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (2*x*(-4641*a^6 + 83538*a^5*b*x^2 + 41769*a^4*b^2*x^4 + 30940*a^3*b^3*x^6 +
 16065*a^2*b^4*x^8 + 4914*a*b^5*x^10 + 663*b^6*x^12))/(13923*(d*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 100, normalized size = 0.79

$$\frac{2\left(-4641a^6d^{12}+83538a^5bd^{12}x^2+41769a^4b^2d^{12}x^4+30940a^3b^3d^{12}x^6+16065a^2b^4d^{12}x^8+4914ab^5d^{12}x^{10}+663b^6d^{12}x^{12}\right)}{13923d^{13}(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (2*(-4641*a^6*d^12 + 83538*a^5*b*d^12*x^2 + 41769*a^4*b^2*d^12*x^4 + 30940*
a^3*b^3*d^12*x^6 + 16065*a^2*b^4*d^12*x^8 + 4914*a*b^5*d^12*x^10 + 663*b^6*
d^12*x^12))/(13923*d^13*(d*x)^(3/2))
```

fricas [A] time = 1.31, size = 78, normalized size = 0.61

$$\frac{2\left(663\,b^6x^{12}+4914\,ab^5x^{10}+16065\,a^2b^4x^8+30940\,a^3b^3x^6+41769\,a^4b^2x^4+83538\,a^5bx^2-4641\,a^6\right)\sqrt{dx}}{13923\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^6*x^12 + 4914*a*b^5*x^10 + 16065*a^2*b^4*x^8 + 30940*a^3*b^3
*x^6 + 41769*a^4*b^2*x^4 + 83538*a^5*b*x^2 - 4641*a^6)*sqrt(d*x)/(d^3*x^2)
```

giac [A] time = 0.17, size = 130, normalized size = 1.02

$$-\frac{2\left(\frac{4641\,a^6 d}{\sqrt{dx}\,x}-\frac{663\,\sqrt{dx}\,b^6 d^{210}x^{10}+4914\,\sqrt{dx}\,ab^5 d^{210}x^8+16065\,\sqrt{dx}\,a^2b^4d^{210}x^6+30940\,\sqrt{dx}\,a^3b^3d^{210}x^4+41769\,\sqrt{dx}\,a^4b^2d^{210}x^2+83538\,\sqrt{dx}\,a^5bd^{210}}{d^{210}}\right)}{13923\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/13923*(4641*a^6*d/(sqrt(d*x)*x) - (663*sqrt(d*x)*b^6*d^210*x^10 + 4914*s
qrt(d*x)*a*b^5*d^210*x^8 + 16065*sqrt(d*x)*a^2*b^4*d^210*x^6 + 30940*sqrt(d
*x)*a^3*b^3*d^210*x^4 + 41769*sqrt(d*x)*a^4*b^2*d^210*x^2 + 83538*sqrt(d*x)
*a^5*b*d^210)/d^210)/d^3
```

maple [A] time = 0.01, size = 74, normalized size = 0.58

$$-\frac{2\left(-663b^6x^{12}-4914a\,b^5x^{10}-16065a^2b^4x^8-30940a^3b^3x^6-41769a^4b^2x^4-83538a^5b\,x^2+4641a^6\right)x}{13923\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x)
```

```
[Out] -2/13923*(-663*b^6*x^12-4914*a*b^5*x^10-16065*a^2*b^4*x^8-30940*a^3*b^3*x^6
-41769*a^4*b^2*x^4-83538*a^5*b*x^2+4641*a^6)*x/(d*x)^(5/2)
```

maxima [A] time = 1.39, size = 108, normalized size = 0.85

$$-\frac{2\left(\frac{4641\,a^6}{(dx)^{\frac{3}{2}}}-\frac{663\,(dx)^{\frac{21}{2}}\,b^6+4914\,(dx)^{\frac{17}{2}}\,ab^5d^2+16065\,(dx)^{\frac{13}{2}}\,a^2b^4d^4+30940\,(dx)^{\frac{9}{2}}\,a^3b^3d^6+41769\,(dx)^{\frac{5}{2}}\,a^4b^2d^8+83538\,\sqrt{dx}\,a^5bd^{10}}{d^{12}}\right)}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/13923*(4641*a^6/(d*x)^(3/2) - (663*(d*x)^(21/2)*b^6 + 4914*(d*x)^(17/2)*
a*b^5*d^2 + 16065*(d*x)^(13/2)*a^2*b^4*d^4 + 30940*(d*x)^(9/2)*a^3*b^3*d^6
+ 41769*(d*x)^(5/2)*a^4*b^2*d^8 + 83538*sqrt(d*x)*a^5*b*d^10)/d^12)/d
```

mupad [B] time = 0.04, size = 103, normalized size = 0.81

$$\frac{2\,b^6\,(d\,x)^{21/2}}{21\,d^{13}}-\frac{2\,a^6}{3\,d\,(d\,x)^{3/2}}+\frac{6\,a^4\,b^2\,(d\,x)^{5/2}}{d^5}+\frac{40\,a^3\,b^3\,(d\,x)^{9/2}}{9\,d^7}+\frac{30\,a^2\,b^4\,(d\,x)^{13/2}}{13\,d^9}+\frac{12\,a^5\,b\,\sqrt{d\,x}}{d^3}+\frac{12\,a\,b^5\,(d\,x)^{17/2}}{17\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(5/2),x)
```

```
[Out] (2*b^6*(d*x)^(21/2))/(21*d^13) - (2*a^6)/(3*d*(d*x)^(3/2)) + (6*a^4*b^2*(d*
x)^(5/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/2)
)/(13*d^9) + (12*a^5*b*(d*x)^(1/2))/d^3 + (12*a*b^5*(d*x)^(17/2))/(17*d^11)
```

sympy [A] time = 3.57, size = 128, normalized size = 1.01

$$-\frac{2a^6}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{12a^5b\sqrt{x}}{d^{\frac{5}{2}}}+\frac{6a^4b^2x^{\frac{5}{2}}}{d^{\frac{5}{2}}}+\frac{40a^3b^3x^{\frac{9}{2}}}{9d^{\frac{5}{2}}}+\frac{30a^2b^4x^{\frac{13}{2}}}{13d^{\frac{5}{2}}}+\frac{12ab^5x^{\frac{17}{2}}}{17d^{\frac{5}{2}}}+\frac{2b^6x^{\frac{21}{2}}}{21d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(5/2),x)
```

```
[Out] -2*a**6/(3*d**(5/2)*x**(3/2)) + 12*a**5*b*sqrt(x)/d**(5/2) + 6*a**4*b**2*x*
*(5/2)/d**(5/2) + 40*a**3*b**3*x**(9/2)/(9*d**(5/2)) + 30*a**2*b**4*x**(13/
2)/(13*d**(5/2)) + 12*a*b**5*x**(17/2)/(17*d**(5/2)) + 2*b**6*x**(21/2)/(21
*d**(5/2))
```

3.507 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^6}{5d(dx)^{5/2}}-\frac{12a^5b}{d^3\sqrt{dx}}+\frac{10a^4b^2(dx)^{3/2}}{d^5}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{10a^4b^2(dx)^{3/2}}{d^5}-\frac{12a^5b}{d^3\sqrt{dx}}-\frac{2a^6}{5d(dx)^{5/2}}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (-2*a^6)/(5*d*(d*x)^(5/2)) - (12*a^5*b)/(d^3*Sqrt[d*x]) + (10*a^4*b^2*(d*x)
^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7) + (30*a^2*b^4*(d*x)^(11/2))/
(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11) + (2*b^6*(d*x)^(19/2))/(19*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{7/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{(dx)^{7/2}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{7/2}} + \frac{6a^5b^7}{d^2(dx)^{3/2}} + \frac{15a^4b^8\sqrt{dx}}{d^4} + \frac{20a^3b^9(dx)^{5/2}}{d^6} + \frac{15a^2b^{10}(dx)^{9/2}}{d^8} + \frac{6ab^{11}(dx)^{13/2}}{d^{10}} + \frac{b^{12}(dx)^{17/}}{d^{12}}\right.}{b^6}$$
$$= -\frac{2a^6}{5d(dx)^{5/2}} - \frac{12a^5b}{d^3\sqrt{dx}} + \frac{10a^4b^2(dx)^{3/2}}{d^5} + \frac{40a^3b^3(dx)^{7/2}}{7d^7} + \frac{30a^2b^4(dx)^{11/2}}{11d^9} + \frac{4ab^5(d}{5d^1}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.65

$$\frac{2\sqrt{dx}\left(-1463a^6-43890a^5bx^2+36575a^4b^2x^4+20900a^3b^3x^6+9975a^2b^4x^8+2926ab^5x^{10}+385b^6x^{12}\right)}{7315d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-1463*a^6 - 43890*a^5*b*x^2 + 36575*a^4*b^2*x^4 + 20900*a^3*b
^3*x^6 + 9975*a^2*b^4*x^8 + 2926*a*b^5*x^10 + 385*b^6*x^12))/(7315*d^4*x^3)
```

IntegrateAlgebraic [A] time = 0.06, size = 100, normalized size = 0.79

$$\frac{2\left(-1463a^6d^{12}-43890a^5bd^{12}x^2+36575a^4b^2d^{12}x^4+20900a^3b^3d^{12}x^6+9975a^2b^4d^{12}x^8+2926ab^5d^{12}x^{10}+385b^6d^{12}x^{12}\right)}{7315d^{13}(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (2*(-1463*a^6*d^12 - 43890*a^5*b*d^12*x^2 + 36575*a^4*b^2*d^12*x^4 + 20900*
a^3*b^3*d^12*x^6 + 9975*a^2*b^4*d^12*x^8 + 2926*a*b^5*d^12*x^10 + 385*b^6*d
^12*x^12))/(7315*d^13*(d*x)^(5/2))
```

fricas [A] time = 0.89, size = 78, normalized size = 0.61

$$\frac{2\left(385\,b^6x^{12}+2926\,ab^5x^{10}+9975\,a^2b^4x^8+20900\,a^3b^3x^6+36575\,a^4b^2x^4-43890\,a^5bx^2-1463\,a^6\right)\sqrt{dx}}{7315\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^6*x^12 + 2926*a*b^5*x^10 + 9975*a^2*b^4*x^8 + 20900*a^3*b^3*x
^6 + 36575*a^4*b^2*x^4 - 43890*a^5*b*x^2 - 1463*a^6)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.17, size = 133, normalized size = 1.05

$$-\frac{2\left(\frac{1463\left(30\,a^5bd^3x^2+a^6d^3\right)}{\sqrt{dx}\,d^2x^2}-\frac{385\,\sqrt{dx}\,b^6d^{171}x^9+2926\,\sqrt{dx}\,ab^5d^{171}x^7+9975\,\sqrt{dx}\,a^2b^4d^{171}x^5+20900\,\sqrt{dx}\,a^3b^3d^{171}x^3+36575\,\sqrt{dx}\,a^4b^2d^{171}x}{d^{171}}\right)}{7315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^3*x^2 + a^6*d^3)/(sqrt(d*x)*d^2*x^2) - (385*sqrt(
d*x)*b^6*d^171*x^9 + 2926*sqrt(d*x)*a*b^5*d^171*x^7 + 9975*sqrt(d*x)*a^2*b^
4*d^171*x^5 + 20900*sqrt(d*x)*a^3*b^3*d^171*x^3 + 36575*sqrt(d*x)*a^4*b^2*d
^171*x)/d^171)/d^4
```

maple [A] time = 0.01, size = 74, normalized size = 0.58

$$-\frac{2\left(-385b^6x^{12}-2926a\,b^5x^{10}-9975a^2b^4x^8-20900a^3b^3x^6-36575a^4b^2x^4+43890a^5b\,x^2+1463a^6\right)x}{7315\,(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x)
```

```
[Out] -2/7315*(-385*b^6*x^12-2926*a*b^5*x^10-9975*a^2*b^4*x^8-20900*a^3*b^3*x^6-3
6575*a^4*b^2*x^4+43890*a^5*b*x^2+1463*a^6)*x/(d*x)^(7/2)
```

maxima [A] time = 1.35, size = 114, normalized size = 0.90

$$-\frac{2\left(\frac{1463\left(30\,a^5bd^2x^2+a^6d^2\right)}{(dx)^{\frac{5}{2}}d^2}-\frac{385\,(dx)^{\frac{19}{2}}b^6+2926\,(dx)^{\frac{15}{2}}ab^5d^2+9975\,(dx)^{\frac{11}{2}}a^2b^4d^4+20900\,(dx)^{\frac{7}{2}}a^3b^3d^6+36575\,(dx)^{\frac{3}{2}}a^4b^2d^8}{d^{12}}\right)}{7315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^2*x^2 + a^6*d^2)/((d*x)^(5/2)*d^2) - (385*(d*x)^(
19/2)*b^6 + 2926*(d*x)^(15/2)*a*b^5*d^2 + 9975*(d*x)^(11/2)*a^2*b^4*d^4 + 2
0900*(d*x)^(7/2)*a^3*b^3*d^6 + 36575*(d*x)^(3/2)*a^4*b^2*d^8)/d^12)/d
```

mupad [B] time = 0.04, size = 107, normalized size = 0.84

$$\frac{2\,b^6\,(d\,x)^{19/2}}{19\,d^{13}}-\frac{\frac{2\,a^6\,d^2}{5}+12\,b\,a^5\,d^2\,x^2}{d^3\,(d\,x)^{5/2}}+\frac{10\,a^4\,b^2\,(d\,x)^{3/2}}{d^5}+\frac{40\,a^3\,b^3\,(d\,x)^{7/2}}{7\,d^7}+\frac{30\,a^2\,b^4\,(d\,x)^{11/2}}{11\,d^9}+\frac{4\,a\,b^5\,(d\,x)^{15/2}}{5\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(7/2),x)
```

```
[Out] (2*b^6*(d*x)^(19/2))/(19*d^13) - ((2*a^6*d^2)/5 + 12*a^5*b*d^2*x^2)/(d^3*(d
*x)^(5/2)) + (10*a^4*b^2*(d*x)^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7
) + (30*a^2*b^4*(d*x)^(11/2))/(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11)
```

sympy [A] time = 4.56, size = 128, normalized size = 1.01

$$-\frac{2a^6}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{12a^5b}{d^{\frac{7}{2}}\sqrt{x}}+\frac{10a^4b^2x^{\frac{3}{2}}}{d^{\frac{7}{2}}}+\frac{40a^3b^3x^{\frac{7}{2}}}{7d^{\frac{7}{2}}}+\frac{30a^2b^4x^{\frac{11}{2}}}{11d^{\frac{7}{2}}}+\frac{4ab^5x^{\frac{15}{2}}}{5d^{\frac{7}{2}}}+\frac{2b^6x^{\frac{19}{2}}}{19d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(7/2),x)
```

```
[Out] -2*a**6/(5*d**(7/2)*x**(5/2)) - 12*a**5*b/(d**(7/2)*sqrt(x)) + 10*a**4*b**2
*x**(3/2)/d**(7/2) + 40*a**3*b**3*x**(7/2)/(7*d**(7/2)) + 30*a**2*b**4*x**(
11/2)/(11*d**(7/2)) + 4*a*b**5*x**(15/2)/(5*d**(7/2)) + 2*b**6*x**(19/2)/(1
9*d**(7/2))
```

3.508 $\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=316

$$-\frac{9a^{5/4}d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}-$$

Rubi **[A]** time = 0.38, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9a^{5/4}d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,b^{13/4}}-\frac{9ad^5\sqrt{dx}}{2b^3}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{9d^3(dx)^{5/2}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-9*a*d^5*Sqrt[d*x])/(2*b^3) + (9*d^3*(d*x)^(5/2))/(10*b^2) - (d*(d*x)^(9/2
))/(2*b*(a + b*x^2)) - (9*a^(5/4)*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4))
- (9*a^(5/4)*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*S
qrt[2]*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx \\
&= -\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{1}{4}\left(9d^2\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx \\
&= \frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}-\frac{\left(9ad^4\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{4b} \\
&= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{\left(9a^2d^6\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4b^2} \\
&= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{\left(9a^2d^5\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{2b^2} \\
&= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{\left(9a^{3/2}d^4\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4b^2} \\
&= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}-\frac{\left(9a^{5/4}d^{11/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx,\right.}{8\sqrt{2}\,b^{13/4}} \\
&= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}-\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\right.}{8\sqrt{2}\,b^{13/4}} \\
&= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}-\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11}}{}
\end{aligned}
$$

Mathematica [A] time = 0.34, size = 235, normalized size = 0.74

$$
\frac{d^5\sqrt{dx}\left(-45\sqrt{2}\,a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+45\sqrt{2}\,a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-90\sqrt{2}\,a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+90\sqrt{2}\,a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+\frac{8\sqrt[4]{b}\sqrt{x}\left(-45a^2-36abx^2+4b^2x^4\right)}{a+bx^2}\right)}{80b^{13/4}\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^5*Sqrt[d*x]*((8*b^(1/4)*Sqrt[x]*(-45*a^2 - 36*a*b*x^2 + 4*b^2*x^4))/(a +
 b*x^2) - 90*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 90*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 45*Sqr
t[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 4
5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
]))/(80*b^(13/4)*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.50, size = 218, normalized size = 0.69

$$-\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{-45a^2d^7\sqrt{dx}-36abd^5(dx)^{5/2}+4b^2d^3(dx)^{9/2}}{10b^3\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-45*a^2*d^7*Sqrt[d*x] - 36*a*b*d^5*(d*x)^(5/2) + 4*b^2*d^3*(d*x)^(9/2))/(1
0*b^3*(a*d^2 + b*d^2*x^2)) - (9*a^(5/4)*d^(11/2)*ArcTan[((a^(1/4)*Sqrt[d])/
(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(4*S
qrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(4*Sqrt[2]*b^(13/4))
```

fricas [A] time = 2.07, size = 283, normalized size = 0.90

$$\frac{180\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^4x^2+ab^3\right)\arctan\left(-\frac{\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{10}d^5-\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{a^2d^{11}x+\sqrt{-\frac{a^5d^{22}}{b^{13}}}\,b^6}\,b^{10}}{a^5d^{22}}\right)+45\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^4x^2+ab^3\right)\log\left(9\sqrt{dx}\,ad^5+9\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)-45\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^4x^2+ab^3\right)\log\left(9\sqrt{dx}\,ad^5-9\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)+4\left(4b^2d^5x^4-36abd^5x^2-45a^2d^5\right)\sqrt{dx}}{40\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/40*(180*(-a^5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*arctan(-((-a^5*d^22/b^13
)^(3/4)*sqrt(d*x)*a*b^10*d^5 - (-a^5*d^22/b^13)^(3/4)*sqrt(a^2*d^11*x + sqr
t(-a^5*d^22/b^13)*b^6)*b^10)/(a^5*d^22)) + 45*(-a^5*d^22/b^13)^(1/4)*(b^4*x
^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 + 9*(-a^5*d^22/b^13)^(1/4)*b^3) - 45*(-a^
5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 - 9*(-a^5*d^22/b
^13)^(1/4)*b^3) + 4*(4*b^2*d^5*x^4 - 36*a*b*d^5*x^2 - 45*a^2*d^5)*sqrt(d*x)
)/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.21, size = 297, normalized size = 0.94

$$-\frac{1}{80}d^5\left(\frac{40\sqrt{dx}\,a^2d^2}{\left(bd^2x^2+ad^2\right)b^3}-\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4}+\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4}-\frac{32\left(\sqrt{dx}\,b^8d^{10}x^2-10\sqrt{dx}\,ab^7d^{10}\right)}{b^{10}d^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/80*d^5*(40*sqrt(d*x)*a^2*d^2/((b*d^2*x^2 + a*d^2)*b^3) - 90*sqrt(2)*(a*b
^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/b^4 - 90*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^4 - 45*sqrt(2)*(a
*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/b^4 + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*
sqrt(d*x) + sqrt(a*d^2/b))/b^4 - 32*(sqrt(d*x)*b^8*d^10*x^2 - 10*sqrt(d*x)*
a*b^7*d^10)/(b^10*d^10))
```

maple [A] time = 0.02, size = 242, normalized size = 0.77

$$-\frac{\sqrt{dx}\, a^2 d^7}{2\left(b\,d^2x^2+d^2a\right)b^3}+\frac{9\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^3}+\frac{9\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^3}+\frac{9\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^5\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16b^3}-\frac{4\sqrt{dx}\,a\,d^5}{b^3}+\frac{2\,(dx)^{\frac{5}{2}}\,d^3}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/5*d^3*(d*x)^(5/2)/b^2-4*a*d^5*(d*x)^(1/2)/b^3-1/2*d^7/b^3*a^2*(d*x)^(1/2)
/(b*d^2*x^2+a*d^2)+9/16*d^5/b^3*a*(d^2*a/b)^(1/4)*2^(1/2)*ln((d*x+(d^2*a/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2*a/b)^(1/2))/(d*x-(d^2*a/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(d^2*a/b)^(1/2)))+9/8*d^5/b^3*a*(d^2*a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(d^2*a/b)^(1/4)*(d*x)^(1/2)+1)+9/8*d^5/b^3*a*(d^2*a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(d^2*a/b)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.04, size = 300, normalized size = 0.95

$$-\frac{\frac{40\sqrt{dx}\,a^2d^8}{b^4d^2x^2+ab^3d^2}-\frac{45\left(\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^7\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)a^2}{b^3}-\frac{32\left((dx)^{\frac{5}{2}}bd^4-10\sqrt{dx}\,ad^6\right)}{b^3}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/80*(40*sqrt(d*x)*a^2*d^8/(b^4*d^2*x^2 + a*b^3*d^2) - 45*(sqrt(2)*d^8*log
(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt
(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^7*arctan(1
/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
```

```
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^7*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a
)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^2/b^3 - 32*((d*x)^(5/2)*
b*d^4 - 10*sqrt(d*x)*a*d^6)/b^3)/d
```

mupad [B] time = 4.27, size = 129, normalized size = 0.41

$$\frac{2\,d^3\,{\left(d\,x\right)}^{5/2}}{5\,b^2}-\frac{9\,{\left(-a\right)}^{5/4}\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{13/4}}-\frac{a^2\,d^7\,\sqrt{d\,x}}{2\,\left(b^4\,d^2\,x^2+a\,b^3\,d^2\right)}-\frac{4\,a\,d^5\,\sqrt{d\,x}}{b^3}+\frac{{\left(-a\right)}^{5/4}\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,9{}\mathrm{i}}{4\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (2*d^3*(d*x)^(5/2))/(5*b^2) - (9*(-a)^(5/4)*d^(11/2)*atan((b^(1/4)*(d*x)^(1
/2))/((-a)^(1/4)*d^(1/2))))/(4*b^(13/4)) + ((-a)^(5/4)*d^(11/2)*atan((b^(1/
4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*9i)/(4*b^(13/4)) - (a^2*d^7*(d*x)^
(1/2))/(2*(a*b^3*d^2 + b^4*d^2*x^2)) - (4*a*d^5*(d*x)^(1/2))/b^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{11}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(11/2)/(a + b*x**2)**2, x)
```

3.509 $\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=298

$$-\frac{7a^{3/4}d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7a^{3}}{\ }$$

Rubi [A] time = 0.30, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7a^{3/4}d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,b^{11/4}}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}+\frac{7d^3(dx)^{3/2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (7*d^3*(d*x)^(3/2))/(6*b^2) - (d*(d*x)^(7/2))/(2*b*(a + b*x^2)) + (7*a^(3/4
)*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqr
t[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]
*b^(11/4)) + (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} + \frac{1}{4}\left(7d^2\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} - \frac{\left(7ad^4\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4b}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} - \frac{\left(7ad^3\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2b}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} + \frac{\left(7ad^3\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4b^{3/2}} - \frac{\left(7ad^3\right)\text{Subst}\Big(}{}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} - \frac{\left(7a^{3/4}d^{9/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\,b^{11/4}} - \frac{\left(7a\right.}{}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} - \frac{7a^{3/4}d^{9/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\,b^{11/4}} + \frac{7a}{}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} + \frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{11/4}} - \frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{4\sqrt{2}\,b^{11/4}}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.21

$$-\frac{2d^4x\sqrt{dx}\left(7\left(a+bx^2\right){}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)-7a-bx^2\right)}{3b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-2*d^4*x*Sqrt[d*x]*(-7*a - b*x^2 + 7*(a + b*x^2)*Hypergeometric2F1[3/4, 2,
 7/4, -((b*x^2)/a)]))/(3*b^2*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.51, size = 200, normalized size = 0.67

$$\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{4\sqrt{2}b^{11/4}}+\frac{7ad^5(dx)^{3/2}+4bd^3(dx)^{7/2}}{6b^2\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (7*a*d^5*(d*x)^(3/2) + 4*b*d^3*(d*x)^(7/2))/(6*b^2*(a*d^2 + b*d^2*x^2)) + (
7*a^(3/4)*d^(9/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sq
rt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(4*Sqrt[2]*b^(11/4)) + (7*a^(3/4)*d
^(9/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqr
t[b]*d*x)])/(4*Sqrt[2]*b^(11/4))
```

fricas [A] time = 1.64, size = 283, normalized size = 0.95

$$\frac{84\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\arctan\left(-\frac{\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^3d^{13}-\sqrt{a^4d^{27}x-\sqrt{-\frac{a^3d^{18}}{b^{11}}}\,a^3b^5d^{18}}\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}b^3}{a^3d^{18}}\right)-21\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\log\left(343\sqrt{dx}\,a^2d^{13}+343\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{3}{4}}b^8\right)+21\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\log\left(343\sqrt{dx}\,a^2d^{13}-343\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{3}{4}}b^8\right)+4\left(4\,bd^4x^3+7\,ad^4x\right)\sqrt{dx}}{24\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(84*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((-a^3*d^18/b^11)
^(1/4)*sqrt(d*x)*a^2*b^3*d^13 - sqrt(a^4*d^27*x - sqrt(-a^3*d^18/b^11)*a^3*
b^5*d^18)*(-a^3*d^18/b^11)^(1/4)*b^3)/(a^3*d^18)) - 21*(-a^3*d^18/b^11)^(1/
4)*(b^3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*d^13 + 343*(-a^3*d^18/b^11)^(3/4
)*b^8) + 21*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*
d^13 - 343*(-a^3*d^18/b^11)^(3/4)*b^8) + 4*(4*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d
*x))/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.19, size = 277, normalized size = 0.93

$$\frac{1}{48}\left(\frac{24\sqrt{dx}\,ad^2x}{\left(bd^2x^2+ad^2\right)b^2}+\frac{32\sqrt{dx}\,x}{b^2}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5d}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5d}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^5d}-\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^5d}\right)d^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/48*(24*sqrt(d*x)*a*d^2*x/((b*d^2*x^2 + a*d^2)*b^2) + 32*sqrt(d*x)*x/b^2 -
 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) +
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*d) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^
5*d) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(
d*x) + sqrt(a*d^2/b))/(b^5*d) - 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt
(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*d))*d^4
```

maple [A] time = 0.02, size = 226, normalized size = 0.76

$$\frac{(dx)^{\frac{3}{2}}\,a\,d^5}{2\left(b\,d^2x^2+d^2a\right)b^2}-\frac{7\sqrt{2}\,a\,d^5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^3}-\frac{7\sqrt{2}\,a\,d^5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^3}-\frac{7\sqrt{2}\,a\,d^5\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^3}+\frac{2\,(dx)^{\frac{3}{2}}\,d^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/3*d^3*(d*x)^(3/2)/b^2+1/2*d^5*a/b^2*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)-7/16*d^
5*a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))
)-7/8*d^5*a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x
)^(1/2)+1)-7/8*d^5*a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(
1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.12, size = 273, normalized size = 0.92

$$\frac{\frac{24\,(dx)^{\frac{3}{2}}\,ad^6}{b^3d^2x^2+ab^2d^2}-\frac{21\,ad^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^2}+\frac{32\,(dx)^{\frac{3}{2}}d^4}{b^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/48*(24*(d*x)^(3/2)*a*d^6/(b^3*d^2*x^2 + a*b^2*d^2) - 21*a*d^6*(2*sqrt(2)*
arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sq
rt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sq
rt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d
*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^
(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^2 + 32*(d*x)^(3/2)*d^4/b^2)/d
```

mupad [B] time = 0.12, size = 112, normalized size = 0.38

$$\frac{2\,d^3\,{\left(d\,x\right)}^{3/2}}{3\,b^2}+\frac{7\,{\left(-a\right)}^{3/4}\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{11/4}}+\frac{a\,d^5\,{\left(d\,x\right)}^{3/2}}{2\,\left(b^3\,d^2\,x^2+a\,b^2\,d^2\right)}+\frac{{\left(-a\right)}^{3/4}\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,7{}\mathrm{i}}{4\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (2*d^3*(d*x)^(3/2))/(3*b^2) + (7*(-a)^(3/4)*d^(9/2)*atan((b^(1/4)*(d*x)^(1/
2))/((-a)^(1/4)*d^(1/2))))/(4*b^(11/4)) + ((-a)^(3/4)*d^(9/2)*atan((b^(1/4)
*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*7i)/(4*b^(11/4)) + (a*d^5*(d*x)^(3/2
))/(2*(a*b^2*d^2 + b^3*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{9}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**2, x)
```

3.510 $\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=298

$$\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{a}}{}$$

Rubi [A] time = 0.29, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,b^{9/4}}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5d^3\sqrt{dx}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (5*d^3*Sqrt[d*x])/(2*b^2) - (d*(d*x)^(5/2))/(2*b*(a + b*x^2)) + (5*a^(1/4)*
d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[
2]*b^(9/4)) - (5*a^(1/4)*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(
9/4)) - (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{1}{4}\left(5d^2\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5ad^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4b}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5ad^3\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{2b}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5\sqrt{a}\,d^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4b}-\frac{\left(5\sqrt{a}\,d^2\right)\text{Subs}}{}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{\left(5\sqrt[4]{a}\,d^{7/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{dx}\right)}{8\sqrt{2}\,b^{9/4}}+\frac{\left(5\sqrt[4]{a}\right.}{}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}}{}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,b^{9/4}}$$

Mathematica [A] time = 0.16, size = 244, normalized size = 0.82

$$\frac{d^3\sqrt{dx}\left(\frac{32b^{5/4}x^2}{a+bx^2}+\frac{40a\sqrt[4]{b}}{a+bx^2}+\frac{5\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}-\frac{5\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}+\frac{10\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{x}}-\frac{10\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{x}}\right)}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^3*Sqrt[d*x]*((40*a*b^(1/4))/(a + b*x^2) + (32*b^(5/4)*x^2)/(a + b*x^2) +
 (10*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x]
 - (10*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[
x] + (5*Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/Sqrt[x] - (5*Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/Sqrt[x]))/(16*b^(9/4))
```

IntegrateAlgebraic [A] time = 0.48, size = 200, normalized size = 0.67

$$\frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{4\sqrt{2}\,b^{9/4}}+\frac{5ad^5\sqrt{dx}+4bd^3(dx)^{5/2}}{2b^2\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (5*a*d^5*Sqrt[d*x] + 4*b*d^3*(d*x)^(5/2))/(2*b^2*(a*d^2 + b*d^2*x^2)) + (5*
a^(1/4)*d^(7/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt
[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(4*Sqrt[2]*b^(9/4)) - (5*a^(1/4)*d^(7
/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b
]*d*x)])/(4*Sqrt[2]*b^(9/4))
```

fricas [A] time = 1.13, size = 247, normalized size = 0.83

$$-\frac{20\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\arctan\left(-\frac{\left(-\frac{ad^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{dx}\,b^7d^3-\sqrt{d^7x+\sqrt{-\frac{ad^{14}}{b^9}}\,b^4}\left(-\frac{ad^{14}}{b^9}\right)^{\frac{3}{4}}b^7}{ad^{14}}\right)+5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\log\left(5\sqrt{dx}\,d^3+5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)-5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\log\left(5\sqrt{dx}\,d^3-5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)-4\left(4\,bd^3x^2+5\,ad^3\right)\sqrt{dx}}{8\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(20*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((-a*d^14/b^9)^(3/4)
*sqrt(d*x)*b^7*d^3 - sqrt(d^7*x + sqrt(-a*d^14/b^9)*b^4)*(-a*d^14/b^9)^(3/4
)*b^7)/(a*d^14)) + 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*sqrt(d*x)*
```

```
d^3 + 5*(-a*d^14/b^9)^(1/4)*b^2) - 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*
log(5*sqrt(d*x)*d^3 - 5*(-a*d^14/b^9)^(1/4)*b^2) - 4*(4*b*d^3*x^2 + 5*a*d^3
)*sqrt(d*x))/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.19, size = 263, normalized size = 0.88

$$\frac{1}{16}\,d^{3}\left(\frac{8\,\sqrt{dx}\,ad^{2}}{(bd^{2}x^{2}+ad^{2})b^{2}}-\frac{10\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{3}}-\frac{10\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{3}}-\frac{5\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{3}}+\frac{5\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{3}}+\frac{32\,\sqrt{dx}}{b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/16*d^3*(8*sqrt(d*x)*a*d^2/((b*d^2*x^2 + a*d^2)*b^2) - 10*sqrt(2)*(a*b^3*d
^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2
/b)^(1/4))/b^3 - 10*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 - 5*sqrt(2)*(a*b^3*d^2)
^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 5
*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/b^3 + 32*sqrt(d*x)/b^2)
```

maple [A] time = 0.02, size = 223, normalized size = 0.75

$$\frac{\sqrt{dx}\;a\,d^{5}}{2\left(b\,d^{2}x^{2}+d^{2}a\right)b^{2}}-\frac{5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\;d^{3}\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^{2}}-\frac{5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\;d^{3}\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}}-\frac{5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\;d^{3}\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{16b^{2}}+\frac{2\sqrt{dx}\;d^{3}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2*d^3*(d*x)^(1/2)/b^2+1/2*d^5/b^2*a*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)-5/16*d^3/
b^2*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-5/
8*d^3/b^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2
)+1)-5/8*d^3/b^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*
x)^(1/2)-1)
```

maxima [A] time = 3.03, size = 282, normalized size = 0.95

$$\frac{\frac{8\,\sqrt{dx}\,ad^{6}}{b^{3}d^{2}x^{2}+ab^{2}d^{2}}+\frac{32\,\sqrt{dx}\,d^{4}}{b^{2}}-\frac{5\left(\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}\,d^{5}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\,\sqrt{2}\,d^{5}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)a}{b^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*sqrt(d*x)*a*d^6/(b^3*d^2*x^2 + a*b^2*d^2) + 32*sqrt(d*x)*d^4/b^2 -
5*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sq
rt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt
(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sq
rt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a/b^2)/d
```

mupad [B] time = 0.12, size = 112, normalized size = 0.38

$$\frac{2\,d^3\,\sqrt{d\,x}}{b^2}-\frac{5\,{\left(-a\right)}^{1/4}\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{9/4}}+\frac{a\,d^5\,\sqrt{d\,x}}{2\,\left(b^3\,d^2\,x^2+a\,b^2\,d^2\right)}+\frac{{\left(-a\right)}^{1/4}\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,5{}\mathrm{i}}{4\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (2*d^3*(d*x)^(1/2))/b^2 - (5*(-a)^(1/4)*d^(7/2)*atan((b^(1/4)*(d*x)^(1/2))/
((-a)^(1/4)*d^(1/2))))/(4*b^(9/4)) + ((-a)^(1/4)*d^(7/2)*atan((b^(1/4)*(d*x
)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*5i)/(4*b^(9/4)) + (a*d^5*(d*x)^(1/2))/(2*
(a*b^2*d^2 + b^3*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**2, x)
```

3.511 $\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=281

$$\frac{3d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Rubi [A] time = 0.28, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -(d*(d*x)^(3/2))/(2*b*(a + b*x^2)) - (3*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*d^(5/2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)
*b^(7/4)) + (3*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*d^(5/2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sq
rt[2]*a^(1/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{1}{4}\left(3d^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{1}{2}(3d)\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} - \frac{(3d)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{b}} + \frac{(3d)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x\right.}{4\sqrt{b}}$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{\left(3d^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}} + \frac{\left(3d^{5/2}\right)\text{Subst}\left(\int -\right.}{8}$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{3d^{5/2}\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}} - \frac{3d^{5/2}\log\left(\sqrt{a}\,\sqrt{d}\right.}{}$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} - \frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}} + \frac{3d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}} + \frac{3d^{5/2}\log}{}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.19

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right){}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)-a\right)}{ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a + (a + b*x^2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)
/a)]))/(a*b*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.48, size = 183, normalized size = 0.65

$$-\frac{3d^{5/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\,\sqrt{d}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{d}\,\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}-\frac{d^3(dx)^{3/2}}{2b\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*(d^3*(d*x)^(3/2))/(b*(a*d^2 + b*d^2*x^2)) - (3*d^(5/2)*ArcTan[((a^(1/4
)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[
d*x]])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*d^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(4*Sqrt[2]*a^(1/4)*b^(
7/4))
```

fricas [A] time = 1.57, size = 247, normalized size = 0.88

$$-\frac{4\sqrt{dx}\,d^2x+12\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\sqrt{dx}\,b^2d^7-\sqrt{d^{15}x-\sqrt{-\frac{d^{10}}{ab^7}}\,ab^3d^{10}}\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}b^2}{d^{10}}\right)-3\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\log\left(27\sqrt{dx}\,d^7+27\left(-\frac{d^{10}}{ab^7}\right)^{\frac{3}{4}}ab^5\right)+3\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\log\left(27\sqrt{dx}\,d^7-27\left(-\frac{d^{10}}{ab^7}\right)^{\frac{3}{4}}ab^5\right)}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*sqrt(d*x)*d^2*x + 12*(b^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*arctan(-
((-d^10/(a*b^7))^(1/4)*sqrt(d*x)*b^2*d^7 - sqrt(d^15*x - sqrt(-d^10/(a*b^7)
)*a*b^3*d^10)*(-d^10/(a*b^7))^(1/4)*b^2)/d^10) - 3*(b^2*x^2 + a*b)*(-d^10/(
a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 + 27*(-d^10/(a*b^7))^(3/4)*a*b^5) + 3*(b
^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 - 27*(-d^10/(a*b^7
))^(3/4)*a*b^5))/(b^2*x^2 + a*b)
```

giac [A] time = 0.23, size = 277, normalized size = 0.99

$$-\frac{1}{16}\left(\frac{8\sqrt{dx}\,d^2x}{\left(bd^2x^2+ad^2\right)b}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4d}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4d}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^4d}-\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^4d}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*(8*sqrt(d*x)*d^2*x/((b*d^2*x^2 + a*d^2)*b) - 6*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1
```

```
/4))/(a*b^4*d) - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4*d) + 3*sqrt(2)*(a*b^3
*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
*b^4*d) - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/(a*b^4*d))*d^2
```

maple [A] time = 0.02, size = 209, normalized size = 0.74

$$-\frac{(dx)^{\frac{3}{2}} d^3}{2\left(b\,d^2x^2+d^2a\right)b}+\frac{3\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}} b^2}+\frac{3\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}} b^2}+\frac{3\sqrt{2}\,d^3\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*d^3/b*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+3/16*d^3/b^2/(a/b*d^2)^(1/4)*2^(1/
2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+3/8*d^3/b^2/(a/b*d^2)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+3/8*d^3/b^2/(a/b*d^2
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.10, size = 256, normalized size = 0.91

$$-\frac{\frac{8\,(dx)^{\frac{3}{2}} d^4}{b^2d^2x^2+abd^2}-\frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(d*x)^(3/2)*d^4/(b^2*d^2*x^2 + a*b*d^2) - 3*d^4*(2*sqrt(2)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*s
qrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4))
+ sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(
a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b)/d
```

mupad [B] time = 4.25, size = 92, normalized size = 0.33

$$\frac{3\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}-\frac{3\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}-\frac{d^3\,(d\,x)^{3/2}}{2\,b\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (3*d^(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(1/4)*
b^(7/4)) - (3*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4
*(-a)^(1/4)*b^(7/4)) - (d^3*(d*x)^(3/2))/(2*b*(a*d^2 + b*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^{\frac{5}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**2, x)
```

3.512 $\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=281

$$-\frac{d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d^{3/2}\tan^{-1}\left(1\right.}{4\sqrt{2}\,a}$$

Rubi [A] time = 0.26, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -(d*Sqrt[d*x])/(2*b*(a + b*x^2)) - (d^(3/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/
4)) - (d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3
/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} + \frac{1}{4}d^2\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)}\,dx$$
$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} + \frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)$$
$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} + \frac{\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{a}} + \frac{\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{a}}$$
$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} - \frac{d^{3/2}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{d^{3/2}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}+\sqrt{2}}\right.}{8\sqrt{2}\,a}$$
$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} - \frac{d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\right.}{8}$$
$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} - \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{d^{3/2}\log\left(\right.}{}$$

Mathematica [A] time = 0.17, size = 210, normalized size = 0.75

$$\frac{(dx)^{3/2}\left(-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{8\sqrt[4]{b}\sqrt{x}}{a+bx^2}\right)}{16b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(3/2)*((-8*b^(1/4)*Sqrt[x])/(a + b*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
```

```
^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4)))/(16*b^(5/4)*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.47, size = 183, normalized size = 0.65

$$-\frac{d^{3/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d^3\sqrt{dx}}{2b\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*(d^3*Sqrt[d*x])/(b*(a*d^2 + b*d^2*x^2)) - (d^(3/2)*ArcTan[((a^(1/4)*Sq
rt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]
])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
```

fricas [A] time = 1.19, size = 234, normalized size = 0.83

$$\frac{4\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^2b^4d\left(-\frac{d^6}{a^3b^5}\right)^{\frac{3}{4}}-\sqrt{a^2b^2\sqrt{-\frac{d^6}{a^3b^5}}+d^3x}\,a^2b^4\left(-\frac{d^6}{a^3b^5}\right)^{\frac{3}{4}}}{d^6}\right)+\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}+\sqrt{dx}\,d\right)-\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\log\left(-ab\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}+\sqrt{dx}\,d\right)-4\sqrt{dx}\,d}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^2*b^4*d*
(-d^6/(a^3*b^5))^(3/4) - sqrt(a^2*b^2*sqrt(-d^6/(a^3*b^5)) + d^3*x)*a^2*b^4
*(-d^6/(a^3*b^5))^(3/4))/d^6) + (b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*log(
a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - (b^2*x^2 + a*b)*(-d^6/(a^3*b^5)
)^(1/4)*log(-a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - 4*sqrt(d*x)*d)/(b^
2*x^2 + a*b)
```

giac [A] time = 0.23, size = 261, normalized size = 0.93

$$-\frac{1}{16}\,d\left(\frac{8\sqrt{dx}\,d^2}{\left(bd^2x^2+ad^2\right)b}-\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^2}-\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^2}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^2}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*d*(8*sqrt(d*x)*d^2/((b*d^2*x^2 + a*d^2)*b) - 2*sqrt(2)*(a*b^3*d^2)^(1
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1
/4))/(a*b^2) - 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^2) - sqrt(2)*(a*b^3*d^2)^
(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^2)
+ sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a*b^2))
```

maple [A] time = 0.01, size = 212, normalized size = 0.75

$$-\frac{\sqrt{dx}\, d^{3}}{2\left(b\, d^{2} x^{2}+d^{2} a\right) b}+\frac{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}}{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{8 a b}+\frac{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}}{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{8 a b}+\frac{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d \ln\left(\frac{dx+\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{dx}\, \sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}{dx-\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{dx}\, \sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}\right)}{16 a b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*d^3/b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+1/16*d/b*(a/b*d^2)^(1/4)/a*2^(1/2)
*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1/8*d/b*(a/b*d^2)^(1/4)/a*2^(
1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+1/8*d/b*(a/b*d^2)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.06, size = 265, normalized size = 0.94

$$-\frac{\frac{8\sqrt{dx}\, d^{4}}{b^{2} d^{2} x^{2}+a b d^{2}}-\frac{\frac{\sqrt{2}\, d^{4} \log\left(\sqrt{b}\, dx+\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(a d^{2}\right)^{\frac{3}{4}} b^{\frac{1}{4}}}-\frac{\sqrt{2}\, d^{4} \log\left(\sqrt{b}\, dx-\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(a d^{2}\right)^{\frac{3}{4}} b^{\frac{1}{4}}}+\frac{2\sqrt{2}\, d^{3} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\, \sqrt{b}\right)}{2\sqrt{\sqrt{a}\, \sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\, \sqrt{b}\, d}\, \sqrt{a}}+\frac{2\sqrt{2}\, d^{3} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2\sqrt{dx}\, \sqrt{b}\right)}{2\sqrt{\sqrt{a}\, \sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\, \sqrt{b}\, d}\, \sqrt{a}}}{b}}{16\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*sqrt(d*x)*d^4/(b^2*d^2*x^2 + a*b*d^2) - (sqrt(2)*d^4*log(sqrt(b)*d
*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^
(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)
*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)
*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*
d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b)/d
```

mupad [B] time = 4.35, size = 92, normalized size = 0.33

$$-\frac{d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}-\frac{d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}-\frac{d^3\,\sqrt{d\,x}}{2\,b\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] - (d^(3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(3/4)*
b^(5/4)) - (d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(
-a)^(3/4)*b^(5/4)) - (d^3*(d*x)^(1/2))/(2*b*(a*d^2 + b*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**2, x)
```

3.513 $\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\tan^{-1}\left(1-\right.}{4\sqrt{2}a^{5/4}}$$

Rubi [A] time = 0.27, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d*x)^(3/2)/(2*a*d*(a + b*x^2)) - (Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4)) + (Sqrt[d]*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4
)) + (Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - (Sqrt[d]*Log[Sqrt[a]*Sqrt[d]
 + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/
4)*b^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} + \frac{b\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a}$$
$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} + \frac{b\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2ad}$$
$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4ad} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x,\right.}{4ad}$$
$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}+\sqrt{2}}\right.}{8\sqrt{2}\,a^{5}}$$
$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} + \frac{\sqrt{d}\,\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}} - \frac{\sqrt{d}\,\log\left(\sqrt{a}\,\sqrt{d}+\right.}{8}$$
$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} - \frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{\sqrt{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{\sqrt{d}\,\log\left(\right.}{}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.11

$$\frac{2x\sqrt{dx}\;{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (2*x*Sqrt[d*x]*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^2)
```

IntegrateAlgebraic [A] time = 0.46, size = 181, normalized size = 0.64

$$-\frac{\sqrt{d}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{d(dx)^{3/2}}{2a\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d*(d*x)^(3/2))/(2*a*(a*d^2 + b*d^2*x^2)) - (Sqrt[d]*ArcTan[((a^(1/4)*Sqrt[
d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/
(4*Sqrt[2]*a^(5/4)*b^(3/4)) - (Sqrt[d]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(4*Sqrt[2]*a^(5/4)*b^(3/4))
```

fricas [A] time = 1.60, size = 232, normalized size = 0.82

$$-\frac{4\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac14}\arctan\left(-\frac{\sqrt{dx}\,abd\left(-\frac{d^2}{a^5b^3}\right)^{\frac14}-\sqrt{-a^3bd^2\sqrt{-\frac{d^2}{a^5b^3}}+d^3x}\,ab\left(-\frac{d^2}{a^5b^3}\right)^{\frac14}}{d^2}\right)-\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac14}\log\left(a^4b^2\left(-\frac{d^2}{a^5b^3}\right)^{\frac34}+\sqrt{dx}\,d\right)+\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac14}\log\left(-a^4b^2\left(-\frac{d^2}{a^5b^3}\right)^{\frac34}+\sqrt{dx}\,d\right)-4\sqrt{dx}\,x}{8\left(abx^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*arctan(-(sqrt(d*x)*a*b*d*(-d
^2/(a^5*b^3))^(1/4) - sqrt(-a^3*b*d^2*sqrt(-d^2/(a^5*b^3)) + d^3*x)*a*b*(-d
^2/(a^5*b^3))^(1/4))/d^2) - (a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*log(a^4*
b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) + (a*b*x^2 + a^2)*(-d^2/(a^5*b^3)
)^(1/4)*log(-a^4*b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) - 4*sqrt(d*x)*x)
/(a*b*x^2 + a^2)
```

giac [A] time = 0.19, size = 264, normalized size = 0.93

$$\frac{\frac{8\sqrt{dx}\,d^3x}{\left(bd^2x^2+ad^2\right)a}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{a^2b^3}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac34}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{a^2b^3}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac34}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^3}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac34}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^3}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/16*(8*sqrt(d*x)*d^3*x/((b*d^2*x^2 + a*d^2)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/
```

```
4))/(a^2*b^3) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) - sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*
b^3) + sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(a^2*b^3))/d
```

maple [A] time = 0.01, size = 210, normalized size = 0.74

$$\frac{\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} ab}+\frac{\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} ab}+\frac{\sqrt{2}\, d \ln\left(\frac{dx-\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^{2}}{b}}}{dx+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^{2}}{b}}}\right)}{16\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} ab}+\frac{(dx)^{\frac{3}{2}} d}{2\left(b d^{2} x^{2}+d^{2} a\right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*d*(d*x)^(3/2)/a/(b*d^2*x^2+a*d^2)+1/16*d/a/b/(a/b*d^2)^(1/4)*2^(1/2)*ln
((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1/8*d/a/b/(a/b*d^2)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+1/8*d/a/b/(a/b*d^2)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.06, size = 255, normalized size = 0.90

$$\frac{\frac{8 (dx)^{\frac{3}{2}} d^{2}}{abd^{2}x^{2}+a^{2}d^{2}}+\frac{d^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\, d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\, d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\, dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(ad^{2}\right)^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\, dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(ad^{2}\right)^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{a}}{16\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*(d*x)^(3/2)*d^2/(a*b*d^2*x^2 + a^2*d^2) + d^2*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt
(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(
2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + s
qrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*
d)/((a*d^2)^(1/4)*b^(3/4)))/a)/d
```

mupad [B] time = 0.11, size = 90, normalized size = 0.32

$$\frac{\sqrt{d}\,\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{5/4}\,b^{3/4}}-\frac{\sqrt{d}\,\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{5/4}\,b^{3/4}}+\frac{d\,(d\,x)^{3/2}}{2\,a\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(5/4)*b
^(3/4)) - (d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a
)^(5/4)*b^(3/4)) + (d*(d*x)^(3/2))/(2*a*(a*d^2 + b*d^2*x^2))
```

sympy [A] time = 6.67, size = 78, normalized size = 0.28

$$\frac{2d^3\,(dx)^{\frac{3}{2}}}{4a^2d^4+4abd^4x^2}+2d^3\operatorname{RootSum}\left(65536t^4a^5b^3d^{10}+1,\left(t\mapsto t\log\left(4096t^3a^4b^2d^8+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*d**3*(d*x)**(3/2)/(4*a**2*d**4 + 4*a*b*d**4*x**2) + 2*d**3*RootSum(65536*
_t**4*a**5*b**3*d**10 + 1, Lambda(_t, _t*log(4096*_t**3*a**4*b**2*d**8 + sq
rt(d*x))))
```

3.514 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=283

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{}}$$

Rubi [A] time = 0.26, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] Sqrt[d*x]/(2*a*d*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt
[d]) - (3*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)
*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{(3b)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2ad}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{(3b)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^{3/2}d^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\right.}{4a^{3/2}d^2}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{dx}\right)}{8a^{3/2}\sqrt{b}} + \frac{3\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}\,d}{\sqrt{b}}+\ldots}\right.}{8a\ldots}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} - \frac{3\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}} + \frac{3\log\left(\sqrt{a}\,\sqrt{d}\right.}{}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}} - \frac{3\log\left(\sqrt{\right.}{}$$

Mathematica [A] time = 0.17, size = 211, normalized size = 0.75

$$\frac{\sqrt{x}\left(\frac{8a^{3/4}\sqrt{x}}{a+bx^2}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}\right)}{16a^{7/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (Sqrt[x]*((8*a^(3/4)*Sqrt[x])/(a + b*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)
```

```
*Sqrt[x])/a^(1/4)])/b^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(16*a^(7/4)*Sqrt[d*x])
```

IntegrateAlgebraic [A] time = 0.40, size = 181, normalized size = 0.64

$$-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{d\sqrt{dx}}{2a\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (d*Sqrt[d*x])/(2*a*(a*d^2 + b*d^2*x^2)) - (3*ArcTan[((a^(1/4)*Sqrt[d])/(Sqr
t[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(4*Sqrt[
2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*S
qrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d])
```

fricas [A] time = 2.31, size = 232, normalized size = 0.82

$$\frac{12\left(abdx^2+a^2d\right)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4d^2\sqrt{-\frac{1}{a^7bd^2}}+dx}\,a^5bd\left(-\frac{1}{a^7bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^5bd\left(-\frac{1}{a^7bd^2}\right)^{\frac{3}{4}}\right)+3\left(abdx^2+a^2d\right)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\log\left(a^2d\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-3\left(abdx^2+a^2d\right)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\log\left(-a^2d\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\sqrt{dx}}{8\left(abdx^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(12*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*arctan(sqrt(a^4*d^2*sqrt
(-1/(a^7*b*d^2)) + d*x)*a^5*b*d*(-1/(a^7*b*d^2))^(3/4) - sqrt(d*x)*a^5*b*d*
(-1/(a^7*b*d^2))^(3/4)) + 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*log(
a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) - 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*
b*d^2))^(1/4)*log(-a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) + 4*sqrt(d*x))
/(a*b*d*x^2 + a^2*d)
```

giac [A] time = 0.19, size = 269, normalized size = 0.95

$$\frac{\sqrt{dx}\,d}{2\left(bd^2x^2+ad^2\right)a}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^2bd}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^2bd}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16a^2bd}-\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*d/((b*d^2*x^2 + a*d^2)*a) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a
^2*b*d) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b*d) + 3/16*sqrt(2)*(a*b^3*d
^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2
*b*d) - 3/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a^2*b*d)
```

maple [A] time = 0.01, size = 207, normalized size = 0.73

$$\frac{\sqrt{dx}\, d}{2\left(b\,d^2x^2+d^2a\right)a}+\frac{3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^2d}+\frac{3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^2d}+\frac{3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x)
```

```
[Out] 1/2*d*(d*x)^(1/2)/a/(b*d^2*x^2+a*d^2)+3/16/d/a^2*(a/b*d^2)^(1/4)*2^(1/2)*ln
((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+3/8/d/a^2*(a/b*d^2)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+3/8/d/a^2*(a/b*d^2)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.00, size = 261, normalized size = 0.92

$$\frac{\frac{8\sqrt{dx}\,d^2}{abd^2x^2+a^2d^2}+\frac{3\left(\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*sqrt(d*x)*d^2/(a*b*d^2*x^2 + a^2*d^2) + 3*(sqrt(2)*d^2*log(sqrt(b)*
d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b
^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*
d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))
/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a)/d
```

mupad [B] time = 0.10, size = 90, normalized size = 0.32

$$\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{7/4}\,b^{1/4}\,\sqrt{d}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{7/4}\,b^{1/4}\,\sqrt{d}}+\frac{d\,\sqrt{d\,x}}{2\,a\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (3*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(7/4)*b^(1/4)*
d^(1/2)) + (3*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(7
/4)*b^(1/4)*d^(1/2)) + (d*(d*x)^(1/2))/(2*a*(a*d^2 + b*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**2), x)
```

3.515 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=300

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\tan^{-1}}{4\sqrt{}}$$

Rubi [A] time = 0.32, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}}-\frac{5}{2a^2d\sqrt{dx}}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -5/(2*a^2*d*Sqrt[d*x]) + 1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*b^(1/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(9/4)*d^
(3/2)) - (5*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(4*Sqrt[2]*a^(9/4)*d^(3/2)) - (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*
Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
 + (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} + \frac{(5b)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} - \frac{\left(5b^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a^2d^2}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} - \frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2a^2d^3}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} + \frac{\left(5b^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^2d^3} -$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} - \frac{\left(5\sqrt[4]{b}\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x,\right.}{8\sqrt{2}\,a^{9/4}d^{3/2}}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} - \frac{5\sqrt[4]{b}\,\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{8\sqrt{2}\,a^{9/4}d^{3/2}}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} + \frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}} - \frac{5\sqrt[4]{b}\,\tan^{-1}}{4\sqrt{}}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.10

$$-\frac{2x\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^2(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-1/4, 2, 3/4, -((b*x^2)/a)])/(a^2*(d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.48, size = 199, normalized size = 0.66

$$\frac{5\sqrt[4]{b}\,\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{-4ad^2-5bd^2x^2}{2a^2d\sqrt{dx}\,\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-4*a*d^2 - 5*b*d^2*x^2)/(2*a^2*d*Sqrt[d*x]*(a*d^2 + b*d^2*x^2)) + (5*b^(1/
4)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[
2]*a^(1/4)))/Sqrt[d*x]])/(4*Sqrt[2]*a^(9/4)*d^(3/2)) + (5*b^(1/4)*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(4*S
qrt[2]*a^(9/4)*d^(3/2))
```

fricas [A] time = 0.76, size = 276, normalized size = 0.92

$$\frac{20\left(a^2bd^2x^3+a^3d^2x\right)\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\sqrt{dx}\,a^2bd\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}-\sqrt{-15625a^5bd^4\sqrt{-\frac{b}{a^9d^6}}+15625b^2dx}\,a^2d\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}}{125b}\right)-5\left(a^2bd^2x^3+a^3d^2x\right)\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}\log\left(125a^7d^5\left(-\frac{b}{a^9d^6}\right)^{\frac{3}{4}}+125\sqrt{dx}\,b\right)+5\left(a^2bd^2x^3+a^3d^2x\right)\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}\log\left(-125a^7d^5\left(-\frac{b}{a^9d^6}\right)^{\frac{3}{4}}+125\sqrt{dx}\,b\right)-4\left(5bx^2+4a\right)\sqrt{dx}}{8\left(a^2bd^2x^3+a^3d^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(20*(a^2*b*d^2*x^3 + a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*arctan(-1/125*(125
*sqrt(d*x)*a^2*b*d*(-b/(a^9*d^6))^(1/4) - sqrt(-15625*a^5*b*d^4*sqrt(-b/(a^
9*d^6)) + 15625*b^2*d*x)*a^2*d*(-b/(a^9*d^6))^(1/4))/b) - 5*(a^2*b*d^2*x^3
+ a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(125*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 12
5*sqrt(d*x)*b) + 5*(a^2*b*d^2*x^3 + a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(-12
5*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 125*sqrt(d*x)*b) - 4*(5*b*x^2 + 4*a)*sqrt(
d*x))/(a^2*b*d^2*x^3 + a^3*d^2*x)
```

giac [A] time = 0.18, size = 294, normalized size = 0.98

$$-\frac{\frac{8\left(5bd^2x^2+4ad^2\right)}{\left(\sqrt{dx}\,bd^2x^2+\sqrt{dx}\,ad^2\right)a^2}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}-\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^2d^2}+\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^2d^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)*a
^2) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2*d^2) + 10*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^3*b^2*d^2) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2*d^2) + 5*sqrt(2)*(a*b^3*d^2)
^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^
2*d^2))/d
```

maple [A] time = 0.02, size = 223, normalized size = 0.74

$$-\frac{(dx)^{\frac{3}{2}}\,b}{2\left(b\,d^2x^2+d^2a\right)a^2d}-\frac{5\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^2d}-\frac{5\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^2d}-\frac{5\sqrt{2}\;\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^2d}-\frac{2}{\sqrt{dx}\;a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/d*b/a^2*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)-5/16/d/a^2/(a/b*d^2)^(1/4)*2^(1/
2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-5/8/d/a^2/(a/b*d^2)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-5/8/d/a^2/(a/b*d^2)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/a^2/d/(d*x)^(1/
2)
```

maxima [A] time = 3.12, size = 268, normalized size = 0.89

$$-\frac{\frac{8\left(5bd^2x^2+4ad^2\right)}{(dx)^{\frac{5}{2}}a^2b+\sqrt{dx}\,a^3d^2}+\frac{5b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((d*x)^(5/2)*a^2*b + sqrt(d*x)*a^3*d^2) +
5*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2
)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((
a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^2)/d
```

mupad [B] time = 0.12, size = 102, normalized size = 0.34

$$\frac{5\,{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{9/4}\,d^{3/2}}-\frac{5\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{9/4}\,d^{3/2}}-\frac{\frac{2\,d}{a}+\frac{5\,b\,d\,x^2}{2\,a^2}}{b\,{\left(d\,x\right)}^{5/2}+a\,d^2\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (5*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(4*a^(9/4)
*d^(3/2)) - (5*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))
/(4*a^(9/4)*d^(3/2)) - ((2*d)/a + (5*b*d*x^2)/(2*a^2))/(b*(d*x)^(5/2) + a*d
^2*(d*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**2), x)
```

3.516 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=300

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}+\frac{7b^{3/4}\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Rubi [A] time = 0.30, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -7/(6*a^2*d*(d*x)^(3/2)) + 1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7*b^(3/4)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(11/
4)*d^(5/2)) - (7*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(4*Sqrt[2]*a^(11/4)*d^(5/2)) + (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sq
rt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d
^(5/2)) - (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{(7b)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} - \frac{\left(7b^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4a^2d^2}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2a^2d^3}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^{5/2}d^4}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{\left(7b^{3/4}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx}{8\sqrt{2}\,a^{11/4}d^{5/2}}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{7b^{3/4}\log\left(\sqrt{a}\sqrt{d} + \sqrt{b}\sqrt{d}\,x - \sqrt{2}\sqrt[4]{a}}{8\sqrt{2}\,a^{11/4}d^{5/2}}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}} - \frac{7b^{3/4}\,\text{ta}}{}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.11

$$-\frac{2x\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^2(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-3/4, 2, 1/4, -((b*x^2)/a)])/(3*a^2*(d*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.47, size = 199, normalized size = 0.66

$$\frac{7b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}}+\frac{-4ad^2-7bd^2x^2}{6a^2d(dx)^{3/2}\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-4*a*d^2 - 7*b*d^2*x^2)/(6*a^2*d*(d*x)^(3/2)*(a*d^2 + b*d^2*x^2)) + (7*b^(
3/4)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqr
t[2]*a^(1/4)))/Sqrt[d*x]])/(4*Sqrt[2]*a^(11/4)*d^(5/2)) - (7*b^(3/4)*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(
4*Sqrt[2]*a^(11/4)*d^(5/2))
```

fricas [A] time = 1.00, size = 300, normalized size = 1.00

$$-\frac{84\left(a^2bd^3x^4+a^3d^3x^2\right)\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^8bd^7\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^6d^6\sqrt{-\frac{b^3}{a^{11}d^{10}}}+b^2dx}\,a^8d^7\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+21\left(a^2bd^3x^4+a^3d^3x^2\right)\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}\log\left(7a^3d^3\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}+7\sqrt{dx}\,b\right)-21\left(a^2bd^3x^4+a^3d^3x^2\right)\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}\log\left(-7a^3d^3\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}+7\sqrt{dx}\,b\right)+4\left(7bx^2+4a\right)\sqrt{dx}}{24\left(a^2bd^3x^4+a^3d^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/24*(84*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*arctan(-(s
qrt(d*x)*a^8*b*d^7*(-b^3/(a^11*d^10))^(3/4) - sqrt(a^6*d^6*sqrt(-b^3/(a^11*
d^10)) + b^2*d*x)*a^8*d^7*(-b^3/(a^11*d^10))^(3/4))/b^3) + 21*(a^2*b*d^3*x^
4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*log(7*a^3*d^3*(-b^3/(a^11*d^10))^
(1/4) + 7*sqrt(d*x)*b) - 21*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10)
)^(1/4)*log(-7*a^3*d^3*(-b^3/(a^11*d^10))^(1/4) + 7*sqrt(d*x)*b) + 4*(7*b*x
^2 + 4*a)*sqrt(d*x))/(a^2*b*d^3*x^4 + a^3*d^3*x^2)
```

giac [A] time = 0.19, size = 276, normalized size = 0.92

$$-\frac{\sqrt{dx}\,b}{2\left(bd^2x^2+ad^2\right)a^2d}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^3d^3}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^3d^3}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16a^3d^3}+\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16a^3d^3}-\frac{2}{3\sqrt{dx}\,a^2d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d*x)*b/((b*d^2*x^2 + a*d^2)*a^2*d) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4
)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a^3*d^3) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*d^3) - 7/16*sqrt(2)*(a*
b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^3*d^3) + 7/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*d^3) - 2/3/(sqrt(d*x)*a^2*d^2*x)
```

maple [A] time = 0.02, size = 226, normalized size = 0.75

$$-\frac{\sqrt{dx}\;b}{2\left(b\,d^2x^2+d^2a\right)a^2d}-\frac{2}{3\left(dx\right)^{\frac{3}{2}}a^2d}-\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^3d^3}-\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^3d^3}-\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\;b\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16a^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/d/a^2*b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)-7/16/d^3/a^3*b*(a/b*d^2)^(1/4)*2
^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-7/8/d^3/a^3*b*(a/b*d^2)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-7/8/d^3/a^3*b*
(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/3/a
^2/d/(d*x)^(3/2)
```

maxima [A] time = 3.08, size = 275, normalized size = 0.92

$$-\frac{\frac{8\left(7bd^2x^2+4ad^2\right)}{(dx)^{\frac{7}{2}}a^2b+(dx)^{\frac{3}{2}}a^3d^2}+\frac{21\left(\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}}+\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}d}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}d}\right)}{a^2}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/48*(8*(7*b*d^2*x^2 + 4*a*d^2)/((d*x)^(7/2)*a^2*b + (d*x)^(3/2)*a^3*d^2)
+ 21*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*d*x - sqrt(2)
*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)*b*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt
(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d))/a^2)/d
```

mupad [B] time = 4.40, size = 102, normalized size = 0.34

$$\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{11/4}\,d^{5/2}}-\frac{\frac{2\,d}{3\,a}+\frac{7\,b\,d\,x^2}{6\,a^2}}{b\,{\left(d\,x\right)}^{7/2}+a\,d^2\,{\left(d\,x\right)}^{3/2}}+\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{11/4}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (7*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(4*a^(11/4)
*d^(5/2)) - ((2*d)/(3*a) + (7*b*d*x^2)/(6*a^2))/(b*(d*x)^(7/2) + a*d^2*(d*x
)^(3/2)) + (7*(-b)^(3/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))
/(4*a^(11/4)*d^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx\right)^{\frac{5}{2}} \left(a + bx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**2), x)
```

3.517 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=318

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Rubi [A] time = 0.34, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{13/4}d^{7/2}}+\frac{9b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{13/4}d^{7/2}}+\frac{9b}{2a^3d^3\sqrt{dx}}-\frac{9}{10a^2d(dx)^{5/2}}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -9/(10*a^2*d*(d*x)^(5/2)) + (9*b)/(2*a^3*d^3*Sqrt[d*x]) + 1/(2*a*d*(d*x)^(5
/2)*(a + b*x^2)) - (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b
^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(8*Sqrt[2]*a^(13/4)*d^(7/2)) - (9*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(13/4)*
d^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{(9b)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{\left(9b^2\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{4a^2d^2}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^3\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a^3d^4}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^3\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2}{d}\cdots}\cdots\right.}{2a^3d^5\cdots}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{\left(9b^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}\,\cdots}{ab\cdots}\cdots\right.}{4a^3\cdots}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^{5/4}\right)\text{Subst}\left(\int \frac{\cdots}{-\frac{\sqrt{a}\cdots}{\sqrt{\cdots}}}\cdots\right.}{8\sqrt{2}\cdots}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{9b^{5/4}\log\left(\sqrt{a}\sqrt{d}+\cdots\right.}{8\sqrt{\cdots}}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}\cdots}\right.}{4\sqrt{2}\,a^{13/4}d^{7/2}\cdots}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.12

$$-\frac{2\sqrt{dx}\ {}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^2d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-2*Sqrt[d*x]*Hypergeometric2F1[-5/4, 2, -1/4, -((b*x^2)/a)])/(5*a^2*d^4*x^
3)
```

IntegrateAlgebraic [A] time = 0.48, size = 213, normalized size = 0.67

$$-\frac{9b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{4\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{4\sqrt{2}\,a^{13/4}d^{7/2}}+\frac{-4a^2d^4+36abd^4x^2+45b^2d^4x^4}{10a^3d^3(dx)^{5/2}\left(ad^2+bd^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-4*a^2*d^4 + 36*a*b*d^4*x^2 + 45*b^2*d^4*x^4)/(10*a^3*d^3*(d*x)^(5/2)*(a*d
^2 + b*d^2*x^2)) - (9*b^(5/4)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) -
 (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(4*Sqrt[2]*a^(13/4)*d^(
7/2)) - (9*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqr
t[a]*d + Sqrt[b]*d*x)])/(4*Sqrt[2]*a^(13/4)*d^(7/2))
```

fricas [A] time = 0.79, size = 323, normalized size = 1.02

$$-\frac{180\left(a^3bd^4x^5+a^4d^4x^3\right)\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\sqrt{dx}\,a^3b^4d^3\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-531441\,a^7b^5d^8\sqrt{-\frac{b^5}{a^{13}d^{14}}}+531441\,b^8dx}\,a^3d^3\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}}{729\,b^5}\right)-45\left(a^3bd^4x^5+a^4d^4x^3\right)\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}\log\left(729\,a^{10}d^{11}\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{3}{4}}+729\sqrt{dx}\,b^4\right)+45\left(a^3bd^4x^5+a^4d^4x^3\right)\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}\log\left(-729\,a^{10}d^{11}\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{3}{4}}+729\sqrt{dx}\,b^4\right)-4\left(45\,b^2x^4+36\,abx^2-4\,a^2\right)\sqrt{dx}}{40\left(a^3bd^4x^5+a^4d^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/40*(180*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*arctan(-1
/729*(729*sqrt(d*x)*a^3*b^4*d^3*(-b^5/(a^13*d^14))^(1/4) - sqrt(-531441*a^7
*b^5*d^8*sqrt(-b^5/(a^13*d^14)) + 531441*b^8*d*x)*a^3*d^3*(-b^5/(a^13*d^14)
)^(1/4))/b^5) - 45*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*l
og(729*a^10*d^11*(-b^5/(a^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) + 45*(a^3*b*
d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*log(-729*a^10*d^11*(-b^5/(a
^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) - 4*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)
*sqrt(d*x))/(a^3*b*d^4*x^5 + a^4*d^4*x^3)
```

giac [A] time = 0.18, size = 307, normalized size = 0.97

$$\frac{\sqrt{dx}\,b^2x}{2\left(bd^2x^2+ad^2\right)a^3d^2}+\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4bd^5}+\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4bd^5}-\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^4bd^5}+\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^4bd^5}+\frac{2\left(10\,bd^2x^2-ad^2\right)}{5\sqrt{dx}\,a^3d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*b^2*x/((b*d^2*x^2 + a*d^2)*a^3*d^2) + 9/8*sqrt(2)*(a*b^3*d^2)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^4*b*d^5) + 9/8*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d^5) - 9/16*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^4*b*d^5) + 9/16*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d^5) + 2/5*(10*b*d^2*x^2 -
a*d^2)/(sqrt(d*x)*a^3*d^5*x^2)
```

maple [A] time = 0.02, size = 242, normalized size = 0.76

$$-\frac{2}{5\left(dx\right)^{\frac{5}{2}} a^{2} d}+\frac{\left(dx\right)^{\frac{3}{2}} b^{2}}{2\left(b\,d^{2} x^{2}+d^{2} a\right) a^{3} d^{3}}+\frac{9\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} a^{3} d^{3}}+\frac{9\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} a^{3} d^{3}}+\frac{9\sqrt{2}\,b\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{16\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} a^{3} d^{3}}+\frac{4b}{\sqrt{dx}\,a^{3} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2/d^3*b^2/a^3*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+9/16/d^3*b/a^3/(a/b*d^2)^(1/4
)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x
+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+9/8/d^3*b/a^3/(a/b*d
^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+9/8/d^3*b/a
^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/
5/a^2/d/(d*x)^(5/2)+4*b/a^3/d^3/(d*x)^(1/2)
```

maxima [A] time = 3.01, size = 290, normalized size = 0.91

$$\frac{\frac{8\left(45\,b^{2}d^{4}x^{4}+36\,abd^{4}x^{2}-4\,a^{2}d^{4}\right)}{\left(dx\right)^{\frac{9}{2}}a^{3}bd^{2}+\left(dx\right)^{\frac{5}{2}}a^{4}d^{4}}+\frac{45\,b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^{3}d^{2}}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/80*(8*(45*b^2*d^4*x^4 + 36*a*b*d^4*x^2 - 4*a^2*d^4)/((d*x)^(9/2)*a^3*b*d^
2 + (d*x)^(5/2)*a^4*d^4) + 45*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(s
qrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)
^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a
```

```
)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d
*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^
(3/4)))/(a^3*d^2))/d
```

mupad [B] time = 4.35, size = 113, normalized size = 0.36

$$\frac{\frac{9\,b^2\,d\,x^4}{2\,a^3}-\frac{2\,d}{5\,a}+\frac{18\,b\,d\,x^2}{5\,a^2}}{b\,{\left(d\,x\right)}^{9/2}+a\,d^2\,{\left(d\,x\right)}^{5/2}}-\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{13/4}\,d^{7/2}}+\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{13/4}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] ((9*b^2*d*x^4)/(2*a^3) - (2*d)/(5*a) + (18*b*d*x^2)/(5*a^2))/(b*(d*x)^(9/2)
 + a*d^2*(d*x)^(5/2)) - (9*(-b)^(5/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4
)*d^(1/2))))/(4*a^(13/4)*d^(7/2)) + (9*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(
1/2))/(a^(1/4)*d^(1/2))))/(4*a^(13/4)*d^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**2), x)
```

3.518 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx$

Optimal. Leaf size=368

$$-\frac{663a^{5/4}d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{21/4}}+\frac{663a^{5/4}d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{256\sqrt{2}\,b^{21/4}}$$

Rubi [A] time = 0.42, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{663a^{5/4}d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{21/4}}+\frac{663a^{5/4}d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{21/4}}-\frac{663a^{5/4}d^{19/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,b^{21/4}}+\frac{663a^{5/4}d^{19/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,b^{21/4}}-\frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}-\frac{663ad^9\sqrt{dx}}{64b^5}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}+\frac{663d^7(dx)^{5/2}}{320b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-663*a*d^9*Sqrt[d*x])/(64*b^5) + (663*d^7*(d*x)^(5/2))/(320*b^4) - (d*(d*x
)^(17/2))/(6*b*(a + b*x^2)^3) - (17*d^3*(d*x)^(13/2))/(48*b^2*(a + b*x^2)^2
) - (221*d^5*(d*x)^(9/2))/(192*b^3*(a + b*x^2)) - (663*a^(5/4)*d^(19/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(21/
4)) + (663*a^(5/4)*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(128*Sqrt[2]*b^(21/4)) - (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*
b^(21/4)) + (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(21/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(17b^2d^2\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{96}\left(221d^4\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} + \frac{\left(663d^6\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} - \frac{\left(663ad^8\right)\int \frac{}{a}}{128b}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right.}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right.}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right.}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right.}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right.}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right.}$$

Mathematica [A] time = 0.23, size = 347, normalized size = 0.94

$$\frac{d^9\sqrt{dx}\left(\frac{-69615\sqrt{2}a^{5/4}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+69615\sqrt{2}a^{5/4}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+139230\sqrt{2}a^{5/4}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)-848640a^4\sqrt[4]{b}\sqrt{x}-2036736a^3b^{5/4}x^{5/2}+106080a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-1584128a^2b^{9/4}x^{9/2}+185640a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2-365568ab^{13/4}x^{13/2}+21504b^{17/4}x^{17/2}}{\left(a+bx^2\right)^3}-139230\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{53760b^{21/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^9*Sqrt[d*x]*(-139230*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] + (-848640*a^4*b^(1/4)*Sqrt[x] - 2036736*a^3*b^(5/4)*x^(5/2) - 1
584128*a^2*b^(9/4)*x^(9/2) - 365568*a*b^(13/4)*x^(13/2) + 21504*b^(17/4)*x^
(17/2) + 106080*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2) + 185640*a^2*b^(1/4)*Sqrt[x
]*(a + b*x^2)^2 + 139230*Sqrt[2]*a^(5/4)*(a + b*x^2)^3*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)] - 69615*Sqrt[2]*a^(5/4)*(a + b*x^2)^3*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 69615*Sqrt[2]*a^(5/4)*(a
+ b*x^2)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a +
 b*x^2)^3))/(53760*b^(21/4)*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.94, size = 222, normalized size = 0.60

$$-\frac{663a^{5/4}d^{19/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}b^{21/4}}+\frac{663a^{5/4}d^{19/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x}\right)}{128\sqrt{2}b^{21/4}}-\frac{d^9\sqrt{dx}\left(9945a^4+27846a^3bx^2+24973a^2b^2x^4+6528ab^3x^6-384b^4x^8\right)}{960b^5\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/960*(d^9*Sqrt[d*x]*(9945*a^4 + 27846*a^3*b*x^2 + 24973*a^2*b^2*x^4 + 652
8*a*b^3*x^6 - 384*b^4*x^8))/(b^5*(a + b*x^2)^3) - (663*a^(5/4)*d^(19/2)*Arc
Tan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(
1/4)))/Sqrt[d*x]])/(128*Sqrt[2]*b^(21/4)) + (663*a^(5/4)*d^(19/2)*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x])/(Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x)])/
(128*Sqrt[2]*b^(21/4))
```

fricas [A] time = 1.00, size = 399, normalized size = 1.08

$$\frac{39780\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^8x^6+3ab^7x^4+3a^2b^6x^2+a^3b^5\right)\arctan\left(-\frac{\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{dx}ab^{16}d^9-\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{a^2d^{19}x+\sqrt{-\frac{a^5d^{38}}{b^{21}}}b^{10}}b^{16}}{a^5d^{38}}\right)+9945\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^8x^6+3ab^7x^4+3a^2b^6x^2+a^3b^5\right)\log\left(663\sqrt{dx}ad^9+663\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}b^5\right)-9945\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^8x^6+3ab^7x^4+3a^2b^6x^2+a^3b^5\right)\log\left(663\sqrt{dx}ad^9-663\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}b^5\right)+4\left(384b^4d^9x^8-6528ab^3d^9x^6-24973a^2b^2d^9x^4-27846a^3bd^9x^2-9945a^4d^9\right)\sqrt{dx}}{3840\left(b^8x^6+3ab^7x^4+3a^2b^6x^2+a^3b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3840*(39780*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2
 + a^3*b^5)*arctan(-((-a^5*d^38/b^21)^(3/4)*sqrt(d*x)*a*b^16*d^9 - (-a^5*d^
```

```
38/b^21)^(3/4)*sqrt(a^2*d^19*x + sqrt(-a^5*d^38/b^21)*b^10)*b^16)/(a^5*d^38
)) + 9945*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a
^3*b^5)*log(663*sqrt(d*x)*a*d^9 + 663*(-a^5*d^38/b^21)^(1/4)*b^5) - 9945*(-
a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)*log(
663*sqrt(d*x)*a*d^9 - 663*(-a^5*d^38/b^21)^(1/4)*b^5) + 4*(384*b^4*d^9*x^8
- 6528*a*b^3*d^9*x^6 - 24973*a^2*b^2*d^9*x^4 - 27846*a^3*b*d^9*x^2 - 9945*a
^4*d^9)*sqrt(d*x))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

giac [A] time = 0.22, size = 336, normalized size = 0.91

$$\frac{1}{7680}d^9\left(\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{9945\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6}-\frac{9945\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6}-\frac{40\left(617\sqrt{dx}a^2b^2d^6x^4+1038\sqrt{dx}a^3bd^6x^2+453\sqrt{dx}a^4d^6\right)}{\left(bd^2x^2+ad^2\right)^3b^5}+\frac{3072\left(\sqrt{dx}b^{16}d^{10}x^2-20\sqrt{dx}ab^{15}d^{10}\right)}{b^{20}d^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/7680*d^9*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^6 + 19890*sqrt(2)*(a*b^3*d
^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/b^6 + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6 - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)
*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6 - 40*(6
17*sqrt(d*x)*a^2*b^2*d^6*x^4 + 1038*sqrt(d*x)*a^3*b*d^6*x^2 + 453*sqrt(d*x)
*a^4*d^6)/((b*d^2*x^2 + a*d^2)^3*b^5) + 3072*(sqrt(d*x)*b^16*d^10*x^2 - 20*
sqrt(d*x)*a*b^15*d^10)/(b^20*d^10))
```

maple [A] time = 0.02, size = 306, normalized size = 0.83

$$-\frac{151\sqrt{dx}\,a^4d^{15}}{64\left(bd^2x^2+d^2a\right)^3b^5}-\frac{173\left(dx\right)^{\frac{5}{2}}a^3d^{13}}{32\left(bd^2x^2+d^2a\right)^3b^4}-\frac{617\left(dx\right)^{\frac{9}{2}}a^2d^{11}}{192\left(bd^2x^2+d^2a\right)^3b^3}+\frac{663\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ad^9\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256b^5}+\frac{663\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ad^9\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}+1\right)}{256b^5}+\frac{663\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ad^9\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)}{512b^5}-\frac{8\sqrt{dx}\,ad^9}{b^5}+\frac{2\left(dx\right)^{\frac{5}{2}}d^7}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/5*d^7*(d*x)^(5/2)/b^4-8*a*d^9*(d*x)^(1/2)/b^5-617/192*d^11/b^3*a^2/(b*d^2
*x^2+a*d^2)^3*(d*x)^(9/2)-173/32*d^13/b^4*a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(5/
2)-151/64*d^15/b^5*a^4/(b*d^2*x^2+a*d^2)^3*(d*x)^(1/2)+663/512*d^9/b^5*a*(a
/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)
^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+663/256*
d^9/b^5*a*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2
)+1)+663/256*d^9/b^5*a*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/
4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.10, size = 361, normalized size = 0.98

$$-\frac{\frac{40\left(617\,(dx)^{\frac{9}{2}}a^2b^2d^{12}+1038\,(dx)^{\frac{5}{2}}a^3bd^{14}+453\,\sqrt{dx}\,a^4d^{16}\right)}{b^8d^6x^6+3\,ab^7d^6x^4+3\,a^2b^6d^6x^2+a^3b^5d^6}-\frac{9945\left(\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}\,d^{11}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\,\sqrt{2}\,d^{11}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)a^2}{b^5}-\frac{3072\left((dx)^{\frac{5}{2}}bd^8-20\,\sqrt{dx}\,ad^{10}\right)}{b^5}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/7680*(40*(617*(d*x)^(9/2)*a^2*b^2*d^12 + 1038*(d*x)^(5/2)*a^3*b*d^14 + 4
53*sqrt(d*x)*a^4*d^16)/(b^8*d^6*x^6 + 3*a*b^7*d^6*x^4 + 3*a^2*b^6*d^6*x^2 +
 a^3*b^5*d^6) - 9945*(sqrt(2)*d^12*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*
sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^12*log(s
qrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^
(3/4)*b^(1/4)) + 2*sqrt(2)*d^11*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(a)) + 2*sqrt(2)*d^11*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(a)))*a^2/b^5 - 3072*((d*x)^(5/2)*b*d^8 - 20*sqrt(d*x)*a*d^10)/b^5
)/d
```

mupad [B] time = 0.13, size = 188, normalized size = 0.51

$$\frac{2\,d^7\,{\left(d\,x\right)}^{5/2}}{5\,b^4}-\frac{\frac{151\,a^4\,d^{15}\,\sqrt{d\,x}}{64}+\frac{617\,a^2\,b^2\,d^{11}\,{\left(d\,x\right)}^{9/2}}{192}+\frac{173\,a^3\,b\,d^{13}\,{\left(d\,x\right)}^{5/2}}{32}}{a^3\,b^5\,d^6+3\,a^2\,b^6\,d^6\,x^2+3\,a\,b^7\,d^6\,x^4+b^8\,d^6\,x^6}-\frac{663\,{\left(-a\right)}^{5/4}\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,b^{21/4}}-\frac{8\,a\,d^9\,\sqrt{d\,x}}{b^5}+\frac{{\left(-a\right)}^{5/4}\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,663{}\mathrm{i}}{128\,b^{21/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*d^7*(d*x)^(5/2))/(5*b^4) - ((151*a^4*d^15*(d*x)^(1/2))/64 + (617*a^2*b^2
*d^11*(d*x)^(9/2))/192 + (173*a^3*b*d^13*(d*x)^(5/2))/32)/(a^3*b^5*d^6 + b^
8*d^6*x^6 + 3*a*b^7*d^6*x^4 + 3*a^2*b^6*d^6*x^2) - (663*(-a)^(5/4)*d^(19/2)
*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(21/4)) + ((-a)^(
5/4)*d^(19/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*663i)/(12
8*b^(21/4)) - (8*a*d^9*(d*x)^(1/2))/b^5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

3.519 $\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=350

$$-\frac{385a^{3/4}d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{19/4}}+\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{19/4}}$$

Rubi [A] time = 0.39, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{385a^{3/4}d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{19/4}}+\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{19/4}}+\frac{385a^{3/4}d^{17/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,b^{19/4}}-\frac{385a^{3/4}d^{17/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,b^{19/4}}-\frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)}-\frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}+\frac{385d^7(dx)^{3/2}}{192b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (385*d^7*(d*x)^(3/2))/(192*b^4) - (d*(d*x)^(15/2))/(6*b*(a + b*x^2)^3) - (5
*d^3*(d*x)^(11/2))/(16*b^2*(a + b*x^2)^2) - (55*d^5*(d*x)^(7/2))/(64*b^3*(a
 + b*x^2)) + (385*a^(3/4)*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) - (385*a^(3/4)*d^(17/2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) -
(385*a^(3/4)*d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(19/4)) + (385*a^(3/4)*d^(17/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(256*Sqrt[2]*b^(19/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{4}\left(5b^2d^2\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} + \frac{1}{32}\left(55d^4\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} + \frac{\left(385d^6\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} - \frac{\left(385ad^8\right)\int \cdots}{128b \cdots}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} - \frac{\left(385ad^7\right)\text{Su}\cdots}{\cdots}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} + \frac{\left(385ad^7\right)\text{Su}\cdots}{\cdots}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} - \frac{\left(385a^{3/4}d^{17/2}\cdots\right.}{\cdots}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} - \frac{385a^{3/4}d^{17/2}\cdots}{\cdots}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} + \frac{385a^{3/4}d^{17/2}\cdots}{1\cdots}$$

Mathematica [C] time = 0.03, size = 87, normalized size = 0.25

$$-\frac{2d^8x\sqrt{dx}\left(-77a^3-99a^2bx^2-45ab^2x^4+77\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-3b^3x^6\right)}{9b^4\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-2*d^8*x*Sqrt[d*x]*(-77*a^3 - 99*a^2*b*x^2 - 45*a*b^2*x^4 - 3*b^3*x^6 + 77
*(a + b*x^2)^3*Hypergeometric2F1[3/4, 4, 7/4, -((b*x^2)/a)]))/(9*b^4*(a + b
*x^2)^3)
```

IntegrateAlgebraic [A] time = 0.89, size = 212, normalized size = 0.61

$$\frac{385a^{3/4}d^{17/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}b^{19/4}}+\frac{385a^{3/4}d^{17/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x}\right)}{128\sqrt{2}b^{19/4}}+\frac{d^8\sqrt{dx}\left(385a^3x+990a^2bx^3+765ab^2x^5+128b^3x^7\right)}{192b^4\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^8*Sqrt[d*x]*(385*a^3*x + 990*a^2*b*x^3 + 765*a*b^2*x^5 + 128*b^3*x^7))/(
192*b^4*(a + b*x^2)^3) + (385*a^(3/4)*d^(17/2)*ArcTan[((a^(1/4)*Sqrt[d])/(S
qrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*S
qrt[2]*b^(19/4)) + (385*a^(3/4)*d^(17/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x])/(Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x)])/(128*Sqrt[2]*b^(19/4))
```

fricas [A] time = 2.46, size = 399, normalized size = 1.14

$$\frac{4620\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\left(b^7x^6+3ab^6x^4+3a^2b^5x^2+a^3b^4\right)\arctan\left(-\frac{\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^5d^{25}-\sqrt{a^4d^{51}x-\sqrt{-\frac{a^3d^{34}}{b^{19}}}\,a^3b^9d^{34}}\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}b^5}{a^3d^{34}}\right)-1155\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\left(b^7x^6+3ab^6x^4+3a^2b^5x^2+a^3b^4\right)\log\left(57066625\sqrt{dx}\,a^2d^{25}+57066625\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{3}{4}}b^{14}\right)+1155\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\left(b^7x^6+3ab^6x^4+3a^2b^5x^2+a^3b^4\right)\log\left(57066625\sqrt{dx}\,a^2d^{25}-57066625\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{3}{4}}b^{14}\right)+4\left(128b^3d^8x^7+765ab^2d^8x^5+990a^2bd^8x^3+385a^3d^8x\right)\sqrt{dx}}{768\left(b^7x^6+3ab^6x^4+3a^2b^5x^2+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(4620*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
 a^3*b^4)*arctan(-((-a^3*d^34/b^19)^(1/4)*sqrt(d*x)*a^2*b^5*d^25 - sqrt(a^4
*d^51*x - sqrt(-a^3*d^34/b^19)*a^3*b^9*d^34)*(-a^3*d^34/b^19)^(1/4)*b^5)/(a
^3*d^34)) - 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*
x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 + 57066625*(-a^3*d^34/b^19)^
(3/4)*b^14) + 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^
5*x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 - 57066625*(-a^3*d^34/b^19
```

```
)^(3/4)*b^14) + 4*(128*b^3*d^8*x^7 + 765*a*b^2*d^8*x^5 + 990*a^2*b*d^8*x^3
+ 385*a^3*d^8*x)*sqrt(d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^
4)
```

giac [A] time = 0.24, size = 316, normalized size = 0.90

$$\frac{1}{1536}\,d^{8}\left(\frac{1024\,\sqrt{dx}\,x}{b^{4}}-\frac{2310\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{7}d}-\frac{2310\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{7}d}+\frac{1155\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{7}d}-\frac{1155\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{7}d}+\frac{8\left(381\,\sqrt{dx}\,ab^{2}d^{6}x^{5}+606\,\sqrt{dx}\,a^{2}bd^{6}x^{3}+257\,\sqrt{dx}\,a^{3}d^{6}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{3}b^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^8*(1024*sqrt(d*x)*x/b^4 - 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*d)
- 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*d) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)
*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^7*d) - 115
5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(b^7*d) + 8*(381*sqrt(d*x)*a*b^2*d^6*x^5 + 606*sqrt(d*x)*a^2*
b*d^6*x^3 + 257*sqrt(d*x)*a^3*d^6*x)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

maple [A] time = 0.02, size = 290, normalized size = 0.83

$$\frac{257\,(dx)^{\frac{3}{2}}\,a^{3}d^{13}}{192\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{4}}+\frac{101\,(dx)^{\frac{7}{2}}\,a^{2}d^{11}}{32\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{3}}+\frac{127\,(dx)^{\frac{11}{2}}\,a\,d^{9}}{64\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{2}}-\frac{385\sqrt{2}\,a\,d^{9}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{256\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}b^{5}}-\frac{385\sqrt{2}\,a\,d^{9}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{256\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}b^{5}}-\frac{385\sqrt{2}\,a\,d^{9}\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{512\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}b^{5}}+\frac{2\,(dx)^{\frac{3}{2}}\,d^{7}}{3b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/3*d^7*(d*x)^(3/2)/b^4+127/64*d^9*a/b^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(11/2)+1
01/32*d^11*a^2/b^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(7/2)+257/192*d^13*a^3/b^4/(b*
d^2*x^2+a*d^2)^3*(d*x)^(3/2)-385/512*d^9*a/b^5/(a/b*d^2)^(1/4)*2^(1/2)*ln((
d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-385/256*d^9*a/b^5/(a/b*d^2)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-385/256*d^9*a/b^5/(a/
b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.08, size = 334, normalized size = 0.95

$$-\frac{\frac{1155\,ad^{10}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^{4}}-\frac{1024\,(dx)^{\frac{3}{2}}d^{8}}{b^{4}}-\frac{8\left(381\,(dx)^{\frac{11}{2}}ab^{2}d^{10}+606\,(dx)^{\frac{7}{2}}a^{2}bd^{12}+257\,(dx)^{\frac{3}{2}}a^{3}d^{14}\right)}{b^{7}d^{6}x^{6}+3\,ab^{6}d^{6}x^{4}+3\,a^{2}b^{5}d^{6}x^{2}+a^{3}b^{4}d^{6}}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(1155*a*d^10*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*
sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4
) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^4 -
 1024*(d*x)^(3/2)*d^8/b^4 - 8*(381*(d*x)^(11/2)*a*b^2*d^10 + 606*(d*x)^(7/2
)*a^2*b*d^12 + 257*(d*x)^(3/2)*a^3*d^14)/(b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3
*a^2*b^5*d^6*x^2 + a^3*b^4*d^6))/d
```

mupad [B] time = 4.33, size = 171, normalized size = 0.49

$$\frac{\frac{257\,a^3\,d^{13}\,{\left(d\,x\right)}^{3/2}}{192}+\frac{101\,a^2\,b\,d^{11}\,{\left(d\,x\right)}^{7/2}}{32}+\frac{127\,a\,b^2\,d^9\,{\left(d\,x\right)}^{11/2}}{64}}{a^3\,b^4\,d^6+3\,a^2\,b^5\,d^6\,x^2+3\,a\,b^6\,d^6\,x^4+b^7\,d^6\,x^6}+\frac{2\,d^7\,{\left(d\,x\right)}^{3/2}}{3\,b^4}+\frac{385\,{\left(-a\right)}^{3/4}\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,b^{19/4}}+\frac{{\left(-a\right)}^{3/4}\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,385{}\mathrm{i}}{128\,b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((257*a^3*d^13*(d*x)^(3/2))/192 + (101*a^2*b*d^11*(d*x)^(7/2))/32 + (127*a*
b^2*d^9*(d*x)^(11/2))/64)/(a^3*b^4*d^6 + b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3*
a^2*b^5*d^6*x^2) + (2*d^7*(d*x)^(3/2))/(3*b^4) + (385*(-a)^(3/4)*d^(17/2)*a
tan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(19/4)) + ((-a)^(3/
4)*d^(17/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*385i)/(128*
b^(19/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

3.520 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=350

$$\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{17/4}}-\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{17/4}}$$

Rubi [A] time = 0.38, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}+\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{17/4}}-\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{17/4}}+\frac{195\sqrt[4]{a}\,d^{15/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,b^{17/4}}-\frac{195\sqrt[4]{a}\,d^{15/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,b^{17/4}}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}+\frac{195d^7\sqrt{dx}}{64b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (195*d^7*Sqrt[d*x])/(64*b^4) - (d*(d*x)^(13/2))/(6*b*(a + b*x^2)^3) - (13*d
^3*(d*x)^(9/2))/(48*b^2*(a + b*x^2)^2) - (39*d^5*(d*x)^(5/2))/(64*b^3*(a +
b*x^2)) + (195*a^(1/4)*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) + (19
5*a^(1/4)*d^(15/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(25
6*Sqrt[2]*b^(17/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{12}\left(13b^2d^2\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}+\frac{1}{32}\left(39d^4\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{\left(195d^6\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{\left(195ad^8\right)\int \frac{}{\sqrt{dx}(}}{128b^3}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{\left(195ad^7\right)\text{Subst}}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{\left(195\sqrt{a}\,d^6\right)\text{Sub}}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{\left(195\sqrt[4]{a}\,d^{15/2}\right)\text{S}}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{195\sqrt[4]{a}\,d^{15/2}\log}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{195\sqrt[4]{a}\,d^{15/2}\tan}{128\sqrt{}}$$

Mathematica [A] time = 0.14, size = 324, normalized size = 0.93

$$\frac{d^7\sqrt{dx}\left(\frac{49920a^3\sqrt[4]{b}}{(a+bx^2)^3}+\frac{119808a^2b^{5/4}x^2}{(a+bx^2)^3}-\frac{6240a^2\sqrt[4]{b}}{(a+bx^2)^2}+\frac{21504b^{13/4}x^6}{(a+bx^2)^3}+\frac{93184ab^{9/4}x^4}{(a+bx^2)^3}-\frac{10920a\sqrt[4]{b}}{a+bx^2}+\frac{4095\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}-\frac{4095\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}+\frac{8190\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{x}}-\frac{8190\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{x}}\right)}{10752b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^7*Sqrt[d*x]*((49920*a^3*b^(1/4))/(a + b*x^2)^3 + (119808*a^2*b^(5/4)*x^2
)/(a + b*x^2)^3 + (93184*a*b^(9/4)*x^4)/(a + b*x^2)^3 + (21504*b^(13/4)*x^6
)/(a + b*x^2)^3 - (6240*a^2*b^(1/4))/(a + b*x^2)^2 - (10920*a*b^(1/4))/(a +
 b*x^2) + (8190*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/Sqrt[x] - (8190*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/Sqrt[x] + (4095*Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x] - (4095*Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x]))/(10752*b^(17/4))
```

IntegrateAlgebraic [A] time = 0.87, size = 227, normalized size = 0.65

$$\frac{d^7\sqrt{dx}\left(585a^3d^6+1638a^2bd^6x^2+1469ab^2d^6x^4+384b^3d^6x^6\right)}{192b^4\left(ad^2+bd^2x^2\right)^3}+\frac{195\sqrt[4]{a}\,d^{15/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}\,b^{17/4}}-\frac{195\sqrt[4]{a}\,d^{15/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}\,b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^7*Sqrt[d*x]*(585*a^3*d^6 + 1638*a^2*b*d^6*x^2 + 1469*a*b^2*d^6*x^4 + 384
*b^3*d^6*x^6))/(192*b^4*(a*d^2 + b*d^2*x^2)^3) + (195*a^(1/4)*d^(15/2)*ArcT
an[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1
/4)))/Sqrt[d*x]])/(128*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(128*
Sqrt[2]*b^(17/4))
```

fricas [A] time = 0.83, size = 363, normalized size = 1.04

$$-\frac{2340\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^7x^6+3ab^6x^4+3a^2b^5x^2+a^3b^4\right)\arctan\left(-\frac{\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{ad^{30}}{b^{17}}}\,b^8}\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{3}{4}}b^{13}}{ad^{30}}\right)+585\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^7x^6+3ab^6x^4+3a^2b^5x^2+a^3b^4\right)\log\left(195\sqrt{dx}\,d^7+195\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}b^4\right)-585\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^7x^6+3ab^6x^4+3a^2b^5x^2+a^3b^4\right)\log\left(195\sqrt{dx}\,d^7-195\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}b^4\right)-4\left(384b^3d^7x^6+1469ab^2d^7x^4+1638a^2bd^7x^2+585a^3d^7\right)\sqrt{dx}}{768\left(b^7x^6+3ab^6x^4+3a^2b^5x^2+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(2340*(-a*d^30/b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
a^3*b^4)*arctan(-((-a*d^30/b^17)^(3/4)*sqrt(d*x)*b^13*d^7 - sqrt(d^15*x + s
qrt(-a*d^30/b^17)*b^8)*(-a*d^30/b^17)^(3/4)*b^13)/(a*d^30)) + 585*(-a*d^30/
```

```
b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqrt(
d*x)*d^7 + 195*(-a*d^30/b^17)^(1/4)*b^4) - 585*(-a*d^30/b^17)^(1/4)*(b^7*x^
6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqrt(d*x)*d^7 - 195*(-a*
d^30/b^17)^(1/4)*b^4) - 4*(384*b^3*d^7*x^6 + 1469*a*b^2*d^7*x^4 + 1638*a^2*
b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2
+ a^3*b^4)
```

giac [A] time = 0.20, size = 302, normalized size = 0.86

$$-\frac{1}{1536}\,d^{7}\left(\frac{1170\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{5}}+\frac{1170\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{5}}+\frac{585\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{5}}-\frac{585\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{5}}-\frac{3072\,\sqrt{dx}}{b^{4}}-\frac{8\left(317\,\sqrt{dx}\,ab^{2}d^{6}x^{4}+486\,\sqrt{dx}\,a^{2}bd^{6}x^{2}+201\,\sqrt{dx}\,a^{3}d^{6}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{3}b^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 + 1170*sqrt(2)*(a*b^3*d^2)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/b^5 + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1
/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 3072*sqrt(d*x)/b
^4 - 8*(317*sqrt(d*x)*a*b^2*d^6*x^4 + 486*sqrt(d*x)*a^2*b*d^6*x^2 + 201*sqr
t(d*x)*a^3*d^6)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

maple [A] time = 0.02, size = 287, normalized size = 0.82

$$\frac{67\sqrt{dx}\,a^{3}d^{13}}{64\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{4}}+\frac{81\left(dx\right)^{\frac{5}{2}}a^{2}d^{11}}{32\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{3}}+\frac{317\left(dx\right)^{\frac{9}{2}}a\,d^{9}}{192\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{2}}-\frac{195\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{7}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{256b^{4}}-\frac{195\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{7}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{256b^{4}}-\frac{195\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{7}\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{512b^{4}}+\frac{2\sqrt{dx}\,d^{7}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2*d^7*(d*x)^(1/2)/b^4+317/192*d^9/b^2*a/(b*d^2*x^2+a*d^2)^3*(d*x)^(9/2)+81/
32*d^11/b^3*a^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(5/2)+67/64*d^13/b^4*a^3/(b*d^2*x
^2+a*d^2)^3*(d*x)^(1/2)-195/512*d^7/b^4*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-195/256*d^7/b^4*(a/b*d^2)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-195/256*d^7/b^4*(a/b*d^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.14, size = 343, normalized size = 0.98

$$\frac{\frac{3072\,\sqrt{dx}\,d^8}{b^4}+\frac{8\left(317\,(dx)^{\frac{9}{2}}ab^2d^{10}+486\,(dx)^{\frac{5}{2}}a^2bd^{12}+201\,\sqrt{dx}\,a^3d^{14}\right)}{b^7d^6x^6+3\,ab^6d^6x^4+3\,a^2b^5d^6x^2+a^3b^4d^6}-\frac{585\left(\frac{\sqrt{2}\,d^{10}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\,\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}\,b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{10}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\,\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}\,b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}\,d^9\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}\,b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\,\sqrt{2}\,d^9\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}\,b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)a}{b^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(3072*sqrt(d*x)*d^8/b^4 + 8*(317*(d*x)^(9/2)*a*b^2*d^10 + 486*(d*x)^
(5/2)*a^2*b*d^12 + 201*sqrt(d*x)*a^3*d^14)/(b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 +
 3*a^2*b^5*d^6*x^2 + a^3*b^4*d^6) - 585*(sqrt(2)*d^10*log(sqrt(b)*d*x + sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) -
 sqrt(2)*d^10*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + s
qrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^9*arctan(1/2*sqrt(2)*(sqrt(
2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(s
qrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^9*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sq
rt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a/b^4)/d
```

mupad [B] time = 4.30, size = 171, normalized size = 0.49

$$\frac{\frac{67\,a^3\,d^{13}\,\sqrt{d\,x}}{64}+\frac{81\,a^2\,b\,d^{11}\,{(d\,x)}^{5/2}}{32}+\frac{317\,a\,b^2\,d^9\,{(d\,x)}^{9/2}}{192}}{a^3\,b^4\,d^6+3\,a^2\,b^5\,d^6\,x^2+3\,a\,b^6\,d^6\,x^4+b^7\,d^6\,x^6}+\frac{2\,d^7\,\sqrt{d\,x}}{b^4}-\frac{195\,{(-a)}^{1/4}\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,b^{17/4}}+\frac{{(-a)}^{1/4}\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{(-a)}^{1/4}\,\sqrt{d}}\right)\,195{}\mathrm{i}}{128\,b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((67*a^3*d^13*(d*x)^(1/2))/64 + (81*a^2*b*d^11*(d*x)^(5/2))/32 + (317*a*b^2
*d^9*(d*x)^(9/2))/192)/(a^3*b^4*d^6 + b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3*a^2
*b^5*d^6*x^2) + (2*d^7*(d*x)^(1/2))/b^4 - (195*(-a)^(1/4)*d^(15/2)*atan((b^
(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(17/4)) + ((-a)^(1/4)*d^(1
5/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*195i)/(128*b^(17/4
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

3.521 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=333

$$\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\sqrt[4]{a}\,b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\sqrt[4]{a}\,b^{15/4}}-\frac{77d^{13/2}}{1}$$

Rubi [A] time = 0.35, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}+\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\sqrt[4]{a}\,b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\sqrt[4]{a}\,b^{15/4}}-\frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\sqrt[4]{a}\,b^{15/4}}+\frac{77d^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\sqrt[4]{a}\,b^{15/4}}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(11/2))/(6*b*(a + b*x^2)^3) - (11*d^3*(d*x)^(7/2))/(48*b^2*(a + b
*x^2)^2) - (77*d^5*(d*x)^(3/2))/(192*b^3*(a + b*x^2)) - (77*d^(13/2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(1/4)*b
^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(128*Sqrt[2]*a^(1/4)*b^(15/4)) + (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/
4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/4)*b^(15/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{12}\left(11b^2d^2\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}+\frac{1}{96}\left(77d^4\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}+\frac{\left(77d^6\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}+\frac{\left(77d^5\right)\operatorname{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx\right.}{64b^2}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{\left(77d^5\right)\operatorname{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\right.}{128b^{5/2}}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}+\frac{\left(77d^{13/2}\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-}\right.}{256\sqrt{2}\sqrt[4]{a}}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}+\frac{77d^{13/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\right.}{256\sqrt{2}\sqrt[4]{a}}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{128\sqrt{2}\sqrt[4]{a}\,b^{15/4}}$$

Mathematica [C] time = 0.03, size = 83, normalized size = 0.25

$$\frac{2d^6x\sqrt{dx}\left(77\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(77a^2+99abx^2+45b^2x^4\right)\right)}{45ab^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*d^6*x*Sqrt[d*x]*(-(a*(77*a^2 + 99*a*b*x^2 + 45*b^2*x^4)) + 77*(a + b*x^2
)^3*Hypergeometric2F1[3/4, 4, 7/4, -((b*x^2)/a)]))/(45*a*b^3*(a + b*x^2)^3)
```

IntegrateAlgebraic [A] time = 0.86, size = 213, normalized size = 0.64

$$-\frac{d^7(dx)^{3/2}\left(77a^2d^4+198abd^4x^2+153b^2d^4x^4\right)}{192b^3\left(ad^2+bd^2x^2\right)^3}-\frac{77d^{13/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{77d^{13/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/192*(d^7*(d*x)^(3/2)*(77*a^2*d^4 + 198*a*b*d^4*x^2 + 153*b^2*d^4*x^4))/(
b^3*(a*d^2 + b*d^2*x^2)^3) - (77*d^(13/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2
]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*Sqrt[2
]*a^(1/4)*b^(15/4)) - (77*d^(13/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]
*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(1/4)*b^(15/4))
```

fricas [A] time = 1.89, size = 370, normalized size = 1.11

$$-\frac{924\left(b^6x^6+3ab^5x^4+3a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^4d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{ab^{15}}}\,ab^7d^{26}}\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}b^4}{d^{26}}\right)-231\left(b^6x^6+3ab^5x^4+3a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\log\left(456533\sqrt{dx}\,d^{19}+456533\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{3}{4}}ab^{11}\right)+231\left(b^6x^6+3ab^5x^4+3a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\log\left(456533\sqrt{dx}\,d^{19}-456533\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{3}{4}}ab^{11}\right)+4\left(153b^2d^6x^5+198abd^6x^3+77a^2d^6x\right)\sqrt{dx}}{768\left(b^6x^6+3ab^5x^4+3a^2b^4x^2+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(924*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^1
5))^(1/4)*arctan(-((-d^26/(a*b^15))^(1/4)*sqrt(d*x)*b^4*d^19 - sqrt(d^39*x
- sqrt(-d^26/(a*b^15))*a*b^7*d^26)*(-d^26/(a*b^15))^(1/4)*b^4)/d^26) - 231*
(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^15))^(1/4)*lo
g(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a*b^15))^(3/4)*a*b^11) + 231*(b^6*
x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^15))^(1/4)*log(456
533*sqrt(d*x)*d^19 - 456533*(-d^26/(a*b^15))^(3/4)*a*b^11) + 4*(153*b^2*d^6
```

```
*x^5 + 198*a*b*d^6*x^3 + 77*a^2*d^6*x)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 +
3*a^2*b^4*x^2 + a^3*b^3)
```

giac [A] time = 0.20, size = 314, normalized size = 0.94

$$-\frac{1}{1536}d^6\left(\frac{8\left(153\sqrt{dx}\,b^2d^6x^5+198\sqrt{dx}\,abd^6x^3+77\sqrt{dx}\,a^2d^6x\right)}{\left(bd^2x^2+ad^2\right)^3b^3}-\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6d}-\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6d}+\frac{231\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^6d}-\frac{231\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^6d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*d^6*(8*(153*sqrt(d*x)*b^2*d^6*x^5 + 198*sqrt(d*x)*a*b*d^6*x^3 + 77*
sqrt(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*b^3) - 462*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/(a*b^6*d) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(
2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6*d) + 231*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a*b^6*d) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6*d))
```

maple [A] time = 0.02, size = 271, normalized size = 0.81

$$-\frac{77\left(dx\right)^{\frac{3}{2}}a^2d^{11}}{192\left(b\,d^2x^2+d^2a\right)^3b^3}-\frac{33\left(dx\right)^{\frac{7}{2}}a\,d^9}{32\left(b\,d^2x^2+d^2a\right)^3b^2}-\frac{51\left(dx\right)^{\frac{11}{2}}d^7}{64\left(b\,d^2x^2+d^2a\right)^3b}+\frac{77\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^4}+\frac{77\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^4}+\frac{77\sqrt{2}\,d^7\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{512\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -51/64*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(11/2)-33/32*d^9/(b*d^2*x^2+a*d^2)^3
/b^2*a*(d*x)^(7/2)-77/192*d^11/(b*d^2*x^2+a*d^2)^3/b^3*a^2*(d*x)^(3/2)+77/5
12*d^7/b^4/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))+77/256*d^7/b^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*
(d*x)^(1/2)+1)+77/256*d^7/b^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d
^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.44, size = 317, normalized size = 0.95

$$\frac{\frac{231d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^3}-\frac{8\left(153\left(dx\right)^{\frac{11}{2}}b^2d^8+198\left(dx\right)^{\frac{7}{2}}abd^{10}+77\left(dx\right)^{\frac{3}{2}}a^2d^{12}\right)}{b^6d^6x^6+3\,ab^5d^6x^4+3\,a^2b^4d^6x^2+a^3b^3d^6}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(231*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*
sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2
*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(
b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + s
qrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^
2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^3 - 8*(1
53*(d*x)^(11/2)*b^2*d^8 + 198*(d*x)^(7/2)*a*b*d^10 + 77*(d*x)^(3/2)*a^2*d^1
2)/(b^6*d^6*x^6 + 3*a*b^5*d^6*x^4 + 3*a^2*b^4*d^6*x^2 + a^3*b^3*d^6))/d
```

mupad [B] time = 0.11, size = 153, normalized size = 0.46

$$\frac{77\,d^{13/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{1/4}\,b^{15/4}}-\frac{\frac{51\,d^7\,(d\,x)^{11/2}}{64\,b}+\frac{77\,a^2\,d^{11}\,(d\,x)^{3/2}}{192\,b^3}+\frac{33\,a\,d^9\,(d\,x)^{7/2}}{32\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{77\,d^{13/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{1/4}\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (77*d^(13/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(1
/4)*b^(15/4)) - ((51*d^7*(d*x)^(11/2))/(64*b) + (77*a^2*d^11*(d*x)^(3/2))/(
192*b^3) + (33*a*d^9*(d*x)^(7/2))/(32*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*
b*d^6*x^2 + 3*a*b^2*d^6*x^4) - (77*d^(13/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-
a)^(1/4)*d^(1/2))))/(128*(-a)^(1/4)*b^(15/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{13}{2}}}{\left(a + bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(13/2)/(a + b*x**2)**4, x)
```

3.522 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=333

$$-\frac{15d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{15d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{15d^{11/}}{}$$

Rubi [A] time = 0.34, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{15d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{15d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{15d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{15d^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(9/2))/(6*b*(a + b*x^2)^3) - (3*d^3*(d*x)^(5/2))/(16*b^2*(a + b*x
^2)^2) - (15*d^5*Sqrt[d*x])/(64*b^3*(a + b*x^2)) - (15*d^(11/2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(3/4)*b^(13/
4)) + (15*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(128*Sqrt[2]*a^(3/4)*b^(13/4)) - (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt
[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^
(13/4)) + (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{4}\left(3b^2d^2\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}+\frac{1}{32}\left(15d^4\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}+\frac{\left(15d^6\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}+\frac{\left(15d^5\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x\right.}{64b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}+\frac{\left(15d^4\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,d\right.}{128\sqrt{a}\,b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{\left(15d^{11/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}}{}}\right.}{256\sqrt{2}\,a^{3/4}b}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{15d^{11/2}\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\right.}{256\sqrt{2}\,a^{3/4}}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{15d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,a^{3/4}b^{13/4}}$$

Mathematica [A] time = 0.13, size = 299, normalized size = 0.90

$$\frac{d^5\sqrt{dx}\left(-\frac{315\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}+\frac{315\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}-\frac{630\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}\sqrt{x}}+\frac{630\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}\sqrt{x}}-\frac{3840a^2\sqrt[4]{b}}{(a+bx^2)^3}-\frac{9216ab^{5/4}x^2}{(a+bx^2)^3}-\frac{7168b^{9/4}x^4}{(a+bx^2)^3}+\frac{840\sqrt[4]{b}}{a+bx^2}+\frac{480a\sqrt[4]{b}}{(a+bx^2)^2}\right)}{10752b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^5*Sqrt[d*x]*((-3840*a^2*b^(1/4))/(a + b*x^2)^3 - (9216*a*b^(5/4)*x^2)/(a
 + b*x^2)^3 - (7168*b^(9/4)*x^4)/(a + b*x^2)^3 + (480*a*b^(1/4))/(a + b*x^2
)^2 + (840*b^(1/4))/(a + b*x^2) - (630*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) + (630*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) - (315*Sqrt[2]*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x]) + (315*Sqrt[2]*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x
])))/(10752*b^(13/4))
```

IntegrateAlgebraic [A] time = 0.81, size = 213, normalized size = 0.64

$$-\frac{15d^{11/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}a^{3/4}b^{13/4}}+\frac{15d^{11/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}a^{3/4}b^{13/4}}-\frac{d^7\sqrt{dx}\left(45a^2d^4+126abd^4x^2+113b^2d^4x^4\right)}{192b^3\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/192*(d^7*Sqrt[d*x]*(45*a^2*d^4 + 126*a*b*d^4*x^2 + 113*b^2*d^4*x^4))/(b^
3*(a*d^2 + b*d^2*x^2)^3) - (15*d^(11/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*
b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*Sqrt[2]*
a^(3/4)*b^(13/4)) + (15*d^(11/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*S
qrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(3/4)*b^(13/4))
```

fricas [A] time = 1.77, size = 373, normalized size = 1.12

$$\frac{180\left(b^6x^6+3ab^5x^4+3a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,a^2b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{d^{22}}{a^3b^{13}}}a^2b^6}\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{3}{4}}a^2b^{10}}{d^{22}}\right)+45\left(b^6x^6+3ab^5x^4+3a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(15\sqrt{dx}\,d^5+15\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}ab^3\right)-45\left(b^6x^6+3ab^5x^4+3a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(15\sqrt{dx}\,d^5-15\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}ab^3\right)-4\left(113b^2d^5x^4+126abd^5x^2+45a^2d^5\right)\sqrt{dx}}{768\left(b^6x^6+3ab^5x^4+3a^2b^4x^2+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(180*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^
13))^(1/4)*arctan(-((-d^22/(a^3*b^13))^(3/4)*sqrt(d*x)*a^2*b^10*d^5 - sqrt(
```

```
d^11*x + sqrt(-d^22/(a^3*b^13))*a^2*b^6)*(-d^22/(a^3*b^13))^(3/4)*a^2*b^10)
/d^22) + 45*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b
^13))^(1/4)*log(15*sqrt(d*x)*d^5 + 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 45*
(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^13))^(1/4)*
log(15*sqrt(d*x)*d^5 - 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 4*(113*b^2*d^5*
x^4 + 126*a*b*d^5*x^2 + 45*a^2*d^5)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 + 3*a
^2*b^4*x^2 + a^3*b^3)
```

giac [A] time = 0.21, size = 301, normalized size = 0.90

$$\frac{1}{1536}\,d^{5}\left(\frac{90\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{4}}+\frac{90\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{4}}+\frac{45\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\,\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{4}}-\frac{45\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\,\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{4}}-\frac{8\left(113\,\sqrt{dx}\,b^{2}d^{6}x^{4}+126\,\sqrt{dx}\,abd^{6}x^{2}+45\,\sqrt{dx}\,a^{2}d^{6}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{3}b^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^5*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4) + 90*sqrt(2)*(a*b^3*d^2)^
(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a*b^4) + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^
(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log
(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 8*(113*
sqrt(d*x)*b^2*d^6*x^4 + 126*sqrt(d*x)*a*b*d^6*x^2 + 45*sqrt(d*x)*a^2*d^6)/(
(b*d^2*x^2 + a*d^2)^3*b^3))
```

maple [A] time = 0.02, size = 280, normalized size = 0.84

$$-\frac{15\sqrt{dx}\,a^{2}d^{11}}{64\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{3}}-\frac{21\left(dx\right)^{\frac{5}{2}}a\,d^{9}}{32\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{2}}-\frac{113\left(dx\right)^{\frac{9}{2}}d^{7}}{192\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b}+\frac{15\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{5}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{256a\,b^{3}}+\frac{15\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{5}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{256a\,b^{3}}+\frac{15\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{5}\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{512a\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -113/192*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(9/2)-21/32*d^9/(b*d^2*x^2+a*d^2)^
3/b^2*a*(d*x)^(5/2)-15/64*d^11/(b*d^2*x^2+a*d^2)^3/b^3*a^2*(d*x)^(1/2)+15/5
12*d^5/b^3*(a/b*d^2)^(1/4)/a*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(
1/2)))+15/256*d^5/b^3*(a/b*d^2)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1
/4)*(d*x)^(1/2)+1)+15/256*d^5/b^3*(a/b*d^2)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/
(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.13, size = 326, normalized size = 0.98

$$-\frac{\frac{8\left(113\,(dx)^{\frac{9}{2}}b^{2}d^{8}+126\,(dx)^{\frac{5}{2}}abd^{10}+45\,\sqrt{dx}\,a^{2}d^{12}\right)}{b^{6}d^{6}x^{6}+3\,ab^{5}d^{6}x^{4}+3\,a^{2}b^{4}d^{6}x^{2}+a^{3}b^{3}d^{6}}-\frac{45\left(\frac{\sqrt{2}\,d^{8}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{8}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^{7}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^{7}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{b^{3}}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(8*(113*(d*x)^(9/2)*b^2*d^8 + 126*(d*x)^(5/2)*a*b*d^10 + 45*sqrt(d*
x)*a^2*d^12)/(b^6*d^6*x^6 + 3*a*b^5*d^6*x^4 + 3*a^2*b^4*d^6*x^2 + a^3*b^3*d
^6) - 45*(sqrt(2)*d^8*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - s
qrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4))
 + 2*sqrt(2)*d^7*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt
(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
 2*sqrt(2)*d^7*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b
^3)/d
```

mupad [B] time = 4.29, size = 153, normalized size = 0.46

$$-\frac{\frac{113\,d^7\,{(d\,x)}^{9/2}}{192\,b}+\frac{15\,a^2\,d^{11}\,\sqrt{d\,x}}{64\,b^3}+\frac{21\,a\,d^9\,{(d\,x)}^{5/2}}{32\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{15\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,{(-a)}^{3/4}\,b^{13/4}}-\frac{15\,d^{11/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,{(-a)}^{3/4}\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] - ((113*d^7*(d*x)^(9/2))/(192*b) + (15*a^2*d^11*(d*x)^(1/2))/(64*b^3) + (21
*a*d^9*(d*x)^(5/2))/(32*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*
a*b^2*d^6*x^4) - (15*d^(11/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(128*(-a)^(3/4)*b^(13/4)) - (15*d^(11/2)*atanh((b^(1/4)*(d*x)^(1/2))/(
(-a)^(1/4)*d^(1/2))))/(128*(-a)^(3/4)*b^(13/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{11}{2}}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
[Out] Integral((d*x)**(11/2)/(a + b*x**2)**4, x)
```

3.523 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=336

$$\frac{7d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{7d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{7d^{9/2}\tan^{-1}}{128}$$

Rubi [A] time = 0.35, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{7d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{7d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{7d^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(7/2))/(6*b*(a + b*x^2)^3) - (7*d^3*(d*x)^(3/2))/(48*b^2*(a + b*x
^2)^2) + (7*d^3*(d*x)^(3/2))/(64*a*b^2*(a + b*x^2)) - (7*d^(9/2)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(5/4)*b^(11
/4)) + (7*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(128*Sqrt[2]*a^(5/4)*b^(11/4)) + (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11
/4)) - (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(7b^2d^2\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{32}\left(7d^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{\left(7d^4\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128ab}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,\ldots\right.}{64ab}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} - \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,d\ldots\right.}{128ab^{3/2}}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{\left(7d^{9/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{\ldots}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{\ldots}}{\ldots}}\ldots\right.}{256\sqrt{2}\,a^{5/4}\ldots}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{7d^{9/2}\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\ldots\right.}{256\sqrt{2}\,a^{5/4}\ldots}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} - \frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)\ldots}{128\sqrt{2}\,a^{5/4}b^{11/4}}$$

Mathematica [C] time = 0.02, size = 74, normalized size = 0.22

$$\frac{2d^4x\sqrt{dx}\left(7\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\left(7a+9bx^2\right)\right)}{45a^2b^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*d^4*x*Sqrt[d*x]*(-(a^2*(7*a + 9*b*x^2)) + 7*(a + b*x^2)^3*Hypergeometric
2F1[3/4, 4, 7/4, -((b*x^2)/a)]))/(45*a^2*b^2*(a + b*x^2)^3)
```

IntegrateAlgebraic [A] time = 0.87, size = 221, normalized size = 0.66

$$-\frac{7d^{9/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}a^{5/4}b^{11/4}}-\frac{7d^{9/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}a^{5/4}b^{11/4}}+\frac{-7a^2d^9(dx)^{3/2}-18abd^7(dx)^{7/2}+21b^2d^5(dx)^{11/2}}{192ab^2\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-7*a^2*d^9*(d*x)^(3/2) - 18*a*b*d^7*(d*x)^(7/2) + 21*b^2*d^5*(d*x)^(11/2))
/(192*a*b^2*(a*d^2 + b*d^2*x^2)^3) - (7*d^(9/2)*ArcTan[((a^(1/4)*Sqrt[d])/(
Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*
Sqrt[2]*a^(5/4)*b^(11/4)) - (7*d^(9/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(5/4)*b^(11/4))
```

fricas [A] time = 1.15, size = 390, normalized size = 1.16

$$-\frac{84\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\sqrt{dx}ab^3d^{13}-\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{a^5b^{11}}}a^3b^5d^{18}}\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}ab^3}{d^{18}}\right)-21\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\log\left(343\sqrt{dx}d^{13}+343\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{3}{4}}a^4b^8\right)+21\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\log\left(343\sqrt{dx}d^{13}-343\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{3}{4}}a^4b^8\right)-4\left(21b^2d^4x^5-18abd^4x^3-7a^2d^4x\right)\sqrt{dx}}{768\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(84*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^18/(a^
5*b^11))^(1/4)*arctan(-((-d^18/(a^5*b^11))^(1/4)*sqrt(d*x)*a*b^3*d^13 - sqr
t(d^27*x - sqrt(-d^18/(a^5*b^11))*a^3*b^5*d^18)*(-d^18/(a^5*b^11))^(1/4)*a*
b^3)/d^18) - 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^1
8/(a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 + 343*(-d^18/(a^5*b^11))^(3/4)*a
^4*b^8) + 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^18/(
a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 - 343*(-d^18/(a^5*b^11))^(3/4)*a^4*
b^8) - 4*(21*b^2*d^4*x^5 - 18*a*b*d^4*x^3 - 7*a^2*d^4*x)*sqrt(d*x))/(a*b^5*
x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)
```

giac [A] time = 0.21, size = 317, normalized size = 0.94

$$\frac{1}{1536}d^4\left(\frac{8\left(21\sqrt{dx}b^2d^6x^5-18\sqrt{dx}abd^6x^3-7\sqrt{dx}a^2d^6x\right)}{\left(bd^2x^2+ad^2\right)^3ab^2}+\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5d}+\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5d}-\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^5d}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^5d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^4*(8*(21*sqrt(d*x)*b^2*d^6*x^5 - 18*sqrt(d*x)*a*b*d^6*x^3 - 7*sqrt
(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a*b^2) + 42*sqrt(2)*(a*b^3*d^2)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/
4))/(a^2*b^5*d) + 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5*d) - 21*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^2*b^5*d) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5*d))
```

maple [A] time = 0.02, size = 277, normalized size = 0.82

$$-\frac{7 (dx)^{\frac{3}{2}} a d^9}{192 \left(b d^2 x^2+d^2 a\right)^3 b^2}-\frac{3 (dx)^{\frac{7}{2}} d^7}{32 \left(b d^2 x^2+d^2 a\right)^3 b}+\frac{7 (dx)^{\frac{11}{2}} d^5}{64 \left(b d^2 x^2+d^2 a\right)^3 a}+\frac{7\sqrt{2}\, d^5 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a b^3}+\frac{7\sqrt{2}\, d^5 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{256 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a b^3}+\frac{7\sqrt{2}\, d^5 \ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)}{512 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 7/64*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(11/2)-3/32*d^7/(b*d^2*x^2+a*d^2)^3/b*
(d*x)^(7/2)-7/192*d^9/(b*d^2*x^2+a*d^2)^3/b^2*a*(d*x)^(3/2)+7/512*d^5/a/b^3
/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d
^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+7/256
*d^5/a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/
2)+1)+7/256*d^5/a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4
)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.02, size = 323, normalized size = 0.96

$$\frac{\frac{21 d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{ab^2}+\frac{8\left(21 (dx)^{\frac{11}{2}} b^2 d^6-18 (dx)^{\frac{7}{2}} abd^8-7 (dx)^{\frac{3}{2}} a^2 d^{10}\right)}{ab^5 d^6 x^6+3 a^2 b^4 d^6 x^4+3 a^3 b^3 d^6 x^2+a^4 b^2 d^6}}{1536 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
```

```
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sq
rt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a*b^2) + 8
*(21*(d*x)^(11/2)*b^2*d^6 - 18*(d*x)^(7/2)*a*b*d^8 - 7*(d*x)^(3/2)*a^2*d^10
)/(a*b^5*d^6*x^6 + 3*a^2*b^4*d^6*x^4 + 3*a^3*b^3*d^6*x^2 + a^4*b^2*d^6))/d
```

mupad [B] time = 4.26, size = 150, normalized size = 0.45

$$\frac{7\,d^{9/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{5/4}\,b^{11/4}}-\frac{7\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{5/4}\,b^{11/4}}-\frac{\frac{3\,d^7\,(d\,x)^{7/2}}{32\,b}-\frac{7\,d^5\,(d\,x)^{11/2}}{64\,a}+\frac{7\,a\,d^9\,(d\,x)^{3/2}}{192\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (7*d^(9/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(5/
4)*b^(11/4)) - (7*d^(9/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))
/(128*(-a)^(5/4)*b^(11/4)) - ((3*d^7*(d*x)^(7/2))/(32*b) - (7*d^5*(d*x)^(11
/2))/(64*a) + (7*a*d^9*(d*x)^(3/2))/(192*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a
^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^{\frac{9}{2}}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**4, x)
```

3.524 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=336

$$-\frac{5d^{7/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^{7/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{5d^{7/2}\tan^{-}}{128}$$

Rubi [A] time = 0.34, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5d^{7/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^{7/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{5d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(5/2))/(6*b*(a + b*x^2)^3) - (5*d^3*Sqrt[d*x])/(48*b^2*(a + b*x^2
)^2) + (5*d^3*Sqrt[d*x])/(192*a*b^2*(a + b*x^2)) - (5*d^(7/2)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(7/4)*b^(9/4))
 + (5*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(1
28*Sqrt[2]*a^(7/4)*b^(9/4)) - (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4)) +
 (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^4}\,dx \\
&= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{12}\left(5b^2d^2\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^3}\,dx \\
&= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}+\frac{1}{96}\left(5d^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx \\
&= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}+\frac{\left(5d^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128ab} \\
&= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}+\frac{\left(5d^3\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,d \cdots\right.}{64ab} \\
&= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}+\frac{\left(5d^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x \cdots}{ab+\frac{b^2x^4}{d^2}}\cdots\right.}{128a^{3/2}b} \\
&= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}-\frac{\left(5d^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\,\sqrt[4]{\cdots}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\cdots}\cdots\right.}{256\sqrt{2}\,a\cdots} \\
&= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}-\frac{5d^{7/2}\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\cdots\right.}{256\sqrt{2}\,a\cdots} \\
&= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}-\frac{5d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{d\cdots}}{\sqrt[4]{a}\,\sqrt{d}}\cdots\right.}{128\sqrt{2}\,a^{7/4}b^{9/4}}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 279, normalized size = 0.83

$$
\frac{d^3\sqrt{dx}\left(-\frac{105\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}+\frac{105\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}-\frac{210\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}\sqrt{x}}+\frac{210\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}\sqrt{x}}+\frac{280\sqrt[4]{b}}{a^2+abx^2}-\frac{3072b^{5/4}x^2}{\left(a+bx^2\right)^3}+\frac{160\sqrt[4]{b}}{\left(a+bx^2\right)^2}-\frac{1280a\sqrt[4]{b}}{\left(a+bx^2\right)^3}\right)}{10752b^{9/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^3*Sqrt[d*x]*((-1280*a*b^(1/4))/(a + b*x^2)^3 - (3072*b^(5/4)*x^2)/(a + b
*x^2)^3 + (160*b^(1/4))/(a + b*x^2)^2 + (280*b^(1/4))/(a^2 + a*b*x^2) - (21
0*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*Sqrt[x])
+ (210*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*Sqrt
[x]) - (105*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(a^(7/4)*Sqrt[x]) + (105*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x])))/(10752*b^(9/4))
```

IntegrateAlgebraic [A] time = 0.86, size = 221, normalized size = 0.66

$$-\frac{5d^{7/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}a^{7/4}b^{9/4}}+\frac{-15a^2d^9\sqrt{dx}-42abd^7(dx)^{5/2}+5b^2d^5(dx)^{9/2}}{192ab^2\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-15*a^2*d^9*Sqrt[d*x] - 42*a*b*d^7*(d*x)^(5/2) + 5*b^2*d^5*(d*x)^(9/2))/(1
92*a*b^2*(a*d^2 + b*d^2*x^2)^3) - (5*d^(7/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqr
t[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*Sqr
t[2]*a^(7/4)*b^(9/4)) + (5*d^(7/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]
*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(7/4)*b^(9/4))
```

fricas [A] time = 0.91, size = 389, normalized size = 1.16

$$\frac{60\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^5b^7d^3\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{3}{4}}-\sqrt{a^4b^4\sqrt{-\frac{d^{14}}{a^7b^9}}+d^7x}\,a^5b^7\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{3}{4}}}{d^{14}}\right)+15\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\log\left(5a^2b^2\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}+5\sqrt{dx}d^3\right)-15\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\log\left(-5a^2b^2\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}+5\sqrt{dx}d^3\right)+4\left(5b^2d^3x^4-42abd^3x^2-15a^2d^3\right)\sqrt{dx}}{768\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(60*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7
*b^9))^(1/4)*arctan(-(sqrt(d*x)*a^5*b^7*d^3*(-d^14/(a^7*b^9))^(3/4) - sqrt(
a^4*b^4*sqrt(-d^14/(a^7*b^9)) + d^7*x)*a^5*b^7*(-d^14/(a^7*b^9))^(3/4))/d^1
4) + 15*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7*b
^9))^(1/4)*log(5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) - 15*(a
*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7*b^9))^(1/4)
*log(-5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) + 4*(5*b^2*d^3*x
^4 - 42*a*b*d^3*x^2 - 15*a^2*d^3)*sqrt(d*x))/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3
*a^3*b^3*x^2 + a^4*b^2)
```

giac [A] time = 0.24, size = 304, normalized size = 0.90

$$\frac{1}{1536}\,d^{3}\left(\frac{30\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{3}}+\frac{30\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{3}}+\frac{15\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{3}}-\frac{15\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{3}}+\frac{8\left(5\,\sqrt{dx}\,b^{2}d^{6}x^{4}-42\,\sqrt{dx}\,abd^{6}x^{2}-15\,\sqrt{dx}\,a^{2}d^{6}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{3}ab^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^3*(30*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) + 30*sqrt(2)*(a*b^3*d^2
)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a^2*b^3) + 15*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3) - 15*sqrt(2)*(a*b^3*d^2)^(1/
4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3) +
 8*(5*sqrt(d*x)*b^2*d^6*x^4 - 42*sqrt(d*x)*a*b*d^6*x^2 - 15*sqrt(d*x)*a^2*d
^6)/((b*d^2*x^2 + a*d^2)^3*a*b^2))
```

maple [A] time = 0.02, size = 277, normalized size = 0.82

$$-\frac{5\sqrt{dx}\,a\,d^{9}}{64\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{2}}-\frac{7\left(dx\right)^{\frac{5}{2}}d^{7}}{32\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b}+\frac{5\left(dx\right)^{\frac{9}{2}}d^{5}}{192\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}a}+\frac{5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{256a^{2}b^{2}}+\frac{5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{256a^{2}b^{2}}+\frac{5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{3}\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{512a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 5/192*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(9/2)-7/32*d^7/(b*d^2*x^2+a*d^2)^3/b*
(d*x)^(5/2)-5/64*d^9/(b*d^2*x^2+a*d^2)^3/b^2*a*(d*x)^(1/2)+5/512*d^3/a^2/b^
2*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+5/25
6*d^3/a^2/b^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^
(1/2)+1)+5/256*d^3/a^2/b^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)
^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 2.96, size = 332, normalized size = 0.99

$$\frac{\frac{8\left(5\left(dx\right)^{\frac{9}{2}}b^{2}d^{6}-42\left(dx\right)^{\frac{5}{2}}abd^{8}-15\,\sqrt{dx}\,a^{2}d^{10}\right)}{ab^{5}d^{6}x^{6}+3\,a^{2}b^{4}d^{6}x^{4}+3\,a^{3}b^{3}d^{6}x^{2}+a^{4}b^{2}d^{6}}+\frac{15\left(\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}\,d^{5}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\,\sqrt{2}\,d^{5}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{ab^{2}}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(5*(d*x)^(9/2)*b^2*d^6 - 42*(d*x)^(5/2)*a*b*d^8 - 15*sqrt(d*x)*a^
2*d^10)/(a*b^5*d^6*x^6 + 3*a^2*b^4*d^6*x^4 + 3*a^3*b^3*d^6*x^2 + a^4*b^2*d^
6) + 15*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4))
+ 2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a
*b^2))/d
```

mupad [B] time = 4.26, size = 150, normalized size = 0.45

$$\frac{5\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{7/4}\,b^{9/4}}-\frac{\frac{7\,d^7\,{\left(d\,x\right)}^{5/2}}{32\,b}-\frac{5\,d^5\,{\left(d\,x\right)}^{9/2}}{192\,a}+\frac{5\,a\,d^9\,\sqrt{d\,x}}{64\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{5\,d^{7/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{7/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (5*d^(7/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(7/4
)*b^(9/4)) - ((7*d^7*(d*x)^(5/2))/(32*b) - (5*d^5*(d*x)^(9/2))/(192*a) + (5
*a*d^9*(d*x)^(1/2))/(64*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*
a*b^2*d^6*x^4) + (5*d^(7/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2)
)))/(128*(-a)^(7/4)*b^(9/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**4, x)
```

3.525 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{5d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\tan^{-1}}{128}$$

Rubi [A] time = 0.35, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{5d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)}+\frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(3/2))/(6*b*(a + b*x^2)^3) + (d*(d*x)^(3/2))/(16*a*b*(a + b*x^2)^
2) + (5*d*(d*x)^(3/2))/(64*a^2*b*(a + b*x^2)) - (5*d^(5/2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(9/4)*b^(7/4)) +
(5*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*
Sqrt[2]*a^(9/4)*b^(7/4)) + (5*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4)) - (5
*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{4}\left(b^2d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{\left(5bd^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{32a}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} + \frac{\left(5d^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128a^2}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} + \frac{(5d)\,\mathrm{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x\right.}{64a^2}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} - \frac{(5d)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx\right.}{128a^2\sqrt{b}}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} + \frac{\left(5d^{5/2}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\ldots}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\ldots}{}}\right.}{256\sqrt{2}\,a^{9/4}}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} + \frac{5d^{5/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\ldots\right.}{256\sqrt{2}\,a^{9/\ldots}}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} - \frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{9/4}b^{7/4}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.18

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-a^3\right)}{9a^3b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a^3 + (a + b*x^2)^3*Hypergeometric2F1[3/4, 4, 7/4, -((b*
x^2)/a)]))/(9*a^3*b*(a + b*x^2)^3)
```

IntegrateAlgebraic [A] time = 0.79, size = 213, normalized size = 0.64

$$-\frac{5d^{5/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{128\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{(dx)^{3/2}\left(-5a^2d^7+42abd^7x^2+15b^2d^7x^4\right)}{192a^2b\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((d*x)^(3/2)*(-5*a^2*d^7 + 42*a*b*d^7*x^2 + 15*b^2*d^7*x^4))/(192*a^2*b*(a*
d^2 + b*d^2*x^2)^3) - (5*d^(5/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)
) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*Sqrt[2]*a^(9/4)
*b^(7/4)) - (5*d^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/
(Sqrt[a]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(9/4)*b^(7/4))
```

fricas [A] time = 0.88, size = 396, normalized size = 1.18

$$-\frac{60\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\sqrt{dx}\,a^2b^2d^7\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^5b^3d^{10}\sqrt{-\frac{d^{10}}{a^9b^7}}+15625\,d^{15}x}\,a^2b^2\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}}{125\,d^{10}}\right)-15\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}\log\left(125\,a^7b^5\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{3}{4}}+125\sqrt{dx}\,d^7\right)+15\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}\log\left(-125\,a^7b^5\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{3}{4}}+125\sqrt{dx}\,d^7\right)-4\left(15\,b^2d^2x^5+42\,abd^2x^3-5\,a^2d^2x\right)\sqrt{dx}}{768\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(60*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^10/(a^
9*b^7))^(1/4)*arctan(-1/125*(125*sqrt(d*x)*a^2*b^2*d^7*(-d^10/(a^9*b^7))^(1
/4) - sqrt(-15625*a^5*b^3*d^10*sqrt(-d^10/(a^9*b^7)) + 15625*d^15*x)*a^2*b^
2*(-d^10/(a^9*b^7))^(1/4))/d^10) - 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*
b^2*x^2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(125*a^7*b^5*(-d^10/(a^9*b^7))^
(3/4) + 125*sqrt(d*x)*d^7) + 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^
2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(-125*a^7*b^5*(-d^10/(a^9*b^7))^(3/4)
 + 125*sqrt(d*x)*d^7) - 4*(15*b^2*d^2*x^5 + 42*a*b*d^2*x^3 - 5*a^2*d^2*x)*s
qrt(d*x))/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)
```

giac [A] time = 0.22, size = 317, normalized size = 0.95

$$\frac{1}{1536}\,d^2\left(\frac{8\left(15\sqrt{dx}\,b^2d^6x^5+42\sqrt{dx}\,abd^6x^3-5\sqrt{dx}\,a^2d^6x\right)}{\left(bd^2x^2+ad^2\right)^3a^2b}+\frac{30\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4d}+\frac{30\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4d}-\frac{15\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^4d}+\frac{15\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")

[Out] 1/1536*d^2*(8*(15*sqrt(d*x)*b^2*d^6*x^5 + 42*sqrt(d*x)*a*b*d^6*x^3 - 5*sqrt(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a^2*b) + 30*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4*d) + 30*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4*d) - 15*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4*d) + 15*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4*d))

maple [A] time = 0.02, size = 277, normalized size = 0.83

$$-\frac{5 (dx)^{\frac{3}{2}} d^7}{192 \left(b d^2 x^2+d^2 a\right)^3 b}+\frac{7 (dx)^{\frac{7}{2}} d^5}{32 \left(b d^2 x^2+d^2 a\right)^3 a}+\frac{5 (dx)^{\frac{11}{2}} b d^3}{64 \left(b d^2 x^2+d^2 a\right)^3 a^2}+\frac{5\sqrt{2}\, d^3 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a^2 b^2}+\frac{5\sqrt{2}\, d^3 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{256 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a^2 b^2}+\frac{5\sqrt{2}\, d^3 \ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)}{512 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)

[Out] 5/64*d^3/(b*d^2*x^2+a*d^2)^3/a^2*b*(d*x)^(11/2)+7/32*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(7/2)-5/192*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(3/2)+5/512*d^3/a^2/b^2/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+5/256*d^3/a^2/b^2/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+5/256*d^3/a^2/b^2/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)

maxima [A] time = 3.02, size = 323, normalized size = 0.96

$$\frac{\frac{15 d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{a^2 b}+\frac{8\left(15 (dx)^{\frac{11}{2}} b^2 d^4+42 (dx)^{\frac{7}{2}} a b d^6-5 (dx)^{\frac{3}{2}} a^2 d^8\right)}{a^2 b^4 d^6 x^6+3 a^3 b^3 d^6 x^4+3 a^4 b^2 d^6 x^2+a^5 b d^6}}{1536 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")

[Out] 1/1536*(15*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*

```
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sq
rt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*b) + 8
*(15*(d*x)^(11/2)*b^2*d^4 + 42*(d*x)^(7/2)*a*b*d^6 - 5*(d*x)^(3/2)*a^2*d^8)
/(a^2*b^4*d^6*x^6 + 3*a^3*b^3*d^6*x^4 + 3*a^4*b^2*d^6*x^2 + a^5*b*d^6))/d
```

mupad [B] time = 4.23, size = 149, normalized size = 0.44

$$\frac{\frac{7\,d^5\,{\left(d\,x\right)}^{7/2}}{32\,a}-\frac{5\,d^7\,{\left(d\,x\right)}^{3/2}}{192\,b}+\frac{5\,b\,d^3\,{\left(d\,x\right)}^{11/2}}{64\,a^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{5\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{9/4}\,b^{7/4}}-\frac{5\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{9/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((7*d^5*(d*x)^(7/2))/(32*a) - (5*d^7*(d*x)^(3/2))/(192*b) + (5*b*d^3*(d*x)^
(11/2))/(64*a^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^
4) + (5*d^(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)
^(9/4)*b^(7/4)) - (5*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(128*(-a)^(9/4)*b^(7/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(a + bx^2\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**4, x)
```

3.526 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=335

$$-\frac{7d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{7d^{3/2}\tan^{-1}}{128}$$

Rubi [A] time = 0.35, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{7d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)}+\frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*Sqrt[d*x])/(6*b*(a + b*x^2)^3) + (d*Sqrt[d*x])/(48*a*b*(a + b*x^2)^2) +
 (7*d*Sqrt[d*x])/(192*a^2*b*(a + b*x^2)) - (7*d^(3/2)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d
^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt
[2]*a^(11/4)*b^(5/4)) - (7*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d
^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(b^2d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{\left(7bd^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{96a}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} + \frac{\left(7d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128a^2}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} + \frac{(7d)\,\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx \ldots\right.}{64a^2}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} + \frac{7\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx \ldots\right.}{128a^{5/2} \ldots}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} - \frac{\left(7d^{3/2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{2}\ldots}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\ldots}\ldots\right.}{256\sqrt{2}\,a \ldots}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} - \frac{7d^{3/2}\log\left(\sqrt{a}\,\sqrt{d} + \sqrt{b}\ldots\right.}{256\sqrt{2}\,a\ldots}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} - \frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{\ldots}}{\sqrt[4]{a}\,\sqrt{d}}\ldots\right.}{128\sqrt{2}\,a^{11/4}b^{5/4}}$$

Mathematica [A] time = 0.13, size = 260, normalized size = 0.78

$$\frac{d\sqrt{dx}\left(-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{11/4}\sqrt{x}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{11/4}\sqrt{x}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}\sqrt{x}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{11/4}\sqrt{x}}+\frac{56\sqrt[4]{b}}{a^2\left(a+bx^2\right)}+\frac{32\sqrt[4]{b}}{a\left(a+bx^2\right)^2}-\frac{256\sqrt[4]{b}}{\left(a+bx^2\right)^3}\right)}{1536b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*Sqrt[d*x]*((-256*b^(1/4))/(a + b*x^2)^3 + (32*b^(1/4))/(a*(a + b*x^2)^2)
 + (56*b^(1/4))/(a^2*(a + b*x^2)) - (42*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) + (42*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) - (21*Sqrt[2]*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt[x]) + (21*Sqrt[2]
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt
[x])))/(1536*b^(5/4))
```

IntegrateAlgebraic [A] time = 0.78, size = 213, normalized size = 0.64

$$-\frac{7d^{3/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\sqrt{dx}\left(-21a^2d^7+18abd^7x^2+7b^2d^7x^4\right)}{192a^2b\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (Sqrt[d*x]*(-21*a^2*d^7 + 18*a*b*d^7*x^2 + 7*b^2*d^7*x^4))/(192*a^2*b*(a*d^
2 + b*d^2*x^2)^3) - (7*d^(3/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4))
- (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*Sqrt[2]*a^(11/4)*
b^(5/4)) + (7*d^(3/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(
Sqrt[a]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

fricas [A] time = 1.68, size = 373, normalized size = 1.11

$$\frac{84\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^8b^4d\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{3}{4}}-\sqrt{a^6b^2\sqrt{-\frac{d^6}{a^{11}b^5}}+d^3x}a^8b^4\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{3}{4}}}{d^6}\right)+21\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}\log\left(7a^3b\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}+7\sqrt{dx}d\right)-21\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}\log\left(-7a^3b\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}+7\sqrt{dx}d\right)+4\left(7b^2dx^4+18abdx^2-21a^2d\right)\sqrt{dx}}{768\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(84*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11
*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^8*b^4*d*(-d^6/(a^11*b^5))^(3/4) - sqrt(a^
6*b^2*sqrt(-d^6/(a^11*b^5)) + d^3*x)*a^8*b^4*(-d^6/(a^11*b^5))^(3/4))/d^6)
+ 21*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^5)
)^(1/4)*log(7*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) - 21*(a^2*b^4*
x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^5))^(1/4)*log(-7
*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) + 4*(7*b^2*d*x^4 + 18*a*b*d
*x^2 - 21*a^2*d)*sqrt(d*x))/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 +
a^5*b)
```

giac [A] time = 0.20, size = 302, normalized size = 0.90

$$\frac{1}{1536}\,d\left(\frac{42\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{2}}+\frac{42\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{2}}+\frac{21\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{3}b^{2}}-\frac{21\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{3}b^{2}}+\frac{8\left(7\,\sqrt{dx}b^{2}d^{6}x^{4}+18\,\sqrt{dx}abd^{6}x^{2}-21\,\sqrt{dx}a^{2}d^{6}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{3}a^{2}b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d*(42*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2) + 42*sqrt(2)*(a*b^3*d^2)^
(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^3*b^2) + 21*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2) - 21*sqrt(2)*(a*b^3*d^2)^(1/4)
*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2) + 8
*(7*sqrt(d*x)*b^2*d^6*x^4 + 18*sqrt(d*x)*a*b*d^6*x^2 - 21*sqrt(d*x)*a^2*d^6
)/((b*d^2*x^2 + a*d^2)^3*a^2*b))
```

maple [A] time = 0.02, size = 271, normalized size = 0.81

$$-\frac{7\sqrt{dx}\,d^{7}}{64\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b}+\frac{3\left(dx\right)^{\frac{5}{2}}d^{5}}{32\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}a}+\frac{7\left(dx\right)^{\frac{9}{2}}b\,d^{3}}{192\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}a^{2}}+\frac{7\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{256a^{3}b}+\frac{7\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{256a^{3}b}+\frac{7\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{512a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 7/192*d^3/(b*d^2*x^2+a*d^2)^3/a^2*b*(d*x)^(9/2)+3/32*d^5/(b*d^2*x^2+a*d^2)^
3/a*(d*x)^(5/2)-7/64*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(1/2)+7/512*d/a^3/b*(a
/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)
^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+7/256*d/
a^3/b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)
+7/256*d/a^3/b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)
^(1/2)-1)
```

maxima [A] time = 3.02, size = 332, normalized size = 0.99

$$\frac{\frac{8\left(7\left(dx\right)^{\frac{9}{2}}b^{2}d^{4}+18\left(dx\right)^{\frac{5}{2}}abd^{6}-21\,\sqrt{dx}a^{2}d^{8}\right)}{a^{2}b^{4}d^{6}x^{6}+3\,a^{3}b^{3}d^{6}x^{4}+3\,a^{4}b^{2}d^{6}x^{2}+a^{5}bd^{6}}+\frac{21\left(\frac{\sqrt{2}d^{4}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^{4}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}d^{3}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}+\frac{2\,\sqrt{2}d^{3}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}\right)}{a^{2}b}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(7*(d*x)^(9/2)*b^2*d^4 + 18*(d*x)^(5/2)*a*b*d^6 - 21*sqrt(d*x)*a^
2*d^8)/(a^2*b^4*d^6*x^6 + 3*a^3*b^3*d^6*x^4 + 3*a^4*b^2*d^6*x^2 + a^5*b*d^6
) + 21*(sqrt(2)*d^4*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/
4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2
*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*
x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^
2*b))/d
```

mupad [B] time = 4.27, size = 149, normalized size = 0.44

$$\frac{\frac{3\,d^5\,{\left(d\,x\right)}^{5/2}}{32\,a}-\frac{7\,d^7\,\sqrt{d\,x}}{64\,b}+\frac{7\,b\,d^3\,{\left(d\,x\right)}^{9/2}}{192\,a^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{7\,d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{11/4}\,b^{5/4}}-\frac{7\,d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{11/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((3*d^5*(d*x)^(5/2))/(32*a) - (7*d^7*(d*x)^(1/2))/(64*b) + (7*b*d^3*(d*x)^(
9/2))/(192*a^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4
) - (7*d^(3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^
(11/4)*b^(5/4)) - (7*d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(128*(-a)^(11/4)*b^(5/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx\right)^{\frac{3}{2}}}{\left(a + bx^{2}\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**4, x)
```

3.527 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{15\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\ \text{ta}}{12}$$

Rubi [A] time = 0.35, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{15\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{15\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)}+\frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*(d*x)^(3/2))/(16*a^2*d*(a + b*x^2)^2
) + (15*(d*x)^(3/2))/(64*a^3*d*(a + b*x^2)) - (15*Sqrt[d]*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(13/4)*b^(3/4)) +
(15*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128
*Sqrt[2]*a^(13/4)*b^(3/4)) + (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
- (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{\left(3b^3\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{4a}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{\left(15b^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{32a^2}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{(15b)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128a^3}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{(15b)\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,d\right.}{64a^3d}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} - \frac{\left(15\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{}}{ab+\frac{b^2}{d}}\right.}{128a^3d}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{\left(15\sqrt{d}\right)\text{Subst}\left(\int \frac{\sqrt{2}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-}\right.}{256\sqrt{2}\,a}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{15\sqrt{d}\,\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\right.}{256\sqrt{2}\,a}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} - \frac{15\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{}}{\sqrt[4]{a}\,\sqrt{d}}\right.}{128\sqrt{2}\,a^{13/4}b^{3/4}}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.10

$$\frac{2x\sqrt{dx}\ {}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*Sqrt[d*x]*Hypergeometric2F1[3/4, 4, 7/4, -((b*x^2)/a)])/(3*a^4)
```

IntegrateAlgebraic [A] time = 0.46, size = 210, normalized size = 0.63

$$-\frac{15\sqrt{d}\ \tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\ \sqrt{d}}{\sqrt{2}\ \sqrt[4]{b}}-\frac{\sqrt[4]{b}\ \sqrt{d}\,x}{\sqrt{2}\ \sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\ \tanh^{-1}\left(\frac{\sqrt{2}\ \sqrt[4]{a}\ \sqrt[4]{b}\ \sqrt{d}\ \sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{128\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{(dx)^{3/2}\left(113a^2d^5+126abd^5x^2+45b^2d^5x^4\right)}{192a^3\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((d*x)^(3/2)*(113*a^2*d^5 + 126*a*b*d^5*x^2 + 45*b^2*d^5*x^4))/(192*a^3*(a*
d^2 + b*d^2*x^2)^3) - (15*Sqrt[d]*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4
)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*Sqrt[2]*a^(13/
4)*b^(3/4)) - (15*Sqrt[d]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x
])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(13/4)*b^(3/4))
```

fricas [A] time = 1.99, size = 359, normalized size = 1.07

$$-\frac{180\left(a^3b^3x^6+3a^4b^2x^4+3a^5bx^2+a^6\right)\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{3375\sqrt{dx}\,a^3bd\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}-\sqrt{-11390625a^7bd^2\sqrt{-\frac{d^2}{a^{13}b^3}}+11390625d^3x}\,a^3b\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}}{3375d^2}\right)-45\left(a^3b^3x^6+3a^4b^2x^4+3a^5bx^2+a^6\right)\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}\log\left(3375a^{10}b^2\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{3}{4}}+3375\sqrt{dx}\,d\right)+45\left(a^3b^3x^6+3a^4b^2x^4+3a^5bx^2+a^6\right)\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}\log\left(-3375a^{10}b^2\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{3}{4}}+3375\sqrt{dx}\,d\right)-4\left(45b^2x^5+126abx^3+113a^2x\right)\sqrt{dx}}{768\left(a^3b^3x^6+3a^4b^2x^4+3a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(180*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-d^2/(a^13*b
^3))^(1/4)*arctan(-1/3375*(3375*sqrt(d*x)*a^3*b*d*(-d^2/(a^13*b^3))^(1/4) -
 sqrt(-11390625*a^7*b*d^2*sqrt(-d^2/(a^13*b^3)) + 11390625*d^3*x)*a^3*b*(-d
^2/(a^13*b^3))^(1/4))/d^2) - 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2
+ a^6)*(-d^2/(a^13*b^3))^(1/4)*log(3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) +
3375*sqrt(d*x)*d) + 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-
d^2/(a^13*b^3))^(1/4)*log(-3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) + 3375*sqr
t(d*x)*d) - 4*(45*b^2*x^5 + 126*a*b*x^3 + 113*a^2*x)*sqrt(d*x))/(a^3*b^3*x^
6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

giac [A] time = 0.22, size = 302, normalized size = 0.90

$$\frac{\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}-\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^3}+\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^3}+\frac{8\left(45\sqrt{dx}\,b^2d^7x^5+126\sqrt{dx}\,abd^7x^3+113\sqrt{dx}\,a^2d^7x\right)}{\left(bd^2x^2+ad^2\right)^3a^3}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 90*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^4*b^3) - 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^
(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) + 45*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) + 8*(
45*sqrt(d*x)*b^2*d^7*x^5 + 126*sqrt(d*x)*a*b*d^7*x^3 + 113*sqrt(d*x)*a^2*d^
7*x)/((b*d^2*x^2 + a*d^2)^3*a^3))/d
```

maple [A] time = 0.02, size = 272, normalized size = 0.81

$$\frac{113\,(dx)^{\frac{3}{2}}d^5}{192\left(b\,d^2x^2+d^2a\right)^3a}+\frac{21\,(dx)^{\frac{7}{2}}\,b\,d^3}{32\left(b\,d^2x^2+d^2a\right)^3a^2}+\frac{15\,(dx)^{\frac{11}{2}}\,b^2d}{64\left(b\,d^2x^2+d^2a\right)^3a^3}+\frac{15\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^3b}+\frac{15\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^3b}+\frac{15\sqrt{2}\,d\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{512\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 15/64*d/(b*d^2*x^2+a*d^2)^3/a^3*b^2*(d*x)^(11/2)+21/32*d^3/(b*d^2*x^2+a*d^2
)^3/a^2*b*(d*x)^(7/2)+113/192*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(3/2)+15/512*
d/a^3/b/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))
)+15/256*d/a^3/b/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*
x)^(1/2)+1)+15/256*d/a^3/b/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)
^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 2.98, size = 317, normalized size = 0.95

$$\frac{\frac{8\left(45\,(dx)^{\frac{11}{2}}b^2d^2+126\,(dx)^{\frac{7}{2}}abd^4+113\,(dx)^{\frac{3}{2}}a^2d^6\right)}{a^3b^3d^6x^6+3\,a^4b^2d^6x^4+3\,a^5bd^6x^2+a^6d^6}+\frac{45\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^3}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(45*(d*x)^(11/2)*b^2*d^2 + 126*(d*x)^(7/2)*a*b*d^4 + 113*(d*x)^(3
/2)*a^2*d^6)/(a^3*b^3*d^6*x^6 + 3*a^4*b^2*d^6*x^4 + 3*a^5*b*d^6*x^2 + a^6*d
^6) + 45*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) +
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sq
rt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
 - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt
(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^
(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^3)/d
```

mupad [B] time = 0.10, size = 150, normalized size = 0.45

$$\frac{\frac{113\,d^5\,{\left(d\,x\right)}^{3/2}}{192\,a}+\frac{21\,b\,d^3\,{\left(d\,x\right)}^{7/2}}{32\,a^2}+\frac{15\,b^2\,d\,{\left(d\,x\right)}^{11/2}}{64\,a^3}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{15\,\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{13/4}\,b^{3/4}}+\frac{15\,\sqrt{d}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{13/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((113*d^5*(d*x)^(3/2))/(192*a) + (21*b*d^3*(d*x)^(7/2))/(32*a^2) + (15*b^2*
d*(d*x)^(11/2))/(64*a^3))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^
2*d^6*x^4) - (15*d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/
(128*(-a)^(13/4)*b^(3/4)) + (15*d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(
1/4)*d^(1/2))))/(128*(-a)^(13/4)*b^(3/4))
```

sympy [A] time = 28.99, size = 252, normalized size = 0.75

$$\frac{226a^2d^{11}\left(dx\right)^{\frac{3}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+\frac{252abd^9\left(dx\right)^{\frac{7}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+\frac{90b^2d^7\left(dx\right)^{\frac{11}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+2d^7\operatorname{RootSum}\left(68719476736t^4a^{13}b^3d^{26}+50625,\left(t\mapsto t\log\left(\frac{134217728t^3a^{10}b^2d^{20}}{3375}+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 226*a**2*d**11*(d*x)**(3/2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 + 1152
*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6) + 252*a*b*d**9*(d*x)**(7/
2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 + 1152*a**4*b**2*d**12*x**4 + 3
84*a**3*b**3*d**12*x**6) + 90*b**2*d**7*(d*x)**(11/2)/(384*a**6*d**12 + 115
2*a**5*b*d**12*x**2 + 1152*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6)
 + 2*d**7*RootSum(68719476736*_t**4*a**13*b**3*d**26 + 50625, Lambda(_t, _t
*log(134217728*_t**3*a**10*b**2*d**20/3375 + sqrt(d*x))))
```

3.528 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$-\frac{77\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}-\frac{77\tan^{-1}\left(1-\right.}{128\sqrt{2}\,a^{15/}}$$

Rubi [A] time = 0.35, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)}+\frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2}-\frac{77\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}-\frac{77\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*Sqrt[d*x])/(48*a^2*d*(a + b*x^2)^2) +
 (77*Sqrt[d*x])/(192*a^3*d*(a + b*x^2)) - (77*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (77*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(
15/4)*b^(1/4)*Sqrt[d]) - (77*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (7
7*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{\left(11b^3\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx}{12a}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{\left(77b^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{96a^2}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\int \frac{1}{\sqrt{dx}\left(ab+b^2x\ldots\right.}}{128a^3}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\,\text{Subst}\left(\int \frac{\ldots}{ab\ldots}\right.}{64a\ldots}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\,\text{Subst}\left(\int \frac{\sqrt{a}\ldots}{a\ldots}\right.}{128\ldots}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{77\,\text{Subst}\left(\int \frac{\ldots}{\frac{\sqrt{a}\,d}{\sqrt{b}}-\ldots}\right.}{25\ldots}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} - \frac{77\log\left(\sqrt{a}\sqrt{d}+\ldots\right.}{256\ldots}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} - \frac{77\tan^{-1}\left(1-\frac{\sqrt{2}\,\ldots}{\sqrt[4]{\ldots}}\right.}{128\sqrt{2}\,a^{15/4}\sqrt[4]{b}\ldots}$$

Mathematica [A] time = 0.11, size = 253, normalized size = 0.76

$$\frac{\sqrt{x}\left(\frac{256a^{11/4}\sqrt{x}}{(a+bx^2)^3}+\frac{352a^{7/4}\sqrt{x}}{(a+bx^2)^2}+\frac{616a^{3/4}\sqrt{x}}{a+bx^2}-\frac{231\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{231\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}-\frac{462\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{462\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}\right)}{1536a^{15/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (Sqrt[x]*((256*a^(11/4)*Sqrt[x])/(a + b*x^2)^3 + (352*a^(7/4)*Sqrt[x])/(a +
 b*x^2)^2 + (616*a^(3/4)*Sqrt[x])/(a + b*x^2) - (462*Sqrt[2]*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (462*Sqrt[2]*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (231*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (231*Sqrt[2]*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(1536*a^(15/4)*Sqrt[d
*x])
```

IntegrateAlgebraic [A] time = 0.44, size = 210, normalized size = 0.63

$$-\frac{77\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{\sqrt{dx}\left(153a^2d^5+198abd^5x^2+77b^2d^5x^4\right)}{192a^3\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (Sqrt[d*x]*(153*a^2*d^5 + 198*a*b*d^5*x^2 + 77*b^2*d^5*x^4))/(192*a^3*(a*d^
2 + b*d^2*x^2)^3) - (77*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1
/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*Sqrt[2]*a^(15/4)*b^(1/4)
*Sqrt[d]) + (77*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a
]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d])
```

fricas [A] time = 0.65, size = 357, normalized size = 1.07

$$\frac{924\left(a^3b^3dx^6+3a^4b^2dx^4+3a^5bdx^2+a^6d\right)\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^8d^2\sqrt{-\frac{1}{a^{15}bd^2}}+dx}\,a^{11}bd\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^{11}bd\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{3}{4}}\right)+231\left(a^3b^3dx^6+3a^4b^2dx^4+3a^5bdx^2+a^6d\right)\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}\log\left(a^4d\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-231\left(a^3b^3dx^6+3a^4b^2dx^4+3a^5bdx^2+a^6d\right)\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}\log\left(-a^4d\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\left(77b^2x^4+198abx^2+153a^2\right)\sqrt{dx}}{768\left(a^3b^3dx^6+3a^4b^2dx^4+3a^5bdx^2+a^6d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(924*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a
^15*b*d^2))^(1/4)*arctan(sqrt(a^8*d^2*sqrt(-1/(a^15*b*d^2)) + d*x)*a^11*b*d
*(-1/(a^15*b*d^2))^(3/4) - sqrt(d*x)*a^11*b*d*(-1/(a^15*b*d^2))^(3/4)) + 23
```

```
1*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^15*b*d^2
))^(1/4)*log(a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) - 231*(a^3*b^3*d*x^
6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^15*b*d^2))^(1/4)*log(-a
^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) + 4*(77*b^2*x^4 + 198*a*b*x^2 + 1
53*a^2)*sqrt(d*x))/(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d
)
```

giac [A] time = 0.27, size = 308, normalized size = 0.92

$$\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^4bd}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^4bd}+\frac{77\sqrt{dx}\,b^2d^5x^4+198\sqrt{dx}\,abd^5x^2+153\sqrt{dx}\,a^2d^5}{192\left(bd^2x^2+ad^2\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 77/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d) + 77/256*sqrt(2)*(a*b^3*d^2)^(1
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^4*b*d) + 77/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d) - 77/512*sqrt(2)*(a*b^3*d^2)
^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*
d) + 1/192*(77*sqrt(d*x)*b^2*d^5*x^4 + 198*sqrt(d*x)*a*b*d^5*x^2 + 153*sqrt
(d*x)*a^2*d^5)/((b*d^2*x^2 + a*d^2)^3*a^3)
```

maple [A] time = 0.02, size = 269, normalized size = 0.80

$$\frac{51\sqrt{dx}\,d^5}{64\left(b\,d^2x^2+d^2a\right)^3a}+\frac{33\,(dx)^{\frac{5}{2}}\,b\,d^3}{32\left(b\,d^2x^2+d^2a\right)^3a^2}+\frac{77\,(dx)^{\frac{9}{2}}\,b^2d}{192\left(b\,d^2x^2+d^2a\right)^3a^3}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256a^4d}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{256a^4d}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{512a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x)
```

```
[Out] 77/192*d/(b*d^2*x^2+a*d^2)^3/a^3*b^2*(d*x)^(9/2)+33/32*d^3/(b*d^2*x^2+a*d^2
)^3/a^2*b*(d*x)^(5/2)+51/64*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(1/2)+77/512/d/
a^4*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+77
/256/d/a^4*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/
2)+1)+77/256/d/a^4*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(
d*x)^(1/2)-1)
```

maxima [A] time = 3.09, size = 322, normalized size = 0.96

$$\frac{\frac{8\left(77\,(dx)^{\frac{9}{2}}b^2d^2+198\,(dx)^{\frac{5}{2}}abd^4+153\,\sqrt{dx}\,a^2d^6\right)}{a^3b^3d^6x^6+3\,a^4b^2d^6x^4+3\,a^5bd^6x^2+a^6d^6}+\frac{231\left(\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\,\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{a^3}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(77*(d*x)^(9/2)*b^2*d^2 + 198*(d*x)^(5/2)*a*b*d^4 + 153*sqrt(d*x)
*a^2*d^6)/(a^3*b^3*d^6*x^6 + 3*a^4*b^2*d^6*x^4 + 3*a^5*b*d^6*x^2 + a^6*d^6)
 + 231*(sqrt(2)*d^2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/
4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
 2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x
)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*s
qrt(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a^3)/d
```

mupad [B] time = 4.28, size = 150, normalized size = 0.45

$$\frac{\frac{51\,d^5\,\sqrt{d\,x}}{64\,a}+\frac{33\,b\,d^3\,{(d\,x)}^{5/2}}{32\,a^2}+\frac{77\,b^2\,d\,{(d\,x)}^{9/2}}{192\,a^3}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{77\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,{(-a)}^{15/4}\,b^{1/4}\,\sqrt{d}}+\frac{77\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,{(-a)}^{15/4}\,b^{1/4}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((51*d^5*(d*x)^(1/2))/(64*a) + (33*b*d^3*(d*x)^(5/2))/(32*a^2) + (77*b^2*d*
(d*x)^(9/2))/(192*a^3))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*
d^6*x^4) + (77*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^
(15/4)*b^(1/4)*d^(1/2)) + (77*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/
2))))/(128*(-a)^(15/4)*b^(1/4)*d^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**4), x)
```

3.529 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=352

$$-\frac{195\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{17/4}d^{3/2}}+\frac{195}{}$$

Rubi [A] time = 0.40, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{195\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{17/4}d^{3/2}}-\frac{195\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{17/4}d^{3/2}}+\frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)}+\frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2}-\frac{195}{64a^4d\sqrt{dx}}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -195/(64*a^4*d*Sqrt[d*x]) + 1/(6*a*d*Sqrt[d*x]*(a + b*x^2)^3) + 13/(48*a^2*
d*Sqrt[d*x]*(a + b*x^2)^2) + 39/(64*a^3*d*Sqrt[d*x]*(a + b*x^2)) + (195*b^(
1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2
]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sq
rt[2]*a^(17/4)*d^(3/2)) + (195*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(17/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{\left(13b^3\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^3}\,dx}{12a}$$

$$= \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{\left(39b^2\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^2}\,dx}{32a^2}$$

$$= \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)} + \frac{(195\cdots}{\cdots}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(\cdots}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(\cdots}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(\cdots}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(\cdots}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(\cdots}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(\cdots}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.09

$$-\frac{2x\,{}_2F_1\left(-\frac{1}{4},4;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^4(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-1/4, 4, 3/4, -((b*x^2)/a)])/(a^4*(d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.83, size = 227, normalized size = 0.64

$$\frac{195\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}a^{17/4}d^{3/2}}+\frac{-384a^3d^6-1469a^2bd^6x^2-1638ab^2d^6x^4-585b^3d^6x^6}{192a^4d\sqrt{dx}\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-384*a^3*d^6 - 1469*a^2*b*d^6*x^2 - 1638*a*b^2*d^6*x^4 - 585*b^3*d^6*x^6)/
(192*a^4*d*Sqrt[d*x]*(a*d^2 + b*d^2*x^2)^3) + (195*b^(1/4)*ArcTan[((a^(1/4)
*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d
*x]])/(128*Sqrt[2]*a^(17/4)*d^(3/2)) + (195*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(17
/4)*d^(3/2))
```

fricas [A] time = 1.67, size = 410, normalized size = 1.16

$$\frac{2340\left(a^4b^3d^2x^7+3a^5b^2d^2x^5+3a^6bd^2x^3+a^7d^2x\right)\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{7414875\sqrt{dx}a^4bd\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}-\sqrt{-54980371265625a^9bd^4\sqrt{-\frac{b}{a^{17}d^6}}+54980371265625b^2dx}\,a^4d\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}}{7414875b}\right)-585\left(a^4b^3d^2x^7+3a^5b^2d^2x^5+3a^6bd^2x^3+a^7d^2x\right)\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}\log\left(7414875a^{13}d^5\left(-\frac{b}{a^{17}d^6}\right)^{\frac{3}{4}}+7414875\sqrt{dx}b\right)+585\left(a^4b^3d^2x^7+3a^5b^2d^2x^5+3a^6bd^2x^3+a^7d^2x\right)\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}\log\left(-7414875a^{13}d^5\left(-\frac{b}{a^{17}d^6}\right)^{\frac{3}{4}}+7414875\sqrt{dx}b\right)-4\left(585b^3x^6+1638ab^2x^4+1469a^2bx^2+384a^3\right)\sqrt{dx}}{768\left(a^4b^3d^2x^7+3a^5b^2d^2x^5+3a^6bd^2x^3+a^7d^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(2340*(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^
2*x)*(-b/(a^17*d^6))^(1/4)*arctan(-1/7414875*(7414875*sqrt(d*x)*a^4*b*d*(-b
/(a^17*d^6))^(1/4) - sqrt(-54980371265625*a^9*b*d^4*sqrt(-b/(a^17*d^6)) + 5
4980371265625*b^2*d*x)*a^4*d*(-b/(a^17*d^6))^(1/4))/b) - 585*(a^4*b^3*d^2*x
^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*(-b/(a^17*d^6))^(1/4)
*log(7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x)*b) + 585*(a
^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*(-b/(a^17
*d^6))^(1/4)*log(-7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x
```

```
)*b) - 4*(585*b^3*x^6 + 1638*a*b^2*x^4 + 1469*a^2*b*x^2 + 384*a^3)*sqrt(d*x
))/(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)
```

giac [A] time = 0.20, size = 327, normalized size = 0.93

$$-\frac{\frac{3072}{\sqrt{dx}\,a^4}+\frac{8\left(201\sqrt{dx}\,b^3d^5x^5+486\sqrt{dx}\,ab^2d^5x^3+317\sqrt{dx}\,a^2bd^5x\right)}{\left(bd^2x^2+ad^2\right)^3a^4}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2d^2}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2d^2}-\frac{585\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^2d^2}+\frac{585\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^2d^2}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*(3072/(sqrt(d*x)*a^4) + 8*(201*sqrt(d*x)*b^3*d^5*x^5 + 486*sqrt(d*x
)*a*b^2*d^5*x^3 + 317*sqrt(d*x)*a^2*b*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^4) +
1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
+ 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2*d^2) + 1170*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^5*b^2*d^2) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2*d^2) + 585*sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*
b^2*d^2))/d
```

maple [A] time = 0.02, size = 285, normalized size = 0.81

$$-\frac{317\left(dx\right)^{\frac{3}{2}}b\,d^3}{192\left(b\,d^2x^2+d^2a\right)^3a^2}-\frac{81\left(dx\right)^{\frac{7}{2}}b^2d}{32\left(b\,d^2x^2+d^2a\right)^3a^3}-\frac{67\left(dx\right)^{\frac{11}{2}}b^3}{64\left(b\,d^2x^2+d^2a\right)^3a^4d}-\frac{195\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^4d}-\frac{195\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^4d}-\frac{195\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{512\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^4d}-\frac{2}{\sqrt{dx}\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -67/64/d*b^3/a^4/(b*d^2*x^2+a*d^2)^3*(d*x)^(11/2)-81/32*d*b^2/a^3/(b*d^2*x^
2+a*d^2)^3*(d*x)^(7/2)-317/192*d^3*b/a^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(3/2)-19
5/512/d/a^4/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1
/2)))-195/256/d/a^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*
(d*x)^(1/2)+1)-195/256/d/a^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^
2)^(1/4)*(d*x)^(1/2)-1)-2/a^4/d/(d*x)^(1/2)
```

maxima [A] time = 3.07, size = 328, normalized size = 0.93

$$-\frac{\frac{8\left(585\,b^{3}d^{6}x^{6}+1638\,ab^{2}d^{6}x^{4}+1469\,a^{2}bd^{6}x^{2}+384\,a^{3}d^{6}\right)}{(dx)^{\frac{13}{2}}a^{4}b^{3}+3\,(dx)^{\frac{9}{2}}a^{5}b^{2}d^{2}+3\,(dx)^{\frac{5}{2}}a^{6}bd^{4}+\sqrt{dx}\,a^{7}d^{6}}+\frac{585\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^{4}}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(8*(585*b^3*d^6*x^6 + 1638*a*b^2*d^6*x^4 + 1469*a^2*b*d^6*x^2 + 384
*a^3*d^6)/((d*x)^(13/2)*a^4*b^3 + 3*(d*x)^(9/2)*a^5*b^2*d^2 + 3*(d*x)^(5/2)
*a^6*b*d^4 + sqrt(d*x)*a^7*d^6) + 585*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(
sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(
sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqr
t(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1
/4)*b^(3/4)))/a^4)/d
```

mupad [B] time = 0.14, size = 166, normalized size = 0.47

$$\frac{195\,{(-b)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{17/4}\,d^{3/2}}-\frac{195\,{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{17/4}\,d^{3/2}}-\frac{\frac{2\,d^5}{a}+\frac{1469\,b\,d^5\,x^2}{192\,a^2}+\frac{273\,b^2\,d^5\,x^4}{32\,a^3}+\frac{195\,b^3\,d^5\,x^6}{64\,a^4}}{b^3\,{(d\,x)}^{13/2}+a^3\,d^6\,\sqrt{d\,x}+3\,a^2\,b\,d^4\,{(d\,x)}^{5/2}+3\,a\,b^2\,d^2\,{(d\,x)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] (195*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(
17/4)*d^(3/2)) - (195*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(
1/2))))/(128*a^(17/4)*d^(3/2)) - ((2*d^5)/a + (1469*b*d^5*x^2)/(192*a^2) +
(273*b^2*d^5*x^4)/(32*a^3) + (195*b^3*d^5*x^6)/(64*a^4))/(b^3*(d*x)^(13/2)
+ a^3*d^6*(d*x)^(1/2) + 3*a^2*b*d^4*(d*x)^(5/2) + 3*a*b^2*d^2*(d*x)^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^{2}\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**4), x)
```

3.530 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=352

$$\frac{385b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}-\frac{385b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}+\frac{385b^{3/4}}{}$$

Rubi [A] time = 0.39, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{385b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}-\frac{385b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}+\frac{385b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{19/4}d^{5/2}}-\frac{385b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{19/4}d^{5/2}}+\frac{55}{64a^3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2}-\frac{385}{192a^4d(dx)^{3/2}}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -385/(192*a^4*d*(d*x)^(3/2)) + 1/(6*a*d*(d*x)^(3/2)*(a + b*x^2)^3) + 5/(16*
a^2*d*(d*x)^(3/2)*(a + b*x^2)^2) + 55/(64*a^3*d*(d*x)^(3/2)*(a + b*x^2)) +
(385*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(19/4)*d^(5/2)) + (385*b^(3/4)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(256*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(19/4)*d^(
5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{\left(5b^3\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^3}\,dx}{4a}$$

$$= \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{\left(55b^2\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^2}}{32a^2}$$

$$= \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(a+bx^2\right)}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.09

$$-\frac{2x\,{}_2F_1\left(-\frac{3}{4},4;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^4(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-3/4, 4, 1/4, -((b*x^2)/a)])/(3*a^4*(d*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.82, size = 227, normalized size = 0.64

$$\frac{385b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}a^{19/4}d^{5/2}}-\frac{385b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}a^{19/4}d^{5/2}}+\frac{-128a^3d^6-765a^2bd^6x^2-990ab^2d^6x^4-385b^3d^6x^6}{192a^4d(dx)^{3/2}\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-128*a^3*d^6 - 765*a^2*b*d^6*x^2 - 990*a*b^2*d^6*x^4 - 385*b^3*d^6*x^6)/(1
92*a^4*d*(d*x)^(3/2)*(a*d^2 + b*d^2*x^2)^3) + (385*b^(3/4)*ArcTan[((a^(1/4)
*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d
*x]])/(128*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(128*Sqrt[2]*a^(19
/4)*d^(5/2))
```

fricas [A] time = 0.89, size = 434, normalized size = 1.23

$$-\frac{4620\left(a^4b^3d^3x^8+3a^5b^2d^3x^6+3a^6bd^3x^4+a^7d^3x^2\right)\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{14}bd^7\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{10}d^6\sqrt{-\frac{b^3}{a^{19}d^{10}}}+b^2dx}\,a^{14}d^7\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+1155\left(a^4b^3d^3x^8+3a^5b^2d^3x^6+3a^6bd^3x^4+a^7d^3x^2\right)\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}\log\left(385a^5d^3\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}+385\sqrt{dx}b\right)-1155\left(a^4b^3d^3x^8+3a^5b^2d^3x^6+3a^6bd^3x^4+a^7d^3x^2\right)\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}\log\left(-385a^5d^3\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}+385\sqrt{dx}b\right)+4\left(385b^3x^6+990ab^2x^4+765a^2bx^2+128a^3\right)\sqrt{dx}}{768\left(a^4b^3d^3x^8+3a^5b^2d^3x^6+3a^6bd^3x^4+a^7d^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(4620*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d
^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*arctan(-(sqrt(d*x)*a^14*b*d^7*(-b^3/(a^19*
d^10))^(3/4) - sqrt(a^10*d^6*sqrt(-b^3/(a^19*d^10)) + b^2*d*x)*a^14*d^7*(-b
^3/(a^19*d^10))^(3/4))/b^3) + 1155*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3
*a^6*b*d^3*x^4 + a^7*d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(385*a^5*d^3*(-b^
3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) - 1155*(a^4*b^3*d^3*x^8 + 3*a^5*b^2
*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(-385
*a^5*d^3*(-b^3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) + 4*(385*b^3*x^6 + 990
```

```
*a*b^2*x^4 + 765*a^2*b*x^2 + 128*a^3)*sqrt(d*x))/(a^4*b^3*d^3*x^8 + 3*a^5*b
^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x^2)
```

giac [A] time = 0.22, size = 308, normalized size = 0.88

$$-\frac{385\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^5d^3}-\frac{385\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^5d^3}-\frac{385\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^5d^3}+\frac{385\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^5d^3}-\frac{385\,b^3d^6x^6+990\,ab^2d^6x^4+765\,a^2bd^6x^2+128\,a^3d^6}{192\left(\sqrt{dx}\,bd^2x^2+\sqrt{dx}\,ad^2\right)^3a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -385/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*d^3) - 385/256*sqrt(2)*(a*b^3*d^2)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/(a^5*d^3) - 385/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*d^3) + 385/512*sqrt(2)*(a*b^3
*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
^5*d^3) - 1/192*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6*x^2 +
128*a^3*d^6)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)^3*a^4*d)
```

maple [A] time = 0.02, size = 288, normalized size = 0.82

$$-\frac{127\sqrt{dx}\,b\,d^3}{64\left(b\,d^2x^2+d^2a\right)^3a^2}-\frac{101\,(dx)^{\frac{5}{2}}\,b^2d}{32\left(b\,d^2x^2+d^2a\right)^3a^3}-\frac{257\,(dx)^{\frac{9}{2}}\,b^3}{192\left(b\,d^2x^2+d^2a\right)^3a^4d}-\frac{2}{3\,(dx)^{\frac{3}{2}}\,a^4d}-\frac{385\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256a^5d^3}-\frac{385\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}+1\right)}{256a^5d^3}-\frac{385\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)}{512a^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -257/192/d/a^4*b^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(9/2)-101/32*d/a^3*b^2/(b*d^2*
x^2+a*d^2)^3*(d*x)^(5/2)-127/64*d^3/a^2*b/(b*d^2*x^2+a*d^2)^3*(d*x)^(1/2)-3
85/512/d^3/a^5*b*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^
2)^(1/2)))-385/256/d^3/a^5*b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^
2)^(1/4)*(d*x)^(1/2)+1)-385/256/d^3/a^5*b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/3/a^4/d/(d*x)^(3/2)
```

maxima [A] time = 3.07, size = 335, normalized size = 0.95

$$-\frac{\frac{8\left(385\,b^3d^6x^6+990\,ab^2d^6x^4+765\,a^2bd^6x^2+128\,a^3d^6\right)}{(dx)^{\frac{15}{2}}a^4b^3+3\,(dx)^{\frac{11}{2}}a^5b^2d^2+3\,(dx)^{\frac{7}{2}}a^6bd^4+(dx)^{\frac{3}{2}}a^7d^6}+\frac{1155\left(\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}}+\frac{2\,\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}\,d}+\frac{2\,\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}\,d}\right)}{a^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(8*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6*x^2 + 128*a
^3*d^6)/((d*x)^(15/2)*a^4*b^3 + 3*(d*x)^(11/2)*a^5*b^2*d^2 + 3*(d*x)^(7/2)*
a^6*b*d^4 + (d*x)^(3/2)*a^7*d^6) + 1155*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x +
sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2
)*b^(3/4)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(
a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)
*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt
(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(a)*d))/a^4)/d
```

mupad [B] time = 4.25, size = 166, normalized size = 0.47

$$\frac{385\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{19/4}\,d^{5/2}}-\frac{\frac{2\,d^5}{3\,a}+\frac{255\,b\,d^5\,x^2}{64\,a^2}+\frac{165\,b^2\,d^5\,x^4}{32\,a^3}+\frac{385\,b^3\,d^5\,x^6}{192\,a^4}}{b^3\,{\left(d\,x\right)}^{15/2}+a^3\,d^6\,{\left(d\,x\right)}^{3/2}+3\,a^2\,b\,d^4\,{\left(d\,x\right)}^{7/2}+3\,a\,b^2\,d^2\,{\left(d\,x\right)}^{11/2}}+\frac{385\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{19/4}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] (385*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(1
9/4)*d^(5/2)) - ((2*d^5)/(3*a) + (255*b*d^5*x^2)/(64*a^2) + (165*b^2*d^5*x^
4)/(32*a^3) + (385*b^3*d^5*x^6)/(192*a^4))/(b^3*(d*x)^(15/2) + a^3*d^6*(d*x
)^(3/2) + 3*a^2*b*d^4*(d*x)^(7/2) + 3*a*b^2*d^2*(d*x)^(11/2)) + (385*(-b)^(
3/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(19/4)*d^(5/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx\right)^{\frac{5}{2}} \left(a + bx^{2}\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**4), x)
```

3.531 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=370

$$\frac{663b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{21/4}d^{7/2}}-\frac{663b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{21/4}d^{7/2}}-\frac{663b^{5}}{}$$

Rubi [A] time = 0.45, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{663b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{21/4}d^{7/2}}-\frac{663b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{21/4}d^{7/2}}-\frac{663b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{21/4}d^{7/2}}+\frac{663b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{21/4}d^{7/2}}+\frac{663b}{64a^5d^3\sqrt{dx}}+\frac{221}{192a^3d(dx)^{5/2}\left(a+bx^2\right)}+\frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2}-\frac{663}{320a^4d(dx)^{5/2}}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -663/(320*a^4*d*(d*x)^(5/2)) + (663*b)/(64*a^5*d^3*Sqrt[d*x]) + 1/(6*a*d*(d
*x)^(5/2)*(a + b*x^2)^3) + 17/(48*a^2*d*(d*x)^(5/2)*(a + b*x^2)^2) + 221/(1
92*a^3*d*(d*x)^(5/2)*(a + b*x^2)) - (663*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(21/4)*d^(7/2)) + (663*b^(5
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]
*a^(21/4)*d^(7/2)) + (663*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2)) - (663*
b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{\left(17b^3\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^3}\,dx}{12a}$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{\left(221b^2\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^2}}{96a^2}$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{221}{192a^3d(dx)^{5/2}\left(a+bx^2\right)}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{}{192a^3}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.10

$$-\frac{2\sqrt{dx}\ {}_2F_1\left(-\frac{5}{4},4;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^4d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-2*Sqrt[d*x]*Hypergeometric2F1[-5/4, 4, -1/4, -((b*x^2)/a)])/(5*a^4*d^4*x^
3)
```

IntegrateAlgebraic [A] time = 0.86, size = 241, normalized size = 0.65

$$-\frac{663b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{128\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{128\sqrt{2}a^{21/4}d^{7/2}}+\frac{-384a^4d^8+6528a^3bd^8x^2+24973a^2b^2d^8x^4+27846ab^3d^8x^6+9945b^4d^8x^8}{960a^5d^3(dx)^{5/2}\left(ad^2+bd^2x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-384*a^4*d^8 + 6528*a^3*b*d^8*x^2 + 24973*a^2*b^2*d^8*x^4 + 27846*a*b^3*d^
8*x^6 + 9945*b^4*d^8*x^8)/(960*a^5*d^3*(d*x)^(5/2)*(a*d^2 + b*d^2*x^2)^3) -
 (663*b^(5/4)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d
]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(128*Sqrt[2]*a^(21/4)*d^(7/2)) - (663*b
^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqr
t[b]*d*x)])/(128*Sqrt[2]*a^(21/4)*d^(7/2))
```

fricas [A] time = 2.02, size = 457, normalized size = 1.24

$$-\frac{39780\left(a^5b^3d^4x^9+3a^6b^2d^4x^7+3a^7bd^4x^5+a^8d^4x^3\right)\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{291434247\sqrt{dx}a^5b^4d^3\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-84933920324457009a^{11}b^5d^8\sqrt{-\frac{b^5}{a^{21}d^{14}}}+84933920324457009b^8dx}\,a^5d^3\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}}{291434247b^5}\right)-9945\left(a^5b^3d^4x^9+3a^6b^2d^4x^7+3a^7bd^4x^5+a^8d^4x^3\right)\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}\log\left(291434247a^{16}d^{11}\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{3}{4}}+291434247\sqrt{dx}b^4\right)+9945\left(a^5b^3d^4x^9+3a^6b^2d^4x^7+3a^7bd^4x^5+a^8d^4x^3\right)\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}\log\left(-291434247a^{16}d^{11}\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{3}{4}}+291434247\sqrt{dx}b^4\right)-4\left(9945b^4x^8+27846ab^3x^6+24973a^2b^2x^4+6528a^3bx^2-384a^4\right)\sqrt{dx}}{3840\left(a^5b^3d^4x^9+3a^6b^2d^4x^7+3a^7bd^4x^5+a^8d^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3840*(39780*(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8
*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*arctan(-1/291434247*(291434247*sqrt(d*x)
*a^5*b^4*d^3*(-b^5/(a^21*d^14))^(1/4) - sqrt(-84933920324457009*a^11*b^5*d^
8*sqrt(-b^5/(a^21*d^14)) + 84933920324457009*b^8*d*x)*a^5*d^3*(-b^5/(a^21*d
^14))^(1/4))/b^5) - 9945*(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4
*x^5 + a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*log(291434247*a^16*d^11*(-b^5/
(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4) + 9945*(a^5*b^3*d^4*x^9 + 3*a
^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*lo
g(-291434247*a^16*d^11*(-b^5/(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4)
```

```
- 4*(9945*b^4*x^8 + 27846*a*b^3*x^6 + 24973*a^2*b^2*x^4 + 6528*a^3*b*x^2 -
384*a^4)*sqrt(d*x))/(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5
+ a^8*d^4*x^3)
```

giac [A] time = 0.22, size = 349, normalized size = 0.94

$$\frac{663\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5}+\frac{663\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5}-\frac{663\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^6bd^5}+\frac{663\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^6bd^5}+\frac{453\,\sqrt{dx}\,b^4d^5x^5+1038\,\sqrt{dx}\,ab^3d^5x^3+617\,\sqrt{dx}\,a^2b^2d^5x}{192\left(bd^2x^2+ad^2\right)^3a^5d^3}+\frac{2\left(20\,bd^2x^2-ad^2\right)}{5\,\sqrt{dx}\,a^5d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 663/256*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d^5) + 663/256*sqrt(2)*(a*b^3*d^2
)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a^6*b*d^5) - 663/512*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d^5) + 663/512*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^6*b*d^5) + 1/192*(453*sqrt(d*x)*b^4*d^5*x^5 + 1038*sqrt(d*x)*a*b^3*d^
5*x^3 + 617*sqrt(d*x)*a^2*b^2*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^5*d^3) + 2/5*
(20*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^5*d^5*x^2)
```

maple [A] time = 0.03, size = 304, normalized size = 0.82

$$\frac{617\,(dx)^{\frac{3}{2}}\,b^2d}{192\left(b\,d^2x^2+d^2a\right)^3a^3}+\frac{173\,(dx)^{\frac{7}{2}}\,b^3}{32\left(b\,d^2x^2+d^2a\right)^3a^4d}+\frac{151\,(dx)^{\frac{11}{2}}\,b^4}{64\left(b\,d^2x^2+d^2a\right)^3a^5d^3}-\frac{2}{5\,(dx)^{\frac{5}{2}}\,a^4d}+\frac{663\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^5d^3}+\frac{663\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^5d^3}+\frac{663\sqrt{2}\;b\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{512\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^5d^3}+\frac{8b}{\sqrt{dx}\,a^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 151/64/d^3*b^4/a^5/(b*d^2*x^2+a*d^2)^3*(d*x)^(11/2)+173/32/d*b^3/a^4/(b*d^2
*x^2+a*d^2)^3*(d*x)^(7/2)+617/192*d*b^2/a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(3/2)
+663/512/d^3*b/a^5/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+663/256/d^3*b/a^5/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*
d^2)^(1/4)*(d*x)^(1/2)+1)+663/256/d^3*b/a^5/(a/b*d^2)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/5/a^4/d/(d*x)^(5/2)+8*b/a^5/d^3/(d
*x)^(1/2)
```

maxima [A] time = 3.17, size = 350, normalized size = 0.95

$$\frac{\frac{8\left(9945\,b^{4}d^{8}x^{8}+27846\,ab^{3}d^{8}x^{6}+24973\,a^{2}b^{2}d^{8}x^{4}+6528\,a^{3}bd^{8}x^{2}-384\,a^{4}d^{8}\right)}{(dx)^{\frac{17}{2}}a^{5}b^{3}d^{2}+3\,(dx)^{\frac{13}{2}}a^{6}b^{2}d^{4}+3\,(dx)^{\frac{9}{2}}a^{7}bd^{6}+(dx)^{\frac{5}{2}}a^{8}d^{8}}+\frac{9945\,b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^{5}d^{2}}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/7680*(8*(9945*b^4*d^8*x^8 + 27846*a*b^3*d^8*x^6 + 24973*a^2*b^2*d^8*x^4 +
 6528*a^3*b*d^8*x^2 - 384*a^4*d^8)/((d*x)^(17/2)*a^5*b^3*d^2 + 3*(d*x)^(13/
2)*a^6*b^2*d^4 + 3*(d*x)^(9/2)*a^7*b*d^6 + (d*x)^(5/2)*a^8*d^8) + 9945*b^2*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*
sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(
b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*lo
g(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^
2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*
x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^5*d^2))/d
```

mupad [B] time = 4.33, size = 179, normalized size = 0.48

$$\frac{\frac{34\,b\,d^{5}\,x^{2}}{5\,a^{2}}-\frac{2\,d^{5}}{5\,a}+\frac{24973\,b^{2}\,d^{5}\,x^{4}}{960\,a^{3}}+\frac{4641\,b^{3}\,d^{5}\,x^{6}}{160\,a^{4}}+\frac{663\,b^{4}\,d^{5}\,x^{8}}{64\,a^{5}}}{b^{3}\,(d\,x)^{17/2}+a^{3}\,d^{6}\,(d\,x)^{5/2}+3\,a^{2}\,b\,d^{4}\,(d\,x)^{9/2}+3\,a\,b^{2}\,d^{2}\,(d\,x)^{13/2}}-\frac{663\,(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{21/4}\,d^{7/2}}+\frac{663\,(-b)^{5/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{21/4}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((34*b*d^5*x^2)/(5*a^2) - (2*d^5)/(5*a) + (24973*b^2*d^5*x^4)/(960*a^3) + (
4641*b^3*d^5*x^6)/(160*a^4) + (663*b^4*d^5*x^8)/(64*a^5))/(b^3*(d*x)^(17/2)
 + a^3*d^6*(d*x)^(5/2) + 3*a^2*b*d^4*(d*x)^(9/2) + 3*a*b^2*d^2*(d*x)^(13/2)
) - (663*(-b)^(5/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*
a^(21/4)*d^(7/2)) + (663*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)
*d^(1/2))))/(128*a^(21/4)*d^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left(a+bx^{2}\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**4), x)
```

3.532 $\int \frac{(dx)^{27/2}}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx$

Optimal. Leaf size=420

$$-\frac{69615a^{5/4}d^{27/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{}\right.}{16384\sqrt{2}\,b^{29/4}}$$

Rubi [A] time = 0.53, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{69615a^{5/4}d^{27/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{29/4}}-\frac{69615a^{5/4}d^{27/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,b^{29/4}}-\frac{7735d^9(dx)^{9/2}}{4096b^5\left(a+bx^2\right)}-\frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4}-\frac{69615ad^{13}\sqrt{dx}}{4096b^7}-\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5}+\frac{13923d^{11}(dx)^{5/2}}{4096b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-69615*a*d^13*Sqrt[d*x])/(4096*b^7) + (13923*d^11*(d*x)^(5/2))/(4096*b^6)
- (d*(d*x)^(25/2))/(10*b*(a + b*x^2)^5) - (5*d^3*(d*x)^(21/2))/(32*b^2*(a +
 b*x^2)^4) - (35*d^5*(d*x)^(17/2))/(128*b^3*(a + b*x^2)^3) - (595*d^7*(d*x)
^(13/2))/(1024*b^4*(a + b*x^2)^2) - (7735*d^9*(d*x)^(9/2))/(4096*b^5*(a + b
*x^2)) - (69615*a^(5/4)*d^(27/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(27/2)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4))
- (69615*a^(5/4)*d^(27/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(2
7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*b^(29/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{27/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{27/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{4}\left(5b^4d^2\right)\int \frac{(dx)^{23/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(105b^2d^4\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(595d^6\right)\int \frac{(dx)^{15/\cdots}}{\left(ab+b^2x\cdots\right.}$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} - \frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{(773\cdots}{\cdots}$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} - \frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{77\cdots}{409\cdots}$$

$$= \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} - \frac{595d^{\cdots}}{1024b^4\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

Mathematica [A] time = 0.18, size = 432, normalized size = 1.03

$$\frac{d^{13}\sqrt{dx}\left(-3828825\sqrt{2}a^{5/4}\left(a+bx^2\right)^5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+3828825\sqrt{2}a^{5/4}\left(a+bx^2\right)^5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-7657650\sqrt{2}a^{5/4}\left(a+bx^2\right)^5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+7657650\sqrt{2}a^{5/4}\left(a+bx^2\right)^5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)-54312960a^6\sqrt[4]{b}\sqrt{x}-217251840a^5b^{5/4}x^{5/2}+3394560a^5\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-362086400a^4b^{9/4}x^{9/2}+4243200a^4\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2-306380800a^3b^{13/4}x^{13/2}+5834400a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3-126156800a^2b^{17/4}x^{17/2}+10210200a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^4-18022400ab^{21/4}x^{21/2}+720896b^{25/4}x^{25/2}\right)}{1802240b^{29/4}\sqrt{x}\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^13*Sqrt[d*x]*(-54312960*a^6*b^(1/4)*Sqrt[x] - 217251840*a^5*b^(5/4)*x^(5
/2) - 362086400*a^4*b^(9/4)*x^(9/2) - 306380800*a^3*b^(13/4)*x^(13/2) - 126
156800*a^2*b^(17/4)*x^(17/2) - 18022400*a*b^(21/4)*x^(21/2) + 720896*b^(25/
4)*x^(25/2) + 3394560*a^5*b^(1/4)*Sqrt[x]*(a + b*x^2) + 4243200*a^4*b^(1/4)
*Sqrt[x]*(a + b*x^2)^2 + 5834400*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 + 102102
00*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^4 - 7657650*Sqrt[2]*a^(5/4)*(a + b*x^2)^
5*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 7657650*Sqrt[2]*a^(5/4)*(
a + b*x^2)^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 3828825*Sqrt[2
]*a^(5/4)*(a + b*x^2)^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x] + 3828825*Sqrt[2]*a^(5/4)*(a + b*x^2)^5*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(1802240*b^(29/4)*Sqrt[x]*(a + b*x^2)^5)
```

IntegrateAlgebraic [A] time = 1.47, size = 244, normalized size = 0.58

$$-\frac{69615a^{5/4}d^{27/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x}\right)}{8192\sqrt{2}b^{29/4}}-\frac{d^{13}\sqrt{dx}\left(348075a^6+1670760a^5bx^2+3171350a^4b^2x^4+2951200a^3b^3x^6+1317575a^2b^4x^8+204800ab^5x^{10}-8192b^6x^{12}\right)}{20480b^7\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20480*(d^13*Sqrt[d*x]*(348075*a^6 + 1670760*a^5*b*x^2 + 3171350*a^4*b^2*
x^4 + 2951200*a^3*b^3*x^6 + 1317575*a^2*b^4*x^8 + 204800*a*b^5*x^10 - 8192*
b^6*x^12))/(b^7*(a + b*x^2)^5) - (69615*a^(5/4)*d^(27/2)*ArcTan[((a^(1/4)*S
qrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x
]])/(8192*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(27/2)*ArcTanh[(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x])/(Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x)])/(8192*Sqrt[2
]*b^(29/4))
```

fricas [A] time = 0.95, size = 515, normalized size = 1.23

$$\frac{1392300\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}\left(b^{12}x^{10}+5ab^{11}x^8+10a^2b^{10}x^6+10a^3b^9x^4+5a^4b^8x^2+a^5b^7\right)\arctan\left(-\frac{\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{3}{4}}\sqrt{dx}ab^{22}d^{13}-\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{3}{4}}\sqrt{a^2d^{27}x+\sqrt{-\frac{a^5d^{54}}{b^{29}}}b^{14}}b^{22}}{a^5d^{54}}\right)+348075\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}\left(b^{12}x^{10}+5ab^{11}x^8+10a^2b^{10}x^6+10a^3b^9x^4+5a^4b^8x^2+a^5b^7\right)\log\left(69615\sqrt{dx}ad^{13}+69615\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}b^7\right)-348075\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}\left(b^{12}x^{10}+5ab^{11}x^8+10a^2b^{10}x^6+10a^3b^9x^4+5a^4b^8x^2+a^5b^7\right)\log\left(69615\sqrt{dx}ad^{13}-69615\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}b^7\right)+4\left(8192b^6d^{13}x^{12}-204800ab^5d^{13}x^{10}-1317575a^2b^4d^{13}x^8-2951200a^3b^3d^{13}x^6-3171350a^4b^2d^{13}x^4-1670760a^5bd^{13}x^2-348075a^6d^{13}\right)\sqrt{dx}}{81920\left(b^{12}x^{10}+5ab^{11}x^8+10a^2b^{10}x^6+10a^3b^9x^4+5a^4b^8x^2+a^5b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(1392300*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*
b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*arctan(-((-a^5*d^54/b^
```

```
29)^(3/4)*sqrt(d*x)*a*b^22*d^13 - (-a^5*d^54/b^29)^(3/4)*sqrt(a^2*d^27*x +
sqrt(-a^5*d^54/b^29)*b^14)*b^22)/(a^5*d^54)) + 348075*(-a^5*d^54/b^29)^(1/4
)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*
x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13 + 69615*(-a^5*d^54/b^29)^(1/4)*b^
7) - 348075*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*
x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13
- 69615*(-a^5*d^54/b^29)^(1/4)*b^7) + 4*(8192*b^6*d^13*x^12 - 204800*a*b^5*
d^13*x^10 - 1317575*a^2*b^4*d^13*x^8 - 2951200*a^3*b^3*d^13*x^6 - 3171350*a
^4*b^2*d^13*x^4 - 1670760*a^5*b*d^13*x^2 - 348075*a^6*d^13)*sqrt(d*x))/(b^1
2*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 +
a^5*b^7)
```

giac [A] time = 0.24, size = 374, normalized size = 0.89

$$\frac{1}{163840}\,d^{13}\left(\frac{696150\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8}+\frac{696150\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8}+\frac{348075\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^8}-\frac{348075\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^8}-\frac{8\left(170695\,\sqrt{dx}\,a^2b^4d^{10}x^8+575520\,\sqrt{dx}\,a^3b^3d^{10}x^6+754710\,\sqrt{dx}\,a^4b^2d^{10}x^4+450152\,\sqrt{dx}\,a^5bd^{10}x^2+102315\,\sqrt{dx}\,a^6d^{10}\right)}{\left(bd^2x^2+ad^2\right)^5b^7}+\frac{65536\left(\sqrt{dx}\,b^{24}d^{10}x^2-30\,\sqrt{dx}\,ab^{23}d^{10}\right)}{b^{30}d^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^13*(696150*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(
2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^8 + 696150*sqrt(2)*(a*
b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x)
)/(a*d^2/b)^(1/4))/b^8 + 348075*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^8 - 348075*sqrt(2)*(a*b^3*d
^2)^(1/4)*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^
8 - 8*(170695*sqrt(d*x)*a^2*b^4*d^10*x^8 + 575520*sqrt(d*x)*a^3*b^3*d^10*x^
6 + 754710*sqrt(d*x)*a^4*b^2*d^10*x^4 + 450152*sqrt(d*x)*a^5*b*d^10*x^2 + 1
02315*sqrt(d*x)*a^6*d^10)/((b*d^2*x^2 + a*d^2)^5*b^7) + 65536*(sqrt(d*x)*b^
24*d^10*x^2 - 30*sqrt(d*x)*a*b^23*d^10)/(b^30*d^10))
```

maple [A] time = 0.03, size = 370, normalized size = 0.88

$$-\frac{20463\sqrt{dx}\,a^6d^{23}}{4096\left(b\,d^2x^2+d^2a\right)^5b^7}-\frac{56269\left(dx\right)^{\frac{5}{2}}a^5d^{21}}{2560\left(b\,d^2x^2+d^2a\right)^5b^6}-\frac{75471\left(dx\right)^{\frac{9}{2}}a^4d^{19}}{2048\left(b\,d^2x^2+d^2a\right)^5b^5}-\frac{3597\left(dx\right)^{\frac{13}{2}}a^3d^{17}}{128\left(b\,d^2x^2+d^2a\right)^5b^4}-\frac{34139\left(dx\right)^{\frac{17}{2}}a^2d^{15}}{4096\left(b\,d^2x^2+d^2a\right)^5b^3}+\frac{69615\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{13}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384b^7}+\frac{69615\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{13}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384b^7}+\frac{69615\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{13}\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768b^7}-\frac{12\sqrt{dx}\,a\,d^{13}}{b^7}+\frac{2\left(dx\right)^{\frac{5}{2}}d^{11}}{5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/5*d^11*(d*x)^(5/2)/b^6-12*a*d^13*(d*x)^(1/2)/b^7-20463/4096*d^23/b^7*a^6/
(b*d^2*x^2+a*d^2)^5*(d*x)^(1/2)-56269/2560*d^21/b^6*a^5/(b*d^2*x^2+a*d^2)^5
*(d*x)^(5/2)-75471/2048*d^19/b^5*a^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(9/2)-3597/1
28*d^17/b^4*a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(13/2)-34139/4096*d^15/b^3*a^2/(b
*d^2*x^2+a*d^2)^5*(d*x)^(17/2)+69615/32768*d^13/b^7*a*(a/b*d^2)^(1/4)*2^(1/
```

```
2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+69615/16384*d^13/b^7*a*(a/b
*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+69615/163
84*d^13/b^7*a*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^
(1/2)-1)
```

maxima [A] time = 3.18, size = 421, normalized size = 1.00

$$-\frac{\frac{8\left(170695\,(dx)^{\frac{17}{2}}a^{2}b^{4}d^{16}+575520\,(dx)^{\frac{13}{2}}a^{3}b^{3}d^{18}+754710\,(dx)^{\frac{9}{2}}a^{4}b^{2}d^{20}+450152\,(dx)^{\frac{5}{2}}a^{5}bd^{22}+102315\,\sqrt{dx}\,a^{6}d^{24}\right)}{b^{12}d^{10}x^{10}+5\,ab^{11}d^{10}x^{8}+10\,a^{2}b^{10}d^{10}x^{6}+10\,a^{3}b^{9}d^{10}x^{4}+5\,a^{4}b^{8}d^{10}x^{2}+a^{5}b^{7}d^{10}}-\frac{348075\left(\frac{\sqrt{2}d^{16}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^{16}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}d^{15}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}+\frac{2\sqrt{2}d^{15}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}\right)a^{2}}{b^{7}}-\frac{65536\left((dx)^{\frac{5}{2}}bd^{12}-30\sqrt{dx}\,ad^{14}\right)}{b^{7}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/163840*(8*(170695*(d*x)^(17/2)*a^2*b^4*d^16 + 575520*(d*x)^(13/2)*a^3*b^
3*d^18 + 754710*(d*x)^(9/2)*a^4*b^2*d^20 + 450152*(d*x)^(5/2)*a^5*b*d^22 +
102315*sqrt(d*x)*a^6*d^24)/(b^12*d^10*x^10 + 5*a*b^11*d^10*x^8 + 10*a^2*b^1
0*d^10*x^6 + 10*a^3*b^9*d^10*x^4 + 5*a^4*b^8*d^10*x^2 + a^5*b^7*d^10) - 348
075*(sqrt(2)*d^16*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^16*log(sqrt(b)*d*x - sqrt
(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
2*sqrt(2)*d^15*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2
*sqrt(2)*d^15*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^
2/b^7 - 65536*((d*x)^(5/2)*b*d^12 - 30*sqrt(d*x)*a*d^14)/b^7)/d
```

mupad [B] time = 4.40, size = 248, normalized size = 0.59

$$\frac{2\,d^{11}\,{\left(d\,x\right)}^{5/2}}{5\,b^6}-\frac{\frac{20463\,a^6\,d^{23}\,\sqrt{d\,x}}{4096}+\frac{75471\,a^4\,b^2\,d^{19}\,{\left(d\,x\right)}^{9/2}}{2048}+\frac{3597\,a^3\,b^3\,d^{17}\,{\left(d\,x\right)}^{13/2}}{128}+\frac{34139\,a^2\,b^4\,d^{15}\,{\left(d\,x\right)}^{17/2}}{4096}+\frac{56269\,a^5\,b\,d^{21}\,{\left(d\,x\right)}^{5/2}}{2560}}{a^5\,b^7\,d^{10}+5\,a^4\,b^8\,d^{10}\,x^2+10\,a^3\,b^9\,d^{10}\,x^4+10\,a^2\,b^{10}\,d^{10}\,x^6+5\,a\,b^{11}\,d^{10}\,x^8+b^{12}\,d^{10}\,x^{10}}-\frac{69615\,{\left(-a\right)}^{5/4}\,d^{27/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,b^{29/4}}-\frac{12\,a\,d^{13}\,\sqrt{d\,x}}{b^7}+\frac{{\left(-a\right)}^{5/4}\,d^{27/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,69615{}\mathrm{i}}{8192\,b^{29/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(27/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*d^11*(d*x)^(5/2))/(5*b^6) - ((20463*a^6*d^23*(d*x)^(1/2))/4096 + (75471*
a^4*b^2*d^19*(d*x)^(9/2))/2048 + (3597*a^3*b^3*d^17*(d*x)^(13/2))/128 + (34
139*a^2*b^4*d^15*(d*x)^(17/2))/4096 + (56269*a^5*b*d^21*(d*x)^(5/2))/2560)/
(a^5*b^7*d^10 + b^12*d^10*x^10 + 5*a*b^11*d^10*x^8 + 5*a^4*b^8*d^10*x^2 + 1
0*a^3*b^9*d^10*x^4 + 10*a^2*b^10*d^10*x^6) - (69615*(-a)^(5/4)*d^(27/2)*ata
n((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(29/4)) + ((-a)^(5/4
)*d^(27/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*69615i)/(819
2*b^(29/4)) - (12*a*d^13*(d*x)^(1/2))/b^7
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(27/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.533 $\int \frac{(dx)^{25/2}}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx$

Optimal. Leaf size=402

$$-\frac{33649a^{3/4}d^{25/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{\right.}}{16384\sqrt{2}\,b^{27/4}}$$

Rubi [A] time = 0.47, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{33649a^{3/4}d^{25/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,b^{27/4}}-\frac{33649a^{3/4}d^{25/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,b^{27/4}}-\frac{4807d^9(dx)^{7/2}}{4096b^5\left(a+bx^2\right)}-\frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}+\frac{33649d^{11}(dx)^{3/2}}{12288b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (33649*d^11*(d*x)^(3/2))/(12288*b^6) - (d*(d*x)^(23/2))/(10*b*(a + b*x^2)^5
) - (23*d^3*(d*x)^(19/2))/(160*b^2*(a + b*x^2)^4) - (437*d^5*(d*x)^(15/2))/
(1920*b^3*(a + b*x^2)^3) - (437*d^7*(d*x)^(11/2))/(1024*b^4*(a + b*x^2)^2)
- (4807*d^9*(d*x)^(7/2))/(4096*b^5*(a + b*x^2)) + (33649*a^(3/4)*d^(25/2)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(
27/4)) - (33649*a^(3/4)*d^(25/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(27/4)) - (33649*a^(3/4)*d^(25/2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(1638
4*Sqrt[2]*b^(27/4)) + (33649*a^(3/4)*d^(25/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(27/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{25/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{25/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(23b^4d^2\right)\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(437b^2d^4\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(437d^6\right)\int \frac{(dx)\ldots}{\left(ab+\ldots\right.}$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2} + \ldots$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2} - \ldots$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \ldots$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \ldots$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \ldots$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \ldots$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \ldots$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \ldots$$

Mathematica [C] time = 0.03, size = 109, normalized size = 0.27

$$-\frac{2d^{12}x\sqrt{dx}\left(-168245a^5-408595a^4bx^2-482885a^3b^2x^4-289731a^2b^3x^6-76245ab^4x^8+168245\left(a+bx^2\right)^5{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-3315b^5x^{10}\right)}{9945b^6\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-2*d^12*x*Sqrt[d*x]*(-168245*a^5 - 408595*a^4*b*x^2 - 482885*a^3*b^2*x^4 -
 289731*a^2*b^3*x^6 - 76245*a*b^4*x^8 - 3315*b^5*x^10 + 168245*(a + b*x^2)^
5*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)]))/(9945*b^6*(a + b*x^2)^5)
```

IntegrateAlgebraic [A] time = 1.46, size = 234, normalized size = 0.58

$$\frac{33649a^{3/4}d^{25/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x}\right)}{8192\sqrt{2}b^{27/4}}+\frac{d^{12}\sqrt{dx}\left(168245a^5x+769120a^4bx^3+1367810a^3b^2x^5+1157176a^2b^3x^7+437345ab^4x^9+40960b^5x^{11}\right)}{61440b^6\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^12*Sqrt[d*x]*(168245*a^5*x + 769120*a^4*b*x^3 + 1367810*a^3*b^2*x^5 + 11
57176*a^2*b^3*x^7 + 437345*a*b^4*x^9 + 40960*b^5*x^11))/(61440*b^6*(a + b*x
^2)^5) + (33649*a^(3/4)*d^(25/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)
) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*b^(27/
4)) + (33649*a^(3/4)*d^(25/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x])/(
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x)])/(8192*Sqrt[2]*b^(27/4))
```

fricas [A] time = 2.34, size = 515, normalized size = 1.28

$$\frac{2018940\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5ab^{10}x^8+10a^2b^9x^6+10a^3b^8x^4+5a^4b^7x^2+a^5b^6\right)\arctan\left(-\frac{\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}\sqrt{dx}a^2b^7d^{37}-\sqrt{a^4d^{75}x-\sqrt{-\frac{a^3d^{50}}{b^{27}}}a^3b^{13}d^{50}}\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}b^7}{a^3d^{50}}\right)-504735\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5ab^{10}x^8+10a^2b^9x^6+10a^3b^8x^4+5a^4b^7x^2+a^5b^6\right)\log\left(38099255258449\sqrt{dx}a^2d^{37}+38099255258449\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{3}{4}}b^{20}\right)+504735\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5ab^{10}x^8+10a^2b^9x^6+10a^3b^8x^4+5a^4b^7x^2+a^5b^6\right)\log\left(38099255258449\sqrt{dx}a^2d^{37}-38099255258449\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{3}{4}}b^{20}\right)+4\left(40960b^5d^{12}x^{11}+437345ab^4d^{12}x^9+1157176a^2b^3d^{12}x^7+1367810a^3b^2d^{12}x^5+769120a^4bd^{12}x^3+168245a^5d^{12}x\right)\sqrt{dx}}{245760\left(b^{11}x^{10}+5ab^{10}x^8+10a^2b^9x^6+10a^3b^8x^4+5a^4b^7x^2+a^5b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(2018940*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2
*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*arctan(-((-a^3*d^50/b^
27)^(1/4)*sqrt(d*x)*a^2*b^7*d^37 - sqrt(a^4*d^75*x - sqrt(-a^3*d^50/b^27)*a
^3*b^13*d^50)*(-a^3*d^50/b^27)^(1/4)*b^7)/(a^3*d^50)) - 504735*(-a^3*d^50/b
^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*
a^4*b^7*x^2 + a^5*b^6)*log(38099255258449*sqrt(d*x)*a^2*d^37 + 380992552584
49*(-a^3*d^50/b^27)^(3/4)*b^20) + 504735*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10
+ 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
*log(38099255258449*sqrt(d*x)*a^2*d^37 - 38099255258449*(-a^3*d^50/b^27)^(3
```

```
/4)*b^20) + 4*(40960*b^5*d^12*x^11 + 437345*a*b^4*d^12*x^9 + 1157176*a^2*b^
3*d^12*x^7 + 1367810*a^3*b^2*d^12*x^5 + 769120*a^4*b*d^12*x^3 + 168245*a^5*
d^12*x)*sqrt(d*x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*
x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

giac [A] time = 0.22, size = 354, normalized size = 0.88

$$\frac{1}{491520}\,d^{12}\left(\frac{327680\,\sqrt{dx}\,x}{b^6}-\frac{1009470\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^9d}-\frac{1009470\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^9d}+\frac{504735\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^9d}-\frac{504735\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^9d}+\frac{8\left(232545\,\sqrt{dx}\,ab^4d^{10}x^9+747576\,\sqrt{dx}\,a^2b^3d^{10}x^7+958210\,\sqrt{dx}\,a^3b^2d^{10}x^5+564320\,\sqrt{dx}\,a^4bd^{10}x^3+127285\,\sqrt{dx}\,a^5d^{10}x\right)}{\left(bd^2x^2+ad^2\right)^5b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^12*(327680*sqrt(d*x)*x/b^6 - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
(b^9*d) - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^9*d) + 504735*sqrt(2)*(a*b
^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(b^9*d) - 504735*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4
)*sqrt(d*x) + sqrt(a*d^2/b))/(b^9*d) + 8*(232545*sqrt(d*x)*a*b^4*d^10*x^9 +
 747576*sqrt(d*x)*a^2*b^3*d^10*x^7 + 958210*sqrt(d*x)*a^3*b^2*d^10*x^5 + 56
4320*sqrt(d*x)*a^4*b*d^10*x^3 + 127285*sqrt(d*x)*a^5*d^10*x)/((b*d^2*x^2 +
a*d^2)^5*b^6))
```

maple [A] time = 0.03, size = 354, normalized size = 0.88

$$\frac{25457\,(dx)^{\frac{3}{2}}\,a^5d^{21}}{12288\left(b\,d^2x^2+d^2a\right)^5b^6}+\frac{3527\,(dx)^{\frac{7}{2}}\,a^4d^{19}}{384\left(b\,d^2x^2+d^2a\right)^5b^5}+\frac{95821\,(dx)^{\frac{11}{2}}\,a^3d^{17}}{6144\left(b\,d^2x^2+d^2a\right)^5b^4}+\frac{31149\,(dx)^{\frac{15}{2}}\,a^2d^{15}}{2560\left(b\,d^2x^2+d^2a\right)^5b^3}+\frac{15503\,(dx)^{\frac{19}{2}}\,a\,d^{13}}{4096\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{33649\sqrt{2}\,a\,d^{13}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^7}-\frac{33649\sqrt{2}\,a\,d^{13}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^7}-\frac{33649\sqrt{2}\,a\,d^{13}\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^7}+\frac{2\,(dx)^{\frac{3}{2}}\,d^{11}}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/3*d^11*(d*x)^(3/2)/b^6+25457/12288*d^21*a^5/b^6/(b*d^2*x^2+a*d^2)^5*(d*x)
^(3/2)+3527/384*d^19*a^4/b^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(7/2)+95821/6144*d^1
7*a^3/b^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(11/2)+31149/2560*d^15*a^2/b^3/(b*d^2*x
^2+a*d^2)^5*(d*x)^(15/2)+15503/4096*d^13*a/b^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(1
9/2)-33649/32768*d^13*a/b^7/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2)))-33649/16384*d^13*a/b^7/(a/b*d^2)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-33649/16384*d^13*a/b^7/(a/b*d^2)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.20, size = 394, normalized size = 0.98

$$-\frac{\frac{504735\,ad^{14}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^{6}}-\frac{327680\left(dx\right)^{\frac{3}{2}}d^{12}}{b^{6}}-\frac{8\left(232545\left(dx\right)^{\frac{19}{2}}ab^{4}d^{14}+747576\left(dx\right)^{\frac{15}{2}}a^{2}b^{3}d^{16}+958210\left(dx\right)^{\frac{11}{2}}a^{3}b^{2}d^{18}+564320\left(dx\right)^{\frac{7}{2}}a^{4}bd^{20}+127285\left(dx\right)^{\frac{3}{2}}a^{5}d^{22}\right)}{b^{11}d^{10}x^{10}+5\,ab^{10}d^{10}x^{8}+10\,a^{2}b^{9}d^{10}x^{6}+10\,a^{3}b^{8}d^{10}x^{4}+5\,a^{4}b^{7}d^{10}x^{2}+a^{5}b^{6}d^{10}}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/491520*(504735*a*d^14*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/
4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sq
rt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^
(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)
*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^
(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt
(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b
^6 - 327680*(d*x)^(3/2)*d^12/b^6 - 8*(232545*(d*x)^(19/2)*a*b^4*d^14 + 7475
76*(d*x)^(15/2)*a^2*b^3*d^16 + 958210*(d*x)^(11/2)*a^3*b^2*d^18 + 564320*(d
*x)^(7/2)*a^4*b*d^20 + 127285*(d*x)^(3/2)*a^5*d^22)/(b^11*d^10*x^10 + 5*a*b
^10*d^10*x^8 + 10*a^2*b^9*d^10*x^6 + 10*a^3*b^8*d^10*x^4 + 5*a^4*b^7*d^10*x
^2 + a^5*b^6*d^10))/d
```

mupad [B] time = 0.24, size = 231, normalized size = 0.57

$$\frac{\frac{25457\,a^5\,d^{21}\,{\left(d\,x\right)}^{3/2}}{12288}+\frac{95821\,a^3\,b^2\,d^{17}\,{\left(d\,x\right)}^{11/2}}{6144}+\frac{31149\,a^2\,b^3\,d^{15}\,{\left(d\,x\right)}^{15/2}}{2560}+\frac{3527\,a^4\,b\,d^{19}\,{\left(d\,x\right)}^{7/2}}{384}+\frac{15503\,a\,b^4\,d^{13}\,{\left(d\,x\right)}^{19/2}}{4096}}{a^5\,b^6\,d^{10}+5\,a^4\,b^7\,d^{10}\,x^2+10\,a^3\,b^8\,d^{10}\,x^4+10\,a^2\,b^9\,d^{10}\,x^6+5\,a\,b^{10}\,d^{10}\,x^8+b^{11}\,d^{10}\,x^{10}}+\frac{2\,d^{11}\,{\left(d\,x\right)}^{3/2}}{3\,b^6}+\frac{33649\,{\left(-a\right)}^{3/4}\,d^{25/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,b^{27/4}}+\frac{{\left(-a\right)}^{3/4}\,d^{25/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,33649{}\mathrm{i}}{8192\,b^{27/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(25/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((25457*a^5*d^21*(d*x)^(3/2))/12288 + (95821*a^3*b^2*d^17*(d*x)^(11/2))/614
4 + (31149*a^2*b^3*d^15*(d*x)^(15/2))/2560 + (3527*a^4*b*d^19*(d*x)^(7/2))/
384 + (15503*a*b^4*d^13*(d*x)^(19/2))/4096)/(a^5*b^6*d^10 + b^11*d^10*x^10
+ 5*a*b^10*d^10*x^8 + 5*a^4*b^7*d^10*x^2 + 10*a^3*b^8*d^10*x^4 + 10*a^2*b^9
*d^10*x^6) + (2*d^11*(d*x)^(3/2))/(3*b^6) + (33649*(-a)^(3/4)*d^(25/2)*atan
((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(27/4)) + ((-a)^(3/4)
*d^(25/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*33649i)/(8192
*b^(27/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(25/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.534 $\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=402

$$\frac{13923\sqrt[4]{a}\,d^{23/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{25/4}}-\frac{13923\sqrt[4]{a}\,d^{23/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\right.}{16384\sqrt{2}\,b^{25/4}}$$

Rubi [A] time = 0.49, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{13923d^9(dx)^{5/2}}{20480b^5\left(a+bx^2\right)}-\frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2}-\frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3}-\frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4}+\frac{13923\sqrt[4]{a}\,d^{23/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{25/4}}-\frac{13923\sqrt[4]{a}\,d^{23/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{25/4}}+\frac{13923\sqrt[4]{a}\,d^{23/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,b^{25/4}}-\frac{13923\sqrt[4]{a}\,d^{23/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,b^{25/4}}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}+\frac{13923d^{11}\sqrt{dx}}{4096b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (13923*d^11*Sqrt[d*x])/(4096*b^6) - (d*(d*x)^(21/2))/(10*b*(a + b*x^2)^5) -
 (21*d^3*(d*x)^(17/2))/(160*b^2*(a + b*x^2)^4) - (119*d^5*(d*x)^(13/2))/(64
0*b^3*(a + b*x^2)^3) - (1547*d^7*(d*x)^(9/2))/(5120*b^4*(a + b*x^2)^2) - (1
3923*d^9*(d*x)^(5/2))/(20480*b^5*(a + b*x^2)) + (13923*a^(1/4)*d^(23/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25
/4)) - (13923*a^(1/4)*d^(23/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25/4)) + (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*
Sqrt[2]*b^(25/4)) - (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*S
qrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(25/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{(dx)^{23/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(21b^4d^2\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(357b^2d^4\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} + \frac{\left(1547d^6\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^3}\,dx}{1280}$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} -$$

Mathematica [A] time = 0.30, size = 408, normalized size = 1.01

$$\frac{d^{11}\sqrt{dx}\left(\frac{10862592a^5\sqrt[4]{b}\sqrt{x}+43450368a^4b^{5/4}x^{5/2}-678912a^4\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+72417280a^3b^{9/4}x^{9/2}-848640a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+61276160a^2b^{13/4}x^{13/2}-1166880a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+25231360ab^{17/4}x^{17/2}-2042040a\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^4+765765\sqrt{2}\sqrt[4]{a}\left(a+bx^2\right)^5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-765765\sqrt{2}\sqrt[4]{a}\left(a+bx^2\right)^5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-1531530\sqrt{2}\sqrt[4]{a}\left(a+bx^2\right)^5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+3604480b^{21/4}x^{21/2}}{\left(a+bx^2\right)^5}+1531530\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right)}{1802240b^{25/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^11*Sqrt[d*x]*(1531530*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)] + (10862592*a^5*b^(1/4)*Sqrt[x] + 43450368*a^4*b^(5/4)*x^(5/2)
+ 72417280*a^3*b^(9/4)*x^(9/2) + 61276160*a^2*b^(13/4)*x^(13/2) + 25231360*
a*b^(17/4)*x^(17/2) + 3604480*b^(21/4)*x^(21/2) - 678912*a^4*b^(1/4)*Sqrt[x
]*(a + b*x^2) - 848640*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 - 1166880*a^2*b^(1
/4)*Sqrt[x]*(a + b*x^2)^3 - 2042040*a*b^(1/4)*Sqrt[x]*(a + b*x^2)^4 - 15315
30*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)] + 765765*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x] - 765765*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a + b*x^2)^5))/(1802240
*b^(25/4)*Sqrt[x])
```

IntegrateAlgebraic [A] time = 1.27, size = 233, normalized size = 0.58

$$\frac{d^{11}\sqrt{dx}\left(69615a^5+334152a^4bx^2+634270a^3b^2x^4+590240a^2b^3x^6+263515ab^4x^8+40960b^5x^{10}\right)}{20480b^6\left(a+bx^2\right)^5}+\frac{13923\sqrt[4]{a}\,d^{23/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}\,b^{25/4}}-\frac{13923\sqrt[4]{a}\,d^{23/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x}\right)}{8192\sqrt{2}\,b^{25/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^11*Sqrt[d*x]*(69615*a^5 + 334152*a^4*b*x^2 + 634270*a^3*b^2*x^4 + 590240
*a^2*b^3*x^6 + 263515*a*b^4*x^8 + 40960*b^5*x^10))/(20480*b^6*(a + b*x^2)^5
) + (13923*a^(1/4)*d^(23/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (
b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*b^(25/4)) -
 (13923*a^(1/4)*d^(23/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x])/(Sqrt[
a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x)])/(8192*Sqrt[2]*b^(25/4))
```

fricas [A] time = 1.76, size = 479, normalized size = 1.19

$$-\frac{278460\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5ab^{10}x^8+10a^2b^9x^6+10a^3b^8x^4+5a^4b^7x^2+a^5b^6\right)\arctan\left(-\frac{\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{19}d^{11}-\sqrt{d^{23}x+\sqrt{-\frac{ad^{46}}{b^{25}}}\,b^{12}}\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{3}{4}}b^{19}}{ad^{46}}\right)+69615\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5ab^{10}x^8+10a^2b^9x^6+10a^3b^8x^4+5a^4b^7x^2+a^5b^6\right)\log\left(13923\sqrt{dx}\,d^{11}+13923\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}b^6\right)-69615\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5ab^{10}x^8+10a^2b^9x^6+10a^3b^8x^4+5a^4b^7x^2+a^5b^6\right)\log\left(13923\sqrt{dx}\,d^{11}-13923\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}b^6\right)-4\left(40960b^5d^{11}x^{10}+263515ab^4d^{11}x^8+590240a^2b^3d^{11}x^6+634270a^3b^2d^{11}x^4+334152a^4bd^{11}x^2+69615a^5d^{11}\right)\sqrt{dx}}{81920\left(b^{11}x^{10}+5ab^{10}x^8+10a^2b^9x^6+10a^3b^8x^4+5a^4b^7x^2+a^5b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(278460*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^
9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*arctan(-((-a*d^46/b^25)^(
```

3/4)*sqrt(d*x)*b^19*d^11 - sqrt(d^23*x + sqrt(-a*d^46/b^25)*b^12)*(-a*d^46/b^25)^(3/4)*b^19)/(a*d^46)) + 69615*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(13923*sqrt(d*x)*d^11 + 13923*(-a*d^46/b^25)^(1/4)*b^6) - 69615*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(13923*sqrt(d*x)*d^11 - 13923*(-a*d^46/b^25)^(1/4)*b^6) - 4*(40960*b^5*d^11*x^10 + 263515*a*b^4*d^11*x^8 + 590240*a^2*b^3*d^11*x^6 + 634270*a^3*b^2*d^11*x^4 + 334152*a^4*b*d^11*x^2 + 69615*a^5*d^11)*sqrt(d*x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)

giac [A] time = 0.25, size = 340, normalized size = 0.85

$$-\frac{1}{163840}\,d^{11}\left(\frac{139230\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}+\frac{139230\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}+\frac{69615\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7}-\frac{69615\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7}-\frac{327680\,\sqrt{dx}}{b^6}-\frac{8\left(58715\,\sqrt{dx}\,ab^4d^{10}x^8+180640\,\sqrt{dx}\,a^2b^3d^{10}x^6+224670\,\sqrt{dx}\,a^3b^2d^{10}x^4+129352\,\sqrt{dx}\,a^4bd^{10}x^2+28655\,\sqrt{dx}\,a^5d^{10}\right)}{\left(bd^2x^2+ad^2\right)^5b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")

[Out] -1/163840*d^11*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7 + 139230*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7 + 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 327680*sqrt(d*x)/b^6 - 8*(58715*sqrt(d*x)*a*b^4*d^10*x^8 + 180640*sqrt(d*x)*a^2*b^3*d^10*x^6 + 224670*sqrt(d*x)*a^3*b^2*d^10*x^4 + 129352*sqrt(d*x)*a^4*b*d^10*x^2 + 28655*sqrt(d*x)*a^5*d^10)/((b*d^2*x^2 + a*d^2)^5*b^6))

maple [A] time = 0.03, size = 351, normalized size = 0.87

$$\frac{5731\,\sqrt{dx}\,a^5d^{21}}{4096\left(b\,d^2x^2+d^2a\right)^5b^6}+\frac{16169\,(dx)^{\frac{5}{2}}\,a^4d^{19}}{2560\left(b\,d^2x^2+d^2a\right)^5b^5}+\frac{22467\,(dx)^{\frac{9}{2}}\,a^3d^{17}}{2048\left(b\,d^2x^2+d^2a\right)^5b^4}+\frac{1129\,(dx)^{\frac{13}{2}}\,a^2d^{15}}{128\left(b\,d^2x^2+d^2a\right)^5b^3}+\frac{11743\,(dx)^{\frac{17}{2}}\,a\,d^{13}}{4096\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{13923\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{11}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384b^6}-\frac{13923\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{11}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384b^6}-\frac{13923\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{11}\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768b^6}+\frac{2\sqrt{dx}\,d^{11}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)

[Out] 2*d^11*(d*x)^(1/2)/b^6+5731/4096*d^21/b^6*a^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(1/2)+16169/2560*d^19/b^5*a^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(5/2)+22467/2048*d^17/b^4*a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(9/2)+1129/128*d^15/b^3*a^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(13/2)+11743/4096*d^13/b^2*a/(b*d^2*x^2+a*d^2)^5*(d*x)^(17/2)-13923/32768*d^11/b^6*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-13923/16384*d^11/b^6*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/

```
(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-13923/16384*d^11/b^6*(a/b*d^2)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.23, size = 403, normalized size = 1.00

$$\frac{\frac{327680\sqrt{dx}\,d^{12}}{b^6}+\frac{8\left(58715\,(dx)^{\frac{17}{2}}ab^4d^{14}+180640\,(dx)^{\frac{13}{2}}a^2b^3d^{16}+224670\,(dx)^{\frac{9}{2}}a^3b^2d^{18}+129352\,(dx)^{\frac{5}{2}}a^4bd^{20}+28655\sqrt{dx}\,a^5d^{22}\right)}{b^{11}d^{10}x^{10}+5ab^{10}d^{10}x^8+10a^2b^9d^{10}x^6+10a^3b^8d^{10}x^4+5a^4b^7d^{10}x^2+a^5b^6d^{10}}-\frac{69615\left(\frac{\sqrt{2}d^{14}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^{14}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}d^{13}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}+\frac{2\sqrt{2}d^{13}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}\right)a}{b^6}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(327680*sqrt(d*x)*d^12/b^6 + 8*(58715*(d*x)^(17/2)*a*b^4*d^14 + 18
0640*(d*x)^(13/2)*a^2*b^3*d^16 + 224670*(d*x)^(9/2)*a^3*b^2*d^18 + 129352*(
d*x)^(5/2)*a^4*b*d^20 + 28655*sqrt(d*x)*a^5*d^22)/(b^11*d^10*x^10 + 5*a*b^1
0*d^10*x^8 + 10*a^2*b^9*d^10*x^6 + 10*a^3*b^8*d^10*x^4 + 5*a^4*b^7*d^10*x^2
 + a^5*b^6*d^10) - 69615*(sqrt(2)*d^14*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1
/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^14*l
og(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d
^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^13*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/
4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sq
rt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^13*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/
4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sq
rt(b)*d)*sqrt(a)))*a/b^6)/d
```

mupad [B] time = 4.36, size = 231, normalized size = 0.57

$$\frac{\frac{5731\,a^5\,d^{21}\,\sqrt{d\,x}}{4096}+\frac{22467\,a^3\,b^2\,d^{17}\,{\left(d\,x\right)}^{9/2}}{2048}+\frac{1129\,a^2\,b^3\,d^{15}\,{\left(d\,x\right)}^{13/2}}{128}+\frac{16169\,a^4\,b\,d^{19}\,{\left(d\,x\right)}^{5/2}}{2560}+\frac{11743\,a\,b^4\,d^{13}\,{\left(d\,x\right)}^{17/2}}{4096}}{a^5\,b^6\,d^{10}+5\,a^4\,b^7\,d^{10}\,x^2+10\,a^3\,b^8\,d^{10}\,x^4+10\,a^2\,b^9\,d^{10}\,x^6+5\,a\,b^{10}\,d^{10}\,x^8+b^{11}\,d^{10}\,x^{10}}+\frac{2\,d^{11}\,\sqrt{d\,x}}{b^6}-\frac{13923\,{\left(-a\right)}^{1/4}\,d^{23/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,b^{25/4}}+\frac{{\left(-a\right)}^{1/4}\,d^{23/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,13923{}\mathrm{i}}{8192\,b^{25/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((5731*a^5*d^21*(d*x)^(1/2))/4096 + (22467*a^3*b^2*d^17*(d*x)^(9/2))/2048 +
 (1129*a^2*b^3*d^15*(d*x)^(13/2))/128 + (16169*a^4*b*d^19*(d*x)^(5/2))/2560
 + (11743*a*b^4*d^13*(d*x)^(17/2))/4096)/(a^5*b^6*d^10 + b^11*d^10*x^10 + 5
*a*b^10*d^10*x^8 + 5*a^4*b^7*d^10*x^2 + 10*a^3*b^8*d^10*x^4 + 10*a^2*b^9*d^
10*x^6) + (2*d^11*(d*x)^(1/2))/b^6 - (13923*(-a)^(1/4)*d^(23/2)*atan((b^(1/
4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(25/4)) + ((-a)^(1/4)*d^(23/
2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*13923i)/(8192*b^(25/
4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.535 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=385

$$\frac{4389d^{21/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\sqrt[4]{a}\,b^{23/4}}-\frac{4389d^{21/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\sqrt[4]{a}\,b^{23/4}}-- 4$$

Rubi [A] time = 0.45, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1463d^9(dx)^{3/2}}{4096b^5\left(a+bx^2\right)}-\frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3}-\frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4}+\frac{4389d^{21/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\sqrt[4]{a}\,b^{23/4}}-\frac{4389d^{21/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\sqrt[4]{a}\,b^{23/4}}-\frac{4389d^{21/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\sqrt[4]{a}\,b^{23/4}}+\frac{4389d^{21/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\sqrt[4]{a}\,b^{23/4}}-\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(19/2))/(10*b*(a + b*x^2)^5) - (19*d^3*(d*x)^(15/2))/(160*b^2*(a
+ b*x^2)^4) - (19*d^5*(d*x)^(11/2))/(128*b^3*(a + b*x^2)^3) - (209*d^7*(d*x
)^(7/2))/(1024*b^4*(a + b*x^2)^2) - (1463*d^9*(d*x)^(3/2))/(4096*b^5*(a + b
*x^2)) - (4389*d^(21/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) + (4389*d^(21/2)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) +
(4389*d^(21/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1/4)*b^(23/4)) - (4389*d^(21/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(163
84*Sqrt[2]*a^(1/4)*b^(23/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(19b^4d^2\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(57b^2d^4\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(209d^6\right)\int \cdots$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \cdots$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \cdots$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \cdots$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \cdots$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \cdots$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \cdots$$

Mathematica [C] time = 0.04, size = 104, normalized size = 0.27

$$\frac{2d^9(dx)^{3/2}\left(7315\left(a+bx^2\right)^5\ {}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(7315a^4+17765a^3bx^2+20995a^2b^2x^4+12597ab^3x^6+3315b^4x^8\right)\right)}{3315ab^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^9*(d*x)^(3/2)*(-(a*(7315*a^4 + 17765*a^3*b*x^2 + 20995*a^2*b^2*x^4 + 1
2597*a*b^3*x^6 + 3315*b^4*x^8)) + 7315*(a + b*x^2)^5*Hypergeometric2F1[3/4,
 6, 7/4, -((b*x^2)/a)]))/(3315*a*b^5*(a + b*x^2)^5)
```

IntegrateAlgebraic [A] time = 1.18, size = 223, normalized size = 0.58

$$-\frac{d^{10}\sqrt{dx}\left(7315a^4x+33440a^3bx^3+59470a^2b^2x^5+50312ab^3x^7+19015b^4x^9\right)}{20480b^5\left(a+bx^2\right)^5}-\frac{4389d^{21/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}\sqrt[4]{a}b^{23/4}}-\frac{4389d^{21/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x}\right)}{8192\sqrt{2}\sqrt[4]{a}b^{23/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20480*(d^10*Sqrt[d*x]*(7315*a^4*x + 33440*a^3*b*x^3 + 59470*a^2*b^2*x^5
+ 50312*a*b^3*x^7 + 19015*b^4*x^9))/(b^5*(a + b*x^2)^5) - (4389*d^(21/2)*Ar
cTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^
(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) - (4389*d^(21/2)*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x])/(Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x)
])/(8192*Sqrt[2]*a^(1/4)*b^(23/4))
```

fricas [A] time = 1.04, size = 486, normalized size = 1.26

$$-\frac{87780\left(b^{10}x^{10}+5ab^9x^8+10a^2b^8x^6+10a^3b^7x^4+5a^4b^6x^2+a^5b^5\right)\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}\sqrt{dx}b^6d^{31}-\sqrt{d^{63}x-\sqrt{-\frac{d^{42}}{ab^{23}}}ab^{11}d^{42}}\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}b^6}{d^{42}}\right)-21945\left(b^{10}x^{10}+5ab^9x^8+10a^2b^8x^6+10a^3b^7x^4+5a^4b^6x^2+a^5b^5\right)\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}\log\left(84546715869\sqrt{dx}d^{31}+84546715869\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{3}{4}}ab^{17}\right)+21945\left(b^{10}x^{10}+5ab^9x^8+10a^2b^8x^6+10a^3b^7x^4+5a^4b^6x^2+a^5b^5\right)\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}\log\left(84546715869\sqrt{dx}d^{31}-84546715869\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{3}{4}}ab^{17}\right)+4\left(19015b^4d^{10}x^9+50312ab^3d^{10}x^7+59470a^2b^2d^{10}x^5+33440a^3bd^{10}x^3+7315a^4d^{10}x\right)\sqrt{dx}}{81920\left(b^{10}x^{10}+5ab^9x^8+10a^2b^8x^6+10a^3b^7x^4+5a^4b^6x^2+a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(87780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*arctan(-((-d^42/(a*b^23))
^(1/4)*sqrt(d*x)*b^6*d^31 - sqrt(d^63*x - sqrt(-d^42/(a*b^23))*a*b^11*d^42)
*(-d^42/(a*b^23))^(1/4)*b^6)/d^42) - 21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^
2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4
)*log(84546715869*sqrt(d*x)*d^31 + 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^1
7) + 21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a
^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*log(84546715869*sqrt(d*x)*d^31
```

```
 - 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^17) + 4*(19015*b^4*d^10*x^9 + 503
12*a*b^3*d^10*x^7 + 59470*a^2*b^2*d^10*x^5 + 33440*a^3*b*d^10*x^3 + 7315*a^
4*d^10*x)*sqrt(d*x))/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7
*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)
```

giac [A] time = 0.21, size = 352, normalized size = 0.91

$$\frac{1}{163840}\,d^{10}\left(\frac{43890\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^8d}+\frac{43890\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^8d}-\frac{21945\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^8d}+\frac{21945\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^8d}-\frac{8\left(19015\,\sqrt{dx}\,b^4d^{10}x^9+50312\,\sqrt{dx}\,ab^3d^{10}x^7+59470\,\sqrt{dx}\,a^2b^2d^{10}x^5+33440\,\sqrt{dx}\,a^3bd^{10}x^3+7315\,\sqrt{dx}\,a^4d^{10}x\right)}{\left(bd^2x^2+ad^2\right)^5b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^10*(43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^8*d) + 43890*sqrt(2)*(
a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x)
)/(a*d^2/b)^(1/4))/(a*b^8*d) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^8*d) + 21945*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a*b^8*d) - 8*(19015*sqrt(d*x)*b^4*d^10*x^9 + 50312*sqrt(d*x)*a*b^3*d^1
0*x^7 + 59470*sqrt(d*x)*a^2*b^2*d^10*x^5 + 33440*sqrt(d*x)*a^3*b*d^10*x^3 +
 7315*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

maple [A] time = 0.02, size = 335, normalized size = 0.87

$$-\frac{1463\left(dx\right)^{\frac{3}{2}}a^4d^{19}}{4096\left(b\,d^2x^2+d^2a\right)^5b^5}-\frac{209\left(dx\right)^{\frac{7}{2}}a^3d^{17}}{128\left(b\,d^2x^2+d^2a\right)^5b^4}-\frac{5947\left(dx\right)^{\frac{11}{2}}a^2d^{15}}{2048\left(b\,d^2x^2+d^2a\right)^5b^3}-\frac{6289\left(dx\right)^{\frac{15}{2}}a\,d^{13}}{2560\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{3803\left(dx\right)^{\frac{19}{2}}d^{11}}{4096\left(b\,d^2x^2+d^2a\right)^5b}+\frac{4389\sqrt{2}\,d^{11}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^6}+\frac{4389\sqrt{2}\,d^{11}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^6}+\frac{4389\sqrt{2}\,d^{11}\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1463/4096*d^19/(b*d^2*x^2+a*d^2)^5/b^5*a^4*(d*x)^(3/2)-209/128*d^17/(b*d^2
*x^2+a*d^2)^5/b^4*a^3*(d*x)^(7/2)-5947/2048*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^
2*(d*x)^(11/2)-6289/2560*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(15/2)-3803/4
096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(19/2)+4389/32768*d^11/b^6/(a/b*d^2)^(
1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4389/16384*d^11/b
^6/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+43
89/16384*d^11/b^6/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d
*x)^(1/2)-1)
```

maxima [A] time = 3.16, size = 377, normalized size = 0.98

$$\frac{\frac{21945\,d^{12}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^{5}}-\frac{8\left(19015\,(dx)^{\frac{19}{2}}b^{4}d^{12}+50312\,(dx)^{\frac{15}{2}}ab^{3}d^{14}+59470\,(dx)^{\frac{11}{2}}a^{2}b^{2}d^{16}+33440\,(dx)^{\frac{7}{2}}a^{3}bd^{18}+7315\,(dx)^{\frac{3}{2}}a^{4}d^{20}\right)}{b^{10}d^{10}x^{10}+5\,ab^{9}d^{10}x^{8}+10\,a^{2}b^{8}d^{10}x^{6}+10\,a^{3}b^{7}d^{10}x^{4}+5\,a^{4}b^{6}d^{10}x^{2}+a^{5}b^{5}d^{10}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(21945*d^12*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*
sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4
) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^5 -
 8*(19015*(d*x)^(19/2)*b^4*d^12 + 50312*(d*x)^(15/2)*a*b^3*d^14 + 59470*(d*
x)^(11/2)*a^2*b^2*d^16 + 33440*(d*x)^(7/2)*a^3*b*d^18 + 7315*(d*x)^(3/2)*a^
4*d^20)/(b^10*d^10*x^10 + 5*a*b^9*d^10*x^8 + 10*a^2*b^8*d^10*x^6 + 10*a^3*b
^7*d^10*x^4 + 5*a^4*b^6*d^10*x^2 + a^5*b^5*d^10))/d
```

mupad [B] time = 0.21, size = 213, normalized size = 0.55

$$\frac{4389\,d^{21/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{1/4}\,b^{23/4}}-\frac{\frac{3803\,d^{11}\,(d\,x)^{19/2}}{4096\,b}+\frac{5947\,a^2\,d^{15}\,(d\,x)^{11/2}}{2048\,b^3}+\frac{209\,a^3\,d^{17}\,(d\,x)^{7/2}}{128\,b^4}+\frac{1463\,a^4\,d^{19}\,(d\,x)^{3/2}}{4096\,b^5}+\frac{6289\,a\,d^{13}\,(d\,x)^{15/2}}{2560\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{4389\,d^{21/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{1/4}\,b^{23/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (4389*d^(21/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(1/4)*b^(23/4)) - ((3803*d^11*(d*x)^(19/2))/(4096*b) + (5947*a^2*d^15*(d*x
)^(11/2))/(2048*b^3) + (209*a^3*d^17*(d*x)^(7/2))/(128*b^4) + (1463*a^4*d^1
9*(d*x)^(3/2))/(4096*b^5) + (6289*a*d^13*(d*x)^(15/2))/(2560*b^2))/(a^5*d^1
0 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x
^4 + 10*a^2*b^3*d^10*x^6) - (4389*d^(21/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a
)^(1/4)*d^(1/2))))/(8192*(-a)^(1/4)*b^(23/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.536 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=385

$$-\frac{663d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}-\frac{663}{}$$

Rubi [A] time = 0.45, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{663d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}-\frac{663d^{19/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{3/4}b^{21/4}}-\frac{663d^9\sqrt{dx}}{4096b^5\left(a+bx^2\right)}-\frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2}-\frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(17/2))/(10*b*(a + b*x^2)^5) - (17*d^3*(d*x)^(13/2))/(160*b^2*(a
+ b*x^2)^4) - (221*d^5*(d*x)^(9/2))/(1920*b^3*(a + b*x^2)^3) - (663*d^7*(d*
x)^(5/2))/(5120*b^4*(a + b*x^2)^2) - (663*d^9*Sqrt[d*x])/(4096*b^5*(a + b*x
^2)) - (663*d^(19/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) - (663
*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqr
t[2]*a^(3/4)*b^(21/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(17b^4d^2\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(221b^2d^4\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{\left(663d^6\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^3}\,d}{1280}$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

Mathematica [A] time = 0.20, size = 381, normalized size = 0.99

$$\frac{d^9\sqrt{dx}\left(-\frac{765765\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}+\frac{765765\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}-\frac{1531530\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}\sqrt{x}}+\frac{1531530\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}\sqrt{x}}-\frac{10862592a^4\sqrt[4]{b}}{(a+bx^2)^5}-\frac{43450368a^3b^{5/4}x^2}{(a+bx^2)^5}+\frac{678912a^3\sqrt[4]{b}}{(a+bx^2)^4}-\frac{72417280a^2b^{9/4}x^4}{(a+bx^2)^5}+\frac{848640a^2\sqrt[4]{b}}{(a+bx^2)^3}-\frac{25231360b^{17/4}x^8}{(a+bx^2)^5}-\frac{61276160ab^{13/4}x^6}{(a+bx^2)^5}+\frac{2042040\sqrt[4]{b}}{a+bx^2}+\frac{1166880a\sqrt[4]{b}}{(a+bx^2)^2}\right)}{37847040b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^9*Sqrt[d*x]*((-10862592*a^4*b^(1/4))/(a + b*x^2)^5 - (43450368*a^3*b^(5/
4)*x^2)/(a + b*x^2)^5 - (72417280*a^2*b^(9/4)*x^4)/(a + b*x^2)^5 - (6127616
0*a*b^(13/4)*x^6)/(a + b*x^2)^5 - (25231360*b^(17/4)*x^8)/(a + b*x^2)^5 + (
678912*a^3*b^(1/4))/(a + b*x^2)^4 + (848640*a^2*b^(1/4))/(a + b*x^2)^3 + (1
166880*a*b^(1/4))/(a + b*x^2)^2 + (2042040*b^(1/4))/(a + b*x^2) - (1531530*
Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) +
(1531530*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sq
rt[x]) - (765765*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(a^(3/4)*Sqrt[x]) + (765765*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x])))/(37847040*b^(21/4))
```

IntegrateAlgebraic [A] time = 1.22, size = 222, normalized size = 0.58

$$-\frac{663d^{19/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x}\right)}{8192\sqrt{2}a^{3/4}b^{21/4}}-\frac{d^9\sqrt{dx}\left(9945a^4+47736a^3bx^2+90610a^2b^2x^4+84320ab^3x^6+37645b^4x^8\right)}{61440b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/61440*(d^9*Sqrt[d*x]*(9945*a^4 + 47736*a^3*b*x^2 + 90610*a^2*b^2*x^4 + 8
4320*a*b^3*x^6 + 37645*b^4*x^8))/(b^5*(a + b*x^2)^5) - (663*d^(19/2)*ArcTan
[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4
)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*ArcTanh[(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x])/(Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x)])/(8
192*Sqrt[2]*a^(3/4)*b^(21/4))
```

fricas [A] time = 1.69, size = 489, normalized size = 1.27

$$\frac{39780\left(b^{10}x^{10}+5ab^9x^8+10a^2b^8x^6+10a^3b^7x^4+5a^4b^6x^2+a^5b^5\right)\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac{3}{4}}\sqrt{dx}a^2b^{16}d^9-\sqrt{d^{19}x+\sqrt{-\frac{d^{38}}{a^3b^{21}}}a^2b^{10}}\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac{3}{4}}a^2b^{16}}{d^{38}}\right)+9945\left(b^{10}x^{10}+5ab^9x^8+10a^2b^8x^6+10a^3b^7x^4+5a^4b^6x^2+a^5b^5\right)\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac{1}{4}}\log\left(663\sqrt{dx}d^9+663\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac{1}{4}}ab^5\right)-9945\left(b^{10}x^{10}+5ab^9x^8+10a^2b^8x^6+10a^3b^7x^4+5a^4b^6x^2+a^5b^5\right)\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac{1}{4}}\log\left(663\sqrt{dx}d^9-663\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac{1}{4}}ab^5\right)-4\left(37645b^4d^9x^8+84320ab^3d^9x^6+90610a^2b^2d^9x^4+47736a^3bd^9x^2+9945a^4d^9\right)\sqrt{dx}}{245760\left(b^{10}x^{10}+5ab^9x^8+10a^2b^8x^6+10a^3b^7x^4+5a^4b^6x^2+a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(39780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*arctan(-((-d^38/(a^3*b^
```

```
21))^(3/4)*sqrt(d*x)*a^2*b^16*d^9 - sqrt(d^19*x + sqrt(-d^38/(a^3*b^21))*a^
2*b^10)*(-d^38/(a^3*b^21))^(3/4)*a^2*b^16)/d^38) + 9945*(b^10*x^10 + 5*a*b^
9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(
a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 + 663*(-d^38/(a^3*b^21))^(1/4)*a*b^5
) - 9945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4
*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 - 663*(-
d^38/(a^3*b^21))^(1/4)*a*b^5) - 4*(37645*b^4*d^9*x^8 + 84320*a*b^3*d^9*x^6
+ 90610*a^2*b^2*d^9*x^4 + 47736*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b
^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 +
a^5*b^5)
```

giac [A] time = 0.24, size = 339, normalized size = 0.88

$$\frac{1}{491520} d^9 \left(\frac{19890 \sqrt{2} \left(ab^3d^2\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}} + 2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6} + \frac{19890 \sqrt{2} \left(ab^3d^2\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}} - 2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6} + \frac{9945 \sqrt{2} \left(ab^3d^2\right)^{\frac{1}{4}} \log\left(dx + \sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx} + \sqrt{\frac{ad^2}{b}}\right)}{ab^6} - \frac{9945 \sqrt{2} \left(ab^3d^2\right)^{\frac{1}{4}} \log\left(dx - \sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx} + \sqrt{\frac{ad^2}{b}}\right)}{ab^6} - \frac{8\left(37645\sqrt{dx}\,b^4d^{10}x^8 + 84320\sqrt{dx}\,ab^3d^{10}x^6 + 90610\sqrt{dx}\,a^2b^2d^{10}x^4 + 47736\sqrt{dx}\,a^3bd^{10}x^2 + 9945\sqrt{dx}\,a^4d^{10}\right)}{\left(bd^2x^2 + ad^2\right)^5 b^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^9*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6) + 19890*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a*b^6) + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6) - 9945*sqrt(2)*(a*b^3*d^
2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^
6) - 8*(37645*sqrt(d*x)*b^4*d^10*x^8 + 84320*sqrt(d*x)*a*b^3*d^10*x^6 + 906
10*sqrt(d*x)*a^2*b^2*d^10*x^4 + 47736*sqrt(d*x)*a^3*b*d^10*x^2 + 9945*sqrt(
d*x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

maple [A] time = 0.03, size = 344, normalized size = 0.89

$$-\frac{663\sqrt{dx}\,a^4d^{19}}{4096\left(b\,d^2x^2 + d^2a\right)^5 b^5} - \frac{1989\left(dx\right)^{\frac{5}{2}} a^3d^{17}}{2560\left(b\,d^2x^2 + d^2a\right)^5 b^4} - \frac{9061\left(dx\right)^{\frac{9}{2}} a^2d^{15}}{6144\left(b\,d^2x^2 + d^2a\right)^5 b^3} - \frac{527\left(dx\right)^{\frac{13}{2}} a\,d^{13}}{384\left(b\,d^2x^2 + d^2a\right)^5 b^2} - \frac{7529\left(dx\right)^{\frac{17}{2}} d^{11}}{12288\left(b\,d^2x^2 + d^2a\right)^5 b} + \frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^9 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}} - 1\right)}{16384a\,b^5} + \frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^9 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}} + 1\right)}{16384a\,b^5} + \frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^9 \ln\left(\frac{dx + \left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2} + \sqrt{\frac{a\,d^2}{b}}}{dx - \left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2} + \sqrt{\frac{a\,d^2}{b}}}\right)}{32768a\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -663/4096*d^19/(b*d^2*x^2+a*d^2)^5/b^5*a^4*(d*x)^(1/2)-1989/2560*d^17/(b*d^
2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(5/2)-9061/6144*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a
^2*(d*x)^(9/2)-527/384*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(13/2)-7529/122
88*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(17/2)+663/32768*d^9/b^5*(a/b*d^2)^(1/4
)/a*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d
*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+663/16384*d^9/b^5*
```

```
(a/b*d^2)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+663
/16384*d^9/b^5*(a/b*d^2)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*
x)^(1/2)-1)
```

maxima [A] time = 3.11, size = 386, normalized size = 1.00

$$-\frac{\frac{8\left(37645\,(dx)^{\frac{17}{2}}b^{4}d^{12}+84320\,(dx)^{\frac{13}{2}}ab^{3}d^{14}+90610\,(dx)^{\frac{9}{2}}a^{2}b^{2}d^{16}+47736\,(dx)^{\frac{5}{2}}a^{3}bd^{18}+9945\,\sqrt{dx}\,a^{4}d^{20}\right)}{b^{10}d^{10}x^{10}+5\,ab^{9}d^{10}x^{8}+10\,a^{2}b^{8}d^{10}x^{6}+10\,a^{3}b^{7}d^{10}x^{4}+5\,a^{4}b^{6}d^{10}x^{2}+a^{5}b^{5}d^{10}}-\frac{9945\left(\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^{11}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^{11}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}\right)}{b^{5}}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/491520*(8*(37645*(d*x)^(17/2)*b^4*d^12 + 84320*(d*x)^(13/2)*a*b^3*d^14 +
 90610*(d*x)^(9/2)*a^2*b^2*d^16 + 47736*(d*x)^(5/2)*a^3*b*d^18 + 9945*sqrt(
d*x)*a^4*d^20)/(b^10*d^10*x^10 + 5*a*b^9*d^10*x^8 + 10*a^2*b^8*d^10*x^6 + 1
0*a^3*b^7*d^10*x^4 + 5*a^4*b^6*d^10*x^2 + a^5*b^5*d^10) - 9945*(sqrt(2)*d^1
2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((
a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^12*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4
)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^11*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^11*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b^5)/d
```

mupad [B] time = 4.27, size = 213, normalized size = 0.55

$$-\frac{\frac{7529\,d^{11}\,{(d\,x)}^{17/2}}{12288\,b}+\frac{9061\,a^2\,d^{15}\,{(d\,x)}^{9/2}}{6144\,b^3}+\frac{1989\,a^3\,d^{17}\,{(d\,x)}^{5/2}}{2560\,b^4}+\frac{663\,a^4\,d^{19}\,\sqrt{d\,x}}{4096\,b^5}+\frac{527\,a\,d^{13}\,{(d\,x)}^{13/2}}{384\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{663\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{3/4}\,b^{21/4}}-\frac{663\,d^{19/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{3/4}\,b^{21/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] - ((7529*d^11*(d*x)^(17/2))/(12288*b) + (9061*a^2*d^15*(d*x)^(9/2))/(6144*b
^3) + (1989*a^3*d^17*(d*x)^(5/2))/(2560*b^4) + (663*a^4*d^19*(d*x)^(1/2))/(
4096*b^5) + (527*a*d^13*(d*x)^(13/2))/(384*b^2))/(a^5*d^10 + b^5*d^10*x^10
+ 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^
10*x^6) - (663*d^(19/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(3/4)*b^(21/4)) - (663*d^(19/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)
^(1/4)*d^(1/2))))/(8192*(-a)^(3/4)*b^(21/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.537 $\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=388

$$\frac{231d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{5/4}b^{19/4}}-\frac{231d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{5/4}b^{19/4}}-\frac{231d^{1}\cdots}{\cdots}$$

Rubi [A] time = 0.45, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{231d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{5/4}b^{19/4}}-\frac{231d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{5/4}b^{19/4}}-\frac{231d^{17/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{5/4}b^{19/4}}+\frac{231d^{17/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{5/4}b^{19/4}}+\frac{231d^7(dx)^{3/2}}{4096ab^4\left(a+bx^2\right)}-\frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(15/2))/(10*b*(a + b*x^2)^5) - (3*d^3*(d*x)^(11/2))/(32*b^2*(a +
b*x^2)^4) - (11*d^5*(d*x)^(7/2))/(128*b^3*(a + b*x^2)^3) - (77*d^7*(d*x)^(3
/2))/(1024*b^4*(a + b*x^2)^2) + (231*d^7*(d*x)^(3/2))/(4096*a*b^4*(a + b*x^
2)) - (231*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*d^(17/2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*
d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(16384*Sqrt[2]*a^(5/4)*b^(19/4)) - (231*d^(17/2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(5/4)*b^(19/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{4}\left(3b^4d^2\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(33b^2d^4\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(77d^6\right)\int \frac{(dx)^{5/}}{(ab+b^2}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{(2}{}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

Mathematica [C] time = 0.03, size = 96, normalized size = 0.25

$$\frac{2d^8x\sqrt{dx}\left(385\left(a+bx^2\right)^5\,{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\left(385a^3+935a^2bx^2+1105ab^2x^4+663b^3x^6\right)\right)}{3315a^2b^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^8*x*Sqrt[d*x]*(-(a^2*(385*a^3 + 935*a^2*b*x^2 + 1105*a*b^2*x^4 + 663*b
^3*x^6)) + 385*(a + b*x^2)^5*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)]))
/(3315*a^2*b^4*(a + b*x^2)^5)
```

IntegrateAlgebraic [A] time = 1.03, size = 226, normalized size = 0.58

$$-\frac{231d^{17/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{5/4}b^{19/4}}-\frac{231d^{17/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x}\right)}{8192\sqrt{2}a^{5/4}b^{19/4}}-\frac{d^8\sqrt{dx}\left(385a^4x+1760a^3bx^3+3130a^2b^2x^5+2648ab^3x^7-1155b^4x^9\right)}{20480ab^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20480*(d^8*Sqrt[d*x]*(385*a^4*x + 1760*a^3*b*x^3 + 3130*a^2*b^2*x^5 + 26
48*a*b^3*x^7 - 1155*b^4*x^9))/(a*b^4*(a + b*x^2)^5) - (231*d^(17/2)*ArcTan[
((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)
))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) - (231*d^(17/2)*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d*x])/(Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x)])/(81
92*Sqrt[2]*a^(5/4)*b^(19/4))
```

fricas [A] time = 1.05, size = 506, normalized size = 1.30

$$-\frac{4620\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}\sqrt{dx}ab^5d^{25}-\sqrt{d^{51}x-\sqrt{-\frac{d^{34}}{a^5b^{19}}}a^3b^9d^{34}}\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}ab^5}{d^{34}}\right)-1155\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}\log\left(12326391\sqrt{dx}d^{25}+12326391\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{3}{4}}a^4b^{14}\right)+1155\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}\log\left(12326391\sqrt{dx}d^{25}-12326391\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{3}{4}}a^4b^{14}\right)-4\left(1155b^4d^8x^9-2648ab^3d^8x^7-3130a^2b^2d^8x^5-1760a^3bd^8x^3-385a^4d^8x\right)\sqrt{dx}}{81920\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(4620*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^
4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*arctan(-((-d^34/(a^5*
b^19))^(1/4)*sqrt(d*x)*a*b^5*d^25 - sqrt(d^51*x - sqrt(-d^34/(a^5*b^19))*a^
3*b^9*d^34)*(-d^34/(a^5*b^19))^(1/4)*a*b^5)/d^34) - 1155*(a*b^9*x^10 + 5*a^
2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^
34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*x)*d^25 + 12326391*(-d^34/(a^5*b^1
9))^(3/4)*a^4*b^14) + 1155*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 1
```

0*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*log(12326
391*sqrt(d*x)*d^25 - 12326391*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14) - 4*(1155*
b^4*d^8*x^9 - 2648*a*b^3*d^8*x^7 - 3130*a^2*b^2*d^8*x^5 - 1760*a^3*b*d^8*x^
3 - 385*a^4*d^8*x)*sqrt(d*x))/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6
+ 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)

giac [A] time = 0.21, size = 355, normalized size = 0.91

$$\frac{1}{163840}\,d^{8}\left(\frac{2310\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{7}d}+\frac{2310\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{7}d}-\frac{1155\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{7}d}+\frac{1155\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{7}d}+\frac{8\left(1155\,\sqrt{dx}\,b^{4}d^{10}x^{9}-2648\,\sqrt{dx}\,ab^{3}d^{10}x^{7}-3130\,\sqrt{dx}\,a^{2}b^{2}d^{10}x^{5}-1760\,\sqrt{dx}\,a^{3}bd^{10}x^{3}-385\,\sqrt{dx}\,a^{4}d^{10}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{5}ab^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")

[Out] 1/163840*d^8*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^7*d) + 2310*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^2*b^7*d) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^7*d) + 1155*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^7*d) + 8*(1155*sqrt(d*x)*b^4*d^10*x^9 - 2648*sqrt(d*x)*a*b^3*d^
10*x^7 - 3130*sqrt(d*x)*a^2*b^2*d^10*x^5 - 1760*sqrt(d*x)*a^3*b*d^10*x^3 -
385*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a*b^4))

maple [A] time = 0.03, size = 341, normalized size = 0.88

$$-\frac{77\left(dx\right)^{\frac{3}{2}}a^{3}d^{17}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{4}}-\frac{11\left(dx\right)^{\frac{7}{2}}a^{2}d^{15}}{128\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{3}}-\frac{313\left(dx\right)^{\frac{11}{2}}a\,d^{13}}{2048\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{2}}-\frac{331\left(dx\right)^{\frac{15}{2}}d^{11}}{2560\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b}+\frac{231\left(dx\right)^{\frac{19}{2}}d^{9}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a}+\frac{231\sqrt{2}\,d^{9}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{16384\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a\,b^{5}}+\frac{231\sqrt{2}\,d^{9}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{16384\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a\,b^{5}}+\frac{231\sqrt{2}\,d^{9}\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{32768\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)

[Out] -77/4096*d^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(3/2)-11/128*d^15/(b*d^2*x^
2+a*d^2)^5/b^3*a^2*(d*x)^(7/2)-313/2048*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x
)^(11/2)-331/2560*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(15/2)+231/4096*d^9/(b*d
^2*x^2+a*d^2)^5/a*(d*x)^(19/2)+231/32768*d^9/a/b^5/(a/b*d^2)^(1/4)*2^(1/2)*
ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+231/16384*d^9/a/b^5/(a/b*d^2)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+231/16384*d^9/a
/b^5/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)

maxima [A] time = 3.04, size = 383, normalized size = 0.99

$$\frac{\frac{1155\, d^{10}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{ab^{4}}+\frac{8\left(1155\,(dx)^{\frac{19}{2}}b^{4}d^{10}-2648\,(dx)^{\frac{15}{2}}ab^{3}d^{12}-3130\,(dx)^{\frac{11}{2}}a^{2}b^{2}d^{14}-1760\,(dx)^{\frac{7}{2}}a^{3}bd^{16}-385\,(dx)^{\frac{3}{2}}a^{4}d^{18}\right)}{ab^{9}d^{10}x^{10}+5\,a^{2}b^{8}d^{10}x^{8}+10\,a^{3}b^{7}d^{10}x^{6}+10\,a^{4}b^{6}d^{10}x^{4}+5\,a^{5}b^{5}d^{10}x^{2}+a^{6}b^{4}d^{10}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(1155*d^10*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^
(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)
*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(
a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a*b^4
) + 8*(1155*(d*x)^(19/2)*b^4*d^10 - 2648*(d*x)^(15/2)*a*b^3*d^12 - 3130*(d*
x)^(11/2)*a^2*b^2*d^14 - 1760*(d*x)^(7/2)*a^3*b*d^16 - 385*(d*x)^(3/2)*a^4*
d^18)/(a*b^9*d^10*x^10 + 5*a^2*b^8*d^10*x^8 + 10*a^3*b^7*d^10*x^6 + 10*a^4*
b^6*d^10*x^4 + 5*a^5*b^5*d^10*x^2 + a^6*b^4*d^10))/d
```

mupad [B] time = 4.29, size = 210, normalized size = 0.54

$$\frac{231\, d^{17/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{5/4}\,b^{19/4}}-\frac{231\, d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{5/4}\,b^{19/4}}-\frac{\frac{331\,d^{11}\,(d\,x)^{15/2}}{2560\,b}-\frac{231\,d^{9}\,(d\,x)^{19/2}}{4096\,a}+\frac{11\,a^{2}\,d^{15}\,(d\,x)^{7/2}}{128\,b^{3}}+\frac{77\,a^{3}\,d^{17}\,(d\,x)^{3/2}}{4096\,b^{4}}+\frac{313\,a\,d^{13}\,(d\,x)^{11/2}}{2048\,b^{2}}}{a^{5}\,d^{10}+5\,a^{4}\,b\,d^{10}\,x^{2}+10\,a^{3}\,b^{2}\,d^{10}\,x^{4}+10\,a^{2}\,b^{3}\,d^{10}\,x^{6}+5\,a\,b^{4}\,d^{10}\,x^{8}+b^{5}\,d^{10}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (231*d^(17/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(5/4)*b^(19/4)) - (231*d^(17/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(
1/2))))/(8192*(-a)^(5/4)*b^(19/4)) - ((331*d^11*(d*x)^(15/2))/(2560*b) - (2
31*d^9*(d*x)^(19/2))/(4096*a) + (11*a^2*d^15*(d*x)^(7/2))/(128*b^3) + (77*a
^3*d^17*(d*x)^(3/2))/(4096*b^4) + (313*a*d^13*(d*x)^(11/2))/(2048*b^2))/(a^
5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d
^10*x^4 + 10*a^2*b^3*d^10*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.538 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=388

$$-\frac{117d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{117d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{7/4}b^{17/4}}-\frac{117}{}$$

Rubi [A] time = 0.45, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{117d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{117d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{7/4}b^{17/4}}-\frac{117d^{15/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{117d^{15/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{39d^7\sqrt{dx}}{4096ab^4\left(a+bx^2\right)}-\frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(13/2))/(10*b*(a + b*x^2)^5) - (13*d^3*(d*x)^(9/2))/(160*b^2*(a +
 b*x^2)^4) - (39*d^5*(d*x)^(5/2))/(640*b^3*(a + b*x^2)^3) - (39*d^7*Sqrt[d*
x])/(1024*b^4*(a + b*x^2)^2) + (39*d^7*Sqrt[d*x])/(4096*a*b^4*(a + b*x^2))
- (117*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]])/
(8192*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]])/(8192*Sqrt[2]*a^(7/4)*b^(17/4)) - (117*d^(1
5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*
a^(7/4)*b^(17/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(13b^4d^2\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(117b^2d^4\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(39d^6\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x\right.}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{(39}{}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{}{40}$$

Mathematica [A] time = 0.26, size = 359, normalized size = 0.93

$$\frac{d^7\sqrt{dx}\left(-\frac{45045\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}+\frac{45045\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}-\frac{90090\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}\sqrt{x}}+\frac{90090\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}\sqrt{x}}-\frac{638976a^3\sqrt[4]{b}}{(a+bx^2)^5}-\frac{2555904a^2b^{5/4}x^2}{(a+bx^2)^5}+\frac{120120\sqrt[4]{b}}{a^2+abx^2}+\frac{39936a^2\sqrt[4]{b}}{(a+bx^2)^4}-\frac{3604480b^{13/4}x^6}{(a+bx^2)^5}-\frac{4259840ab^{9/4}x^4}{(a+bx^2)^5}+\frac{68640\sqrt[4]{b}}{(a+bx^2)^2}+\frac{49920a\sqrt[4]{b}}{(a+bx^2)^3}\right)}{12615680b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^7*Sqrt[d*x]*((-638976*a^3*b^(1/4))/(a + b*x^2)^5 - (2555904*a^2*b^(5/4)*
x^2)/(a + b*x^2)^5 - (4259840*a*b^(9/4)*x^4)/(a + b*x^2)^5 - (3604480*b^(13
/4)*x^6)/(a + b*x^2)^5 + (39936*a^2*b^(1/4))/(a + b*x^2)^4 + (49920*a*b^(1/
4))/(a + b*x^2)^3 + (68640*b^(1/4))/(a + b*x^2)^2 + (120120*b^(1/4))/(a^2 +
 a*b*x^2) - (90090*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
a^(7/4)*Sqrt[x]) + (90090*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(a^(7/4)*Sqrt[x]) - (45045*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x]) + (45045*Sqrt[2]*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x])))/(12615680
*b^(17/4))
```

IntegrateAlgebraic [A] time = 1.29, size = 244, normalized size = 0.63

$$-\frac{117d^{15/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{7/4}b^{17/4}}+\frac{117d^{15/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{7/4}b^{17/4}}-\frac{d^9\sqrt{dx}\left(585a^4d^8+2808a^3bd^8x^2+5330a^2b^2d^8x^4+4960ab^3d^8x^6-195b^4d^8x^8\right)}{20480ab^4\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20480*(d^9*Sqrt[d*x]*(585*a^4*d^8 + 2808*a^3*b*d^8*x^2 + 5330*a^2*b^2*d^
8*x^4 + 4960*a*b^3*d^8*x^6 - 195*b^4*d^8*x^8))/(a*b^4*(a*d^2 + b*d^2*x^2)^5
) - (117*d^(15/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sq
rt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(7/4)*b^(17/4)) + (
117*d^(15/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d
 + Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^(7/4)*b^(17/4))
```

fricas [A] time = 1.01, size = 505, normalized size = 1.30

$$\frac{2340\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}a^5b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{d^{30}}{a^7b^{17}}}a^4b^8}\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{3}{4}}a^5b^{13}}{d^{30}}\right)+585\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}\log\left(117\sqrt{dx}d^7+117\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}a^2b^4\right)-585\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}\log\left(117\sqrt{dx}d^7-117\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}a^2b^4\right)+4\left(195b^4d^7x^8-4960ab^3d^7x^6-5330a^2b^2d^7x^4-2808a^3bd^7x^2-585a^4d^7\right)\sqrt{dx}}{81920\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(2340*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4
 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*arctan(-((-d^30/(a^7*b
```

```
^17))^(3/4)*sqrt(d*x)*a^5*b^13*d^7 - sqrt(d^15*x + sqrt(-d^30/(a^7*b^17))*a
^4*b^8)*(-d^30/(a^7*b^17))^(3/4)*a^5*b^13)/d^30) + 585*(a*b^9*x^10 + 5*a^2*
b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30
/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 + 117*(-d^30/(a^7*b^17))^(1/4)*a^2
*b^4) - 585*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 +
 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 -
117*(-d^30/(a^7*b^17))^(1/4)*a^2*b^4) + 4*(195*b^4*d^7*x^8 - 4960*a*b^3*d^7
*x^6 - 5330*a^2*b^2*d^7*x^4 - 2808*a^3*b*d^7*x^2 - 585*a^4*d^7)*sqrt(d*x))/
(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x
^2 + a^6*b^4)
```

giac [A] time = 0.22, size = 342, normalized size = 0.88

$$\frac{1}{163840}\,d^{7}\left(\frac{1170\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{5}}+\frac{1170\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{5}}+\frac{585\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{5}}-\frac{585\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{5}}+\frac{8\left(195\,\sqrt{dx}\,b^{4}d^{10}x^{8}-4960\,\sqrt{dx}\,ab^{3}d^{10}x^{6}-5330\,\sqrt{dx}\,a^{2}b^{2}d^{10}x^{4}-2808\,\sqrt{dx}\,a^{3}bd^{10}x^{2}-585\,\sqrt{dx}\,a^{4}d^{10}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{5}ab^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) + 1170*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^2*b^5) + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5) - 585*sqrt(2)*(a*b^3*
d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
2*b^5) + 8*(195*sqrt(d*x)*b^4*d^10*x^8 - 4960*sqrt(d*x)*a*b^3*d^10*x^6 - 53
30*sqrt(d*x)*a^2*b^2*d^10*x^4 - 2808*sqrt(d*x)*a^3*b*d^10*x^2 - 585*sqrt(d*
x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

maple [A] time = 0.02, size = 341, normalized size = 0.88

$$-\frac{117\sqrt{dx}\,a^{3}d^{17}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{4}}-\frac{351\left(dx\right)^{\frac{5}{2}}a^{2}d^{15}}{2560\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{3}}-\frac{533\left(dx\right)^{\frac{9}{2}}a\,d^{13}}{2048\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{2}}-\frac{31\left(dx\right)^{\frac{13}{2}}d^{11}}{128\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b}+\frac{39\left(dx\right)^{\frac{17}{2}}d^{9}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a}+\frac{117\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{7}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{16384a^{2}b^{4}}+\frac{117\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{7}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{16384a^{2}b^{4}}+\frac{117\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{7}\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{32768a^{2}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -117/4096*d^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(1/2)-351/2560*d^15/(b*d^2
*x^2+a*d^2)^5/b^3*a^2*(d*x)^(5/2)-533/2048*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(
d*x)^(9/2)-31/128*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(13/2)+39/4096*d^9/(b*d^
2*x^2+a*d^2)^5/a*(d*x)^(17/2)+117/32768*d^7/a^2/b^4*(a/b*d^2)^(1/4)*2^(1/2)
*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+117/16384*d^7/a^2/b^4*(a/b*d^
```

```
2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+117/16384*d^
7/a^2/b^4*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2
)-1)
```

maxima [A] time = 3.22, size = 392, normalized size = 1.01

$$\frac{8\left(195\,(dx)^{\frac{17}{2}}b^4d^{10}-4960\,(dx)^{\frac{13}{2}}ab^3d^{12}-5330\,(dx)^{\frac{9}{2}}a^2b^2d^{14}-2808\,(dx)^{\frac{5}{2}}a^3bd^{16}-585\,\sqrt{dx}\,a^4d^{18}\right)}{ab^9d^{10}x^{10}+5\,a^2b^8d^{10}x^8+10\,a^3b^7d^{10}x^6+10\,a^4b^6d^{10}x^4+5\,a^5b^5d^{10}x^2+a^6b^4d^{10}}+\frac{585\left(\frac{\frac{\sqrt{2}d^{10}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^{10}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}d^9\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}+\frac{2\sqrt{2}d^9\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}}{ab^4}\right)}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(195*(d*x)^(17/2)*b^4*d^10 - 4960*(d*x)^(13/2)*a*b^3*d^12 - 533
0*(d*x)^(9/2)*a^2*b^2*d^14 - 2808*(d*x)^(5/2)*a^3*b*d^16 - 585*sqrt(d*x)*a^
4*d^18)/(a*b^9*d^10*x^10 + 5*a^2*b^8*d^10*x^8 + 10*a^3*b^7*d^10*x^6 + 10*a^
4*b^6*d^10*x^4 + 5*a^5*b^5*d^10*x^2 + a^6*b^4*d^10) + 585*(sqrt(2)*d^10*log
(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) - sqrt(2)*d^10*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^9*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^9*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a*b^4))/d
```

mupad [B] time = 0.13, size = 210, normalized size = 0.54

$$\frac{117\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{7/4}\,b^{17/4}}-\frac{\frac{31\,d^{11}\,{(d\,x)}^{13/2}}{128\,b}-\frac{39\,d^9\,{(d\,x)}^{17/2}}{4096\,a}+\frac{351\,a^2\,d^{15}\,{(d\,x)}^{5/2}}{2560\,b^3}+\frac{117\,a^3\,d^{17}\,\sqrt{d\,x}}{4096\,b^4}+\frac{533\,a\,d^{13}\,{(d\,x)}^{9/2}}{2048\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{117\,d^{15/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{7/4}\,b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (117*d^(15/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^
(7/4)*b^(17/4)) - ((31*d^11*(d*x)^(13/2))/(128*b) - (39*d^9*(d*x)^(17/2))/(
4096*a) + (351*a^2*d^15*(d*x)^(5/2))/(2560*b^3) + (117*a^3*d^17*(d*x)^(1/2)
)/(4096*b^4) + (533*a*d^13*(d*x)^(9/2))/(2048*b^2))/(a^5*d^10 + b^5*d^10*x^
10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3
*d^10*x^6) + (117*d^(15/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))
))/(8192*(-a)^(7/4)*b^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.539 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=391

$$\frac{77d^{13/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{9/4}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{9/4}b^{15/4}}-\frac{77d^{13/}}{}$$

Rubi [A] time = 0.48, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^5(dx)^{3/2}}{4096a^2b^3\left(a+bx^2\right)}+\frac{77d^{13/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{9/4}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{9/4}b^{15/4}}-\frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{8192\sqrt{2}\,a^{9/4}b^{15/4}}+\frac{77d^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{9/4}b^{15/4}}+\frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(11/2))/(10*b*(a + b*x^2)^5) - (11*d^3*(d*x)^(7/2))/(160*b^2*(a +
 b*x^2)^4) - (77*d^5*(d*x)^(3/2))/(1920*b^3*(a + b*x^2)^3) + (77*d^5*(d*x)^
(3/2))/(5120*a*b^3*(a + b*x^2)^2) + (77*d^5*(d*x)^(3/2))/(4096*a^2*b^3*(a +
 b*x^2)) - (77*d^(13/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (7
7*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(9/4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqr
t[2]*a^(9/4)*b^(15/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(11b^4d^2\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(77b^2d^4\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{\left(77d^6\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{1280}$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \cdots$$

Mathematica [C] time = 0.03, size = 85, normalized size = 0.22

$$\frac{2d^6x\sqrt{dx}\left(77\left(a+bx^2\right)^5{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-a^3\left(77a^2+187abx^2+221b^2x^4\right)\right)}{1989a^3b^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^6*x*Sqrt[d*x]*(-(a^3*(77*a^2 + 187*a*b*x^2 + 221*b^2*x^4)) + 77*(a + b
*x^2)^5*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)]))/(1989*a^3*b^3*(a + b
*x^2)^5)
```

IntegrateAlgebraic [A] time = 1.30, size = 244, normalized size = 0.62

$$-\frac{77d^{13/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{9/4}b^{15/4}}-\frac{77d^{13/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{9/4}b^{15/4}}-\frac{d^7(dx)^{3/2}\left(385a^4d^8+1760a^3bd^8x^2+3130a^2b^2d^8x^4-5544ab^3d^8x^6-1155b^4d^8x^8\right)}{61440a^2b^3\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/61440*(d^7*(d*x)^(3/2)*(385*a^4*d^8 + 1760*a^3*b*d^8*x^2 + 3130*a^2*b^2*
d^8*x^4 - 5544*a*b^3*d^8*x^6 - 1155*b^4*d^8*x^8))/(a^2*b^3*(a*d^2 + b*d^2*x
^2)^5) - (77*d^(13/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4
)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(9/4)*b^(15/4))
 - (77*d^(13/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a
]*d + Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^(9/4)*b^(15/4))
```

fricas [A] time = 1.76, size = 518, normalized size = 1.32

$$-\frac{4620\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}a^2b^4d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{a^9b^{15}}}a^5b^7d^{26}}\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{1}{4}}a^2b^4}{d^{26}}\right)-1155\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{1}{4}}\log\left(456533\sqrt{dx}d^{19}+456533\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{3}{4}}a^7b^{11}\right)+1155\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{1}{4}}\log\left(456533\sqrt{dx}d^{19}-456533\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{3}{4}}a^7b^{11}\right)-4\left(1155b^4d^6x^9+5544ab^3d^6x^7-3130a^2b^2d^6x^5-1760a^3bd^6x^3-385a^4d^6x\right)\sqrt{dx}}{245760\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(4620*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5
*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*arctan(-((-d^26/(a
^9*b^15))^(1/4)*sqrt(d*x)*a^2*b^4*d^19 - sqrt(d^39*x - sqrt(-d^26/(a^9*b^15
))*a^5*b^7*d^26)*(-d^26/(a^9*b^15))^(1/4)*a^2*b^4)/d^26) - 1155*(a^2*b^8*x^
10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*
b^3)*(-d^26/(a^9*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a^
9*b^15))^(3/4)*a^7*b^11) + 1155*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*
```

```
x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*lo
g(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a^9*b^15))^(3/4)*a^7*b^11) - 4*(11
55*b^4*d^6*x^9 + 5544*a*b^3*d^6*x^7 - 3130*a^2*b^2*d^6*x^5 - 1760*a^3*b*d^6
*x^3 - 385*a^4*d^6*x)*sqrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6
*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)
```

giac [A] time = 0.23, size = 355, normalized size = 0.91

$$\frac{1}{491520}d^6\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^6d}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^6d}-\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^6d}+\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^6d}+\frac{8\left(1155\sqrt{dx}\,b^4d^{10}x^9+5544\sqrt{dx}\,ab^3d^{10}x^7-3130\sqrt{dx}\,a^2b^2d^{10}x^5-1760\sqrt{dx}\,a^3bd^{10}x^3-385\sqrt{dx}\,a^4d^{10}x\right)}{\left(bd^2x^2+ad^2\right)^5a^2b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^6*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^6*d) + 2310*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^3*b^6*d) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^6*d) + 1155*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^3*b^6*d) + 8*(1155*sqrt(d*x)*b^4*d^10*x^9 + 5544*sqrt(d*x)*a*b^3*d^
10*x^7 - 3130*sqrt(d*x)*a^2*b^2*d^10*x^5 - 1760*sqrt(d*x)*a^3*b*d^10*x^3 -
385*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

maple [A] time = 0.02, size = 339, normalized size = 0.87

$$-\frac{77\left(dx\right)^{\frac{3}{2}}a^2d^{15}}{12288\left(b\,d^2x^2+d^2a\right)^5b^3}-\frac{11\left(dx\right)^{\frac{7}{2}}a\,d^{13}}{384\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{313\left(dx\right)^{\frac{11}{2}}d^{11}}{6144\left(b\,d^2x^2+d^2a\right)^5b}+\frac{231\left(dx\right)^{\frac{15}{2}}d^9}{2560\left(b\,d^2x^2+d^2a\right)^5a}+\frac{77\left(dx\right)^{\frac{19}{2}}b\,d^7}{4096\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{77\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^2b^4}+\frac{77\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^2b^4}+\frac{77\sqrt{2}\,d^7\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/12288*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(3/2)-11/384*d^13/(b*d^2*x
^2+a*d^2)^5/b^2*a*(d*x)^(7/2)-313/6144*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(11
/2)+231/2560*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(15/2)+77/4096*d^7/(b*d^2*x^2+
a*d^2)^5/a^2*b*(d*x)^(19/2)+77/32768*d^7/a^2/b^4/(a/b*d^2)^(1/4)*2^(1/2)*ln
((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+77/16384*d^7/a^2/b^4/(a/b*d^2)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+77/16384*d^7/a^2
/b^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.00, size = 385, normalized size = 0.98

$$\frac{\frac{1155\,d^8\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^2b^3}+\frac{8\left(1155\,(dx)^{\frac{19}{2}}b^4d^8+5544\,(dx)^{\frac{15}{2}}ab^3d^{10}-3130\,(dx)^{\frac{11}{2}}a^2b^2d^{12}-1760\,(dx)^{\frac{7}{2}}a^3bd^{14}-385\,(dx)^{\frac{3}{2}}a^4d^{16}\right)}{a^2b^8d^{10}x^{10}+5\,a^3b^7d^{10}x^8+10\,a^4b^6d^{10}x^6+10\,a^5b^5d^{10}x^4+5\,a^6b^4d^{10}x^2+a^7b^3d^{10}}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/491520*(1155*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(
1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*
d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
- 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sq
rt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a
*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*b^
3) + 8*(1155*(d*x)^(19/2)*b^4*d^8 + 5544*(d*x)^(15/2)*a*b^3*d^10 - 3130*(d*
x)^(11/2)*a^2*b^2*d^12 - 1760*(d*x)^(7/2)*a^3*b*d^14 - 385*(d*x)^(3/2)*a^4*
d^16)/(a^2*b^8*d^10*x^10 + 5*a^3*b^7*d^10*x^8 + 10*a^4*b^6*d^10*x^6 + 10*a^
5*b^5*d^10*x^4 + 5*a^6*b^4*d^10*x^2 + a^7*b^3*d^10))/d
```

mupad [B] time = 4.32, size = 208, normalized size = 0.53

$$\frac{77\,d^{13/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{9/4}\,b^{15/4}}-\frac{\frac{313\,d^{11}\,(d\,x)^{11/2}}{6144\,b}-\frac{231\,d^9\,(d\,x)^{15/2}}{2560\,a}+\frac{77\,a^2\,d^{15}\,(d\,x)^{3/2}}{12288\,b^3}+\frac{11\,a\,d^{13}\,(d\,x)^{7/2}}{384\,b^2}-\frac{77\,b\,d^7\,(d\,x)^{19/2}}{4096\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{77\,d^{13/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{9/4}\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (77*d^(13/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(
9/4)*b^(15/4)) - ((313*d^11*(d*x)^(11/2))/(6144*b) - (231*d^9*(d*x)^(15/2))
/(2560*a) + (77*a^2*d^15*(d*x)^(3/2))/(12288*b^3) + (11*a*d^13*(d*x)^(7/2))
/(384*b^2) - (77*b*d^7*(d*x)^(19/2))/(4096*a^2))/(a^5*d^10 + b^5*d^10*x^10
+ 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^
10*x^6) - (77*d^(13/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(9/4)*b^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.540 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{63d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{11/4}b^{13/4}}+\frac{63d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{11/4}b^{13/4}}-\frac{63d^{11/}}{}$$

Rubi [A] time = 0.47, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{21d^5\sqrt{dx}}{4096a^2b^3\left(a+bx^2\right)}-\frac{63d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{11/4}b^{13/4}}+\frac{63d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{11/4}b^{13/4}}-\frac{63d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{11/4}b^{13/4}}+\frac{63d^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{11/4}b^{13/4}}+\frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2}-\frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3}-\frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(9/2))/(10*b*(a + b*x^2)^5) - (9*d^3*(d*x)^(5/2))/(160*b^2*(a + b
*x^2)^4) - (3*d^5*Sqrt[d*x])/(128*b^3*(a + b*x^2)^3) + (3*d^5*Sqrt[d*x])/(1
024*a*b^3*(a + b*x^2)^2) + (21*d^5*Sqrt[d*x])/(4096*a^2*b^3*(a + b*x^2)) -
(63*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(81
92*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(11/4)*b^(13/4)) - (63*d^(11/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(
11/4)*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(9b^4d^2\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(9b^2d^4\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(3d^6\right)\int \frac{1}{\sqrt{dx}\,\left(ab\right.}$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} +$$

Mathematica [A] time = 0.18, size = 337, normalized size = 0.86

$$\frac{d^5\sqrt{dx}\left(-\frac{3465\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}+\frac{3465\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}-\frac{6930\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}\sqrt{x}}+\frac{6930\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{11/4}\sqrt{x}}+\frac{9240\sqrt[4]{b}}{a^2(a+bx^2)}-\frac{49152a^2\sqrt[4]{b}}{(a+bx^2)^5}-\frac{196608ab^{5/4}x^2}{(a+bx^2)^5}-\frac{327680b^{9/4}x^4}{(a+bx^2)^5}+\frac{5280\sqrt[4]{b}}{a(a+bx^2)^2}+\frac{3840\sqrt[4]{b}}{(a+bx^2)^3}+\frac{3072a\sqrt[4]{b}}{(a+bx^2)^4}\right)}{1802240b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^5*Sqrt[d*x]*((-49152*a^2*b^(1/4))/(a + b*x^2)^5 - (196608*a*b^(5/4)*x^2)
/(a + b*x^2)^5 - (327680*b^(9/4)*x^4)/(a + b*x^2)^5 + (3072*a*b^(1/4))/(a +
 b*x^2)^4 + (3840*b^(1/4))/(a + b*x^2)^3 + (5280*b^(1/4))/(a*(a + b*x^2)^2)
 + (9240*b^(1/4))/(a^2*(a + b*x^2)) - (6930*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) + (6930*Sqrt[2]*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) - (3465*Sqrt[2]*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt[x]) + (34
65*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(
11/4)*Sqrt[x])))/(1802240*b^(13/4))
```

IntegrateAlgebraic [A] time = 1.27, size = 244, normalized size = 0.62

$$-\frac{63d^{11/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{11/4}b^{13/4}}+\frac{63d^{11/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{11/4}b^{13/4}}-\frac{d^7\sqrt{dx}\left(315a^4d^8+1512a^3bd^8x^2+2870a^2b^2d^8x^4-480ab^3d^8x^6-105b^4d^8x^8\right)}{20480a^2b^3\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20480*(d^7*Sqrt[d*x]*(315*a^4*d^8 + 1512*a^3*b*d^8*x^2 + 2870*a^2*b^2*d^
8*x^4 - 480*a*b^3*d^8*x^6 - 105*b^4*d^8*x^8))/(a^2*b^3*(a*d^2 + b*d^2*x^2)^
5) - (63*d^(11/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sq
rt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(11/4)*b^(13/4)) +
(63*d^(11/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d
 + Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^(11/4)*b^(13/4))
```

fricas [A] time = 3.18, size = 513, normalized size = 1.31

$$\frac{1260\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^8b^{10}d^5\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{3}{4}}-\sqrt{a^6b^6\sqrt{-\frac{d^{22}}{a^{11}b^{13}}}+d^{11}x}\,a^8b^{10}\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{3}{4}}}{d^{22}}\right)+315\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{1}{4}}\log\left(63a^3b^3\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{1}{4}}+63\sqrt{dx}d^5\right)-315\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{1}{4}}\log\left(-63a^3b^3\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{1}{4}}+63\sqrt{dx}d^5\right)+4\left(105b^4d^5x^8+480ab^3d^5x^6-2870a^2b^2d^5x^4-1512a^3bd^5x^2-315a^4d^5\right)\sqrt{dx}}{81920\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(1260*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x
^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*arctan(-(sqrt(d*x)*
```

```
a^8*b^10*d^5*(-d^22/(a^11*b^13))^(3/4) - sqrt(a^6*b^6*sqrt(-d^22/(a^11*b^13
)) + d^11*x)*a^8*b^10*(-d^22/(a^11*b^13))^(3/4))/d^22) + 315*(a^2*b^8*x^10
+ 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3
)*(-d^22/(a^11*b^13))^(1/4)*log(63*a^3*b^3*(-d^22/(a^11*b^13))^(1/4) + 63*s
qrt(d*x)*d^5) - 315*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5
*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*log(-63*a^3*b
^3*(-d^22/(a^11*b^13))^(1/4) + 63*sqrt(d*x)*d^5) + 4*(105*b^4*d^5*x^8 + 480
*a*b^3*d^5*x^6 - 2870*a^2*b^2*d^5*x^4 - 1512*a^3*b*d^5*x^2 - 315*a^4*d^5)*s
qrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 +
 5*a^6*b^4*x^2 + a^7*b^3)
```

giac [A] time = 0.21, size = 342, normalized size = 0.87

$$\frac{1}{163840}\,d^5\left(\frac{630\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}+\frac{630\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}+\frac{315\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^4}-\frac{315\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^4}+\frac{8\left(105\,\sqrt{dx}\,b^4d^{10}x^8+480\,\sqrt{dx}\,ab^3d^{10}x^6-2870\,\sqrt{dx}\,a^2b^2d^{10}x^4-1512\,\sqrt{dx}\,a^3bd^{10}x^2-315\,\sqrt{dx}\,a^4d^{10}\right)}{\left(bd^2x^2+ad^2\right)^5a^2b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^5*(630*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4) + 630*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^3*b^4) + 315*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4) - 315*sqrt(2)*(a*b^3*d^
2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*
b^4) + 8*(105*sqrt(d*x)*b^4*d^10*x^8 + 480*sqrt(d*x)*a*b^3*d^10*x^6 - 2870*
sqrt(d*x)*a^2*b^2*d^10*x^4 - 1512*sqrt(d*x)*a^3*b*d^10*x^2 - 315*sqrt(d*x)*
a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

maple [A] time = 0.02, size = 339, normalized size = 0.87

$$-\frac{63\sqrt{dx}\,a^2d^{15}}{4096\left(b\,d^2x^2+d^2a\right)^5b^3}-\frac{189\,(dx)^{\frac{5}{2}}\,a\,d^{13}}{2560\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{287\,(dx)^{\frac{9}{2}}\,d^{11}}{2048\left(b\,d^2x^2+d^2a\right)^5b}+\frac{3\,(dx)^{\frac{13}{2}}\,d^9}{128\left(b\,d^2x^2+d^2a\right)^5a}+\frac{21\,(dx)^{\frac{17}{2}}\,b\,d^7}{4096\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{63\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384a^3b^3}+\frac{63\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384a^3b^3}+\frac{63\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^5\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -63/4096*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(1/2)-189/2560*d^13/(b*d^2*
x^2+a*d^2)^5/b^2*a*(d*x)^(5/2)-287/2048*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(9
/2)+3/128*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(13/2)+21/4096*d^7/(b*d^2*x^2+a*d
^2)^5/a^2*b*(d*x)^(17/2)+63/32768*d^5/a^3/b^3*(a/b*d^2)^(1/4)*2^(1/2)*ln((d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+63/16384*d^5/a^3/b^3*(a/b*d^2)^(1/4
```

)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+63/16384*d^5/a^3/b^3*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)

maxima [A] time = 3.16, size = 394, normalized size = 1.01

$$\frac{\frac{8\left(105\,(dx)^{\frac{17}{2}}b^{4}d^{8}+480\,(dx)^{\frac{13}{2}}ab^{3}d^{10}-2870\,(dx)^{\frac{9}{2}}a^{2}b^{2}d^{12}-1512\,(dx)^{\frac{5}{2}}a^{3}bd^{14}-315\,\sqrt{dx}\,a^{4}d^{16}\right)}{a^{2}b^{8}d^{10}x^{10}+5\,a^{3}b^{7}d^{10}x^{8}+10\,a^{4}b^{6}d^{10}x^{6}+10\,a^{5}b^{5}d^{10}x^{4}+5\,a^{6}b^{4}d^{10}x^{2}+a^{7}b^{3}d^{10}}+\frac{315\left(\frac{\sqrt{2}d^{8}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^{8}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}d^{7}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{a}}+\frac{2\,\sqrt{2}d^{7}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{a}}\right)}{a^{2}b^{3}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")

[Out] 1/163840*(8*(105*(d*x)^(17/2)*b^4*d^8 + 480*(d*x)^(13/2)*a*b^3*d^10 - 2870*(d*x)^(9/2)*a^2*b^2*d^12 - 1512*(d*x)^(5/2)*a^3*b*d^14 - 315*sqrt(d*x)*a^4*d^16)/(a^2*b^8*d^10*x^10 + 5*a^3*b^7*d^10*x^8 + 10*a^4*b^6*d^10*x^6 + 10*a^5*b^5*d^10*x^4 + 5*a^6*b^4*d^10*x^2 + a^7*b^3*d^10) + 315*(sqrt(2)*d^8*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^7*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^7*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^2*b^3))/d

mupad [B] time = 4.23, size = 208, normalized size = 0.53

$$-\frac{\frac{287\,d^{11}\,{\left(d\,x\right)}^{9/2}}{2048\,b}-\frac{3\,d^9\,{\left(d\,x\right)}^{13/2}}{128\,a}+\frac{63\,a^2\,d^{15}\,\sqrt{d\,x}}{4096\,b^3}+\frac{189\,a\,d^{13}\,{\left(d\,x\right)}^{5/2}}{2560\,b^2}-\frac{21\,b\,d^7\,{\left(d\,x\right)}^{17/2}}{4096\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{63\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{11/4}\,b^{13/4}}-\frac{63\,d^{11/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{11/4}\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)

[Out] - ((287*d^11*(d*x)^(9/2))/(2048*b) - (3*d^9*(d*x)^(13/2))/(128*a) + (63*a^2*d^15*(d*x)^(1/2))/(4096*b^3) + (189*a*d^13*(d*x)^(5/2))/(2560*b^2) - (21*b*d^7*(d*x)^(17/2))/(4096*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (63*d^(11/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(11/4)*b^(13/4)) - (63*d^(11/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(11/4)*b^(13/4))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.541 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=394

$$\frac{63d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{63d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{63d^{9/2}\tan\ldots}{8192\ldots}$$

Rubi [A] time = 0.46, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{63d^3(dx)^{3/2}}{4096a^3b^2\left(a+bx^2\right)}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}+\frac{63d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{63d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{63d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{13/4}b^{11/4}}+\frac{63d^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{13/4}b^{11/4}}+\frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3}-\frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(7/2))/(10*b*(a + b*x^2)^5) - (7*d^3*(d*x)^(3/2))/(160*b^2*(a + b
*x^2)^4) + (7*d^3*(d*x)^(3/2))/(640*a*b^2*(a + b*x^2)^3) + (63*d^3*(d*x)^(3
/2))/(5120*a^2*b^2*(a + b*x^2)^2) + (63*d^3*(d*x)^(3/2))/(4096*a^3*b^2*(a +
 b*x^2)) - (63*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (63*d^(9/2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (6
3*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(13/4)*b^(11/4)) - (63*d^(9/2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(13/4)*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(7b^4d^2\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(21b^2d^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{\left(63bd^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,d}{1280a}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.15

$$\frac{2d^4x\sqrt{dx}\left(\frac{7\,{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)}{a^4}+\frac{-7a-17bx^2}{\left(a+bx^2\right)^5}\right)}{221b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^4*x*Sqrt[d*x]*((-7*a - 17*b*x^2)/(a + b*x^2)^5 + (7*Hypergeometric2F1[
3/4, 6, 7/4, -((b*x^2)/a)])/a^4))/(221*b^2)
```

IntegrateAlgebraic [A] time = 1.17, size = 244, normalized size = 0.62

$$-\frac{63d^{9/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{63d^{9/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{8192\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{d^5(dx)^{3/2}\left(105a^4d^8+480a^3bd^8x^2-2870a^2b^2d^8x^4-1512ab^3d^8x^6-315b^4d^8x^8\right)}{20480a^3b^2\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20480*(d^5*(d*x)^(3/2)*(105*a^4*d^8 + 480*a^3*b*d^8*x^2 - 2870*a^2*b^2*d
^8*x^4 - 1512*a*b^3*d^8*x^6 - 315*b^4*d^8*x^8))/(a^3*b^2*(a*d^2 + b*d^2*x^2
)^5) - (63*d^(9/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*S
qrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) -
 (63*d^(9/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d
 + Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^(13/4)*b^(11/4))
```

fricas [A] time = 3.27, size = 520, normalized size = 1.32

$$-\frac{1260\left(a^3b^7x^{10}+5a^4b^6x^8+10a^5b^5x^6+10a^6b^4x^4+5a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{250047\sqrt{dx}\,a^3b^3d^{13}\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}-\sqrt{-62523502209\,a^7b^5d^{18}\sqrt{-\frac{d^{18}}{a^{13}b^{11}}}+62523502209\,d^{27}x}\,a^3b^3\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}}{250047\,d^{18}}\right)-315\left(a^3b^7x^{10}+5a^4b^6x^8+10a^5b^5x^6+10a^6b^4x^4+5a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}\log\left(250047\,a^{10}b^8\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{3}{4}}+250047\sqrt{dx}\,d^{13}\right)+315\left(a^3b^7x^{10}+5a^4b^6x^8+10a^5b^5x^6+10a^6b^4x^4+5a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}\log\left(-250047\,a^{10}b^8\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{3}{4}}+250047\sqrt{dx}\,d^{13}\right)-4\left(315\,b^4d^4x^9+1512\,ab^3d^4x^7+2870\,a^2b^2d^4x^5-480\,a^3bd^4x^3-105\,a^4d^4x\right)\sqrt{dx}}{81920\left(a^3b^7x^{10}+5a^4b^6x^8+10a^5b^5x^6+10a^6b^4x^4+5a^7b^3x^2+a^8b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1260*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*
x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*arctan(-1/250047*(
250047*sqrt(d*x)*a^3*b^3*d^13*(-d^18/(a^13*b^11))^(1/4) - sqrt(-62523502209
*a^7*b^5*d^18*sqrt(-d^18/(a^13*b^11)) + 62523502209*d^27*x)*a^3*b^3*(-d^18/
(a^13*b^11))^(1/4))/d^18) - 315*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*
x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*l
og(250047*a^10*b^8*(-d^18/(a^13*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) + 315
```

```
*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^
3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*log(-250047*a^10*b^8*(-d^18/(a^1
3*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) - 4*(315*b^4*d^4*x^9 + 1512*a*b^3*d
^4*x^7 + 2870*a^2*b^2*d^4*x^5 - 480*a^3*b*d^4*x^3 - 105*a^4*d^4*x)*sqrt(d*x
))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*
b^3*x^2 + a^8*b^2)
```

giac [A] time = 0.26, size = 355, normalized size = 0.90

$$\frac{1}{163840}\,d^4\left(\frac{630\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5d}+\frac{630\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5d}-\frac{315\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^5d}+\frac{315\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^5d}+\frac{8\left(315\sqrt{dx}\,b^4d^{10}x^9+1512\sqrt{dx}\,ab^3d^{10}x^7+2870\sqrt{dx}\,a^2b^2d^{10}x^5-480\sqrt{dx}\,a^3bd^{10}x^3-105\sqrt{dx}\,a^4d^{10}x\right)}{\left(bd^2x^2+ad^2\right)^5a^3b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^4*(630*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^5*d) + 630*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^4*b^5*d) - 315*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^5*d) + 315*sqrt(2)*(a*
b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^4*b^5*d) + 8*(315*sqrt(d*x)*b^4*d^10*x^9 + 1512*sqrt(d*x)*a*b^3*d^10*x^
7 + 2870*sqrt(d*x)*a^2*b^2*d^10*x^5 - 480*sqrt(d*x)*a^3*b*d^10*x^3 - 105*sq
rt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

maple [A] time = 0.03, size = 339, normalized size = 0.86

$$-\frac{21\,(dx)^{\frac{3}{2}}\,a\,d^{13}}{4096\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{3\,(dx)^{\frac{7}{2}}\,d^{11}}{128\left(b\,d^2x^2+d^2a\right)^5b}+\frac{287\,(dx)^{\frac{11}{2}}\,d^9}{2048\left(b\,d^2x^2+d^2a\right)^5a}+\frac{189\,(dx)^{\frac{15}{2}}\,b\,d^7}{2560\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{63\,(dx)^{\frac{19}{2}}\,b^2d^5}{4096\left(b\,d^2x^2+d^2a\right)^5a^3}+\frac{63\sqrt{2}\,d^5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^3b^3}+\frac{63\sqrt{2}\,d^5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^3b^3}+\frac{63\sqrt{2}\,d^5\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -21/4096*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(3/2)-3/128*d^11/(b*d^2*x^2+a
*d^2)^5/b*(d*x)^(7/2)+287/2048*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(11/2)+189/2
560*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(15/2)+63/4096*d^5/(b*d^2*x^2+a*d^2
)^5/a^3*b^2*(d*x)^(19/2)+63/32768*d^5/a^3/b^3/(a/b*d^2)^(1/4)*2^(1/2)*ln((d
*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+63/16384*d^5/a^3/b^3/(a/b*d^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+63/16384*d^5/a^3/b^
3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.17, size = 385, normalized size = 0.98

$$\frac{\frac{315\, d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^3b^2}+\frac{8\left(315\,(dx)^{\frac{19}{2}}b^4d^6+1512\,(dx)^{\frac{15}{2}}ab^3d^8+2870\,(dx)^{\frac{11}{2}}a^2b^2d^{10}-480\,(dx)^{\frac{7}{2}}a^3bd^{12}-105\,(dx)^{\frac{3}{2}}a^4d^{14}\right)}{a^3b^7d^{10}x^{10}+5\,a^4b^6d^{10}x^8+10\,a^5b^5d^{10}x^6+10\,a^6b^4d^{10}x^4+5\,a^7b^3d^{10}x^2+a^8b^2d^{10}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(315*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1
/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d
)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) -
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqr
t(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
 sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*
d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^3*b^2
) + 8*(315*(d*x)^(19/2)*b^4*d^6 + 1512*(d*x)^(15/2)*a*b^3*d^8 + 2870*(d*x)^
(11/2)*a^2*b^2*d^10 - 480*(d*x)^(7/2)*a^3*b*d^12 - 105*(d*x)^(3/2)*a^4*d^14
)/(a^3*b^7*d^10*x^10 + 5*a^4*b^6*d^10*x^8 + 10*a^5*b^5*d^10*x^6 + 10*a^6*b^
4*d^10*x^4 + 5*a^7*b^3*d^10*x^2 + a^8*b^2*d^10))/d
```

mupad [B] time = 0.12, size = 207, normalized size = 0.53

$$\frac{\frac{287\,d^9\,(d\,x)^{11/2}}{2048\,a}-\frac{3\,d^{11}\,(d\,x)^{7/2}}{128\,b}+\frac{63\,b^2\,d^5\,(d\,x)^{19/2}}{4096\,a^3}-\frac{21\,a\,d^{13}\,(d\,x)^{3/2}}{4096\,b^2}+\frac{189\,b\,d^7\,(d\,x)^{15/2}}{2560\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{63\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{13/4}\,b^{11/4}}+\frac{63\,d^{9/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{13/4}\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((287*d^9*(d*x)^(11/2))/(2048*a) - (3*d^11*(d*x)^(7/2))/(128*b) + (63*b^2*d
^5*(d*x)^(19/2))/(4096*a^3) - (21*a*d^13*(d*x)^(3/2))/(4096*b^2) + (189*b*d
^7*(d*x)^(15/2))/(2560*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 +
 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (63*d^(9/2
)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(13/4)*b^(11
/4)) + (63*d^(9/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192
*(-a)^(13/4)*b^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.542 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx$

Optimal. Leaf size=394

$$-\frac{77d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}-\frac{77d^{7/2}\text{ t}}{81}$$

Rubi [A] time = 0.45, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77d^3\sqrt{dx}}{12288a^3b^2\left(a+bx^2\right)}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}-\frac{77d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}-\frac{77d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(5/2))/(10*b*(a + b*x^2)^5) - (d^3*Sqrt[d*x])/(32*b^2*(a + b*x^2)
^4) + (d^3*Sqrt[d*x])/(384*a*b^2*(a + b*x^2)^3) + (11*d^3*Sqrt[d*x])/(3072*
a^2*b^2*(a + b*x^2)^2) + (77*d^3*Sqrt[d*x])/(12288*a^3*b^2*(a + b*x^2)) - (
77*d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192
*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)) - (77*d^(7/2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(15/4)*b^
(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{4}\left(b^4d^2\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(b^2d^4\right)\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{\left(11bd^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}}{768a}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

Mathematica [A] time = 0.17, size = 317, normalized size = 0.80

$$\frac{d^3\sqrt{dx}\left(-\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{15/4}\sqrt{x}}+\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{15/4}\sqrt{x}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{15/4}\sqrt{x}}+\frac{2310\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{15/4}\sqrt{x}}+\frac{3080\sqrt[4]{b}}{a^3(a+bx^2)}+\frac{1760\sqrt[4]{b}}{a^2(a+bx^2)^2}-\frac{65536b^{5/4}x^2}{(a+bx^2)^5}+\frac{1280\sqrt[4]{b}}{a(a+bx^2)^3}+\frac{1024\sqrt[4]{b}}{(a+bx^2)^4}-\frac{16384a\sqrt[4]{b}}{(a+bx^2)^5}\right)}{491520b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^3*Sqrt[d*x]*((-16384*a*b^(1/4))/(a + b*x^2)^5 - (65536*b^(5/4)*x^2)/(a +
 b*x^2)^5 + (1024*b^(1/4))/(a + b*x^2)^4 + (1280*b^(1/4))/(a*(a + b*x^2)^3)
 + (1760*b^(1/4))/(a^2*(a + b*x^2)^2) + (3080*b^(1/4))/(a^3*(a + b*x^2)) -
(2310*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(15/4)*Sqrt
[x]) + (2310*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(15/
4)*Sqrt[x]) - (1155*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(a^(15/4)*Sqrt[x]) + (1155*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(15/4)*Sqrt[x])))/(491520*b^(9/4))
```

IntegrateAlgebraic [A] time = 1.15, size = 244, normalized size = 0.62

$$-\frac{77d^{7/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}-\frac{d^5\sqrt{dx}\left(1155a^4d^8+5544a^3bd^8x^2-3130a^2b^2d^8x^4-1760ab^3d^8x^6-385b^4d^8x^8\right)}{61440a^3b^2\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/61440*(d^5*Sqrt[d*x]*(1155*a^4*d^8 + 5544*a^3*b*d^8*x^2 - 3130*a^2*b^2*d
^8*x^4 - 1760*a*b^3*d^8*x^6 - 385*b^4*d^8*x^8))/(a^3*b^2*(a*d^2 + b*d^2*x^2
)^5) - (77*d^(7/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*S
qrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)) +
(77*d^(7/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d
+ Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^(15/4)*b^(9/4))
```

fricas [A] time = 1.17, size = 513, normalized size = 1.30

$$\frac{4620\left(a^3b^7x^{10}+5a^4b^6x^8+10a^5b^5x^6+10a^6b^4x^4+5a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{11}b^7d^3\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{3}{4}}-\sqrt{a^8b^4\sqrt{-\frac{d^{14}}{a^{15}b^9}}+d^7x}\,a^{11}b^7\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{3}{4}}}{d^{14}}\right)+1155\left(a^3b^7x^{10}+5a^4b^6x^8+10a^5b^5x^6+10a^6b^4x^4+5a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{1}{4}}\log\left(77a^4b^2\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{1}{4}}+77\sqrt{dx}d^3\right)-1155\left(a^3b^7x^{10}+5a^4b^6x^8+10a^5b^5x^6+10a^6b^4x^4+5a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{1}{4}}\log\left(-77a^4b^2\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{1}{4}}+77\sqrt{dx}d^3\right)+4\left(385b^4d^3x^8+1760ab^3d^3x^6+3130a^2b^2d^3x^4-5544a^3bd^3x^2-1155a^4d^3\right)\sqrt{dx}}{245760\left(a^3b^7x^{10}+5a^4b^6x^8+10a^5b^5x^6+10a^6b^4x^4+5a^7b^3x^2+a^8b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(4620*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*
x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*arctan(-(sqrt(d*x)*
a^11*b^7*d^3*(-d^14/(a^15*b^9))^(3/4) - sqrt(a^8*b^4*sqrt(-d^14/(a^15*b^9))
```

```
 + d^7*x)*a^11*b^7*(-d^14/(a^15*b^9))^(3/4))/d^14) + 1155*(a^3*b^7*x^10 + 5
*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(
-d^14/(a^15*b^9))^(1/4)*log(77*a^4*b^2*(-d^14/(a^15*b^9))^(1/4) + 77*sqrt(d
*x)*d^3) - 1155*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4
*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*log(-77*a^4*b^2*(-
d^14/(a^15*b^9))^(1/4) + 77*sqrt(d*x)*d^3) + 4*(385*b^4*d^3*x^8 + 1760*a*b^
3*d^3*x^6 + 3130*a^2*b^2*d^3*x^4 - 5544*a^3*b*d^3*x^2 - 1155*a^4*d^3)*sqrt(
d*x))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a
^7*b^3*x^2 + a^8*b^2)
```

giac [A] time = 0.22, size = 342, normalized size = 0.87

$$\frac{1}{491520}d^3\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}+\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^3}-\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^3}+\frac{8\left(385\sqrt{dx}\,b^4d^{10}x^8+1760\sqrt{dx}\,ab^3d^{10}x^6+3130\sqrt{dx}\,a^2b^2d^{10}x^4-5544\sqrt{dx}\,a^3bd^{10}x^2-1155\sqrt{dx}\,a^4d^{10}\right)}{\left(bd^2x^2+ad^2\right)^5a^3b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^3*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 2310*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^4*b^3) + 1155*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) - 1155*sqrt(2)*(a*b^
3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^4*b^3) + 8*(385*sqrt(d*x)*b^4*d^10*x^8 + 1760*sqrt(d*x)*a*b^3*d^10*x^6 +
3130*sqrt(d*x)*a^2*b^2*d^10*x^4 - 5544*sqrt(d*x)*a^3*b*d^10*x^2 - 1155*sqrt
(d*x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

maple [A] time = 0.02, size = 339, normalized size = 0.86

$$-\frac{77\sqrt{dx}\,a\,d^{13}}{4096\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{231\,(dx)^{\frac{5}{2}}\,d^{11}}{2560\left(b\,d^2x^2+d^2a\right)^5b}+\frac{313\,(dx)^{\frac{9}{2}}\,d^9}{6144\left(b\,d^2x^2+d^2a\right)^5a}+\frac{11\,(dx)^{\frac{13}{2}}\,b\,d^7}{384\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{77\,(dx)^{\frac{17}{2}}\,b^2d^5}{12288\left(b\,d^2x^2+d^2a\right)^5a^3}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384a^4b^2}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384a^4b^2}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/4096*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(1/2)-231/2560*d^11/(b*d^2*x^
2+a*d^2)^5/b*(d*x)^(5/2)+313/6144*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(9/2)+11/
384*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(13/2)+77/12288*d^5/(b*d^2*x^2+a*d^
2)^5/a^3*b^2*(d*x)^(17/2)+77/32768*d^3/a^4/b^2*(a/b*d^2)^(1/4)*2^(1/2)*ln((
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+77/16384*d^3/a^4/b^2*(a/b*d^2)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+77/16384*d^3/a^4/b
^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.15, size = 394, normalized size = 1.00

$$\frac{\frac{8\left(385\,(dx)^{\frac{17}{2}} b^4 d^6+1760\,(dx)^{\frac{13}{2}} a b^3 d^8+3130\,(dx)^{\frac{9}{2}} a^2 b^2 d^{10}-5544\,(dx)^{\frac{5}{2}} a^3 b d^{12}-1155\,\sqrt{dx}\, a^4 d^{14}\right)}{a^3 b^7 d^{10} x^{10}+5\,a^4 b^6 d^{10} x^8+10\,a^5 b^5 d^{10} x^6+10\,a^6 b^4 d^{10} x^4+5\,a^7 b^3 d^{10} x^2+a^8 b^2 d^{10}}+\frac{1155\left(\frac{\sqrt{2}\, d^6 \log\left(\sqrt{b}\, dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{3}{4}} b^{\frac{1}{4}}}-\frac{\sqrt{2}\, d^6 \log\left(\sqrt{b}\, dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{3}{4}} b^{\frac{1}{4}}}+\frac{2\sqrt{2}\, d^5 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\, d}\,\sqrt{a}}+\frac{2\sqrt{2}\, d^5 \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\, d}\,\sqrt{a}}\right)}{a^3 b^2}}{491520\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/491520*(8*(385*(d*x)^(17/2)*b^4*d^6 + 1760*(d*x)^(13/2)*a*b^3*d^8 + 3130*
(d*x)^(9/2)*a^2*b^2*d^10 - 5544*(d*x)^(5/2)*a^3*b*d^12 - 1155*sqrt(d*x)*a^4
*d^14)/(a^3*b^7*d^10*x^10 + 5*a^4*b^6*d^10*x^8 + 10*a^5*b^5*d^10*x^6 + 10*a
^6*b^4*d^10*x^4 + 5*a^7*b^3*d^10*x^2 + a^8*b^2*d^10) + 1155*(sqrt(2)*d^6*lo
g(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^
2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^5*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^5*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^3*b^2))/d
```

mupad [B] time = 4.27, size = 207, normalized size = 0.53

$$\frac{\frac{313\,d^9\,(d\,x)^{9/2}}{6144\,a}-\frac{231\,d^{11}\,(d\,x)^{5/2}}{2560\,b}+\frac{77\,b^2\,d^5\,(d\,x)^{17/2}}{12288\,a^3}-\frac{77\,a\,d^{13}\,\sqrt{d\,x}}{4096\,b^2}+\frac{11\,b\,d^7\,(d\,x)^{13/2}}{384\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{77\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{15/4}\,b^{9/4}}+\frac{77\,d^{7/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{15/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((313*d^9*(d*x)^(9/2))/(6144*a) - (231*d^11*(d*x)^(5/2))/(2560*b) + (77*b^2
*d^5*(d*x)^(17/2))/(12288*a^3) - (77*a*d^13*(d*x)^(1/2))/(4096*b^2) + (11*b
*d^7*(d*x)^(13/2))/(384*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2
+ 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) + (77*d^(7/
2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(15/4)*b^(9
/4)) + (77*d^(7/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192
*(-a)^(15/4)*b^(9/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(a+bx^2\right)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**6, x)
```

3.543 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=389

$$\frac{117d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{17/4}b^{7/4}}-\frac{117d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{17/4}b^{7/4}}-\frac{117d^{5/2}}{8}$$

Rubi [A] time = 0.46, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{17/4}b^{7/4}}-\frac{117d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{17/4}b^{7/4}}-\frac{117d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{17/4}b^{7/4}}+\frac{117d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{17/4}b^{7/4}}+\frac{117d(dx)^{3/2}}{4096a^4b\left(a+bx^2\right)}+\frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}+\frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3}+\frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4}-\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(3/2))/(10*b*(a + b*x^2)^5) + (3*d*(d*x)^(3/2))/(160*a*b*(a + b*x
^2)^4) + (13*d*(d*x)^(3/2))/(640*a^2*b*(a + b*x^2)^3) + (117*d*(d*x)^(3/2))
/(5120*a^3*b*(a + b*x^2)^2) + (117*d*(d*x)^(3/2))/(4096*a^4*b*(a + b*x^2))
- (117*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(17/4)*b^(7/4)) - (117*d^(5/2)*Log[Sqrt[a]*Sqrt[d] +
Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1
7/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(3b^4d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{\left(39b^3d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{320a}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{\left(117b^2d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)}}{1280a^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.12

$$\frac{2d(dx)^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)}{a^5}-\frac{1}{(a+bx^2)^5}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-(a + b*x^2)^(-5) + Hypergeometric2F1[3/4, 6, 7/4, -((b*x
^2)/a)]/a^5))/(17*b)
```

IntegrateAlgebraic [A] time = 1.11, size = 241, normalized size = 0.62

$$-\frac{117d^{5/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{17/4}b^{7/4}}-\frac{(dx)^{3/2}\left(195a^4d^{11}-4960a^3bd^{11}x^2-5330a^2b^2d^{11}x^4-2808ab^3d^{11}x^6-585b^4d^{11}x^8\right)}{20480a^4b\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20480*((d*x)^(3/2)*(195*a^4*d^11 - 4960*a^3*b*d^11*x^2 - 5330*a^2*b^2*d^
11*x^4 - 2808*a*b^3*d^11*x^6 - 585*b^4*d^11*x^8))/(a^4*b*(a*d^2 + b*d^2*x^2
)^5) - (117*d^(5/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*
Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(17/4)*b^(7/4)) -
 (117*d^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*
d + Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^(17/4)*b^(7/4))
```

fricas [A] time = 0.70, size = 512, normalized size = 1.32

$$-\frac{2340\left(a^4b^6x^{10}+5a^5b^5x^8+10a^6b^4x^6+10a^7b^3x^4+5a^8b^2x^2+a^9b\right)\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\sqrt{dx}a^4b^2d^7\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{1}{4}}-\sqrt{-2565164201769a^9b^3d^{10}\sqrt{-\frac{d^{10}}{a^{17}b^7}}+2565164201769d^{15}x}\,a^4b^2\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{1}{4}}}{1601613d^{10}}\right)-585\left(a^4b^6x^{10}+5a^5b^5x^8+10a^6b^4x^6+10a^7b^3x^4+5a^8b^2x^2+a^9b\right)\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{1}{4}}\log\left(1601613a^{13}b^5\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{3}{4}}+1601613\sqrt{dx}d^7\right)+585\left(a^4b^6x^{10}+5a^5b^5x^8+10a^6b^4x^6+10a^7b^3x^4+5a^8b^2x^2+a^9b\right)\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{1}{4}}\log\left(-1601613a^{13}b^5\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{3}{4}}+1601613\sqrt{dx}d^7\right)-4\left(585b^4d^2x^9+2808ab^3d^2x^7+5330a^2b^2d^2x^5+4960a^3bd^2x^3-195a^4d^2x\right)\sqrt{dx}}{81920\left(a^4b^6x^{10}+5a^5b^5x^8+10a^6b^4x^6+10a^7b^3x^4+5a^8b^2x^2+a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(2340*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*
x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*arctan(-1/1601613*(16
01613*sqrt(d*x)*a^4*b^2*d^7*(-d^10/(a^17*b^7))^(1/4) - sqrt(-2565164201769*
a^9*b^3*d^10*sqrt(-d^10/(a^17*b^7)) + 2565164201769*d^15*x)*a^4*b^2*(-d^10/
(a^17*b^7))^(1/4))/d^10) - 585*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x
^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(1
601613*a^13*b^5*(-d^10/(a^17*b^7))^(3/4) + 1601613*sqrt(d*x)*d^7) + 585*(a^
```

```
4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^
2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(-1601613*a^13*b^5*(-d^10/(a^17*b^7)
)^(3/4) + 1601613*sqrt(d*x)*d^7) - 4*(585*b^4*d^2*x^9 + 2808*a*b^3*d^2*x^7
+ 5330*a^2*b^2*d^2*x^5 + 4960*a^3*b*d^2*x^3 - 195*a^4*d^2*x)*sqrt(d*x))/(a^
4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^
2 + a^9*b)
```

giac [A] time = 0.22, size = 355, normalized size = 0.91

$$\frac{1}{163840}\,d^2\left(\frac{1170\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4d}+\frac{1170\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4d}-\frac{585\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^4d}+\frac{585\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^4d}+\frac{8\left(585\,\sqrt{dx}\,b^4d^{10}x^9+2808\,\sqrt{dx}\,ab^3d^{10}x^7+5330\,\sqrt{dx}\,a^2b^2d^{10}x^5+4960\,\sqrt{dx}\,a^3bd^{10}x^3-195\,\sqrt{dx}\,a^4d^{10}x\right)}{\left(bd^2x^2+ad^2\right)^5a^4b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^2*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^4*d) + 1170*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^5*b^4*d) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqr
t(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^4*d) + 585*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^5*b^4*d) + 8*(585*sqrt(d*x)*b^4*d^10*x^9 + 2808*sqrt(d*x)*a*b^3*d^10*
x^7 + 5330*sqrt(d*x)*a^2*b^2*d^10*x^5 + 4960*sqrt(d*x)*a^3*b*d^10*x^3 - 195
*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^4*b))
```

maple [A] time = 0.02, size = 341, normalized size = 0.88

$$-\frac{39\,(dx)^{\frac{3}{2}}\,d^{11}}{4096\left(b\,d^2x^2+d^2a\right)^5b}+\frac{31\,(dx)^{\frac{7}{2}}\,d^9}{128\left(b\,d^2x^2+d^2a\right)^5a}+\frac{533\,(dx)^{\frac{11}{2}}\,b\,d^7}{2048\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{351\,(dx)^{\frac{15}{2}}\,b^2d^5}{2560\left(b\,d^2x^2+d^2a\right)^5a^3}+\frac{117\,(dx)^{\frac{19}{2}}\,b^3d^3}{4096\left(b\,d^2x^2+d^2a\right)^5a^4}+\frac{117\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^4b^2}+\frac{117\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^4b^2}+\frac{117\sqrt{2}\,d^3\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -39/4096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(3/2)+31/128*d^9/(b*d^2*x^2+a*d^2
)^5/a*(d*x)^(7/2)+533/2048*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(11/2)+351/2
560*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(15/2)+117/4096*d^3/(b*d^2*x^2+a*
d^2)^5/a^4*b^3*(d*x)^(19/2)+117/32768*d^3/a^4/b^2/(a/b*d^2)^(1/4)*2^(1/2)*l
n((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+117/16384*d^3/a^4/b^2/(a/b*d^2)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+117/16384*d^3/
a^4/b^2/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-
1)
```

maxima [A] time = 3.16, size = 383, normalized size = 0.98

$$\frac{\frac{8\left(585 (dx)^{\frac{19}{2}} b^4 d^4 + 2808 (dx)^{\frac{15}{2}} a b^3 d^6 + 5330 (dx)^{\frac{11}{2}} a^2 b^2 d^8 + 4960 (dx)^{\frac{7}{2}} a^3 b d^{10} - 195 (dx)^{\frac{3}{2}} a^4 d^{12}\right)}{a^4 b^6 d^{10} x^{10} + 5 a^5 b^5 d^{10} x^8 + 10 a^6 b^4 d^{10} x^6 + 10 a^7 b^3 d^{10} x^4 + 5 a^8 b^2 d^{10} x^2 + a^9 b d^{10}} + \frac{585 d^4 \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac14} b^{\frac14} + 2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac14} b^{\frac14} - 2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}dx + \sqrt{2}(ad^2)^{\frac14}\sqrt{dx} b^{\frac14} + \sqrt{a}d\right)}{(ad^2)^{\frac14} b^{\frac34}} + \frac{\sqrt{2}\log\left(\sqrt{b}dx - \sqrt{2}(ad^2)^{\frac14}\sqrt{dx}b^{\frac14} + \sqrt{a}d\right)}{(ad^2)^{\frac14}b^{\frac34}}\right)}{a^4 b}}{163840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(585*(d*x)^(19/2)*b^4*d^4 + 2808*(d*x)^(15/2)*a*b^3*d^6 + 5330*
(d*x)^(11/2)*a^2*b^2*d^8 + 4960*(d*x)^(7/2)*a^3*b*d^10 - 195*(d*x)^(3/2)*a^
4*d^12)/(a^4*b^6*d^10*x^10 + 5*a^5*b^5*d^10*x^8 + 10*a^6*b^4*d^10*x^6 + 10*
a^7*b^3*d^10*x^4 + 5*a^8*b^2*d^10*x^2 + a^9*b*d^10) + 585*d^4*(2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt
(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x
 + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3
/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^4*b))/d
```

mupad [B] time = 4.28, size = 209, normalized size = 0.54

$$\frac{\frac{31\,d^9\,(d\,x)^{7/2}}{128\,a}-\frac{39\,d^{11}\,(d\,x)^{3/2}}{4096\,b}+\frac{351\,b^2\,d^5\,(d\,x)^{15/2}}{2560\,a^3}+\frac{117\,b^3\,d^3\,(d\,x)^{19/2}}{4096\,a^4}+\frac{533\,b\,d^7\,(d\,x)^{11/2}}{2048\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{117\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{17/4}\,b^{7/4}}-\frac{117\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{17/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((31*d^9*(d*x)^(7/2))/(128*a) - (39*d^11*(d*x)^(3/2))/(4096*b) + (351*b^2*d
^5*(d*x)^(15/2))/(2560*a^3) + (117*b^3*d^3*(d*x)^(19/2))/(4096*a^4) + (533*
b*d^7*(d*x)^(11/2))/(2048*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^
2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) + (117*d^
(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(17/4)*b
^(7/4)) - (117*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(17/4)*b^(7/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(a+bx^2\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**6, x)
```

3.544 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=389

$$-\frac{231d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}+\frac{231d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}-\frac{231d^{3}}{}$$

Rubi [A] time = 0.50, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{231d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}+\frac{231d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}-\frac{231d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{19/4}b^{5/4}}+\frac{231d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{19/4}b^{5/4}}+\frac{77d\sqrt{dx}}{4096a^4b\left(a+bx^2\right)}+\frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}+\frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}-\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*Sqrt[d*x])/(10*b*(a + b*x^2)^5) + (d*Sqrt[d*x])/(160*a*b*(a + b*x^2)^4)
 + (d*Sqrt[d*x])/(128*a^2*b*(a + b*x^2)^3) + (11*d*Sqrt[d*x])/(1024*a^3*b*(
a + b*x^2)^2) + (77*d*Sqrt[d*x])/(4096*a^4*b*(a + b*x^2)) - (231*d^(3/2)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1
9/4)*b^(5/4)) + (231*d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4
)*Sqrt[d])])/(8192*Sqrt[2]*a^(19/4)*b^(5/4)) - (231*d^(3/2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[
2]*a^(19/4)*b^(5/4)) + (231*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(19/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(b^4d^2\right)\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{\left(3b^3d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{64a}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{\left(11b^2d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2\right.}}{256a^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.17, size = 298, normalized size = 0.77

$$\frac{d\sqrt{dx}\left(-\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{19/4}\sqrt{x}}+\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{19/4}\sqrt{x}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{19/4}\sqrt{x}}+\frac{2310\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{19/4}\sqrt{x}}+\frac{3080\sqrt[4]{b}}{a^4(a+bx^2)}+\frac{1760\sqrt[4]{b}}{a^3(a+bx^2)^2}+\frac{1280\sqrt[4]{b}}{a^2(a+bx^2)^3}+\frac{1024\sqrt[4]{b}}{a(a+bx^2)^4}-\frac{16384\sqrt[4]{b}}{(a+bx^2)^5}\right)}{163840b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d*Sqrt[d*x]*((-16384*b^(1/4))/(a + b*x^2)^5 + (1024*b^(1/4))/(a*(a + b*x^2
)^4) + (1280*b^(1/4))/(a^2*(a + b*x^2)^3) + (1760*b^(1/4))/(a^3*(a + b*x^2)
^2) + (3080*b^(1/4))/(a^4*(a + b*x^2)) - (2310*Sqrt[2]*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(19/4)*Sqrt[x]) + (2310*Sqrt[2]*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(19/4)*Sqrt[x]) - (1155*Sqrt[2]*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(19/4)*Sqrt[x]) +
(1155*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
a^(19/4)*Sqrt[x])))/(163840*b^(5/4))
```

IntegrateAlgebraic [A] time = 0.79, size = 241, normalized size = 0.62

$$-\frac{231d^{3/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{19/4}b^{5/4}}+\frac{231d^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{19/4}b^{5/4}}-\frac{\sqrt{dx}\left(1155a^4d^{11}-2648a^3bd^{11}x^2-3130a^2b^2d^{11}x^4-1760ab^3d^{11}x^6-385b^4d^{11}x^8\right)}{20480a^4b\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20480*(Sqrt[d*x]*(1155*a^4*d^11 - 2648*a^3*b*d^11*x^2 - 3130*a^2*b^2*d^1
1*x^4 - 1760*a*b^3*d^11*x^6 - 385*b^4*d^11*x^8))/(a^4*b*(a*d^2 + b*d^2*x^2)
^5) - (231*d^(3/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*S
qrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(19/4)*b^(5/4)) +
(231*d^(3/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d
 + Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^(19/4)*b^(5/4))
```

fricas [A] time = 1.26, size = 485, normalized size = 1.25

$$\frac{4620\left(a^4b^6x^{10}+5a^5b^5x^8+10a^6b^4x^6+10a^7b^3x^4+5a^8b^2x^2+a^9b\right)\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac14}\arctan\left(-\frac{\sqrt{dx}a^{14}b^4d\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac34}-\sqrt{a^{10}b^2\sqrt{-\frac{d^6}{a^{19}b^5}}+d^3x}\,a^{14}b^4\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac34}}{d^6}\right)+1155\left(a^4b^6x^{10}+5a^5b^5x^8+10a^6b^4x^6+10a^7b^3x^4+5a^8b^2x^2+a^9b\right)\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac14}\log\left(231a^5b\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac14}+231\sqrt{dx}d\right)-1155\left(a^4b^6x^{10}+5a^5b^5x^8+10a^6b^4x^6+10a^7b^3x^4+5a^8b^2x^2+a^9b\right)\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac14}\log\left(-231a^5b\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac14}+231\sqrt{dx}d\right)+4\left(385b^4dx^8+1760ab^3dx^6+3130a^2b^2dx^4+2648a^3bdx^2-1155a^4d\right)\sqrt{dx}}{81920\left(a^4b^6x^{10}+5a^5b^5x^8+10a^6b^4x^6+10a^7b^3x^4+5a^8b^2x^2+a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(4620*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x
^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^14
*b^4*d*(-d^6/(a^19*b^5))^(3/4) - sqrt(a^10*b^2*sqrt(-d^6/(a^19*b^5)) + d^3*
```

```
x)*a^14*b^4*(-d^6/(a^19*b^5))^(3/4))/d^6) + 1155*(a^4*b^6*x^10 + 5*a^5*b^5*
x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*
b^5))^(1/4)*log(231*a^5*b*(-d^6/(a^19*b^5))^(1/4) + 231*sqrt(d*x)*d) - 1155
*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^
2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*log(-231*a^5*b*(-d^6/(a^19*b^5))^(1/
4) + 231*sqrt(d*x)*d) + 4*(385*b^4*d*x^8 + 1760*a*b^3*d*x^6 + 3130*a^2*b^2*
d*x^4 + 2648*a^3*b*d*x^2 - 1155*a^4*d)*sqrt(d*x))/(a^4*b^6*x^10 + 5*a^5*b^5
*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)
```

giac [A] time = 0.21, size = 340, normalized size = 0.87

$$\frac{1}{163840}\,d\left(\frac{2310\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{5}b^{2}}+\frac{2310\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{5}b^{2}}+\frac{1155\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{5}b^{2}}-\frac{1155\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{5}b^{2}}+\frac{8\left(385\,\sqrt{dx}\,b^{4}d^{10}x^{8}+1760\,\sqrt{dx}\,ab^{3}d^{10}x^{6}+3130\,\sqrt{dx}\,a^{2}b^{2}d^{10}x^{4}+2648\,\sqrt{dx}\,a^{3}bd^{10}x^{2}-1155\,\sqrt{dx}\,a^{4}d^{10}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{4}b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2) + 2310*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^5*b^2) + 1155*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2) - 1155*sqrt(2)*(a*b^3*
d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
5*b^2) + 8*(385*sqrt(d*x)*b^4*d^10*x^8 + 1760*sqrt(d*x)*a*b^3*d^10*x^6 + 31
30*sqrt(d*x)*a^2*b^2*d^10*x^4 + 2648*sqrt(d*x)*a^3*b*d^10*x^2 - 1155*sqrt(d
*x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a^4*b))
```

maple [A] time = 0.02, size = 335, normalized size = 0.86

$$-\frac{231\sqrt{dx}\,d^{11}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b}+\frac{331\left(dx\right)^{\frac{5}{2}}d^{9}}{2560\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a}+\frac{313\left(dx\right)^{\frac{9}{2}}b\,d^{7}}{2048\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{2}}+\frac{11\left(dx\right)^{\frac{13}{2}}b^{2}d^{5}}{128\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{3}}+\frac{77\left(dx\right)^{\frac{17}{2}}b^{3}d^{3}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{4}}+\frac{231\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{16384a^{5}b}+\frac{231\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{16384a^{5}b}+\frac{231\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{32768a^{5}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -231/4096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(1/2)+331/2560*d^9/(b*d^2*x^2+a*
d^2)^5/a*(d*x)^(5/2)+313/2048*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(9/2)+11/
128*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(13/2)+77/4096*d^3/(b*d^2*x^2+a*d
^2)^5/a^4*b^3*(d*x)^(17/2)+231/32768*d/a^5/b*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*
x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+231/16384*d/a^5/b*(a/b*d^2)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+231/16384*d/a^5/b*(a/b*
d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.23, size = 392, normalized size = 1.01

$$\frac{\frac{8\left(385\,(dx)^{\frac{17}{2}}b^4d^4+1760\,(dx)^{\frac{13}{2}}ab^3d^6+3130\,(dx)^{\frac{9}{2}}a^2b^2d^8+2648\,(dx)^{\frac{5}{2}}a^3bd^{10}-1155\,\sqrt{dx}\,a^4d^{12}\right)}{a^4b^6d^{10}x^{10}+5\,a^5b^5d^{10}x^8+10\,a^6b^4d^{10}x^6+10\,a^7b^3d^{10}x^4+5\,a^8b^2d^{10}x^2+a^9bd^{10}}+\frac{1155\left(\frac{\sqrt{2}d^4\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^4\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{a}}+\frac{2\sqrt{2}d^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{a}}\right)}{a^4b}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(385*(d*x)^(17/2)*b^4*d^4 + 1760*(d*x)^(13/2)*a*b^3*d^6 + 3130*
(d*x)^(9/2)*a^2*b^2*d^8 + 2648*(d*x)^(5/2)*a^3*b*d^10 - 1155*sqrt(d*x)*a^4*
d^12)/(a^4*b^6*d^10*x^10 + 5*a^5*b^5*d^10*x^8 + 10*a^6*b^4*d^10*x^6 + 10*a^
7*b^3*d^10*x^4 + 5*a^8*b^2*d^10*x^2 + a^9*b*d^10) + 1155*(sqrt(2)*d^4*log(s
qrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^
(3/4)*b^(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d
*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*
sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^4*b))/d
```

mupad [B] time = 0.13, size = 209, normalized size = 0.54

$$\frac{\frac{331\,d^9\,(d\,x)^{5/2}}{2560\,a}-\frac{231\,d^{11}\,\sqrt{d\,x}}{4096\,b}+\frac{11\,b^2\,d^5\,(d\,x)^{13/2}}{128\,a^3}+\frac{77\,b^3\,d^3\,(d\,x)^{17/2}}{4096\,a^4}+\frac{313\,b\,d^7\,(d\,x)^{9/2}}{2048\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{231\,d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{19/4}\,b^{5/4}}-\frac{231\,d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{19/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((331*d^9*(d*x)^(5/2))/(2560*a) - (231*d^11*(d*x)^(1/2))/(4096*b) + (11*b^2
*d^5*(d*x)^(13/2))/(128*a^3) + (77*b^3*d^3*(d*x)^(17/2))/(4096*a^4) + (313*
b*d^7*(d*x)^(9/2))/(2048*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2
 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (231*d^(
3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(19/4)*b^
(5/4)) - (231*d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8
192*(-a)^(19/4)*b^(5/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**6, x)
```

3.545 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=387

$$\frac{663\sqrt{d}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{21/4}b^{3/4}}-\frac{663\sqrt{d}}{8}$$

Rubi [A] time = 0.49, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{663\sqrt{d}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{8192\sqrt{2}\,a^{21/4}b^{3/4}}+\frac{663\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{21/4}b^{3/4}}+\frac{663(dx)^{3/2}}{4096a^5d\left(a+bx^2\right)}+\frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}+\frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3}+\frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d*x)^(3/2)/(10*a*d*(a + b*x^2)^5) + (17*(d*x)^(3/2))/(160*a^2*d*(a + b*x^2
)^4) + (221*(d*x)^(3/2))/(1920*a^3*d*(a + b*x^2)^3) + (663*(d*x)^(3/2))/(51
20*a^4*d*(a + b*x^2)^2) + (663*(d*x)^(3/2))/(4096*a^5*d*(a + b*x^2)) - (663
*Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*S
qrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(21/4)*b^(3/4)) - (663*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(21/4)*b
^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{\left(17b^5\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{\left(221b^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{320a^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{\left(663b^3\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)}}{1280a^3}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.08

$$\frac{2x\sqrt{dx}\ {}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*Sqrt[d*x]*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)])/(3*a^6)
```

IntegrateAlgebraic [A] time = 0.57, size = 238, normalized size = 0.61

$$-\frac{663\sqrt{d}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{21/4}b^{3/4}}+\frac{(dx)^{3/2}\left(37645a^4d^9+84320a^3bd^9x^2+90610a^2b^2d^9x^4+47736ab^3d^9x^6+9945b^4d^9x^8\right)}{61440a^5\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((d*x)^(3/2)*(37645*a^4*d^9 + 84320*a^3*b*d^9*x^2 + 90610*a^2*b^2*d^9*x^4 +
 47736*a*b^3*d^9*x^6 + 9945*b^4*d^9*x^8))/(61440*a^5*(a*d^2 + b*d^2*x^2)^5)
 - (663*Sqrt[d]*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt
[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(21/4)*b^(3/4)) - (66
3*Sqrt[d]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d +
Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^(21/4)*b^(3/4))
```

fricas [A] time = 1.00, size = 469, normalized size = 1.21

$$-\frac{39780\left(a^5b^5x^{10}+5a^6b^4x^8+10a^7b^3x^6+10a^8b^2x^4+5a^9bx^2+a^{10}\right)\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{291434247\sqrt{dx}a^5bd\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{1}{4}}-\sqrt{-84933920324457009a^{11}bd^2\sqrt{-\frac{d^2}{a^{21}b^3}}+84933920324457009d^3x}\,a^5b\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{1}{4}}}{291434247d^2}\right)-9945\left(a^5b^5x^{10}+5a^6b^4x^8+10a^7b^3x^6+10a^8b^2x^4+5a^9bx^2+a^{10}\right)\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{1}{4}}\log\left(291434247a^{16}b^2\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{3}{4}}+291434247\sqrt{dx}d\right)+9945\left(a^5b^5x^{10}+5a^6b^4x^8+10a^7b^3x^6+10a^8b^2x^4+5a^9bx^2+a^{10}\right)\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{1}{4}}\log\left(-291434247a^{16}b^2\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{3}{4}}+291434247\sqrt{dx}d\right)-4\left(9945b^4x^9+47736ab^3x^7+90610a^2b^2x^5+84320a^3bx^3+37645a^4x\right)\sqrt{dx}}{245760\left(a^5b^5x^{10}+5a^6b^4x^8+10a^7b^3x^6+10a^8b^2x^4+5a^9bx^2+a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(39780*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^
2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*arctan(-1/291434247*(29
1434247*sqrt(d*x)*a^5*b*d*(-d^2/(a^21*b^3))^(1/4) - sqrt(-84933920324457009
*a^11*b*d^2*sqrt(-d^2/(a^21*b^3)) + 84933920324457009*d^3*x)*a^5*b*(-d^2/(a
^21*b^3))^(1/4))/d^2) - 9945*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6
 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*log(2914342
47*a^16*b^2*(-d^2/(a^21*b^3))^(3/4) + 291434247*sqrt(d*x)*d) + 9945*(a^5*b^
5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^
10)*(-d^2/(a^21*b^3))^(1/4)*log(-291434247*a^16*b^2*(-d^2/(a^21*b^3))^(3/4)
 + 291434247*sqrt(d*x)*d) - 4*(9945*b^4*x^9 + 47736*a*b^3*x^7 + 90610*a^2*b
```

```
^2*x^5 + 84320*a^3*b*x^3 + 37645*a^4*x)*sqrt(d*x))/(a^5*b^5*x^10 + 5*a^6*b^
4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)
```

giac [A] time = 0.22, size = 340, normalized size = 0.88

$$\frac{\frac{19890\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^3}+\frac{19890\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^3}-\frac{9945\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^6b^3}+\frac{9945\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^6b^3}+\frac{8\left(9945\,\sqrt{dx}\,b^4d^{11}x^9+47736\,\sqrt{dx}\,ab^3d^{11}x^7+90610\,\sqrt{dx}\,a^2b^2d^{11}x^5+84320\,\sqrt{dx}\,a^3bd^{11}x^3+37645\,\sqrt{dx}\,a^4d^{11}x\right)}{\left(bd^2x^2+ad^2\right)^5a^5}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*(19890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b^3) + 19890*sqrt(2)*(a*b^3
*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^6*b^3) - 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^3) + 9945*sqrt(2)*(a*b^3*
d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
6*b^3) + 8*(9945*sqrt(d*x)*b^4*d^11*x^9 + 47736*sqrt(d*x)*a*b^3*d^11*x^7 +
90610*sqrt(d*x)*a^2*b^2*d^11*x^5 + 84320*sqrt(d*x)*a^3*b*d^11*x^3 + 37645*s
qrt(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^5))/d
```

maple [A] time = 0.03, size = 336, normalized size = 0.87

$$\frac{7529\,(dx)^{\frac{3}{2}}\,d^9}{12288\left(b\,d^2x^2+d^2a\right)^5a}+\frac{527\,(dx)^{\frac{7}{2}}\,b\,d^7}{384\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{9061\,(dx)^{\frac{11}{2}}\,b^2d^5}{6144\left(b\,d^2x^2+d^2a\right)^5a^3}+\frac{1989\,(dx)^{\frac{15}{2}}\,b^3d^3}{2560\left(b\,d^2x^2+d^2a\right)^5a^4}+\frac{663\,(dx)^{\frac{19}{2}}\,b^4d}{4096\left(b\,d^2x^2+d^2a\right)^5a^5}+\frac{663\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^5b}+\frac{663\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^5b}+\frac{663\sqrt{2}\,d\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 7529/12288*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(3/2)+527/384*d^7/(b*d^2*x^2+a*d
^2)^5/a^2*b*(d*x)^(7/2)+9061/6144*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(11
/2)+1989/2560*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3*(d*x)^(15/2)+663/4096*d/(b*d^
2*x^2+a*d^2)^5/a^5*b^4*(d*x)^(19/2)+663/32768*d/a^5/b/(a/b*d^2)^(1/4)*2^(1/
2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+663/16384*d/a^5/b/(a/b*d^2)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+663/16384*d/a^
5/b/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.19, size = 377, normalized size = 0.97

$$\frac{\frac{8\left(9945\,(dx)^{\frac{19}{2}}\,b^4d^2+47736\,(dx)^{\frac{15}{2}}\,ab^3d^4+90610\,(dx)^{\frac{11}{2}}\,a^2b^2d^6+84320\,(dx)^{\frac{7}{2}}\,a^3bd^8+37645\,(dx)^{\frac{3}{2}}\,a^4d^{10}\right)}{a^5b^5d^{10}x^{10}+5\,a^6b^4d^{10}x^8+10\,a^7b^3d^{10}x^6+10\,a^8b^2d^{10}x^4+5\,a^9bd^{10}x^2+a^{10}d^{10}}+\frac{9945\,d^2\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^5}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/491520*(8*(9945*(d*x)^(19/2)*b^4*d^2 + 47736*(d*x)^(15/2)*a*b^3*d^4 + 906
10*(d*x)^(11/2)*a^2*b^2*d^6 + 84320*(d*x)^(7/2)*a^3*b*d^8 + 37645*(d*x)^(3/
2)*a^4*d^10)/(a^5*b^5*d^10*x^10 + 5*a^6*b^4*d^10*x^8 + 10*a^7*b^3*d^10*x^6
+ 10*a^8*b^2*d^10*x^4 + 5*a^9*b*d^10*x^2 + a^10*d^10) + 9945*d^2*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/s
qrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(s
qrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*
d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b
^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^5)/d
```

mupad [B] time = 4.25, size = 210, normalized size = 0.54

$$\frac{\frac{7529\,d^9\,(d\,x)^{3/2}}{12288\,a}+\frac{9061\,b^2\,d^5\,(d\,x)^{11/2}}{6144\,a^3}+\frac{1989\,b^3\,d^3\,(d\,x)^{15/2}}{2560\,a^4}+\frac{527\,b\,d^7\,(d\,x)^{7/2}}{384\,a^2}+\frac{663\,b^4\,d\,(d\,x)^{19/2}}{4096\,a^5}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{663\,\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{21/4}\,b^{3/4}}+\frac{663\,\sqrt{d}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{21/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((7529*d^9*(d*x)^(3/2))/(12288*a) + (9061*b^2*d^5*(d*x)^(11/2))/(6144*a^3)
+ (1989*b^3*d^3*(d*x)^(15/2))/(2560*a^4) + (527*b*d^7*(d*x)^(7/2))/(384*a^2
) + (663*b^4*d*(d*x)^(19/2))/(4096*a^5))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*
b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6)
- (663*d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(21/4)*b^(3/4)) + (663*d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(
1/2))))/(8192*(-a)^(21/4)*b^(3/4))
```

sympy [A] time = 89.24, size = 547, normalized size = 1.41

$$\frac{75290a^4d^{19}(dx)^{\frac{3}{2}}}{122880a^{10}d^{20}+614400a^9bd^{20}x^2+1228800a^8b^2d^{20}x^4+1228800a^7b^3d^{20}x^6+614400a^6b^4d^{20}x^8+122880a^5b^5d^{20}x^{10}}+\frac{168640a^3bd^{17}(dx)^{\frac{7}{2}}}{122880a^{10}d^{20}+614400a^9bd^{20}x^2+1228800a^8b^2d^{20}x^4+1228800a^7b^3d^{20}x^6+614400a^6b^4d^{20}x^8+122880a^5b^5d^{20}x^{10}}+\frac{181220a^2b^2d^{15}(dx)^{\frac{11}{2}}}{122880a^{10}d^{20}+614400a^9bd^{20}x^2+1228800a^8b^2d^{20}x^4+1228800a^7b^3d^{20}x^6+614400a^6b^4d^{20}x^8+122880a^5b^5d^{20}x^{10}}+\frac{95472ab^3d^{13}(dx)^{\frac{15}{2}}}{122880a^{10}d^{20}+614400a^9bd^{20}x^2+1228800a^8b^2d^{20}x^4+1228800a^7b^3d^{20}x^6+614400a^6b^4d^{20}x^8+122880a^5b^5d^{20}x^{10}}+\frac{19890b^4d^{11}(dx)^{\frac{19}{2}}}{122880a^{10}d^{20}+614400a^9bd^{20}x^2+1228800a^8b^2d^{20}x^4+1228800a^7b^3d^{20}x^6+614400a^6b^4d^{20}x^8+122880a^5b^5d^{20}x^{10}}+2d^{11}\operatorname{RootSum}\left(1152921504606846976t^4a^{21}b^3d^{42}+193220905761,\left(t\mapsto t\log\left(\frac{35184372088832t^3a^{16}b^2d^{32}}{291434247}+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 75290*a**4*d**19*(d*x)**(3/2)/(122880*a**10*d**20 + 614400*a**9*b*d**20*x**
2 + 1228800*a**8*b**2*d**20*x**4 + 1228800*a**7*b**3*d**20*x**6 + 614400*a*
*6*b**4*d**20*x**8 + 122880*a**5*b**5*d**20*x**10) + 168640*a**3*b*d**17*(d
*x)**(7/2)/(122880*a**10*d**20 + 614400*a**9*b*d**20*x**2 + 1228800*a**8*b*
*2*d**20*x**4 + 1228800*a**7*b**3*d**20*x**6 + 614400*a**6*b**4*d**20*x**8
+ 122880*a**5*b**5*d**20*x**10) + 181220*a**2*b**2*d**15*(d*x)**(11/2)/(122
```

```
880*a**10*d**20 + 614400*a**9*b*d**20*x**2 + 1228800*a**8*b**2*d**20*x**4 +
 1228800*a**7*b**3*d**20*x**6 + 614400*a**6*b**4*d**20*x**8 + 122880*a**5*b
**5*d**20*x**10) + 95472*a*b**3*d**13*(d*x)**(15/2)/(122880*a**10*d**20 + 6
14400*a**9*b*d**20*x**2 + 1228800*a**8*b**2*d**20*x**4 + 1228800*a**7*b**3*
d**20*x**6 + 614400*a**6*b**4*d**20*x**8 + 122880*a**5*b**5*d**20*x**10) +
19890*b**4*d**11*(d*x)**(19/2)/(122880*a**10*d**20 + 614400*a**9*b*d**20*x*
*2 + 1228800*a**8*b**2*d**20*x**4 + 1228800*a**7*b**3*d**20*x**6 + 614400*a
**6*b**4*d**20*x**8 + 122880*a**5*b**5*d**20*x**10) + 2*d**11*RootSum(11529
21504606846976*_t**4*a**21*b**3*d**42 + 193220905761, Lambda(_t, _t*log(351
84372088832*_t**3*a**16*b**2*d**32/291434247 + sqrt(d*x))))
```

3.546 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=387

$$-\frac{4389\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{23/4}\,\sqrt[4]{b}\,\sqrt{d}}+\frac{4389\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{23/4}\,\sqrt[4]{b}\,\sqrt{d}}-\frac{4389\tan^{-}}{8192\sqrt{}}$$

Rubi [A] time = 0.50, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{1463\sqrt{dx}}{4096a^5d\left(a+bx^2\right)}+\frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2}+\frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3}+\frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4}-\frac{4389\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{23/4}\,\sqrt[4]{b}\,\sqrt{d}}+\frac{4389\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{23/4}\,\sqrt[4]{b}\,\sqrt{d}}-\frac{4389\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{8192\sqrt{2}\,a^{23/4}\,\sqrt[4]{b}\,\sqrt{d}}+\frac{4389\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{23/4}\,\sqrt[4]{b}\,\sqrt{d}}+\frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] Sqrt[d*x]/(10*a*d*(a + b*x^2)^5) + (19*Sqrt[d*x])/(160*a^2*d*(a + b*x^2)^4)
 + (19*Sqrt[d*x])/(128*a^3*d*(a + b*x^2)^3) + (209*Sqrt[d*x])/(1024*a^4*d*(
a + b*x^2)^2) + (1463*Sqrt[d*x])/(4096*a^5*d*(a + b*x^2)) - (4389*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(23/4)*b^
(1/4)*Sqrt[d]) + (4389*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) - (4389*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^
(23/4)*b^(1/4)*Sqrt[d]) + (4389*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{\left(19b^5\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{\left(57b^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{64a^2}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{\left(209b^3\right)\int \frac{1}{\sqrt{dx}\cdots}}{256\cdots}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+\cdots\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+\cdots\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+\cdots\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+\cdots\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+\cdots\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+\cdots\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+\cdots\right.}$$

Mathematica [A] time = 0.16, size = 295, normalized size = 0.76

$$\frac{\sqrt{x}\left(\frac{16384a^{19/4}\sqrt{x}}{(a+bx^2)^5}+\frac{19456a^{15/4}\sqrt{x}}{(a+bx^2)^4}+\frac{24320a^{11/4}\sqrt{x}}{(a+bx^2)^3}+\frac{33440a^{7/4}\sqrt{x}}{(a+bx^2)^2}+\frac{58520a^{3/4}\sqrt{x}}{a+bx^2}-\frac{21945\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{21945\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}-\frac{43890\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{43890\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}\right)}{163840a^{23/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (Sqrt[x]*((16384*a^(19/4)*Sqrt[x])/(a + b*x^2)^5 + (19456*a^(15/4)*Sqrt[x])
/(a + b*x^2)^4 + (24320*a^(11/4)*Sqrt[x])/(a + b*x^2)^3 + (33440*a^(7/4)*Sq
rt[x])/(a + b*x^2)^2 + (58520*a^(3/4)*Sqrt[x])/(a + b*x^2) - (43890*Sqrt[2]
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (43890*Sqrt[2]*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (21945*Sqrt[2]*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (21945*Sqr
t[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/
(163840*a^(23/4)*Sqrt[d*x])
```

IntegrateAlgebraic [A] time = 0.56, size = 238, normalized size = 0.61

$$-\frac{4389\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{4389\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{\sqrt{dx}\left(19015a^4d^9+50312a^3bd^9x^2+59470a^2b^2d^9x^4+33440ab^3d^9x^6+7315b^4d^9x^8\right)}{20480a^5\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (Sqrt[d*x]*(19015*a^4*d^9 + 50312*a^3*b*d^9*x^2 + 59470*a^2*b^2*d^9*x^4 + 3
3440*a*b^3*d^9*x^6 + 7315*b^4*d^9*x^8))/(20480*a^5*(a*d^2 + b*d^2*x^2)^5) -
 (4389*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(S
qrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) + (438
9*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*
d*x)])/(8192*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d])
```

fricas [A] time = 1.23, size = 475, normalized size = 1.23

$$\frac{87780\left(a^5b^5dx^{10}+5a^6b^4dx^8+10a^7b^3dx^6+10a^8b^2dx^4+5a^9bdx^2+a^{10}d\right)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{12}d^2\sqrt{-\frac{1}{a^{23}bd^2}}+dx}\,a^{17}bd\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^{17}bd\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{3}{4}}\right)+21945\left(a^5b^5dx^{10}+5a^6b^4dx^8+10a^7b^3dx^6+10a^8b^2dx^4+5a^9bdx^2+a^{10}d\right)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\log\left(a^6d\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-21945\left(a^5b^5dx^{10}+5a^6b^4dx^8+10a^7b^3dx^6+10a^8b^2dx^4+5a^9bdx^2+a^{10}d\right)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\log\left(-a^6d\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\left(7315b^4x^8+33440ab^3x^6+59470a^2b^2x^4+50312a^3bx^2+19015a^4\right)\sqrt{dx}}{81920\left(a^5b^5dx^{10}+5a^6b^4dx^8+10a^7b^3dx^6+10a^8b^2dx^4+5a^9bdx^2+a^{10}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/81920*(87780*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^
8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*arctan(sqrt(a
^12*d^2*sqrt(-1/(a^23*b*d^2)) + d*x)*a^17*b*d*(-1/(a^23*b*d^2))^(3/4) - sqr
```

```
t(d*x)*a^17*b*d*(-1/(a^23*b*d^2))^(3/4)) + 21945*(a^5*b^5*d*x^10 + 5*a^6*b^
4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1
/(a^23*b*d^2))^(1/4)*log(a^6*d*(-1/(a^23*b*d^2))^(1/4) + sqrt(d*x)) - 21945
*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 +
5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*log(-a^6*d*(-1/(a^23*b*d^2)
)^(1/4) + sqrt(d*x)) + 4*(7315*b^4*x^8 + 33440*a*b^3*x^6 + 59470*a^2*b^2*x^
4 + 50312*a^3*b*x^2 + 19015*a^4)*sqrt(d*x))/(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x
^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)
```

giac [A] time = 0.19, size = 346, normalized size = 0.89

$$\frac{4389\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^{6}bd}+\frac{4389\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^{6}bd}+\frac{4389\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{32768\,a^{6}bd}-\frac{4389\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{32768\,a^{6}bd}+\frac{7315\sqrt{dx}\,b^{4}d^{9}x^{8}+33440\sqrt{dx}\,ab^{3}d^{9}x^{6}+59470\sqrt{dx}\,a^{2}b^{2}d^{9}x^{4}+50312\sqrt{dx}\,a^{3}bd^{9}x^{2}+19015\sqrt{dx}\,a^{4}d^{9}}{20480\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4389/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d) + 4389/16384*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^6*b*d) + 4389/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d) - 4389/32768*sqr
t(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^6*b*d) + 1/20480*(7315*sqrt(d*x)*b^4*d^9*x^8 + 33440*sqrt(d*x)*
a*b^3*d^9*x^6 + 59470*sqrt(d*x)*a^2*b^2*d^9*x^4 + 50312*sqrt(d*x)*a^3*b*d^9
*x^2 + 19015*sqrt(d*x)*a^4*d^9)/((b*d^2*x^2 + a*d^2)^5*a^5)
```

maple [A] time = 0.03, size = 333, normalized size = 0.86

$$\frac{3803\sqrt{dx}\,d^{9}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a}+\frac{6289\left(dx\right)^{\frac{5}{2}}b\,d^{7}}{2560\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{2}}+\frac{5947\left(dx\right)^{\frac{9}{2}}b^{2}d^{5}}{2048\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{3}}+\frac{209\left(dx\right)^{\frac{13}{2}}b^{3}d^{3}}{128\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{4}}+\frac{1463\left(dx\right)^{\frac{17}{2}}b^{4}d}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{5}}+\frac{4389\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{16384a^{6}d}+\frac{4389\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{16384a^{6}d}+\frac{4389\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{32768a^{6}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x)
```

```
[Out] 3803/4096*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(1/2)+6289/2560*d^7/(b*d^2*x^2+a*
d^2)^5/a^2*b*(d*x)^(5/2)+5947/2048*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(9
/2)+209/128*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3*(d*x)^(13/2)+1463/4096*d/(b*d^2
*x^2+a*d^2)^5/a^5*b^4*(d*x)^(17/2)+4389/32768/d/a^6*(a/b*d^2)^(1/4)*2^(1/2)
*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4389/16384/d/a^6*(a/b*d^2)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+4389/16384/d/a^6*
(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.11, size = 382, normalized size = 0.99

$$\frac{\frac{8\left(7315\,(dx)^{\frac{17}{2}}b^4d^2+33440\,(dx)^{\frac{13}{2}}ab^3d^4+59470\,(dx)^{\frac{9}{2}}a^2b^2d^6+50312\,(dx)^{\frac{5}{2}}a^3bd^8+19015\,\sqrt{dx}\,a^4d^{10}\right)}{a^5b^5d^{10}x^{10}+5\,a^6b^4d^{10}x^8+10\,a^7b^3d^{10}x^6+10\,a^8b^2d^{10}x^4+5\,a^9bd^{10}x^2+a^{10}d^{10}}+\frac{21945\left(\frac{\sqrt{2}d^2\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^2\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}+\frac{2\sqrt{2}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}\right)}{a^5}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(7315*(d*x)^(17/2)*b^4*d^2 + 33440*(d*x)^(13/2)*a*b^3*d^4 + 594
70*(d*x)^(9/2)*a^2*b^2*d^6 + 50312*(d*x)^(5/2)*a^3*b*d^8 + 19015*sqrt(d*x)*
a^4*d^10)/(a^5*b^5*d^10*x^10 + 5*a^6*b^4*d^10*x^8 + 10*a^7*b^3*d^10*x^6 + 1
0*a^8*b^2*d^10*x^4 + 5*a^9*b*d^10*x^2 + a^10*d^10) + 21945*(sqrt(2)*d^2*log
(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt
(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d*arctan(1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sq
rt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a^5)/d
```

mupad [B] time = 4.29, size = 210, normalized size = 0.54

$$\frac{\frac{3803\,d^9\,\sqrt{d\,x}}{4096\,a}+\frac{5947\,b^2\,d^5\,{\left(d\,x\right)}^{9/2}}{2048\,a^3}+\frac{209\,b^3\,d^3\,{\left(d\,x\right)}^{13/2}}{128\,a^4}+\frac{6289\,b\,d^7\,{\left(d\,x\right)}^{5/2}}{2560\,a^2}+\frac{1463\,b^4\,d\,{\left(d\,x\right)}^{17/2}}{4096\,a^5}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{4389\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{23/4}\,b^{1/4}\,\sqrt{d}}+\frac{4389\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{23/4}\,b^{1/4}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((3803*d^9*(d*x)^(1/2))/(4096*a) + (5947*b^2*d^5*(d*x)^(9/2))/(2048*a^3) +
(209*b^3*d^3*(d*x)^(13/2))/(128*a^4) + (6289*b*d^7*(d*x)^(5/2))/(2560*a^2)
+ (1463*b^4*d*(d*x)^(17/2))/(4096*a^5))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b
*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) +
 (4389*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(23/4)*
b^(1/4)*d^(1/2)) + (4389*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))
/(8192*(-a)^(23/4)*b^(1/4)*d^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**6), x)
```

3.547 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=404

$$-\frac{13923\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}+ 1$$

Rubi [A] time = 0.53, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{13923\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{8192\sqrt{2}\,a^{25/4}d^{3/2}}-\frac{13923\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{25/4}d^{3/2}}+\frac{13923}{20480a^5d\sqrt{dx}\left(a+bx^2\right)}+\frac{1547}{5120a^4d\sqrt{dx}\left(a+bx^2\right)^2}+\frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3}+\frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4}-\frac{13923}{4096a^6d\sqrt{dx}}+\frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -13923/(4096*a^6*d*Sqrt[d*x]) + 1/(10*a*d*Sqrt[d*x]*(a + b*x^2)^5) + 21/(16
0*a^2*d*Sqrt[d*x]*(a + b*x^2)^4) + 119/(640*a^3*d*Sqrt[d*x]*(a + b*x^2)^3)
+ 1547/(5120*a^4*d*Sqrt[d*x]*(a + b*x^2)^2) + 13923/(20480*a^5*d*Sqrt[d*x]*
(a + b*x^2)) + (13923*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/2)) - (13923*b^(1/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/
2)) - (13923*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(25/4)*d^(3/2)) + (13923*b^(1/4)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(16384*Sqrt[2]*a^(25/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{\left(21b^5\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{\left(357b^4\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^4}}{320a^2}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.07

$$-\frac{2x\,{}_2F_1\left(-\frac{1}{4},6;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^6(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-1/4, 6, 3/4, -((b*x^2)/a)])/(a^6*(d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 1.32, size = 255, normalized size = 0.63

$$\frac{13923\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{25/4}d^{3/2}}+\frac{-40960a^5d^{10}-263515a^4bd^{10}x^2-590240a^3b^2d^{10}x^4-634270a^2b^3d^{10}x^6-334152ab^4d^{10}x^8-69615b^5d^{10}x^{10}}{20480a^6d\sqrt{dx}\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-40960*a^5*d^10 - 263515*a^4*b*d^10*x^2 - 590240*a^3*b^2*d^10*x^4 - 634270
*a^2*b^3*d^10*x^6 - 334152*a*b^4*d^10*x^8 - 69615*b^5*d^10*x^10)/(20480*a^6
*d*Sqrt[d*x]*(a*d^2 + b*d^2*x^2)^5) + (13923*b^(1/4)*ArcTan[((a^(1/4)*Sqrt[
d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/
(8192*Sqrt[2]*a^(25/4)*d^(3/2)) + (13923*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^(25/4
)*d^(3/2))
```

fricas [A] time = 1.13, size = 544, normalized size = 1.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(278460*(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7
+ 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)
*arctan(-1/2698972561467*(2698972561467*sqrt(d*x)*a^6*b*d*(-b/(a^25*d^6))^(
1/4) - sqrt(-7284452887551739093192089*a^13*b*d^4*sqrt(-b/(a^25*d^6)) + 728
4452887551739093192089*b^2*d*x)*a^6*d*(-b/(a^25*d^6))^(1/4))/b) - 69615*(a^
6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^
5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)*log(2698972561467*
a^19*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*sqrt(d*x)*b) + 69615*(a^6*b^
5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 +
```

```
5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)*log(-2698972561467*a^1
9*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*sqrt(d*x)*b) - 4*(69615*b^5*x^1
0 + 334152*a*b^4*x^8 + 634270*a^2*b^3*x^6 + 590240*a^3*b^2*x^4 + 263515*a^4
*b*x^2 + 40960*a^5)*sqrt(d*x))/(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a
^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)
```

giac [A] time = 0.20, size = 365, normalized size = 0.90

$$-\frac{\frac{327680}{\sqrt{dx}\,a^{6}}+\frac{139230\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{7}b^{2}d^{2}}+\frac{139230\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{7}b^{2}d^{2}}-\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{7}b^{2}d^{2}}+\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{7}b^{2}d^{2}}+\frac{8\left(28655\,\sqrt{dx}\,b^{5}d^{9}x^{9}+129352\,\sqrt{dx}\,ab^{4}d^{9}x^{7}+224670\,\sqrt{dx}\,a^{2}b^{3}d^{9}x^{5}+180640\,\sqrt{dx}\,a^{3}b^{2}d^{9}x^{3}+58715\,\sqrt{dx}\,a^{4}bd^{9}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{6}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/163840*(327680/(sqrt(d*x)*a^6) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*
b^2*d^2) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b^2*d^2) - 69615*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^7*b^2*d^2) + 69615*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*b^2*d^2) + 8*(28655*sqrt(d*x)*b
^5*d^9*x^9 + 129352*sqrt(d*x)*a*b^4*d^9*x^7 + 224670*sqrt(d*x)*a^2*b^3*d^9*
x^5 + 180640*sqrt(d*x)*a^3*b^2*d^9*x^3 + 58715*sqrt(d*x)*a^4*b*d^9*x)/((b*d
^2*x^2 + a*d^2)^5*a^6))/d
```

maple [A] time = 0.03, size = 349, normalized size = 0.86

$$-\frac{11743\,(dx)^{\frac{3}{2}}\,b\,d^{7}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{2}}-\frac{1129\,(dx)^{\frac{7}{2}}\,b^{2}d^{5}}{128\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{3}}-\frac{22467\,(dx)^{\frac{11}{2}}\,b^{3}d^{3}}{2048\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{4}}-\frac{16169\,(dx)^{\frac{15}{2}}\,b^{4}d}{2560\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{5}}-\frac{5731\,(dx)^{\frac{19}{2}}\,b^{5}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{6}d}-\frac{13923\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{16384\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{6}d}-\frac{13923\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{16384\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{6}d}-\frac{13923\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{32768\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{6}d}-\frac{2}{\sqrt{dx}\,a^{6}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -11743/4096*d^7*b/a^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(3/2)-1129/128*d^5*b^2/a^3/
(b*d^2*x^2+a*d^2)^5*(d*x)^(7/2)-22467/2048*d^3*b^3/a^4/(b*d^2*x^2+a*d^2)^5*
(d*x)^(11/2)-16169/2560*d*b^4/a^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(15/2)-5731/409
6/d*b^5/a^6/(b*d^2*x^2+a*d^2)^5*(d*x)^(19/2)-13923/32768/d/a^6/(a/b*d^2)^(1
/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-13923/16384/d/a^6/
(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-13923
/16384/d/a^6/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(
1/2)-1)-2/a^6/d/(d*x)^(1/2)
```

maxima [A] time = 3.23, size = 388, normalized size = 0.96

$$-\frac{\frac{8\left(69615\,b^5 d^{10} x^{10}+334152\,a b^4 d^{10} x^8+634270\,a^2 b^3 d^{10} x^6+590240\,a^3 b^2 d^{10} x^4+263515\,a^4 b d^{10} x^2+40960\,a^5 d^{10}\right)}{(dx)^{\frac{21}{2}} a^6 b^5+5\,(dx)^{\frac{17}{2}} a^7 b^4 d^2+10\,(dx)^{\frac{13}{2}} a^8 b^3 d^4+10\,(dx)^{\frac{9}{2}} a^9 b^2 d^6+5\,(dx)^{\frac{5}{2}} a^{10} b d^8+\sqrt{dx}\, a^{11} d^{10}}+\frac{69615\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{a^6}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/163840*(8*(69615*b^5*d^10*x^10 + 334152*a*b^4*d^10*x^8 + 634270*a^2*b^3*
d^10*x^6 + 590240*a^3*b^2*d^10*x^4 + 263515*a^4*b*d^10*x^2 + 40960*a^5*d^10
)/((d*x)^(21/2)*a^6*b^5 + 5*(d*x)^(17/2)*a^7*b^4*d^2 + 10*(d*x)^(13/2)*a^8*
b^3*d^4 + 10*(d*x)^(9/2)*a^9*b^2*d^6 + 5*(d*x)^(5/2)*a^10*b*d^8 + sqrt(d*x)
*a^11*d^10) + 69615*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/
4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)
*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/
4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)
*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^6)
/d
```

mupad [B] time = 0.21, size = 226, normalized size = 0.56

$$\frac{13923\,{(-b)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{25/4}\,d^{3/2}}-\frac{13923\,{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{25/4}\,d^{3/2}}-\frac{\frac{2\,d^9}{a}+\frac{52703\,b\,d^9\,x^2}{4096\,a^2}+\frac{3689\,b^2\,d^9\,x^4}{128\,a^3}+\frac{63427\,b^3\,d^9\,x^6}{2048\,a^4}+\frac{41769\,b^4\,d^9\,x^8}{2560\,a^5}+\frac{13923\,b^5\,d^9\,x^{10}}{4096\,a^6}}{b^5\,{(d\,x)}^{21/2}+a^5\,d^{10}\,\sqrt{d\,x}+10\,a^3\,b^2\,d^6\,{(d\,x)}^{9/2}+10\,a^2\,b^3\,d^4\,{(d\,x)}^{13/2}+5\,a^4\,b\,d^8\,{(d\,x)}^{5/2}+5\,a\,b^4\,d^2\,{(d\,x)}^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] (13923*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*
a^(25/4)*d^(3/2)) - (13923*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4
)*d^(1/2))))/(8192*a^(25/4)*d^(3/2)) - ((2*d^9)/a + (52703*b*d^9*x^2)/(4096
*a^2) + (3689*b^2*d^9*x^4)/(128*a^3) + (63427*b^3*d^9*x^6)/(2048*a^4) + (41
769*b^4*d^9*x^8)/(2560*a^5) + (13923*b^5*d^9*x^10)/(4096*a^6))/(b^5*(d*x)^(
21/2) + a^5*d^10*(d*x)^(1/2) + 10*a^3*b^2*d^6*(d*x)^(9/2) + 10*a^2*b^3*d^4*
(d*x)^(13/2) + 5*a^4*b*d^8*(d*x)^(5/2) + 5*a*b^4*d^2*(d*x)^(17/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} \left(a + bx^2\right)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**6), x)
```

3.548 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=404

$$\frac{33649b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{27/4}d^{5/2}}+\ldots$$

Rubi [A] time = 0.51, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{33649b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{27/4}d^{5/2}}+\frac{33649b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{27/4}d^{5/2}}+\frac{4807}{4096a^5d(dx)^{3/2}\left(a+bx^2\right)}+\frac{437}{1024a^4d(dx)^{3/2}\left(a+bx^2\right)^2}+\frac{437}{1920a^3d(dx)^{3/2}\left(a+bx^2\right)^3}+\frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4}-\frac{33649}{12288a^6d(dx)^{3/2}}+\frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -33649/(12288*a^6*d*(d*x)^(3/2)) + 1/(10*a*d*(d*x)^(3/2)*(a + b*x^2)^5) + 2
3/(160*a^2*d*(d*x)^(3/2)*(a + b*x^2)^4) + 437/(1920*a^3*d*(d*x)^(3/2)*(a +
b*x^2)^3) + 437/(1024*a^4*d*(d*x)^(3/2)*(a + b*x^2)^2) + 4807/(4096*a^5*d*(
d*x)^(3/2)*(a + b*x^2)) + (33649*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(27/4)*d^(5/2)) - (33649*b^(3/4)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(
27/4)*d^(5/2)) + (33649*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2)) - (3364
9*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{\left(23b^5\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{\left(437b^4\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^{2}\right.}}{320a^2}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\left(a+\right.}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\left(a+\right.}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\left(a+\right.}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} +$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} +$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} +$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} +$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} +$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} +$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.08

$$-\frac{2x\,{}_2F_1\left(-\frac{3}{4},6;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^6(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-3/4, 6, 1/4, -((b*x^2)/a)])/(3*a^6*(d*x)^(5/2))
```

IntegrateAlgebraic [A] time = 1.33, size = 255, normalized size = 0.63

$$\frac{33649b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}a^{27/4}d^{5/2}}+\frac{-40960a^5d^{10}-437345a^4bd^{10}x^2-1157176a^3b^2d^{10}x^4-1367810a^2b^3d^{10}x^6-769120ab^4d^{10}x^8-168245b^5d^{10}x^{10}}{61440a^6d(dx)^{3/2}\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-40960*a^5*d^10 - 437345*a^4*b*d^10*x^2 - 1157176*a^3*b^2*d^10*x^4 - 13678
10*a^2*b^3*d^10*x^6 - 769120*a*b^4*d^10*x^8 - 168245*b^5*d^10*x^10)/(61440*
a^6*d*(d*x)^(3/2)*(a*d^2 + b*d^2*x^2)^5) + (33649*b^(3/4)*ArcTan[((a^(1/4)*
Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*
x]])/(8192*Sqrt[2]*a^(27/4)*d^(5/2)) - (33649*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(8192*Sqrt[2]*a^
(27/4)*d^(5/2))
```

fricas [A] time = 1.46, size = 568, normalized size = 1.41

$$-\frac{2018940\left(a^6b^5d^3x^{12}+5a^7b^4d^3x^{10}+10a^8b^3d^3x^8+10a^9b^2d^3x^6+5a^{10}bd^3x^4+a^{11}d^3x^2\right)\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{20}bd^7\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{14}d^6\sqrt{-\frac{b^3}{a^{27}d^{10}}}+b^2dx}\,a^{20}d^7\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+504735\left(a^6b^5d^3x^{12}+5a^7b^4d^3x^{10}+10a^8b^3d^3x^8+10a^9b^2d^3x^6+5a^{10}bd^3x^4+a^{11}d^3x^2\right)\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}\log\left(33649a^7d^3\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}+33649\sqrt{dx}b\right)-504735\left(a^6b^5d^3x^{12}+5a^7b^4d^3x^{10}+10a^8b^3d^3x^8+10a^9b^2d^3x^6+5a^{10}bd^3x^4+a^{11}d^3x^2\right)\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}\log\left(-33649a^7d^3\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}+33649\sqrt{dx}b\right)+4\left(168245b^5x^{10}+769120ab^4x^8+1367810a^2b^3x^6+1157176a^3b^2x^4+437345a^4bx^2+40960a^5\right)\sqrt{dx}}{245760\left(a^6b^5d^3x^{12}+5a^7b^4d^3x^{10}+10a^8b^3d^3x^8+10a^9b^2d^3x^6+5a^{10}bd^3x^4+a^{11}d^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(2018940*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*
x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^1
0))^(1/4)*arctan(-(sqrt(d*x)*a^20*b*d^7*(-b^3/(a^27*d^10))^(3/4) - sqrt(a^1
4*d^6*sqrt(-b^3/(a^27*d^10)) + b^2*d*x)*a^20*d^7*(-b^3/(a^27*d^10))^(3/4))/
b^3) + 504735*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*x^8 +
 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^10))^(
1/4)*log(33649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b) - 5047
35*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*x^8 + 10*a^9*b^2
*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^10))^(1/4)*log(-3
```

```
3649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b) + 4*(168245*b^5*
x^10 + 769120*a*b^4*x^8 + 1367810*a^2*b^3*x^6 + 1157176*a^3*b^2*x^4 + 43734
5*a^4*b*x^2 + 40960*a^5)*sqrt(d*x))/(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10
+ 10*a^8*b^3*d^3*x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2
)
```

giac [A] time = 0.20, size = 356, normalized size = 0.88

$$-\frac{33649\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^7d^3}+\frac{33649\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^7d^3}-\frac{2}{3\sqrt{dx}\,a^6d^2x}-\frac{127285\sqrt{dx}\,b^5d^8x^8+564320\sqrt{dx}\,ab^4d^8x^6+958210\sqrt{dx}\,a^2b^3d^8x^4+747576\sqrt{dx}\,a^3b^2d^8x^2+232545\sqrt{dx}\,a^4bd^8}{61440\left(bd^2x^2+ad^2\right)^5a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -33649/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/16384*sqrt(2)*(a*
b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x
+ sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*d^3) + 33649/3276
8*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^7*d^3) - 2/3/(sqrt(d*x)*a^6*d^2*x) - 1/61440*(127285*sqrt(
d*x)*b^5*d^8*x^8 + 564320*sqrt(d*x)*a*b^4*d^8*x^6 + 958210*sqrt(d*x)*a^2*b^
3*d^8*x^4 + 747576*sqrt(d*x)*a^3*b^2*d^8*x^2 + 232545*sqrt(d*x)*a^4*b*d^8)/
((b*d^2*x^2 + a*d^2)^5*a^6*d)
```

maple [A] time = 0.03, size = 352, normalized size = 0.87

$$-\frac{15503\sqrt{dx}\;b\,d^7}{4096\left(b\,d^2x^2+d^2a\right)^5a^2}-\frac{31149\,(dx)^{\frac{5}{2}}\,b^2d^5}{2560\left(b\,d^2x^2+d^2a\right)^5a^3}-\frac{95821\,(dx)^{\frac{9}{2}}\,b^3d^3}{6144\left(b\,d^2x^2+d^2a\right)^5a^4}-\frac{3527\,(dx)^{\frac{13}{2}}\,b^4d}{384\left(b\,d^2x^2+d^2a\right)^5a^5}-\frac{25457\,(dx)^{\frac{17}{2}}\,b^5}{12288\left(b\,d^2x^2+d^2a\right)^5a^6d}-\frac{2}{3\,(dx)^{\frac{3}{2}}\,a^6d}-\frac{33649\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{16384a^7d^3}-\frac{33649\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{16384a^7d^3}-\frac{33649\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\;b\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{32768a^7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -15503/4096*d^7/a^2*b/(b*d^2*x^2+a*d^2)^5*(d*x)^(1/2)-31149/2560*d^5/a^3*b^
2/(b*d^2*x^2+a*d^2)^5*(d*x)^(5/2)-95821/6144*d^3/a^4*b^3/(b*d^2*x^2+a*d^2)^
5*(d*x)^(9/2)-3527/384*d/a^5*b^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(13/2)-25457/122
88/d/a^6*b^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(17/2)-33649/32768/d^3/a^7*b*(a/b*d^
2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2
))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-33649/16384/d
^3/a^7*b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)
+1)-33649/16384/d^3/a^7*b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^
(1/4)*(d*x)^(1/2)-1)-2/3/a^6/d/(d*x)^(3/2)
```

maxima [A] time = 3.23, size = 395, normalized size = 0.98

$$-\frac{\frac{8 \left(168245 \, b^{5} d^{10} x^{10} + 769120 \, a b^{4} d^{10} x^{8} + 1367810 \, a^{2} b^{3} d^{10} x^{6} + 1157176 \, a^{3} b^{2} d^{10} x^{4} + 437345 \, a^{4} b d^{10} x^{2} + 40960 \, a^{5} d^{10}\right)}{\left(dx\right)^{\frac{23}{2}} a^{6} b^{5} + 5 \left(dx\right)^{\frac{19}{2}} a^{7} b^{4} d^{2} + 10 \left(dx\right)^{\frac{15}{2}} a^{8} b^{3} d^{4} + 10 \left(dx\right)^{\frac{11}{2}} a^{9} b^{2} d^{6} + 5 \left(dx\right)^{\frac{7}{2}} a^{10} b d^{8} + \left(dx\right)^{\frac{3}{2}} a^{11} d^{10}} + \frac{504735 \left(\frac{\sqrt{2} b^{\frac{3}{4}} \log\left(\sqrt{b} dx + \sqrt{2} \left(ad^{2}\right)^{\frac{1}{4}} \sqrt{dx} b^{\frac{1}{4}} + \sqrt{a} d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}} - \frac{\sqrt{2} b^{\frac{3}{4}} \log\left(\sqrt{b} dx - \sqrt{2} \left(ad^{2}\right)^{\frac{1}{4}} \sqrt{dx} b^{\frac{1}{4}} + \sqrt{a} d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}} + \frac{2 \sqrt{2} b \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(ad^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}} + 2 \sqrt{dx} \sqrt{b}\right)}{2 \sqrt{\sqrt{a} \sqrt{b} d}}\right)}{\sqrt{\sqrt{a} \sqrt{b} d} \sqrt{a} d} + \frac{2 \sqrt{2} b \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(ad^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}} - 2 \sqrt{dx} \sqrt{b}\right)}{2 \sqrt{\sqrt{a} \sqrt{b} d}}\right)}{\sqrt{\sqrt{a} \sqrt{b} d} \sqrt{a} d}\right)}{a^{6}}}{491520 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/491520*(8*(168245*b^5*d^10*x^10 + 769120*a*b^4*d^10*x^8 + 1367810*a^2*b^
3*d^10*x^6 + 1157176*a^3*b^2*d^10*x^4 + 437345*a^4*b*d^10*x^2 + 40960*a^5*d
^10)/((d*x)^(23/2)*a^6*b^5 + 5*(d*x)^(19/2)*a^7*b^4*d^2 + 10*(d*x)^(15/2)*a
^8*b^3*d^4 + 10*(d*x)^(11/2)*a^9*b^2*d^6 + 5*(d*x)^(7/2)*a^10*b*d^8 + (d*x)
^(3/2)*a^11*d^10) + 504735*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^
2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log
(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)
^(3/4) + 2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a
)*d) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*s
qrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)
*d))/a^6)/d
```

mupad [B] time = 4.46, size = 226, normalized size = 0.56

$$\frac{33649 \, {\left(-b\right)}^{3/4} \, \mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4} \sqrt{d\,x}}{a^{1/4} \sqrt{d}}\right)}{8192 \, a^{27/4} \, d^{5/2}} - \frac{\frac{2\,d^9}{3\,a} + \frac{87469\,b\,d^9\,x^2}{12288\,a^2} + \frac{144647\,b^2\,d^9\,x^4}{7680\,a^3} + \frac{136781\,b^3\,d^9\,x^6}{6144\,a^4} + \frac{4807\,b^4\,d^9\,x^8}{384\,a^5} + \frac{33649\,b^5\,d^9\,x^{10}}{12288\,a^6}}{b^5\,{\left(d\,x\right)}^{23/2} + a^5\,d^{10}\,{\left(d\,x\right)}^{3/2} + 10\,a^3\,b^2\,d^6\,{\left(d\,x\right)}^{11/2} + 10\,a^2\,b^3\,d^4\,{\left(d\,x\right)}^{15/2} + 5\,a^4\,b\,d^8\,{\left(d\,x\right)}^{7/2} + 5\,a\,b^4\,d^2\,{\left(d\,x\right)}^{19/2}} + \frac{33649 \, {\left(-b\right)}^{3/4} \, \mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4} \sqrt{d\,x}}{a^{1/4} \sqrt{d}}\right)}{8192 \, a^{27/4} \, d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] (33649*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a
^(27/4)*d^(5/2)) - ((2*d^9)/(3*a) + (87469*b*d^9*x^2)/(12288*a^2) + (144647
*b^2*d^9*x^4)/(7680*a^3) + (136781*b^3*d^9*x^6)/(6144*a^4) + (4807*b^4*d^9*
x^8)/(384*a^5) + (33649*b^5*d^9*x^10)/(12288*a^6))/(b^5*(d*x)^(23/2) + a^5*
d^10*(d*x)^(3/2) + 10*a^3*b^2*d^6*(d*x)^(11/2) + 10*a^2*b^3*d^4*(d*x)^(15/2
) + 5*a^4*b*d^8*(d*x)^(7/2) + 5*a*b^4*d^2*(d*x)^(19/2)) + (33649*(-b)^(3/4)
*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a^(27/4)*d^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.549 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=422

$$\frac{69615b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{29/4}d^{7/2}}-\frac{69}{}$$

Rubi [A] time = 0.55, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{69615b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{29/4}d^{7/2}}+\frac{69615b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{29/4}d^{7/2}}+\frac{69615b}{4096a^7d^3\sqrt{dx}}+\frac{7735}{4096a^5d(dx)^{5/2}\left(a+bx^2\right)}+\frac{595}{1024a^4d(dx)^{5/2}\left(a+bx^2\right)^2}+\frac{35}{128a^3d(dx)^{5/2}\left(a+bx^2\right)^3}+\frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4}-\frac{13923}{4096a^6d(dx)^{5/2}}+\frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -13923/(4096*a^6*d*(d*x)^(5/2)) + (69615*b)/(4096*a^7*d^3*Sqrt[d*x]) + 1/(1
0*a*d*(d*x)^(5/2)*(a + b*x^2)^5) + 5/(32*a^2*d*(d*x)^(5/2)*(a + b*x^2)^4) +
 35/(128*a^3*d*(d*x)^(5/2)*(a + b*x^2)^3) + 595/(1024*a^4*d*(d*x)^(5/2)*(a
+ b*x^2)^2) + 7735/(4096*a^5*d*(d*x)^(5/2)*(a + b*x^2)) - (69615*b^(5/4)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(2
9/4)*d^(7/2)) + (69615*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) + (69615*b^(5/4)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*S
qrt[2]*a^(29/4)*d^(7/2)) - (69615*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqr
t[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(29/4)*d^(7/2
))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{\left(5b^5\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^5}\,dx}{4a}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{\left(105b^4\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x\right.}}{64a^2}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}\left(a+bx\right.}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}\left(a+bx\right.}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}\left(a+bx\right.}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{}{12}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(dx)^5}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(dx)^5}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(dx)^5}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(dx)^5}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(dx)^5}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.09

$$-\frac{2\sqrt{dx}\,{}_2F_1\left(-\frac{5}{4},6;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^6d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-2*Sqrt[d*x]*Hypergeometric2F1[-5/4, 6, -1/4, -((b*x^2)/a)])/(5*a^6*d^4*x^
3)
```

IntegrateAlgebraic [A] time = 1.37, size = 269, normalized size = 0.64

$$-\frac{69615b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{8192\sqrt{2}\,a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{8192\sqrt{2}\,a^{29/4}d^{7/2}}+\frac{-8192a^6d^{12}+204800a^5bd^{12}x^2+1317575a^4b^2d^{12}x^4+2951200a^3b^3d^{12}x^6+3171350a^2b^4d^{12}x^8+1670760ab^5d^{12}x^{10}+348075b^6d^{12}x^{12}}{20480a^7d^3(dx)^{5/2}\left(ad^2+bd^2x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-8192*a^6*d^12 + 204800*a^5*b*d^12*x^2 + 1317575*a^4*b^2*d^12*x^4 + 295120
0*a^3*b^3*d^12*x^6 + 3171350*a^2*b^4*d^12*x^8 + 1670760*a*b^5*d^12*x^10 + 3
48075*b^6*d^12*x^12)/(20480*a^7*d^3*(d*x)^(5/2)*(a*d^2 + b*d^2*x^2)^5) - (6
9615*b^(5/4)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]
*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) - (69615
*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + S
qrt[b]*d*x)])/(8192*Sqrt[2]*a^(29/4)*d^(7/2))
```

fricas [A] time = 2.52, size = 591, normalized size = 1.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1392300*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x
^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^4*x^3)*(-b^5/(a^29*d^1
4))^(1/4)*arctan(-1/337371570183375*(337371570183375*sqrt(d*x)*a^7*b^4*d^3*
(-b^5/(a^29*d^14))^(1/4) - sqrt(-113819576367995923331126390625*a^15*b^5*d^
8*sqrt(-b^5/(a^29*d^14)) + 113819576367995923331126390625*b^8*d*x)*a^7*d^3*
(-b^5/(a^29*d^14))^(1/4))/b^5) - 348075*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x
^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^
4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log(337371570183375*a^22*d^11*(-b^5/(a^29*d
^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) + 348075*(a^7*b^5*d^4*x^13 + 5
```

```
*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4
*x^5 + a^12*d^4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log(-337371570183375*a^22*d^1
1*(-b^5/(a^29*d^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) - 4*(348075*b^6
*x^12 + 1670760*a*b^5*x^10 + 3171350*a^2*b^4*x^8 + 2951200*a^3*b^3*x^6 + 13
17575*a^4*b^2*x^4 + 204800*a^5*b*x^2 - 8192*a^6)*sqrt(d*x))/(a^7*b^5*d^4*x^
13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11
*b*d^4*x^5 + a^12*d^4*x^3)
```

giac [A] time = 0.20, size = 362, normalized size = 0.86

$$\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^{8}bd^{5}}+\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^{8}bd^{5}}-\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{32768\,a^{8}bd^{5}}+\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{32768\,a^{8}bd^{5}}+\frac{348075\,b^{6}d^{12}x^{12}+1670760\,ab^{5}d^{12}x^{10}+3171350\,a^{2}b^{4}d^{12}x^{8}+2951200\,a^{3}b^{3}d^{12}x^{6}+1317575\,a^{4}b^{2}d^{12}x^{4}+204800\,a^{5}bd^{12}x^{2}-8192\,a^{6}d^{12}}{20480\left(\sqrt{dx}\,bd^{2}x^{2}+\sqrt{dx}\,ad^{2}\right)^{5}a^{7}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 69615/16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^8*b*d^5) + 69615/16384*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^8*b*d^5) - 69615/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d
*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^8*b*d^5) + 69615
/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(a^8*b*d^5) + 1/20480*(348075*b^6*d^12*x^12 + 1670760*a*
b^5*d^12*x^10 + 3171350*a^2*b^4*d^12*x^8 + 2951200*a^3*b^3*d^12*x^6 + 13175
75*a^4*b^2*d^12*x^4 + 204800*a^5*b*d^12*x^2 - 8192*a^6*d^12)/((sqrt(d*x)*b*
d^2*x^2 + sqrt(d*x)*a*d^2)^5*a^7*d^3)
```

maple [A] time = 0.04, size = 368, normalized size = 0.87

$$\frac{34139\left(dx\right)^{\frac{3}{2}}b^{2}d^{5}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{3}}+\frac{3597\left(dx\right)^{\frac{7}{2}}b^{3}d^{3}}{128\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{4}}+\frac{75471\left(dx\right)^{\frac{11}{2}}b^{4}d}{2048\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{5}}+\frac{56269\left(dx\right)^{\frac{15}{2}}b^{5}}{2560\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{6}d}+\frac{20463\left(dx\right)^{\frac{19}{2}}b^{6}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{7}d^{3}}-\frac{2}{5\left(dx\right)^{\frac{5}{2}}a^{6}d}+\frac{69615\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{16384\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{7}d^{3}}+\frac{69615\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{16384\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{7}d^{3}}+\frac{69615\sqrt{2}\,b\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{32768\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{7}d^{3}}+\frac{12b}{\sqrt{dx}\,a^{7}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 34139/4096*d^5*b^2/a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(3/2)+3597/128*d^3*b^3/a^4
/(b*d^2*x^2+a*d^2)^5*(d*x)^(7/2)+75471/2048*d*b^4/a^5/(b*d^2*x^2+a*d^2)^5*(
d*x)^(11/2)+56269/2560/d*b^5/a^6/(b*d^2*x^2+a*d^2)^5*(d*x)^(15/2)+20463/409
6/d^3*b^6/a^7/(b*d^2*x^2+a*d^2)^5*(d*x)^(19/2)+69615/32768/d^3*b/a^7/(a/b*d
^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+69615/16384/
d^3*b/a^7/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2
)+1)+69615/16384/d^3*b/a^7/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)
^(1/4)*(d*x)^(1/2)-1)-2/5/a^6/d/(d*x)^(5/2)+12*b/a^7/d^3/(d*x)^(1/2)
```

maxima [A] time = 3.32, size = 410, normalized size = 0.97

$$\frac{\frac{8\left(348075\,b^{6}d^{12}x^{12}+1670760\,ab^{5}d^{12}x^{10}+3171350\,a^{2}b^{4}d^{12}x^{8}+2951200\,a^{3}b^{3}d^{12}x^{6}+1317575\,a^{4}b^{2}d^{12}x^{4}+204800\,a^{5}bd^{12}x^{2}-8192\,a^{6}d^{12}\right)}{(dx)^{\frac{25}{2}}a^{7}b^{5}d^{2}+5\,(dx)^{\frac{21}{2}}a^{8}b^{4}d^{4}+10\,(dx)^{\frac{17}{2}}a^{9}b^{3}d^{6}+10\,(dx)^{\frac{13}{2}}a^{10}b^{2}d^{8}+5\,(dx)^{\frac{9}{2}}a^{11}bd^{10}+(dx)^{\frac{5}{2}}a^{12}d^{12}}+\frac{348075\,b^{2}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^{7}d^{2}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(348075*b^6*d^12*x^12 + 1670760*a*b^5*d^12*x^10 + 3171350*a^2*b
^4*d^12*x^8 + 2951200*a^3*b^3*d^12*x^6 + 1317575*a^4*b^2*d^12*x^4 + 204800*
a^5*b*d^12*x^2 - 8192*a^6*d^12)/((d*x)^(25/2)*a^7*b^5*d^2 + 5*(d*x)^(21/2)*
a^8*b^4*d^4 + 10*(d*x)^(17/2)*a^9*b^3*d^6 + 10*(d*x)^(13/2)*a^10*b^2*d^8 +
5*(d*x)^(9/2)*a^11*b*d^10 + (d*x)^(5/2)*a^12*d^12) + 348075*b^2*(2*sqrt(2)*
arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sq
rt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sq
rt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d
*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^
(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^7*d^2))/d
```

mupad [B] time = 0.27, size = 239, normalized size = 0.57

$$\frac{\frac{10\,b\,d^{9}\,x^{2}}{a^{2}}-\frac{2\,d^{9}}{5\,a}+\frac{263515\,b^{2}\,d^{9}\,x^{4}}{4096\,a^{3}}+\frac{18445\,b^{3}\,d^{9}\,x^{6}}{128\,a^{4}}+\frac{317135\,b^{4}\,d^{9}\,x^{8}}{2048\,a^{5}}+\frac{41769\,b^{5}\,d^{9}\,x^{10}}{512\,a^{6}}+\frac{69615\,b^{6}\,d^{9}\,x^{12}}{4096\,a^{7}}}{b^{5}\,(d\,x)^{25/2}+a^{5}\,d^{10}\,(d\,x)^{5/2}+10\,a^{3}\,b^{2}\,d^{6}\,(d\,x)^{13/2}+10\,a^{2}\,b^{3}\,d^{4}\,(d\,x)^{17/2}+5\,a^{4}\,b\,d^{8}\,(d\,x)^{9/2}+5\,a\,b^{4}\,d^{2}\,(d\,x)^{21/2}}-\frac{69615\,(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{29/4}\,d^{7/2}}+\frac{69615\,(-b)^{5/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{29/4}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((10*b*d^9*x^2)/a^2 - (2*d^9)/(5*a) + (263515*b^2*d^9*x^4)/(4096*a^3) + (18
445*b^3*d^9*x^6)/(128*a^4) + (317135*b^4*d^9*x^8)/(2048*a^5) + (41769*b^5*d
^9*x^10)/(512*a^6) + (69615*b^6*d^9*x^12)/(4096*a^7))/(b^5*(d*x)^(25/2) + a
^5*d^10*(d*x)^(5/2) + 10*a^3*b^2*d^6*(d*x)^(13/2) + 10*a^2*b^3*d^4*(d*x)^(1
7/2) + 5*a^4*b*d^8*(d*x)^(9/2) + 5*a*b^4*d^2*(d*x)^(21/2)) - (69615*(-b)^(5
/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a^(29/4)*d^(7/2
)) + (69615*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(
8192*a^(29/4)*d^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.550 $\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (2*b*
(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^{5/2}+\frac{b^2(dx)^{9/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)} + \frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(11a+7bx^2\right)}{77\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(11*a + 7*b*x^2))/(77*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 59.99, size = 69, normalized size = 0.74

$$\frac{2\left(ad^2+bd^2x^2\right)\left(11ad^2(dx)^{7/2}+7b(dx)^{11/2}\right)}{77d^5\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(11*a*d^2*(d*x)^(7/2) + 7*b*(d*x)^(11/2)))/(77*d^5*S
qrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.96, size = 26, normalized size = 0.28

$$\frac{2}{77}\left(7\,bd^2x^5+11\,ad^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b*d^2*x^5 + 11*a*d^2*x^3)*sqrt(d*x)
```

giac [A] time = 0.18, size = 45, normalized size = 0.48

$$\frac{2}{11}\sqrt{dx}\,bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}\,ad^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/11*sqrt(d*x)*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)*a*d^2*x^3*sgn(b*x^2
 + a)
```

maple [A] time = 0.00, size = 39, normalized size = 0.42

$$\frac{2\left(7b\,x^2+11a\right)(dx)^{\frac{5}{2}}\sqrt{\left(b\,x^2+a\right)^2}\;x}{77\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 2/77*x*(7*b*x^2+11*a)*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.32, size = 25, normalized size = 0.27

$$\frac{2\left(7\,(dx)^{\frac{11}{2}}\,b+11\,(dx)^{\frac{7}{2}}\,ad^2\right)}{77\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/77*(7*(d*x)^(11/2)*b + 11*(d*x)^(7/2)*a*d^2)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{5/2}\sqrt{\left(b\,x^2+a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(5/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.551 $\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*b*
(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^{3/2}+\frac{b^2(dx)^{7/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)} + \frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(9a+5bx^2\right)}{45\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(9*a + 5*b*x^2))/(45*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 31.60, size = 69, normalized size = 0.74

$$\frac{2\left(ad^2+bd^2x^2\right)\left(9ad^2(dx)^{5/2}+5b(dx)^{9/2}\right)}{45d^5\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(9*a*d^2*(d*x)^(5/2) + 5*b*(d*x)^(9/2)))/(45*d^5*Sqr
t[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.56, size = 22, normalized size = 0.24

$$\frac{2}{45}\left(5\,bdx^4+9\,adx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b*d*x^4 + 9*a*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.16, size = 42, normalized size = 0.45

$$\frac{2}{45}\left(5\sqrt{dx}\,bx^4\mathrm{sgn}\left(bx^2+a\right)+9\sqrt{dx}\,ax^2\mathrm{sgn}\left(bx^2+a\right)\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(5*sqrt(d*x)*b*x^4*sgn(b*x^2 + a) + 9*sqrt(d*x)*a*x^2*sgn(b*x^2 + a))*
d
```

maple [A] time = 0.00, size = 39, normalized size = 0.42

$$\frac{2\left(5b\,x^2+9a\right)(dx)^{\frac{3}{2}}\sqrt{\left(b\,x^2+a\right)^2}\;x}{45\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 2/45*x*(5*b*x^2+9*a)*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.25, size = 25, normalized size = 0.27

$$\frac{2\left(5\,(dx)^{\frac{9}{2}}\,b+9\,(dx)^{\frac{5}{2}}\,ad^2\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*(d*x)^(9/2)*b + 9*(d*x)^(5/2)*a*d^2)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\sqrt{\left(b\,x^2+a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(3/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*((b*x**2+a)**2)**(1/2),x)
[Out] Timed out
```

3.552 $\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3(a+bx^2)}+\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)}$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3(a+bx^2)}+\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (2*b*
(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\,(ab+b^2x^2)\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab\sqrt{dx}+\frac{b^2(dx)^{5/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)} + \frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\,(a+bx^2)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.47

$$\frac{2\sqrt{dx}\sqrt{(a+bx^2)^2}\,(7ax+3bx^3)}{21\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(7*a*x + 3*b*x^3))/(21*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 22.91, size = 69, normalized size = 0.74

$$\frac{2\,(ad^2+bd^2x^2)\,(7ad^2(dx)^{3/2}+3b(dx)^{7/2})}{21d^5\sqrt{\frac{(ad^2+bd^2x^2)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (2*(a*d^2 + b*d^2*x^2)*(7*a*d^2*(d*x)^(3/2) + 3*b*(d*x)^(7/2)))/(21*d^5*Sqr
t[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.12, size = 18, normalized size = 0.19

$$\frac{2}{21}\left(3\,bx^3+7\,ax\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*b*x^3 + 7*a*x)*sqrt(d*x)
```

giac [A] time = 0.17, size = 37, normalized size = 0.40

$$\frac{2}{7}\,\sqrt{dx}\,bx^3\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\,\sqrt{dx}\,ax\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 2/7*sqrt(d*x)*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*x)*a*x*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 39, normalized size = 0.42

$$\frac{2\left(3b\,x^2+7a\right)\sqrt{dx}\,\sqrt{\left(b\,x^2+a\right)^2}\,x}{21\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x)
[Out] 2/21*x*(3*b*x^2+7*a)*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.40, size = 25, normalized size = 0.27

$$\frac{2\left(3\,\left(dx\right)^{\frac{7}{2}}b+7\,\left(dx\right)^{\frac{3}{2}}ad^2\right)}{21\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 2/21*(3*(d*x)^(7/2)*b + 7*(d*x)^(3/2)*a*d^2)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\sqrt{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*((a + b*x^2)^2)^(1/2),x)
[Out] int((d*x)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 133.05, size = 27, normalized size = 0.29

$$\frac{2a\left(dx\right)^{\frac{3}{2}}}{3d}+\frac{2b\left(dx\right)^{\frac{7}{2}}}{7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] 2*a*(d*x)**(3/2)/(3*d) + 2*b*(d*x)**(7/2)/(7*d**3)
```

3.553 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=91

$$\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*b*(d*x
)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{\sqrt{dx}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{\sqrt{dx}}+\frac{b^2(dx)^{3/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)} + \frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.47

$$\frac{2\sqrt{\left(a+bx^2\right)^2}\left(5ax+bx^3\right)}{5\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(5*a*x + b*x^3))/(5*Sqrt[d*x]*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 23.99, size = 68, normalized size = 0.75

$$\frac{2\left(ad^2+bd^2x^2\right)\left(5ad^2\sqrt{dx}+b(dx)^{5/2}\right)}{5d^5\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(5*a*d^2*Sqrt[d*x] + b*(d*x)^(5/2)))/(5*d^5*Sqrt[(a*
d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.71, size = 19, normalized size = 0.21

$$\frac{2\left(bx^2+5\,a\right)\sqrt{dx}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b*x^2 + 5*a)*sqrt(d*x)/d
```

giac [A] time = 0.17, size = 40, normalized size = 0.44

$$\frac{2\left(\sqrt{dx}\,bx^2\mathrm{sgn}\left(bx^2+a\right)+5\sqrt{dx}\,a\mathrm{sgn}\left(bx^2+a\right)\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
[Out] 2/5*(sqrt(d*x)*b*x^2*sgn(b*x^2 + a) + 5*sqrt(d*x)*a*sgn(b*x^2 + a))/d
```

maple [A] time = 0.00, size = 38, normalized size = 0.42

$$\frac{2\left(b\,x^2+5a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x}{5\left(b\,x^2+a\right)\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x)
[Out] 2/5*x*(b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(1/2)
```

maxima [A] time = 1.37, size = 24, normalized size = 0.26

$$\frac{2\left(5\sqrt{dx}\,a+\frac{(dx)^{\frac{5}{2}}b}{d^2}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
[Out] 2/5*(5*sqrt(d*x)*a + (d*x)^(5/2)*b/d^2)/d
```

mupad [B] time = 4.36, size = 47, normalized size = 0.52

$$\frac{\left(\frac{2\,x^3}{5}+\frac{2\,a\,x}{b}\right)\sqrt{\left(b\,x^2+a\right)^2}}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(1/2),x)
```

```
[Out] (((2*x^3)/5 + (2*a*x)/b)*((a + b*x^2)^2)^(1/2))/(x^2*(d*x)^(1/2) + (a*(d*x)
^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(a+bx^2\right)^2}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/sqrt(d*x), x)
```

3.554 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*b*(d*
x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{3/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{3/2}}+\frac{b^2\sqrt{dx}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)}+\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.47

$$\frac{2x\,(bx^2-3a)\sqrt{(a+bx^2)^2}}{3(dx)^{3/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-3*a + b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(3/2)*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 26.01, size = 67, normalized size = 0.74

$$\frac{2\,(bd^2x^2-3ad^2)\,(ad^2+bd^2x^2)}{3d^5\sqrt{dx}\sqrt{\frac{(ad^2+bd^2x^2)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (2*(-3*a*d^2 + b*d^2*x^2)*(a*d^2 + b*d^2*x^2))/(3*d^5*Sqrt[d*x]*Sqrt[(a*d^2
 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.89, size = 22, normalized size = 0.24

$$\frac{2\,(bx^2-3\,a)\sqrt{dx}}{3\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x^2 - 3*a)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.16, size = 41, normalized size = 0.45

$$\frac{2\left(\frac{\sqrt{dx}\,bx\mathrm{sgn}\left(bx^2+a\right)}{d}-\frac{3\,a\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
[Out] 2/3*(sqrt(d*x)*b*x*sgn(b*x^2 + a)/d - 3*a*sgn(b*x^2 + a)/sqrt(d*x))/d
```

maple [A] time = 0.00, size = 39, normalized size = 0.43

$$-\frac{2\left(-b\,x^2+3a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x}{3\left(b\,x^2+a\right)(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x)
[Out] -2/3*x*(-b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(3/2)
```

maxima [A] time = 1.36, size = 25, normalized size = 0.27

$$-\frac{2\left(\frac{3\,a}{\sqrt{dx}}-\frac{(dx)^{\frac{3}{2}}b}{d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
[Out] -2/3*(3*a/sqrt(d*x) - (d*x)^(3/2)*b/d^2)/d
```

mupad [B] time = 4.35, size = 52, normalized size = 0.57

$$\frac{\left(\frac{2\,x^2}{3\,d}-\frac{2\,a}{b\,d}\right)\sqrt{\left(b\,x^2+a\right)^2}}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(3/2),x)
```

```
[Out] (((2*x^2)/(3*d) - (2*a)/(b*d))*((a + b*x^2)^2)^(1/2))/(x^2*(d*x)^(1/2) + (a
*(d*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

3.555 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (2*b
*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{5/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{5/2}}+\frac{b^2}{d^2\sqrt{dx}}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.46

$$-\frac{2x\left(a-3bx^2\right)\sqrt{\left(a+bx^2\right)^2}}{3(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(a - 3*b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(5/2)*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 25.42, size = 68, normalized size = 0.75

$$\frac{2\left(ad^2+bd^2x^2\right)\left(3bd^2x^2-ad^2\right)}{3d^5(dx)^{3/2}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(-(a*d^2) + 3*b*d^2*x^2))/(3*d^5*(d*x)^(3/2)*Sqrt[(a
*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.92, size = 23, normalized size = 0.25

$$\frac{2\left(3\,bx^2-a\right)\sqrt{dx}}{3\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="fricas")
```

[Out] 2/3*(3*b*x^2 - a)*sqrt(d*x)/(d^3*x^2)

giac [A] time = 0.16, size = 42, normalized size = 0.46

$$\frac{2\left(3\sqrt{dx}\,b\mathrm{sgn}\left(bx^2+a\right)-\frac{ad\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}\,x}\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="giac")

[Out] 2/3*(3*sqrt(d*x)*b*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a)/(sqrt(d*x)*x))/d^3

maple [A] time = 0.00, size = 37, normalized size = 0.41

$$-\frac{2\left(-3b\,x^2+a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x}{3\left(b\,x^2+a\right)(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x)

[Out] -2/3*x*(-3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(5/2)

maxima [A] time = 1.40, size = 24, normalized size = 0.26

$$-\frac{2\left(\frac{a}{(dx)^{\frac{3}{2}}}-\frac{3\sqrt{dx}\,b}{d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="maxima")

[Out] -2/3*(a/(d*x)^(3/2) - 3*sqrt(d*x)*b/d^2)/d

mupad [B] time = 4.38, size = 53, normalized size = 0.58

$$\frac{\left(\frac{2\,x^2}{d^2}-\frac{2\,a}{3\,b\,d^2}\right)\sqrt{\left(b\,x^2+a\right)^2}}{x^3\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(5/2),x)
```

```
[Out] (((2*x^2)/d^2 - (2*a)/(3*b*d^2))*((a + b*x^2)^2)^(1/2))/(x^3*(d*x)^(1/2) +
(a*x*(d*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

3.556 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}} dx$

Optimal. Leaf size=91

$$-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (2*b
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{7/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{7/2}}+\frac{b^2}{d^2(dx)^{3/2}}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)} - \frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.46

$$-\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(a+5bx^2\right)}{5(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

```
[Out] (-2*x*Sqrt[(a + b*x^2)^2]*(a + 5*b*x^2))/(5*(d*x)^(7/2)*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 29.29, size = 67, normalized size = 0.74

$$-\frac{2\left(ad^2+bd^2x^2\right)\left(ad^2+5bd^2x^2\right)}{5d^5(dx)^{5/2}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

```
[Out] (-2*(a*d^2 + b*d^2*x^2)*(a*d^2 + 5*b*d^2*x^2))/(5*d^5*(d*x)^(5/2)*Sqrt[(a*d
^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.19, size = 21, normalized size = 0.23

$$-\frac{2\left(5\,bx^2+a\right)\sqrt{dx}}{5\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(5*b*x^2 + a)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.18, size = 44, normalized size = 0.48

$$-\frac{2\left(5\,bd^3x^2\mathrm{sgn}\left(bx^2+a\right)+ad^3\mathrm{sgn}\left(bx^2+a\right)\right)}{5\,\sqrt{dx}\,d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*b*d^3*x^2*sgn(b*x^2 + a) + a*d^3*sgn(b*x^2 + a))/(sqrt(d*x)*d^6*x^2
)
```

maple [A] time = 0.00, size = 37, normalized size = 0.41

$$-\frac{2\left(5b\,x^2+a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x}{5\left(b\,x^2+a\right)(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x)
```

```
[Out] -2/5*x*(5*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(7/2)
```

maxima [A] time = 1.27, size = 25, normalized size = 0.27

$$-\frac{2\left(5\,bd^2x^2+ad^2\right)}{5\,(dx)^{\frac{5}{2}}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*b*d^2*x^2 + a*d^2)/((d*x)^(5/2)*d^3)
```

mupad [B] time = 4.32, size = 56, normalized size = 0.62

$$-\frac{\left(\frac{2\,x^2}{d^3}+\frac{2\,a}{5\,b\,d^3}\right)\sqrt{\left(b\,x^2+a\right)^2}}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(7/2),x)
```

```
[Out] -((((2*x^2)/d^3 + (2*a)/(5*b*d^3))*((a + b*x^2)^2)^(1/2))/(x^4*(d*x)^(1/2) +
 (a*x^2*(d*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

3.557 $\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=195

$$\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{7/2}}{7}$$

Rubi [A] time = 0.06, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
(2*a*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2))
+ (2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (dx)^{5/2} \left(ab+b^2x^2\right)^3 dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3(dx)^{5/2} + \frac{3a^2b^4(dx)^{9/2}}{d^2} + \frac{3ab^5(dx)^{13/2}}{d^4} + \frac{b^6(dx)^{17/2}}{d^6}\right)}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)} + \frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)} + \frac{2ab}{}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2} \left(1045a^3+1995a^2bx^2+1463ab^2x^4+385b^3x^6\right)}{7315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^
2*x^4 + 385*b^3*x^6))/(7315*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 117.05, size = 105, normalized size = 0.54

$$\frac{2\left(ad^2+bd^2x^2\right)\left(1045a^3d^6(dx)^{7/2}+1995a^2bd^4(dx)^{11/2}+1463ab^2d^2(dx)^{15/2}+385b^3(dx)^{19/2}\right)}{7315d^9\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(1045*a^3*d^6*(d*x)^(7/2) + 1995*a^2*b*d^4*(d*x)^(11
/2) + 1463*a*b^2*d^2*(d*x)^(15/2) + 385*b^3*(d*x)^(19/2)))/(7315*d^9*Sqrt[(
a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.65, size = 54, normalized size = 0.28

$$\frac{2}{7315}\left(385\,b^3d^2x^9+1463\,ab^2d^2x^7+1995\,a^2bd^2x^5+1045\,a^3d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^3*d^2*x^9 + 1463*a*b^2*d^2*x^7 + 1995*a^2*b*d^2*x^5 + 1045*a^
3*d^2*x^3)*sqrt(d*x)
```

giac [A] time = 0.16, size = 99, normalized size = 0.51

$$\frac{2}{19}\sqrt{dx}\,b^3d^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{5}\sqrt{dx}\,ab^2d^2x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{11}\sqrt{dx}\,a^2bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}\,a^3d^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*sqrt(d*x)*b^3*d^2*x^9*sgn(b*x^2 + a) + 2/5*sqrt(d*x)*a*b^2*d^2*x^7*sgn
(b*x^2 + a) + 6/11*sqrt(d*x)*a^2*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)*a
^3*d^2*x^3*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 61, normalized size = 0.31

$$\frac{2\left(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3\right)(dx)^{\frac{5}{2}}\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x}{7315\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 2/7315*x*(385*b^3*x^6+1463*a*b^2*x^4+1995*a^2*b*x^2+1045*a^3)*(d*x)^(5/2)*(
(b*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.46, size = 83, normalized size = 0.43

$$\frac{2}{285}\left(15\,b^3d^{\frac{5}{2}}x^3+19\,ab^2d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{4}{165}\left(11\,ab^2d^{\frac{5}{2}}x^3+15\,a^2bd^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+\frac{2}{77}\left(7\,a^2bd^{\frac{5}{2}}x^3+11\,a^3d^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/285*(15*b^3*d^(5/2)*x^3 + 19*a*b^2*d^(5/2)*x)*x^(13/2) + 4/165*(11*a*b^2*
d^(5/2)*x^3 + 15*a^2*b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^2*b*d^(5/2)*x^3 + 11*
a^3*d^(5/2)*x)*x^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{5/2}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}} \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(5/2)*((a + b*x**2)**2)**(3/2), x)
```

3.558 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=195

$$\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{5/2}\sqrt{\ }}{5d}$$

Rubi [A] time = 0.06, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*
a^2*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (dx)^{3/2} \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3(dx)^{3/2} + \frac{3a^2b^4(dx)^{7/2}}{d^2} + \frac{3ab^5(dx)^{11/2}}{d^4} + \frac{b^6(dx)^{15/2}}{d^6}\right)}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)} + \frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)} + \frac{6ab^2}{}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(663a^3+1105a^2bx^2+765ab^2x^4+195b^3x^6\right)}{3315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*
x^4 + 195*b^3*x^6))/(3315*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 114.65, size = 105, normalized size = 0.54

$$\frac{2\left(ad^2+bd^2x^2\right)\left(663a^3d^6(dx)^{5/2}+1105a^2bd^4(dx)^{9/2}+765ab^2d^2(dx)^{13/2}+195b^3(dx)^{17/2}\right)}{3315d^9\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(663*a^3*d^6*(d*x)^(5/2) + 1105*a^2*b*d^4*(d*x)^(9/2
) + 765*a*b^2*d^2*(d*x)^(13/2) + 195*b^3*(d*x)^(17/2)))/(3315*d^9*Sqrt[(a*d
^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.04, size = 46, normalized size = 0.24

$$\frac{2}{3315}\left(195\,b^3dx^8+765\,ab^2dx^6+1105\,a^2bdx^4+663\,a^3dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^3*d*x^8 + 765*a*b^2*d*x^6 + 1105*a^2*b*d*x^4 + 663*a^3*d*x^2)
*sqrt(d*x)
```

giac [A] time = 0.21, size = 90, normalized size = 0.46

$$\frac{2}{3315}\left(195\,\sqrt{dx}\,b^3x^8\mathrm{sgn}\left(bx^2+a\right)+765\,\sqrt{dx}\,ab^2x^6\mathrm{sgn}\left(bx^2+a\right)+1105\,\sqrt{dx}\,a^2bx^4\mathrm{sgn}\left(bx^2+a\right)+663\,\sqrt{dx}\,a^3x^2\mathrm{sgn}\left(bx^2+a\right)\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3315*(195*sqrt(d*x)*b^3*x^8*sgn(b*x^2 + a) + 765*sqrt(d*x)*a*b^2*x^6*sgn(
b*x^2 + a) + 1105*sqrt(d*x)*a^2*b*x^4*sgn(b*x^2 + a) + 663*sqrt(d*x)*a^3*x^
2*sgn(b*x^2 + a))*d
```

maple [A] time = 0.01, size = 61, normalized size = 0.31

$$\frac{2\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)(dx)^{\frac{3}{2}}\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x}{3315\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 2/3315*x*(195*b^3*x^6+765*a*b^2*x^4+1105*a^2*b*x^2+663*a^3)*(d*x)^(3/2)*((b
*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.41, size = 83, normalized size = 0.43

$$\frac{2}{221}\left(13\,b^3d^{\frac{3}{2}}x^3+17\,ab^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{4}{117}\left(9\,ab^2d^{\frac{3}{2}}x^3+13\,a^2bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\frac{2}{45}\left(5\,a^2bd^{\frac{3}{2}}x^3+9\,a^3d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/221*(13*b^3*d^(3/2)*x^3 + 17*a*b^2*d^(3/2)*x)*x^(11/2) + 4/117*(9*a*b^2*d
^(3/2)*x^3 + 13*a^2*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^2*b*d^(3/2)*x^3 + 9*a^
3*d^(3/2)*x)*x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(\left(a + bx^2 \right)^2 \right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**(3/2), x)
```

3.559 $\int \sqrt{dx} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=195

$$\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}$$

Rubi [A] time = 0.05, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3\sqrt{dx}+\frac{3a^2b^4(dx)^{5/2}}{d^2}+\frac{3ab^5(dx)^{9/2}}{d^4}+\frac{b^6(dx)^{13/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{6ab^2(}{}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.34

$$\frac{2\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(385a^3x+495a^2bx^3+315ab^2x^5+77b^3x^7\right)}{1155\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(385*a^3*x + 495*a^2*b*x^3 + 315*a*b^2*x^5
 + 77*b^3*x^7))/(1155*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 83.78, size = 96, normalized size = 0.49

$$\frac{2(dx)^{3/2}\left(ad^2+bd^2x^2\right)\left(385a^3d^6+495a^2bd^6x^2+315ab^2d^6x^4+77b^3d^6x^6\right)}{1155d^9\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*(d*x)^(3/2)*(a*d^2 + b*d^2*x^2)*(385*a^3*d^6 + 495*a^2*b*d^6*x^2 + 315*a
*b^2*d^6*x^4 + 77*b^3*d^6*x^6))/(1155*d^9*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.77, size = 40, normalized size = 0.21

$$\frac{2}{1155}\left(77\,b^3x^7+315\,ab^2x^5+495\,a^2bx^3+385\,a^3x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(d*x)
```

giac [A] time = 0.16, size = 85, normalized size = 0.44

$$\frac{2}{15}\sqrt{dx}\,b^3x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{11}\sqrt{dx}\,ab^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{7}\sqrt{dx}\,a^2bx^3\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}\,a^3x\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(d*x)*b^3*x^7*sgn(b*x^2 + a) + 6/11*sqrt(d*x)*a*b^2*x^5*sgn(b*x^2
+ a) + 6/7*sqrt(d*x)*a^2*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*x)*a^3*x*sgn(b*x
^2 + a)
```

maple [A] time = 0.01, size = 61, normalized size = 0.31

$$\frac{2\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\sqrt{dx}\;x}{1155\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x)
```

```
[Out] 2/1155*x*(77*b^3*x^6+315*a*b^2*x^4+495*a^2*b*x^2+385*a^3)*((b*x^2+a)^2)^(3/
2)*(d*x)^(1/2)/(b*x^2+a)^3
```

maxima [A] time = 1.44, size = 83, normalized size = 0.43

$$\frac{2}{165}\left(11\,b^3\sqrt{d}\,x^3+15\,ab^2\sqrt{d}\,x\right)x^{\frac{9}{2}}+\frac{4}{77}\left(7\,ab^2\sqrt{d}\,x^3+11\,a^2b\sqrt{d}\,x\right)x^{\frac{5}{2}}+\frac{2}{21}\left(3\,a^2b\sqrt{d}\,x^3+7\,a^3\sqrt{d}\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/165*(11*b^3*sqrt(d)*x^3 + 15*a*b^2*sqrt(d)*x)*x^(9/2) + 4/77*(7*a*b^2*sqr
t(d)*x^3 + 11*a^2*b*sqrt(d)*x)*x^(5/2) + 2/21*(3*a^2*b*sqrt(d)*x^3 + 7*a^3*
sqrt(d)*x)*sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**(3/2), x)
```

3.560 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=193

$$\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2a^3\sqrt{dx}\,\sqrt{a^2 \cdots}}{d\left(\cdots\right.}$$

Rubi [A] time = 0.05, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*a^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (6*a^2*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2)) + (2*a*b
^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) + (2*b^
3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{\sqrt{dx}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{\sqrt{dx}}+\frac{3a^2b^4(dx)^{3/2}}{d^2}+\frac{3ab^5(dx)^{7/2}}{d^4}+\frac{b^6(dx)^{11/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{2a^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2ab^2(dx)^{9/2}\sqrt{\ }}{3d^5}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.34

$$\frac{2\sqrt{\left(a+bx^2\right)^2}\left(195a^3x+117a^2bx^3+65ab^2x^5+15b^3x^7\right)}{195\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(195*a^3*x + 117*a^2*b*x^3 + 65*a*b^2*x^5 + 15*b^3*x
^7))/(195*Sqrt[d*x]*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 52.72, size = 105, normalized size = 0.54

$$\frac{2\left(ad^2+bd^2x^2\right)\left(195a^3d^6\sqrt{dx}+117a^2bd^4(dx)^{5/2}+65ab^2d^2(dx)^{9/2}+15b^3(dx)^{13/2}\right)}{195d^9\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(195*a^3*d^6*Sqrt[d*x] + 117*a^2*b*d^4*(d*x)^(5/2) +
 65*a*b^2*d^2*(d*x)^(9/2) + 15*b^3*(d*x)^(13/2)))/(195*d^9*Sqrt[(a*d^2 + b*
d^2*x^2)^2/d^4])
```

fricas [A] time = 2.01, size = 42, normalized size = 0.22

$$\frac{2\left(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3\right)\sqrt{dx}}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(d*x)/d
```

giac [A] time = 0.17, size = 89, normalized size = 0.46

$$\frac{2\left(15\sqrt{dx}\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+65\sqrt{dx}\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+117\sqrt{dx}\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+195\sqrt{dx}\,a^3\mathrm{sgn}\left(bx^2+a\right)\right)}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/195*(15*sqrt(d*x)*b^3*x^6*sgn(b*x^2 + a) + 65*sqrt(d*x)*a*b^2*x^4*sgn(b*x
^2 + a) + 117*sqrt(d*x)*a^2*b*x^2*sgn(b*x^2 + a) + 195*sqrt(d*x)*a^3*sgn(b*
x^2 + a))/d
```

maple [A] time = 0.01, size = 61, normalized size = 0.32

$$\frac{2\left(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x}{195\left(b\,x^2+a\right)^3\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] 2/195*x*(15*b^3*x^6+65*a*b^2*x^4+117*a^2*b*x^2+195*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3/(d*x)^(1/2)
```

maxima [A] time = 1.46, size = 87, normalized size = 0.45

$$\frac{2\left(5\left(9\,b^3\sqrt{d}\,x^3+13\,ab^2\sqrt{d}\,x\right)x^{\frac{7}{2}}+26\left(5\,ab^2\sqrt{d}\,x^3+9\,a^2b\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{117\left(a^2b\sqrt{d}\,x^3+5\,a^3\sqrt{d}\,x\right)}{\sqrt{x}}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/585*(5*(9*b^3*sqrt(d)*x^3 + 13*a*b^2*sqrt(d)*x)*x^(7/2) + 26*(5*a*b^2*sqr
t(d)*x^3 + 9*a^2*b*sqrt(d)*x)*x^(3/2) + 117*(a^2*b*sqrt(d)*x^3 + 5*a^3*sqrt
(d)*x)/sqrt(x))/d
```

mupad [B] time = 4.50, size = 76, normalized size = 0.39

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{6\,a^2\,x^3}{5}+\frac{2\,b^2\,x^7}{13}+\frac{2\,a^3\,x}{b}+\frac{2\,a\,b\,x^5}{3}\right)}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((6*a^2*x^3)/5 + (2*b^2*x^7)/13 + (2*a^3
*x)/b + (2*a*b*x^5)/3))/(x^2*(d*x)^(1/2) + (a*(d*x)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/sqrt(d*x), x)
```

3.561 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2a\ldots}}{d\sqrt{dx}\,(\ldots}$$

Rubi [A] time = 0.06, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*a^2
*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*b^
2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \frac{\left(ab+b^2x^2\right)^3}{(dx)^{3/2}}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \left(\frac{a^3b^3}{(dx)^{3/2}}+\frac{3a^2b^4\sqrt{dx}}{d^2}+\frac{3ab^5(dx)^{5/2}}{d^4}+\frac{b^6(dx)^{9/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{7/2}\sqrt{a^2}}{7d^5\left(a\right.}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.35

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-77a^3+77a^2bx^2+33ab^2x^4+7b^3x^6\right)}{77(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-77*a^3 + 77*a^2*b*x^2 + 33*a*b^2*x^4 + 7*b^3*x^6
))/(77*(d*x)^(3/2)*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 41.07, size = 96, normalized size = 0.50

$$\frac{2\left(ad^2+bd^2x^2\right)\left(-77a^3d^6+77a^2bd^6x^2+33ab^2d^6x^4+7b^3d^6x^6\right)}{77d^9\sqrt{dx}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(-77*a^3*d^6 + 77*a^2*b*d^6*x^2 + 33*a*b^2*d^6*x^4 +
 7*b^3*d^6*x^6))/(77*d^9*Sqrt[d*x]*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.02, size = 45, normalized size = 0.24

$$\frac{2\left(7\,b^3x^6+33\,ab^2x^4+77\,a^2bx^2-77\,a^3\right)\sqrt{dx}}{77\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.17, size = 102, normalized size = 0.53

$$-\frac{2\left(\frac{77\,a^{3}\mathrm{sgn}\left(bx^{2}+a\right)}{\sqrt{dx}}-\frac{7\,\sqrt{dx}\,b^{3}d^{65}x^{5}\mathrm{sgn}\left(bx^{2}+a\right)+33\,\sqrt{dx}\,ab^{2}d^{65}x^{3}\mathrm{sgn}\left(bx^{2}+a\right)+77\,\sqrt{dx}\,a^{2}bd^{65}x\mathrm{sgn}\left(bx^{2}+a\right)}{d^{66}}\right)}{77\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/77*(77*a^3*sgn(b*x^2 + a)/sqrt(d*x) - (7*sqrt(d*x)*b^3*d^65*x^5*sgn(b*x^
2 + a) + 33*sqrt(d*x)*a*b^2*d^65*x^3*sgn(b*x^2 + a) + 77*sqrt(d*x)*a^2*b*d^
65*x*sgn(b*x^2 + a))/d^66)/d
```

maple [A] time = 0.01, size = 61, normalized size = 0.32

$$-\frac{2\left(-7b^{3}x^{6}-33a\,b^{2}x^{4}-77a^{2}b\,x^{2}+77a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x}{77\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x)
```

```
[Out] -2/77*x*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3/(d*x)^(3/2)
```

maxima [A] time = 1.42, size = 87, normalized size = 0.46

$$\frac{2\left(3\left(7\,b^{3}\sqrt{d}\,x^{3}+11\,ab^{2}\sqrt{d}\,x\right)x^{\frac{5}{2}}+22\left(3\,ab^{2}\sqrt{d}\,x^{3}+7\,a^{2}b\sqrt{d}\,x\right)\sqrt{x}+\frac{77\left(a^{2}b\sqrt{d}\,x^{3}-3\,a^{3}\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}\right)}{231\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*(3*(7*b^3*sqrt(d)*x^3 + 11*a*b^2*sqrt(d)*x)*x^(5/2) + 22*(3*a*b^2*sqr
t(d)*x^3 + 7*a^2*b*sqrt(d)*x)*sqrt(x) + 77*(a^2*b*sqrt(d)*x^3 - 3*a^3*sqrt(
d)*x)/x^(3/2))/d^2
```

mupad [B] time = 4.53, size = 87, normalized size = 0.46

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,a^2\,x^2}{d}-\frac{2\,a^3}{b\,d}+\frac{2\,b^2\,x^6}{11\,d}+\frac{6\,a\,b\,x^4}{7\,d}\right)}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(3/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*a^2*x^2)/d - (2*a^3)/(b*d) + (2*b^2*
x^6)/(11*d) + (6*a*b*x^4)/(7*d)))/(x^2*(d*x)^(1/2) + (a*(d*x)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(3/2), x)
```

3.562 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Rubi [A] time = 0.05, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (6
*a^2*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*
b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2)) + (2*b
^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{(dx)^{5/2}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{5/2}}+\frac{3a^2b^4}{d^2\sqrt{dx}}+\frac{3ab^5(dx)^{3/2}}{d^4}+\frac{b^6(dx)^{7/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{5/2}\sqrt{a^2+}}{5d^5\left(a+\right.}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.34

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-15a^3+135a^2bx^2+27ab^2x^4+5b^3x^6\right)}{45(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-15*a^3 + 135*a^2*b*x^2 + 27*a*b^2*x^4 + 5*b^3*x^
6))/(45*(d*x)^(5/2)*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 34.47, size = 96, normalized size = 0.50

$$\frac{2\left(ad^2+bd^2x^2\right)\left(-15a^3d^6+135a^2bd^6x^2+27ab^2d^6x^4+5b^3d^6x^6\right)}{45d^9(dx)^{3/2}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(-15*a^3*d^6 + 135*a^2*b*d^6*x^2 + 27*a*b^2*d^6*x^4
+ 5*b^3*d^6*x^6))/(45*d^9*(d*x)^(3/2)*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.70, size = 45, normalized size = 0.23

$$\frac{2\left(5\,b^3x^6+27\,ab^2x^4+135\,a^2bx^2-15\,a^3\right)\sqrt{dx}}{45\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)*sqrt(d*x)/(d^3*x^2
)
```

giac [A] time = 0.17, size = 105, normalized size = 0.54

$$-\frac{2\left(\frac{15\,a^{3}d\operatorname{sgn}\left(bx^{2}+a\right)}{\sqrt{dx}\,x}-\frac{5\,\sqrt{dx}\,b^{3}d^{36}x^{4}\operatorname{sgn}\left(bx^{2}+a\right)+27\,\sqrt{dx}\,ab^{2}d^{36}x^{2}\operatorname{sgn}\left(bx^{2}+a\right)+135\,\sqrt{dx}\,a^{2}bd^{36}\operatorname{sgn}\left(bx^{2}+a\right)}{d^{36}}\right)}{45\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/45*(15*a^3*d*sgn(b*x^2 + a)/(sqrt(d*x)*x) - (5*sqrt(d*x)*b^3*d^36*x^4*sg
n(b*x^2 + a) + 27*sqrt(d*x)*a*b^2*d^36*x^2*sgn(b*x^2 + a) + 135*sqrt(d*x)*a
^2*b*d^36*sgn(b*x^2 + a))/d^36)/d^3
```

maple [A] time = 0.01, size = 61, normalized size = 0.32

$$-\frac{2\left(-5b^{3}x^{6}-27a\,b^{2}x^{4}-135a^{2}b\,x^{2}+15a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x}{45\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x)
```

```
[Out] -2/45*x*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)*((b*x^2+a)^2)^(3/2)/
(b*x^2+a)^3/(d*x)^(5/2)
```

maxima [A] time = 1.47, size = 86, normalized size = 0.45

$$\frac{2\left(\left(5\,b^{3}\sqrt{d}\,x^{3}+9\,ab^{2}\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{18\left(ab^{2}\sqrt{d}\,x^{3}+5\,a^{2}b\sqrt{d}\,x\right)}{\sqrt{x}}+\frac{15\left(3\,a^{2}b\sqrt{d}\,x^{3}-a^{3}\sqrt{d}\,x\right)}{x^{\frac{5}{2}}}\right)}{45\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45*((5*b^3*sqrt(d)*x^3 + 9*a*b^2*sqrt(d)*x)*x^(3/2) + 18*(a*b^2*sqrt(d)*x
^3 + 5*a^2*b*sqrt(d)*x)/sqrt(x) + 15*(3*a^2*b*sqrt(d)*x^3 - a^3*sqrt(d)*x)/
x^(5/2))/d^3
```

mupad [B] time = 4.49, size = 88, normalized size = 0.46

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{6\,a^2\,x^2}{d^2}-\frac{2\,a^3}{3\,b\,d^2}+\frac{2\,b^2\,x^6}{9\,d^2}+\frac{6\,a\,b\,x^4}{5\,d^2}\right)}{x^3\,\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(5/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((6*a^2*x^2)/d^2 - (2*a^3)/(3*b*d^2) + (
2*b^2*x^6)/(9*d^2) + (6*a*b*x^4)/(5*d^2)))/(x^3*(d*x)^(1/2) + (a*x*(d*x)^(1
/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(5/2), x)
```

3.563 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\left(a+bx^2\right)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}+\frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b}}{5d(dx)^{5/2}\left(a+bx\right.}$$

Rubi [A] time = 0.06, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\left(a+bx^2\right)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (6
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (2*a*
b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{(dx)^{7/2}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{7/2}}+\frac{3a^2b^4}{d^2(dx)^{3/2}}+\frac{3ab^5\sqrt{dx}}{d^4}+\frac{b^6(dx)^{5/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)} - \frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)} + \frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx}}{d^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.35

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-7a^3-105a^2bx^2+35ab^2x^4+5b^3x^6\right)}{35(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-7*a^3 - 105*a^2*b*x^2 + 35*a*b^2*x^4 + 5*b^3*x^6
))/(35*(d*x)^(7/2)*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 28.58, size = 96, normalized size = 0.50

$$\frac{2\left(ad^2+bd^2x^2\right)\left(-7a^3d^6-105a^2bd^6x^2+35ab^2d^6x^4+5b^3d^6x^6\right)}{35d^9(dx)^{5/2}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(-7*a^3*d^6 - 105*a^2*b*d^6*x^2 + 35*a*b^2*d^6*x^4 +
 5*b^3*d^6*x^6))/(35*d^9*(d*x)^(5/2)*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 2.07, size = 45, normalized size = 0.24

$$\frac{2\left(5\,b^3x^6+35\,ab^2x^4-105\,a^2bx^2-7\,a^3\right)\sqrt{dx}}{35\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.22, size = 107, normalized size = 0.56

$$-\frac{2\left(\frac{7\left(15\,a^{2}bd^{3}x^{2}\mathrm{sgn}\left(bx^{2}+a\right)+a^{3}d^{3}\mathrm{sgn}\left(bx^{2}+a\right)\right)}{\sqrt{dx}\,d^{2}x^{2}}-\frac{5\left(\sqrt{dx}\,b^{3}d^{21}x^{3}\mathrm{sgn}\left(bx^{2}+a\right)+7\,\sqrt{dx}\,ab^{2}d^{21}x\mathrm{sgn}\left(bx^{2}+a\right)\right)}{d^{21}}\right)}{35\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/35*(7*(15*a^2*b*d^3*x^2*sgn(b*x^2 + a) + a^3*d^3*sgn(b*x^2 + a))/(sqrt(d
*x)*d^2*x^2) - 5*(sqrt(d*x)*b^3*d^21*x^3*sgn(b*x^2 + a) + 7*sqrt(d*x)*a*b^2
*d^21*x*sgn(b*x^2 + a))/d^21)/d^4
```

maple [A] time = 0.01, size = 61, normalized size = 0.32

$$-\frac{2\left(-5b^{3}x^{6}-35a\,b^{2}x^{4}+105a^{2}b\,x^{2}+7a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x}{35\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x)
```

```
[Out] -2/35*x*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3/(d*x)^(7/2)
```

maxima [A] time = 1.45, size = 86, normalized size = 0.45

$$\frac{2\left(5\left(3\,b^{3}\sqrt{d}\,x^{3}+7\,ab^{2}\sqrt{d}\,x\right)\sqrt{x}+\frac{70\left(ab^{2}\sqrt{d}\,x^{3}-3\,a^{2}b\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}-\frac{21\left(5\,a^{2}b\sqrt{d}\,x^{3}+a^{3}\sqrt{d}\,x\right)}{x^{\frac{7}{2}}}\right)}{105\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/105*(5*(3*b^3*sqrt(d)*x^3 + 7*a*b^2*sqrt(d)*x)*sqrt(x) + 70*(a*b^2*sqrt(d
)*x^3 - 3*a^2*b*sqrt(d)*x)/x^(3/2) - 21*(5*a^2*b*sqrt(d)*x^3 + a^3*sqrt(d)*
x)/x^(7/2))/d^4
```

mupad [B] time = 4.53, size = 91, normalized size = 0.48

$$-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,a^3}{5\,b\,d^3}+\frac{6\,a^2\,x^2}{d^3}-\frac{2\,b^2\,x^6}{7\,d^3}-\frac{2\,a\,b\,x^4}{d^3}\right)}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(7/2),x)
```

```
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*a^3)/(5*b*d^3) + (6*a^2*x^2)/d^3 -
(2*b^2*x^6)/(7*d^3) - (2*a*b*x^4)/d^3))/(x^4*(d*x)^(1/2) + (a*x^2*(d*x)^(1/
2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(7/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(7/2), x)
```

3.564 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2a^5(dx$$

Rubi [A] time = 0.08, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}+\frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
 (4*a^3*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(27/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(27*d^1
1*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int (dx)^{5/2}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int\left(a^5b^5(dx)^{5/2}+\frac{5a^4b^6(dx)^{9/2}}{d^2}+\frac{10a^3b^7(dx)^{13/2}}{d^4}+\frac{10a^2b^8(dx)}{d^6}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{4a}{}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.30

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(129789a^5+412965a^4bx^2+605682a^3b^2x^4+478170a^2b^3x^6+197505ab^4x^8+33649b^5x^{10}\right)}{908523\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(129789*a^5 + 412965*a^4*b*x^2 + 60568
2*a^3*b^2*x^4 + 478170*a^2*b^3*x^6 + 197505*a*b^4*x^8 + 33649*b^5*x^10))/(9
08523*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 126.12, size = 141, normalized size = 0.47

$$\frac{2\left(ad^2+bd^2x^2\right)\left(129789a^5d^{10}(dx)^{7/2}+412965a^4bd^8(dx)^{11/2}+605682a^3b^2d^6(dx)^{15/2}+478170a^2b^3d^4(dx)^{19/2}+197505ab^4d^2(dx)^{23/2}+33649b^5(dx)^{27/2}\right)}{908523d^{13}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(129789*a^5*d^10*(d*x)^(7/2) + 412965*a^4*b*d^8*(d*x
)^(11/2) + 605682*a^3*b^2*d^6*(d*x)^(15/2) + 478170*a^2*b^3*d^4*(d*x)^(19/2
) + 197505*a*b^4*d^2*(d*x)^(23/2) + 33649*b^5*(d*x)^(27/2)))/(908523*d^13*S
qrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.58, size = 82, normalized size = 0.28

$$\frac{2}{908523}\left(33649\,b^5d^2x^{13}+197505\,ab^4d^2x^{11}+478170\,a^2b^3d^2x^9+605682\,a^3b^2d^2x^7+412965\,a^4bd^2x^5+129789\,a^5d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^5*d^2*x^13 + 197505*a*b^4*d^2*x^11 + 478170*a^2*b^3*d^2*x
^9 + 605682*a^3*b^2*d^2*x^7 + 412965*a^4*b*d^2*x^5 + 129789*a^5*d^2*x^3)*sq
rt(d*x)
```

giac [A] time = 0.17, size = 153, normalized size = 0.52

$$\frac{2}{27}\sqrt{dx}\,b^5d^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{23}\sqrt{dx}\,ab^4d^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{20}{19}\sqrt{dx}\,a^2b^3d^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{4}{3}\sqrt{dx}\,a^3b^2d^2x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{11}\sqrt{dx}\,a^4bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}\,a^5d^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(d*x)*b^5*d^2*x^13*sgn(b*x^2 + a) + 10/23*sqrt(d*x)*a*b^4*d^2*x^11
*sgn(b*x^2 + a) + 20/19*sqrt(d*x)*a^2*b^3*d^2*x^9*sgn(b*x^2 + a) + 4/3*sqrt
(d*x)*a^3*b^2*d^2*x^7*sgn(b*x^2 + a) + 10/11*sqrt(d*x)*a^4*b*d^2*x^5*sgn(b*
x^2 + a) + 2/7*sqrt(d*x)*a^5*d^2*x^3*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$\frac{2\left(33649b^5x^{10}+197505a\,b^4x^8+478170a^2b^3x^6+605682a^3b^2x^4+412965a^4b\,x^2+129789a^5\right)(dx)^{\frac{5}{2}}\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{908523\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 2/908523*x*(33649*b^5*x^10+197505*a*b^4*x^8+478170*a^2*b^3*x^6+605682*a^3*b
^2*x^4+412965*a^4*b*x^2+129789*a^5)*(d*x)^(5/2)*((b*x^2+a)^2)^(5/2)/(b*x^2+
a)^5
```

maxima [A] time = 1.47, size = 147, normalized size = 0.49

$$\frac{2}{621}\left(23\,b^5d^{\frac{5}{2}}x^3+27\,ab^4d^{\frac{5}{2}}x\right)x^{\frac{21}{2}}+\frac{8}{437}\left(19\,ab^4d^{\frac{5}{2}}x^3+23\,a^2b^3d^{\frac{5}{2}}x\right)x^{\frac{17}{2}}+\frac{4}{95}\left(15\,a^2b^3d^{\frac{5}{2}}x^3+19\,a^3b^2d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{8}{165}\left(11\,a^3b^2d^{\frac{5}{2}}x^3+15\,a^4bd^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+\frac{2}{77}\left(7\,a^4bd^{\frac{5}{2}}x^3+11\,a^5d^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/621*(23*b^5*d^(5/2)*x^3 + 27*a*b^4*d^(5/2)*x)*x^(21/2) + 8/437*(19*a*b^4*
d^(5/2)*x^3 + 23*a^2*b^3*d^(5/2)*x)*x^(17/2) + 4/95*(15*a^2*b^3*d^(5/2)*x^3
 + 19*a^3*b^2*d^(5/2)*x)*x^(13/2) + 8/165*(11*a^3*b^2*d^(5/2)*x^3 + 15*a^4*
b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^4*b*d^(5/2)*x^3 + 11*a^5*d^(5/2)*x)*x^(5/2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{5/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.565 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{2a^5(dx}{}$$

Rubi [A] time = 0.08, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)}+\frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(25/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(25*d^1
1*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5(dx)^{3/2}+\frac{5a^4b^6(dx)^{7/2}}{d^2}+\frac{10a^3b^7(dx)^{11/2}}{d^4}+\frac{10a^2b^8(dx)}{d^6}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{20a}{}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.30

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(69615a^5+193375a^4bx^2+267750a^3b^2x^4+204750a^2b^3x^6+82875ab^4x^8+13923b^5x^{10}\right)}{348075\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(69615*a^5 + 193375*a^4*b*x^2 + 267750
*a^3*b^2*x^4 + 204750*a^2*b^3*x^6 + 82875*a*b^4*x^8 + 13923*b^5*x^10))/(348
075*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 119.00, size = 141, normalized size = 0.47

$$\frac{2\left(ad^2+bd^2x^2\right)\left(69615a^5d^{10}(dx)^{5/2}+193375a^4bd^8(dx)^{9/2}+267750a^3b^2d^6(dx)^{13/2}+204750a^2b^3d^4(dx)^{17/2}+82875ab^4d^2(dx)^{21/2}+13923b^5(dx)^{25/2}\right)}{348075d^{13}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(69615*a^5*d^10*(d*x)^(5/2) + 193375*a^4*b*d^8*(d*x)
^(9/2) + 267750*a^3*b^2*d^6*(d*x)^(13/2) + 204750*a^2*b^3*d^4*(d*x)^(17/2)
+ 82875*a*b^4*d^2*(d*x)^(21/2) + 13923*b^5*(d*x)^(25/2)))/(348075*d^13*Sqrt
[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.10, size = 70, normalized size = 0.24

$$\frac{2}{348075}\left(13923\,b^5dx^{12}+82875\,ab^4dx^{10}+204750\,a^2b^3dx^8+267750\,a^3b^2dx^6+193375\,a^4bdx^4+69615\,a^5dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^5*d*x^12 + 82875*a*b^4*d*x^10 + 204750*a^2*b^3*d*x^8 + 26
7750*a^3*b^2*d*x^6 + 193375*a^4*b*d*x^4 + 69615*a^5*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.20, size = 138, normalized size = 0.46

$$\frac{2}{348075}\left(13923\,\sqrt{dx}\,b^5x^{12}\mathrm{sgn}\left(bx^2+a\right)+82875\,\sqrt{dx}\,ab^4x^{10}\mathrm{sgn}\left(bx^2+a\right)+204750\,\sqrt{dx}\,a^2b^3x^8\mathrm{sgn}\left(bx^2+a\right)+267750\,\sqrt{dx}\,a^3b^2x^6\mathrm{sgn}\left(bx^2+a\right)+193375\,\sqrt{dx}\,a^4bx^4\mathrm{sgn}\left(bx^2+a\right)+69615\,\sqrt{dx}\,a^5x^2\mathrm{sgn}\left(bx^2+a\right)\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/348075*(13923*sqrt(d*x)*b^5*x^12*sgn(b*x^2 + a) + 82875*sqrt(d*x)*a*b^4*x
^10*sgn(b*x^2 + a) + 204750*sqrt(d*x)*a^2*b^3*x^8*sgn(b*x^2 + a) + 267750*s
qrt(d*x)*a^3*b^2*x^6*sgn(b*x^2 + a) + 193375*sqrt(d*x)*a^4*b*x^4*sgn(b*x^2
+ a) + 69615*sqrt(d*x)*a^5*x^2*sgn(b*x^2 + a))*d
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$\frac{2\left(13923b^5x^{10}+82875a\,b^4x^8+204750a^2b^3x^6+267750a^3b^2x^4+193375a^4b\,x^2+69615a^5\right)(dx)^{\frac{3}{2}}\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{348075\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 2/348075*x*(13923*b^5*x^10+82875*a*b^4*x^8+204750*a^2*b^3*x^6+267750*a^3*b^
2*x^4+193375*a^4*b*x^2+69615*a^5)*(d*x)^(3/2)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)
^5
```

maxima [A] time = 1.42, size = 147, normalized size = 0.49

$$\frac{2}{525}\left(21\,b^5d^{\frac{3}{2}}x^3+25\,ab^4d^{\frac{3}{2}}x\right)x^{\frac{19}{2}}+\frac{8}{357}\left(17\,ab^4d^{\frac{3}{2}}x^3+21\,a^2b^3d^{\frac{3}{2}}x\right)x^{\frac{15}{2}}+\frac{12}{221}\left(13\,a^2b^3d^{\frac{3}{2}}x^3+17\,a^3b^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{8}{117}\left(9\,a^3b^2d^{\frac{3}{2}}x^3+13\,a^4bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\frac{2}{45}\left(5\,a^4bd^{\frac{3}{2}}x^3+9\,a^5d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/525*(21*b^5*d^(3/2)*x^3 + 25*a*b^4*d^(3/2)*x)*x^(19/2) + 8/357*(17*a*b^4*
d^(3/2)*x^3 + 21*a^2*b^3*d^(3/2)*x)*x^(15/2) + 12/221*(13*a^2*b^3*d^(3/2)*x
^3 + 17*a^3*b^2*d^(3/2)*x)*x^(11/2) + 8/117*(9*a^3*b^2*d^(3/2)*x^3 + 13*a^4
*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^4*b*d^(3/2)*x^3 + 9*a^5*d^(3/2)*x)*x^(3/2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{3/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**(5/2), x)
```

3.566 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9\left(a+bx^2\right)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7\left(a+bx^2\right)}+\frac{2a^5(dx)^{\cdots}}{}$$

Rubi [A] time = 0.08, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9\left(a+bx^2\right)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2
)) + (4*a^2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^7*(a + b
*x^2)) + (10*a*b^4*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^9*(a
 + b*x^2)) + (2*b^5*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^11*
(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(ab+b^2x^2\right)^5dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5\sqrt{dx}+\frac{5a^4b^6(dx)^{5/2}}{d^2}+\frac{10a^3b^7(dx)^{9/2}}{d^4}+\frac{10a^2b^8(dx)^{13/2}}{d^6}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{20a^3}{}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.30

$$\frac{2\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(33649a^5x+72105a^4bx^3+91770a^3b^2x^5+67298a^2b^3x^7+26565ab^4x^9+4389b^5x^{11}\right)}{100947\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(33649*a^5*x + 72105*a^4*b*x^3 + 91770*a^3
*b^2*x^5 + 67298*a^2*b^3*x^7 + 26565*a*b^4*x^9 + 4389*b^5*x^11))/(100947*(a
 + b*x^2))
```

IntegrateAlgebraic [A] time = 120.78, size = 141, normalized size = 0.47

$$\frac{2\left(ad^2+bd^2x^2\right)\left(33649a^5d^{10}(dx)^{3/2}+72105a^4bd^8(dx)^{7/2}+91770a^3b^2d^6(dx)^{11/2}+67298a^2b^3d^4(dx)^{15/2}+26565ab^4d^2(dx)^{19/2}+4389b^5(dx)^{23/2}\right)}{100947d^{13}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(33649*a^5*d^10*(d*x)^(3/2) + 72105*a^4*b*d^8*(d*x)^
(7/2) + 91770*a^3*b^2*d^6*(d*x)^(11/2) + 67298*a^2*b^3*d^4*(d*x)^(15/2) + 2
6565*a*b^4*d^2*(d*x)^(19/2) + 4389*b^5*(d*x)^(23/2)))/(100947*d^13*Sqrt[(a*
d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 2.78, size = 62, normalized size = 0.21

$$\frac{2}{100947}\left(4389\,b^5x^{11}+26565\,ab^4x^9+67298\,a^2b^3x^7+91770\,a^3b^2x^5+72105\,a^4bx^3+33649\,a^5x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/100947*(4389*b^5*x^11 + 26565*a*b^4*x^9 + 67298*a^2*b^3*x^7 + 91770*a^3*b
^2*x^5 + 72105*a^4*b*x^3 + 33649*a^5*x)*sqrt(d*x)
```

giac [A] time = 0.20, size = 133, normalized size = 0.45

$$\frac{2}{23}\sqrt{dx}\,b^5x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{19}\sqrt{dx}\,ab^4x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{4}{3}\sqrt{dx}\,a^2b^3x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{20}{11}\sqrt{dx}\,a^3b^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}\sqrt{dx}\,a^4bx^3\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}\,a^5x\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/23*sqrt(d*x)*b^5*x^11*sgn(b*x^2 + a) + 10/19*sqrt(d*x)*a*b^4*x^9*sgn(b*x^
2 + a) + 4/3*sqrt(d*x)*a^2*b^3*x^7*sgn(b*x^2 + a) + 20/11*sqrt(d*x)*a^3*b^2
*x^5*sgn(b*x^2 + a) + 10/7*sqrt(d*x)*a^4*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*
x)*a^5*x*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$\frac{2\left(4389b^5x^{10}+26565a\,b^4x^8+67298a^2b^3x^6+91770a^3b^2x^4+72105a^4b\,x^2+33649a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\sqrt{dx}\;x}{100947\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x)
```

```
[Out] 2/100947*x*(4389*b^5*x^10+26565*a*b^4*x^8+67298*a^2*b^3*x^6+91770*a^3*b^2*x
^4+72105*a^4*b*x^2+33649*a^5)*((b*x^2+a)^2)^(5/2)*(d*x)^(1/2)/(b*x^2+a)^5
```

maxima [A] time = 1.53, size = 147, normalized size = 0.49

$$\frac{2}{437}\left(19\,b^5\sqrt{d}\,x^3+23\,ab^4\sqrt{d}\,x\right)x^{\frac{17}{2}}+\frac{8}{285}\left(15\,ab^4\sqrt{d}\,x^3+19\,a^2b^3\sqrt{d}\,x\right)x^{\frac{13}{2}}+\frac{4}{55}\left(11\,a^2b^3\sqrt{d}\,x^3+15\,a^3b^2\sqrt{d}\,x\right)x^{\frac{9}{2}}+\frac{8}{77}\left(7\,a^3b^2\sqrt{d}\,x^3+11\,a^4b\sqrt{d}\,x\right)x^{\frac{5}{2}}+\frac{2}{21}\left(3\,a^4b\sqrt{d}\,x^3+7\,a^5\sqrt{d}\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/437*(19*b^5*sqrt(d)*x^3 + 23*a*b^4*sqrt(d)*x)*x^(17/2) + 8/285*(15*a*b^4*
sqrt(d)*x^3 + 19*a^2*b^3*sqrt(d)*x)*x^(13/2) + 4/55*(11*a^2*b^3*sqrt(d)*x^3
 + 15*a^3*b^2*sqrt(d)*x)*x^(9/2) + 8/77*(7*a^3*b^2*sqrt(d)*x^3 + 11*a^4*b*s
qrt(d)*x)*x^(5/2) + 2/21*(3*a^4*b*sqrt(d)*x^3 + 7*a^5*sqrt(d)*x)*sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**(5/2), x)
```

3.567 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=293

$$\frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2a^5\sqrt{d}}{}$$

Rubi [A] time = 0.08, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^5\left(a+bx^2\right)}+\frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2a^5\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

```
[Out] (2*a^5*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*a^4*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (20*a^3*
b^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^5*(a + b*x^2)) + (20*
a^2*b^3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
+ (10*a*b^4*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^9*(a + b*x^
2)) + (2*b^5*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^11*(a + b*
x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{\sqrt{dx}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{\sqrt{dx}}+\frac{5a^4b^6(dx)^{3/2}}{d^2}+\frac{10a^3b^7(dx)^{7/2}}{d^4}+\frac{10a^2b^8(dx)^{11/2}}{d^6}+\frac{5ab^9(dx)^{15}}{d^8}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{2a^5\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}+\frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{9/}}{9d}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.30

$$\frac{2\sqrt{\left(a+bx^2\right)^2}\left(13923a^5x+13923a^4bx^3+15470a^3b^2x^5+10710a^2b^3x^7+4095ab^4x^9+663b^5x^{11}\right)}{13923\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(13923*a^5*x + 13923*a^4*b*x^3 + 15470*a^3*b^2*x^5 +
 10710*a^2*b^3*x^7 + 4095*a*b^4*x^9 + 663*b^5*x^11))/(13923*Sqrt[d*x]*(a +
b*x^2))
```

IntegrateAlgebraic [A] time = 125.05, size = 141, normalized size = 0.48

$$\frac{2\left(ad^2+bd^2x^2\right)\left(13923a^5d^{10}\sqrt{dx}+13923a^4bd^8(dx)^{5/2}+15470a^3b^2d^6(dx)^{9/2}+10710a^2b^3d^4(dx)^{13/2}+4095ab^4d^2(dx)^{17/2}+663b^5(dx)^{21/2}\right)}{13923d^{13}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(13923*a^5*d^10*Sqrt[d*x] + 13923*a^4*b*d^8*(d*x)^(5
/2) + 15470*a^3*b^2*d^6*(d*x)^(9/2) + 10710*a^2*b^3*d^4*(d*x)^(13/2) + 4095
*a*b^4*d^2*(d*x)^(17/2) + 663*b^5*(d*x)^(21/2)))/(13923*d^13*Sqrt[(a*d^2 +
b*d^2*x^2)^2/d^4])
```

fricas [A] time = 2.24, size = 64, normalized size = 0.22

$$\frac{2\left(663\,b^5x^{10}+4095\,ab^4x^8+10710\,a^2b^3x^6+15470\,a^3b^2x^4+13923\,a^4bx^2+13923\,a^5\right)\sqrt{dx}}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^5*x^10 + 4095*a*b^4*x^8 + 10710*a^2*b^3*x^6 + 15470*a^3*b^2*
x^4 + 13923*a^4*b*x^2 + 13923*a^5)*sqrt(d*x)/d
```

giac [A] time = 0.17, size = 137, normalized size = 0.47

$$\frac{2\left(663\,\sqrt{dx}\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+4095\,\sqrt{dx}\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+10710\,\sqrt{dx}\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+15470\,\sqrt{dx}\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+13923\,\sqrt{dx}\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+13923\,\sqrt{dx}\,a^5\mathrm{sgn}\left(bx^2+a\right)\right)}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/13923*(663*sqrt(d*x)*b^5*x^10*sgn(b*x^2 + a) + 4095*sqrt(d*x)*a*b^4*x^8*s
gn(b*x^2 + a) + 10710*sqrt(d*x)*a^2*b^3*x^6*sgn(b*x^2 + a) + 15470*sqrt(d*x
)*a^3*b^2*x^4*sgn(b*x^2 + a) + 13923*sqrt(d*x)*a^4*b*x^2*sgn(b*x^2 + a) + 1
3923*sqrt(d*x)*a^5*sgn(b*x^2 + a))/d
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$\frac{2\left(663b^5x^{10}+4095a\,b^4x^8+10710a^2b^3x^6+15470a^3b^2x^4+13923a^4b\,x^2+13923a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{13923\left(b\,x^2+a\right)^5\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] 2/13923*x*(663*b^5*x^10+4095*a*b^4*x^8+10710*a^2*b^3*x^6+15470*a^3*b^2*x^4+
13923*a^4*b*x^2+13923*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(1/2)
```

maxima [A] time = 1.45, size = 151, normalized size = 0.52

$$\frac{2\left(195\left(17\,b^5\sqrt{d}\,x^3+21\,ab^4\sqrt{d}\,x\right)x^{\frac{15}{2}}+1260\left(13\,ab^4\sqrt{d}\,x^3+17\,a^2b^3\sqrt{d}\,x\right)x^{\frac{11}{2}}+3570\left(9\,a^2b^3\sqrt{d}\,x^3+13\,a^3b^2\sqrt{d}\,x\right)x^{\frac{7}{2}}+6188\left(5\,a^3b^2\sqrt{d}\,x^3+9\,a^4b\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{13923\left(a^4b\sqrt{d}\,x^3+5\,a^5\sqrt{d}\,x\right)}{\sqrt{x}}\right)}{69615\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/69615*(195*(17*b^5*sqrt(d)*x^3 + 21*a*b^4*sqrt(d)*x)*x^(15/2) + 1260*(13*
a*b^4*sqrt(d)*x^3 + 17*a^2*b^3*sqrt(d)*x)*x^(11/2) + 3570*(9*a^2*b^3*sqrt(d
)*x^3 + 13*a^3*b^2*sqrt(d)*x)*x^(7/2) + 6188*(5*a^3*b^2*sqrt(d)*x^3 + 9*a^4
*b*sqrt(d)*x)*x^(3/2) + 13923*(a^4*b*sqrt(d)*x^3 + 5*a^5*sqrt(d)*x)/sqrt(x)
)/d
```

mupad [B] time = 4.57, size = 112, normalized size = 0.38

$$\frac{2\,x\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(5731\,a^4+8192\,a^3\,b\,x^2+7278\,a^2\,b^2\,x^4+3432\,a\,b^3\,x^6+663\,b^4\,x^8\right)}{13923\,\sqrt{d\,x}}+\frac{16384\,a^5\,x\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13923\,\sqrt{d\,x}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] (2*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(5731*a^4 + 663*b^4*x^8 + 8192*a^3*b
*x^2 + 3432*a*b^3*x^6 + 7278*a^2*b^2*x^4))/(13923*(d*x)^(1/2)) + (16384*a^5
*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13923*(d*x)^(1/2)*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/sqrt(d*x), x)
```

3.568 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}\left(a+bx^2\right)}+\frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}-\frac{2a^5\sqrt{a^2}}{d\sqrt{}}$$

Rubi [A] time = 0.08, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}\left(a+bx^2\right)}+\frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (10*a^
4*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^
2)) + (2*a*b^4*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^11*(a +
b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{(dx)^{3/2}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{3/2}}+\frac{5a^4b^6\sqrt{dx}}{d^2}+\frac{10a^3b^7(dx)^{5/2}}{d^4}+\frac{10a^2b^8(dx)^{9/2}}{d^6}+\frac{5ab^9(dx)^{13/2}}{d^8}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}+\frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{7/2}\sqrt{}}{7d^5\left(\right.}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.30

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-4389a^5+7315a^4bx^2+6270a^3b^2x^4+3990a^2b^3x^6+1463ab^4x^8+231b^5x^{10}\right)}{4389(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-4389*a^5 + 7315*a^4*b*x^2 + 6270*a^3*b^2*x^4 + 3
990*a^2*b^3*x^6 + 1463*a*b^4*x^8 + 231*b^5*x^10))/(4389*(d*x)^(3/2)*(a + b*
x^2))
```

IntegrateAlgebraic [A] time = 94.08, size = 124, normalized size = 0.42

$$\frac{2\left(ad^2+bd^2x^2\right)\left(-4389a^5d^{10}+7315a^4bd^{10}x^2+6270a^3b^2d^{10}x^4+3990a^2b^3d^{10}x^6+1463ab^4d^{10}x^8+231b^5d^{10}x^{10}\right)}{4389d^{13}\sqrt{dx}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(-4389*a^5*d^10 + 7315*a^4*b*d^10*x^2 + 6270*a^3*b^2
*d^10*x^4 + 3990*a^2*b^3*d^10*x^6 + 1463*a*b^4*d^10*x^8 + 231*b^5*d^10*x^10
))/(4389*d^13*Sqrt[d*x]*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.86, size = 67, normalized size = 0.23

$$\frac{2\left(231\,b^5x^{10}+1463\,ab^4x^8+3990\,a^2b^3x^6+6270\,a^3b^2x^4+7315\,a^4bx^2-4389\,a^5\right)\sqrt{dx}}{4389\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/4389*(231*b^5*x^10 + 1463*a*b^4*x^8 + 3990*a^2*b^3*x^6 + 6270*a^3*b^2*x^4
 + 7315*a^4*b*x^2 - 4389*a^5)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.19, size = 156, normalized size = 0.53

$$-\frac{2\left(\frac{4389\,a^5\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}-\frac{231\,\sqrt{dx}\,b^5d^{189}x^9\mathrm{sgn}\left(bx^2+a\right)+1463\,\sqrt{dx}\,ab^4d^{189}x^7\mathrm{sgn}\left(bx^2+a\right)+3990\,\sqrt{dx}\,a^2b^3d^{189}x^5\mathrm{sgn}\left(bx^2+a\right)+6270\,\sqrt{dx}\,a^3b^2d^{189}x^3\mathrm{sgn}\left(bx^2+a\right)+7315\,\sqrt{dx}\,a^4bd^{189}x\mathrm{sgn}\left(bx^2+a\right)}{d^{190}}\right)}{4389\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/4389*(4389*a^5*sgn(b*x^2 + a)/sqrt(d*x) - (231*sqrt(d*x)*b^5*d^189*x^9*s
gn(b*x^2 + a) + 1463*sqrt(d*x)*a*b^4*d^189*x^7*sgn(b*x^2 + a) + 3990*sqrt(d
*x)*a^2*b^3*d^189*x^5*sgn(b*x^2 + a) + 6270*sqrt(d*x)*a^3*b^2*d^189*x^3*sgn
(b*x^2 + a) + 7315*sqrt(d*x)*a^4*b*d^189*x*sgn(b*x^2 + a))/d^190)/d
```

maple [A] time = 0.00, size = 83, normalized size = 0.28

$$-\frac{2\left(-231b^5x^{10}-1463a\,b^4x^8-3990a^2b^3x^6-6270a^3b^2x^4-7315a^4b\,x^2+4389a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{4389\left(b\,x^2+a\right)^5(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x)
```

```
[Out] -2/4389*x*(-231*b^5*x^10-1463*a*b^4*x^8-3990*a^2*b^3*x^6-6270*a^3*b^2*x^4-7
315*a^4*b*x^2+4389*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(3/2)
```

maxima [A] time = 1.50, size = 151, normalized size = 0.51

$$\frac{2\left(77\left(15\,b^5\sqrt{d}\,x^3+19\,ab^4\sqrt{d}\,x\right)x^{\frac{13}{2}}+532\left(11\,ab^4\sqrt{d}\,x^3+15\,a^2b^3\sqrt{d}\,x\right)x^{\frac{9}{2}}+1710\left(7\,a^2b^3\sqrt{d}\,x^3+11\,a^3b^2\sqrt{d}\,x\right)x^{\frac{5}{2}}+4180\left(3\,a^3b^2\sqrt{d}\,x^3+7\,a^4b\sqrt{d}\,x\right)\sqrt{x}+\frac{7315\left(a^4b\sqrt{d}\,x^3-3\,a^5\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}\right)}{21945\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21945*(77*(15*b^5*sqrt(d)*x^3 + 19*a*b^4*sqrt(d)*x)*x^(13/2) + 532*(11*a*
b^4*sqrt(d)*x^3 + 15*a^2*b^3*sqrt(d)*x)*x^(9/2) + 1710*(7*a^2*b^3*sqrt(d)*x
^3 + 11*a^3*b^2*sqrt(d)*x)*x^(5/2) + 4180*(3*a^3*b^2*sqrt(d)*x^3 + 7*a^4*b*
sqrt(d)*x)*sqrt(x) + 7315*(a^4*b*sqrt(d)*x^3 - 3*a^5*sqrt(d)*x)/x^(3/2))/d^
2
```

mupad [B] time = 4.54, size = 116, normalized size = 0.39

$$\frac{2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(3803\,a^4+3512\,a^3\,b\,x^2+2758\,a^2\,b^2\,x^4+1232\,a\,b^3\,x^6+231\,b^4\,x^8\right)}{4389\,d\,\sqrt{d\,x}}-\frac{16384\,a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4389\,d\,\sqrt{d\,x}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(3/2),x)
```

```
[Out] (2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(3803*a^4 + 231*b^4*x^8 + 3512*a^3*b*x
^2 + 1232*a*b^3*x^6 + 2758*a^2*b^2*x^4))/(4389*d*(d*x)^(1/2)) - (16384*a^5*
(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4389*d*(d*x)^(1/2)*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(3/2), x)
```

3.569 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=293

$$\frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}-\frac{2a^5\sqrt{a^2}}{3d(dx}$$

Rubi [A] time = 0.08, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\left(a+bx^2\right)}+\frac{10a^4b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (1
0*a^4*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (4*a
^3*b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (20
*a^2*b^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2)) +
 (10*a*b^4*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^9*(a + b*x^2
)) + (2*b^5*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^11*(a + b*x
^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{(dx)^{5/2}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{5/2}}+\frac{5a^4b^6}{d^2\sqrt{dx}}+\frac{10a^3b^7(dx)^{3/2}}{d^4}+\frac{10a^2b^8(dx)^{7/2}}{d^6}+\frac{5ab^9(dx)^{11/2}}{d^8}+\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{10a^4b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{a^2}}{d^5\left(a+\right.}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.30

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-663a^5+9945a^4bx^2+3978a^3b^2x^4+2210a^2b^3x^6+765ab^4x^8+117b^5x^{10}\right)}{1989(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-663*a^5 + 9945*a^4*b*x^2 + 3978*a^3*b^2*x^4 + 22
10*a^2*b^3*x^6 + 765*a*b^4*x^8 + 117*b^5*x^10))/(1989*(d*x)^(5/2)*(a + b*x^
2))
```

IntegrateAlgebraic [A] time = 74.82, size = 124, normalized size = 0.42

$$\frac{2\left(ad^2+bd^2x^2\right)\left(-663a^5d^{10}+9945a^4bd^{10}x^2+3978a^3b^2d^{10}x^4+2210a^2b^3d^{10}x^6+765ab^4d^{10}x^8+117b^5d^{10}x^{10}\right)}{1989d^{13}(dx)^{3/2}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(-663*a^5*d^10 + 9945*a^4*b*d^10*x^2 + 3978*a^3*b^2*
d^10*x^4 + 2210*a^2*b^3*d^10*x^6 + 765*a*b^4*d^10*x^8 + 117*b^5*d^10*x^10))
/(1989*d^13*(d*x)^(3/2)*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.94, size = 67, normalized size = 0.23

$$\frac{2\left(117\,b^5x^{10}+765\,ab^4x^8+2210\,a^2b^3x^6+3978\,a^3b^2x^4+9945\,a^4bx^2-663\,a^5\right)\sqrt{dx}}{1989\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*b^5*x^10 + 765*a*b^4*x^8 + 2210*a^2*b^3*x^6 + 3978*a^3*b^2*x^4
+ 9945*a^4*b*x^2 - 663*a^5)*sqrt(d*x)/(d^3*x^2)
```

giac [A] time = 0.19, size = 159, normalized size = 0.54

$$-\frac{2\left(\frac{663\,a^5 d\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}\,x} - \frac{117\,\sqrt{dx}\,b^5 d^{136} x^8 \mathrm{sgn}\left(bx^2+a\right)+765\,\sqrt{dx}\,ab^4 d^{136} x^6 \mathrm{sgn}\left(bx^2+a\right)+2210\,\sqrt{dx}\,a^2 b^3 d^{136} x^4 \mathrm{sgn}\left(bx^2+a\right)+3978\,\sqrt{dx}\,a^3 b^2 d^{136} x^2 \mathrm{sgn}\left(bx^2+a\right)+9945\,\sqrt{dx}\,a^4 b d^{136} \mathrm{sgn}\left(bx^2+a\right)}{d^{136}}\right)}{1989\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/1989*(663*a^5*d*sgn(b*x^2 + a)/(sqrt(d*x)*x) - (117*sqrt(d*x)*b^5*d^136*
x^8*sgn(b*x^2 + a) + 765*sqrt(d*x)*a*b^4*d^136*x^6*sgn(b*x^2 + a) + 2210*sq
rt(d*x)*a^2*b^3*d^136*x^4*sgn(b*x^2 + a) + 3978*sqrt(d*x)*a^3*b^2*d^136*x^2
*sgn(b*x^2 + a) + 9945*sqrt(d*x)*a^4*b*d^136*sgn(b*x^2 + a))/d^136)/d^3
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$-\frac{2\left(-117b^5x^{10}-765a\,b^4x^8-2210a^2b^3x^6-3978a^3b^2x^4-9945a^4b\,x^2+663a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{1989\left(b\,x^2+a\right)^5(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x)
```

```
[Out] -2/1989*x*(-117*b^5*x^10-765*a*b^4*x^8-2210*a^2*b^3*x^6-3978*a^3*b^2*x^4-99
45*a^4*b*x^2+663*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(5/2)
```

maxima [A] time = 1.54, size = 151, normalized size = 0.52

$$\frac{2\left(45\left(13\,b^5\sqrt{d}\,x^3+17\,ab^4\sqrt{d}\,x\right)x^{\frac{11}{2}}+340\left(9\,ab^4\sqrt{d}\,x^3+13\,a^2b^3\sqrt{d}\,x\right)x^{\frac{7}{2}}+1326\left(5\,a^2b^3\sqrt{d}\,x^3+9\,a^3b^2\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{7956\left(a^3b^2\sqrt{d}\,x^3+5\,a^4b\sqrt{d}\,x\right)}{\sqrt{x}}+\frac{3315\left(3\,a^4b\sqrt{d}\,x^3-a^5\sqrt{d}\,x\right)}{x^{\frac{5}{2}}}\right)}{9945\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9945*(45*(13*b^5*sqrt(d)*x^3 + 17*a*b^4*sqrt(d)*x)*x^(11/2) + 340*(9*a*b^
4*sqrt(d)*x^3 + 13*a^2*b^3*sqrt(d)*x)*x^(7/2) + 1326*(5*a^2*b^3*sqrt(d)*x^3
 + 9*a^3*b^2*sqrt(d)*x)*x^(3/2) + 7956*(a^3*b^2*sqrt(d)*x^3 + 5*a^4*b*sqrt(
d)*x)/sqrt(x) + 3315*(3*a^4*b*sqrt(d)*x^3 - a^5*sqrt(d)*x)/x^(5/2))/d^3
```

mupad [B] time = 4.56, size = 116, normalized size = 0.40

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{10\,a^4\,x^2}{d^2}-\frac{2\,a^5}{3\,b\,d^2}+\frac{2\,b^4\,x^{10}}{17\,d^2}+\frac{4\,a^3\,b\,x^4}{d^2}+\frac{10\,a\,b^3\,x^8}{13\,d^2}+\frac{20\,a^2\,b^2\,x^6}{9\,d^2}\right)}{x^3\,\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(5/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((10*a^4*x^2)/d^2 - (2*a^5)/(3*b*d^2) +
(2*b^4*x^10)/(17*d^2) + (4*a^3*b*x^4)/d^2 + (10*a*b^3*x^8)/(13*d^2) + (20*a
^2*b^2*x^6)/(9*d^2)))/(x^3*(d*x)^(1/2) + (a*x*(d*x)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(5/2), x)
```

3.570 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}-\frac{2a^5\sqrt{a^2}}{5d(dx}$$

Rubi [A] time = 0.08, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}-\frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (1
0*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2)
) + (10*a*b^4*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^11*(a +
b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{(dx)^{7/2}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{7/2}}+\frac{5a^4b^6}{d^2(dx)^{3/2}}+\frac{10a^3b^7\sqrt{dx}}{d^4}+\frac{10a^2b^8(dx)^{5/2}}{d^6}+\frac{5ab^9(dx)^{9/2}}{d^8}+\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)} - \frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)} + \frac{20a^3b^2(dx)^{3/2}\sqrt{a^2+2}}{3d^5\left(a+bx\right.}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.30

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-231a^5-5775a^4bx^2+3850a^3b^2x^4+1650a^2b^3x^6+525ab^4x^8+77b^5x^{10}\right)}{1155(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-231*a^5 - 5775*a^4*b*x^2 + 3850*a^3*b^2*x^4 + 16
50*a^2*b^3*x^6 + 525*a*b^4*x^8 + 77*b^5*x^10))/(1155*(d*x)^(7/2)*(a + b*x^2
))
```

IntegrateAlgebraic [A] time = 61.31, size = 124, normalized size = 0.42

$$\frac{2\left(ad^2+bd^2x^2\right)\left(-231a^5d^{10}-5775a^4bd^{10}x^2+3850a^3b^2d^{10}x^4+1650a^2b^3d^{10}x^6+525ab^4d^{10}x^8+77b^5d^{10}x^{10}\right)}{1155d^{13}(dx)^{5/2}\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (2*(a*d^2 + b*d^2*x^2)*(-231*a^5*d^10 - 5775*a^4*b*d^10*x^2 + 3850*a^3*b^2*
d^10*x^4 + 1650*a^2*b^3*d^10*x^6 + 525*a*b^4*d^10*x^8 + 77*b^5*d^10*x^10))/
(1155*d^13*(d*x)^(5/2)*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.81, size = 67, normalized size = 0.23

$$\frac{2\left(77\,b^5x^{10}+525\,ab^4x^8+1650\,a^2b^3x^6+3850\,a^3b^2x^4-5775\,a^4bx^2-231\,a^5\right)\sqrt{dx}}{1155\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^5*x^10 + 525*a*b^4*x^8 + 1650*a^2*b^3*x^6 + 3850*a^3*b^2*x^4 -
 5775*a^4*b*x^2 - 231*a^5)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.20, size = 162, normalized size = 0.55

$$-\frac{2\left(\frac{231\left(25\,a^4bd^3x^2\mathrm{sgn}\left(bx^2+a\right)+a^5d^3\mathrm{sgn}\left(bx^2+a\right)\right)}{\sqrt{dx}\,d^2x^2}-\frac{77\,\sqrt{dx}\,b^5d^{105}x^7\mathrm{sgn}\left(bx^2+a\right)+525\,\sqrt{dx}\,ab^4d^{105}x^5\mathrm{sgn}\left(bx^2+a\right)+1650\,\sqrt{dx}\,a^2b^3d^{105}x^3\mathrm{sgn}\left(bx^2+a\right)+3850\,\sqrt{dx}\,a^3b^2d^{105}x\mathrm{sgn}\left(bx^2+a\right)}{d^{105}}\right)}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/1155*(231*(25*a^4*b*d^3*x^2*sgn(b*x^2 + a) + a^5*d^3*sgn(b*x^2 + a))/(sq
rt(d*x)*d^2*x^2) - (77*sqrt(d*x)*b^5*d^105*x^7*sgn(b*x^2 + a) + 525*sqrt(d*
x)*a*b^4*d^105*x^5*sgn(b*x^2 + a) + 1650*sqrt(d*x)*a^2*b^3*d^105*x^3*sgn(b*
x^2 + a) + 3850*sqrt(d*x)*a^3*b^2*d^105*x*sgn(b*x^2 + a))/d^105)/d^4
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$-\frac{2\left(-77b^5x^{10}-525a\,b^4x^8-1650a^2b^3x^6-3850a^3b^2x^4+5775a^4b\,x^2+231a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{1155\left(b\,x^2+a\right)^5(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x)
```

```
[Out] -2/1155*x*(-77*b^5*x^10-525*a*b^4*x^8-1650*a^2*b^3*x^6-3850*a^3*b^2*x^4+577
5*a^4*b*x^2+231*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(7/2)
```

maxima [A] time = 1.55, size = 150, normalized size = 0.51

$$\frac{2\left(7\left(11\,b^5\sqrt{d}\,x^3+15\,ab^4\sqrt{d}\,x\right)x^{\frac{9}{2}}+60\left(7\,ab^4\sqrt{d}\,x^3+11\,a^2b^3\sqrt{d}\,x\right)x^{\frac{5}{2}}+330\left(3\,a^2b^3\sqrt{d}\,x^3+7\,a^3b^2\sqrt{d}\,x\right)\sqrt{x}+\frac{1540\left(a^3b^2\sqrt{d}\,x^3-3\,a^4b\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}-\frac{231\left(5\,a^4b\sqrt{d}\,x^3+a^5\sqrt{d}\,x\right)}{x^{\frac{7}{2}}}\right)}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(7*(11*b^5*sqrt(d)*x^3 + 15*a*b^4*sqrt(d)*x)*x^(9/2) + 60*(7*a*b^4*s
qrt(d)*x^3 + 11*a^2*b^3*sqrt(d)*x)*x^(5/2) + 330*(3*a^2*b^3*sqrt(d)*x^3 + 7
*a^3*b^2*sqrt(d)*x)*sqrt(x) + 1540*(a^3*b^2*sqrt(d)*x^3 - 3*a^4*b*sqrt(d)*x
)/x^(3/2) - 231*(5*a^4*b*sqrt(d)*x^3 + a^5*sqrt(d)*x)/x^(7/2))/d^4
```

mupad [B] time = 4.72, size = 118, normalized size = 0.40

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,b^4\,x^{10}}{15\,d^3}-\frac{10\,a^4\,x^2}{d^3}-\frac{2\,a^5}{5\,b\,d^3}+\frac{20\,a^3\,b\,x^4}{3\,d^3}+\frac{10\,a\,b^3\,x^8}{11\,d^3}+\frac{20\,a^2\,b^2\,x^6}{7\,d^3}\right)}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(7/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*b^4*x^10)/(15*d^3) - (10*a^4*x^2)/d^
3 - (2*a^5)/(5*b*d^3) + (20*a^3*b*x^4)/(3*d^3) + (10*a*b^3*x^8)/(11*d^3) +
(20*a^2*b^2*x^6)/(7*d^3)))/(x^4*(d*x)^(1/2) + (a*x^2*(d*x)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(7/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(7/2), x)
```

3.571 $\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=457

$$-\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.32, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (-2*a*d^3*Sqrt[d*x]*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2
*d*(d*x)^(5/2)*(a + b*x^2))/(5*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4
)*d^(7/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqr
t[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4)*d^(7/2)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a^2d^4\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b\right.}}{b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(2a^2d^3\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int\right.}{b^2\sqrt{a^2+2abx^2+}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a^{3/2}d^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int\right.}{b^2\sqrt{a^2+2abx^2}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a^{5/4}d^{7/2}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\right.}{2\sqrt{2}\,b^{13/4}\sqrt{a^2}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}\right.}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(1-\right.}{\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b}}$$

Mathematica [A] time = 0.09, size = 238, normalized size = 0.52

$$\frac{d^3\sqrt{dx}\left(a+bx^2\right)\left(-5\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-10\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+10\sqrt{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)-40a\sqrt[4]{b}\sqrt{x}+8b^{5/4}x^{5/2}\right)}{20b^{9/4}\sqrt{x}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (d^3*Sqrt[d*x]*(a + b*x^2)*(-40*a*b^(1/4)*Sqrt[x] + 8*b^(5/4)*x^(5/2) - 10*
Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10*Sqrt[2]*
a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 5*Sqrt[2]*a^(5/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*a^(5/
4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(20*b^(9/4)
*Sqrt[x]*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 54.46, size = 217, normalized size = 0.47

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{a^{5/4}d^{7/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{\sqrt{2}b^{9/4}}+\frac{a^{5/4}d^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{\sqrt{2}b^{9/4}}+\frac{2d\sqrt{dx}\left(bd^2x^2-5ad^2\right)}{5b^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((2*d*Sqrt[d*x]*(-5*a*d^2 + b*d^2*x^2))/(5*b^2) - (a^(
5/4)*d^(7/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]
*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(Sqrt[2]*b^(9/4)) + (a^(5/4)*d^(7/2)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)]
)/(Sqrt[2]*b^(9/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.49, size = 223, normalized size = 0.49

$$\frac{20\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\arctan\left(-\frac{\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^7d^3-\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{a^2d^7x+\sqrt{-\frac{a^5d^{14}}{b^9}}\,b^4}\,b^7}{a^5d^{14}}\right)+5\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\log\left(\sqrt{dx}\,ad^3+\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)-5\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\log\left(\sqrt{dx}\,ad^3-\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)+4\left(bd^3x^2-5\,ad^3\right)\sqrt{dx}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*(-a^5*d^14/b^9)^(1/4)*b^2*arctan(-((-a^5*d^14/b^9)^(3/4)*sqrt(d*x)
*a*b^7*d^3 - (-a^5*d^14/b^9)^(3/4)*sqrt(a^2*d^7*x + sqrt(-a^5*d^14/b^9)*b^4
)*b^7)/(a^5*d^14)) + 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 + (-a^
5*d^14/b^9)^(1/4)*b^2) - 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 -
(-a^5*d^14/b^9)^(1/4)*b^2) + 4*(b*d^3*x^2 - 5*a*d^3)*sqrt(d*x))/b^2
```

giac [A] time = 0.20, size = 273, normalized size = 0.60

$$\frac{1}{20}\,d^{3}\left(\frac{10\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{3}}+\frac{10\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{3}}+\frac{5\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{3}}-\frac{5\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{3}}+\frac{8\left(\sqrt{dx}\,b^{4}d^{10}x^{2}-5\,\sqrt{dx}\,ab^{3}d^{10}\right)}{b^{5}d^{10}}\right)\operatorname{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*d^3*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 + 10*sqrt(2)*(a*b^3*d^2)^(1/4
)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/b^3 + 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)
*sqrt(d*x) + sqrt(a*d^2/b))/b^3 - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x - s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 8*(sqrt(d*x)*b^4*d^
10*x^2 - 5*sqrt(d*x)*a*b^3*d^10)/(b^5*d^10))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 239, normalized size = 0.52

$$\frac{\left(b\,x^{2}+a\right)\left(10\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+10\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{2}\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)-40\sqrt{dx}\,a\,d^{2}+8\left(dx\right)^{\frac{5}{2}}b\right)d}{20\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/20*(b*x^2+a)*d*(5*a*d^2*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a/b*d^2)^(1/2)))+10*a*d^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+10*a*d^2*(a/b*d^2)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+8*(d*x)^(5/2)*b
-40*d^2*a*(d*x)^(1/2))/((b*x^2+a)^2)^(1/2)/b^2
```

maxima [A] time = 2.94, size = 266, normalized size = 0.58

$$\frac{\frac{5\left(\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}\,d^{5}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\,\sqrt{2}\,d^{5}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)a^{2}}{b^{2}}+\frac{8\left(\left(dx\right)^{\frac{5}{2}}bd^{2}-5\,\sqrt{dx}\,ad^{4}\right)}{b^{2}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(5*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4))
+ 2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^
2/b^2 + 8*((d*x)^(5/2)*b*d^2 - 5*sqrt(d*x)*a*d^4)/b^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{7/2}}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(7/2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.572 $\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$\frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{\right.}{2\sqrt{2}\,b^{7/4}}$$

Rubi [A] time = 0.29, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1112, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*d*(d*x)^(3/2)*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3
/4)*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2
)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/4)*d^(5/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2ad\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ad\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad\left(a\right.\right.}{}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a^{3/4}d^{5/2}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{\right.}{2\sqrt{2}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\right.}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{3/4}d^{5/2}\left(a+bx\right.}{\sqrt{2}\,b^{7/4}\sqrt{}}$$

Mathematica [A] time = 0.05, size = 110, normalized size = 0.27

$$\frac{(dx)^{5/2}\left(a+bx^2\right)\left(3(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)-3(-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+2b^{3/4}x^{3/2}\right)}{3b^{7/4}x^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(5/2)*(a + b*x^2)*(2*b^(3/4)*x^(3/2) + 3*(-a)^(3/4)*ArcTan[(b^(1/4)*
Sqrt[x])/(-a)^(1/4)] - 3*(-a)^(3/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)]))
/(3*b^(7/4)*x^(5/2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 42.66, size = 201, normalized size = 0.49

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{a^{3/4}d^{5/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{\sqrt{2}\,b^{7/4}}+\frac{a^{3/4}d^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{\sqrt{2}\,b^{7/4}}+\frac{2d(dx)^{3/2}}{3b}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((2*d*(d*x)^(3/2))/(3*b) + (a^(3/4)*d^(5/2)*ArcTan[((a
^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/
Sqrt[d*x]])/(Sqrt[2]*b^(7/4)) + (a^(3/4)*d^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(Sqrt[2]*b^(7/4))))/(
d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.84, size = 219, normalized size = 0.53

$$\frac{4\sqrt{dx}\,d^2x+12\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^2d^7-\sqrt{a^4d^{15}x-\sqrt{-\frac{a^3d^{10}}{b^7}}\,a^3b^3d^{10}}\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b^2}{a^3d^{10}}\right)-3\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}\,a^2d^7+\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{3}{4}}b^5\right)+3\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}\,a^2d^7-\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{3}{4}}b^5\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(d*x)*d^2*x + 12*(-a^3*d^10/b^7)^(1/4)*b*arctan(-((-a^3*d^10/b^7
)^(1/4)*sqrt(d*x)*a^2*b^2*d^7 - sqrt(a^4*d^15*x - sqrt(-a^3*d^10/b^7)*a^3*b
^3*d^10)*(-a^3*d^10/b^7)^(1/4)*b^2)/(a^3*d^10)) - 3*(-a^3*d^10/b^7)^(1/4)*b
*log(sqrt(d*x)*a^2*d^7 + (-a^3*d^10/b^7)^(3/4)*b^5) + 3*(-a^3*d^10/b^7)^(1/
4)*b*log(sqrt(d*x)*a^2*d^7 - (-a^3*d^10/b^7)^(3/4)*b^5))/b
```

giac [A] time = 0.20, size = 254, normalized size = 0.62

$$\frac{1}{12}\,d^2\left(\frac{8\sqrt{dx}\,x}{b}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4d}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4d}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4d}-\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4d}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*d^2*(8*sqrt(d*x)*x/b - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*d) - 6*sqrt(2
)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d
*x))/(a*d^2/b)^(1/4))/(b^4*d) + 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*d) - 3*sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*
d))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 221, normalized size = 0.54

$$-\frac{\left(b\,x^{2}+a\right)\left(6\sqrt{2}\,a\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+6\sqrt{2}\,a\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+3\sqrt{2}\,a\,d^{2}\ln\left(-\frac{-dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)-8\,(dx)^{\frac{3}{2}}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}b\right)d}{12\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/12*(b*x^2+a)*d*(3*a*d^2*2^(1/2)*ln(-((a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
-d*x-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1
/2)))+6*a*d^2*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2
)^(1/4))+6*a*d^2*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))-8*(d*x)^(3/2)*b*(a/b*d^2)^(1/4))/((b*x^2+a)^2)^(1/2)/b^2/(a/b*d
^2)^(1/4)
```

maxima [A] time = 2.97, size = 241, normalized size = 0.58

$$-\frac{\frac{3\,ad^{4}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b}-\frac{8\,(dx)^{\frac{3}{2}}d^{2}}{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
```

```
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sq
rt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b - 8*(d*x)
^(3/2)*d^2/b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{5/2}}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(5/2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.573 $\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=410

$$\frac{2d\sqrt{dx}\,\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\right.}{2\sqrt{2}\,b^{5/4}\sqrt{}}$$

Rubi [A] time = 0.28, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d\sqrt{dx}\,\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*d*Sqrt[d*x]*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*
d^(3/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(a
 + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[
2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2ad\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{a}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{a}\,\left(\right.\right.}{}\\
&= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt[4]{a}\,d^{3/2}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{d\right.}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right.}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right.}{\sqrt{2}\,b^{5/4}\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 221, normalized size = 0.54

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+8\sqrt[4]{b}\sqrt{x}\right)}{4b^{5/4}x^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(8*b^(1/4)*Sqrt[x] + 2*Sqrt[2]*a^(1/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*
```

```
b^(1/4)*Sqrt[x])/a^(1/4)] + Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(4*b^(5/4)*x^(3/2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 36.81, size = 200, normalized size = 0.49

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{\sqrt[4]{a}\,d^{3/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\,d^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{\sqrt{2}\,b^{5/4}}+\frac{2d\sqrt{dx}}{b}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((2*d*Sqrt[d*x])/b + (a^(1/4)*d^(3/2)*ArcTan[((a^(1/4)
*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d
*x]])/(Sqrt[2]*b^(5/4)) - (a^(1/4)*d^(3/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(Sqrt[2]*b^(5/4))))/(d^2*Sq
rt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.91, size = 170, normalized size = 0.41

$$-\frac{4\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{ad^6}{b^5}\right)^{\frac{3}{4}}\sqrt{dx}\,b^4d-\sqrt{d^3x+\sqrt{-\frac{ad^6}{b^5}}\,b^2}\left(-\frac{ad^6}{b^5}\right)^{\frac{3}{4}}b^4}{ad^6}\right)+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}\,d+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\right)-\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}\,d-\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\right)-4\sqrt{dx}\,d}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-a*d^6/b^5)^(1/4)*b*arctan(-((-a*d^6/b^5)^(3/4)*sqrt(d*x)*b^4*d -
sqrt(d^3*x + sqrt(-a*d^6/b^5)*b^2)*(-a*d^6/b^5)^(3/4)*b^4)/(a*d^6)) + (-a*d
^6/b^5)^(1/4)*b*log(sqrt(d*x)*d + (-a*d^6/b^5)^(1/4)*b) - (-a*d^6/b^5)^(1/4
)*b*log(sqrt(d*x)*d - (-a*d^6/b^5)^(1/4)*b) - 4*sqrt(d*x)*d)/b
```

giac [A] time = 0.24, size = 238, normalized size = 0.58

$$-\frac{1}{4}d\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^2}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^2}-\frac{8\sqrt{dx}}{b}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^2 + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^
2 + sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/b^2 - sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^2 - 8*sqrt(d*x)/b)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 214, normalized size = 0.52

$$-\frac{\left(b x^2+a\right)\left(2\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac14}}{\left(\frac{a d^2}{b}\right)^{\frac14}}\right)+2\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac14}}{\left(\frac{a d^2}{b}\right)^{\frac14}}\right)+\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)-8\sqrt{dx}\right)d}{4\sqrt{\left(b x^2+a\right)^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)*d*((a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d
^2)^(1/2)))+2*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)
^(1/4))/(a/b*d^2)^(1/4))+2*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))-8*(d*x)^(1/2))/((b*x^2+a)^2)^(1/2)/b
```

maxima [A] time = 3.10, size = 250, normalized size = 0.61

$$\frac{\frac{8\sqrt{dx}\,d^2}{b}-\frac{\left(\frac{\sqrt{2}\,d^4\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac14}\sqrt{dx}\,b^{\frac14}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac34}b^{\frac14}}-\frac{\sqrt{2}\,d^4\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac14}\sqrt{dx}\,b^{\frac14}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac34}b^{\frac14}}+\frac{2\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac14}b^{\frac14}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac14}b^{\frac14}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)a}{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*sqrt(d*x)*d^2/b - (sqrt(2)*d^4*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^4*log
(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
```

```
^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(a)))*a/b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{3/2}}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/((a + b*x^2)^2)^(1/2),x)
[Out] int((d*x)^(3/2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
[Out] Timed out
```

3.574 $\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=368

$$\frac{\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.25, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1112, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (Sqrt[d
]*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[
d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{\sqrt{b}\,d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,\right.}{\sqrt{b}\,d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(\sqrt{d}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt{d}\left(ab+b^2x^2\right)\right)\text{Subs}}{2\sqrt{2}\sqrt[4]{a}\,b^{7}}$$

$$= \frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{\right.}}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}$$

$$= -\frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{d}}{}$$

<u>Mathematica</u> [A] time = 0.04, size = 85, normalized size = 0.23

$$\frac{\sqrt{dx}\left(a+bx^2\right)\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+\tanh^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)\right)}{\sqrt[4]{-a}\,b^{3/4}\sqrt{x}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[d*x]*(a + b*x^2)*(ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + ArcTanh[(a*b
^(1/4)*Sqrt[x])/(-a)^(5/4)]))/((-a)^(1/4)*b^(3/4)*Sqrt[x]*Sqrt[(a + b*x^2)^
2])
```

IntegrateAlgebraic [A] time = 34.53, size = 188, normalized size = 0.51

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{\sqrt{d}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(-((Sqrt[d]*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)
) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(Sqrt[2]*a^(1/4)*b^(
3/4))) - (Sqrt[d]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt
[a]*d + Sqrt[b]*d*x)])/(Sqrt[2]*a^(1/4)*b^(3/4))))/(d^2*Sqrt[(a*d^2 + b*d^2
*x^2)^2/d^4])
```

fricas [A] time = 1.70, size = 173, normalized size = 0.47

$$-2\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,bd\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}-\sqrt{-abd^2\sqrt{-\frac{d^2}{ab^3}}+d^3x}\,b\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}}{d^2}\right)+\frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}\,d\right)-\frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}\,d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-d^2/(a*b^3))^(1/4)*arctan(-(sqrt(d*x)*b*d*(-d^2/(a*b^3))^(1/4) - sqrt(
-a*b*d^2*sqrt(-d^2/(a*b^3)) + d^3*x)*b*(-d^2/(a*b^3))^(1/4))/d^2) + 1/2*(-d
^2/(a*b^3))^(1/4)*log(a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d) - 1/2*(-d^2
/(a*b^3))^(1/4)*log(-a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d)
```

giac [A] time = 0.19, size = 242, normalized size = 0.66

$$\frac{\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^3}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^3}\right)\mathrm{sgn}\left(bx^2+a\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(
a*b^3) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a*b^3) + sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3))*sgn(b*x^2 + a)/d
```

maple [A] time = 0.01, size = 183, normalized size = 0.50

$$\frac{\left(b\,x^2+a\right)\sqrt{2}\,\left(2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+\ln\left(-\frac{-dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\right)d}{4\sqrt{\left(b\,x^2+a\right)^2}\,\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)*d/b/(a/b*d^2)^(1/4)*2^(1/2)*(ln(-(-d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^
(1/4))/(a/b*d^2)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b
*d^2)^(1/4)))
```

maxima [A] time = 3.00, size = 216, normalized size = 0.59

$$\frac{1}{4}\,d\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx+\sqrt{2}\,\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx-\sqrt{2}\,\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}\,b^{\frac{3}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt
(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) +
 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)
*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt
(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/
((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*s
qrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x}}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 57.26, size = 41, normalized size = 0.11

$$2d\,\mathrm{RootSum}\left(256t^4ab^3d^2+1,\left(t\mapsto t\log\left(64t^3ab^2d^2+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] 2*d*RootSum(256*_t**4*a*b**3*d**2 + 1, Lambda(_t, _t*log(64*_t**3*a*b**2*d*
*2 + sqrt(d*x))))
```

3.575 $\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

<u>**Optimal**</u>. Leaf size=368

$$-\frac{\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\cdots$$

<u>**Rubi**</u> **[A]** time = 0.24, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1112, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + ((a + b
*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a
^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

<u>Rule 329</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{\sqrt{a}\,d^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\right.}{\sqrt{a}\,d^2\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \right.}{2\sqrt{a}\,b^{3/2}\sqrt{a^2}}$$

$$= -\frac{\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

$$= -\frac{\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} - $$

Mathematica [A] time = 0.05, size = 178, normalized size = 0.48

$$-\frac{\sqrt{x}\left(a+bx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/2*(Sqrt[x]*(a + b*x^2)*(2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
- 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x]))/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d*x]*Sqrt[(a + b*x^2)
^2])
```

IntegrateAlgebraic [A] time = 30.85, size = 187, normalized size = 0.51

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(-(ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1
/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]]/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[
d])) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqr
t[b]*d*x)]/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d])))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2
)^2/d^4])
```

fricas [A] time = 1.33, size = 165, normalized size = 0.45

$$2\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2d^2\sqrt{-\frac{1}{a^3bd^2}}+dx}\,a^2bd\left(-\frac{1}{a^3bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^2bd\left(-\frac{1}{a^3bd^2}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-\frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(a^3*b*d^2))^(1/4)*arctan(sqrt(a^2*d^2*sqrt(-1/(a^3*b*d^2)) + d*x)*a^
2*b*d*(-1/(a^3*b*d^2))^(3/4) - sqrt(d*x)*a^2*b*d*(-1/(a^3*b*d^2))^(3/4)) +
1/2*(-1/(a^3*b*d^2))^(1/4)*log(a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x)) - 1/
2*(-1/(a^3*b*d^2))^(1/4)*log(-a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x))
```

giac [A] time = 0.26, size = 251, normalized size = 0.68

$$\frac{1}{4}\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{abd}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{abd}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b*d) + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(
a*b*d) + sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a*b*d) - sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b*d))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 182, normalized size = 0.49

$$\frac{\left(b x^2+a\right)\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)\right)}{4\sqrt{\left(b x^2+a\right)^2}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/d*(a/b*d^2)^(1/4)/a*2^(1/2)*(ln((d*x+(a/b
*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1
/4))/(a/b*d^2)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d
^2)^(1/4)))
```

maxima [A] time = 3.06, size = 226, normalized size = 0.61

$$\frac{\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*d^2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2
)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*
sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt
(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt
(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d\,x}\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*((a + b*x^2)^2)^(1/2)),x)
[Out] int(1/((d*x)^(1/2)*((a + b*x^2)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*sqrt((a + b*x**2)**2)), x)
```

3.576 $\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$-\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(\sqrt{2}\right.}{2\sqrt{2}\,a^{5/4}d^{3}}$$

Rubi [A] time = 0.28, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\,\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}\,\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(a*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sq
rt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(5
/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*S
qrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right.}{ad^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt{b}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\right.}{ad^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx,x\right.}{2\sqrt{2}\,a^{5/4}b^{3/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\right.}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{b}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{b}\left(\right.}{\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.12

$$-\frac{2x\left(a+bx^2\right)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^2}{a}\right)}{a(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*x*(a + b*x^2)*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^2)/a)])/(a*(d*x)^(
3/2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 34.07, size = 201, normalized size = 0.49

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{\sqrt{2}a^{5/4}d^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{\sqrt{2}a^{5/4}d^{3/2}}-\frac{2}{ad\sqrt{dx}}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(-2/(a*d*Sqrt[d*x]) + (b^(1/4)*ArcTan[((a^(1/4)*Sqrt[d
])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(
Sqrt[2]*a^(5/4)*d^(3/2)) + (b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(Sqrt[2]*a^(5/4)*d^(3/2))))/(d^2*S
qrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.50, size = 198, normalized size = 0.48

$$\frac{4\,ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,abd\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}-\sqrt{-a^3bd^4\sqrt{-\frac{b}{a^5d^6}}+b^2dx}\,ad\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}}{b}\right)-ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\log\left(a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}+\sqrt{dx}\,b\right)+ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\log\left(-a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}+\sqrt{dx}\,b\right)-4\sqrt{dx}}{2\,ad^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*d^2*x*(-b/(a^5*d^6))^(1/4)*arctan(-(sqrt(d*x)*a*b*d*(-b/(a^5*d^6))
^(1/4) - sqrt(-a^3*b*d^4*sqrt(-b/(a^5*d^6)) + b^2*d*x)*a*d*(-b/(a^5*d^6))^(
1/4))/b) - a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(a^4*d^5*(-b/(a^5*d^6))^(3/4) +
sqrt(d*x)*b) + a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(-a^4*d^5*(-b/(a^5*d^6))^(3/
4) + sqrt(d*x)*b) - 4*sqrt(d*x))/(a*d^2*x)
```

giac [A] time = 0.23, size = 264, normalized size = 0.64

$$-\frac{\left(\frac{8}{\sqrt{dx}\,a}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^2d^2}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^2d^2}\right)\mathrm{sgn}\left(bx^2+a\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(8/(sqrt(d*x)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) + 2*sqrt
(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt
(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x +
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*d^2) + sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^2*d^2))*sgn(b*x^2 + a)/d
```

maple [A] time = 0.01, size = 224, normalized size = 0.54

$$-\frac{\left(b\,x^{2}+a\right)\left(2\sqrt{2}\,\sqrt{dx}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+2\sqrt{2}\,\sqrt{dx}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+\sqrt{2}\,\sqrt{dx}\,\ln\left(-\frac{-dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+8\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\right)}{4\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)/d*(2^(1/2)*(d*x)^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))+2*2^(1/2)*(d*x)^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))+2*2^(1/2)*(d*x)^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d
^2)^(1/4))/(a/b*d^2)^(1/4))+8*(a/b*d^2)^(1/4))/((b*x^2+a)^2)^(1/2)/a/(a/b*d
^2)^(1/4)/(d*x)^(1/2)
```

maxima [A] time = 3.26, size = 234, normalized size = 0.57

$$-\frac{\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a}+\frac{8}{\sqrt{dx}a}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sq
rt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*
```

```
x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sq
rt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d
)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)
*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a + 8/(sqrt(d*x)*a
))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{3/2}\,\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*((a + b*x^2)^2)^(1/2)),x)
[Out] int(1/((d*x)^(3/2)*((a + b*x^2)^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
[Out] Timed out
```

3.577 $\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=414

$$-\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}\left(a+bx^2\right)\log\left(\right.}{2\sqrt{2}\,a^{7}}$$

Rubi [A] time = 0.28, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(3*a*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(
3/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*
a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/4)*(a + b*x^2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
 b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{\right.}{ad^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x,\right.}{a^{3/2}d^4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,d\right.}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,d^{5/2}\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{\right.}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{3}}{}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.13

$$-\frac{2x\left(a+bx^2\right){}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*x*(a + b*x^2)*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^2)/a)])/(3*a*(d*x)
^(5/2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 44.93, size = 204, normalized size = 0.49

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{\sqrt{2}a^{7/4}d^{5/2}}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{\sqrt{2}a^{7/4}d^{5/2}}-\frac{2}{3ad(dx)^{3/2}}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(-2/(3*a*d*(d*x)^(3/2)) + (b^(3/4)*ArcTan[((a^(1/4)*Sq
rt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]
])/(Sqrt[2]*a^(7/4)*d^(5/2)) - (b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(Sqrt[2]*a^(7/4)*d^(5/2))))/(d
^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.96, size = 227, normalized size = 0.55

$$-\frac{12\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^5bd^7\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^4d^6\sqrt{-\frac{b^3}{a^7d^{10}}}+b^2dx}\,a^5d^7\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+3\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\log\left(a^2d^3\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}\,b\right)-3\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\log\left(-a^2d^3\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}\,b\right)+4\sqrt{dx}}{6\,ad^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*arctan(-(sqrt(d*x)*a^5*b*d^7*(-b
^3/(a^7*d^10))^(3/4) - sqrt(a^4*d^6*sqrt(-b^3/(a^7*d^10)) + b^2*d*x)*a^5*d^
7*(-b^3/(a^7*d^10))^(3/4))/b^3) + 3*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*log(a
^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) - 3*a*d^3*x^2*(-b^3/(a^7*d^10
))^(1/4)*log(-a^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) + 4*sqrt(d*x))
/(a*d^3*x^2)
```

giac [A] time = 0.24, size = 256, normalized size = 0.62

$$-\frac{1}{12}\left(\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2d^3}-\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2d^3}+\frac{8}{\sqrt{dx}\,ad^2x}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*d^3) + 6*sqrt(2)*(a*b^3*d^2)^(1/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a^2*d^3) + 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4
)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*
x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) + 8/(sqrt(
d*x)*a*d^2*x))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 239, normalized size = 0.58

$$-\frac{\left(b x^2+a\right)\left(8 a d^2+6\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\,(dx)^{\frac{3}{2}} b \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+6\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\,(dx)^{\frac{3}{2}} b \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+3\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\,(dx)^{\frac{3}{2}} b \ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)\right)}{12\sqrt{\left(b x^2+a\right)^2}\,(dx)^{\frac{3}{2}} a^2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/12*(b*x^2+a)/d^3*(3*b*(a/b*d^2)^(1/4)*2^(1/2)*(d*x)^(3/2)*ln((d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+6*b*(a/b*d^2)^(1/4)*2^(1/2)*(d*x)^(3/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+6*b*(a/b*d^2)^(1/
4)*2^(1/2)*(d*x)^(3/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))+8*d^2*a)/((b*x^2+a)^2)^(1/2)/a^2/(d*x)^(3/2)
```

maxima [A] time = 3.08, size = 242, normalized size = 0.58

$$-\frac{\frac{3\left(\frac{\sqrt{2}\, b^{\frac{3}{4}} \log\left(\sqrt{b}\, dx+\sqrt{2}\left(a d^2\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(a d^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\, b^{\frac{3}{4}} \log\left(\sqrt{b}\, dx-\sqrt{2}\left(a d^2\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(a d^2\right)^{\frac{3}{4}}}+\frac{2\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(a d^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\, d}\,\sqrt{a}\, d}+\frac{2\sqrt{2}\, b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(a d^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\, d}\,\sqrt{a}\, d}\right)}{a}+\frac{8}{(dx)^{\frac{3}{2}} a}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)
*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*d*x - sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)
*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))
```

```
/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/
sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d))/a + 8/((d*x)^
(3/2)*a))/d
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{5/2}\,\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*((a + b*x^2)^2)^(1/2)),x)
[Out] int(1/((d*x)^(5/2)*((a + b*x^2)^2)^(1/2)), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
[Out] Timed out
```

3.578 $\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=459

$$\frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\right.}{2\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.33, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(5*a*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*b
*(a + b*x^2))/(a^2*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x
^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(
9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x^2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*
Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4)*(a + b*
x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(2*Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab\right.\right.}{a^2d^4\sqrt{}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(2b^2\left(ab\right.\right.}{}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b^{3/2}\left(ab\right.\right.}{}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt[4]{b}\left(ab\right.\right.}{}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{5/4}\left(a+\right.}{}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{5/4}\left(a+\right.}{\sqrt{2}\,a^{9/}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.11

$$-\frac{2x\left(a+bx^2\right)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a(dx)^{7/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*x*(a + b*x^2)*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^2)/a)])/(5*a*(d*x
)^(7/2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 59.45, size = 220, normalized size = 0.48

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{\sqrt{2}a^{9/4}d^{7/2}}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{\sqrt{2}a^{9/4}d^{7/2}}-\frac{2\left(ad^2-5bd^2x^2\right)}{5a^2d^3(dx)^{5/2}}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-2*(a*d^2 - 5*b*d^2*x^2))/(5*a^2*d^3*(d*x)^(5/2)) -
(b^(5/4)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/
(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(Sqrt[2]*a^(9/4)*d^(7/2)) - (b^(5/4)*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(S
qrt[2]*a^(9/4)*d^(7/2)))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.51, size = 253, normalized size = 0.55

$$-\frac{20\,a^2d^4x^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^2b^4d^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}-\sqrt{-a^5b^5d^8\sqrt{-\frac{b^5}{a^9d^{14}}}+b^8dx}\,a^2d^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}}{b^5}\right)-5\,a^2d^4x^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}\log\left(a^7d^{11}\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{3}{4}}+\sqrt{dx}\,b^4\right)+5\,a^2d^4x^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}\log\left(-a^7d^{11}\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{3}{4}}+\sqrt{dx}\,b^4\right)-4\left(5\,bx^2-a\right)\sqrt{dx}}{10\,a^2d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*d^4*x^3*(-b^5/(a^9*d^14))^(1/4)*arctan(-(sqrt(d*x)*a^2*b^4*d^
3*(-b^5/(a^9*d^14))^(1/4) - sqrt(-a^5*b^5*d^8*sqrt(-b^5/(a^9*d^14)) + b^8*d
*x)*a^2*d^3*(-b^5/(a^9*d^14))^(1/4))/b^5) - 5*a^2*d^4*x^3*(-b^5/(a^9*d^14))
^(1/4)*log(a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)*b^4) + 5*a^2*d^4*x^
3*(-b^5/(a^9*d^14))^(1/4)*log(-a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)
*b^4) - 4*(5*b*x^2 - a)*sqrt(d*x))/(a^2*d^4*x^3)
```

giac [A] time = 0.32, size = 284, normalized size = 0.62

$$\frac{1}{20}\left(\frac{10\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}bd^{5}}+\frac{10\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}bd^{5}}-\frac{5\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{3}bd^{5}}+\frac{5\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{3}bd^{5}}+\frac{8\left(5bd^{2}x^{2}-ad^{2}\right)}{\sqrt{dx}\,a^{2}d^{5}x^{2}}\right)\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*(10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d^5) + 10*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^3*b*d^5) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) + 5*sqrt(2)*(a*b^3*d^2)^(3/4)
*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) +
 8*(5*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^2*d^5*x^2))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 251, normalized size = 0.55

$$\frac{\left(bx^{2}+a\right)\left(40\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}bd^{2}x^{2}-8\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}ad^{2}+10\sqrt{2}\,(dx)^{\frac{5}{2}}b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+10\sqrt{2}\,(dx)^{\frac{5}{2}}b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+5\sqrt{2}\,(dx)^{\frac{5}{2}}b\ln\left(-\frac{-dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^{2}}{b}}}{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)\right)}{20\sqrt{\left(bx^{2}+a\right)^{2}}\,\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}(dx)^{\frac{5}{2}}a^{2}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/20*(b*x^2+a)/d^3*(5*b*2^(1/2)*(d*x)^(5/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)
^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
/b*d^2)^(1/2)))+10*b*2^(1/2)*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d
^2)^(1/4))/(a/b*d^2)^(1/4))+10*b*2^(1/2)*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^
(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+40*b*(a/b*d^2)^(1/4)*d^2*x^2-8*d^2*
a*(a/b*d^2)^(1/4))/((b*x^2+a)^2)^(1/2)/a^2/(a/b*d^2)^(1/4)/(d*x)^(5/2)
```

maxima [A] time = 3.03, size = 259, normalized size = 0.56

$$\frac{\frac{5b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^{2}d^{2}}+\frac{8\left(5bd^{2}x^{2}-ad^{2}\right)}{(dx)^{\frac{5}{2}}a^{2}d^{2}}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(5*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) +
2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt
(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqr
t(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
- sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(
a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(
1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*d^2) + 8*
(5*b*d^2*x^2 - a*d^2)/((d*x)^(5/2)*a^2*d^2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{7/2}\,\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/((d*x)^(7/2)*((a + b*x^2)^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.579 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=551

$$-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}\left(a+bx^2\right.}{80b^3\sqrt{a^2+2abx^2+b^2}}$$

Rubi [A] time = 0.40, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}\left(a+bx^2\right)}{80b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-13*d^3*(d*x)^(9/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
13/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a*d^7*Sqrt[
d*x]*(a + b*x^2))/(16*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*d^5*(d*x)
^(5/2)*(a + b*x^2))/(80*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)
*d^(15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])]])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*a^(5/4)*
d^(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])]])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (117*a^(5/4)*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(117d^4\left(ab+\right.\right.}{32b^2\sqrt{a^2 -}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{117d^5(dx)^{5/2}}{80b^3\sqrt{a^2+2a}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}}{16b^4\sqrt{a^2+2a}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}}{16b^4\sqrt{a^2+2a}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}}{16b^4\sqrt{a^2+2a}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}}{16b^4\sqrt{a^2+2a}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}}{16b^4\sqrt{a^2+2a}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}}{16b^4\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.16, size = 498, normalized size = 0.90

$$\frac{d^7\sqrt{dx}\left(-585\sqrt{2}a^{5/4}b^2x^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+585\sqrt{2}a^{5/4}b^2x^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-1170\sqrt{2}a^{9/4}bx^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+1170\sqrt{2}a^{9/4}bx^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-1170\sqrt{2}a^{5/4}\left(a+bx^2\right)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+1170\sqrt{2}a^{5/4}\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)-585\sqrt{2}a^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+585\sqrt{2}a^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-4680a^3\sqrt[4]{b}\sqrt{x}-8424a^2b^{5/4}x^{5/2}-3328ab^{9/4}x^{9/2}+256b^{13/4}x^{13/2}\right)}{640b^{17/4}\sqrt{x}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d^7*Sqrt[d*x]*(-4680*a^3*b^(1/4)*Sqrt[x] - 8424*a^2*b^(5/4)*x^(5/2) - 3328
*a*b^(9/4)*x^(9/2) + 256*b^(13/4)*x^(13/2) - 1170*Sqrt[2]*a^(5/4)*(a + b*x^
2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 1170*Sqrt[2]*a^(5/4)*(
a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 585*Sqrt[2]*a^
(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 1170*Sq
rt[2]*a^(9/4)*b*x^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x] - 585*Sqrt[2]*a^(5/4)*b^2*x^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x] + 585*Sqrt[2]*a^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x] + 1170*Sqrt[2]*a^(9/4)*b*x^2*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 585*Sqrt[2]*a^(5/4)*b^2*x^4*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(640*b^(17/4)*Sqrt[x]
*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 111.37, size = 269, normalized size = 0.49

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{117a^{5/4}d^{15/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}b^{17/4}}+\frac{117a^{5/4}d^{15/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{32\sqrt{2}b^{17/4}}+\frac{d^5\sqrt{dx}\left(-585a^3d^6-1053a^2bd^6x^2-416ab^2d^6x^4+32b^3d^6x^6\right)}{80b^4\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((d^5*Sqrt[d*x]*(-585*a^3*d^6 - 1053*a^2*b*d^6*x^2 - 4
16*a*b^2*d^6*x^4 + 32*b^3*d^6*x^6))/(80*b^4*(a*d^2 + b*d^2*x^2)^2) - (117*a
^(5/4)*d^(15/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt
[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(32*Sqrt[2]*b^(17/4)) + (117*a^(5/4)*
d^(15/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + S
qrt[b]*d*x)])/(32*Sqrt[2]*b^(17/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.02, size = 341, normalized size = 0.62

$$\frac{2340\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^6x^4+2ab^5x^2+a^2b^4\right)\arctan\left(-\frac{\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}ab^{13}d^7-\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{a^2d^{15}x+\sqrt{-\frac{a^5d^{30}}{b^{17}}}b^8}b^{13}}{a^5d^{30}}\right)+585\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^6x^4+2ab^5x^2+a^2b^4\right)\log\left(117\sqrt{dx}ad^7+117\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}b^4\right)-585\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^6x^4+2ab^5x^2+a^2b^4\right)\log\left(117\sqrt{dx}ad^7-117\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}b^4\right)+4\left(32b^3d^7x^6-416ab^2d^7x^4-1053a^2bd^7x^2-585a^3d^7\right)\sqrt{dx}}{320\left(b^6x^4+2ab^5x^2+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/320*(2340*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*arctan
(-((-a^5*d^30/b^17)^(3/4)*sqrt(d*x)*a*b^13*d^7 - (-a^5*d^30/b^17)^(3/4)*sqr
t(a^2*d^15*x + sqrt(-a^5*d^30/b^17)*b^8)*b^13)/(a^5*d^30)) + 585*(-a^5*d^30
/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 + 11
7*(-a^5*d^30/b^17)^(1/4)*b^4) - 585*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b
^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 - 117*(-a^5*d^30/b^17)^(1/4)*b^4)
 + 4*(32*b^3*d^7*x^6 - 416*a*b^2*d^7*x^4 - 1053*a^2*b*d^7*x^2 - 585*a^3*d^7
)*sqrt(d*x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

giac [A] time = 0.35, size = 419, normalized size = 0.76

$$\frac{1}{640}\,d^{7}\left(\frac{1170\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{1170\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{585\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{585\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{40\left(25\,\sqrt{dx}\,a^{2}bd^{4}x^{2}+21\,\sqrt{dx}\,a^{3}d^{4}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{2}b^{4}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{256\left(\sqrt{dx}\,b^{12}d^{10}x^{2}-15\,\sqrt{dx}\,ab^{11}d^{10}\right)}{b^{15}d^{10}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/640*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*d^4*x^2 + a*d^4)) +
 1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*d^4*x^2 + a*d^4)) + 585*sqrt
(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(b^5*sgn(b*d^4*x^2 + a*d^4)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*a*lo
g(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*sgn(b*d^4*x
^2 + a*d^4)) - 40*(25*sqrt(d*x)*a^2*b*d^4*x^2 + 21*sqrt(d*x)*a^3*d^4)/((b*d
^2*x^2 + a*d^2)^2*b^4*sgn(b*d^4*x^2 + a*d^4)) + 256*(sqrt(d*x)*b^12*d^10*x^
2 - 15*sqrt(d*x)*a*b^11*d^10)/(b^15*d^10*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 737, normalized size = 1.34

$$\frac{\left(1170\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ab^{2}d^{2}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+1170\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ab^{2}d^{2}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+585\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ab^{2}d^{2}x^{4}\ln\left(\frac{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}{dx-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)-3840\sqrt{dx}\,ab^{2}d^{2}x^{4}+2340\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}bd^{2}x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+2340\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}bd^{2}x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+1170\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}bd^{2}x^{2}\ln\left(\frac{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}{dx-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)-7680\sqrt{dx}\,a^{2}bd^{2}x^{2}+256\left(dx\right)^{\frac{5}{2}}b^{3}x^{4}+1170\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{3}d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+1170\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{3}d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+585\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{3}d^{2}\ln\left(\frac{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}{dx-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)-4680\sqrt{dx}\,a^{3}d^{2}+512\left(dx\right)^{\frac{5}{2}}ab^{2}x^{2}-744\left(dx\right)^{\frac{5}{2}}a^{2}b\right)\left(bx^{2}+a\right)d^{5}}{640\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{3}{2}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/640*(1170*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(
a/b*d^2)^(1/4)*2^(1/2)*x^4*a*b^2*d^2+585*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a*b^2*d^2+1170*(a/b*d^2)^(1/4)*2^(1/
```

```
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a*b^2*
d^2+256*(d*x)^(5/2)*x^4*b^3+2340*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4
))/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*x^2*a^2*b*d^2+1170*(a/b*d^2)^(1
/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d
*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^2*b*d^2+2340
*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))*x^2*a^2*b*d^2+512*(d*x)^(5/2)*x^2*a*b^2-3840*(d*x)^(1/2)*x^4*a*
b^2*d^2+1170*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*
(a/b*d^2)^(1/4)*2^(1/2)*a^3*d^2+585*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^
2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^3*d^2+1170*(a/b*d^2)^(1/4)*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^3*d^2-744*(d*x)^(5/
2)*a^2*b-7680*(d*x)^(1/2)*x^2*a^2*b*d^2-4680*(d*x)^(1/2)*a^3*d^2)*d^5*(b*x^
2+a)/b^4/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^{2} d^{\frac{15}{2}} x^{\frac{5}{2}}}{2 \left(a b^{4} x^{2} + a^{2} b^{3} + \left(b^{5} x^{2} + a b^{4}\right) x^{2}\right)} - 2 \, a d^{\frac{15}{2}} \int \frac{x^{\frac{3}{2}}}{b^{4} x^{2} + a b^{3}}\,{d x} + d^{\frac{15}{2}} \int \frac{x^{\frac{7}{2}}}{b^{3} x^{2} + a b^{2}}\,{d x} + \frac{21 \, {\left(\frac{2 \, \sqrt{2} a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} + 2 \, \sqrt{b} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}}} + \frac{2 \, \sqrt{2} a^{\frac{3}{2}} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} - 2 \, \sqrt{b} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}}} + \frac{\sqrt{2} a^{\frac{5}{4}} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2} a^{\frac{5}{4}} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{b^{\frac{1}{4}}}\right)} d^{\frac{15}{2}}}{128 \, b^{4}} - \frac{17 \, a^{2} b d^{\frac{15}{2}} x^{\frac{5}{2}} + 21 \, a^{3} d^{\frac{15}{2}} \sqrt{x}}{16 \, {\left(b^{6} x^{4} + 2 \, a b^{5} x^{2} + a^{2} b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*d^(15/2)*x^(5/2)/(a*b^4*x^2 + a^2*b^3 + (b^5*x^2 + a*b^4)*x^2) - 2
*a*d^(15/2)*integrate(x^(3/2)/(b^4*x^2 + a*b^3), x) + d^(15/2)*integrate(x^
(7/2)/(b^3*x^2 + a*b^2), x) + 21/128*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(s
qrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + s
qrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b
^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/b^(1/4))*d^(15/2)/b^4 - 1/16*(17*a^2*b*d^(15/2)*x^(5/2) + 21*a^3*
d^(15/2)*sqrt(x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x\right)}^{15/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Timed out
```

3.580 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\, dx$

Optimal. Leaf size=504

$$-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\text{l}}{64\sqrt{2}}$$

Rubi [A] time = 0.37, antiderivative size = 504, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-11*d^3*(d*x)^(7/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
11/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^5*(d*x)^(3
/2)*(a + b*x^2))/(48*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*a^(3/4)*d^(
13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13
/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13/2
)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7
7*a^(3/4)*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(77d^4\left(ab+b\right.\right.}{32b^2\sqrt{a^2+}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}}{48b^3\sqrt{a^2+2a}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}}{48b^3\sqrt{a^2+2a}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}}{48b^3\sqrt{a^2+2a}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}}{48b^3\sqrt{a^2+2a}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}}{48b^3\sqrt{a^2+2a}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}}{48b^3\sqrt{a^2+2a}}$$

Mathematica [C] time = 0.03, size = 88, normalized size = 0.17

$$-\frac{2d^5(dx)^{3/2}\left(-77a^2+77\left(a+bx^2\right)^2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)-55abx^2-5b^2x^4\right)}{15b^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-2*d^5*(d*x)^(3/2)*(-77*a^2 - 55*a*b*x^2 - 5*b^2*x^4 + 77*(a + b*x^2)^2*Hy
pergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)]))/(15*b^3*(a + b*x^2)*Sqrt[(a +
b*x^2)^2])
```

IntegrateAlgebraic [A] time = 97.89, size = 255, normalized size = 0.51

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{77a^{3/4}d^{13/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}\,b^{15/4}}+\frac{77a^{3/4}d^{13/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{32\sqrt{2}\,b^{15/4}}+\frac{d^5(dx)^{3/2}\left(77a^2d^4+121abd^4x^2+32b^2d^4x^4\right)}{48b^3\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((d^5*(d*x)^(3/2)*(77*a^2*d^4 + 121*a*b*d^4*x^2 + 32*b
^2*d^4*x^4))/(48*b^3*(a*d^2 + b*d^2*x^2)^2) + (77*a^(3/4)*d^(13/2)*ArcTan[(
(a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4))
)/Sqrt[d*x]])/(32*Sqrt[2]*b^(15/4)) + (77*a^(3/4)*d^(13/2)*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(32*Sqrt[2]
*b^(15/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.95, size = 341, normalized size = 0.68

$$\frac{924\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\left(b^5x^4+2ab^4x^2+a^2b^3\right)\arctan\left(-\frac{\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^4d^{19}-\sqrt{a^4d^{39}x-\sqrt{-\frac{a^3d^{26}}{b^{15}}}\,a^3b^7d^{26}}\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}b^4}{a^3d^{26}}\right)-231\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\left(b^5x^4+2ab^4x^2+a^2b^3\right)\log\left(456533\sqrt{dx}\,a^2d^{19}+456533\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{3}{4}}b^{11}\right)+231\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\left(b^5x^4+2ab^4x^2+a^2b^3\right)\log\left(456533\sqrt{dx}\,a^2d^{19}-456533\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{3}{4}}b^{11}\right)+4\left(32b^2d^6x^5+121abd^6x^3+77a^2d^6x\right)\sqrt{dx}}{192\left(b^5x^4+2ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/192*(924*(-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*arctan(
-((-a^3*d^26/b^15)^(1/4)*sqrt(d*x)*a^2*b^4*d^19 - sqrt(a^4*d^39*x - sqrt(-a
```

```
^3*d^26/b^15)*a^3*b^7*d^26)*(-a^3*d^26/b^15)^(1/4)*b^4)/(a^3*d^26)) - 231*(
-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(d*x
)*a^2*d^19 + 456533*(-a^3*d^26/b^15)^(3/4)*b^11) + 231*(-a^3*d^26/b^15)^(1/
4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(d*x)*a^2*d^19 - 456533
*(-a^3*d^26/b^15)^(3/4)*b^11) + 4*(32*b^2*d^6*x^5 + 121*a*b*d^6*x^3 + 77*a^
2*d^6*x)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.41, size = 399, normalized size = 0.79

$$\frac{1}{384}\,d^6\left(\frac{256\,\sqrt{dx}\,x}{b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{24\left(19\,\sqrt{dx}\,abd^4x^3+15\,\sqrt{dx}\,a^2d^4x\right)}{\left(bd^2x^2+ad^2\right)^2b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{462\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{462\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{231\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{231\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*d^6*(256*sqrt(d*x)*x/(b^3*sgn(b*d^4*x^2 + a*d^4)) + 24*(19*sqrt(d*x)*
a*b*d^4*x^3 + 15*sqrt(d*x)*a^2*d^4*x)/((b*d^2*x^2 + a*d^2)^2*b^3*sgn(b*d^4*
x^2 + a*d^4)) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*d*sgn(b*d^4*x^2 + a*d^4
)) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*d*sgn(b*d^4*x^2 + a*d^4)) + 231*s
qrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(b^6*d*sgn(b*d^4*x^2 + a*d^4)) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*d*sgn(b*d^
4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 679, normalized size = 1.35

$$\frac{\left(-462\sqrt{2}\,ab^2d^4x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-462\sqrt{2}\,ab^2d^4x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-231\sqrt{2}\,ab^2d^4x^4\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)-924\sqrt{2}\,a^2bd^4x^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-924\sqrt{2}\,a^2bd^4x^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-462\sqrt{2}\,a^2bd^4x^2\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+256\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{3}{2}}b^3d^2x^4-462\sqrt{2}\,a^3d^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-462\sqrt{2}\,a^3d^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-231\sqrt{2}\,a^3d^4\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+512\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{3}{2}}ab^2d^2x^2+616\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{3}{2}}a^2bd^2+456\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{7}{2}}ab^2\right)\left(bx^2+a\right)d^3}{384\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/384*(256*(a/b*d^2)^(1/4)*(d*x)^(3/2)*x^4*b^3*d^2-231*2^(1/2)*ln(-(-d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a*b^2*d^4-462*2^(1/2)*arctan((2^(1/2
)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a*b^2*d^4-462*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a*b^2*d^4+
456*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a*b^2+512*(a/b*d^2)^(1/4)*(d*x)^(3/2)*x^2*a
*b^2*d^2-462*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2
)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^2
*b*d^4-924*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(
1/4))*x^2*a^2*b*d^4-924*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
```

```
)/(a/b*d^2)^(1/4))*x^2*a^2*b*d^4+616*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^2*b*d^2-
231*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/
(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^3*d^4-462*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^3*d^4-4
62*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^
3*d^4)*d^3*(b*x^2+a)/(a/b*d^2)^(1/4)/b^4/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2 d^{\frac{13}{2}} x^{\frac{3}{2}}}{2\left(ab^4x^2 + a^2b^3 + \left(b^5x^2 + ab^4\right)x^2\right)} - 2\,ad^{\frac{13}{2}}\int \frac{\sqrt{x}}{b^4x^2 + ab^3}\,dx + d^{\frac{13}{2}}\int \frac{x^{\frac{5}{2}}}{b^3x^2 + ab^2}\,dx + \frac{19\,ad^{\frac{13}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} - 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}x + \sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}x + \sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,b^3} + \frac{19\,abd^{\frac{13}{2}}x^{\frac{7}{2}} + 23\,a^2d^{\frac{13}{2}}x^{\frac{3}{2}}}{16\left(b^5x^4 + 2\,ab^4x^2 + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*d^(13/2)*x^(3/2)/(a*b^4*x^2 + a^2*b^3 + (b^5*x^2 + a*b^4)*x^2) - 2
*a*d^(13/2)*integrate(sqrt(x)/(b^4*x^2 + a*b^3), x) + d^(13/2)*integrate(x^
(5/2)/(b^3*x^2 + a*b^2), x) + 19/128*a*d^(13/2)*(2*sqrt(2)*arctan(1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b
))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^3 + 1/16*(19*a*b*d^(13/2)*x^(7/2) +
 23*a^2*d^(13/2)*x^(3/2))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{13/2}}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

3.581 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=504

$$-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}\,d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{\right.}{64\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.37, antiderivative size = 504, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}\,d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{a}\,d^{11/2}\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}\,d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{32\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{a}\,d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{32\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-9*d^3*(d*x)^(5/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(9
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^5*Sqrt[d*x]*
(a + b*x^2))/(16*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(3
2*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^(1/4)*d^(11/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*
Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2)*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^
(1/4)*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(45d^4\left(ab+b^2x\right.\right.}{32b^2\sqrt{a^2+2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a\right.}{16b^3\sqrt{a^2+2ab}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a\right.}{16b^3\sqrt{a^2+2ab}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a\right.}{16b^3\sqrt{a^2+2ab}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a\right.}{16b^3\sqrt{a^2+2ab}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a\right.}{16b^3\sqrt{a^2+2ab}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a\right.}{16b^3\sqrt{a^2+2ab}}$$

Mathematica [A] time = 0.17, size = 484, normalized size = 0.96

$$\frac{15a^2(dx)^{11/2}\left(a+bx^2\right)}{4b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{45\sqrt[4]{a}\,(dx)^{11/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} - \frac{45\sqrt[4]{a}\,(dx)^{11/2}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{45\sqrt[4]{a}\,(dx)^{11/2}\left(a+bx^2\right)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} - \frac{45\sqrt[4]{a}\,(dx)^{11/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} - \frac{15a(dx)^{11/2}\left(a+bx^2\right)^2}{16b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{6a(dx)^{11/2}\left(a+bx^2\right)}{b^2x^3\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{2(dx)^{11/2}\left(a+bx^2\right)}{bx\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (15*a^2*(d*x)^(11/2)*(a + b*x^2))/(4*b^3*x^5*((a + b*x^2)^2)^(3/2)) + (6*a*
(d*x)^(11/2)*(a + b*x^2))/(b^2*x^3*((a + b*x^2)^2)^(3/2)) + (2*(d*x)^(11/2)
*(a + b*x^2))/(b*x*((a + b*x^2)^2)^(3/2)) - (15*a*(d*x)^(11/2)*(a + b*x^2)^
2)/(16*b^3*x^5*((a + b*x^2)^2)^(3/2)) + (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2
)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*b^(13/4)*x^(
11/2)*((a + b*x^2)^2)^(3/2)) - (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*b^(13/4)*x^(11/2)*((a
 + b*x^2)^2)^(3/2)) + (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*b^(13/4)*x^(11/2)
*((a + b*x^2)^2)^(3/2)) - (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*b^(13/4)*x^(1
1/2)*((a + b*x^2)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 93.38, size = 260, normalized size = 0.52

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{45a^2d^9\sqrt{dx}+81abd^7(dx)^{5/2}+32b^2d^5(dx)^{9/2}}{16b^3\left(ad^2+bd^2x^2\right)^2}+\frac{45\sqrt[4]{a}\,d^{11/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}\,b^{13/4}}-\frac{45\sqrt[4]{a}\,d^{11/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{32\sqrt{2}\,b^{13/4}}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((45*a^2*d^9*Sqrt[d*x] + 81*a*b*d^7*(d*x)^(5/2) + 32*b
^2*d^5*(d*x)^(9/2))/(16*b^3*(a*d^2 + b*d^2*x^2)^2) + (45*a^(1/4)*d^(11/2)*A
rcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a
^(1/4)))/Sqrt[d*x]])/(32*Sqrt[2]*b^(13/4)) - (45*a^(1/4)*d^(11/2)*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(32*
Sqrt[2]*b^(13/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.01, size = 305, normalized size = 0.61

$$-\frac{180\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\arctan\left(-\frac{\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{ad^{22}}{b^{13}}}\,b^6}\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{3}{4}}b^{10}}{ad^{22}}\right)+45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\log\left(45\sqrt{dx}\,d^5+45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)-45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\log\left(45\sqrt{dx}\,d^5-45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)-4\left(32\,b^2d^5x^4+81\,abd^5x^2+45\,a^2d^5\right)\sqrt{dx}}{64\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(180*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*arctan(-(
(-a*d^22/b^13)^(3/4)*sqrt(d*x)*b^10*d^5 - sqrt(d^11*x + sqrt(-a*d^22/b^13)*
b^6)*(-a*d^22/b^13)^(3/4)*b^10)/(a*d^22)) + 45*(-a*d^22/b^13)^(1/4)*(b^5*x^
4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sqrt(d*x)*d^5 + 45*(-a*d^22/b^13)^(1/4)*b
^3) - 45*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sqrt
(d*x)*d^5 - 45*(-a*d^22/b^13)^(1/4)*b^3) - 4*(32*b^2*d^5*x^4 + 81*a*b*d^5*x
^2 + 45*a^2*d^5)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.31, size = 385, normalized size = 0.76

$$-\frac{1}{128}\,d^{5}\left(\frac{90\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{4}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{90\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{4}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{45\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{4}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{45\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{4}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{256\,\sqrt{dx}}{b^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{8\left(17\,\sqrt{dx}\,abd^{4}x^{2}+13\,\sqrt{dx}\,a^{2}d^{4}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{2}b^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^5*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*d^4*x^2 + a*d^4)) + 90
*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b
^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(b^4*sgn(b*d^4*x^2 + a*d^4)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*sgn(b*d^4*x^2 + a*d^4))
- 256*sqrt(d*x)/(b^3*sgn(b*d^4*x^2 + a*d^4)) - 8*(17*sqrt(d*x)*a*b*d^4*x^2
+ 13*sqrt(d*x)*a^2*d^4)/((b*d^2*x^2 + a*d^2)^2*b^3*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 696, normalized size = 1.38

$$-\frac{\left(90\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}d^{2}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+90\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}d^{2}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+45\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}d^{2}x^{4}\ln\left(\frac{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}{dx-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)-256\sqrt{dx}\,b^{2}d^{2}x^{4}+180\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,abd^{2}x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+180\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,abd^{2}x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+90\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,abd^{2}x^{2}\ln\left(\frac{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}{dx-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)-512\sqrt{dx}\,abd^{2}x^{2}+90\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+90\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+45\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{2}\ln\left(\frac{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}{dx-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)-360\sqrt{dx}\,a^{2}d^{2}-136\left(dx\right)^{\frac{5}{2}}ab\right)\left(bx^{2}+a\right)d^{3}}{128\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128*(45*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))*x^4*b^2*d^2+90*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/
b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*b^2*d^2+90*(a/b*d^2)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*b^2*d^2+90*(a/
b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^
(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a*b*d
^2+180*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))
/(a/b*d^2)^(1/4))*x^2*a*b*d^2+180*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^2-256*(d*x)^(1/2)*x^
```

```
4*b^2*d^2+45*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(
1/2)))*a^2*d^2+90*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*
d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^2+90*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^2-136*(d*x)^(5/2)*
a*b-512*(d*x)^(1/2)*x^2*a*b*d^2-360*(d*x)^(1/2)*a^2*d^2)*d^3*(b*x^2+a)/b^3/
((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ad^{\frac{11}{2}}x^{\frac{5}{2}}}{2\left(ab^3x^2+a^2b^2+\left(b^4x^2+ab^3\right)x^2\right)}+d^{\frac{11}{2}}\int\frac{x^{\frac{3}{2}}}{b^3x^2+ab^2}\,dx-\frac{13\left(\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}\right)d^{\frac{11}{2}}}{128\,b^3}+\frac{9\,abd^{\frac{11}{2}}x^{\frac{5}{2}}+13\,a^2d^{\frac{11}{2}}\sqrt{x}}{16\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*d^(11/2)*x^(5/2)/(a*b^3*x^2 + a^2*b^2 + (b^4*x^2 + a*b^3)*x^2) + d^(1
1/2)*integrate(x^(3/2)/(b^3*x^2 + a*b^2), x) - 13/128*(2*sqrt(2)*sqrt(a)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqr
t(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(
b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/b^(1/4))*d^(11/2)/b^3 + 1/16*(9*a*b*d^(11/2)*x^(
5/2) + 13*a^2*d^(11/2)*sqrt(x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{11/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

3.582 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=458

$$-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}\right.}{64\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.33, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1112, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{32\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{32\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-7*d^3*(d*x)^(3/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(7
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9/2)*(a + b
*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2
]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9
/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^4\left(ab+b^2x\right.\right.}{32b^2\sqrt{a^2+2\ldots}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^3\left(ab+b^2x\right.\right.}{16b\ldots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(21d^3\left(ab+b^2x\right.\right.}{32b\ldots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^{9/2}\left(ab+b\ldots\right.\right.}{64\ldots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{21d^{9/2}\left(a+bx^2\ldots\right.}{64\ldots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21d^{9/2}\left(a+bx^2\right)\ldots}{32\sqrt{2}\sqrt[4]{a}\,b^{11/4}\ldots}$$

Mathematica [C] time = 0.03, size = 84, normalized size = 0.18

$$\frac{2d^3(dx)^{3/2}\left(7\left(a+bx^2\right)^2 {}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(7a+5bx^2\right)\right)}{5ab^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*d^3*(d*x)^(3/2)*(-(a*(7*a + 5*b*x^2)) + 7*(a + b*x^2)^2*Hypergeometric2F
1[3/4, 3, 7/4, -((b*x^2)/a)]))/(5*a*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 89.46, size = 242, normalized size = 0.53

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{21d^{9/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}}-\frac{21d^{9/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}}+\frac{-7ad^7(dx)^{3/2}-11bd^5(dx)^{7/2}}{16b^2\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-7*a*d^7*(d*x)^(3/2) - 11*b*d^5*(d*x)^(7/2))/(16*b^2
*(a*d^2 + b*d^2*x^2)^2) - (21*d^(9/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^
(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(32*Sqrt[2]*a^(
1/4)*b^(11/4)) - (21*d^(9/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[
d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(32*Sqrt[2]*a^(1/4)*b^(11/4))))/(d^2*Sqrt
[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.73, size = 312, normalized size = 0.68

$$-\frac{84\left(b^4x^4+2ab^3x^2+a^2b^2\right)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^3d^{13}-\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{ab^{11}}}\,ab^5d^{18}}\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}b^3}{d^{18}}\right)-21\left(b^4x^4+2ab^3x^2+a^2b^2\right)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\log\left(9261\sqrt{dx}\,d^{13}+9261\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{3}{4}}ab^8\right)+21\left(b^4x^4+2ab^3x^2+a^2b^2\right)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\log\left(9261\sqrt{dx}\,d^{13}-9261\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{3}{4}}ab^8\right)+4\left(11\,bd^4x^3+7\,ad^4x\right)\sqrt{dx}}{64\left(b^4x^4+2ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(84*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*arctan(-
((-d^18/(a*b^11))^(1/4)*sqrt(d*x)*b^3*d^13 - sqrt(d^27*x - sqrt(-d^18/(a*b^
```

```
11))*a*b^5*d^18)*(-d^18/(a*b^11))^(1/4)*b^3)/d^18) - 21*(b^4*x^4 + 2*a*b^3*
x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*log(9261*sqrt(d*x)*d^13 + 9261*(-d^18
/(a*b^11))^(3/4)*a*b^8) + 21*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^
11))^(1/4)*log(9261*sqrt(d*x)*d^13 - 9261*(-d^18/(a*b^11))^(3/4)*a*b^8) + 4
*(11*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.32, size = 380, normalized size = 0.83

$$-\frac{1}{128}\,d^4\left(\frac{8\left(11\sqrt{dx}\,bd^4x^3+7\sqrt{dx}\,ad^4x\right)}{\left(bd^2x^2+ad^2\right)^2 b^2\operatorname{sgn}\left(bd^4x^2+ad^4\right)}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5d\operatorname{sgn}\left(bd^4x^2+ad^4\right)}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5d\operatorname{sgn}\left(bd^4x^2+ad^4\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^5d\operatorname{sgn}\left(bd^4x^2+ad^4\right)}-\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^5d\operatorname{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^4*(8*(11*sqrt(d*x)*b*d^4*x^3 + 7*sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 +
a*d^2)^2*b^2*sgn(b*d^4*x^2 + a*d^4)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5
*d*sgn(b*d^4*x^2 + a*d^4)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*d*sgn(b*
d^4*x^2 + a*d^4)) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*d*sgn(b*d^4*x^2 + a*d^4)) - 21*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a*b^5*d*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 612, normalized size = 1.34

$$-\frac{\left(-42\sqrt{2}\,b^2d^4x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-42\sqrt{2}\,b^2d^4x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-21\sqrt{2}\,b^2d^4x^4\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)-84\sqrt{2}\,abd^4x^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-84\sqrt{2}\,abd^4x^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-42\sqrt{2}\,abd^4x^2\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)-42\sqrt{2}\,a^2d^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-42\sqrt{2}\,a^2d^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-21\sqrt{2}\,a^2d^4\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+56\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{3}{2}}abd^2+88\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{7}{2}}b^2\right)\left(bx^2+a\right)d}{128\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128*(-21*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)
^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*b^2*
d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)*x^4*b^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))*x^4*b^2*d^4+88*(a/b*d^2)^(1/4)*(d*x)^(7/2)*b^2-42*2^(1/2)*ln(-(
-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a*b*d^4-84*2^(1/2)*arctan((2^
(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^4-84*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^4+5
6*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a*b*d^2-21*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*
(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/
```

2)+(a/b*d^2)^(1/2)))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(
a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^4)*d*(b*x^2+a)/(a/b*d^2)^(1/4)/b^3/(
(b*x^2+a)^2)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ad^{\frac{9}{2}}x^{\frac{3}{2}}}{2\left(ab^{3}x^{2}+a^{2}b^{2}+\left(b^{4}x^{2}+ab^{3}\right)x^{2}\right)}+d^{\frac{9}{2}}\int\frac{\sqrt{x}}{b^{3}x^{2}+ab^{2}}\,dx-\frac{11\,d^{\frac{9}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,b^{2}}-\frac{11\,bd^{\frac{9}{2}}x^{\frac{7}{2}}+15\,ad^{\frac{9}{2}}x^{\frac{3}{2}}}{16\left(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")

[Out] 1/2*a*d^(9/2)*x^(3/2)/(a*b^3*x^2 + a^2*b^2 + (b^4*x^2 + a*b^3)*x^2) + d^(9/
2)*integrate(sqrt(x)/(b^3*x^2 + a*b^2), x) - 11/128*d^(9/2)*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^2 - 1/16*(11*b*d^(9/2)*
x^(7/2) + 15*a*d^(9/2)*x^(3/2))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x\right)}^{9/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)

[Out] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx\right)^{\frac{9}{2}}}{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)

[Out] Integral((d*x)**(9/2)/((a + b*x**2)**2)**(3/2), x)

3.583 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=458

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{64\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.32, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*d^3*Sqrt[d*x])/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(5/2
))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^
2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(3/4
)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*
Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^4\left(ab+b^2x^2 \cdots\right.\right.}{32b^2\sqrt{a^2 \cdots}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^3\left(ab+b^2x^2 \cdots\right.\right.}{16b^2 \cdots}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^2\left(ab+b^2x^2 \cdots\right.\right.}{32\sqrt{\cdots}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(5d^{7/2}\left(ab+b^2 \cdots\right.\right.}{64 \cdots}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^{7/2}\left(a+bx^2\right) \cdots}{64 \cdots}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^{7/2}\left(a+bx^2\right) \cdots}{32\sqrt{2}\,a^{3/4}b^{9/4} \cdots}$$

Mathematica [A] time = 0.14, size = 447, normalized size = 0.98

$$-\frac{5(dx)^{7/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{5(dx)^{7/2}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{5(dx)^{7/2}\left(a+bx^2\right)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{5(dx)^{7/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{3/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{5(dx)^{7/2}\left(a+bx^2\right)^2}{48b^2x^3\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{5a(dx)^{7/2}\left(a+bx^2\right)}{12b^2x^3\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{2(dx)^{7/2}\left(a+bx^2\right)}{3bx\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*a*(d*x)^(7/2)*(a + b*x^2))/(12*b^2*x^3*((a + b*x^2)^2)^(3/2)) - (2*(d*x
)^(7/2)*(a + b*x^2))/(3*b*x*((a + b*x^2)^2)^(3/2)) + (5*(d*x)^(7/2)*(a + b*
x^2)^2)/(48*b^2*x^3*((a + b*x^2)^2)^(3/2)) - (5*(d*x)^(7/2)*(a + b*x^2)^3*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*x
^(7/2)*((a + b*x^2)^2)^(3/2)) + (5*(d*x)^(7/2)*(a + b*x^2)^3*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*x^(7/2)*((a +
b*x^2)^2)^(3/2)) - (5*(d*x)^(7/2)*(a + b*x^2)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*x^(7/2)*((a +
b*x^2)^2)^(3/2)) + (5*(d*x)^(7/2)*(a + b*x^2)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*x^(7/2)*((a +
b*x^2)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 79.88, size = 242, normalized size = 0.53

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{5d^{7/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}}+\frac{5d^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{32\sqrt{2}a^{3/4}b^{9/4}}+\frac{-5ad^7\sqrt{dx}-9bd^5(dx)^{5/2}}{16b^2\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-5*a*d^7*Sqrt[d*x] - 9*b*d^5*(d*x)^(5/2))/(16*b^2*(a
*d^2 + b*d^2*x^2)^2) - (5*d^(7/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4
)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(32*Sqrt[2]*a^(3/4)
*b^(9/4)) + (5*d^(7/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/
(Sqrt[a]*d + Sqrt[b]*d*x)])/(32*Sqrt[2]*a^(3/4)*b^(9/4))))/(d^2*Sqrt[(a*d^2
 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.43, size = 315, normalized size = 0.69

$$\frac{20\left(b^4x^4+2ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{3}{4}}\sqrt{dx}a^2b^7d^3-\sqrt{d^7x+\sqrt{-\frac{d^{14}}{a^3b^9}}a^2b^4}\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{3}{4}}a^2b^7}{d^{14}}\right)+5\left(b^4x^4+2ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\log\left(5\sqrt{dx}d^3+5\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}ab^2\right)-5\left(b^4x^4+2ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\log\left(5\sqrt{dx}d^3-5\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}ab^2\right)-4\left(9bd^3x^2+5ad^3\right)\sqrt{dx}}{64\left(b^4x^4+2ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*arctan(-
((-d^14/(a^3*b^9))^(3/4)*sqrt(d*x)*a^2*b^7*d^3 - sqrt(d^7*x + sqrt(-d^14/(a
^3*b^9))*a^2*b^4)*(-d^14/(a^3*b^9))^(3/4)*a^2*b^7)/d^14) + 5*(b^4*x^4 + 2*a
*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*log(5*sqrt(d*x)*d^3 + 5*(-d^14/
(a^3*b^9))^(1/4)*a*b^2) - 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b
^9))^(1/4)*log(5*sqrt(d*x)*d^3 - 5*(-d^14/(a^3*b^9))^(1/4)*a*b^2) - 4*(9*b*
d^3*x^2 + 5*a*d^3)*sqrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.31, size = 367, normalized size = 0.80

$$\frac{1}{128}\,d^3\left(\frac{10\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{10\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{5\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^3\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{5\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^3\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{8\left(9\,\sqrt{dx}\,bd^4x^2+5\,\sqrt{dx}\,ad^4\right)}{\left(bd^2x^2+ad^2\right)^2b^2\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d^3*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/
b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*d^4*x^2 + a*d^4)) + 1
0*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*d^4*x^2 + a*d^4)) + 5*sqrt(2)*(a
*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a*b^3*sgn(b*d^4*x^2 + a*d^4)) - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3*sgn(b*d^4*x^2 + a*d
^4)) - 8*(9*sqrt(d*x)*b*d^4*x^2 + 5*sqrt(d*x)*a*d^4)/((b*d^2*x^2 + a*d^2)^2
*b^2*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 666, normalized size = 1.45

$$\frac{\left(10\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+10\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+5\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2d^2x^4\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+20\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ab\,d^2x^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+20\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ab\,d^2x^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+10\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ab\,d^2x^2\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+10\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^2d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+10\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^2d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+5\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^2d^2\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)-40\sqrt{dx}\,a^2d^2-72\,(dx)^{\frac{5}{2}}\,ab\right)\left(b x^2+a\right)d}{128\left(\left(b x^2+a\right)^2\right)^{\frac{3}{2}}a b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(5*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2)))+10*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x^4*arctan((2^(1/2)*(d*x)
^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+10*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2
*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+10*(a/b*
```

```
d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))
+20*(a/b*d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))+20*(a/b*d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*arctan((2
^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+5*(a/b*d^2)^(1/4)*2^(1
/2)*a^2*d^2*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d
*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+10*(a/b*d^2)^(1/4)
*2^(1/2)*a^2*d^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/
4))+10*(a/b*d^2)^(1/4)*2^(1/2)*a^2*d^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2
)^(1/4))/(a/b*d^2)^(1/4))-72*(d*x)^(5/2)*a*b-40*(d*x)^(1/2)*a^2*d^2)*d*(b*x
^2+a)/a/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.23, size = 279, normalized size = 0.61

$$-\frac{d^{\frac{7}{2}} x^{\frac{5}{2}}}{2\left(ab^2x^2 + a^2b + \left(b^3x^2 + ab^2\right)x^2\right)} + \frac{5\,d^3\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} - 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,b^2} - \frac{bd^{\frac{7}{2}}x^{\frac{5}{2}} + 5\,ad^{\frac{7}{2}}\sqrt{x}}{16\left(b^4x^4 + 2\,ab^3x^2 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*d^(7/2)*x^(5/2)/(a*b^2*x^2 + a^2*b + (b^3*x^2 + a*b^2)*x^2) + 5/128*d^
3*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(
2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sqrt(d)*
log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)
) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(3/4)*b^(1/4)))/b^2 - 1/16*(b*d^(7/2)*x^(5/2) + 5*a*d^(7/2)*sqrt(x)
)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{7/2}}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral((d*x)**(7/2)/((a + b*x**2)**2)**(3/2), x)
```

3.584 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=459

$$\frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.33, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (3*d*(d*x)^(3/2))/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(3/2)
)/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^
(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(5/4)
*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*S
qrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b
*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d*x]])/(64*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^2\left(ab+b^2x^2\right)\right.}{32ab\sqrt{a^2+2a}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d\left(ab+b^2x^2\right)\right)}{16ab\surd}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(3d\left(ab+b^2x^2\right)\right)}{32ab^{3/}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^{5/2}\left(ab+b^2x\right.\right.}{64\surd}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3d^{5/2}\left(a+bx^2\right)\mathrm{lo}}{64\surd}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{5/2}\left(a+bx^2\right)\mathrm{ta}}{32\sqrt{2}\,a^{5/4}b^{7/4}\surd}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.16

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)^2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\right)}{5a^2b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a^2 + (a + b*x^2)^2*Hypergeometric2F1[3/4, 3, 7/4, -((b*
x^2)/a)]))/(5*a^2*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 70.17, size = 245, normalized size = 0.53

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{3d^{5/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}-\frac{3d^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{3bd^3(dx)^{7/2}-ad^5(dx)^{3/2}}{16ab\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-(a*d^5*(d*x)^(3/2)) + 3*b*d^3*(d*x)^(7/2))/(16*a*b*
(a*d^2 + b*d^2*x^2)^2) - (3*d^(5/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1
/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(32*Sqrt[2]*a^(5/
4)*b^(7/4)) - (3*d^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]
)/(Sqrt[a]*d + Sqrt[b]*d*x)])/(32*Sqrt[2]*a^(5/4)*b^(7/4))))/(d^2*Sqrt[(a*d
^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.60, size = 326, normalized size = 0.71

$$-\frac{12\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{27\sqrt{dx}\,ab^2d^7\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}-\sqrt{-729a^3b^3d^{10}\sqrt{-\frac{d^{10}}{a^5b^7}}+729d^{15}x}\,ab^2\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}}{27d^{10}}\right)-3\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\log\left(27a^4b^5\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{3}{4}}+27\sqrt{dx}\,d^7\right)+3\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\log\left(-27a^4b^5\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{3}{4}}+27\sqrt{dx}\,d^7\right)-4\left(3bd^2x^3-ad^2x\right)\sqrt{dx}}{64\left(ab^3x^4+2a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*arcta
n(-1/27*(27*sqrt(d*x)*a*b^2*d^7*(-d^10/(a^5*b^7))^(1/4) - sqrt(-729*a^3*b^3
*d^10*sqrt(-d^10/(a^5*b^7)) + 729*d^15*x)*a*b^2*(-d^10/(a^5*b^7))^(1/4))/d^
10) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(27*
a^4*b^5*(-d^10/(a^5*b^7))^(3/4) + 27*sqrt(d*x)*d^7) + 3*(a*b^3*x^4 + 2*a^2*
b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(-27*a^4*b^5*(-d^10/(a^5*b^7))^
(3/4) + 27*sqrt(d*x)*d^7) - 4*(3*b*d^2*x^3 - a*d^2*x)*sqrt(d*x))/(a*b^3*x^4
 + 2*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.32, size = 383, normalized size = 0.83

$$\frac{1}{128}\,d^2\left(\frac{8\left(3\sqrt{dx}\,bd^4x^3-\sqrt{dx}\,ad^4x\right)}{\left(bd^2x^2+ad^2\right)^2 ab\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^4d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^4d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d^2*(8*(3*sqrt(d*x)*b*d^4*x^3 - sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 + a*d^
2)^2*a*b*sgn(b*d^4*x^2 + a*d^4)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*s
qrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*d*
sgn(b*d^4*x^2 + a*d^4)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*d*sgn(b*d^
4*x^2 + a*d^4)) - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*d*sgn(b*d^4*x^2 + a*d^4)) + 3*sqrt
(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/(a^2*b^4*d*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 617, normalized size = 1.34

$$\frac{\left(6\sqrt{2}\,b^2d^4x^4\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+6\sqrt{2}\,b^2d^4x^4\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+3\sqrt{2}\,b^2d^4x^4\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+12\sqrt{2}\,abd^4x^2\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+12\sqrt{2}\,abd^4x^2\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+6\sqrt{2}\,abd^4x^2\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+6\sqrt{2}\,a^2d^4\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+6\sqrt{2}\,a^2d^4\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+3\sqrt{2}\,a^2d^4\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)-8\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{3}{2}}abd^2+24\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{7}{2}}b^2\right)\left(bx^2+a\right)}{128\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(3*2^(1/2)*b^2*d^4*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-
(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))
+6*2^(1/2)*b^2*d^4*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))+6*2^(1/2)*b^2*d^4*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))+24*(a/b*d^2)^(1/4)*(d*x)^(7/2)*b^2+6*2^(1/2)*a*b*d^4*x^2
*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+12*2^(1/2)*a*b*d^4*x^2*arct
an((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+12*2^(1/2)*a*b*d^
4*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))-8*(a/b*
d^2)^(1/4)*(d*x)^(3/2)*a*b*d^2+3*2^(1/2)*a^2*d^4*ln(-(-d*x+(a/b*d^2)^(1/4)*
(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a/b*d^2)^(1/2)))+6*2^(1/2)*a^2*d^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2
)^(1/4))/(a/b*d^2)^(1/4))+6*2^(1/2)*a^2*d^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/
b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d*(b*x^2+a)/(a/b*d^2)^(1/4)/b^2/a/((b*x^2+a
)^2)^(3/2)
```

maxima [A] time = 3.19, size = 272, normalized size = 0.59

$$-\frac{d^{\frac{5}{2}}x^{\frac{3}{2}}}{2\left(ab^{2}x^{2}+a^{2}b+\left(b^{3}x^{2}+ab^{2}\right)x^{2}\right)}+\frac{3\,d^{\frac{5}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,ab}+\frac{3\,bd^{\frac{5}{2}}x^{\frac{7}{2}}+7\,ad^{\frac{5}{2}}x^{\frac{3}{2}}}{16\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*d^(5/2)*x^(3/2)/(a*b^2*x^2 + a^2*b + (b^3*x^2 + a*b^2)*x^2) + 3/128*d^
(5/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b
) + 1/16*(3*b*d^(5/2)*x^(7/2) + 7*a*d^(5/2)*x^(3/2))/(a*b^3*x^4 + 2*a^2*b^2
*x^2 + a^3*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{5/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(3/2), x)
```

3.585 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=459

$$\frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{64\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.33, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d*Sqrt[d*x])/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x])/(4*b
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*
b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*b^(5/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]
*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(64*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(d^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^2\left(ab+b^2x^2 \cdots\right.\right.}{32ab\sqrt{a^2 \cdots}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d\left(ab+b^2x^2\right) \cdots\right.}{16ab \cdots}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3\left(ab+b^2x^2\right)\right) \cdots}{32a^{3/\cdots}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(3d^{3/2}\left(ab+b^2 \cdots\right.\right.}{64 \cdots}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{3/2}\left(a+bx^2\right) \cdots}{64 \cdots}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{3/2}\left(a+bx^2\right) \cdots}{32\sqrt{2}\,a^{7/4}b^{5/4} \cdots}$$

Mathematica [A]   time = 0.21, size = 272, normalized size = 0.59

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(8a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-32a^{7/4}\sqrt[4]{b}\sqrt{x}-3\sqrt{2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+3\sqrt{2}\left(a+bx^2\right)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-6\sqrt{2}\left(a+bx^2\right)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+6\sqrt{2}\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{128a^{7/4}b^{5/4}x^{3/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(-32*a^(7/4)*b^(1/4)*Sqrt[x] + 8*a^(3/4)*b^(1/4)*S
qrt[x]*(a + b*x^2) - 6*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)] + 6*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)] - 3*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] + 3*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(128*a^(7/4)*b^(5/4)*x^(3/2)*((a + b*x^2
)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 73.26, size = 244, normalized size = 0.53

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{3d^{3/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}+\frac{3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}+\frac{bd^3(dx)^{5/2}-3ad^5\sqrt{dx}}{16ab\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-3*a*d^5*Sqrt[d*x] + b*d^3*(d*x)^(5/2))/(16*a*b*(a*d
^2 + b*d^2*x^2)^2) - (3*d^(3/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4))
 - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(32*Sqrt[2]*a^(7/4)*b
^(5/4)) + (3*d^(3/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(S
qrt[a]*d + Sqrt[b]*d*x)])/(32*Sqrt[2]*a^(7/4)*b^(5/4))))/(d^2*Sqrt[(a*d^2 +
 b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.07, size = 308, normalized size = 0.67

$$\frac{12\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^5b^4d\left(-\frac{d^6}{a^7b^5}\right)^{\frac{3}{4}}-\sqrt{a^4b^2\sqrt{-\frac{d^6}{a^7b^5}}+d^3x}\,a^5b^4\left(-\frac{d^6}{a^7b^5}\right)^{\frac{3}{4}}}{d^6}\right)+3\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\log\left(3a^2b\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}+3\sqrt{dx}d\right)-3\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\log\left(-3a^2b\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}+3\sqrt{dx}d\right)+4\left(bdx^2-3ad\right)\sqrt{dx}}{64\left(ab^3x^4+2a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*arctan(
-(sqrt(d*x)*a^5*b^4*d*(-d^6/(a^7*b^5))^(3/4) - sqrt(a^4*b^2*sqrt(-d^6/(a^7*
b^5)) + d^3*x)*a^5*b^4*(-d^6/(a^7*b^5))^(3/4))/d^6) + 3*(a*b^3*x^4 + 2*a^2*
b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*log(3*a^2*b*(-d^6/(a^7*b^5))^(1/4)
+ 3*sqrt(d*x)*d) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(
1/4)*log(-3*a^2*b*(-d^6/(a^7*b^5))^(1/4) + 3*sqrt(d*x)*d) + 4*(b*d*x^2 - 3*
a*d)*sqrt(d*x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.34, size = 367, normalized size = 0.80

$$\frac{1}{128}\,d\left(\frac{6\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{2}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{6\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{2}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{3\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{2}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{3\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{2}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{8\left(\sqrt{dx}\,bd^{4}x^{2}-3\,\sqrt{dx}\,ad^{4}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{2}ab\,\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*d^4*x^2 + a*d^4)) + 6*
sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*
sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*d^4*x^2 + a*d^4)) + 3*sqrt(2)*(a
*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a^2*b^2*sgn(b*d^4*x^2 + a*d^4)) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x -
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*sgn(b*d^4*x^2 +
 a*d^4)) + 8*(sqrt(d*x)*b*d^4*x^2 - 3*sqrt(d*x)*a*d^4)/((b*d^2*x^2 + a*d^2)
^2*a*b*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 668, normalized size = 1.46

$$\frac{\left(6\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}d^{2}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+6\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}d^{2}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+3\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}d^{2}x^{4}\ln\left(\frac{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}{dx-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)+12\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ab\,d^{2}x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+12\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ab\,d^{2}x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+6\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ab\,d^{2}x^{2}\ln\left(\frac{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}{dx-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)+6\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+6\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+3\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{2}\ln\left(\frac{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}{dx-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)-24\sqrt{dx}\,a^{2}d^{2}+8\left(dx\right)^{\frac{5}{2}}ab\right)\left(bx^{2}+a\right)}{128\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{3}{2}}a^{2}b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(3*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2)))+6*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x^4*arctan((2^(1/2)*(d*x)^
(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+6*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x
^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+6*(a/b*d^2
)^(1/4)*2^(1/2)*a*b*d^2*x^2*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+12
*(a/b*d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^
(1/4))/(a/b*d^2)^(1/4))+12*(a/b*d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*arctan((2^(1
/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+3*(a/b*d^2)^(1/4)*2^(1/2)
*a^2*d^2*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+6*(a/b*d^2)^(1/4)*2^(
1/2)*a^2*d^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
6*(a/b*d^2)^(1/4)*2^(1/2)*a^2*d^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/
4))/(a/b*d^2)^(1/4))+8*(d*x)^(5/2)*a*b-24*(d*x)^(1/2)*a^2*d^2)/d*(b*x^2+a)/
b/a^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.22, size = 281, normalized size = 0.61

$$\frac{d^{\frac{3}{2}}x^{\frac{5}{2}}}{2\left(a^2bx^2+a^3+\left(ab^2x^2+a^2b\right)x^2\right)}-\frac{7\,bd^{\frac{3}{2}}x^{\frac{5}{2}}+3\,ad^{\frac{3}{2}}\sqrt{x}}{16\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)}+\frac{3\,d\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*d^(3/2)*x^(5/2)/(a^2*b*x^2 + a^3 + (a*b^2*x^2 + a^2*b)*x^2) - 1/16*(7*b
*d^(3/2)*x^(5/2) + 3*a*d^(3/2)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
 + 3/128*d*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) +
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)
*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4
)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{3/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(3/2), x)
```

3.586 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{64\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.33, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (5*(d*x)^(3/2))/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2)/(4
*a*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x^2)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9
/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9/4)*b
^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqr
t[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(64*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\left(ab+b^2x\right.\right.}{32a^2\sqrt{a^2+}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\left(ab+b^2x\right.\right.}{16a}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(5\left(ab+b^2x\right.\right.}{32a}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\sqrt{d}\left(ab+\right.\right.}{}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{d}\left(a+bx\right.}{6}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5\sqrt{d}\left(a+bx\right.}{32\sqrt{2}\,a^{9/4}b}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.12

$$\frac{2x\sqrt{dx}\left(a+bx^2\right)^3{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^3\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(a + b*x^2)^3*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/
(3*a^3*((a + b*x^2)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 77.41, size = 238, normalized size = 0.52

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{5\sqrt{d}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{(dx)^{3/2}\left(9ad^3+5bd^3x^2\right)}{16a^2\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(((d*x)^(3/2)*(9*a*d^3 + 5*b*d^3*x^2))/(16*a^2*(a*d^2
+ b*d^2*x^2)^2) - (5*Sqrt[d]*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) -
(b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(32*Sqrt[2]*a^(9/4)*b^(3
/4)) - (5*Sqrt[d]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt
[a]*d + Sqrt[b]*d*x)])/(32*Sqrt[2]*a^(9/4)*b^(3/4))))/(d^2*Sqrt[(a*d^2 + b*
d^2*x^2)^2/d^4])
```

fricas [A] time = 0.64, size = 304, normalized size = 0.66

$$-\frac{20\left(a^2b^2x^4+2a^3bx^2+a^4\right)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\sqrt{dx}a^2bd\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}-\sqrt{-15625a^5bd^2\sqrt{-\frac{d^2}{a^9b^3}}+15625d^3xa^2b}\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}}{125d^2}\right)-5\left(a^2b^2x^4+2a^3bx^2+a^4\right)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}\log\left(125a^7b^2\left(-\frac{d^2}{a^9b^3}\right)^{\frac{3}{4}}+125\sqrt{dx}d\right)+5\left(a^2b^2x^4+2a^3bx^2+a^4\right)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}\log\left(-125a^7b^2\left(-\frac{d^2}{a^9b^3}\right)^{\frac{3}{4}}+125\sqrt{dx}d\right)-4\left(5bx^3+9ax\right)\sqrt{dx}}{64\left(a^2b^2x^4+2a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(20*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*arctan(-
1/125*(125*sqrt(d*x)*a^2*b*d*(-d^2/(a^9*b^3))^(1/4) - sqrt(-15625*a^5*b*d^2
```

```
*sqrt(-d^2/(a^9*b^3)) + 15625*d^3*x)*a^2*b*(-d^2/(a^9*b^3))^(1/4))/d^2) - 5
*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*log(125*a^7*b^2*(
-d^2/(a^9*b^3))^(3/4) + 125*sqrt(d*x)*d) + 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a
^4)*(-d^2/(a^9*b^3))^(1/4)*log(-125*a^7*b^2*(-d^2/(a^9*b^3))^(3/4) + 125*sq
rt(d*x)*d) - 4*(5*b*x^3 + 9*a*x)*sqrt(d*x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^
4)
```

giac [A] time = 0.34, size = 368, normalized size = 0.80

$$\frac{\frac{8\left(5\sqrt{dx}\,bd^5x^3+9\sqrt{dx}\,ad^5x\right)}{\left(bd^2x^2+ad^2\right)^2a^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*(8*(5*sqrt(d*x)*b*d^5*x^3 + 9*sqrt(d*x)*a*d^5*x)/((b*d^2*x^2 + a*d^2)
^2*a^2*sgn(b*d^4*x^2 + a*d^4)) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn
(b*d^4*x^2 + a*d^4)) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*d^4*x^
2 + a*d^4)) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)
*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)) + 5*sqrt(2)*(a
*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)))/d
```

maple [B] time = 0.01, size = 617, normalized size = 1.34

$$\frac{\left(10\sqrt{2}\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+10\sqrt{2}\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+5\sqrt{2}\,b^2d^2x^4\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+20\sqrt{2}\,abd^2x^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+20\sqrt{2}\,abd^2x^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+10\sqrt{2}\,abd^2x^2\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+10\sqrt{2}\,a^2d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+10\sqrt{2}\,a^2d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+5\sqrt{2}\,a^2d^2\ln\left(-\frac{-dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+40\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{3}{2}}b^2x^2+72(dx)^{\frac{3}{2}}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}ab\right)\left(bx^2+a\right)}{128\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(5*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1
/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*b^2*d^2
+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x
^4*b^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2
)^(1/4))*x^4*b^2*d^2+40*(a/b*d^2)^(1/4)*(d*x)^(3/2)*x^2*b^2+10*2^(1/2)*ln(-
(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a*b*d^2+20*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^2+20*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^2+
72*(d*x)^(3/2)*a*b*(a/b*d^2)^(1/4)+5*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x
```

```
)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a/b*d^2)^(1/2)))*a^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(
1/4))/(a/b*d^2)^(1/4))*a^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*
d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^2)/d*(b*x^2+a)/(a/b*d^2)^(1/4)/b/a^2/((b
*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.21, size = 265, normalized size = 0.58

$$\frac{\sqrt{d}\,x^{\frac{3}{2}}}{2\left(a^{2}bx^{2}+a^{3}+\left(ab^{2}x^{2}+a^{2}b\right)x^{2}\right)}+\frac{5\,b\sqrt{d}\,x^{\frac{7}{2}}+a\sqrt{d}\,x^{\frac{3}{2}}}{16\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)}+\frac{5\,\sqrt{d}\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d)*x^(3/2)/(a^2*b*x^2 + a^3 + (a*b^2*x^2 + a^2*b)*x^2) + 1/16*(5*b
*sqrt(d)*x^(7/2) + a*sqrt(d)*x^(3/2))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 5
/128*sqrt(d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral(sqrt(d*x)/((a + b*x**2)**2)**(3/2), x)
```

3.587 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\cdots\right.}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.34, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (7*Sqrt[d*x])/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(4*a*d
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4
)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*S
qrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^
(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(64*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7b\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21\left(ab+\right.\right.}{32a^2}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21\left(ab+\right.\right.}{}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21\left(ab+\right.\right.}{}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21\left(ab+\right.\right.}{}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21\left(a+b\right.}{6}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21\left(a+\right.}{32\sqrt{2}\,a^{11}}$$

Mathematica [A] time = 0.19, size = 272, normalized size = 0.59

$$\frac{\sqrt{x}\left(a+bx^2\right)\left(56a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+32a^{7/4}\sqrt[4]{b}\sqrt{x}-21\sqrt{2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+21\sqrt{2}\left(a+bx^2\right)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-42\sqrt{2}\left(a+bx^2\right)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+42\sqrt{2}\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{128a^{11/4}\sqrt[4]{b}\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[x]*(a + b*x^2)*(32*a^(7/4)*b^(1/4)*Sqrt[x] + 56*a^(3/4)*b^(1/4)*Sqrt[
x]*(a + b*x^2) - 42*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)] + 42*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] - 21*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x] + 21*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(128*a^(11/4)*b^(1/4)*Sqrt[d*x]*((a + b*
x^2)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 83.69, size = 238, normalized size = 0.52

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{21\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}}+\frac{21\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}}+\frac{\sqrt{dx}\left(11ad^3+7bd^3x^2\right)}{16a^2\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((Sqrt[d*x]*(11*a*d^3 + 7*b*d^3*x^2))/(16*a^2*(a*d^2 +
 b*d^2*x^2)^2) - (21*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)
*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*Sqr
t[d]) + (21*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d
+ Sqrt[b]*d*x)])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d])))/(d^2*Sqrt[(a*d^2 +
 b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.66, size = 298, normalized size = 0.65

$$\frac{84\left(a^2b^2dx^4+2a^3bdx^2+a^4d\right)\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^6d^2\sqrt{-\frac{1}{a^{11}bd^2}}+dx}\,a^8bd\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^8bd\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{3}{4}}\right)+21\left(a^2b^2dx^4+2a^3bdx^2+a^4d\right)\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}\log\left(a^3d\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-21\left(a^2b^2dx^4+2a^3bdx^2+a^4d\right)\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}\log\left(-a^3d\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\left(7bx^2+11a\right)\sqrt{dx}}{64\left(a^2b^2dx^4+2a^3bdx^2+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(84*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*ar
ctan(sqrt(a^6*d^2*sqrt(-1/(a^11*b*d^2)) + d*x)*a^8*b*d*(-1/(a^11*b*d^2))^(3
/4) - sqrt(d*x)*a^8*b*d*(-1/(a^11*b*d^2))^(3/4)) + 21*(a^2*b^2*d*x^4 + 2*a^
3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*log(a^3*d*(-1/(a^11*b*d^2))^(1/4
) + sqrt(d*x)) - 21*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2
))^(1/4)*log(-a^3*d*(-1/(a^11*b*d^2))^(1/4) + sqrt(d*x)) + 4*(7*b*x^2 + 11*
a)*sqrt(d*x))/(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)
```

giac [A] time = 0.29, size = 374, normalized size = 0.81

$$\frac{7\sqrt{dx}\,bd^3x^2+11\sqrt{dx}\,ad^3}{16\left(bd^2x^2+ad^2\right)^2a^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64a^3bd\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64a^3bd\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128a^3bd\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128a^3bd\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(7*sqrt(d*x)*b*d^3*x^2 + 11*sqrt(d*x)*a*d^3)/((b*d^2*x^2 + a*d^2)^2*a^
2*sgn(b*d^4*x^2 + a*d^4)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b
*d^4*x^2 + a*d^4)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b*d^4*x
^2 + a*d^4)) + 21/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d*sgn(b*d^4*x^2 + a*d^4)) - 21/128
*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/(a^3*b*d*sgn(b*d^4*x^2 + a*d^4))
```

maple [B] time = 0.01, size = 638, normalized size = 1.39

$$\frac{\left(42\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2x^4\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+42\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2x^4\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+21\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2x^4\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+84\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,abx^2\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+84\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,abx^2\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+42\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,abx^2\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+56\sqrt{dx}\,abx^2+42\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+42\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+21\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+88\sqrt{dx}\,a^2\right)\left(bx^2+a\right)}{128\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] 1/128*(42*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(a/
b*d^2)^(1/4)*2^(1/2)*x^4*b^2+42*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*x^4*b^2+21*(a/b*d^2)^(1/4)*2^(1/
2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*b^2+84*arctan((2^(1/2)*
(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*x^2*a
*b+84*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(a/b*d^
2)^(1/4)*2^(1/2)*x^2*a*b+42*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)
```

```
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2)))*x^2*a*b+42*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4
))/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*a^2+42*arctan((2^(1/2)*(d*x)^(1
/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*a^2+21*(a/b*d
^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^2+56*(d*x)
^(1/2)*x^2*a*b+88*(d*x)^(1/2)*a^2)/d*(b*x^2+a)/a^3/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\sqrt{d}\,x^{\frac{5}{2}}}{2\left(a^3bdx^2+a^4d+\left(a^2b^2dx^2+a^3bd\right)x^2\right)}+\frac{15\,bx^{\frac{5}{2}}+11\,a\sqrt{x}}{16\left(a^2b^2\sqrt{d}\,x^4+2\,a^3b\sqrt{d}\,x^2+a^4\sqrt{d}\right)}-\frac{11\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,a^2d}+\int\frac{1}{\left(a^2b\sqrt{d}\,x^2+a^3\sqrt{d}\right)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*b*sqrt(d)*x^(5/2)/(a^3*b*d*x^2 + a^4*d + (a^2*b^2*d*x^2 + a^3*b*d)*x^2
) + 1/16*(15*b*x^(5/2) + 11*a*sqrt(x))/(a^2*b^2*sqrt(d)*x^4 + 2*a^3*b*sqrt(
d)*x^2 + a^4*sqrt(d)) - 11/128*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(
2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + sqrt(2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*d) + integrate(1/
((a^2*b*sqrt(d)*x^2 + a^3*sqrt(d))*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d\,x}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(3/2)), x)
```

3.588 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}-\frac{45\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}\right.}{64\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2ab}}$$

Rubi [A] time = 0.38, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{45\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\,\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{32\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{b}\,\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\left(a+bx^2\right)}{16a^3d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 9/(16*a^2*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*Sqrt[d*x]
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*(a + b*x^2))/(16*a^3*d*
Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x^2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d
^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d^(3
/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[
2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(64*Sqrt[2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right.}}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.10

$$-\frac{2x\left(a+bx^2\right)^3\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^3(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^3*Hypergeometric2F1[-1/4, 3, 3/4, -((b*x^2)/a)])/(a^3*(d*
x)^(3/2)*((a + b*x^2)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 91.79, size = 255, normalized size = 0.50

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{45\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}a^{13/4}d^{3/2}}+\frac{45\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{32\sqrt{2}a^{13/4}d^{3/2}}+\frac{-32a^2d^4-81abd^4x^2-45b^2d^4x^4}{16a^3d\sqrt{dx}\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-32*a^2*d^4 - 81*a*b*d^4*x^2 - 45*b^2*d^4*x^4)/(16*a
^3*d*Sqrt[d*x]*(a*d^2 + b*d^2*x^2)^2) + (45*b^(1/4)*ArcTan[((a^(1/4)*Sqrt[d
])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(
32*Sqrt[2]*a^(13/4)*d^(3/2)) + (45*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(32*Sqrt[2]*a^(13/4)*d^(3/2
))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 2.18, size = 343, normalized size = 0.68

$$\frac{180\left(a^3b^2d^2x^5+2a^4bd^2x^3+a^5d^2x\right)\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\sqrt{dx}a^3bd\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}-\sqrt{-8303765625a^7bd^4\sqrt{-\frac{b}{a^{13}d^6}}+8303765625b^2dx}\,a^3d\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}}{91125b}\right)-45\left(a^3b^2d^2x^5+2a^4bd^2x^3+a^5d^2x\right)\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}\log\left(91125a^{10}d^5\left(-\frac{b}{a^{13}d^6}\right)^{\frac{3}{4}}+91125\sqrt{dx}b\right)+45\left(a^3b^2d^2x^5+2a^4bd^2x^3+a^5d^2x\right)\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}\log\left(-91125a^{10}d^5\left(-\frac{b}{a^{13}d^6}\right)^{\frac{3}{4}}+91125\sqrt{dx}b\right)-4\left(45b^2x^4+81abx^2+32a^2\right)\sqrt{dx}}{64\left(a^3b^2d^2x^5+2a^4bd^2x^3+a^5d^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(180*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6))^(
1/4)*arctan(-1/91125*(91125*sqrt(d*x)*a^3*b*d*(-b/(a^13*d^6))^(1/4) - sqrt(
-8303765625*a^7*b*d^4*sqrt(-b/(a^13*d^6)) + 8303765625*b^2*d*x)*a^3*d*(-b/(
a^13*d^6))^(1/4))/b) - 45*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(
-b/(a^13*d^6))^(1/4)*log(91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(
d*x)*b) + 45*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6)
)^(1/4)*log(-91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*x)*b) - 4*
(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^2*x^5 + 2*a^4*b*d^
2*x^3 + a^5*d^2*x)
```

giac [A] time = 0.31, size = 410, normalized size = 0.81

$$-\frac{\frac{256}{\sqrt{dx}\, a^{3}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{8\left(13\sqrt{dx}\, b^{2}d^{3}x^{3}+17\sqrt{dx}\, abd^{3}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{2}a^{3}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{90\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{2}d^{2}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{90\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{2}d^{2}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{45\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{4}b^{2}d^{2}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{45\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{4}b^{2}d^{2}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*(256/(sqrt(d*x)*a^3*sgn(b*d^4*x^2 + a*d^4)) + 8*(13*sqrt(d*x)*b^2*d^
3*x^3 + 17*sqrt(d*x)*a*b*d^3*x)/((b*d^2*x^2 + a*d^2)^2*a^3*sgn(b*d^4*x^2 +
a*d^4)) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)
) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) - 4
5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^4*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)
^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^
2*d^2*sgn(b*d^4*x^2 + a*d^4)))/d
```

maple [A] time = 0.02, size = 645, normalized size = 1.27

$$-\frac{\left(90\sqrt{2}\,\sqrt{dx}\,b^{2}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)+90\sqrt{2}\,\sqrt{dx}\,b^{2}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)+45\sqrt{2}\,\sqrt{dx}\,b^{2}x^{4}\ln\left(-\frac{-dx+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{a d^{2}}{b}}}{dx+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^{2}}{b}}}\right)+360\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}b^{2}x^{4}+180\sqrt{2}\,\sqrt{dx}\,a b x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)+180\sqrt{2}\,\sqrt{dx}\,a b x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)+90\sqrt{2}\,\sqrt{dx}\,a b x^{2}\ln\left(-\frac{-dx+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{a d^{2}}{b}}}{dx+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^{2}}{b}}}\right)+648\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}a b x^{2}+90\sqrt{2}\,\sqrt{dx}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)+90\sqrt{2}\,\sqrt{dx}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)+45\sqrt{2}\,\sqrt{dx}\,a^{2}\ln\left(-\frac{-dx+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{a d^{2}}{b}}}{dx+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^{2}}{b}}}\right)+256\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}a^{2}\right)\left(b x^{2}+a\right)}{128\sqrt{dx}\,\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}\left(\left(b x^{2}+a\right)^{2}\right)^{\frac{3}{2}}a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128/d*(45*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2
)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d*x)^(
1/2)*x^4*b^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d
^2)^(1/4))*(d*x)^(1/2)*x^4*b^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*
d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(1/2)*x^4*b^2+360*(a/b*d^2)^(1/4)*x^4*b^
2+90*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))
/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d*x)^(1/2)*x^2
```

```
*a*b+180*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/
4))*(d*x)^(1/2)*x^2*a*b+180*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(
1/4))/(a/b*d^2)^(1/4))*(d*x)^(1/2)*x^2*a*b+648*(a/b*d^2)^(1/4)*x^2*a*b+45*2
^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d*x)^(1/2)*a^2+90*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(
1/2)*a^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^
(1/4))*(d*x)^(1/2)*a^2+256*(a/b*d^2)^(1/4)*a^2)*(b*x^2+a)/(d*x)^(1/2)/(a/b*
d^2)^(1/4)/a^3/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^{\frac{3}{2}}}{2\left(a^{3}bd^{\frac{3}{2}}x^{2}+a^{4}d^{\frac{3}{2}}+\left(a^{2}b^{2}d^{\frac{3}{2}}x^{2}+a^{3}bd^{\frac{3}{2}}\right)x^{2}\right)}-\frac{13\,b^{2}x^{\frac{7}{2}}+9\,abx^{\frac{3}{2}}}{16\left(a^{3}b^{2}d^{\frac{3}{2}}x^{4}+2\,a^{4}bd^{\frac{3}{2}}x^{2}+a^{5}d^{\frac{3}{2}}\right)}-\frac{13\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^{3}d^{\frac{3}{2}}}+\int\frac{1}{\left(a^{2}bd^{\frac{3}{2}}x^{2}+a^{3}d^{\frac{3}{2}}\right)x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*b*x^(3/2)/(a^3*b*d^(3/2)*x^2 + a^4*d^(3/2) + (a^2*b^2*d^(3/2)*x^2 + a^
3*b*d^(3/2))*x^2) - 1/16*(13*b^2*x^(7/2) + 9*a*b*x^(3/2))/(a^3*b^2*d^(3/2)*
x^4 + 2*a^4*b*d^(3/2)*x^2 + a^5*d^(3/2)) - 13/128*b*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sq
rt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^3*d^(3/2)) + integrate(1/((a^2
*b*d^(3/2)*x^2 + a^3*d^(3/2))*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d\,x\right)}^{3/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(3/2)), x)
```

3.589 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}+\frac{77b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\right.}{64\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+}}$$

Rubi [A] time = 0.38, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{32\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77\left(a+bx^2\right)}{48a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 11/(16*a^2*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*(a + b*x^2))/(48*a
^3*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(
15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(15/
4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(6
4*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(64*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab+b\right.}}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.11

$$-\frac{2x\left(a+bx^2\right)^3 {}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^3(dx)^{5/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^3*Hypergeometric2F1[-3/4, 3, 1/4, -((b*x^2)/a)])/(3*a^3*(
d*x)^(5/2)*((a + b*x^2)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 95.79, size = 255, normalized size = 0.50

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{77b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}a^{15/4}d^{5/2}}-\frac{77b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{32\sqrt{2}a^{15/4}d^{5/2}}+\frac{-32a^2d^4-121abd^4x^2-77b^2d^4x^4}{48a^3d(dx)^{3/2}\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-32*a^2*d^4 - 121*a*b*d^4*x^2 - 77*b^2*d^4*x^4)/(48*
a^3*d*(d*x)^(3/2)*(a*d^2 + b*d^2*x^2)^2) + (77*b^(3/4)*ArcTan[((a^(1/4)*Sqr
t[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]]
)/(32*Sqrt[2]*a^(15/4)*d^(5/2)) - (77*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(32*Sqrt[2]*a^(15/4)*d^(
5/2))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.81, size = 367, normalized size = 0.73

$$-\frac{924\left(a^3b^2d^3x^6+2a^4bd^3x^4+a^5d^3x^2\right)\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{11}bd^7\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^8d^6\sqrt{-\frac{b^3}{a^{15}d^{10}}}+b^2dx}\,a^{11}d^7\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+231\left(a^3b^2d^3x^6+2a^4bd^3x^4+a^5d^3x^2\right)\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\log\left(77a^4d^3\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}+77\sqrt{dx}b\right)-231\left(a^3b^2d^3x^6+2a^4bd^3x^4+a^5d^3x^2\right)\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\log\left(-77a^4d^3\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}+77\sqrt{dx}b\right)+4\left(77b^2x^4+121abx^2+32a^2\right)\sqrt{dx}}{192\left(a^3b^2d^3x^6+2a^4bd^3x^4+a^5d^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/192*(924*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d
^10))^(1/4)*arctan(-(sqrt(d*x)*a^11*b*d^7*(-b^3/(a^15*d^10))^(3/4) - sqrt(a
^8*d^6*sqrt(-b^3/(a^15*d^10)) + b^2*d*x)*a^11*d^7*(-b^3/(a^15*d^10))^(3/4))
/b^3) + 231*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d
^10))^(1/4)*log(77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) - 231
*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d^10))^(1/4)
*log(-77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) + 4*(77*b^2*x^4
 + 121*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^
5*d^3*x^2)
```

giac [A] time = 0.35, size = 401, normalized size = 0.79

$$-\frac{15\sqrt{dx}\,b^2d^2x^2+19\sqrt{dx}\,abd^2}{16\left(bd^2x^2+ad^2\right)^2a^3d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{64a^4d^3\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac14}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{64a^4d^3\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac14}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128a^4d^3\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac14}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128a^4d^3\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{2}{3\sqrt{dx}\,a^3d^2x\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(15*sqrt(d*x)*b^2*d^2*x^2 + 19*sqrt(d*x)*a*b*d^2)/((b*d^2*x^2 + a*d^2
)^2*a^3*d*sgn(b*d^4*x^2 + a*d^4)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d
^3*sgn(b*d^4*x^2 + a*d^4)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d^3*sgn
(b*d^4*x^2 + a*d^4)) - 77/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*d^3*sgn(b*d^4*x^2 + a*d^4))
+ 77/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a^4*d^3*sgn(b*d^4*x^2 + a*d^4)) - 2/3/(sqrt(d*x)*a^3*
d^2*x*sgn(b*d^4*x^2 + a*d^4))
```

maple [B] time = 0.02, size = 707, normalized size = 1.40

$$-\frac{\left(616ab^2d^2x^4+462(dx)^{\frac32}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{2}\,b^3x^4\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac14}}{\left(\frac{ad^2}{b}\right)^{\frac14}}\right)+462(dx)^{\frac32}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{2}\,b^3x^4\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac14}}{\left(\frac{ad^2}{b}\right)^{\frac14}}\right)+231(dx)^{\frac32}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{2}\,b^3x^4\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+968a^2bd^2x^2+924(dx)^{\frac32}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{2}\,ab^2x^2\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac14}}{\left(\frac{ad^2}{b}\right)^{\frac14}}\right)+924(dx)^{\frac32}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{2}\,ab^2x^2\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac14}}{\left(\frac{ad^2}{b}\right)^{\frac14}}\right)+462(dx)^{\frac32}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{2}\,ab^2x^2\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+256a^3d^2+462(dx)^{\frac32}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{2}\,a^2b\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac14}}{\left(\frac{ad^2}{b}\right)^{\frac14}}\right)+462(dx)^{\frac32}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{2}\,a^2b\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac14}}{\left(\frac{ad^2}{b}\right)^{\frac14}}\right)+231(dx)^{\frac32}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{2}\,a^2b\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)\right)\left(bx^2+a\right)}{384(dx)^{\frac32}\left(\left(bx^2+a\right)^2\right)^{\frac32}a^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/384/d^3*(231*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2)))*x^4*b^3+462*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*b^3+462*(d*x)^
(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/
(a/b*d^2)^(1/4))*x^4*b^3+462*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a*b^2+924*(d*x)^(3/2)*(a/b*d^2)^(1/4
```

```
)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2
*a*b^2+924*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-
(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b^2+231*(d*x)^(3/2)*(a/b*d^2)^(1/4)
*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^2*b+462*(d*x)^(3/2)
*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))*a^2*b+462*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*b+616*a*d^2*b^2*x^4+968*x^
2*a^2*b*d^2+256*a^3*d^2)*(b*x^2+a)/(d*x)^(3/2)/a^4/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2x^{\frac{5}{2}}}{2\left(a^4bd^{\frac{5}{2}}x^2+a^5d^{\frac{5}{2}}+\left(a^3b^2d^{\frac{5}{2}}x^2+a^4bd^{\frac{5}{2}}\right)x^2\right)}-2b\int\frac{1}{\left(a^3bd^{\frac{5}{2}}x^2+a^4d^{\frac{5}{2}}\right)\sqrt{x}}\,dx-\frac{23\,b^2x^{\frac{5}{2}}+19\,ab\sqrt{x}}{16\left(a^3b^2d^{\frac{5}{2}}x^4+2\,a^4bd^{\frac{5}{2}}x^2+a^5d^{\frac{5}{2}}\right)}+\frac{19\left(\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)}{128\,a^3d^{\frac{5}{2}}}+\int\frac{1}{\left(a^2bd^{\frac{5}{2}}x^2+a^3d^{\frac{5}{2}}\right)x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*b^2*x^(5/2)/(a^4*b*d^(5/2)*x^2 + a^5*d^(5/2) + (a^3*b^2*d^(5/2)*x^2 + a
^4*b*d^(5/2))*x^2) - 2*b*integrate(1/((a^3*b*d^(5/2)*x^2 + a^4*d^(5/2))*sqr
t(x)), x) - 1/16*(23*b^2*x^(5/2) + 19*a*b*sqrt(x))/(a^3*b^2*d^(5/2)*x^4 + 2
*a^4*b*d^(5/2)*x^2 + a^5*d^(5/2)) + 19/128*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/a^(3/4))/(a^3*d^(5/2)) + integrate(1/((a^2*b*d^(5/2
)*x^2 + a^3*d^(5/2))*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d\,x\right)}^{5/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}} \left(\left(a + bx^2 \right)^2 \right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(3/2)), x)
```

3.590 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=553

$$\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{64\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+}}$$

Rubi [A] time = 0.43, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117b\left(a+bx^2\right)}{16a^4d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117\left(a+bx^2\right)}{80a^3d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 13/(16*a^2*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*(a + b*x^2))/(80*
a^3*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b*(a + b*x^2))/(1
6*a^4*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^(5/4)*(a + b*
x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]
*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^
(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab+b\right.}}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.10

$$-\frac{2x\left(a+bx^2\right)^3 {}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^3(dx)^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^3*Hypergeometric2F1[-5/4, 3, -1/4, -((b*x^2)/a)])/(5*a^3*
(d*x)^(7/2)*((a + b*x^2)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 90.94, size = 269, normalized size = 0.49

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{117b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{32\sqrt{2}a^{17/4}d^{7/2}}-\frac{117b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{32\sqrt{2}a^{17/4}d^{7/2}}+\frac{-32a^3d^6+416a^2bd^6x^2+1053ab^2d^6x^4+585b^3d^6x^6}{80a^4d^3(dx)^{5/2}\left(ad^2+bd^2x^2\right)^2}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-32*a^3*d^6 + 416*a^2*b*d^6*x^2 + 1053*a*b^2*d^6*x^4
 + 585*b^3*d^6*x^6)/(80*a^4*d^3*(d*x)^(5/2)*(a*d^2 + b*d^2*x^2)^2) - (117*b
^(5/4)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(S
qrt[2]*a^(1/4)))/Sqrt[d*x]])/(32*Sqrt[2]*a^(17/4)*d^(7/2)) - (117*b^(5/4)*A
rcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x
)])/(32*Sqrt[2]*a^(17/4)*d^(7/2))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.81, size = 390, normalized size = 0.71

$$-\frac{2340\left(a^4b^2d^4x^7+2a^5bd^4x^5+a^6d^4x^3\right)\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\sqrt{dx}a^4b^4d^3\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769a^9b^5d^8\sqrt{-\frac{b^5}{a^{17}d^{14}}}+2565164201769b^8dx}\,a^4d^3\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}}{1601613b^5}\right)-585\left(a^4b^2d^4x^7+2a^5bd^4x^5+a^6d^4x^3\right)\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\log\left(1601613a^{13}d^{11}\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{3}{4}}+1601613\sqrt{dx}b^4\right)+585\left(a^4b^2d^4x^7+2a^5bd^4x^5+a^6d^4x^3\right)\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\log\left(-1601613a^{13}d^{11}\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{3}{4}}+1601613\sqrt{dx}b^4\right)-4\left(585b^3x^6+1053ab^2x^4+416a^2bx^2-32a^3\right)\sqrt{dx}}{320\left(a^4b^2d^4x^7+2a^5bd^4x^5+a^6d^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/320*(2340*(a^4*b^2*d^4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*
d^14))^(1/4)*arctan(-1/1601613*(1601613*sqrt(d*x)*a^4*b^4*d^3*(-b^5/(a^17*d
```

```
^14))^(1/4) - sqrt(-2565164201769*a^9*b^5*d^8*sqrt(-b^5/(a^17*d^14)) + 2565
164201769*b^8*d*x)*a^4*d^3*(-b^5/(a^17*d^14))^(1/4))/b^5) - 585*(a^4*b^2*d^
4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(1601613
*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) + 585*(a^4*b^2
*d^4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(-160
1613*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) - 4*(585*b
^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^3)*sqrt(d*x))/(a^4*b^2*d^4*x
^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)
```

giac [A] time = 0.34, size = 432, normalized size = 0.78

$$\frac{21\,\sqrt{dx}\,b^{3}d^{3}x^{3}+25\,\sqrt{dx}\,ab^{2}d^{3}x}{16\left(bd^{2}x^{2}+ad^{2}\right)^{2}a^{4}d^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{117\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{5}bd^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{117\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{5}bd^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{117\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{128\,a^{5}bd^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{117\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{128\,a^{5}bd^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{2\left(15\,bd^{2}x^{2}-ad^{2}\right)}{5\,\sqrt{dx}\,a^{4}d^{5}x^{2}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(21*sqrt(d*x)*b^3*d^3*x^3 + 25*sqrt(d*x)*a*b^2*d^3*x)/((b*d^2*x^2 + a*
d^2)^2*a^4*d^3*sgn(b*d^4*x^2 + a*d^4)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
(a^5*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^
5*b*d^5*sgn(b*d^4*x^2 + a*d^4)) - 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x
 + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*sgn(b*d^4*
x^2 + a*d^4)) + 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/
b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 2/
5*(15*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^4*d^5*x^2*sgn(b*d^4*x^2 + a*d^4))
```

maple [A] time = 0.03, size = 687, normalized size = 1.24

$$\frac{\left(4680\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}b^{3}d^{2}x^{6}+8424\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}ab^{2}d^{2}x^{4}+1170\sqrt{2}\,(dx)^{\frac{5}{2}}b^{3}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+1170\sqrt{2}\,(dx)^{\frac{5}{2}}b^{3}x^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+585\sqrt{2}\,(dx)^{\frac{5}{2}}b^{3}x^{4}\ln\left(-\frac{-dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^{2}}{b}}}{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)+3328\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}a^{2}bd^{2}x^{2}+2340\sqrt{2}\,(dx)^{\frac{5}{2}}ab^{2}x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+2340\sqrt{2}\,(dx)^{\frac{5}{2}}ab^{2}x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+1170\sqrt{2}\,(dx)^{\frac{5}{2}}ab^{2}x^{2}\ln\left(-\frac{-dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^{2}}{b}}}{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)-256\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}a^{3}d^{2}+1170\sqrt{2}\,(dx)^{\frac{5}{2}}a^{2}b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+1170\sqrt{2}\,(dx)^{\frac{5}{2}}a^{2}b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)+585\sqrt{2}\,(dx)^{\frac{5}{2}}a^{2}b\ln\left(-\frac{-dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-\sqrt{\frac{ad^{2}}{b}}}{dx+\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}}\right)\right)\left(bx^{2}+a\right)}{640\,(dx)^{\frac{5}{2}}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{3}{2}}a^{4}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/640/d^3*(585*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d
^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d*x)
^(5/2)*x^4*b^3+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a
/b*d^2)^(1/4))*(d*x)^(5/2)*x^4*b^3+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*x^4*b^3+4680*(a/b*d^2)^(1/4)
*x^6*b^3*d^2+1170*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/
b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d
*x)^(5/2)*x^2*a*b^2+2340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4
))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*x^2*a*b^2+2340*2^(1/2)*arctan((2^(1/2)*(d*x
```

```
)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*x^2*a*b^2+8424*(a/b*d
^2)^(1/4)*x^4*a*b^2*d^2+585*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2
^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^
(1/2)))*(d*x)^(5/2)*a^2*b+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2
)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*a^2*b+1170*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*a^2*b+3328*(a/b*d^2
)^(1/4)*x^2*a^2*b*d^2-256*(a/b*d^2)^(1/4)*a^3*d^2)*(b*x^2+a)/(d*x)^(5/2)/(a
/b*d^2)^(1/4)/a^4/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2x^{\frac{3}{2}}}{2\left(a^4bd^{\frac{7}{2}}x^2+a^5d^{\frac{7}{2}}+\left(a^3b^2d^{\frac{7}{2}}x^2+a^4bd^{\frac{7}{2}}\right)x^2\right)}-2b\int\frac{1}{\left(a^3bd^{\frac{7}{2}}x^2+a^4d^{\frac{7}{2}}\right)x^{\frac{3}{2}}}\,dx+\frac{21\,b^3x^{\frac{7}{2}}+17\,ab^2x^{\frac{3}{2}}}{16\left(a^4b^2d^{\frac{7}{2}}x^4+2\,a^5bd^{\frac{7}{2}}x^2+a^6d^{\frac{7}{2}}\right)}+\frac{21\,b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^4d^{\frac{7}{2}}}+\int\frac{1}{\left(a^2bd^{\frac{7}{2}}x^2+a^3d^{\frac{7}{2}}\right)x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*b^2*x^(3/2)/(a^4*b*d^(7/2)*x^2 + a^5*d^(7/2) + (a^3*b^2*d^(7/2)*x^2 + a
^4*b*d^(7/2))*x^2) - 2*b*integrate(1/((a^3*b*d^(7/2)*x^2 + a^4*d^(7/2))*x^(
3/2)), x) + 1/16*(21*b^3*x^(7/2) + 17*a*b^2*x^(3/2))/(a^4*b^2*d^(7/2)*x^4 +
 2*a^5*b*d^(7/2)*x^2 + a^6*d^(7/2)) + 21/128*b^2*(2*sqrt(2)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1
/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(
b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^4*d^(7/2)) + integrate(1/((a^2*b*
d^(7/2)*x^2 + a^3*d^(7/2))*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d\,x\right)}^{7/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx\right)^{\frac{7}{2}} \left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*((a + b*x**2)**2)**(3/2)), x)
```

3.591 $\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}} \, dx$

Optimal. Leaf size=647

$$-\frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923ad^{11}\sqrt{dx}\left(a+bx^2\right)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1392}{5120b}$$

Rubi [A] time = 0.51, antiderivative size = 647, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{13923ad^{11}\sqrt{dx}\left(a+bx^2\right)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923d^9(dx)^{5/2}\left(a+bx^2\right)}{5120b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{119d^5(dx)^{13/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-1547*d^7*(d*x)^(9/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(21/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(17/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (119*d^
5*(d*x)^(13/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13
923*a*d^11*Sqrt[d*x]*(a + b*x^2))/(1024*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (13923*d^9*(d*x)^(5/2)*(a + b*x^2))/(5120*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(25/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(4096*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{23/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(11 \ldots}{\ldots}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{25\ldots}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\left(a+b\ldots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\left(a+b\ldots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\left(a+b\ldots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\left(a+b\ldots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\left(a+b\ldots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\left(a+b\ldots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\left(a+b\ldots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{\ldots} - \ldots$$

Mathematica [A] time = 0.30, size = 401, normalized size = 0.62

$$\frac{(dx)^{23/2}\left(a+bx^2\right)\left(-765765\sqrt{2}a^{5/4}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+765765\sqrt{2}a^{5/4}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-1531530\sqrt{2}a^{5/4}\left(a+bx^2\right)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+1531530\sqrt{2}a^{5/4}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)-10183680a^5\sqrt[4]{b}\sqrt{x}-32587776a^4b^{5/4}x^{5/2}+848640a^4\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-39829504a^3b^{9/4}x^{9/2}+1166880a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2-21446656a^2b^{13/4}x^{13/2}+2042040a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3-3784704ab^{17/4}x^{17/2}+180224b^{21/4}x^{21/2}\right)}{450560b^{25/4}x^{23/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(23/2)*(a + b*x^2)*(-10183680*a^5*b^(1/4)*Sqrt[x] - 32587776*a^4*b^(
5/4)*x^(5/2) - 39829504*a^3*b^(9/4)*x^(9/2) - 21446656*a^2*b^(13/4)*x^(13/2
) - 3784704*a*b^(17/4)*x^(17/2) + 180224*b^(21/4)*x^(21/2) + 848640*a^4*b^(
1/4)*Sqrt[x]*(a + b*x^2) + 1166880*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 2042
040*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 1531530*Sqrt[2]*a^(5/4)*(a + b*x^2)
^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 1531530*Sqrt[2]*a^(5/4)*
(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 765765*Sqrt[2
]*a^(5/4)*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x] + 765765*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(450560*b^(25/4)*x^(23/2)*((a + b*x^2)^2)^
(5/2))
```

IntegrateAlgebraic [A] time = 1.37, size = 643, normalized size = 0.99

$$\frac{\sqrt{d}\sqrt{x}\left(\frac{13923a^{5/4}d^{23/2}x^8\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2048\sqrt{2}b^{9/4}}+\frac{13923a^{9/4}d^{23/2}x^6\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{512\sqrt{2}b^{13/4}}+\frac{41769a^{13/4}d^{23/2}x^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{1024\sqrt{2}b^{17/4}}+\frac{13923a^{17/4}d^{23/2}x^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{512\sqrt{2}b^{21/4}}+\left(-\frac{13923a^{5/4}d^{23/2}x^8}{2048\sqrt{2}b^{9/4}}-\frac{13923a^{9/4}d^{23/2}x^6}{512\sqrt{2}b^{13/4}}-\frac{41769a^{13/4}d^{23/2}x^4}{1024\sqrt{2}b^{17/4}}-\frac{13923a^{17/4}d^{23/2}x^2}{512\sqrt{2}b^{21/4}}-\frac{13923a^{21/4}d^{23/2}}{2048\sqrt{2}b^{25/4}}\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\frac{13923a^{21/4}d^{23/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2048\sqrt{2}b^{25/4}}-\frac{13923a^5d^{23/2}\sqrt{x}}{1024b^6}-\frac{264537a^4d^{23/2}x^{5/2}}{5120b^5}-\frac{369733a^3d^{23/2}x^{9/2}}{5120b^4}-\frac{220507a^2d^{23/2}x^{13/2}}{5120b^3}-\frac{42ad^{23/2}x^{17/2}}{5b^2}+\frac{2d^{23/2}x^{21/2}}{5b}\right)}{\sqrt{dx}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[x]*((-13923*a^5*d^(23/2)*Sqrt[x])/(1024*b^6) - (264537*a^4*d^
(23/2)*x^(5/2))/(5120*b^5) - (369733*a^3*d^(23/2)*x^(9/2))/(5120*b^4) - (22
0507*a^2*d^(23/2)*x^(13/2))/(5120*b^3) - (42*a*d^(23/2)*x^(17/2))/(5*b^2) +
 (2*d^(23/2)*x^(21/2))/(5*b) + ((-13923*a^(21/4)*d^(23/2))/(2048*Sqrt[2]*b^
(25/4)) - (13923*a^(17/4)*d^(23/2)*x^2)/(512*Sqrt[2]*b^(21/4)) - (41769*a^(
13/4)*d^(23/2)*x^4)/(1024*Sqrt[2]*b^(17/4)) - (13923*a^(9/4)*d^(23/2)*x^6)/
(512*Sqrt[2]*b^(13/4)) - (13923*a^(5/4)*d^(23/2)*x^8)/(2048*Sqrt[2]*b^(9/4)
))*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + (13923
*a^(21/4)*d^(23/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqr
t[b]*x)])/(2048*Sqrt[2]*b^(25/4)) + (13923*a^(17/4)*d^(23/2)*x^2*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(512*Sqrt[2]*b^(21/
4)) + (41769*a^(13/4)*d^(23/2)*x^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
)/(Sqrt[a] + Sqrt[b]*x)])/(1024*Sqrt[2]*b^(17/4)) + (13923*a^(9/4)*d^(23/2)
*x^6*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(512
*Sqrt[2]*b^(13/4)) + (13923*a^(5/4)*d^(23/2)*x^8*ArcTanh[(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(2048*Sqrt[2]*b^(9/4))))/(Sqrt[d*x]
*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.95, size = 457, normalized size = 0.71

$$\frac{278460 \left(-\frac{a^5 d^{46}}{b^{25}}\right)^{\frac{1}{4}} \left(b^{10}x^8+4ab^9x^6+6a^2b^8x^4+4a^3b^7x^2+a^4b^6\right)\arctan\left(-\frac{\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{19}d^{11}-\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{a^2d^{23}x+\sqrt{-\frac{a^5d^{46}}{b^{25}}}\,b^{12}}\,b^{19}}{a^5d^{46}}\right)+69615\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{10}x^8+4ab^9x^6+6a^2b^8x^4+4a^3b^7x^2+a^4b^6\right)\log\left(13923\sqrt{dx}\,ad^{11}+13923\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}b^6\right)-69615\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{10}x^8+4ab^9x^6+6a^2b^8x^4+4a^3b^7x^2+a^4b^6\right)\log\left(13923\sqrt{dx}\,ad^{11}-13923\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}b^6\right)+4\left(2048b^5d^{11}x^{10}-43008ab^4d^{11}x^8-220507a^2b^3d^{11}x^6-369733a^3b^2d^{11}x^4-264537a^4bd^{11}x^2-69615a^5d^{11}\right)\sqrt{dx}}{20480\left(b^{10}x^8+4ab^9x^6+6a^2b^8x^4+4a^3b^7x^2+a^4b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(278460*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*
x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*arctan(-((-a^5*d^46/b^25)^(3/4)*sqrt(d*x)*a*
b^19*d^11 - (-a^5*d^46/b^25)^(3/4)*sqrt(a^2*d^23*x + sqrt(-a^5*d^46/b^25)*b
^12)*b^19)/(a^5*d^46)) + 69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x
^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x)*a*d^11 +
13923*(-a^5*d^46/b^25)^(1/4)*b^6) - 69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8
+ 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x
)*a*d^11 - 13923*(-a^5*d^46/b^25)^(1/4)*b^6) + 4*(2048*b^5*d^11*x^10 - 4300
8*a*b^4*d^11*x^8 - 220507*a^2*b^3*d^11*x^6 - 369733*a^3*b^2*d^11*x^4 - 2645
37*a^4*b*d^11*x^2 - 69615*a^5*d^11)*sqrt(d*x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*
a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
```

giac [A] time = 0.45, size = 457, normalized size = 0.71

$$\frac{1}{40960}d^{11}\left(\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{69615\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{69615\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{40\left(5599\sqrt{dx}\,a^2b^3d^8x^6+14145\sqrt{dx}\,a^3b^2d^8x^4+12357\sqrt{dx}\,a^4bd^8x^2+3683\sqrt{dx}\,a^5d^8\right)}{\left(bd^2x^2+ad^2\right)^4b^6\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{16384\left(\sqrt{dx}\,b^{20}d^{10}x^2-25\sqrt{dx}\,ab^{19}d^{10}\right)}{b^{25}d^{10}\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/40960*d^11*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*d^4*x^2 + a*d^
4)) + 139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*d^4*x^2 + a*d^4)) + 6
9615*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(b^7*sgn(b*d^4*x^2 + a*d^4)) - 69615*sqrt(2)*(a*b^3*d^2)
^(1/4)*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^7*
sgn(b*d^4*x^2 + a*d^4)) - 40*(5599*sqrt(d*x)*a^2*b^3*d^8*x^6 + 14145*sqrt(d
*x)*a^3*b^2*d^8*x^4 + 12357*sqrt(d*x)*a^4*b*d^8*x^2 + 3683*sqrt(d*x)*a^5*d^
8)/((b*d^2*x^2 + a*d^2)^4*b^6*sgn(b*d^4*x^2 + a*d^4)) + 16384*(sqrt(d*x)*b^
20*d^10*x^2 - 25*sqrt(d*x)*a*b^19*d^10)/(b^25*d^10*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.03, size = 1287, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/40960*(477896*(d*x)^(5/2)*a^4*b*d^4+565800*(d*x)^(9/2)*a^3*b^2*d^2-16384
*(d*x)^(5/2)*x^8*b^5*d^4-65536*(d*x)^(5/2)*x^6*a*b^4*d^4+409600*(d*x)^(1/2)
*x^8*a*b^4*d^6-98304*(d*x)^(5/2)*x^4*a^2*b^3*d^4+1638400*(d*x)^(1/2)*x^6*a^
2*b^3*d^6-65536*(d*x)^(5/2)*x^2*a^3*b^2*d^4+2457600*(d*x)^(1/2)*x^4*a^3*b^2
*d^6+1638400*(d*x)^(1/2)*x^2*a^4*b*d^6-69615*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*
x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^5*d^6-139230*(a/b*d^2)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^5*d^6-13
9230*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(
a/b*d^2)^(1/4))*a^5*d^6-556920*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x
)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^4*b*d^6-69615*(a/b*d^2)^(1/
4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*
x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^8*a*b^4*d^6-13923
0*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b
*d^2)^(1/4))*x^8*a*b^4*d^6-139230*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*a*b^4*d^6-278460*(a/b*d^2)
^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))
/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a^2*b^3*d^6
-556920*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))*x^6*a^2*b^3*d^6-556920*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a^2*b^3*d^6-417690
*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d
^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a
^3*b^2*d^6-835380*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*
d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^3*b^2*d^6-278460*(a/b*d^2)^(1/4)*2^(1/2)
*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^4*b*d^6-556920*(a/b*d^2
)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4
))*x^2*a^4*b*d^6-835380*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^3*b^2*d^6+223960*(d*x)^(13/2)*a^2*
b^3+556920*(d*x)^(1/2)*a^5*d^6)*d^5*(b*x^2+a)/b^6/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,ad^{\frac{23}{2}}\int\frac{x^{\frac{3}{2}}}{b^6x^2+ab^5}\,dx+d^{\frac{23}{2}}\int\frac{x^{\frac{7}{2}}}{b^5x^2+ab^4}\,dx+\frac{3683\left(\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}\right)d^{\frac{23}{2}}}{8192\,b^6}-\frac{6925\,a^2b^3d^{\frac{23}{2}}x^{\frac{13}{2}}+23395\,a^3b^2d^{\frac{23}{2}}x^{\frac{9}{2}}+27135\,a^4bd^{\frac{23}{2}}x^{\frac{5}{2}}+11049\,a^5d^{\frac{23}{2}}\sqrt{x}}{3072\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)}-\frac{\left(617\,a^2b^4d^{\frac{23}{2}}x^5+1386\,a^3b^3d^{\frac{23}{2}}x^3+801\,a^4b^2d^{\frac{23}{2}}x\right)x^{\frac{11}{2}}+2\left(519\,a^3b^3d^{\frac{23}{2}}x^5+1182\,a^4b^2d^{\frac{23}{2}}x^3+695\,a^5bd^{\frac{23}{2}}x\right)x^{\frac{7}{2}}+\left(453\,a^4b^2d^{\frac{23}{2}}x^5+1042\,a^5bd^{\frac{23}{2}}x^3+621\,a^6d^{\frac{23}{2}}x\right)x^{\frac{3}{2}}}{192\left(a^3b^8x^6+3\,a^4b^7x^4+3\,a^5b^6x^2+a^6b^5+\left(b^{11}x^6+3\,ab^{10}x^4+3\,a^2b^9x^2+a^3b^8\right)x^6+3\left(ab^{10}x^6+3\,a^2b^9x^4+3\,a^3b^8x^2+a^4b^7\right)x^4+3\left(a^2b^9x^6+3\,a^3b^8x^4+3\,a^4b^7x^2+a^5b^6\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -4*a*d^(23/2)*integrate(x^(3/2)/(b^6*x^2 + a*b^5), x) + d^(23/2)*integrate(
x^(7/2)/(b^5*x^2 + a*b^4), x) + 3683/8192*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/s
```

```
qrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)
) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/b^(1/4))*d^(23/2)/b^6 - 1/3072*(6925*a^2*b^3*d^(23/2)*x^(13/
2) + 23395*a^3*b^2*d^(23/2)*x^(9/2) + 27135*a^4*b*d^(23/2)*x^(5/2) + 11049*
a^5*d^(23/2)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x
^2 + a^4*b^6) - 1/192*((617*a^2*b^4*d^(23/2)*x^5 + 1386*a^3*b^3*d^(23/2)*x^
3 + 801*a^4*b^2*d^(23/2)*x)*x^(11/2) + 2*(519*a^3*b^3*d^(23/2)*x^5 + 1182*a
^4*b^2*d^(23/2)*x^3 + 695*a^5*b*d^(23/2)*x)*x^(7/2) + (453*a^4*b^2*d^(23/2)
*x^5 + 1042*a^5*b*d^(23/2)*x^3 + 621*a^6*d^(23/2)*x)*x^(3/2))/(a^3*b^8*x^6
+ 3*a^4*b^7*x^4 + 3*a^5*b^6*x^2 + a^6*b^5 + (b^11*x^6 + 3*a*b^10*x^4 + 3*a^
2*b^9*x^2 + a^3*b^8)*x^6 + 3*(a*b^10*x^6 + 3*a^2*b^9*x^4 + 3*a^3*b^8*x^2 +
a^4*b^7)*x^4 + 3*(a^2*b^9*x^6 + 3*a^3*b^8*x^4 + 3*a^4*b^7*x^2 + a^5*b^6)*x^
2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{23/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.592 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=600

$$-\frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315d^9(dx)^{3/2}\left(a+bx^2\right)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{1024b}$$

Rubi [A] time = 0.47, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{7315d^9(dx)^{3/2}\left(a+bx^2\right)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{95d^5(dx)^{11/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315a^{3/4}d^{21/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315a^{3/4}d^{21/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315a^{3/4}d^{21/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315a^{3/4}d^{21/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-1045*d^7*(d*x)^(7/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(19/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (19*d^3*(d
*x)^(15/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (95*d^
5*(d*x)^(11/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (73
15*d^9*(d*x)^(3/2)*(a + b*x^2))/(3072*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x
])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(23/4)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) + (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt
[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(19b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(95}{}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{256}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+bx\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+bx\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+bx\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+bx\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+bx\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+bx\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+bx\right.}$$

Mathematica [C] time = 0.05, size = 110, normalized size = 0.18

$$-\frac{2d^9(dx)^{3/2}\left(-1463a^4-2717a^3bx^2-2223a^2b^2x^4-741ab^3x^6+1463\left(a+bx^2\right)^4\,{}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-39b^4x^8\right)}{117b^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-2*d^9*(d*x)^(3/2)*(-1463*a^4 - 2717*a^3*b*x^2 - 2223*a^2*b^2*x^4 - 741*a*
b^3*x^6 - 39*b^4*x^8 + 1463*(a + b*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -(
(b*x^2)/a)]))/(117*b^5*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 1.31, size = 623, normalized size = 1.04

$$\frac{\sqrt{d}\sqrt{x}\left(\frac{7315a^{3/4}d^{21/2}x^8\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2048\sqrt{2}b^{7/4}}+\frac{7315a^{7/4}d^{21/2}x^6\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{512\sqrt{2}b^{11/4}}+\frac{21945a^{11/4}d^{21/2}x^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{1024\sqrt{2}b^{15/4}}+\frac{7315a^{15/4}d^{21/2}x^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{512\sqrt{2}b^{19/4}}+\left(\frac{7315a^{3/4}d^{21/2}x^8}{2048\sqrt{2}b^{7/4}}+\frac{7315a^{7/4}d^{21/2}x^6}{512\sqrt{2}b^{11/4}}+\frac{21945a^{11/4}d^{21/2}x^4}{1024\sqrt{2}b^{15/4}}+\frac{7315a^{15/4}d^{21/2}x^2}{512\sqrt{2}b^{19/4}}+\frac{7315a^{19/4}d^{21/2}}{2048\sqrt{2}b^{23/4}}\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\frac{7315a^{19/4}d^{21/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2048\sqrt{2}b^{23/4}}+\frac{7315a^4d^{21/2}x^{3/2}}{3072b^5}+\frac{26125a^3d^{21/2}x^{7/2}}{3072b^4}+\frac{11115a^2d^{21/2}x^{11/2}}{1024b^3}+\frac{16967ad^{21/2}x^{15/2}}{3072b^2}+\frac{2d^{21/2}x^{19/2}}{3b}\right)}{\sqrt{dx}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[x]*((7315*a^4*d^(21/2)*x^(3/2))/(3072*b^5) + (26125*a^3*d^(21
/2)*x^(7/2))/(3072*b^4) + (11115*a^2*d^(21/2)*x^(11/2))/(1024*b^3) + (16967
*a*d^(21/2)*x^(15/2))/(3072*b^2) + (2*d^(21/2)*x^(19/2))/(3*b) + ((7315*a^(
19/4)*d^(21/2))/(2048*Sqrt[2]*b^(23/4)) + (7315*a^(15/4)*d^(21/2)*x^2)/(512
*Sqrt[2]*b^(19/4)) + (21945*a^(11/4)*d^(21/2)*x^4)/(1024*Sqrt[2]*b^(15/4))
 + (7315*a^(7/4)*d^(21/2)*x^6)/(512*Sqrt[2]*b^(11/4)) + (7315*a^(3/4)*d^(21/
2)*x^8)/(2048*Sqrt[2]*b^(7/4)))*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])] + (7315*a^(19/4)*d^(21/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(2048*Sqrt[2]*b^(23/4)) + (7315*a^(15
/4)*d^(21/2)*x^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[
b]*x)])/(512*Sqrt[2]*b^(19/4)) + (21945*a^(11/4)*d^(21/2)*x^4*ArcTanh[(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(1024*Sqrt[2]*b^(15/4)
) + (7315*a^(7/4)*d^(21/2)*x^6*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(S
qrt[a] + Sqrt[b]*x)])/(512*Sqrt[2]*b^(11/4)) + (7315*a^(3/4)*d^(21/2)*x^8*A
rcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(2048*Sqrt
[2]*b^(7/4))))/(Sqrt[d*x]*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.83, size = 457, normalized size = 0.76

$$\frac{87780\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)\arctan\left(-\frac{\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^6d^{31}-\sqrt{a^4d^{63}x-\sqrt{-\frac{a^3d^{42}}{b^{23}}}\,a^3b^{11}d^{42}}\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}b^6}{a^3d^{42}}\right)-21945\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)\log\left(391419980875\sqrt{dx}\,a^2d^{31}+391419980875\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{3}{4}}b^{17}\right)+21945\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)\log\left(391419980875\sqrt{dx}\,a^2d^{31}-391419980875\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{3}{4}}b^{17}\right)+4\left(2048b^4d^{10}x^9+16967ab^3d^{10}x^7+33345a^2b^2d^{10}x^5+26125a^3bd^{10}x^3+7315a^4d^{10}x\right)\sqrt{dx}}{12288\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(87780*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^
4 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan(-((-a^3*d^42/b^23)^(1/4)*sqrt(d*x)*a^2*
b^6*d^31 - sqrt(a^4*d^63*x - sqrt(-a^3*d^42/b^23)*a^3*b^11*d^42)*(-a^3*d^42
/b^23)^(1/4)*b^6)/(a^3*d^42)) - 21945*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a
*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(391419980875*sqrt(d
*x)*a^2*d^31 + 391419980875*(-a^3*d^42/b^23)^(3/4)*b^17) + 21945*(-a^3*d^42
/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b
^5)*log(391419980875*sqrt(d*x)*a^2*d^31 - 391419980875*(-a^3*d^42/b^23)^(3/
4)*b^17) + 4*(2048*b^4*d^10*x^9 + 16967*a*b^3*d^10*x^7 + 33345*a^2*b^2*d^10
*x^5 + 26125*a^3*b*d^10*x^3 + 7315*a^4*d^10*x)*sqrt(d*x))/(b^9*x^8 + 4*a*b^
8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

giac [A] time = 0.42, size = 437, normalized size = 0.73

$$\frac{1}{24576}\,d^{10}\left(\frac{16384\,\sqrt{dx}\,x}{b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{43890\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{43890\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{21945\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^8d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{21945\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^8d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{8\left(8775\,\sqrt{dx}\,ab^3d^8x^7+21057\,\sqrt{dx}\,a^2b^2d^8x^5+17933\,\sqrt{dx}\,a^3bd^8x^3+5267\,\sqrt{dx}\,a^4d^8x\right)}{\left(bd^2x^2+ad^2\right)^4b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^10*(16384*sqrt(d*x)*x/(b^5*sgn(b*d^4*x^2 + a*d^4)) - 43890*sqrt(2
)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*
x))/(a*d^2/b)^(1/4))/(b^8*d*sgn(b*d^4*x^2 + a*d^4)) - 43890*sqrt(2)*(a*b^3*
d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/(b^8*d*sgn(b*d^4*x^2 + a*d^4)) + 21945*sqrt(2)*(a*b^3*d^2)^(3/
4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*d*sgn(
b*d^4*x^2 + a*d^4)) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*
d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*d*sgn(b*d^4*x^2 + a*d^4)) + 8*
(8775*sqrt(d*x)*a*b^3*d^8*x^7 + 21057*sqrt(d*x)*a^2*b^2*d^8*x^5 + 17933*sqr
t(d*x)*a^3*b*d^8*x^3 + 5267*sqrt(d*x)*a^4*d^8*x)/((b*d^2*x^2 + a*d^2)^4*b^5
*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.03, size = 1171, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(16384*(d*x)^(3/2)*(a/b*d^2)^(1/4)*x^8*b^5*d^6-21945*2^(1/2)*ln(-(-
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^8*a*b^4*d^8-43890*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*a*b^4*d^8-43890
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*
a*b^4*d^8+70200*(d*x)^(15/2)*(a/b*d^2)^(1/4)*a*b^4+65536*(d*x)^(3/2)*(a/b*d
```

```
^2)^(1/4)*x^6*a*b^4*d^6-87780*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))*x^6*a^2*b^3*d^8-175560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d
^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a^2*b^3*d^8-175560*2^(1/2)*arctan((2^(1/2)*
(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a^2*b^3*d^8+168456*(d*x)^
(11/2)*(a/b*d^2)^(1/4)*a^2*b^3*d^2+98304*(d*x)^(3/2)*(a/b*d^2)^(1/4)*x^4*a^
2*b^3*d^6-131670*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b
*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4
*a^3*b^2*d^8-263340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a
/b*d^2)^(1/4))*x^4*a^3*b^2*d^8-263340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(
a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^3*b^2*d^8+143464*(d*x)^(7/2)*(a/b*d^
2)^(1/4)*a^3*b^2*d^4+65536*(d*x)^(3/2)*(a/b*d^2)^(1/4)*x^2*a^3*b^2*d^6-8778
0*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^4*b*d^8-1755
60*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^
2*a^4*b*d^8-175560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))*x^2*a^4*b*d^8+58520*(d*x)^(3/2)*(a/b*d^2)^(1/4)*a^4*b*d^6-219
45*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^5*d^8-43890*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^5*d^8-
43890*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))
*a^5*d^8)*d^3*(b*x^2+a)/(a/b*d^2)^(1/4)/b^6/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,ad^{\frac{21}{2}}\int\frac{\sqrt{x}}{b^6x^2+ab^5}\,dx+d^{\frac{21}{2}}\int\frac{x^{\frac{5}{2}}}{b^5x^2+ab^4}\,dx+\frac{2925\,ad^{\frac{21}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,b^5}+\frac{8775\,ab^3d^{\frac{21}{2}}x^{\frac{15}{2}}+29649\,a^2b^2d^{\frac{21}{2}}x^{\frac{11}{2}}+34285\,a^3bd^{\frac{21}{2}}x^{\frac{7}{2}}+13795\,a^4d^{\frac{21}{2}}x^{\frac{3}{2}}}{3072\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}-\frac{\left(537\,a^2b^4d^{\frac{21}{2}}x^5+1210\,a^3b^3d^{\frac{21}{2}}x^3+705\,a^4b^2d^{\frac{21}{2}}x\right)x^{\frac{9}{2}}+2\left(443\,a^3b^3d^{\frac{21}{2}}x^5+1014\,a^4b^2d^{\frac{21}{2}}x^3+603\,a^5bd^{\frac{21}{2}}x\right)x^{\frac{5}{2}}+\left(381\,a^4b^2d^{\frac{21}{2}}x^5+882\,a^5bd^{\frac{21}{2}}x^3+533\,a^6d^{\frac{21}{2}}x\right)\sqrt{x}}{192\left(a^3b^8x^6+3\,a^4b^7x^4+3\,a^5b^6x^2+a^6b^5+\left(b^{11}x^6+3\,ab^{10}x^4+3\,a^2b^9x^2+a^3b^8\right)x^6+3\left(ab^{10}x^6+3\,a^2b^9x^4+3\,a^3b^8x^2+a^4b^7\right)x^4+3\left(a^2b^9x^6+3\,a^3b^8x^4+3\,a^4b^7x^2+a^5b^6\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -4*a*d^(21/2)*integrate(sqrt(x)/(b^6*x^2 + a*b^5), x) + d^(21/2)*integrate(
x^(5/2)/(b^5*x^2 + a*b^4), x) + 2925/8192*a*d^(21/2)*(2*sqrt(2)*arctan(1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^5 + 1/3072*(8775*a*b^3*d^(21/2
)*x^(15/2) + 29649*a^2*b^2*d^(21/2)*x^(11/2) + 34285*a^3*b*d^(21/2)*x^(7/2)
 + 13795*a^4*d^(21/2)*x^(3/2))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a
^3*b^6*x^2 + a^4*b^5) - 1/192*((537*a^2*b^4*d^(21/2)*x^5 + 1210*a^3*b^3*d^(
21/2)*x^3 + 705*a^4*b^2*d^(21/2)*x)*x^(9/2) + 2*(443*a^3*b^3*d^(21/2)*x^5 +
 1014*a^4*b^2*d^(21/2)*x^3 + 603*a^5*b*d^(21/2)*x)*x^(5/2) + (381*a^4*b^2*d
^(21/2)*x^5 + 882*a^5*b*d^(21/2)*x^3 + 533*a^6*d^(21/2)*x)*sqrt(x))/(a^3*b^
```

```
8*x^6 + 3*a^4*b^7*x^4 + 3*a^5*b^6*x^2 + a^6*b^5 + (b^11*x^6 + 3*a*b^10*x^4
+ 3*a^2*b^9*x^2 + a^3*b^8)*x^6 + 3*(a*b^10*x^6 + 3*a^2*b^9*x^4 + 3*a^3*b^8*
x^2 + a^4*b^7)*x^4 + 3*(a^2*b^9*x^6 + 3*a^3*b^8*x^4 + 3*a^4*b^7*x^2 + a^5*b
^6)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{21/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.593 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=600

$$-\frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{a}\,d^{19/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{\right.}}{4096\sqrt{2}\,b^{21/4}\sqrt{a^2-}}$$

Rubi [A] time = 0.46, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3315d^9\sqrt{dx}\left(a+bx^2\right)}{1024b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{221d^5(dx)^{9/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{a}\,d^{19/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{a}\,d^{19/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{a}\,d^{19/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\,b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{a}\,d^{19/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\,b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-663*d^7*(d*x)^(5/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(17/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (17*d^3*(d*
x)^(13/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (221*d^
5*(d*x)^(9/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
5*d^9*Sqrt[d*x]*(a + b*x^2))/(1024*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a
^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(21/4)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]
*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(17b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(22\cdots}{\cdots}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{768\cdots}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

Mathematica [A] time = 0.28, size = 384, normalized size = 0.64

$$\frac{(dx)^{19/2}\left(a+bx^2\right)\left(10183680a^4\sqrt[4]{b}\sqrt{x}+32587776a^3b^{5/4}x^{5/2}-848640a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+39829504a^2b^{9/4}x^{9/2}-1166880a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+21446656ab^{13/4}x^{13/2}-2042040a\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+765765\sqrt{2}\sqrt[4]{a}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-765765\sqrt{2}\sqrt[4]{a}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+1531530\sqrt{2}\sqrt[4]{a}\left(a+bx^2\right)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-1531530\sqrt{2}\sqrt[4]{a}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+3784704b^{17/4}x^{17/2}\right)}{1892352b^{21/4}x^{19/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(19/2)*(a + b*x^2)*(10183680*a^4*b^(1/4)*Sqrt[x] + 32587776*a^3*b^(5
/4)*x^(5/2) + 39829504*a^2*b^(9/4)*x^(9/2) + 21446656*a*b^(13/4)*x^(13/2) +
 3784704*b^(17/4)*x^(17/2) - 848640*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2) - 11668
80*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 - 2042040*a*b^(1/4)*Sqrt[x]*(a + b*x^2
)^3 + 1531530*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)] - 1531530*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)] + 765765*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 765765*Sqrt[2]*a^(1/4)*(
a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(
1892352*b^(21/4)*x^(19/2)*((a + b*x^2)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 1.18, size = 621, normalized size = 1.04

$$\frac{\sqrt{d}\sqrt{x}\left(-\frac{3315a^{5/4}d^{19/2}x^6\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{512\sqrt{2}b^{9/4}}-\frac{9945a^{9/4}d^{19/2}x^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{1024\sqrt{2}b^{13/4}}-\frac{3315a^{13/4}d^{19/2}x^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{512\sqrt{2}b^{17/4}}+\left(\frac{3315a^{5/4}d^{19/2}x^6}{512\sqrt{2}b^{9/4}}+\frac{9945a^{9/4}d^{19/2}x^4}{1024\sqrt{2}b^{13/4}}+\frac{3315a^{13/4}d^{19/2}x^2}{512\sqrt{2}b^{17/4}}+\frac{3315a^{17/4}d^{19/2}}{2048\sqrt{2}b^{21/4}}+\frac{3315\sqrt[4]{a}d^{19/2}x^8}{2048\sqrt{2}b^{5/4}}\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\frac{3315a^{17/4}d^{19/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2048\sqrt{2}b^{21/4}}+\frac{3315a^4d^{19/2}\sqrt{x}}{1024b^5}+\frac{12597a^3d^{19/2}x^{5/2}}{1024b^4}+\frac{52819a^2d^{19/2}x^{9/2}}{3072b^3}-\frac{3315\sqrt[4]{a}d^{19/2}x^8\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2048\sqrt{2}b^{5/4}}+\frac{31501ad^{19/2}x^{13/2}}{3072b^2}+\frac{2d^{19/2}x^{17/2}}{b}\right)}{\sqrt{dx}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[x]*((3315*a^4*d^(19/2)*Sqrt[x])/(1024*b^5) + (12597*a^3*d^(19
/2)*x^(5/2))/(1024*b^4) + (52819*a^2*d^(19/2)*x^(9/2))/(3072*b^3) + (31501*
a*d^(19/2)*x^(13/2))/(3072*b^2) + (2*d^(19/2)*x^(17/2))/b + ((3315*a^(17/4)
*d^(19/2))/(2048*Sqrt[2]*b^(21/4)) + (3315*a^(13/4)*d^(19/2)*x^2)/(512*Sqrt
[2]*b^(17/4)) + (9945*a^(9/4)*d^(19/2)*x^4)/(1024*Sqrt[2]*b^(13/4)) + (3315
*a^(5/4)*d^(19/2)*x^6)/(512*Sqrt[2]*b^(9/4)) + (3315*a^(1/4)*d^(19/2)*x^8)/
(2048*Sqrt[2]*b^(5/4)))*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])] - (3315*a^(17/4)*d^(19/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])/(Sqrt[a] + Sqrt[b]*x)])/(2048*Sqrt[2]*b^(21/4)) - (3315*a^(13/4)*d^(1
9/2)*x^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
(512*Sqrt[2]*b^(17/4)) - (9945*a^(9/4)*d^(19/2)*x^4*ArcTanh[(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(1024*Sqrt[2]*b^(13/4)) - (3315*
a^(5/4)*d^(19/2)*x^6*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)])/(512*Sqrt[2]*b^(9/4)) - (3315*a^(1/4)*d^(19/2)*x^8*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(2048*Sqrt[2]*b^(5/4)
)))/(Sqrt[d*x]*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.91, size = 421, normalized size = 0.70

$$-\frac{39780\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)\arctan\left(-\frac{\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{16}d^9-\sqrt{d^{19}x+\sqrt{-\frac{ad^{38}}{b^{21}}}\,b^{10}}\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{3}{4}}b^{16}}{ad^{38}}\right)+9945\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)\log\left(3315\sqrt{dx}\,d^9+3315\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}}b^5\right)-9945\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)\log\left(3315\sqrt{dx}\,d^9-3315\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}}b^5\right)-4\left(6144b^4d^9x^8+31501ab^3d^9x^6+52819a^2b^2d^9x^4+37791a^3bd^9x^2+9945a^4d^9\right)\sqrt{dx}}{12288\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(39780*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4
 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan(-((-a*d^38/b^21)^(3/4)*sqrt(d*x)*b^16*d^
9 - sqrt(d^19*x + sqrt(-a*d^38/b^21)*b^10)*(-a*d^38/b^21)^(3/4)*b^16)/(a*d^
38)) + 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4
*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 + 3315*(-a*d^38/b^21)^(1/4)*
b^5) - 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4
*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 - 3315*(-a*d^38/b^21)^(1/4)*
b^5) - 4*(6144*b^4*d^9*x^8 + 31501*a*b^3*d^9*x^6 + 52819*a^2*b^2*d^9*x^4 +
37791*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a
^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

giac [A] time = 0.42, size = 423, normalized size = 0.70

$$-\frac{1}{24576}\,d^{9}\left(\frac{19890\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{6}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{19890\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{6}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{9945\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{6}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{9945\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{6}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{49152\,\sqrt{dx}}{b^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{8\left(6925\,\sqrt{dx}\,ab^{3}d^{8}x^{6}+15955\,\sqrt{dx}\,a^{2}b^{2}d^{8}x^{4}+13215\,\sqrt{dx}\,a^{3}bd^{8}x^{2}+3801\,\sqrt{dx}\,a^{4}d^{8}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{4}b^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*d^9*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*d^4*x^2 + a*d^4))
 + 19890*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*d^4*x^2 + a*d^4)) + 9945*sq
rt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(b^6*sgn(b*d^4*x^2 + a*d^4)) - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*log
(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*sgn(b*d^4*x^
2 + a*d^4)) - 49152*sqrt(d*x)/(b^5*sgn(b*d^4*x^2 + a*d^4)) - 8*(6925*sqrt(d
*x)*a*b^3*d^8*x^6 + 15955*sqrt(d*x)*a^2*b^2*d^8*x^4 + 13215*sqrt(d*x)*a^3*b
*d^8*x^2 + 3801*sqrt(d*x)*a^4*d^8)/((b*d^2*x^2 + a*d^2)^4*b^5*sgn(b*d^4*x^2
 + a*d^4)))
```

maple [B] time = 0.03, size = 1202, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576*(9945*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)
^(1/2)))*x^8*b^4*d^6+19890*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
```

```
/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4*d^6+19890*(a/b*d^2)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4*
d^6+39780*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2
)))*x^6*a*b^3*d^6+79560*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^3*d^6+79560*(a/b*d^2)^(1/4)*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^3
*d^6-49152*(d*x)^(1/2)*x^8*b^4*d^6+59670*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a^2*b^2*d^6+119340*(a/b*d^2)^(1/4)*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^
2*b^2*d^6+119340*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d
^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^2*d^6-55400*(d*x)^(13/2)*a*b^3-196608
*(d*x)^(1/2)*x^6*a*b^3*d^6+39780*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^3*b*d^6+79560*(a/b*d^2)^(1/4)*2^(1/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b*d^6+795
60*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))*x^2*a^3*b*d^6-127640*(d*x)^(9/2)*a^2*b^2*d^2-294912*(d*x)^(1/
2)*x^4*a^2*b^2*d^6+9945*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a/b*d^2)^(1/2)))*a^4*d^6+19890*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*d^6+19890*(a/b*d^2)^(1/4)*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*d^6
-105720*(d*x)^(5/2)*a^3*b*d^4-196608*(d*x)^(1/2)*x^2*a^3*b*d^6-79560*(d*x)^
(1/2)*a^4*d^6)*d^3*(b*x^2+a)/b^5/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{\frac{19}{2}} \int \frac{x^{\frac{3}{2}}}{b^5 x^2 + a b^4}\, dx - \frac{1267 \left(\frac{2\sqrt{2}\sqrt{a} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} + 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\sqrt{a} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} - 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2} a^{\frac{1}{4}} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2} a^{\frac{1}{4}} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{b^{\frac{1}{4}}} \right) d^{\frac{19}{2}}}{8192\, b^5} + \frac{1853\, a b^3 d^{\frac{19}{2}} x^{\frac{13}{2}} + 6515\, a^2 b^2 d^{\frac{19}{2}} x^{\frac{9}{2}} + 8079\, a^3 b d^{\frac{19}{2}} x^{\frac{5}{2}} + 3801\, a^4 d^{\frac{19}{2}} \sqrt{x}}{3072\left(b^9 x^8 + 4 a b^8 x^6 + 6 a^2 b^7 x^4 + 4 a^3 b^6 x^2 + a^4 b^5\right)} + \frac{\left(317\, a b^4 d^{\frac{19}{2}} x^5 + 738\, a^2 b^3 d^{\frac{19}{2}} x^3 + 453\, a^3 b^2 d^{\frac{19}{2}} x\right) x^{\frac{11}{2}} + 2\left(243\, a^2 b^3 d^{\frac{19}{2}} x^5 + 582\, a^3 b^2 d^{\frac{19}{2}} x^3 + 371\, a^4 b d^{\frac{19}{2}} x\right) x^{\frac{7}{2}} + \left(201\, a^3 b^2 d^{\frac{19}{2}} x^5 + 490\, a^4 b d^{\frac{19}{2}} x^3 + 321\, a^5 d^{\frac{19}{2}} x\right) x^{\frac{3}{2}}}{192\left(a^3 b^7 x^6 + 3 a^4 b^6 x^4 + 3 a^5 b^5 x^2 + a^6 b^4 + \left(b^{10} x^6 + 3 a b^9 x^4 + 3 a^2 b^8 x^2 + a^3 b^7\right) x^6 + 3\left(a b^9 x^6 + 3 a^2 b^8 x^4 + 3 a^3 b^7 x^2 + a^4 b^6\right) x^4 + 3\left(a^2 b^8 x^6 + 3 a^3 b^7 x^4 + 3 a^4 b^6 x^2 + a^5 b^5\right) x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] d^(19/2)*integrate(x^(3/2)/(b^5*x^2 + a*b^4), x) - 1267/8192*(2*sqrt(2)*sqr
t(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/s
qrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))*d^(19/2)/b^5 + 1/3072*(1853*a*b^
3*d^(19/2)*x^(13/2) + 6515*a^2*b^2*d^(19/2)*x^(9/2) + 8079*a^3*b*d^(19/2)*x
^(5/2) + 3801*a^4*d^(19/2)*sqrt(x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4
+ 4*a^3*b^6*x^2 + a^4*b^5) + 1/192*((317*a*b^4*d^(19/2)*x^5 + 738*a^2*b^3*d
```

```
^(19/2)*x^3 + 453*a^3*b^2*d^(19/2)*x)*x^(11/2) + 2*(243*a^2*b^3*d^(19/2)*x^
5 + 582*a^3*b^2*d^(19/2)*x^3 + 371*a^4*b*d^(19/2)*x)*x^(7/2) + (201*a^3*b^2
*d^(19/2)*x^5 + 490*a^4*b*d^(19/2)*x^3 + 321*a^5*d^(19/2)*x)*x^(3/2))/(a^3*
b^7*x^6 + 3*a^4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4 + (b^10*x^6 + 3*a*b^9*x^4
 + 3*a^2*b^8*x^2 + a^3*b^7)*x^6 + 3*(a*b^9*x^6 + 3*a^2*b^8*x^4 + 3*a^3*b^7*
x^2 + a^4*b^6)*x^4 + 3*(a^2*b^8*x^6 + 3*a^3*b^7*x^4 + 3*a^4*b^6*x^2 + a^5*b
^5)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{19/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.594 $\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=554

$$-\frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\right.}{4096\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+\right.}$$

Rubi [A] time = 0.42, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{55d^5(dx)^{7/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155d^{17/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155d^{17/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-385*d^7*(d*x)^(3/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(15/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*(d*x
)^(11/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (55*d^5*
(d*x)^(7/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*
d^(17/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*d^(17/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1
155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (1155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(1
9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= -\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} -$$

Mathematica [C] time = 0.05, size = 106, normalized size = 0.19

$$\frac{2d^7(dx)^{3/2}\left(77\left(a+bx^2\right)^4 {}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(77a^3+143a^2bx^2+117ab^2x^4+39b^3x^6\right)\right)}{39ab^4\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d^7*(d*x)^(3/2)*(-(a*(77*a^3 + 143*a^2*b*x^2 + 117*a*b^2*x^4 + 39*b^3*x^
6)) + 77*(a + b*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -((b*x^2)/a)]))/(39*a
*b^4*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 0.98, size = 603, normalized size = 1.09

$$\frac{\sqrt{d}\sqrt{x}\left(-\frac{1155a^{3/4}d^{17/2}x^6\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{512\sqrt{2}b^{7/4}}-\frac{3465a^{7/4}d^{17/2}x^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{1024\sqrt{2}b^{11/4}}-\frac{1155a^{11/4}d^{17/2}x^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{512\sqrt{2}b^{15/4}}+\left(-\frac{1155a^{3/4}d^{17/2}x^6}{512\sqrt{2}b^{7/4}}-\frac{3465a^{7/4}d^{17/2}x^4}{1024\sqrt{2}b^{11/4}}-\frac{1155a^{11/4}d^{17/2}x^2}{512\sqrt{2}b^{15/4}}-\frac{1155a^{15/4}d^{17/2}}{2048\sqrt{2}b^{19/4}}-\frac{1155d^{17/2}x^8}{2048\sqrt{2}\sqrt[4]{a}b^{3/4}}\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\frac{1155a^{15/4}d^{17/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2048\sqrt{2}b^{19/4}}-\frac{385a^3d^{17/2}x^{3/2}}{1024b^4}-\frac{1375a^2d^{17/2}x^{7/2}}{1024b^3}-\frac{1155d^{17/2}x^8\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2048\sqrt{2}\sqrt[4]{a}b^{3/4}}-\frac{1755ad^{17/2}x^{11/2}}{1024b^2}-\frac{893d^{17/2}x^{15/2}}{1024b}\right)}{\sqrt{dx}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[x]*((-385*a^3*d^(17/2)*x^(3/2))/(1024*b^4) - (1375*a^2*d^(17/
2)*x^(7/2))/(1024*b^3) - (1755*a*d^(17/2)*x^(11/2))/(1024*b^2) - (893*d^(17
/2)*x^(15/2))/(1024*b) + ((-1155*a^(15/4)*d^(17/2))/(2048*Sqrt[2]*b^(19/4))
 - (1155*a^(11/4)*d^(17/2)*x^2)/(512*Sqrt[2]*b^(15/4)) - (3465*a^(7/4)*d^(1
7/2)*x^4)/(1024*Sqrt[2]*b^(11/4)) - (1155*a^(3/4)*d^(17/2)*x^6)/(512*Sqrt[2
]*b^(7/4)) - (1155*d^(17/2)*x^8)/(2048*Sqrt[2]*a^(1/4)*b^(3/4)))*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (1155*a^(15/4)*d^(1
7/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(204
8*Sqrt[2]*b^(19/4)) - (1155*a^(11/4)*d^(17/2)*x^2*ArcTanh[(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(512*Sqrt[2]*b^(15/4)) - (3465*a^(
7/4)*d^(17/2)*x^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)])/(1024*Sqrt[2]*b^(11/4)) - (1155*a^(3/4)*d^(17/2)*x^6*ArcTanh[(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(512*Sqrt[2]*b^(7/4))
- (1155*d^(17/2)*x^8*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)])/(2048*Sqrt[2]*a^(1/4)*b^(3/4))))/(Sqrt[d*x]*(a + b*x^2)^3*Sqrt[
(a + b*x^2)^2])
```

fricas [A] time = 1.77, size = 428, normalized size = 0.77

$$-\frac{4620\left(b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^5d^{25}-\sqrt{d^{51}x-\sqrt{-\frac{d^{34}}{ab^{19}}}\,ab^9d^{34}}\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}b^5}{d^{34}}\right)-1155\left(b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\log\left(1540798875\sqrt{dx}\,d^{25}+1540798875\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{3}{4}}ab^{14}\right)+1155\left(b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\log\left(1540798875\sqrt{dx}\,d^{25}-1540798875\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{3}{4}}ab^{14}\right)+4\left(893b^3d^8x^7+1755ab^2d^8x^5+1375a^2bd^8x^3+385a^3d^8x\right)\sqrt{dx}}{4096\left(b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")

[Out] -1/4096*(4620*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^34/(a*b^19))^(1/4)*arctan(-((-d^34/(a*b^19))^(1/4)*sqrt(d*x)*b^5*d^25 - sqrt(d^51*x - sqrt(-d^34/(a*b^19))*a*b^9*d^34)*(-d^34/(a*b^19))^(1/4)*b^5)/d^34) - 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x)*d^25 + 1540798875*(-d^34/(a*b^19))^(3/4)*a*b^14) + 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x)*d^25 - 1540798875*(-d^34/(a*b^19))^(3/4)*a*b^14) + 4*(893*b^3*d^8*x^7 + 1755*a*b^2*d^8*x^5 + 1375*a^2*b*d^8*x^3 + 385*a^3*d^8*x)*sqrt(d*x))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)

giac [A] time = 0.37, size = 418, normalized size = 0.75

$$\frac{1}{8192}d^8\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^7d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^7d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^7d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^7d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{8\left(893\sqrt{dx}\,b^3d^8x^7+1755\sqrt{dx}\,ab^2d^8x^5+1375\sqrt{dx}\,a^2bd^8x^3+385\sqrt{dx}\,a^3d^8x\right)}{\left(bd^2x^2+ad^2\right)^4b^4\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")

[Out] 1/8192*d^8*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*d*sgn(b*d^4*x^2 + a*d^4)) + 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*d*sgn(b*d^4*x^2 + a*d^4)) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^7*d*sgn(b*d^4*x^2 + a*d^4)) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^7*d*sgn(b*d^4*x^2 + a*d^4)) - 8*(893*sqrt(d*x)*b^3*d^8*x^7 + 1755*sqrt(d*x)*a*b^2*d^8*x^5 + 1375*sqrt(d*x)*a^2*b*d^8*x^3 + 385*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*b^4*sgn(b*d^4*x^2 + a*d^4)))

maple [B] time = 0.02, size = 1046, normalized size = 1.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)

[Out] -1/8192*(-1155*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^8*b^4*d^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4*d^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4*d^8+7144*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4-4620*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+

```
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a*b^3*d^8-9240*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b
^3*d^8-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^
(1/4))*x^6*a*b^3*d^8+14040*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2-6930*2^(1
/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a^2*b^2*d^8-13860*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2
*b^2*d^8-13860*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))*x^4*a^2*b^2*d^8+11000*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4-462
0*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^3*b*d^8-9240
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*
a^3*b*d^8-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))*x^2*a^3*b*d^8+3080*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6-1155*2^(
1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^4*d^8-2310*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*d^8-2310*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*d^8
)*d*(b*x^2+a)/(a/b*d^2)^(1/4)/b^5/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{\frac{17}{2}}\int \frac{\sqrt{x}}{b^5x^2+ab^4}\,dx-\frac{893\,d^{\frac{17}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,b^4}-\frac{2679\,b^3d^{\frac{17}{2}}x^{\frac{15}{2}}+9441\,ab^2d^{\frac{17}{2}}x^{\frac{11}{2}}+11645\,a^2bd^{\frac{17}{2}}x^{\frac{7}{2}}+5267\,a^3d^{\frac{17}{2}}x^{\frac{3}{2}}}{3072\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)}+\frac{\left(261\,ab^4d^{\frac{17}{2}}x^5+610\,a^2b^3d^{\frac{17}{2}}x^3+381\,a^3b^2d^{\frac{17}{2}}x\right)x^{\frac{9}{2}}+2\left(191\,a^2b^3d^{\frac{17}{2}}x^5+462\,a^3b^2d^{\frac{17}{2}}x^3+303\,a^4bd^{\frac{17}{2}}x\right)x^{\frac{5}{2}}+\left(153\,a^3b^2d^{\frac{17}{2}}x^5+378\,a^4bd^{\frac{17}{2}}x^3+257\,a^5d^{\frac{17}{2}}x\right)\sqrt{x}}{192\left(a^3b^7x^6+3\,a^4b^6x^4+3\,a^5b^5x^2+a^6b^4+\left(b^{10}x^6+3\,ab^9x^4+3\,a^2b^8x^2+a^3b^7\right)x^6+3\left(ab^9x^6+3\,a^2b^8x^4+3\,a^3b^7x^2+a^4b^6\right)x^4+3\left(a^2b^8x^6+3\,a^3b^7x^4+3\,a^4b^6x^2+a^5b^5\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] d^(17/2)*integrate(sqrt(x)/(b^5*x^2 + a*b^4), x) - 893/8192*d^(17/2)*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 - 1/3072*(26
79*b^3*d^(17/2)*x^(15/2) + 9441*a*b^2*d^(17/2)*x^(11/2) + 11645*a^2*b*d^(17
/2)*x^(7/2) + 5267*a^3*d^(17/2)*x^(3/2))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6
*x^4 + 4*a^3*b^5*x^2 + a^4*b^4) + 1/192*((261*a*b^4*d^(17/2)*x^5 + 610*a^2*
b^3*d^(17/2)*x^3 + 381*a^3*b^2*d^(17/2)*x)*x^(9/2) + 2*(191*a^2*b^3*d^(17/2
)*x^5 + 462*a^3*b^2*d^(17/2)*x^3 + 303*a^4*b*d^(17/2)*x)*x^(5/2) + (153*a^3
*b^2*d^(17/2)*x^5 + 378*a^4*b*d^(17/2)*x^3 + 257*a^5*d^(17/2)*x)*sqrt(x))/(
a^3*b^7*x^6 + 3*a^4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4 + (b^10*x^6 + 3*a*b^9
*x^4 + 3*a^2*b^8*x^2 + a^3*b^7)*x^6 + 3*(a*b^9*x^6 + 3*a^2*b^8*x^4 + 3*a^3*
b^7*x^2 + a^4*b^6)*x^4 + 3*(a^2*b^8*x^6 + 3*a^3*b^7*x^4 + 3*a^4*b^6*x^2 + a
^5*b^5)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{17/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.595 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=554

$$-\frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{256b^3}$$

Rubi [A] time = 0.42, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{39d^5(dx)^{5/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195d^{15/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195d^{15/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-195*d^7*Sqrt[d*x])/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(13/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13*d^3*(d*x)
^(9/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (39*d^5*(d
*x)^(5/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d^(
15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*d^
(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (195*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[
d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(39\cdots}{\cdots}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{256\cdots}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+bx\cdots\right.}$$

Mathematica [A] time = 0.26, size = 366, normalized size = 0.66

$$\frac{(dx)^{15/2}\left(a+bx^2\right)\left(-\frac{45045\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{45045\sqrt{2}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}-\frac{90090\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{90090\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-599040a^3\sqrt[4]{b}\sqrt{x}-1916928a^2b^{5/4}x^{5/2}+49920a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-2342912ab^{9/4}x^{9/2}+120120\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+68640a\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2-1261568b^{13/4}x^{13/2}\right)}{1892352b^{17/4}x^{15/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(15/2)*(a + b*x^2)*(-599040*a^3*b^(1/4)*Sqrt[x] - 1916928*a^2*b^(5/4
)*x^(5/2) - 2342912*a*b^(9/4)*x^(9/2) - 1261568*b^(13/4)*x^(13/2) + 49920*a
^2*b^(1/4)*Sqrt[x]*(a + b*x^2) + 68640*a*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 12
0120*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - (90090*Sqrt[2]*(a + b*x^2)^4*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (90090*Sqrt[2]*(a + b*x^2)^
4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (45045*Sqrt[2]*(
a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^
(3/4) + (45045*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/a^(3/4)))/(1892352*b^(17/4)*x^(15/2)*((a + b*x^2)^2)^
(5/2))
```

IntegrateAlgebraic [A] time = 118.01, size = 269, normalized size = 0.49

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{195d^{15/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{3/4}b^{17/4}}+\frac{195d^{15/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{3/4}b^{17/4}}-\frac{d^9\sqrt{dx}\left(585a^3d^6+2223a^2bd^6x^2+3107ab^2d^6x^4+1853b^3d^6x^6\right)}{3072b^4\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(-1/3072*(d^9*Sqrt[d*x]*(585*a^3*d^6 + 2223*a^2*b*d^6*
x^2 + 3107*a*b^2*d^6*x^4 + 1853*b^3*d^6*x^6))/(b^4*(a*d^2 + b*d^2*x^2)^4) -
 (195*d^(15/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[
d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(3/4)*b^(17/4)) + (195
*d^(15/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d +
Sqrt[b]*d*x)])/(2048*Sqrt[2]*a^(3/4)*b^(17/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x
^2)^2/d^4])
```

fricas [A] time = 1.20, size = 431, normalized size = 0.78

$$\frac{2340\left(b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}a^2b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{d^{30}}{a^3b^{17}}}a^2b^8}\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{3}{4}}a^2b^{13}}{d^{30}}\right)+585\left(b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}\log\left(195\sqrt{dx}d^7+195\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}ab^4\right)-585\left(b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}\log\left(195\sqrt{dx}d^7-195\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}ab^4\right)-4\left(1853b^3d^7x^6+3107ab^2d^7x^4+2223a^2bd^7x^2+585a^3d^7\right)\sqrt{dx}}{12288\left(b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(2340*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*
b^4)*(-d^30/(a^3*b^17))^(1/4)*arctan(-((-d^30/(a^3*b^17))^(3/4)*sqrt(d*x)*a
^2*b^13*d^7 - sqrt(d^15*x + sqrt(-d^30/(a^3*b^17))*a^2*b^8)*(-d^30/(a^3*b^1
7))^(3/4)*a^2*b^13)/d^30) + 585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*
a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^7 + 195
*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x
^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^
7 - 195*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 4*(1853*b^3*d^7*x^6 + 3107*a*b^2*
d^7*x^4 + 2223*a^2*b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x))/(b^8*x^8 + 4*a*b^7*x
^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

giac [A] time = 0.35, size = 405, normalized size = 0.73

$$\frac{1}{24576}\,d^7\left(\frac{1170\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1170\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{585\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{585\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{8\left(1853\,\sqrt{dx}\,b^3d^8x^6+3107\,\sqrt{dx}\,ab^2d^8x^4+2223\,\sqrt{dx}\,a^2bd^8x^2+585\,\sqrt{dx}\,a^3d^8\right)}{\left(bd^2x^2+ad^2\right)^4b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*d^4*x^2 + a*d^4))
 + 1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*d^4*x^2 + a*d^4)) + 585*sq
rt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a*b^5*sgn(b*d^4*x^2 + a*d^4)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*lo
g(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*sgn(b*d^4
*x^2 + a*d^4)) - 8*(1853*sqrt(d*x)*b^3*d^8*x^6 + 3107*sqrt(d*x)*a*b^2*d^8*x
^4 + 2223*sqrt(d*x)*a^2*b*d^8*x^2 + 585*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*
d^2)^4*b^4*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1134, normalized size = 2.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(585*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2)))+1170*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)
*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1170*(a/b*d^2)^(1/4)*2^(1/2)
*b^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
2340*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1
```

```
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+4680*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*
x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+4680*(a/b*d^2)^(1/4)*2^(1/2)*a*b
^3*d^6*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+35
10*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+7020*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(
d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+7020*(a/b*d^2)^(1/4)*2^(1/2)*a
^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)-14824*(d*x)^(13/2)*a*b^3+2340*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4680*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*
d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+4680*
(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)
^(1/4))/(a/b*d^2)^(1/4))-24856*(d*x)^(9/2)*a^2*b^2*d^2+585*(a/b*d^2)^(1/4)*
2^(1/2)*a^4*d^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)
)/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1170*(a/b*d^2)
^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))+1170*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)-
(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))-17784*(d*x)^(5/2)*a^3*b*d^4-4680*(d*x)^(1
/2)*a^4*d^6)*d*(b*x^2+a)/a/b^4/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.76, size = 583, normalized size = 1.05

$$\frac{195\,d^7\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}-\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}\right)}{8192\,b^4}-\frac{15\,b^3d^{\frac{15}{2}}x^{\frac{13}{2}}+65\,ab^2d^{\frac{15}{2}}x^{\frac92}+117\,a^2bd^{\frac{15}{2}}x^{\frac52}+195\,a^3d^{\frac{15}{2}}\sqrt{x}}{1024\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)}-\frac{\left(113\,b^4d^{\frac{15}{2}}x^5+282\,ab^3d^{\frac{15}{2}}x^3+201\,a^2b^2d^{\frac{15}{2}}x\right)x^{\frac{11}{2}}+2\left(63\,ab^3d^{\frac{15}{2}}x^5+174\,a^2b^2d^{\frac{15}{2}}x^3+143\,a^3bd^{\frac{15}{2}}x\right)x^{\frac72}+\left(45\,a^2b^2d^{\frac{15}{2}}x^5+130\,a^3bd^{\frac{15}{2}}x^3+117\,a^4d^{\frac{15}{2}}x\right)x^{\frac32}}{192\left(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3+\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)x^6+3\left(ab^8x^6+3\,a^2b^7x^4+3\,a^3b^6x^2+a^4b^5\right)x^4+3\left(a^2b^7x^6+3\,a^3b^6x^4+3\,a^4b^5x^2+a^5b^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 195/8192*d^7*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(
2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3
/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(
b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4 - 1/1024*(15*b^3*d^(15/2)*x^(13/2) +
 65*a*b^2*d^(15/2)*x^(9/2) + 117*a^2*b*d^(15/2)*x^(5/2) + 195*a^3*d^(15/2)*
sqrt(x))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
- 1/192*((113*b^4*d^(15/2)*x^5 + 282*a*b^3*d^(15/2)*x^3 + 201*a^2*b^2*d^(15
/2)*x)*x^(11/2) + 2*(63*a*b^3*d^(15/2)*x^5 + 174*a^2*b^2*d^(15/2)*x^3 + 143
*a^3*b*d^(15/2)*x)*x^(7/2) + (45*a^2*b^2*d^(15/2)*x^5 + 130*a^3*b*d^(15/2)*
x^3 + 117*a^4*d^(15/2)*x)*x^(3/2))/(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4
*x^2 + a^6*b^3 + (b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)*x^6 + 3*
```

```
(a*b^8*x^6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^4 + 3*(a^2*b^7*x^6
+ 3*a^3*b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{15/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

3.596 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$-\frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-768b$$

Rubi [A] time = 0.43, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^5(dx)^{3/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{13/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{13/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (77*d^5*(d*x)^(3/2))/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(11/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (11*d^3*(d*
x)^(7/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^5*
(d*x)^(3/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (77*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(77}{}$$

$$= -\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{768}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2(a+bx}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2(a+bx}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2(a+bx}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2(a+bx}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2(a+bx}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2(a+bx}$$

Mathematica [C] time = 0.04, size = 97, normalized size = 0.17

$$\frac{2d^5(dx)^{3/2}\left(77\left(a+bx^2\right)^4\,{}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\left(77a^2+143abx^2+117b^2x^4\right)\right)}{585a^2b^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d^5*(d*x)^(3/2)*(-(a^2*(77*a^2 + 143*a*b*x^2 + 117*b^2*x^4)) + 77*(a + b
*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -((b*x^2)/a)]))/(585*a^2*b^3*(a + b*
x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 116.60, size = 272, normalized size = 0.49

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{77d^{13/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d\,x}}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{5/4}b^{15/4}}-\frac{77d^{13/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{5/4}b^{15/4}}-\frac{d^7(dx)^{3/2}\left(77a^3d^6+275a^2bd^6x^2+351ab^2d^6x^4-231b^3d^6x^6\right)}{3072ab^3\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(-1/3072*(d^7*(d*x)^(3/2)*(77*a^3*d^6 + 275*a^2*b*d^6*
x^2 + 351*a*b^2*d^6*x^4 - 231*b^3*d^6*x^6))/(a*b^3*(a*d^2 + b*d^2*x^2)^4) -
 (77*d^(13/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d
]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)) - (77*d
^(13/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sq
rt[b]*d*x)])/(2048*Sqrt[2]*a^(5/4)*b^(15/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2
)^2/d^4])
```

fricas [A] time = 1.66, size = 448, normalized size = 0.80

$$-\frac{924\left(ab^7x^8+4a^2b^6x^6+6a^3b^5x^4+4a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}\,ab^4d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{a^5b^{15}}}a^3b^7d^{26}}\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}ab^4}{d^{26}}\right)-231\left(ab^7x^8+4a^2b^6x^6+6a^3b^5x^4+4a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\log\left(456533\sqrt{dx}\,d^{19}+456533\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{3}{4}}a^4b^{11}\right)+231\left(ab^7x^8+4a^2b^6x^6+6a^3b^5x^4+4a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\log\left(456533\sqrt{dx}\,d^{19}-456533\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{3}{4}}a^4b^{11}\right)-4\left(231b^3d^6x^7-351ab^2d^6x^5-275a^2bd^6x^3-77a^3d^6x\right)\sqrt{dx}}{12288\left(ab^7x^8+4a^2b^6x^6+6a^3b^5x^4+4a^4b^4x^2+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(924*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 +
a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*arctan(-((-d^26/(a^5*b^15))^(1/4)*sqrt(d*
```

```
x)*a*b^4*d^19 - sqrt(d^39*x - sqrt(-d^26/(a^5*b^15))*a^3*b^7*d^26)*(-d^26/(
a^5*b^15))^(1/4)*a*b^4)/d^26) - 231*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*
x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*log(456533*sqrt(d*x
)*d^19 + 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) + 231*(a*b^7*x^8 + 4*a^2
*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4
)*log(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) - 4
*(231*b^3*d^6*x^7 - 351*a*b^2*d^6*x^5 - 275*a^2*b*d^6*x^3 - 77*a^3*d^6*x)*s
qrt(d*x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*
b^3)
```

giac [A] time = 0.36, size = 421, normalized size = 0.76

$$\frac{1}{24576}\,d^{6}\left(\frac{462\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{6}d\,\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{462\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{6}d\,\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{231\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{6}d\,\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{231\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{6}d\,\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{8\left(231\,\sqrt{dx}\,b^{3}d^{8}x^{7}-351\,\sqrt{dx}\,ab^{2}d^{8}x^{5}-275\,\sqrt{dx}\,a^{2}bd^{8}x^{3}-77\,\sqrt{dx}\,a^{3}d^{8}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{4}ab^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^6*(462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*d*sgn(b*d^4*x^2 + a*d^
4)) + 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*d*sgn(b*d^4*x^2 + a*d^4)) -
231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(a^2*b^6*d*sgn(b*d^4*x^2 + a*d^4)) + 231*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b
^6*d*sgn(b*d^4*x^2 + a*d^4)) + 8*(231*sqrt(d*x)*b^3*d^8*x^7 - 351*sqrt(d*x)
*a*b^2*d^8*x^5 - 275*sqrt(d*x)*a^2*b*d^8*x^3 - 77*sqrt(d*x)*a^3*d^8*x)/((b*
d^2*x^2 + a*d^2)^4*a*b^3*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1051, normalized size = 1.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(231*2^(1/2)*b^4*d^8*x^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))+462*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(
a/b*d^2)^(1/4))+462*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))+1848*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4+924*2^(1/2
)*a*b^3*d^8*x^6*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/
2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1848*2^(1/2)
*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
```

```
)+1848*2^(1/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(
a/b*d^2)^(1/4))-2808*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+1386*2^(1/2)*a^
2*b^2*d^8*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2)
)/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+2772*2^(1/2)*a
^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)+2772*2^(1/2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))
/(a/b*d^2)^(1/4))-2200*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+924*2^(1/2)*
a^3*b*d^8*x^2*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2)
)/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1848*2^(1/2)*a
^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
1848*2^(1/2)*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))-616*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+231*2^(1/2)*a^4*d^8
*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+462*2^(1/2)*a^4*d^8*arctan(
(2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+462*2^(1/2)*a^4*d^8*
arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d*(b*x^2+a)/
(a/b*d^2)^(1/4)/b^4/a/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.75, size = 577, normalized size = 1.04

$$\frac{77\,d^{\frac{13}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,ab^{3}}+\frac{77\,b^{3}d^{\frac{13}{2}}x^{\frac{15}{2}}+315\,ab^{2}d^{\frac{13}{2}}x^{\frac{11}{2}}+495\,a^{2}bd^{\frac{13}{2}}x^{\frac{7}{2}}+385\,a^{3}d^{\frac{13}{2}}x^{\frac{3}{2}}}{1024\left(ab^{7}x^{8}+4\,a^{2}b^{6}x^{6}+6\,a^{3}b^{5}x^{4}+4\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)}-\frac{\left(81\,b^{4}d^{\frac{13}{2}}x^{5}+202\,ab^{3}d^{\frac{13}{2}}x^{3}+153\,a^{2}b^{2}d^{\frac{13}{2}}x\right)x^{\frac{9}{2}}+2\left(35\,ab^{3}d^{\frac{13}{2}}x^{5}+102\,a^{2}b^{2}d^{\frac{13}{2}}x^{3}+99\,a^{3}bd^{\frac{13}{2}}x\right)x^{\frac{5}{2}}+\left(21\,a^{2}b^{2}d^{\frac{13}{2}}x^{5}+66\,a^{3}bd^{\frac{13}{2}}x^{3}+77\,a^{4}d^{\frac{13}{2}}x\right)\sqrt{x}}{192\left(a^{3}b^{6}x^{6}+3\,a^{4}b^{5}x^{4}+3\,a^{5}b^{4}x^{2}+a^{6}b^{3}+\left(b^{9}x^{6}+3\,ab^{8}x^{4}+3\,a^{2}b^{7}x^{2}+a^{3}b^{6}\right)x^{6}+3\left(ab^{8}x^{6}+3\,a^{2}b^{7}x^{4}+3\,a^{3}b^{6}x^{2}+a^{4}b^{5}\right)x^{4}+3\left(a^{2}b^{7}x^{6}+3\,a^{3}b^{6}x^{4}+3\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 77/8192*d^(13/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(
2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(
3/4)))/(a*b^3) + 1/1024*(77*b^3*d^(13/2)*x^(15/2) + 315*a*b^2*d^(13/2)*x^(1
1/2) + 495*a^2*b*d^(13/2)*x^(7/2) + 385*a^3*d^(13/2)*x^(3/2))/(a*b^7*x^8 +
4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3) - 1/192*((81*b^4*d
^(13/2)*x^5 + 202*a*b^3*d^(13/2)*x^3 + 153*a^2*b^2*d^(13/2)*x)*x^(9/2) + 2*
(35*a*b^3*d^(13/2)*x^5 + 102*a^2*b^2*d^(13/2)*x^3 + 99*a^3*b*d^(13/2)*x)*x^
(5/2) + (21*a^2*b^2*d^(13/2)*x^5 + 66*a^3*b*d^(13/2)*x^3 + 77*a^4*d^(13/2)*
x)*sqrt(x))/(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3 + (b^9*x
^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)*x^6 + 3*(a*b^8*x^6 + 3*a^2*b^7*
x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^4 + 3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4
*b^5*x^2 + a^5*b^4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{13/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.597 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$-\frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-256b$$

Rubi [A] time = 0.43, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15d^5\sqrt{dx}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{11/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (15*d^5*Sqrt[d*x])/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(9/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^3*(d*x)^(
5/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (15*d^5*Sqrt
[d*x])/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (45*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(15}{}$$

$$= -\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{256}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\left(a+bx\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\left(a+bx\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\left(a+bx\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\left(a+bx\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\left(a+bx\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\left(a+bx\right.}$$

Mathematica [A] time = 0.29, size = 352, normalized size = 0.63

$$\frac{d(dx)^{9/2}\left(a+bx^2\right)\left(-\frac{3465\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}+\frac{3465\sqrt{2}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}-\frac{6930\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6930\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}}-46080a^2\sqrt[4]{b}\sqrt{x}-147456ab^{5/4}x^{5/2}+\frac{9240\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3}{a}+5280\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+3840a\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-180224b^{9/4}x^{9/2}\right)}{630784b^{13/4}x^{9/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (d*(d*x)^(9/2)*(a + b*x^2)*(-46080*a^2*b^(1/4)*Sqrt[x] - 147456*a*b^(5/4)*x
^(5/2) - 180224*b^(9/4)*x^(9/2) + 3840*a*b^(1/4)*Sqrt[x]*(a + b*x^2) + 5280
*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + (9240*b^(1/4)*Sqrt[x]*(a + b*x^2)^3)/a - (
6930*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a
^(7/4) + (6930*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/a^(7/4) - (3465*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3465*Sqrt[2]*(a + b*x^2)^4*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4)))/(630784*b^
(13/4)*x^(9/2)*((a + b*x^2)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 115.50, size = 272, normalized size = 0.49

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{45d^{11/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{7/4}b^{13/4}}+\frac{45d^{11/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{7/4}b^{13/4}}-\frac{d^7\sqrt{dx}\left(45a^3d^6+171a^2bd^6x^2+239ab^2d^6x^4-15b^3d^6x^6\right)}{1024ab^3\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(-1/1024*(d^7*Sqrt[d*x]*(45*a^3*d^6 + 171*a^2*b*d^6*x^
2 + 239*a*b^2*d^6*x^4 - 15*b^3*d^6*x^6))/(a*b^3*(a*d^2 + b*d^2*x^2)^4) - (4
5*d^(11/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x
)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(7/4)*b^(13/4)) + (45*d^(1
1/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[
b]*d*x)])/(2048*Sqrt[2]*a^(7/4)*b^(13/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2
/d^4])
```

fricas [A] time = 1.20, size = 447, normalized size = 0.80

$$\frac{180\left(ab^7x^8+4a^2b^6x^6+6a^3b^5x^4+4a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}a^5b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{d^{22}}{a^7b^{13}}}a^4b^6}\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{3}{4}}a^5b^{10}}{d^{22}}\right)+45\left(ab^7x^8+4a^2b^6x^6+6a^3b^5x^4+4a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}\log\left(45\sqrt{dx}d^5+45\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}a^2b^3\right)-45\left(ab^7x^8+4a^2b^6x^6+6a^3b^5x^4+4a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}\log\left(45\sqrt{dx}d^5-45\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}a^2b^3\right)+4\left(15b^3d^5x^6-239ab^2d^5x^4-171a^2bd^5x^2-45a^3d^5\right)\sqrt{dx}}{4096\left(ab^7x^8+4a^2b^6x^6+6a^3b^5x^4+4a^4b^4x^2+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/4096*(180*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*arctan(-((-d^22/(a^7*b^13))^(3/4)*sqrt(d*x)*a^5*b^10*d^5 - sqrt(d^11*x + sqrt(-d^22/(a^7*b^13))*a^4*b^6)*(-d^22/(a^7*b^13))^(3/4)*a^5*b^10)/d^22) + 45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(d*x)*d^5 + 45*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) - 45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(d*x)*d^5 - 45*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) + 4*(15*b^3*d^5*x^6 - 239*a*b^2*d^5*x^4 - 171*a^2*b*d^5*x^2 - 45*a^3*d^5)*sqrt(d*x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)

giac [A] time = 0.35, size = 408, normalized size = 0.73

$$\frac{1}{8192}\,d^{5}\left(\frac{90\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{4}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{90\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{4}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{45\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{4}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{45\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{4}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{8\left(15\,\sqrt{dx}\,b^{3}d^{8}x^{6}-239\,\sqrt{dx}\,ab^{2}d^{8}x^{4}-171\,\sqrt{dx}\,a^{2}bd^{8}x^{2}-45\,\sqrt{dx}\,a^{3}d^{8}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{4}ab^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")

[Out] 1/8192*d^5*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)) + 90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)) + 8*(15*sqrt(d*x)*b^3*d^8*x^6 - 239*sqrt(d*x)*a*b^2*d^8*x^4 - 171*sqrt(d*x)*a^2*b*d^8*x^2 - 45*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*d^2)^4*a*b^3*sgn(b*d^4*x^2 + a*d^4)))

maple [B] time = 0.02, size = 1136, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)

[Out] 1/8192*(45*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+90*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+90*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+180*(a

```
/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1
/2)))+360*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*x)^(1/2)
+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+360*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^
6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+270*(a/b*d^
2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)
))+540*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)+
(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+540*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x
^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+120*(d*x)^
(13/2)*a*b^3+180*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((d*x+(a/b*d^2)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a/b*d^2)^(1/2)))+360*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((
2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+360*(a/b*d^2)^(1/4)*2
^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)
^(1/4))-1912*(d*x)^(9/2)*a^2*b^2*d^2+45*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*ln(
(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+90*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^
6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+90*(a/b*d^2
)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d
^2)^(1/4))-1368*(d*x)^(5/2)*a^3*b*d^4-360*(d*x)^(1/2)*a^4*d^6)/d*(b*x^2+a)/
b^3/a^2/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.71, size = 595, normalized size = 1.07

$$\frac{45\,d^{5}\left(\frac{2\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,\sqrt{d}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\sqrt{d}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\sqrt{d}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{8192\,ab^{3}}-\frac{35\,b^{3}d^{\frac{11}{2}}x^{\frac{13}{2}}+173\,ab^{2}d^{\frac{11}{2}}x^{\frac{9}{2}}+657\,a^{2}bd^{\frac{11}{2}}x^{\frac{5}{2}}+135\,a^{3}d^{\frac{11}{2}}\sqrt{x}}{3072\left(ab^{7}x^{8}+4\,a^{2}b^{6}x^{6}+6\,a^{3}b^{5}x^{4}+4\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)}+\frac{\left(5\,b^{4}d^{\frac{11}{2}}x^{5}+18\,ab^{3}d^{\frac{11}{2}}x^{3}+45\,a^{2}b^{2}d^{\frac{11}{2}}x\right)x^{\frac{11}{2}}-2\left(21\,ab^{3}d^{\frac{11}{2}}x^{5}+42\,a^{2}b^{2}d^{\frac{11}{2}}x^{3}-11\,a^{3}bd^{\frac{11}{2}}x\right)x^{\frac{7}{2}}-\left(15\,a^{2}b^{2}d^{\frac{11}{2}}x^{5}+38\,a^{3}bd^{\frac{11}{2}}x^{3}-9\,a^{4}d^{\frac{11}{2}}x\right)x^{\frac{3}{2}}}{192\left(a^{4}b^{5}x^{6}+3\,a^{5}b^{4}x^{4}+3\,a^{6}b^{3}x^{2}+a^{7}b^{2}+\left(ab^{8}x^{6}+3\,a^{2}b^{7}x^{4}+3\,a^{3}b^{6}x^{2}+a^{4}b^{5}\right)x^{6}+3\left(a^{2}b^{7}x^{6}+3\,a^{3}b^{6}x^{4}+3\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)x^{4}+3\left(a^{3}b^{6}x^{6}+3\,a^{4}b^{5}x^{4}+3\,a^{5}b^{4}x^{2}+a^{6}b^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 45/8192*d^5*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^3) - 1/3072*(35*b^3*d^(11/2)*x^(13/2
) + 173*a*b^2*d^(11/2)*x^(9/2) + 657*a^2*b*d^(11/2)*x^(5/2) + 135*a^3*d^(11
/2)*sqrt(x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a
^5*b^3) + 1/192*((5*b^4*d^(11/2)*x^5 + 18*a*b^3*d^(11/2)*x^3 + 45*a^2*b^2*d
^(11/2)*x)*x^(11/2) - 2*(21*a*b^3*d^(11/2)*x^5 + 42*a^2*b^2*d^(11/2)*x^3 -
11*a^3*b*d^(11/2)*x)*x^(7/2) - (15*a^2*b^2*d^(11/2)*x^5 + 38*a^3*b*d^(11/2)
*x^3 - 9*a^4*d^(11/2)*x)*x^(3/2))/(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*
x^2 + a^7*b^2 + (a*b^8*x^6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^6 +
```

```
3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x^4 + 3*(a^3*b^6
*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{11/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.598 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=560

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7d^3(dx)^{3/2}}{256ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\cdots$$

Rubi [A] time = 0.42, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7d^3(dx)^{3/2}}{256ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{9/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{9/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{9/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{9/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (35*d^3*(d*x)^(3/2))/(1024*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d
*x)^(7/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(3/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7*d^3*(
d*x)^(3/2))/(256*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d
^(9/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^
(9/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(
9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) - (35*d^(9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

Mathematica [C] time = 0.04, size = 86, normalized size = 0.15

$$\frac{2d^3(dx)^{3/2}\left(7\left(a+bx^2\right)^4\ {}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a^3\left(7a+13bx^2\right)\right)}{117a^3b^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d^3*(d*x)^(3/2)*(-(a^3*(7*a + 13*b*x^2)) + 7*(a + b*x^2)^4*Hypergeometri
c2F1[3/4, 5, 7/4, -((b*x^2)/a)]))/(117*a^3*b^2*(a + b*x^2)^3*Sqrt[(a + b*x^
2)^2])
```

IntegrateAlgebraic [A] time = 106.99, size = 281, normalized size = 0.50

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{35d^{9/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{9/4}b^{11/4}}-\frac{35d^{9/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{9/4}b^{11/4}}+\frac{-35a^3d^{11}(dx)^{3/2}-125a^2bd^9(dx)^{7/2}+399ab^2d^7(dx)^{11/2}+105b^3d^5(dx)^{15/2}}{3072a^2b^2\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-35*a^3*d^11*(d*x)^(3/2) - 125*a^2*b*d^9*(d*x)^(7/2)
 + 399*a*b^2*d^7*(d*x)^(11/2) + 105*b^3*d^5*(d*x)^(15/2))/(3072*a^2*b^2*(a*
d^2 + b*d^2*x^2)^4) - (35*d^(9/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4
)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(9/
4)*b^(11/4)) - (35*d^(9/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*
x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(2048*Sqrt[2]*a^(9/4)*b^(11/4))))/(d^2*Sqrt
[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.68, size = 462, normalized size = 0.82

$$-\frac{420\left(a^2b^6x^8+4a^3b^5x^6+6a^4b^4x^4+4a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{42875\sqrt{dx}a^2b^3d^{13}\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}-\sqrt{-1838265625a^5b^5d^{18}\sqrt{-\frac{d^{18}}{a^9b^{11}}}+1838265625d^{27}x}\,a^2b^3\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}}{42875d^{18}}\right)-105\left(a^2b^6x^8+4a^3b^5x^6+6a^4b^4x^4+4a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}\log\left(42875a^7b^8\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{3}{4}}+42875\sqrt{dx}d^{13}\right)+105\left(a^2b^6x^8+4a^3b^5x^6+6a^4b^4x^4+4a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}\log\left(-42875a^7b^8\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{3}{4}}+42875\sqrt{dx}d^{13}\right)-4\left(105b^3d^4x^7+399ab^2d^4x^5-125a^2bd^4x^3-35a^3d^4x\right)\sqrt{dx}}{12288\left(a^2b^6x^8+4a^3b^5x^6+6a^4b^4x^4+4a^5b^3x^2+a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2
+ a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*arctan(-1/42875*(42875*sqrt(d*x)*a^2*b^
```

```
3*d^13*(-d^18/(a^9*b^11))^(1/4) - sqrt(-1838265625*a^5*b^5*d^18*sqrt(-d^18/
(a^9*b^11)) + 1838265625*d^27*x)*a^2*b^3*(-d^18/(a^9*b^11))^(1/4))/d^18) -
105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
*(-d^18/(a^9*b^11))^(1/4)*log(42875*a^7*b^8*(-d^18/(a^9*b^11))^(3/4) + 4287
5*sqrt(d*x)*d^13) + 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^
5*b^3*x^2 + a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*log(-42875*a^7*b^8*(-d^18/(a^
9*b^11))^(3/4) + 42875*sqrt(d*x)*d^13) - 4*(105*b^3*d^4*x^7 + 399*a*b^2*d^4
*x^5 - 125*a^2*b*d^4*x^3 - 35*a^3*d^4*x)*sqrt(d*x))/(a^2*b^6*x^8 + 4*a^3*b^
5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

giac [A] time = 0.36, size = 421, normalized size = 0.75

$$\frac{1}{24576}d^4\left(\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{105\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^5d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{105\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^5d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{8\left(105\sqrt{dx}\,b^3d^8x^7+399\sqrt{dx}\,ab^2d^8x^5-125\sqrt{dx}\,a^2bd^8x^3-35\sqrt{dx}\,a^3d^8x\right)}{\left(bd^2x^2+ad^2\right)^4a^2b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^4*(210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*d*sgn(b*d^4*x^2 + a*d^
4)) + 210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*d*sgn(b*d^4*x^2 + a*d^4)) -
105*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(a^3*b^5*d*sgn(b*d^4*x^2 + a*d^4)) + 105*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b
^5*d*sgn(b*d^4*x^2 + a*d^4)) + 8*(105*sqrt(d*x)*b^3*d^8*x^7 + 399*sqrt(d*x)
*a*b^2*d^8*x^5 - 125*sqrt(d*x)*a^2*b*d^8*x^3 - 35*sqrt(d*x)*a^3*d^8*x)/((b*
d^2*x^2 + a*d^2)^4*a^2*b^2*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.03, size = 1051, normalized size = 1.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(105*2^(1/2)*b^4*d^8*x^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))+210*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(
a/b*d^2)^(1/4))+210*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))+840*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4+420*2^(1/2)
*a*b^3*d^8*x^6*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2
))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+840*2^(1/2)*a
*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
```

```
840*2^(1/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b
*d^2)^(1/4))+3192*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+630*2^(1/2)*a^2*b^
2*d^8*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1260*2^(1/2)*a^2*b
^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+12
60*2^(1/2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))-1000*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+420*2^(1/2)*a^3*
b*d^8*x^2*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+840*2^(1/2)*a^3*b*
d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+840*2
^(1/2)*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)
^(1/4))-280*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+105*2^(1/2)*a^4*d^8*ln(-(
-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+210*2^(1/2)*a^4*d^8*arctan((2^(1/
2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+210*2^(1/2)*a^4*d^8*arctan
((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d^3*(b*x^2+a)/(a/b
*d^2)^(1/4)/b^3/a^2/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.69, size = 584, normalized size = 1.04

$$\frac{35\,d^{\frac{9}{2}}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,a^{2}b^{2}}+\frac{105\,b^{3}d^{\frac{9}{2}}x^{\frac{15}{2}}+447\,ab^{2}d^{\frac{9}{2}}x^{\frac{11}{2}}+803\,a^{2}bd^{\frac{9}{2}}x^{\frac{7}{2}}+77\,a^{3}d^{\frac{9}{2}}x^{\frac{3}{2}}}{3072\left(a^{2}b^{6}x^{8}+4\,a^{3}b^{5}x^{6}+6\,a^{4}b^{4}x^{4}+4\,a^{5}b^{3}x^{2}+a^{6}b^{2}\right)}-\frac{\left(3\,b^{4}d^{\frac{9}{2}}x^{5}+14\,ab^{3}d^{\frac{9}{2}}x^{3}-21\,a^{2}b^{2}d^{\frac{9}{2}}x\right)x^{\frac{9}{2}}+2\left(25\,ab^{3}d^{\frac{9}{2}}x^{5}+66\,a^{2}b^{2}d^{\frac{9}{2}}x^{3}+9\,a^{3}bd^{\frac{9}{2}}x\right)x^{\frac{5}{2}}+\left(15\,a^{2}b^{2}d^{\frac{9}{2}}x^{5}+54\,a^{3}bd^{\frac{9}{2}}x^{3}+7\,a^{4}d^{\frac{9}{2}}x\right)\sqrt{x}}{192\left(a^{4}b^{5}x^{6}+3\,a^{5}b^{4}x^{4}+3\,a^{6}b^{3}x^{2}+a^{7}b^{2}+\left(ab^{8}x^{6}+3\,a^{2}b^{7}x^{4}+3\,a^{3}b^{6}x^{2}+a^{4}b^{5}\right)x^{6}+3\left(a^{2}b^{7}x^{6}+3\,a^{3}b^{6}x^{4}+3\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)x^{4}+3\left(a^{3}b^{6}x^{6}+3\,a^{4}b^{5}x^{4}+3\,a^{5}b^{4}x^{2}+a^{6}b^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 35/8192*d^(9/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3
/4)))/(a^2*b^2) + 1/3072*(105*b^3*d^(9/2)*x^(15/2) + 447*a*b^2*d^(9/2)*x^(1
1/2) + 803*a^2*b*d^(9/2)*x^(7/2) + 77*a^3*d^(9/2)*x^(3/2))/(a^2*b^6*x^8 + 4
*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) - 1/192*((3*b^4*d^(
9/2)*x^5 + 14*a*b^3*d^(9/2)*x^3 - 21*a^2*b^2*d^(9/2)*x)*x^(9/2) + 2*(25*a*b
^3*d^(9/2)*x^5 + 66*a^2*b^2*d^(9/2)*x^3 + 9*a^3*b*d^(9/2)*x)*x^(5/2) + (15*
a^2*b^2*d^(9/2)*x^5 + 54*a^3*b*d^(9/2)*x^3 + 7*a^4*d^(9/2)*x)*sqrt(x))/(a^4
*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2 + (a*b^8*x^6 + 3*a^2*b^7
*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^6 + 3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^
4*b^5*x^2 + a^5*b^4)*x^4 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 +
 a^6*b^3)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{9/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{9}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(9/2)/((a + b*x**2)**2)**(5/2), x)
```

3.599 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=560

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^3\sqrt{dx}}{768ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-- 8$$

Rubi [A] time = 0.44, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^3\sqrt{dx}}{768ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (35*d^3*Sqrt[d*x])/(3072*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(5/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*Sqrt[
d*x])/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^3*Sqrt[
d*x])/(768*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(204
8*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (35*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5a\cdots}{\cdots}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{76\cdots}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+b\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+b\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+b\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+b\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+b\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+b\cdots\right.}$$

Mathematica [A] time = 0.31, size = 341, normalized size = 0.61

$$\frac{(dx)^{7/2}\left(a+bx^2\right)\left(-49152a^{11/4}b^{5/4}x^{5/2}+3080a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+1760a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+1280a^{11/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-15360a^{15/4}\sqrt[4]{b}\sqrt{x}-1155\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+1155\sqrt{2}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-2310\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+2310\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{270336a^{11/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(7/2)*(a + b*x^2)*(-15360*a^(15/4)*b^(1/4)*Sqrt[x] - 49152*a^(11/4)*
b^(5/4)*x^(5/2) + 1280*a^(11/4)*b^(1/4)*Sqrt[x]*(a + b*x^2) + 1760*a^(7/4)*
b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 3080*a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3
- 2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
- 1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x] + 1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x]))/(270336*a^(11/4)*b^(9/4)*x^(7/2)*((a + b*x^2)^2
)^(5/2))
```

IntegrateAlgebraic [A] time = 102.81, size = 281, normalized size = 0.50

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{35d^{7/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{11/4}b^{9/4}}+\frac{35d^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{11/4}b^{9/4}}+\frac{-105a^3d^{11}\sqrt{dx}-399a^2bd^9(dx)^{5/2}+125ab^2d^7(dx)^{9/2}+35b^3d^5(dx)^{13/2}}{3072a^2b^2\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-105*a^3*d^11*Sqrt[d*x] - 399*a^2*b*d^9*(d*x)^(5/2)
+ 125*a*b^2*d^7*(d*x)^(9/2) + 35*b^3*d^5*(d*x)^(13/2))/(3072*a^2*b^2*(a*d^2
 + b*d^2*x^2)^4) - (35*d^(7/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4))
- (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(11/4)
*b^(9/4)) + (35*d^(7/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])
/(Sqrt[a]*d + Sqrt[b]*d*x)])/(2048*Sqrt[2]*a^(11/4)*b^(9/4))))/(d^2*Sqrt[(a
*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.87, size = 455, normalized size = 0.81

$$\frac{420\left(a^2b^6x^8+4a^3b^5x^6+6a^4b^4x^4+4a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^8b^7d^3\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{3}{4}}-\sqrt{a^6b^4\sqrt{-\frac{d^{14}}{a^{11}b^9}}+d^7x}\,a^8b^7\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{3}{4}}}{d^{14}}\right)+105\left(a^2b^6x^8+4a^3b^5x^6+6a^4b^4x^4+4a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}\log\left(35a^3b^2\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}+35\sqrt{dx}d^3\right)-105\left(a^2b^6x^8+4a^3b^5x^6+6a^4b^4x^4+4a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}\log\left(-35a^3b^2\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}+35\sqrt{dx}d^3\right)+4\left(35b^3d^3x^6+125ab^2d^3x^4-399a^2bd^3x^2-105a^3d^3\right)\sqrt{dx}}{12288\left(a^2b^6x^8+4a^3b^5x^6+6a^4b^4x^4+4a^5b^3x^2+a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 +
 a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*arctan(-(sqrt(d*x)*a^8*b^7*d^3*(-d^14/(a
^11*b^9))^(3/4) - sqrt(a^6*b^4*sqrt(-d^14/(a^11*b^9)) + d^7*x)*a^8*b^7*(-d^
14/(a^11*b^9))^(3/4))/d^14) + 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*
x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(35*a^3*b^2*(-d^
14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) - 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6
 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(-3
5*a^3*b^2*(-d^14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) + 4*(35*b^3*d^3*x^6
+ 125*a*b^2*d^3*x^4 - 399*a^2*b*d^3*x^2 - 105*a^3*d^3)*sqrt(d*x))/(a^2*b^6*
x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

giac [A] time = 0.35, size = 408, normalized size = 0.73

$$\frac{1}{24576}\,d^{3}\left(\frac{210\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{210\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{105\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\,\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{3}b^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{105\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\,\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{3}b^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{8\left(35\,\sqrt{dx}\,b^{3}d^{8}x^{6}+125\,\sqrt{dx}\,ab^{2}d^{8}x^{4}-399\,\sqrt{dx}\,a^{2}bd^{8}x^{2}-105\,\sqrt{dx}\,a^{3}d^{8}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{4}a^{2}b^{2}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^3*(210*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)
) + 210*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)) + 105*
sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqr
t(a*d^2/b))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)) - 105*sqrt(2)*(a*b^3*d^2)^(1/4
)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*sgn
(b*d^4*x^2 + a*d^4)) + 8*(35*sqrt(d*x)*b^3*d^8*x^6 + 125*sqrt(d*x)*a*b^2*d^
8*x^4 - 399*sqrt(d*x)*a^2*b*d^8*x^2 - 105*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 +
a*d^2)^4*a^2*b^2*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1136, normalized size = 2.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(105*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2)))+210*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)*
(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+210*(a/b*d^2)^(1/4)*2^(1/2)*b
^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+42
0*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
```

```
*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))+840*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*x)^(
1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+840*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^
6*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+630*(a/
b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(
1/2)))+1260*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(
1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1260*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2
*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+280*
(d*x)^(13/2)*a*b^3+420*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+840*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ar
ctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+840*(a/b*d^2)^(
1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))+1000*(d*x)^(9/2)*a^2*b^2*d^2+105*(a/b*d^2)^(1/4)*2^(1/2)*a^4*
d^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*
d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+210*(a/b*d^2)^(1/4)*2^(1/2
)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+210
*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4
))/(a/b*d^2)^(1/4))-3192*(d*x)^(5/2)*a^3*b*d^4-840*(d*x)^(1/2)*a^4*d^6)/d^3
*(b*x^2+a)/b^2/a^3/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.81, size = 597, normalized size = 1.07

$$-\frac{77\,b^3d^{\frac{7}{2}}x^{\frac{13}{2}}+803\,ab^2d^{\frac{7}{2}}x^{\frac{9}{2}}+447\,a^2bd^{\frac{7}{2}}x^{\frac{5}{2}}+105\,a^3d^{\frac{7}{2}}\sqrt{x}}{3072\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)}+\frac{\left(7\,b^4d^{\frac{7}{2}}x^5+54\,ab^3d^{\frac{7}{2}}x^3+15\,a^2b^2d^{\frac{7}{2}}x\right)x^{\frac{11}{2}}+2\left(9\,ab^3d^{\frac{7}{2}}x^5+66\,a^2b^2d^{\frac{7}{2}}x^3+25\,a^3bd^{\frac{7}{2}}x\right)x^{\frac{7}{2}}-\left(21\,a^2b^2d^{\frac{7}{2}}x^5-14\,a^3bd^{\frac{7}{2}}x^3-3\,a^4d^{\frac{7}{2}}x\right)x^{\frac{3}{2}}}{192\left(a^5b^4x^6+3\,a^6b^3x^4+3\,a^7b^2x^2+a^8b+\left(a^2b^7x^6+3\,a^3b^6x^4+3\,a^4b^5x^2+a^5b^4\right)x^6+3\left(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3\right)x^4+3\left(a^4b^5x^6+3\,a^5b^4x^4+3\,a^6b^3x^2+a^7b^2\right)x^2\right)}+\frac{35\,d^3\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{8192\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3072*(77*b^3*d^(7/2)*x^(13/2) + 803*a*b^2*d^(7/2)*x^(9/2) + 447*a^2*b*d^
(7/2)*x^(5/2) + 105*a^3*d^(7/2)*sqrt(x))/(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a
^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) + 1/192*((7*b^4*d^(7/2)*x^5 + 54*a*b^
3*d^(7/2)*x^3 + 15*a^2*b^2*d^(7/2)*x)*x^(11/2) + 2*(9*a*b^3*d^(7/2)*x^5 + 6
6*a^2*b^2*d^(7/2)*x^3 + 25*a^3*b*d^(7/2)*x)*x^(7/2) - (21*a^2*b^2*d^(7/2)*x
^5 - 14*a^3*b*d^(7/2)*x^3 - 3*a^4*d^(7/2)*x)*x^(3/2))/(a^5*b^4*x^6 + 3*a^6*
b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b + (a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*
x^2 + a^5*b^4)*x^6 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b
^3)*x^4 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)*x^2) +
35/8192*d^3*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/
```

```
4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{7/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(7/2)/((a + b*x**2)**2)**(5/2), x)
```

3.600 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{9d(dx)^{3/2}}{256a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.45, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{45d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{9d(dx)^{3/2}}{256a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (45*d*(d*x)^(3/2))/(1024*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(3/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(d*x)^(3/2)
)/(32*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (9*d*(d*x)^(3/2)
)/(256*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(5/2)*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
 b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sq
rt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (45*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(9}{}$$

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$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32ab\left(a+bx\right.}$$

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Mathematica [C] time = 0.03, size = 73, normalized size = 0.13

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)^4 {}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a^4\right)}{13a^4b\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a^4 + (a + b*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -((b*
x^2)/a)]))/(13*a^4*b*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

IntegrateAlgebraic [A] time = 104.78, size = 269, normalized size = 0.48

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{45d^{5/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{13/4}b^{7/4}}-\frac{45d^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{13/4}b^{7/4}}-\frac{(dx)^{3/2}\left(15a^3d^9-239a^2bd^9x^2-171ab^2d^9x^4-45b^3d^9x^6\right)}{1024a^3b\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(-1/1024*((d*x)^(3/2)*(15*a^3*d^9 - 239*a^2*b*d^9*x^2
- 171*a*b^2*d^9*x^4 - 45*b^3*d^9*x^6))/(a^3*b*(a*d^2 + b*d^2*x^2)^4) - (45*
d^(5/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(
Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(13/4)*b^(7/4)) - (45*d^(5/2)
*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d
*x)])/(2048*Sqrt[2]*a^(13/4)*b^(7/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4
])
```

fricas [A] time = 1.56, size = 454, normalized size = 0.82

$$-\frac{180\left(a^3b^5x^8+4a^4b^4x^6+6a^5b^3x^4+4a^6b^2x^2+a^7b\right)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\sqrt{dx}a^3b^2d^7\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}-\sqrt{-8303765625a^7b^3d^{10}\sqrt{-\frac{d^{10}}{a^{13}b^7}}+8303765625d^{15}x}\,a^3b^2\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}}{91125d^{10}}\right)-45\left(a^3b^5x^8+4a^4b^4x^6+6a^5b^3x^4+4a^6b^2x^2+a^7b\right)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\log\left(91125a^{10}b^5\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{3}{4}}+91125\sqrt{dx}d^7\right)+45\left(a^3b^5x^8+4a^4b^4x^6+6a^5b^3x^4+4a^6b^2x^2+a^7b\right)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\log\left(-91125a^{10}b^5\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{3}{4}}+91125\sqrt{dx}d^7\right)-4\left(45b^3d^2x^7+171ab^2d^2x^5+239a^2bd^2x^3-15a^3d^2x\right)\sqrt{dx}}{4096\left(a^3b^5x^8+4a^4b^4x^6+6a^5b^3x^4+4a^6b^2x^2+a^7b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4096*(180*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 +
 a^7*b)*(-d^10/(a^13*b^7))^(1/4)*arctan(-1/91125*(91125*sqrt(d*x)*a^3*b^2*d
```

```
^7*(-d^10/(a^13*b^7))^(1/4) - sqrt(-8303765625*a^7*b^3*d^10*sqrt(-d^10/(a^1
3*b^7)) + 8303765625*d^15*x)*a^3*b^2*(-d^10/(a^13*b^7))^(1/4))/d^10) - 45*(
a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^10
/(a^13*b^7))^(1/4)*log(91125*a^10*b^5*(-d^10/(a^13*b^7))^(3/4) + 91125*sqrt
(d*x)*d^7) + 45*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^
2 + a^7*b)*(-d^10/(a^13*b^7))^(1/4)*log(-91125*a^10*b^5*(-d^10/(a^13*b^7))^
(3/4) + 91125*sqrt(d*x)*d^7) - 4*(45*b^3*d^2*x^7 + 171*a*b^2*d^2*x^5 + 239*
a^2*b*d^2*x^3 - 15*a^3*d^2*x)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

giac [A] time = 0.37, size = 421, normalized size = 0.76

$$\frac{1}{8192}d^2\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^4d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^4d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{8\left(45\sqrt{dx}\,b^3d^8x^7+171\sqrt{dx}\,ab^2d^8x^5+239\sqrt{dx}\,a^2bd^8x^3-15\sqrt{dx}\,a^3d^8x\right)}{\left(bd^2x^2+ad^2\right)^4a^3b\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^2*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*d*sgn(b*d^4*x^2 + a*d^4)
) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*d*sgn(b*d^4*x^2 + a*d^4)) - 45*
sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqr
t(a*d^2/b))/(a^4*b^4*d*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)^(3/
4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^4*d*
sgn(b*d^4*x^2 + a*d^4)) + 8*(45*sqrt(d*x)*b^3*d^8*x^7 + 171*sqrt(d*x)*a*b^2
*d^8*x^5 + 239*sqrt(d*x)*a^2*b*d^8*x^3 - 15*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^
2 + a*d^2)^4*a^3*b*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1051, normalized size = 1.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/8192*(45*2^(1/2)*b^4*d^8*x^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2
)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)
))+90*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b
*d^2)^(1/4))+90*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(
1/4))/(a/b*d^2)^(1/4))+360*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4+180*2^(1/2)*a*b
^3*d^8*x^6*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+360*2^(1/2)*a*b^3
*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+360*
```

```
2^(1/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2
)^(1/4))+1368*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+270*2^(1/2)*a^2*b^2*d^
8*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(
a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+540*2^(1/2)*a^2*b^2*d^
8*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+540*2^(
1/2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)
^(1/4))+1912*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+180*2^(1/2)*a^3*b*d^8*
x^2*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+360*2^(1/2)*a^3*b*d^8*x^
2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+360*2^(1/2)
*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)-120*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+45*2^(1/2)*a^4*d^8*ln(-(-d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+90*2^(1/2)*a^4*d^8*arctan((2^(1/2)*(d*x)
^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+90*2^(1/2)*a^4*d^8*arctan((2^(1/2)
*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d^5*(b*x^2+a)/(a/b*d^2)^(1/
4)/b^2/a^3/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.71, size = 582, normalized size = 1.04

$$\frac{135\,b^3d^{\frac{5}{2}}x^{\frac{15}{2}}+657\,ab^2d^{\frac{5}{2}}x^{\frac{11}{2}}+173\,a^2bd^{\frac{5}{2}}x^{\frac{7}{2}}+35\,a^3d^{\frac{5}{2}}x^{\frac{3}{2}}}{3072\left(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b\right)}-\frac{\left(9\,b^4d^{\frac{5}{2}}x^5-38\,ab^3d^{\frac{5}{2}}x^3-15\,a^2b^2d^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+2\left(11\,ab^3d^{\frac{5}{2}}x^5-42\,a^2b^2d^{\frac{5}{2}}x^3-21\,a^3bd^{\frac{5}{2}}x\right)x^{\frac{5}{2}}+\left(45\,a^2b^2d^{\frac{5}{2}}x^5+18\,a^3bd^{\frac{5}{2}}x^3+5\,a^4d^{\frac{5}{2}}x\right)\sqrt{x}}{192\left(a^5b^4x^6+3\,a^6b^3x^4+3\,a^7b^2x^2+a^8b+\left(a^2b^7x^6+3\,a^3b^6x^4+3\,a^4b^5x^2+a^5b^4\right)x^6+3\left(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3\right)x^4+3\left(a^4b^5x^6+3\,a^5b^4x^4+3\,a^6b^3x^2+a^7b^2\right)x^2\right)}+\frac{45\,d^{\frac{5}{2}}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3072*(135*b^3*d^(5/2)*x^(15/2) + 657*a*b^2*d^(5/2)*x^(11/2) + 173*a^2*b*d
^(5/2)*x^(7/2) + 35*a^3*d^(5/2)*x^(3/2))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) - 1/192*((9*b^4*d^(5/2)*x^5 - 38*a*b^3*
d^(5/2)*x^3 - 15*a^2*b^2*d^(5/2)*x)*x^(9/2) + 2*(11*a*b^3*d^(5/2)*x^5 - 42*
a^2*b^2*d^(5/2)*x^3 - 21*a^3*b*d^(5/2)*x)*x^(5/2) + (45*a^2*b^2*d^(5/2)*x^5
 + 18*a^3*b*d^(5/2)*x^3 + 5*a^4*d^(5/2)*x)*sqrt(x))/(a^5*b^4*x^6 + 3*a^6*b^
3*x^4 + 3*a^7*b^2*x^2 + a^8*b + (a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*x^
2 + a^5*b^4)*x^6 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3
)*x^4 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)*x^2) + 45
/8192*d^(5/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(a^3*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{5/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(5/2), x)
```

3.601 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{11d\sqrt{dx}}{768a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.43, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{77d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{11d\sqrt{dx}}{768a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (77*d*Sqrt[d*x])/(3072*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x
])/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*Sqrt[d*x])/(96*
a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11*d*Sqrt[d*x])/(768*
a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*
a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a
^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(1\cdots}{\cdots}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{76\cdots}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+b\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+b\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+b\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+b\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+b\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+b\cdots\right.}$$

Mathematica [A] time = 0.30, size = 324, normalized size = 0.58

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(616a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+352a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+256a^{11/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-3072a^{15/4}\sqrt[4]{b}\sqrt{x}-231\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+231\sqrt{2}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-462\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+462\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{24576a^{15/4}b^{5/4}x^{3/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(-3072*a^(15/4)*b^(1/4)*Sqrt[x] + 256*a^(11/4)*b^(
1/4)*Sqrt[x]*(a + b*x^2) + 352*a^(7/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 616*
a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 462*Sqrt[2]*(a + b*x^2)^4*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 462*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 231*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 231*Sqrt[2]*(a + b*x^2)^4
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*a^(15/
4)*b^(5/4)*x^(3/2)*((a + b*x^2)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 103.79, size = 269, normalized size = 0.48

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{77d^{3/2}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{15/4}b^{5/4}}+\frac{77d^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{15/4}b^{5/4}}-\frac{\sqrt{dx}\left(231a^3d^9-351a^2bd^9x^2-275ab^2d^9x^4-77b^3d^9x^6\right)}{3072a^3b\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(-1/3072*(Sqrt[d*x]*(231*a^3*d^9 - 351*a^2*b*d^9*x^2 -
 275*a*b^2*d^9*x^4 - 77*b^3*d^9*x^6))/(a^3*b*(a*d^2 + b*d^2*x^2)^4) - (77*d
^(3/2)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(S
qrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(15/4)*b^(5/4)) + (77*d^(3/2)*
ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*
x)])/(2048*Sqrt[2]*a^(15/4)*b^(5/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4]
)
```

fricas [A] time = 1.56, size = 429, normalized size = 0.77

$$\frac{924\left(a^3b^5x^8+4a^4b^4x^6+6a^5b^3x^4+4a^6b^2x^2+a^7b\right)\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{11}b^4d\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{3}{4}}-\sqrt{a^8b^2\sqrt{-\frac{d^6}{a^{15}b^5}}+d^3x}\,a^{11}b^4\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{3}{4}}}{d^6}\right)+231\left(a^3b^5x^8+4a^4b^4x^6+6a^5b^3x^4+4a^6b^2x^2+a^7b\right)\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}\log\left(77a^4b\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}+77\sqrt{dx}d\right)-231\left(a^3b^5x^8+4a^4b^4x^6+6a^5b^3x^4+4a^6b^2x^2+a^7b\right)\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}\log\left(-77a^4b\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}+77\sqrt{dx}d\right)+4\left(77b^3dx^6+275ab^2dx^4+351a^2bdx^2-231a^3d\right)\sqrt{dx}}{12288\left(a^3b^5x^8+4a^4b^4x^6+6a^5b^3x^4+4a^6b^2x^2+a^7b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(924*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 +
 a^7*b)*(-d^6/(a^15*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^11*b^4*d*(-d^6/(a^15*b
^5))^(3/4) - sqrt(a^8*b^2*sqrt(-d^6/(a^15*b^5)) + d^3*x)*a^11*b^4*(-d^6/(a^
15*b^5))^(3/4))/d^6) + 231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4
*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(77*a^4*b*(-d^6/(a^15*b^5)
)^(1/4) + 77*sqrt(d*x)*d) - 231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^
4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(-77*a^4*b*(-d^6/(a^1
5*b^5))^(1/4) + 77*sqrt(d*x)*d) + 4*(77*b^3*d*x^6 + 275*a*b^2*d*x^4 + 351*a
^2*b*d*x^2 - 231*a^3*d)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3
*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

giac [A] time = 0.35, size = 406, normalized size = 0.73

$$\frac{1}{24576}d\left(\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{231\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{231\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{8\left(77\sqrt{dx}\,b^3d^8x^6+275\sqrt{dx}\,ab^2d^8x^4+351\sqrt{dx}\,a^2bd^8x^2-231\sqrt{dx}\,a^3d^8\right)}{\left(bd^2x^2+ad^2\right)^4a^3b\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d*(462*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*d^4*x^2 + a*d^4))
+ 462*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*d^4*x^2 + a*d^4)) + 231*sq
rt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^4*b^2*sgn(b*d^4*x^2 + a*d^4)) - 231*sqrt(2)*(a*b^3*d^2)^(1/4)*
log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^2*sgn(b
*d^4*x^2 + a*d^4)) + 8*(77*sqrt(d*x)*b^3*d^8*x^6 + 275*sqrt(d*x)*a*b^2*d^8*
x^4 + 351*sqrt(d*x)*a^2*b*d^8*x^2 - 231*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*
d^2)^4*a^3*b*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1136, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(231*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2)))+462*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)*
(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+462*(a/b*d^2)^(1/4)*2^(1/2)*b
^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+92
4*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
```

```
*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))+1848*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*x)^
(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1848*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*
d^6*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1386*
(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))+2772*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x
)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+2772*(a/b*d^2)^(1/4)*2^(1/2)*a^2*
b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+6
16*(d*x)^(13/2)*a*b^3+924*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((d*x+(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1848*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^
2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1848*(a/b*d
^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))+2200*(d*x)^(9/2)*a^2*b^2*d^2+231*(a/b*d^2)^(1/4)*2^(1/2)
*a^4*d^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+462*(a/b*d^2)^(1/4)*2
^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)+462*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)
^(1/4))/(a/b*d^2)^(1/4))+2808*(d*x)^(5/2)*a^3*b*d^4-1848*(d*x)^(1/2)*a^4*d^
6)/d^5*(b*x^2+a)/b/a^4/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.64, size = 586, normalized size = 1.05

$$-\frac{385\,b^{3}d^{\frac{3}{2}}x^{\frac{13}{2}}+495\,ab^{2}d^{\frac{3}{2}}x^{\frac{9}{2}}+315\,a^{2}bd^{\frac{3}{2}}x^{\frac{5}{2}}+77\,a^{3}d^{\frac{3}{2}}\sqrt{x}}{1024\,\left(a^{3}b^{5}x^{8}+4\,a^{4}b^{4}x^{6}+6\,a^{5}b^{3}x^{4}+4\,a^{6}b^{2}x^{2}+a^{7}b\right)}+\frac{\left(77\,b^{4}d^{\frac{3}{2}}x^{5}+66\,ab^{3}d^{\frac{3}{2}}x^{3}+21\,a^{2}b^{2}d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+2\left(99\,ab^{3}d^{\frac{3}{2}}x^{5}+102\,a^{2}b^{2}d^{\frac{3}{2}}x^{3}+35\,a^{3}bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\left(153\,a^{2}b^{2}d^{\frac{3}{2}}x^{5}+202\,a^{3}bd^{\frac{3}{2}}x^{3}+81\,a^{4}d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}}{192\left(a^{6}b^{3}x^{6}+3\,a^{7}b^{2}x^{4}+3\,a^{8}bx^{2}+a^{9}+\left(a^{3}b^{6}x^{6}+3\,a^{4}b^{5}x^{4}+3\,a^{5}b^{4}x^{2}+a^{6}b^{3}\right)x^{6}+3\left(a^{4}b^{5}x^{6}+3\,a^{5}b^{4}x^{4}+3\,a^{6}b^{3}x^{2}+a^{7}b^{2}\right)x^{4}+3\left(a^{5}b^{4}x^{6}+3\,a^{6}b^{3}x^{4}+3\,a^{7}b^{2}x^{2}+a^{8}b\right)x^{2}\right)}+\frac{77\,d\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{8192\,a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/1024*(385*b^3*d^(3/2)*x^(13/2) + 495*a*b^2*d^(3/2)*x^(9/2) + 315*a^2*b*d
^(3/2)*x^(5/2) + 77*a^3*d^(3/2)*sqrt(x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) + 1/192*((77*b^4*d^(3/2)*x^5 + 66*a*b^3
*d^(3/2)*x^3 + 21*a^2*b^2*d^(3/2)*x)*x^(11/2) + 2*(99*a*b^3*d^(3/2)*x^5 + 1
02*a^2*b^2*d^(3/2)*x^3 + 35*a^3*b*d^(3/2)*x)*x^(7/2) + (153*a^2*b^2*d^(3/2)
*x^5 + 202*a^3*b*d^(3/2)*x^3 + 81*a^4*d^(3/2)*x)*x^(3/2))/(a^6*b^3*x^6 + 3*
a^7*b^2*x^4 + 3*a^8*b*x^2 + a^9 + (a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*
x^2 + a^6*b^3)*x^6 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b
^2)*x^4 + 3*(a^5*b^4*x^6 + 3*a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b)*x^2) + 77
/8192*d*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sq
rt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b
```

```
^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(3/4)*b^(1/4)))/(a^3*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{3/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(5/2), x)
```

3.602 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{4096\sqrt{2}\,a^{17/4}b^{3/4}\sqrt{a^2-}}$$

Rubi [A] time = 0.43, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{39(dx)^{3/2}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\,a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\,a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (195*(d*x)^(3/2))/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2
)/(8*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13*(d*x)^(3/2))/
(96*a^2*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (39*(d*x)^(3/2))
/(256*a^3*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (195*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b^3\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a+\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a+\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a+\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a+\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a+\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a+\cdots\right.}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.10

$$\frac{2x\sqrt{dx}\left(a+bx^2\right)^5{}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^5\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(a + b*x^2)^5*Hypergeometric2F1[3/4, 5, 7/4, -((b*x^2)/a)])/
(3*a^5*((a + b*x^2)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 114.25, size = 266, normalized size = 0.48

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{195\sqrt{d}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{17/4}b^{3/4}}-\frac{195\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{17/4}b^{3/4}}+\frac{(dx)^{3/2}\left(1853a^3d^7+3107a^2bd^7x^2+2223ab^2d^7x^4+585b^3d^7x^6\right)}{3072a^4\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*(((d*x)^(3/2)*(1853*a^3*d^7 + 3107*a^2*b*d^7*x^2 + 222
3*a*b^2*d^7*x^4 + 585*b^3*d^7*x^6))/(3072*a^4*(a*d^2 + b*d^2*x^2)^4) - (195
*Sqrt[d]*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/
(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(17/4)*b^(3/4)) - (195*Sqrt[
d]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]
*d*x)])/(2048*Sqrt[2]*a^(17/4)*b^(3/4))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d
^4])
```

fricas [A] time = 1.90, size = 414, normalized size = 0.74

$$-\frac{2340\left(a^4b^4x^8+4a^5b^3x^6+6a^6b^2x^4+4a^7bx^2+a^8\right)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{7414875\sqrt{dx}a^4bd\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}-\sqrt{-54980371265625a^9bd^2\sqrt{-\frac{d^2}{a^{17}b^3}}+54980371265625d^3x}a^4b\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}}{7414875d^2}\right)-585\left(a^4b^4x^8+4a^5b^3x^6+6a^6b^2x^4+4a^7bx^2+a^8\right)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\log\left(7414875a^{13}b^2\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{3}{4}}+7414875\sqrt{dx}d\right)+585\left(a^4b^4x^8+4a^5b^3x^6+6a^6b^2x^4+4a^7bx^2+a^8\right)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\log\left(-7414875a^{13}b^2\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{3}{4}}+7414875\sqrt{dx}d\right)-4\left(585b^3x^7+2223ab^2x^5+3107a^2bx^3+1853a^3x\right)\sqrt{dx}}{12288\left(a^4b^4x^8+4a^5b^3x^6+6a^6b^2x^4+4a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(2340*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 +
 a^8)*(-d^2/(a^17*b^3))^(1/4)*arctan(-1/7414875*(7414875*sqrt(d*x)*a^4*b*d*
```

```
(-d^2/(a^17*b^3))^(1/4) - sqrt(-54980371265625*a^9*b*d^2*sqrt(-d^2/(a^17*b^
3)) + 54980371265625*d^3*x)*a^4*b*(-d^2/(a^17*b^3))^(1/4))/d^2) - 585*(a^4*
b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*(-d^2/(a^17*b^
3))^(1/4)*log(7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 7414875*sqrt(d*x)*
d) + 585*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*
(-d^2/(a^17*b^3))^(1/4)*log(-7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 741
4875*sqrt(d*x)*d) - 4*(585*b^3*x^7 + 2223*a*b^2*x^5 + 3107*a^2*b*x^3 + 1853
*a^3*x)*sqrt(d*x))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x
^2 + a^8)
```

giac [A] time = 0.38, size = 406, normalized size = 0.73

$$\frac{\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{585\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{585\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{8\left(585\sqrt{dx}\,b^3d^9x^7+2223\sqrt{dx}\,ab^2d^9x^5+3107\sqrt{dx}\,a^2bd^9x^3+1853\sqrt{dx}\,a^3d^9x\right)}{\left(bd^2x^2+ad^2\right)^4a^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}}{24576\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/
b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*sgn(b*d^4*x^2 + a*d^4)) +
 1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*sgn(b*d^4*x^2 + a*d^4)) - 585*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^5*b^3*sgn(b*d^4*x^2 + a*d^4)) + 585*sqrt(2)*(a*b^3*d^2)^(3/4)*
log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^3*sgn(b
*d^4*x^2 + a*d^4)) + 8*(585*sqrt(d*x)*b^3*d^9*x^7 + 2223*sqrt(d*x)*a*b^2*d^
9*x^5 + 3107*sqrt(d*x)*a^2*b*d^9*x^3 + 1853*sqrt(d*x)*a^3*d^9*x)/((b*d^2*x^
2 + a*d^2)^4*a^4*sgn(b*d^4*x^2 + a*d^4)))/d
```

maple [B] time = 0.02, size = 1051, normalized size = 1.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(585*2^(1/2)*b^4*d^8*x^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))+1170*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/
(a/b*d^2)^(1/4))+1170*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*
d^2)^(1/4))/(a/b*d^2)^(1/4))+4680*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4+2340*2^(
1/2)*a*b^3*d^8*x^6*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^
(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4680*2^(1
/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1
```

```
/4))+4680*2^(1/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))+17784*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+3510*2^(1/2
)*a^2*b^2*d^8*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(
1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+7020*2^(1/
2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(
1/4))+7020*2^(1/2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1
/4))/(a/b*d^2)^(1/4))+24856*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+2340*2^
(1/2)*a^3*b*d^8*x^2*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)
^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4680*2^(
1/2)*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(
1/4))+4680*2^(1/2)*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4
))/(a/b*d^2)^(1/4))+14824*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+585*2^(1/2)
*a^4*d^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*
x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1170*2^(1/2)*a^4*d^
8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1170*2^(1/2
)*a^4*d^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d^
7*(b*x^2+a)/(a/b*d^2)^(1/4)/b/a^4/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.79, size = 569, normalized size = 1.02

$$\frac{195\,b^{3}\sqrt{d}\,x^{\frac{15}{2}}+117\,ab^{2}\sqrt{d}\,x^{\frac{11}{2}}+65\,a^{2}b\sqrt{d}\,x^{\frac{7}{2}}+15\,a^{3}\sqrt{d}\,x^{\frac{3}{2}}}{1024\left(a^{4}b^{4}x^{8}+4\,a^{5}b^{3}x^{6}+6\,a^{6}b^{2}x^{4}+4\,a^{7}bx^{2}+a^{8}\right)}+\frac{\left(117\,b^{4}\sqrt{d}\,x^{5}+130\,ab^{3}\sqrt{d}\,x^{3}+45\,a^{2}b^{2}\sqrt{d}\,x\right)x^{\frac{9}{2}}+2\left(143\,ab^{3}\sqrt{d}\,x^{5}+174\,a^{2}b^{2}\sqrt{d}\,x^{3}+63\,a^{3}b\sqrt{d}\,x\right)x^{\frac{5}{2}}+\left(201\,a^{2}b^{2}\sqrt{d}\,x^{5}+282\,a^{3}b\sqrt{d}\,x^{3}+113\,a^{4}\sqrt{d}\,x\right)\sqrt{x}}{192\left(a^{6}b^{3}x^{6}+3\,a^{7}b^{2}x^{4}+3\,a^{8}bx^{2}+a^{9}+\left(a^{3}b^{6}x^{6}+3\,a^{4}b^{5}x^{4}+3\,a^{5}b^{4}x^{2}+a^{6}b^{3}\right)x^{6}+3\left(a^{4}b^{5}x^{6}+3\,a^{5}b^{4}x^{4}+3\,a^{6}b^{3}x^{2}+a^{7}b^{2}\right)x^{4}+3\left(a^{5}b^{4}x^{6}+3\,a^{6}b^{3}x^{4}+3\,a^{7}b^{2}x^{2}+a^{8}b\right)x^{2}\right)}+\frac{195\sqrt{d}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/1024*(195*b^3*sqrt(d)*x^(15/2) + 117*a*b^2*sqrt(d)*x^(11/2) + 65*a^2*b*sq
rt(d)*x^(7/2) + 15*a^3*sqrt(d)*x^(3/2))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^
6*b^2*x^4 + 4*a^7*b*x^2 + a^8) + 1/192*((117*b^4*sqrt(d)*x^5 + 130*a*b^3*sq
rt(d)*x^3 + 45*a^2*b^2*sqrt(d)*x)*x^(9/2) + 2*(143*a*b^3*sqrt(d)*x^5 + 174*
a^2*b^2*sqrt(d)*x^3 + 63*a^3*b*sqrt(d)*x)*x^(5/2) + (201*a^2*b^2*sqrt(d)*x^
5 + 282*a^3*b*sqrt(d)*x^3 + 113*a^4*sqrt(d)*x)*sqrt(x))/(a^6*b^3*x^6 + 3*a^
7*b^2*x^4 + 3*a^8*b*x^2 + a^9 + (a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^
2 + a^6*b^3)*x^6 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2
)*x^4 + 3*(a^5*b^4*x^6 + 3*a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b)*x^2) + 195/
8192*sqrt(d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/a^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.603 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}\right.}{4096\sqrt{2}\,a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2\cdots}}$$

Rubi [A] time = 0.43, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{55\sqrt{dx}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\,a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\,a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (385*Sqrt[d*x])/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(8
*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d*x])/(32*a^2
*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (55*Sqrt[d*x])/(256*a^3
*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)*
b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*(a + b*x^2)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)
*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4
096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11
55*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a^2}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a^2}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a^2}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a^2}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a^2}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a^2}$$

Mathematica [A] time = 0.13, size = 319, normalized size = 0.57

$$\frac{\sqrt{x}\left(a+bx^2\right)\left(3080a^{3/4}\sqrt{x}\left(a+bx^2\right)^3+1760a^{7/4}\sqrt{x}\left(a+bx^2\right)^2+1280a^{11/4}\sqrt{x}\left(a+bx^2\right)+1024a^{15/4}\sqrt{x}-\frac{1155\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{1155\sqrt{2}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}-\frac{2310\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{2310\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}\right)}{8192a^{19/4}\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (Sqrt[x]*(a + b*x^2)*(1024*a^(15/4)*Sqrt[x] + 1280*a^(11/4)*Sqrt[x]*(a + b*
x^2) + 1760*a^(7/4)*Sqrt[x]*(a + b*x^2)^2 + 3080*a^(3/4)*Sqrt[x]*(a + b*x^2
)^3 - (2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/b^(1/4) + (2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/b^(1/4) - (1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (1155*Sqrt[2]*(a + b*x^2)^
4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(81
92*a^(19/4)*Sqrt[d*x]*((a + b*x^2)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 129.58, size = 266, normalized size = 0.48

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{1155\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}}+\frac{1155\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}}+\frac{\sqrt{dx}\left(893a^3d^7+1755a^2bd^7x^2+1375ab^2d^7x^4+385b^3d^7x^6\right)}{1024a^4\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((Sqrt[d*x]*(893*a^3*d^7 + 1755*a^2*b*d^7*x^2 + 1375*a
*b^2*d^7*x^4 + 385*b^3*d^7*x^6))/(1024*a^4*(a*d^2 + b*d^2*x^2)^4) - (1155*A
rcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a
^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]) + (1155*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(
2048*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d])))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^
4])
```

fricas [A] time = 0.84, size = 416, normalized size = 0.75

$$\frac{4620\left(a^4b^4dx^8+4a^5b^3dx^6+6a^6b^2dx^4+4a^7bdx^2+a^8d\right)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{10}d^2\sqrt{-\frac{1}{a^{19}bd^2}}+dx}\,a^{14}bd\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^{14}bd\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{3}{4}}\right)+1155\left(a^4b^4dx^8+4a^5b^3dx^6+6a^6b^2dx^4+4a^7bdx^2+a^8d\right)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}\log\left(a^5d\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-1155\left(a^4b^4dx^8+4a^5b^3dx^6+6a^6b^2dx^4+4a^7bdx^2+a^8d\right)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}\log\left(-a^5d\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\left(385b^3x^6+1375ab^2x^4+1755a^2bx^2+893a^3\right)\sqrt{dx}}{4096\left(a^4b^4dx^8+4a^5b^3dx^6+6a^6b^2dx^4+4a^7bdx^2+a^8d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/4096*(4620*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4 + 4*a^7*b*d
*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*arctan(sqrt(a^10*d^2*sqrt(-1/(a^19*b*
d^2)) + d*x)*a^14*b*d*(-1/(a^19*b*d^2))^(3/4) - sqrt(d*x)*a^14*b*d*(-1/(a^1
9*b*d^2))^(3/4)) + 1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4
+ 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(a^5*d*(-1/(a^19*b*d^2)
)^(1/4) + sqrt(d*x)) - 1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*
x^4 + 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(-a^5*d*(-1/(a^19*b
*d^2))^(1/4) + sqrt(d*x)) + 4*(385*b^3*x^6 + 1375*a*b^2*x^4 + 1755*a^2*b*x^
2 + 893*a^3)*sqrt(d*x))/(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4
+ 4*a^7*b*d*x^2 + a^8*d)
```

giac [A] time = 0.33, size = 412, normalized size = 0.74

$$\frac{1155\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\operatorname{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1155\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\operatorname{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1155\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5bd\operatorname{sgn}\left(bd^4x^2+ad^4\right)}-\frac{1155\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5bd\operatorname{sgn}\left(bd^4x^2+ad^4\right)}+\frac{385\,\sqrt{dx}\,b^3d^7x^6+1375\,\sqrt{dx}\,ab^2d^7x^4+1755\,\sqrt{dx}\,a^2bd^7x^2+893\,\sqrt{dx}\,a^3d^7}{1024\left(bd^2x^2+ad^2\right)^4a^4\operatorname{sgn}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1155/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) + 115
5/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) + 1155/
8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/(a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) - 1155/8192*sqrt(2)*(a*b^
3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) + 1/1024*(385*sqrt(d*x)*b^3*d^7*x^6 + 1375*
sqrt(d*x)*a*b^2*d^7*x^4 + 1755*sqrt(d*x)*a^2*b*d^7*x^2 + 893*sqrt(d*x)*a^3*
d^7)/((b*d^2*x^2 + a*d^2)^4*a^4*sgn(b*d^4*x^2 + a*d^4))
```

maple [B] time = 0.02, size = 1133, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] 1/8192*(1155*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2)))+2310*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)
*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+2310*(a/b*d^2)^(1/4)*2^(1/2)
*b^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
4620*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1
```

```
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+9240*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*
x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+9240*(a/b*d^2)^(1/4)*2^(1/2)*a*b
^3*d^6*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+69
30*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+13860*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*
(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+13860*(a/b*d^2)^(1/4)*2^(1/2)
*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/
4))+3080*(d*x)^(13/2)*a*b^3+4620*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+9240*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b
*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+9240
*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2
)^(1/4))/(a/b*d^2)^(1/4))+11000*(d*x)^(9/2)*a^2*b^2*d^2+1155*(a/b*d^2)^(1/4
)*2^(1/2)*a^4*d^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+2310*(a/b*d^
2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))+2310*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2
)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+14040*(d*x)^(5/2)*a^3*b*d^4+7144*(d*x)^
(1/2)*a^4*d^6)/d^7*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5267\, b^3 x^{\frac{13}{2}} + 11645\, ab^2 x^{\frac{9}{2}} + 9441\, a^2 b x^{\frac{5}{2}} + 2679\, a^3 \sqrt{x}}{3072\left(a^4 b^4 \sqrt{d}\, x^8 + 4\, a^5 b^3 \sqrt{d}\, x^6 + 6\, a^6 b^2 \sqrt{d}\, x^4 + 4\, a^7 b \sqrt{d}\, x^2 + a^8 \sqrt{d}\right)} - \frac{\left(257\, b^5 \sqrt{d}\, x^5 + 378\, ab^4 \sqrt{d}\, x^3 + 153\, a^2 b^3 \sqrt{d}\, x\right) x^{\frac{11}{2}} + 2\left(303\, ab^4 \sqrt{d}\, x^5 + 462\, a^2 b^3 \sqrt{d}\, x^3 + 191\, a^3 b^2 \sqrt{d}\, x\right) x^{\frac{7}{2}} + \left(381\, a^2 b^3 \sqrt{d}\, x^5 + 610\, a^3 b^2 \sqrt{d}\, x^3 + 261\, a^4 b \sqrt{d}\, x\right) x^{\frac{3}{2}}}{192\left(a^7 b^3 d x^6 + 3\, a^8 b^2 d x^4 + 3\, a^9 b d x^2 + a^{10} d + \left(a^4 b^6 d x^6 + 3\, a^5 b^5 d x^4 + 3\, a^6 b^4 d x^2 + a^7 b^3 d\right) x^6 + 3\left(a^5 b^5 d x^6 + 3\, a^6 b^4 d x^4 + 3\, a^7 b^3 d x^2 + a^8 b^2 d\right) x^4 + 3\left(a^6 b^4 d x^6 + 3\, a^7 b^3 d x^4 + 3\, a^8 b^2 d x^2 + a^9 b d\right) x^2\right)} - \frac{893\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}} + 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}} - 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}\, x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{1}{4}}} - \frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}\, x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{1}{4}}}\right)}{8192\, a^4 d} + \int \frac{1}{\left(a^4 b \sqrt{d}\, x^2 + a^5 \sqrt{d}\right)\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3072*(5267*b^3*x^(13/2) + 11645*a*b^2*x^(9/2) + 9441*a^2*b*x^(5/2) + 2679
*a^3*sqrt(x))/(a^4*b^4*sqrt(d)*x^8 + 4*a^5*b^3*sqrt(d)*x^6 + 6*a^6*b^2*sqrt
(d)*x^4 + 4*a^7*b*sqrt(d)*x^2 + a^8*sqrt(d)) - 1/192*((257*b^5*sqrt(d)*x^5
+ 378*a*b^4*sqrt(d)*x^3 + 153*a^2*b^3*sqrt(d)*x)*x^(11/2) + 2*(303*a*b^4*sq
rt(d)*x^5 + 462*a^2*b^3*sqrt(d)*x^3 + 191*a^3*b^2*sqrt(d)*x)*x^(7/2) + (381
*a^2*b^3*sqrt(d)*x^5 + 610*a^3*b^2*sqrt(d)*x^3 + 261*a^4*b*sqrt(d)*x)*x^(3/
2))/(a^7*b^3*d*x^6 + 3*a^8*b^2*d*x^4 + 3*a^9*b*d*x^2 + a^10*d + (a^4*b^6*d*
x^6 + 3*a^5*b^5*d*x^4 + 3*a^6*b^4*d*x^2 + a^7*b^3*d)*x^6 + 3*(a^5*b^5*d*x^6
 + 3*a^6*b^4*d*x^4 + 3*a^7*b^3*d*x^2 + a^8*b^2*d)*x^4 + 3*(a^6*b^4*d*x^6 +
3*a^7*b^3*d*x^4 + 3*a^8*b^2*d*x^2 + a^9*b*d)*x^2) - 893/8192*(2*sqrt(2)*sqr
t(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*sqrt(d)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sqrt(d)*log(sqrt(2)*a^(1
```

```
/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt
(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^
(1/4)))/(a^4*d) + integrate(1/((a^4*b*sqrt(d)*x^2 + a^5*sqrt(d))*sqrt(x)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d\,x}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(5/2)), x)
```

3.604 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{17}{96a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}-\frac{3315\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\right.}{4096\sqrt{2}\,a^{21/4}}$$

Rubi [A] time = 0.49, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3315\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\,\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{2048\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{b}\,\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{2048\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\left(a+bx^2\right)}{1024a^5d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{221}{768a^3d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}+\frac{17}{96a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 663/(1024*a^4*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*Sqrt[
d*x]*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 17/(96*a^2*d*Sqrt[d*x
]*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 221/(768*a^3*d*Sqrt[d*x]
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*(a + b*x^2))/(1024*a^
5*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*b^(1/4)*(a + b*x^2)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^
(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a
^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
5*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(17b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b\right.}}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{17}{96a^2d\sqrt{dx}\left(a+bx^2\right)^2\sqrt{a^2+2a}}$$

$$= \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{17}{96a^2d\sqrt{dx}\left(a+bx^2\right)^2\sqrt{a^2+2a}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.09

$$-\frac{2x\left(a+bx^2\right)^5\,{}_2F_1\left(-\frac{1}{4},5;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^5(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^5*Hypergeometric2F1[-1/4, 5, 3/4, -((b*x^2)/a)])/(a^5*(d*
x)^(3/2)*((a + b*x^2)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 140.89, size = 283, normalized size = 0.47

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{3315\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{21/4}d^{3/2}}+\frac{3315\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{21/4}d^{3/2}}+\frac{-6144a^4d^8-31501a^3bd^8x^2-52819a^2b^2d^8x^4-37791ab^3d^8x^6-9945b^4d^8x^8}{3072a^5d\sqrt{dx}\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-6144*a^4*d^8 - 31501*a^3*b*d^8*x^2 - 52819*a^2*b^2*
d^8*x^4 - 37791*a*b^3*d^8*x^6 - 9945*b^4*d^8*x^8)/(3072*a^5*d*Sqrt[d*x]*(a*
d^2 + b*d^2*x^2)^4) + (3315*b^(1/4)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1
/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(
21/4)*d^(3/2)) + (3315*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqr
t[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(2048*Sqrt[2]*a^(21/4)*d^(3/2))))/(d^2*
Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 0.97, size = 477, normalized size = 0.79

$$\frac{39780\left(a^5b^4d^2x^9+4a^6b^3d^2x^7+6a^7b^2d^2x^5+4a^8bd^2x^3+a^9d^2x\right)\left(-\frac{b}{a^{21}d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{36429280875\sqrt{dx}a^5bd\left(-\frac{b}{a^{21}d^6}\right)^{\frac{1}{4}}-\sqrt{-1327092505069640765625a^{11}bd^4\sqrt{-\frac{b}{a^{21}d^6}}+1327092505069640765625b^2dx}\,a^5d\left(-\frac{b}{a^{21}d^6}\right)^{\frac{1}{4}}}{36429280875b}\right)-9945\left(a^5b^4d^2x^9+4a^6b^3d^2x^7+6a^7b^2d^2x^5+4a^8bd^2x^3+a^9d^2x\right)\left(-\frac{b}{a^{21}d^6}\right)^{\frac{1}{4}}\log\left(36429280875a^{16}d^5\left(-\frac{b}{a^{21}d^6}\right)^{\frac{3}{4}}+36429280875\sqrt{dx}b\right)+9945\left(a^5b^4d^2x^9+4a^6b^3d^2x^7+6a^7b^2d^2x^5+4a^8bd^2x^3+a^9d^2x\right)\left(-\frac{b}{a^{21}d^6}\right)^{\frac{1}{4}}\log\left(-36429280875a^{16}d^5\left(-\frac{b}{a^{21}d^6}\right)^{\frac{3}{4}}+36429280875\sqrt{dx}b\right)-4\left(9945b^4x^8+37791ab^3x^6+52819a^2b^2x^4+31501a^3bx^2+6144a^4\right)\sqrt{dx}}{12288\left(a^5b^4d^2x^9+4a^6b^3d^2x^7+6a^7b^2d^2x^5+4a^8bd^2x^3+a^9d^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/12288*(39780*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4
*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*arctan(-1/36429280875*(36
```

```
429280875*sqrt(d*x)*a^5*b*d*(-b/(a^21*d^6))^(1/4) - sqrt(-13270925050696407
65625*a^11*b*d^4*sqrt(-b/(a^21*d^6)) + 1327092505069640765625*b^2*d*x)*a^5*
d*(-b/(a^21*d^6))^(1/4))/b) - 9945*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6
*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(3
6429280875*a^16*d^5*(-b/(a^21*d^6))^(3/4) + 36429280875*sqrt(d*x)*b) + 9945
*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3
 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(-36429280875*a^16*d^5*(-b/(a^21*d^6
))^(3/4) + 36429280875*sqrt(d*x)*b) - 4*(9945*b^4*x^8 + 37791*a*b^3*x^6 + 5
2819*a^2*b^2*x^4 + 31501*a^3*b*x^2 + 6144*a^4)*sqrt(d*x))/(a^5*b^4*d^2*x^9
+ 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)
```

giac [A] time = 0.37, size = 448, normalized size = 0.74

$$-\frac{\frac{49152}{\sqrt{dx}\,a^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{19890\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{19890\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{9945\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^6b^2d^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{9945\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^6b^2d^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{8\left(3801\,\sqrt{dx}\,b^4d^7x^7+13215\,\sqrt{dx}\,ab^3d^7x^5+15955\,\sqrt{dx}\,a^2b^2d^7x^3+6925\,\sqrt{dx}\,a^3bd^7x\right)}{\left(bd^2x^2+ad^2\right)^4a^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}}{24576\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*(49152/(sqrt(d*x)*a^5*sgn(b*d^4*x^2 + a*d^4)) + 19890*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a
*d^2/b)^(1/4))/(a^6*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) + 19890*sqrt(2)*(a*b^3*
d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/(a^6*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) - 9945*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b
^2*d^2*sgn(b*d^4*x^2 + a*d^4)) + 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^2*d^2*sgn(b*d^4*x^
2 + a*d^4)) + 8*(3801*sqrt(d*x)*b^4*d^7*x^7 + 13215*sqrt(d*x)*a*b^3*d^7*x^5
 + 15955*sqrt(d*x)*a^2*b^2*d^7*x^3 + 6925*sqrt(d*x)*a^3*b*d^7*x)/((b*d^2*x^
2 + a*d^2)^4*a^5*sgn(b*d^4*x^2 + a*d^4)))/d
```

maple [B] time = 0.03, size = 1081, normalized size = 1.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576/d*(9945*(d*x)^(1/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*
2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)
^(1/2)))*x^8*b^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b
*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4+39780*(d*x)^(1/2
)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a*b^3+79560*(d
```

```
*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1
/4))*x^6*a*b^3+79560*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d
^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^3+79560*(a/b*d^2)^(1/4)*x^8*b^4+59670*(
d*x)^(1/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^
(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a^2*b
^2+119340*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/
(a/b*d^2)^(1/4))*x^4*a^2*b^2+119340*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^2+302328*(a/b*d^2)^(1/
4)*x^6*a*b^3+39780*(d*x)^(1/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2
)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^
2)^(1/2)))*x^2*a^3*b+79560*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b+79560*(d*x)^(1/2)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b+422552*(
a/b*d^2)^(1/4)*x^4*a^2*b^2+9945*(d*x)^(1/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/
4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a/b*d^2)^(1/2)))*a^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4+19890*(d*x)^(1/2)*2^(1/2)*arct
an((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4+252008*(a/b*d
^2)^(1/4)*x^2*a^3*b+49152*(a/b*d^2)^(1/4)*a^4)*(b*x^2+a)/(d*x)^(1/2)/(a/b*d
^2)^(1/4)/a^5/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3801\, b^4 x^{\frac{15}{2}} + 8079\, a b^3 x^{\frac{11}{2}} + 6515\, a^2 b^2 x^{\frac{7}{2}} + 1853\, a^3 b x^{\frac{3}{2}}}{3072 \left(a^5 b^4 d^{\frac{3}{2}} x^8 + 4\, a^6 b^3 d^{\frac{3}{2}} x^6 + 6\, a^7 b^2 d^{\frac{3}{2}} x^4 + 4\, a^8 b d^{\frac{3}{2}} x^2 + a^9 d^{\frac{3}{2}}\right)} - \frac{\left(321\, b^5 \sqrt{d}\, x^5 + 490\, a b^4 \sqrt{d}\, x^3 + 201\, a^2 b^3 \sqrt{d}\, x\right) x^{\frac{9}{2}} + 2 \left(371\, a b^4 \sqrt{d}\, x^5 + 582\, a^2 b^3 \sqrt{d}\, x^3 + 243\, a^3 b^2 \sqrt{d}\, x\right) x^{\frac{5}{2}} + \left(453\, a^2 b^3 \sqrt{d}\, x^5 + 738\, a^3 b^2 \sqrt{d}\, x^3 + 317\, a^4 b \sqrt{d}\, x\right) \sqrt{x}}{192 \left(a^7 b^3 d^2 x^6 + 3\, a^8 b^2 d^2 x^4 + 3\, a^9 b d^2 x^2 + a^{10} d^2 + \left(a^4 b^6 d^2 x^6 + 3\, a^5 b^5 d^2 x^4 + 3\, a^6 b^4 d^2 x^2 + a^7 b^3 d^2\right) x^6 + 3 \left(a^5 b^5 d^2 x^6 + 3\, a^6 b^4 d^2 x^4 + 3\, a^7 b^3 d^2 x^2 + a^8 b^2 d^2\right) x^4 + 3 \left(a^6 b^4 d^2 x^6 + 3\, a^7 b^3 d^2 x^4 + 3\, a^8 b^2 d^2 x^2 + a^9 b d^2\right) x^2\right)} - \frac{1267\, b \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} + 2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}}\, \sqrt{b}} + \frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} - 2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}}\, \sqrt{b}} - \frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{8192\, a^5 d^{\frac{3}{2}}} + \int \frac{1}{\left(a^4 b d^{\frac{3}{2}} x^2 + a^5 d^{\frac{3}{2}}\right) x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/3072*(3801*b^4*x^(15/2) + 8079*a*b^3*x^(11/2) + 6515*a^2*b^2*x^(7/2) + 1
853*a^3*b*x^(3/2))/(a^5*b^4*d^(3/2)*x^8 + 4*a^6*b^3*d^(3/2)*x^6 + 6*a^7*b^2
*d^(3/2)*x^4 + 4*a^8*b*d^(3/2)*x^2 + a^9*d^(3/2)) - 1/192*((321*b^5*sqrt(d)
*x^5 + 490*a*b^4*sqrt(d)*x^3 + 201*a^2*b^3*sqrt(d)*x)*x^(9/2) + 2*(371*a*b^
4*sqrt(d)*x^5 + 582*a^2*b^3*sqrt(d)*x^3 + 243*a^3*b^2*sqrt(d)*x)*x^(5/2) +
(453*a^2*b^3*sqrt(d)*x^5 + 738*a^3*b^2*sqrt(d)*x^3 + 317*a^4*b*sqrt(d)*x)*s
qrt(x))/(a^7*b^3*d^2*x^6 + 3*a^8*b^2*d^2*x^4 + 3*a^9*b*d^2*x^2 + a^10*d^2 +
 (a^4*b^6*d^2*x^6 + 3*a^5*b^5*d^2*x^4 + 3*a^6*b^4*d^2*x^2 + a^7*b^3*d^2)*x^
6 + 3*(a^5*b^5*d^2*x^6 + 3*a^6*b^4*d^2*x^4 + 3*a^7*b^3*d^2*x^2 + a^8*b^2*d^
2)*x^4 + 3*(a^6*b^4*d^2*x^6 + 3*a^7*b^3*d^2*x^4 + 3*a^8*b^2*d^2*x^2 + a^9*b
*d^2)*x^2) - 1267/8192*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
```

```
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(
1/4)*b^(3/4)))/(a^5*d^(3/2)) + integrate(1/((a^4*b*d^(3/2)*x^2 + a^5*d^(3/2
))*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{3/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(5/2)), x)
```

3.605 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{19}{96a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}+\frac{7315b^{3/4}\left(a+bx^2\right)\log}{4096\sqrt{2}\,a}$$

Rubi [A] time = 0.48, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315\left(a+bx^2\right)}{3072a^5d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{95}{256a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}+\frac{19}{96a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1045/(1024*a^4*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(3/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 19/(96*a^2*d*(d*
x)^(3/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 95/(256*a^3*d*(d*
x)^(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*(a + b*x^2))/
(3072*a^5*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*(a
 + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*
Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*b^(3/4)*(
a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048
*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*
(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (7315*b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(19b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab\right.}}{16a\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.09

$$-\frac{2x\left(a+bx^2\right)^5\ {}_2F_1\left(-\frac{3}{4},5;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^5(dx)^{5/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^5*Hypergeometric2F1[-3/4, 5, 1/4, -((b*x^2)/a)])/(3*a^5*(
d*x)^(5/2)*((a + b*x^2)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 145.09, size = 283, normalized size = 0.47

$$\frac{\left(ad^2+bd^2x^2\right)\left(\frac{7315b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}a^{23/4}d^{5/2}}-\frac{7315b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}d+\sqrt{b}dx}\right)}{2048\sqrt{2}a^{23/4}d^{5/2}}+\frac{-2048a^4d^8-16967a^3bd^8x^2-33345a^2b^2d^8x^4-26125ab^3d^8x^6-7315b^4d^8x^8}{3072a^5d(dx)^{3/2}\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-2048*a^4*d^8 - 16967*a^3*b*d^8*x^2 - 33345*a^2*b^2*
d^8*x^4 - 26125*a*b^3*d^8*x^6 - 7315*b^4*d^8*x^8)/(3072*a^5*d*(d*x)^(3/2)*(
a*d^2 + b*d^2*x^2)^4) + (7315*b^(3/4)*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^
(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a
^(23/4)*d^(5/2)) - (7315*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*S
qrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)])/(2048*Sqrt[2]*a^(23/4)*d^(5/2))))/(d^
2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 1.10, size = 501, normalized size = 0.83

$$-\frac{87780\left(a^5b^4d^3x^{10}+4a^6b^3d^3x^8+6a^7b^2d^3x^6+4a^8bd^3x^4+a^9d^3x^2\right)\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{17}bd^7\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{12}d^6\sqrt{-\frac{b^3}{a^{23}d^{10}}}+b^2dx}\,a^{17}d^7\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+21945\left(a^5b^4d^3x^{10}+4a^6b^3d^3x^8+6a^7b^2d^3x^6+4a^8bd^3x^4+a^9d^3x^2\right)\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\log\left(7315a^6d^3\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}+7315\sqrt{dx}b\right)-21945\left(a^5b^4d^3x^{10}+4a^6b^3d^3x^8+6a^7b^2d^3x^6+4a^8bd^3x^4+a^9d^3x^2\right)\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\log\left(-7315a^6d^3\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}+7315\sqrt{dx}b\right)+4\left(7315b^4x^8+26125ab^3x^6+33345a^2b^2x^4+16967a^3bx^2+2048a^4\right)\sqrt{dx}}{12288\left(a^5b^4d^3x^{10}+4a^6b^3d^3x^8+6a^7b^2d^3x^6+4a^8bd^3x^4+a^9d^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/12288*(87780*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 +
 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*arctan(-(sqrt(d*x)
```

```
*a^17*b*d^7*(-b^3/(a^23*d^10))^(3/4) - sqrt(a^12*d^6*sqrt(-b^3/(a^23*d^10))
 + b^2*d*x)*a^17*d^7*(-b^3/(a^23*d^10))^(3/4))/b^3) + 21945*(a^5*b^4*d^3*x^
10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)
*(-b^3/(a^23*d^10))^(1/4)*log(7315*a^6*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*
sqrt(d*x)*b) - 21945*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*
x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*log(-7315*a^6
*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b) + 4*(7315*b^4*x^8 + 26125
*a*b^3*x^6 + 33345*a^2*b^2*x^4 + 16967*a^3*b*x^2 + 2048*a^4)*sqrt(d*x))/(a^
5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 +
a^9*d^3*x^2)
```

giac [A] time = 0.74, size = 439, normalized size = 0.73

$$-\frac{7315\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^6d^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{7315\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^6d^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{7315\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^6d^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{7315\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^6d^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{2}{3\,\sqrt{dx}\,a^5d^2x\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{5267\,\sqrt{dx}\,b^4d^6x^6+17933\,\sqrt{dx}\,ab^3d^6x^4+21057\,\sqrt{dx}\,a^2b^2d^6x^2+8775\,\sqrt{dx}\,a^3bd^6}{3072\left(bd^2x^2+ad^2\right)^4a^5d\mathrm{sgn}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -7315/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) - 73
15/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) - 7315
/8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) + 7315/8192*sqrt(2)*(a*b
^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) - 2/3/(sqrt(d*x)*a^5*d^2*x*sgn(b*d^4*x^2 +
 a*d^4)) - 1/3072*(5267*sqrt(d*x)*b^4*d^6*x^6 + 17933*sqrt(d*x)*a*b^3*d^6*x
^4 + 21057*sqrt(d*x)*a^2*b^2*d^6*x^2 + 8775*sqrt(d*x)*a^3*b*d^6)/((b*d^2*x^
2 + a*d^2)^4*a^5*d*sgn(b*d^4*x^2 + a*d^4))
```

maple [B] time = 0.03, size = 1183, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576/d^3*(21945*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a/b*d^2)^(1/2)))*x^8*b^5+43890*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^5+4389
0*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)
^(1/4))/(a/b*d^2)^(1/4))*x^8*b^5+87780*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*
ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)
```

^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a*b^4+175560*(d*x)^(3/2)*(
a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))*x^6*a*b^4+175560*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^4+131670*(d*x)^(3
/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^
4*a^2*b^3+263340*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^3+263340*(d*x)^(3/2)*(a/b
*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^
(1/4))*x^4*a^2*b^3+58520*x^8*a*b^4*d^2+87780*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^
(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^3*b^2+175560*(d*x)
^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))
/(a/b*d^2)^(1/4))*x^2*a^3*b^2+175560*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b^2+209
000*x^6*a^2*b^3*d^2+21945*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*
d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^4*b+43890*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*b+43
890*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))*a^4*b+266760*x^4*a^3*b^2*d^2+135736*x^2*a^4*b*d^
2+16384*a^5*d^2)*(b*x^2+a)/(d*x)^(3/2)/a^6/((b*x^2+a)^2)^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,b\int \frac{1}{\left(a^{5}bd^{\frac{5}{2}}x^{2}+a^{6}d^{\frac{5}{2}}\right)\sqrt{x}}\,dx-\frac{13795\,b^{4}x^{\frac{13}{2}}+34285\,ab^{3}x^{\frac{9}{2}}+29649\,a^{2}b^{2}x^{\frac{5}{2}}+8775\,a^{3}b\sqrt{x}}{3072\left(a^{5}b^{4}d^{\frac{5}{2}}x^{8}+4\,a^{6}b^{3}d^{\frac{5}{2}}x^{6}+6\,a^{7}b^{2}d^{\frac{5}{2}}x^{4}+4\,a^{8}bd^{\frac{5}{2}}x^{2}+a^{9}d^{\frac{5}{2}}\right)}+\frac{\left(533\,b^{6}x^{5}+882\,ab^{5}x^{3}+381\,a^{2}b^{4}x\right)x^{\frac{11}{2}}+2\left(603\,ab^{5}x^{5}+1014\,a^{2}b^{4}x^{3}+443\,a^{3}b^{3}x\right)x^{\frac{7}{2}}+\left(705\,a^{2}b^{4}x^{5}+1210\,a^{3}b^{3}x^{3}+537\,a^{4}b^{2}x\right)x^{\frac{3}{2}}}{192\left(a^{8}b^{3}d^{\frac{5}{2}}x^{6}+3\,a^{9}b^{2}d^{\frac{5}{2}}x^{4}+3\,a^{10}bd^{\frac{5}{2}}x^{2}+a^{11}d^{\frac{5}{2}}+\left(a^{5}b^{6}d^{\frac{5}{2}}x^{6}+3\,a^{6}b^{5}d^{\frac{5}{2}}x^{4}+3\,a^{7}b^{4}d^{\frac{5}{2}}x^{2}+a^{8}b^{3}d^{\frac{5}{2}}\right)x^{6}+3\left(a^{6}b^{5}d^{\frac{5}{2}}x^{6}+3\,a^{7}b^{4}d^{\frac{5}{2}}x^{4}+3\,a^{8}b^{3}d^{\frac{5}{2}}x^{2}+a^{9}b^{2}d^{\frac{5}{2}}\right)x^{4}+3\left(a^{7}b^{4}d^{\frac{5}{2}}x^{6}+3\,a^{8}b^{3}d^{\frac{5}{2}}x^{4}+3\,a^{9}b^{2}d^{\frac{5}{2}}x^{2}+a^{10}bd^{\frac{5}{2}}\right)x^{2}\right)}+\frac{2925\left(\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)}{8192\,a^{5}d^{\frac{5}{2}}}+\int \frac{1}{\left(a^{4}bd^{\frac{5}{2}}x^{2}+a^{5}d^{\frac{5}{2}}\right)x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)

[Out] -4*b*integrate(1/((a^5*b*d^(5/2)*x^2 + a^6*d^(5/2))*sqrt(x)), x) - 1/3072*(
13795*b^4*x^(13/2) + 34285*a*b^3*x^(9/2) + 29649*a^2*b^2*x^(5/2) + 8775*a^3
*b*sqrt(x))/(a^5*b^4*d^(5/2)*x^8 + 4*a^6*b^3*d^(5/2)*x^6 + 6*a^7*b^2*d^(5/2
)*x^4 + 4*a^8*b*d^(5/2)*x^2 + a^9*d^(5/2)) + 1/192*((533*b^6*x^5 + 882*a*b^
5*x^3 + 381*a^2*b^4*x)*x^(11/2) + 2*(603*a*b^5*x^5 + 1014*a^2*b^4*x^3 + 443
*a^3*b^3*x)*x^(7/2) + (705*a^2*b^4*x^5 + 1210*a^3*b^3*x^3 + 537*a^4*b^2*x)*
x^(3/2))/(a^8*b^3*d^(5/2)*x^6 + 3*a^9*b^2*d^(5/2)*x^4 + 3*a^10*b*d^(5/2)*x^
2 + a^11*d^(5/2) + (a^5*b^6*d^(5/2)*x^6 + 3*a^6*b^5*d^(5/2)*x^4 + 3*a^7*b^4
*d^(5/2)*x^2 + a^8*b^3*d^(5/2))*x^6 + 3*(a^6*b^5*d^(5/2)*x^6 + 3*a^7*b^4*d^
(5/2)*x^4 + 3*a^8*b^3*d^(5/2)*x^2 + a^9*b^2*d^(5/2))*x^4 + 3*(a^7*b^4*d^(5/
2)*x^6 + 3*a^8*b^3*d^(5/2)*x^4 + 3*a^9*b^2*d^(5/2)*x^2 + a^10*b*d^(5/2))*x^
2) + 2925/8192*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +

```
2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(
2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3
/4))/(a^5*d^(5/2)) + integrate(1/((a^4*b*d^(5/2)*x^2 + a^5*d^(5/2))*x^(5/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{5/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(5/2)), x)
```

3.606 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=649

$$\frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}\left(a+bx^2\right)\mathrm{l}}{4096\sqrt{2}}$$

Rubi [A] time = 0.53, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13923b\left(a+bx^2\right)}{1024a^6d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923\left(a+bx^2\right)}{5120a^5d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{119}{256a^3d(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1547/(1024*a^4*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(5/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 7/(32*a^2*d*(d*x
)^(5/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 119/(256*a^3*d*(d*
x)^(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*(a + b*x^2))
/(5120*a^5*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*b*(a + b
*x^2))/(1024*a^6*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*b^
(5/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*
b^(5/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1392
3*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sq
rt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}(ab\cdots}}{16a\sqrt{a^2+2abx^2+b^2\cdots}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a^2\cdots}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a^2\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2\cdots}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.08

$$-\frac{2x\left(a+bx^2\right)^5 {}_2F_1\left(-\frac{5}{4},5;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^5(dx)^{7/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^5*Hypergeometric2F1[-5/4, 5, -1/4, -((b*x^2)/a)])/(5*a^5*
(d*x)^(7/2)*((a + b*x^2)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 144.05, size = 297, normalized size = 0.46

$$\frac{\left(ad^2+bd^2x^2\right)\left(-\frac{13923b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{d}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{dx}}\right)}{2048\sqrt{2}\,a^{25/4}d^{7/2}}-\frac{13923b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}}{\sqrt{a}\,d+\sqrt{b}\,dx}\right)}{2048\sqrt{2}\,a^{25/4}d^{7/2}}+\frac{-2048a^5d^{10}+43008a^4bd^{10}x^2+220507a^3b^2d^{10}x^4+369733a^2b^3d^{10}x^6+264537ab^4d^{10}x^8+69615b^5d^{10}x^{10}}{5120a^6d^3(dx)^{5/2}\left(ad^2+bd^2x^2\right)^4}\right)}{d^2\sqrt{\frac{\left(ad^2+bd^2x^2\right)^2}{d^4}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] ((a*d^2 + b*d^2*x^2)*((-2048*a^5*d^10 + 43008*a^4*b*d^10*x^2 + 220507*a^3*b
^2*d^10*x^4 + 369733*a^2*b^3*d^10*x^6 + 264537*a*b^4*d^10*x^8 + 69615*b^5*d
^10*x^10)/(5120*a^6*d^3*(d*x)^(5/2)*(a*d^2 + b*d^2*x^2)^4) - (13923*b^(5/4)
*ArcTan[((a^(1/4)*Sqrt[d])/(Sqrt[2]*b^(1/4)) - (b^(1/4)*Sqrt[d]*x)/(Sqrt[2]
*a^(1/4)))/Sqrt[d*x]])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)) - (13923*b^(5/4)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x])/(Sqrt[a]*d + Sqrt[b]*d*x)]
)/(2048*Sqrt[2]*a^(25/4)*d^(7/2))))/(d^2*Sqrt[(a*d^2 + b*d^2*x^2)^2/d^4])
```

fricas [A] time = 3.46, size = 524, normalized size = 0.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/20480*(278460*(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7
+ 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*arctan(-1/269897
2561467*(2698972561467*sqrt(d*x)*a^6*b^4*d^3*(-b^5/(a^25*d^14))^(1/4) - sqr
```

```
t(-7284452887551739093192089*a^13*b^5*d^8*sqrt(-b^5/(a^25*d^14)) + 72844528
87551739093192089*b^8*d*x)*a^6*d^3*(-b^5/(a^25*d^14))^(1/4))/b^5) - 69615*(
a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*d^4*x^5
+ a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*log(2698972561467*a^19*d^11*(-b^5/
(a^25*d^14))^(3/4) + 2698972561467*sqrt(d*x)*b^4) + 69615*(a^6*b^4*d^4*x^11
 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)*
(-b^5/(a^25*d^14))^(1/4)*log(-2698972561467*a^19*d^11*(-b^5/(a^25*d^14))^(3
/4) + 2698972561467*sqrt(d*x)*b^4) - 4*(69615*b^5*x^10 + 264537*a*b^4*x^8 +
 369733*a^2*b^3*x^6 + 220507*a^3*b^2*x^4 + 43008*a^4*b*x^2 - 2048*a^5)*sqrt
(d*x))/(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*
d^4*x^5 + a^10*d^4*x^3)
```

giac [A] time = 0.36, size = 470, normalized size = 0.72

$$\frac{13923\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^7bd^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{13923\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^7bd^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{13923\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^7bd^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{13923\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^7bd^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{3683\,\sqrt{dx}\,b^5d^7x^7+12357\,\sqrt{dx}\,ab^4d^7x^5+14145\,\sqrt{dx}\,a^2b^3d^7x^3+5599\,\sqrt{dx}\,a^3b^2d^7x}{1024\left(bd^2x^2+ad^2\right)^4a^6d^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{2\left(25\,bd^2x^2-ad^2\right)}{5\,\sqrt{dx}\,a^6d^5x^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) +
13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) -
 13923/8192*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 13923/8192*sqr
t(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 1/1024*(3683*sqrt(d*x)*b^5*d^
7*x^7 + 12357*sqrt(d*x)*a*b^4*d^7*x^5 + 14145*sqrt(d*x)*a^2*b^3*d^7*x^3 + 5
599*sqrt(d*x)*a^3*b^2*d^7*x)/((b*d^2*x^2 + a*d^2)^4*a^6*d^3*sgn(b*d^4*x^2 +
 a*d^4)) + 2/5*(25*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^6*d^5*x^2*sgn(b*d^4*x^2
+ a*d^4))
```

maple [B] time = 0.03, size = 1129, normalized size = 1.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/40960/d^3*(69615*(d*x)^(5/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2
)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^
2)^(1/2)))*x^8*b^5+139230*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(
a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^5+139230*(d*x)^(5/2)*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^5+278460*(d*x)
^(5/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2
```

```
))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a*b^4+556
920*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d
^2)^(1/4))*x^6*a*b^4+556920*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^4+556920*(a/b*d^2)^(1/4)*x^10*b^
5*d^2+417690*(d*x)^(5/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))*x^4*a^2*b^3+835380*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/
b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^3+835380*(d*x)^(5/2)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^3+211629
6*(a/b*d^2)^(1/4)*x^8*a*b^4*d^2+278460*(d*x)^(5/2)*2^(1/2)*ln(-(-d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^3*b^2+556920*(d*x)^(5/2)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b^2+556920
*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)
^(1/4))*x^2*a^3*b^2+2957864*(a/b*d^2)^(1/4)*x^6*a^2*b^3*d^2+69615*(d*x)^(5/
2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^4*b+139230*(d*x
)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4
))*a^4*b+139230*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(
1/4))/(a/b*d^2)^(1/4))*a^4*b+1764056*(a/b*d^2)^(1/4)*x^4*a^3*b^2*d^2+344064
*(a/b*d^2)^(1/4)*x^2*a^4*b*d^2-16384*(a/b*d^2)^(1/4)*a^5*d^2)*(b*x^2+a)/(d*
x)^(5/2)/(a/b*d^2)^(1/4)/a^6/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4b\int \frac{1}{\left(a^5bd^{\frac{7}{2}}x^2+a^6d^{\frac{7}{2}}\right)x^{\frac{3}{2}}}\,dx+\frac{11049\,b^5x^{\frac{15}{2}}+27135\,ab^4x^{\frac{11}{2}}+23395\,a^2b^3x^{\frac{7}{2}}+6925\,a^3b^2x^{\frac{3}{2}}}{3072\left(a^6b^4d^{\frac{7}{2}}x^8+4\,a^7b^3d^{\frac{7}{2}}x^6+6\,a^8b^2d^{\frac{7}{2}}x^4+4\,a^9bd^{\frac{7}{2}}x^2+a^{10}d^{\frac{7}{2}}\right)}+\frac{\left(621\,b^6x^5+1042\,ab^5x^3+453\,a^2b^4x\right)x^{\frac{9}{2}}+2\left(695\,ab^5x^5+1182\,a^2b^4x^3+519\,a^3b^3x\right)x^{\frac{5}{2}}+\left(801\,a^2b^4x^5+1386\,a^3b^3x^3+617\,a^4b^2x\right)\sqrt{x}}{192\left(a^8b^3d^{\frac{7}{2}}x^6+3\,a^9b^2d^{\frac{7}{2}}x^4+3\,a^{10}bd^{\frac{7}{2}}x^2+a^{11}d^{\frac{7}{2}}+\left(a^5b^6d^{\frac{7}{2}}x^6+3\,a^6b^5d^{\frac{7}{2}}x^4+3\,a^7b^4d^{\frac{7}{2}}x^2+a^8b^3d^{\frac{7}{2}}\right)x^6+3\left(a^6b^5d^{\frac{7}{2}}x^6+3\,a^7b^4d^{\frac{7}{2}}x^4+3\,a^8b^3d^{\frac{7}{2}}x^2+a^9b^2d^{\frac{7}{2}}\right)x^4+3\left(a^7b^4d^{\frac{7}{2}}x^6+3\,a^8b^3d^{\frac{7}{2}}x^4+3\,a^9b^2d^{\frac{7}{2}}x^2+a^{10}bd^{\frac{7}{2}}\right)x^2\right)}+\frac{3683\,b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,a^6d^{\frac{7}{2}}}+\int \frac{1}{\left(a^4bd^{\frac{7}{2}}x^2+a^5d^{\frac{7}{2}}\right)x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -4*b*integrate(1/((a^5*b*d^(7/2)*x^2 + a^6*d^(7/2))*x^(3/2)), x) + 1/3072*(
11049*b^5*x^(15/2) + 27135*a*b^4*x^(11/2) + 23395*a^2*b^3*x^(7/2) + 6925*a^
3*b^2*x^(3/2))/(a^6*b^4*d^(7/2)*x^8 + 4*a^7*b^3*d^(7/2)*x^6 + 6*a^8*b^2*d^(
7/2)*x^4 + 4*a^9*b*d^(7/2)*x^2 + a^10*d^(7/2)) + 1/192*((621*b^6*x^5 + 1042
*a*b^5*x^3 + 453*a^2*b^4*x)*x^(9/2) + 2*(695*a*b^5*x^5 + 1182*a^2*b^4*x^3 +
 519*a^3*b^3*x)*x^(5/2) + (801*a^2*b^4*x^5 + 1386*a^3*b^3*x^3 + 617*a^4*b^2
*x)*sqrt(x))/(a^8*b^3*d^(7/2)*x^6 + 3*a^9*b^2*d^(7/2)*x^4 + 3*a^10*b*d^(7/2
)*x^2 + a^11*d^(7/2) + (a^5*b^6*d^(7/2)*x^6 + 3*a^6*b^5*d^(7/2)*x^4 + 3*a^7
*b^4*d^(7/2)*x^2 + a^8*b^3*d^(7/2))*x^6 + 3*(a^6*b^5*d^(7/2)*x^6 + 3*a^7*b^
4*d^(7/2)*x^4 + 3*a^8*b^3*d^(7/2)*x^2 + a^9*b^2*d^(7/2))*x^4 + 3*(a^7*b^4*d
^(7/2)*x^6 + 3*a^8*b^3*d^(7/2)*x^4 + 3*a^9*b^2*d^(7/2)*x^2 + a^10*b*d^(7/2)
)*x^2) + 3683/8192*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(
```

```
b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log
(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) +
 sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/
4)*b^(3/4)))/(a^6*d^(7/2)) + integrate(1/((a^4*b*d^(7/2)*x^2 + a^5*d^(7/2))
*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{7/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(1/((d*x)**(7/2)*((a + b*x**2)**2)**(5/2)), x)
```

3.607 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=150

$$\frac{a^6(dx)^{m+1}}{d(m+1)}+\frac{6a^5b(dx)^{m+3}}{d^3(m+3)}+\frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)}+\frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)}+\frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)}+\frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)}+\frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 270}

$$\frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)}+\frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)}+\frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)}+\frac{6a^5b(dx)^{m+3}}{d^3(m+3)}+\frac{a^6(dx)^{m+1}}{d(m+1)}+\frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)}+\frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*(d*x)^(1 + m))/(d*(1 + m)) + (6*a^5*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
15*a^4*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (20*a^3*b^3*(d*x)^(7 + m))/(d^7*(
7 + m)) + (15*a^2*b^4*(d*x)^(9 + m))/(d^9*(9 + m)) + (6*a*b^5*(d*x)^(11 + m
))/(d^11*(11 + m)) + (b^6*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^m\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{\int (dx)^m\left(ab+b^2x^2\right)^6\,dx}{b^6}$$

$$=\frac{\int\left(a^6b^6(dx)^m+\frac{6a^5b^7(dx)^{2+m}}{d^2}+\frac{15a^4b^8(dx)^{4+m}}{d^4}+\frac{20a^3b^9(dx)^{6+m}}{d^6}+\frac{15a^2b^{10}(dx)^{8+m}}{d^8}+\frac{6}{}\right.}{b^6}$$

$$=\frac{a^6(dx)^{1+m}}{d(1+m)}+\frac{6a^5b(dx)^{3+m}}{d^3(3+m)}+\frac{15a^4b^2(dx)^{5+m}}{d^5(5+m)}+\frac{20a^3b^3(dx)^{7+m}}{d^7(7+m)}+\frac{15a^2b^4(dx)}{d^9(9+m}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.70

$$x(dx)^m\left(\frac{a^6}{m+1}+\frac{6a^5bx^2}{m+3}+\frac{15a^4b^2x^4}{m+5}+\frac{20a^3b^3x^6}{m+7}+\frac{15a^2b^4x^8}{m+9}+\frac{6ab^5x^{10}}{m+11}+\frac{b^6x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x*(d*x)^m*(a^6/(1 + m) + (6*a^5*b*x^2)/(3 + m) + (15*a^4*b^2*x^4)/(5 + m) +
 (20*a^3*b^3*x^6)/(7 + m) + (15*a^2*b^4*x^8)/(9 + m) + (6*a*b^5*x^10)/(11 +
 m) + (b^6*x^12)/(13 + m))
```

IntegrateAlgebraic [F] time = 1.07, size = 0, normalized size = 0.00

$$\int(dx)^m\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3, x]
```

fricas [B] time = 1.99, size = 507, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] ((b^6*m^6 + 36*b^6*m^5 + 505*b^6*m^4 + 3480*b^6*m^3 + 12139*b^6*m^2 + 19524
*b^6*m + 10395*b^6)*x^13 + 6*(a*b^5*m^6 + 38*a*b^5*m^5 + 555*a*b^5*m^4 + 39
40*a*b^5*m^3 + 14039*a*b^5*m^2 + 22902*a*b^5*m + 12285*a*b^5)*x^11 + 15*(a^
2*b^4*m^6 + 40*a^2*b^4*m^5 + 613*a^2*b^4*m^4 + 4528*a^2*b^4*m^3 + 16627*a^2
*b^4*m^2 + 27688*a^2*b^4*m + 15015*a^2*b^4)*x^9 + 20*(a^3*b^3*m^6 + 42*a^3*
b^3*m^5 + 679*a^3*b^3*m^4 + 5292*a^3*b^3*m^3 + 20335*a^3*b^3*m^2 + 34986*a^
3*b^3*m + 19305*a^3*b^3)*x^7 + 15*(a^4*b^2*m^6 + 44*a^4*b^2*m^5 + 753*a^4*b
^2*m^4 + 6280*a^4*b^2*m^3 + 25979*a^4*b^2*m^2 + 47436*a^4*b^2*m + 27027*a^4
*b^2)*x^5 + 6*(a^5*b*m^6 + 46*a^5*b*m^5 + 835*a^5*b*m^4 + 7540*a^5*b*m^3 +
34759*a^5*b*m^2 + 73054*a^5*b*m + 45045*a^5*b)*x^3 + (a^6*m^6 + 48*a^6*m^5
+ 925*a^6*m^4 + 9120*a^6*m^3 + 48259*a^6*m^2 + 129072*a^6*m + 135135*a^6)*x
)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 26
4207*m + 135135)
```

giac [B] time = 0.25, size = 847, normalized size = 5.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^6*m^6*x^13 + 36*(d*x)^m*b^6*m^5*x^13 + 6*(d*x)^m*a*b^5*m^6*x^11
+ 505*(d*x)^m*b^6*m^4*x^13 + 228*(d*x)^m*a*b^5*m^5*x^11 + 3480*(d*x)^m*b^6*
m^3*x^13 + 15*(d*x)^m*a^2*b^4*m^6*x^9 + 3330*(d*x)^m*a*b^5*m^4*x^11 + 12139
*(d*x)^m*b^6*m^2*x^13 + 600*(d*x)^m*a^2*b^4*m^5*x^9 + 23640*(d*x)^m*a*b^5*m
^3*x^11 + 19524*(d*x)^m*b^6*m*x^13 + 20*(d*x)^m*a^3*b^3*m^6*x^7 + 9195*(d*x
)^m*a^2*b^4*m^4*x^9 + 84234*(d*x)^m*a*b^5*m^2*x^11 + 10395*(d*x)^m*b^6*x^13
 + 840*(d*x)^m*a^3*b^3*m^5*x^7 + 67920*(d*x)^m*a^2*b^4*m^3*x^9 + 137412*(d*
x)^m*a*b^5*m*x^11 + 15*(d*x)^m*a^4*b^2*m^6*x^5 + 13580*(d*x)^m*a^3*b^3*m^4*
x^7 + 249405*(d*x)^m*a^2*b^4*m^2*x^9 + 73710*(d*x)^m*a*b^5*x^11 + 660*(d*x)
^m*a^4*b^2*m^5*x^5 + 105840*(d*x)^m*a^3*b^3*m^3*x^7 + 415320*(d*x)^m*a^2*b^
4*m*x^9 + 6*(d*x)^m*a^5*b*m^6*x^3 + 11295*(d*x)^m*a^4*b^2*m^4*x^5 + 406700*
(d*x)^m*a^3*b^3*m^2*x^7 + 225225*(d*x)^m*a^2*b^4*x^9 + 276*(d*x)^m*a^5*b*m^
5*x^3 + 94200*(d*x)^m*a^4*b^2*m^3*x^5 + 699720*(d*x)^m*a^3*b^3*m*x^7 + (d*x
)^m*a^6*m^6*x + 5010*(d*x)^m*a^5*b*m^4*x^3 + 389685*(d*x)^m*a^4*b^2*m^2*x^5
 + 386100*(d*x)^m*a^3*b^3*x^7 + 48*(d*x)^m*a^6*m^5*x + 45240*(d*x)^m*a^5*b*
m^3*x^3 + 711540*(d*x)^m*a^4*b^2*m*x^5 + 925*(d*x)^m*a^6*m^4*x + 208554*(d*
x)^m*a^5*b*m^2*x^3 + 405405*(d*x)^m*a^4*b^2*x^5 + 9120*(d*x)^m*a^6*m^3*x +
438324*(d*x)^m*a^5*b*m*x^3 + 48259*(d*x)^m*a^6*m^2*x + 270270*(d*x)^m*a^5*b
*x^3 + 129072*(d*x)^m*a^6*m*x + 135135*(d*x)^m*a^6*x)/(m^7 + 49*m^6 + 973*m
^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

maple [B] time = 0.01, size = 602, normalized size = 4.01

$$\frac{\left(b^6m^6x^{12}+36b^6m^5x^{12}+6ab^5m^6x^{10}+505b^6m^4x^{12}+228ab^5m^5x^{10}+3480b^6m^3x^{12}+15a^2b^4m^6x^8+3330ab^5m^4x^{10}+12139b^6m^2x^{12}+600a^2b^4m^5x^8+23640ab^5m^3x^{10}+19524b^6m\,x^{12}+20a^3b^3m^6x^6+9195a^2b^4m^4x^8+84234ab^5m^2x^{10}+10395b^6x^{12}+840a^3b^3m^5x^6+67920a^2b^4m^3x^8+137412ab^5m\,x^{10}+15a^4b^2m^6x^4+13580a^3b^3m^4x^6+249405a^2b^4m^2x^8+73710ab^5x^{10}+660a^4b^2m^5x^4+105840a^3b^3m^3x^6+415320a^2b^4m\,x^8+6a^5bm^6x^2+11295a^4b^2m^4x^4+406700a^3b^3m^2x^6+225225a^2b^4x^8+276a^5bm^5x^2+94200a^4b^2m^3x^4+699720a^3b^3m\,x^6+a^6m^6+5010a^5bm^4x^2+389685a^4b^2m^2x^4+386100a^3b^3x^6+48a^6m^5+45240a^5bm^3x^2+711540a^4b^2m\,x^4+925a^6m^4+208554a^5bm^2x^2+405405a^4b^2x^4+9120a^6m^3+438324a^5bm\,x^2+48259a^6m^2+270270a^5bx^2+129072a^6m+135135a^6\right)x\,(dx)^m}{(m+13)(m+11)(m+9)(m+7)(m+5)(m+3)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (d*x)^m*(b^6*m^6*x^12+36*b^6*m^5*x^12+6*a*b^5*m^6*x^10+505*b^6*m^4*x^12+228
*a*b^5*m^5*x^10+3480*b^6*m^3*x^12+15*a^2*b^4*m^6*x^8+3330*a*b^5*m^4*x^10+12
139*b^6*m^2*x^12+600*a^2*b^4*m^5*x^8+23640*a*b^5*m^3*x^10+19524*b^6*m*x^12+
20*a^3*b^3*m^6*x^6+9195*a^2*b^4*m^4*x^8+84234*a*b^5*m^2*x^10+10395*b^6*x^12
+840*a^3*b^3*m^5*x^6+67920*a^2*b^4*m^3*x^8+137412*a*b^5*m*x^10+15*a^4*b^2*m
^6*x^4+13580*a^3*b^3*m^4*x^6+249405*a^2*b^4*m^2*x^8+73710*a*b^5*x^10+660*a^
4*b^2*m^5*x^4+105840*a^3*b^3*m^3*x^6+415320*a^2*b^4*m*x^8+6*a^5*b*m^6*x^2+1
1295*a^4*b^2*m^4*x^4+406700*a^3*b^3*m^2*x^6+225225*a^2*b^4*x^8+276*a^5*b*m^
5*x^2+94200*a^4*b^2*m^3*x^4+699720*a^3*b^3*m*x^6+a^6*m^6+5010*a^5*b*m^4*x^2
+389685*a^4*b^2*m^2*x^4+386100*a^3*b^3*x^6+48*a^6*m^5+45240*a^5*b*m^3*x^2+7
11540*a^4*b^2*m*x^4+925*a^6*m^4+208554*a^5*b*m^2*x^2+405405*a^4*b^2*x^4+912
0*a^6*m^3+438324*a^5*b*m*x^2+48259*a^6*m^2+270270*a^5*b*x^2+129072*a^6*m+13
5135*a^6)*x/(m+13)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.54, size = 144, normalized size = 0.96

$$\frac{b^6 d^m x^{13} x^m}{m+13} + \frac{6\,ab^5 d^m x^{11} x^m}{m+11} + \frac{15\,a^2 b^4 d^m x^9 x^m}{m+9} + \frac{20\,a^3 b^3 d^m x^7 x^m}{m+7} + \frac{15\,a^4 b^2 d^m x^5 x^m}{m+5} + \frac{6\,a^5 b d^m x^3 x^m}{m+3} + \frac{(dx)^{m+1}\,a^6}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] b^6*d^m*x^13*x^m/(m + 13) + 6*a*b^5*d^m*x^11*x^m/(m + 11) + 15*a^2*b^4*d^m*
x^9*x^m/(m + 9) + 20*a^3*b^3*d^m*x^7*x^m/(m + 7) + 15*a^4*b^2*d^m*x^5*x^m/(
m + 5) + 6*a^5*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^6/(d*(m + 1))
```

mupad [B] time = 4.58, size = 540, normalized size = 3.60

$$\frac{a^6\,x\,(d\,x)^m\,\left(m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{b^6\,x^{13}\,(d\,x)^m\,\left(m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{6\,a\,b^5\,x^{11}\,(d\,x)^m\,\left(m^6+38\,m^5+555\,m^4+3940\,m^3+14039\,m^2+22902\,m+12285\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{6\,a^5\,b\,x^3\,(d\,x)^m\,\left(m^6+46\,m^5+835\,m^4+7540\,m^3+34759\,m^2+73054\,m+45045\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{15\,a^2\,b^4\,x^9\,(d\,x)^m\,\left(m^6+40\,m^5+613\,m^4+4528\,m^3+16627\,m^2+27688\,m+15015\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{20\,a^3\,b^3\,x^7\,(d\,x)^m\,\left(m^6+42\,m^5+679\,m^4+5292\,m^3+20335\,m^2+34986\,m+19305\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{15\,a^4\,b^2\,x^5\,(d\,x)^m\,\left(m^6+44\,m^5+753\,m^4+6280\,m^3+25979\,m^2+47436\,m+27027\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x*(d*x)^m*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 +
135135))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6
+ m^7 + 135135) + (b^6*x^13*(d*x)^m*(19524*m + 12139*m^2 + 3480*m^3 + 505*m
^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
+ 973*m^5 + 49*m^6 + m^7 + 135135) + (6*a*b^5*x^11*(d*x)^m*(22902*m + 14039
*m^2 + 3940*m^3 + 555*m^4 + 38*m^5 + m^6 + 12285))/(264207*m + 177331*m^2 +
 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (6*a^5*b*x^3*(d
*x)^m*(73054*m + 34759*m^2 + 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45045))/(2
64207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135
135) + (15*a^2*b^4*x^9*(d*x)^m*(27688*m + 16627*m^2 + 4528*m^3 + 613*m^4 +
40*m^5 + m^6 + 15015))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973
*m^5 + 49*m^6 + m^7 + 135135) + (20*a^3*b^3*x^7*(d*x)^m*(34986*m + 20335*m^
2 + 5292*m^3 + 679*m^4 + 42*m^5 + m^6 + 19305))/(264207*m + 177331*m^2 + 57
379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (15*a^4*b^2*x^5*(d
*x)^m*(47436*m + 25979*m^2 + 6280*m^3 + 753*m^4 + 44*m^5 + m^6 + 27027))/(2
64207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135
135)
```

sympy [A] time = 7.61, size = 3188, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Piecewise(((-a**6/(12*x**12) - 3*a**5*b/(5*x**10) - 15*a**4*b**2/(8*x**8) -
 10*a**3*b**3/(3*x**6) - 15*a**2*b**4/(4*x**4) - 3*a*b**5/x**2 + b**6*log(x
```

```
))/d**13, Eq(m, -13)), ((-a**6/(10*x**10) - 3*a**5*b/(4*x**8) - 5*a**4*b**2
/(2*x**6) - 5*a**3*b**3/x**4 - 15*a**2*b**4/(2*x**2) + 6*a*b**5*log(x) + b*
*6*x**2/2)/d**11, Eq(m, -11)), ((-a**6/(8*x**8) - a**5*b/x**6 - 15*a**4*b**
2/(4*x**4) - 10*a**3*b**3/x**2 + 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6
*x**4/4)/d**9, Eq(m, -9)), ((-a**6/(6*x**6) - 3*a**5*b/(2*x**4) - 15*a**4*b
**2/(2*x**2) + 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2
+ b**6*x**6/6)/d**7, Eq(m, -7)), ((-a**6/(4*x**4) - 3*a**5*b/x**2 + 15*a**4
*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**6 + b**6
*x**8/8)/d**5, Eq(m, -5)), ((-a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**
2*x**2/2 + 5*a**3*b**3*x**4 + 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x
**10/10)/d**3, Eq(m, -3)), ((a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**
4/4 + 10*a**3*b**3*x**6/3 + 15*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x
**12/12)/d, Eq(m, -1)), (a**6*d**m*m**6*x*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**6*d**m*
m**5*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m
**2 + 264207*m + 135135) + 925*a**6*d**m*m**4*x*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9120*a*
*6*d**m*m**3*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 48259*a**6*d**m*m**2*x*x**m/(m**7 + 49*m
**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 129072*a**6*d**m*m*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 5737
9*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**6*d**m*x*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 6*a**5*b*d**m*m**6*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 276*a**5*b*d**m*m**5*x*
*3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
+ 264207*m + 135135) + 5010*a**5*b*d**m*m**4*x**3*x**m/(m**7 + 49*m**6 + 97
3*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 45240
*a**5*b*d**m*m**3*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379
*m**3 + 177331*m**2 + 264207*m + 135135) + 208554*a**5*b*d**m*m**2*x**3*x**
m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 2642
07*m + 135135) + 438324*a**5*b*d**m*m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5
+ 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 270270*a**5*
b*d**m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 15*a**4*b**2*d**m*m**6*x**5*x**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 660*a**4*b**2*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m
**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 11295*a**4*b**2*d**m*
m**4*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 94200*a**4*b**2*d**m*m**3*x**5*x**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1351
35) + 389685*a**4*b**2*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 100
45*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 711540*a**4*b**2*
d**m*m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 405405*a**4*b**2*d**m*x**5*x**m/(m**7 + 49*
```

```
m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + 20*a**3*b**3*d**m*m**6*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 840*a**3*b**3*d**m*m**5
*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m*
*2 + 264207*m + 135135) + 13580*a**3*b**3*d**m*m**4*x**7*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 105840*a**3*b**3*d**m*m**3*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m
**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 406700*a**3*b**3*d**m
*m**2*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 1773
31*m**2 + 264207*m + 135135) + 699720*a**3*b**3*d**m*m*x**7*x**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 386100*a**3*b**3*d**m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 15*a**2*b**4*d**m*m**6*
x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**
2 + 264207*m + 135135) + 600*a**2*b**4*d**m*m**5*x**9*x**m/(m**7 + 49*m**6
+ 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9
195*a**2*b**4*d**m*m**4*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 +
 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 67920*a**2*b**4*d**m*m**3*
x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**
2 + 264207*m + 135135) + 249405*a**2*b**4*d**m*m**2*x**9*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 415320*a**2*b**4*d**m*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 225225*a**2*b**4*d**m*x*
*9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
+ 264207*m + 135135) + 6*a*b**5*d**m*m**6*x**11*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 228*a*b
**5*d**m*m**5*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m*
*3 + 177331*m**2 + 264207*m + 135135) + 3330*a*b**5*d**m*m**4*x**11*x**m/(m
**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
 + 135135) + 23640*a*b**5*d**m*m**3*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 84234*a*b**5*
d**m*m**2*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
 177331*m**2 + 264207*m + 135135) + 137412*a*b**5*d**m*m*x**11*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 73710*a*b**5*d**m*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + b**6*d**m*m**6*x**13*x*
*m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264
207*m + 135135) + 36*b**6*d**m*m**5*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 505*b**6*d**m
*m**4*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 3480*b**6*d**m*m**3*x**13*x**m/(m**7 + 49*m
**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 12139*b**6*d**m*m**2*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 19524*b**6*d**m*m*x**13*x
**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26
```

```
4207*m + 135135) + 10395*b**6*d**m*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 +
10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135), True))
```

3.608 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=104

$$\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 270}

$$\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*(d*x)^(1 + m))/(d*(1 + m)) + (4*a^3*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
6*a^2*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (4*a*b^3*(d*x)^(7 + m))/(d^7*(7 +
m)) + (b^4*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int (dx)^m \left(ab+b^2x^2\right)^4\,dx}{b^4}\\ &= \frac{\int \left(a^4b^4(dx)^m+\frac{4a^3b^5(dx)^{2+m}}{d^2}+\frac{6a^2b^6(dx)^{4+m}}{d^4}+\frac{4ab^7(dx)^{6+m}}{d^6}+\frac{b^8(dx)^{8+m}}{d^8}\right)dx}{b^4}\\ &= \frac{a^4(dx)^{1+m}}{d(1+m)}+\frac{4a^3b(dx)^{3+m}}{d^3(3+m)}+\frac{6a^2b^2(dx)^{5+m}}{d^5(5+m)}+\frac{4ab^3(dx)^{7+m}}{d^7(7+m)}+\frac{b^4(dx)^{9+m}}{d^9(9+m)}\end{aligned}$$

Mathematica [A]   time = 0.04, size = 73, normalized size = 0.70

$$x(dx)^m\left(\frac{a^4}{m+1}+\frac{4a^3bx^2}{m+3}+\frac{6a^2b^2x^4}{m+5}+\frac{4ab^3x^6}{m+7}+\frac{b^4x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x*(d*x)^m*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
(4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

IntegrateAlgebraic [F]   time = 0.26, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a^2+2abx^2+b^2x^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2, x]
```

fricas [B]   time = 0.98, size = 253, normalized size = 2.43

$$\frac{\left(\left(b^4m^4+16\,b^4m^3+86\,b^4m^2+176\,b^4m+105\,b^4\right)x^9+4\left(ab^3m^4+18\,ab^3m^3+104\,ab^3m^2+222\,ab^3m+135\,ab^3\right)x^7+6\left(a^2b^2m^4+20\,a^2b^2m^3+130\,a^2b^2m^2+300\,a^2b^2m+189\,a^2b^2\right)x^5+4\left(a^3bm^4+22\,a^3bm^3+164\,a^3bm^2+458\,a^3bm+315\,a^3b\right)x^3+\left(a^4m^4+24\,a^4m^3+206\,a^4m^2+744\,a^4m+945\,a^4\right)x\right)(dx)^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] ((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*m
^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2*b
^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)*x^
5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3*b)*
x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*(d*x)^m
/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B]   time = 0.18, size = 415, normalized size = 3.99

$$\frac{(dx)^m b^4m^4x^9+16\,(dx)^m b^4m^3x^9+4\,(dx)^m ab^3m^4x^7+86\,(dx)^m b^4m^2x^9+72\,(dx)^m ab^3m^3x^7+176\,(dx)^m b^4mx^9+6\,(dx)^m a^2b^2m^4x^5+416\,(dx)^m ab^3m^2x^7+105\,(dx)^m b^4x^9+120\,(dx)^m a^2b^2m^3x^5+888\,(dx)^m ab^3mx^7+4\,(dx)^m a^3bm^4x^3+780\,(dx)^m a^2b^2m^2x^5+540\,(dx)^m ab^3x^7+88\,(dx)^m a^3bm^3x^3+1800\,(dx)^m a^2b^2mx^5+(dx)^m a^4m^4x+656\,(dx)^m a^3bm^2x^3+1134\,(dx)^m a^2b^2x^5+24\,(dx)^m a^4m^3x+1832\,(dx)^m a^3bmx^3+206\,(dx)^m a^4m^2x+1260\,(dx)^m a^3bx^3+744\,(dx)^m a^4mx+945\,(dx)^m a^4x}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^4*m^4*x^9 + 16*(d*x)^m*b^4*m^3*x^9 + 4*(d*x)^m*a*b^3*m^4*x^7 + 8
6*(d*x)^m*b^4*m^2*x^9 + 72*(d*x)^m*a*b^3*m^3*x^7 + 176*(d*x)^m*b^4*m*x^9 +
```

```
6*(d*x)^m*a^2*b^2*m^4*x^5 + 416*(d*x)^m*a*b^3*m^2*x^7 + 105*(d*x)^m*b^4*x^9
 + 120*(d*x)^m*a^2*b^2*m^3*x^5 + 888*(d*x)^m*a*b^3*m*x^7 + 4*(d*x)^m*a^3*b*
m^4*x^3 + 780*(d*x)^m*a^2*b^2*m^2*x^5 + 540*(d*x)^m*a*b^3*x^7 + 88*(d*x)^m*
a^3*b*m^3*x^3 + 1800*(d*x)^m*a^2*b^2*m*x^5 + (d*x)^m*a^4*m^4*x + 656*(d*x)^
m*a^3*b*m^2*x^3 + 1134*(d*x)^m*a^2*b^2*x^5 + 24*(d*x)^m*a^4*m^3*x + 1832*(d
*x)^m*a^3*b*m*x^3 + 206*(d*x)^m*a^4*m^2*x + 1260*(d*x)^m*a^3*b*x^3 + 744*(d
*x)^m*a^4*m*x + 945*(d*x)^m*a^4*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689
*m + 945)
```

maple [B] time = 0.01, size = 292, normalized size = 2.81

$$\frac{\left(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176b^4m\,x^8+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2b^2m^3x^4+888a\,b^3m\,x^6+4a^3b\,m^4x^2+780a^2b^2m^2x^4+540a\,b^3x^6+88a^3b\,m^3x^2+1800a^2b^2m\,x^4+a^4m^4+656a^3b\,m^2x^2+1134a^2b^2x^4+24a^4m^3+1832a^3bm\,x^2+206a^4m^2+1260a^3b\,x^2+744a^4m+945a^4\right)x\,(dx)^m}{(m+9)\,(m+7)\,(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (d*x)^m*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^3
*m^3*x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+120*
a^2*b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*a*b
^3*x^6+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a
^2*b^2*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m
+945*a^4)*x/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.49, size = 100, normalized size = 0.96

$$\frac{b^4d^mx^9x^m}{m+9}+\frac{4\,ab^3d^mx^7x^m}{m+7}+\frac{6\,a^2b^2d^mx^5x^m}{m+5}+\frac{4\,a^3bd^mx^3x^m}{m+3}+\frac{(dx)^{m+1}\,a^4}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] b^4*d^m*x^9*x^m/(m + 9) + 4*a*b^3*d^m*x^7*x^m/(m + 7) + 6*a^2*b^2*d^m*x^5*x
^m/(m + 5) + 4*a^3*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^4/(d*(m + 1))
```

mupad [B] time = 4.51, size = 263, normalized size = 2.53

$$(d\,x)^m\,\left(\frac{b^4\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{a^4\,x\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{4\,a\,b^3\,x^7\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{4\,a^3\,b\,x^3\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{6\,a^2\,b^2\,x^5\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (d*x)^m*((b^4*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2
+ 230*m^3 + 25*m^4 + m^5 + 945) + (a^4*x*(744*m + 206*m^2 + 24*m^3 + m^4 +
945))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (4*a*b^3*x^7*(222
```

```
*m + 104*m^2 + 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 +
m^5 + 945) + (4*a^3*b*x^3*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m +
 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (6*a^2*b^2*x^5*(300*m + 130*m^2
+ 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))
```

sympy [A] time = 3.20, size = 1321, normalized size = 12.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Piecewise(((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a
*b**3/x**2 + b**4*log(x))/d**9, Eq(m, -9)), ((-a**4/(6*x**6) - a**3*b/x**4
- 3*a**2*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2)/d**7, Eq(m, -7)), ((-a*
*4/(4*x**4) - 2*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**
4/4)/d**5, Eq(m, -5)), ((-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**
2 + a*b**3*x**4 + b**4*x**6/6)/d**3, Eq(m, -3)), ((a**4*log(x) + 2*a**3*b*x
**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x**8/8)/d, Eq(m, -1)), (a
**4*d**m*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 24*a**4*d**m*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 206*a**4*d**m*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 744*a**4*d**m*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 945*a**4*d**m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 4*a**3*b*d**m*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**
3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*d**m*m**3*x**3*x**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*d**m*m**2*x**3*x**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1832*a**3*b*d**m*m*x*
*3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1260*a**3*b
*d**m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 6*a
**2*b**2*d**m*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 120*a**2*b**2*d**m*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + 780*a**2*b**2*d**m*m**2*x**5*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 1800*a**2*b**2*d**m*m*x**5*x**m/(m*
*5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1134*a**2*b**2*d**m*x*
*5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a*b**3*d*
*m*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 7
2*a*b**3*d**m*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 416*a*b**3*d**m*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 888*a*b**3*d**m*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**
3 + 950*m**2 + 1689*m + 945) + 540*a*b**3*d**m*x**7*x**m/(m**5 + 25*m**4 +
230*m**3 + 950*m**2 + 1689*m + 945) + b**4*d**m*m**4*x**9*x**m/(m**5 + 25*m
**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**4*d**m*m**3*x**9*x**m/(m*
*5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**4*d**m*m**2*x**9
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4*d**m
```

```
*m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*b*
*4*d**m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), Tr
ue))
```

3.609 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=58

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {14}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (b^
2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a^2+2abx^2+b^2x^4\right)dx&=\int\left(a^2(dx)^m+\frac{2ab(dx)^{2+m}}{d^2}+\frac{b^2(dx)^{4+m}}{d^4}\right)dx\\&=\frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{b^2(dx)^{5+m}}{d^5(5+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.71

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{2abx^2}{m+3}+\frac{b^2x^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4), x]
```

fricas [A] time = 1.31, size = 87, normalized size = 1.50

$$\frac{\left(\left(b^2m^2 + 4\,b^2m + 3\,b^2\right)x^5 + 2\left(abm^2 + 6\,abm + 5\,ab\right)x^3 + \left(a^2m^2 + 8\,a^2m + 15\,a^2\right)x\right)(dx)^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^2
*m^2 + 8*a^2*m + 15*a^2)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

giac [B] time = 0.16, size = 135, normalized size = 2.33

$$\frac{(dx)^m\,b^2m^2x^5 + 4\,(dx)^m\,b^2mx^5 + 2\,(dx)^m\,abm^2x^3 + 3\,(dx)^m\,b^2x^5 + 12\,(dx)^m\,abmx^3 + (dx)^m\,a^2m^2x + 10\,(dx)^m\,abx^3 + 8\,(dx)^m\,a^2mx + 15\,(dx)^m\,a^2x}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^2*m^2*x^5 + 4*(d*x)^m*b^2*m*x^5 + 2*(d*x)^m*a*b*m^2*x^3 + 3*(d*x
)^m*b^2*x^5 + 12*(d*x)^m*a*b*m*x^3 + (d*x)^m*a^2*m^2*x + 10*(d*x)^m*a*b*x^3
 + 8*(d*x)^m*a^2*m*x + 15*(d*x)^m*a^2*x)/(m^3 + 9*m^2 + 23*m + 15)
```

maple [A] time = 0.01, size = 94, normalized size = 1.62

$$\frac{\left(b^2m^2x^4 + 4b^2m\,x^4 + 2ab\,m^2x^2 + 3b^2x^4 + 12abm\,x^2 + a^2m^2 + 10ab\,x^2 + 8a^2m + 15a^2\right)x\,(dx)^m}{(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] (d*x)^m*(b^2*m^2*x^4+4*b^2*m*x^4+2*a*b*m^2*x^2+3*b^2*x^4+12*a*b*m*x^2+a^2*m
^2+10*a*b*x^2+8*a^2*m+15*a^2)*x/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.40, size = 56, normalized size = 0.97

$$\frac{b^2 d^m x^5 x^m}{m+5} + \frac{2\,abd^m x^3 x^m}{m+3} + \frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^5*x^m/(m + 5) + 2*a*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^2/(d*
(m + 1))
```

mupad [B] time = 4.27, size = 95, normalized size = 1.64

$$(d\,x)^m\,\left(\frac{a^2\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}+\frac{b^2\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{2\,a\,b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (d*x)^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*m
 + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*m
+ 9*m^2 + m^3 + 15))
```

sympy [A] time = 1.01, size = 345, normalized size = 5.95

$$\begin{cases} \frac{-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)}{d^5} & \text{for}\ m=-5 \\ \frac{-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}}{d^3} & \text{for}\ m=-3 \\ \frac{a^2\log(x)+abx^2+\frac{b^2x^4}{4}}{d} & \text{for}\ m=-1 \\ \frac{a^2d^mm^2xx^m}{m^3+9m^2+23m+15}+\frac{8a^2d^mmxx^m}{m^3+9m^2+23m+15}+\frac{15a^2d^mxx^m}{m^3+9m^2+23m+15}+\frac{2abd^mm^2x^3x^m}{m^3+9m^2+23m+15}+\frac{12abd^mmx^3x^m}{m^3+9m^2+23m+15}+\frac{10abd^mx^3x^m}{m^3+9m^2+23m+15}+\frac{b^2d^mm^2x^5x^m}{m^3+9m^2+23m+15}+\frac{4b^2d^mmx^5x^m}{m^3+9m^2+23m+15}+\frac{3b^2d^mx^5x^m}{m^3+9m^2+23m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Piecewise(((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x))/d**5, Eq(m, -5)), ((-a
**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2)/d**3, Eq(m, -3)), ((a**2*log(x)
+ a*b*x**2 + b**2*x**4/4)/d, Eq(m, -1)), (a**2*d**m*m**2*x*x**m/(m**3 + 9*m
**2 + 23*m + 15) + 8*a**2*d**m*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a*
*2*d**m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 2*a*b*d**m*m**2*x**3*x**m/(m**
3 + 9*m**2 + 23*m + 15) + 12*a*b*d**m*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 1
5) + 10*a*b*d**m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + b**2*d**m*m**2*x**
5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*d**m*m*x**5*x**m/(m**3 + 9*m**2
 + 23*m + 15) + 3*b**2*d**m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15), True))
```

3.610 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=313

$$\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+11}}{d^{11}(m+11)\left(a+bx^2\right)}+\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+9}}{d^9(m+9)\left(a+bx^2\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{a^5\sqrt{a^2\cdots}}{d\cdots}$$

Rubi [A] time = 0.12, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 270}

$$\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+9}}{d^9(m+9)\left(a+bx^2\right)}+\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+11}}{d^{11}(m+11)\left(a+bx^2\right)}+\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+1}}{d(m+1)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (5*a^4*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (10*a^3*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5
*(5 + m)*(a + b*x^2)) + (10*a^2*b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(d^7*(7 + m)*(a + b*x^2)) + (5*a*b^4*(d*x)^(9 + m)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(d^9*(9 + m)*(a + b*x^2)) + (b^5*(d*x)^(11 + m)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(d^11*(11 + m)*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (dx)^m \left(ab+b^2x^2\right)^5 dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^5b^5(dx)^m + \frac{5a^4b^6(dx)^{2+m}}{d^2} + \frac{10a^3b^7(dx)^{4+m}}{d^4} + \frac{10a^2b^8(dx)^{6}}{d^6}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)} + \frac{5a^4b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)} + \frac{10a^3}{}$$

Mathematica [A] time = 0.09, size = 111, normalized size = 0.35

$$\frac{x\left(\left(a+bx^2\right)^2\right)^{5/2}(dx)^m\left(\frac{a^5}{m+1}+\frac{5a^4bx^2}{m+3}+\frac{10a^3b^2x^4}{m+5}+\frac{10a^2b^3x^6}{m+7}+\frac{5ab^4x^8}{m+9}+\frac{b^5x^{10}}{m+11}\right)}{\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^2)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (10
*a^3*b^2*x^4)/(5 + m) + (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m) +
(b^5*x^10)/(11 + m)))/(a + b*x^2)^5
```

IntegrateAlgebraic [F] time = 1.61, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2), x]
```

fricas [A] time = 1.71, size = 369, normalized size = 1.18

$$\frac{\left(\left(b^5m^5+25\,b^5m^4+230\,b^5m^3+950\,b^5m^2+1689\,b^5m+945\,b^5\right)x^{11}+5\left(ab^4m^5+27\,ab^4m^4+262\,ab^4m^3+1122\,ab^4m^2+2041\,ab^4m+1155\,ab^4\right)x^9+10\left(a^2b^3m^5+29\,a^2b^3m^4+302\,a^2b^3m^3+1366\,a^2b^3m^2+2577\,a^2b^3m+1485\,a^2b^3\right)x^7+10\left(a^3b^2m^5+31\,a^3b^2m^4+350\,a^3b^2m^3+1730\,a^3b^2m^2+3489\,a^3b^2m+2079\,a^3b^2\right)x^5+5\left(a^4bm^5+33\,a^4bm^4+406\,a^4bm^3+2262\,a^4bm^2+5353\,a^4bm+3465\,a^4b\right)x^3+\left(a^5m^5+35\,a^5m^4+470\,a^5m^3+3010\,a^5m^2+9129\,a^5m+10395\,a^5\right)x\right)(dx)^m}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)*
x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 2041*
a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b^3*
m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b^2*m
^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^2*m +
 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 2262*a^4
*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 470*a^5*m
^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*(d*x)^m/(m^6 + 36*m^5 + 505*
m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

giac [B] time = 0.28, size = 900, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^11*sgn(b*x^2 + a) + 25*(d*x)^m*b^5*m^4*x^11*sgn(b*x^2 +
a) + 5*(d*x)^m*a*b^4*m^5*x^9*sgn(b*x^2 + a) + 230*(d*x)^m*b^5*m^3*x^11*sgn(
b*x^2 + a) + 135*(d*x)^m*a*b^4*m^4*x^9*sgn(b*x^2 + a) + 950*(d*x)^m*b^5*m^2
*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^2*b^3*m^5*x^7*sgn(b*x^2 + a) + 1310*(d*
x)^m*a*b^4*m^3*x^9*sgn(b*x^2 + a) + 1689*(d*x)^m*b^5*m*x^11*sgn(b*x^2 + a)
+ 290*(d*x)^m*a^2*b^3*m^4*x^7*sgn(b*x^2 + a) + 5610*(d*x)^m*a*b^4*m^2*x^9*s
gn(b*x^2 + a) + 945*(d*x)^m*b^5*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^3*b^2*m^
5*x^5*sgn(b*x^2 + a) + 3020*(d*x)^m*a^2*b^3*m^3*x^7*sgn(b*x^2 + a) + 10205*
(d*x)^m*a*b^4*m*x^9*sgn(b*x^2 + a) + 310*(d*x)^m*a^3*b^2*m^4*x^5*sgn(b*x^2
+ a) + 13660*(d*x)^m*a^2*b^3*m^2*x^7*sgn(b*x^2 + a) + 5775*(d*x)^m*a*b^4*x^
9*sgn(b*x^2 + a) + 5*(d*x)^m*a^4*b*m^5*x^3*sgn(b*x^2 + a) + 3500*(d*x)^m*a^
3*b^2*m^3*x^5*sgn(b*x^2 + a) + 25770*(d*x)^m*a^2*b^3*m*x^7*sgn(b*x^2 + a) +
 165*(d*x)^m*a^4*b*m^4*x^3*sgn(b*x^2 + a) + 17300*(d*x)^m*a^3*b^2*m^2*x^5*s
gn(b*x^2 + a) + 14850*(d*x)^m*a^2*b^3*x^7*sgn(b*x^2 + a) + (d*x)^m*a^5*m^5*
x*sgn(b*x^2 + a) + 2030*(d*x)^m*a^4*b*m^3*x^3*sgn(b*x^2 + a) + 34890*(d*x)^
m*a^3*b^2*m*x^5*sgn(b*x^2 + a) + 35*(d*x)^m*a^5*m^4*x*sgn(b*x^2 + a) + 1131
0*(d*x)^m*a^4*b*m^2*x^3*sgn(b*x^2 + a) + 20790*(d*x)^m*a^3*b^2*x^5*sgn(b*x^
2 + a) + 470*(d*x)^m*a^5*m^3*x*sgn(b*x^2 + a) + 26765*(d*x)^m*a^4*b*m*x^3*s
gn(b*x^2 + a) + 3010*(d*x)^m*a^5*m^2*x*sgn(b*x^2 + a) + 17325*(d*x)^m*a^4*b
*x^3*sgn(b*x^2 + a) + 9129*(d*x)^m*a^5*m*x*sgn(b*x^2 + a) + 10395*(d*x)^m*a
^5*x*sgn(b*x^2 + a))/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524
*m + 10395)
```

maple [A] time = 0.01, size = 453, normalized size = 1.45

$$\frac{\left(b^{5} m^{5} x^{10}+25 b^{5} m^{4} x^{10}+5 a \,b^{4} m^{5} x^{8}+230 b^{5} m^{3} x^{10}+135 a \,b^{4} m^{4} x^{8}+950 b^{5} m^{2} x^{10}+10 a^{2} b^{3} m^{5} x^{6}+1310 a \,b^{4} m^{3} x^{8}+1689 b^{5} m \,x^{10}+290 a^{2} b^{3} m^{4} x^{6}+5610 a \,b^{4} m^{2} x^{8}+945 b^{5} x^{10}+10 a^{3} b^{2} m^{5} x^{4}+3020 a^{2} b^{3} m^{3} x^{6}+10205 a \,b^{4} m \,x^{8}+310 a^{3} b^{2} m^{4} x^{4}+13660 a^{2} b^{3} m^{2} x^{6}+5775 a \,b^{4} x^{8}+5 a^{4} b \,m^{5} x^{2}+3500 a^{3} b^{2} m^{3} x^{4}+25770 a^{2} b^{3} m \,x^{6}+165 a^{4} b \,m^{4} x^{2}+17300 a^{3} b^{2} m^{2} x^{4}+14850 a^{2} b^{3} x^{6}+a^{5} m^{5}+2030 a^{4} b \,m^{3} x^{2}+34890 a^{3} b^{2} m \,x^{4}+35 a^{5} m^{4}+11310 a^{4} b \,m^{2} x^{2}+20790 a^{3} b^{2} x^{4}+470 a^{5} m^{3}+26765 a^{4} b m \,x^{2}+3010 a^{5} m^{2}+17325 a^{4} b \,x^{2}+9129 a^{5} m +10395 a^{5}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}} x \left(d x \right)^{m}}{\left(m +11\right) \left(m +9\right) \left(m +7\right) \left(m +5\right) \left(m +3\right) \left(m +1\right) \left(b \,x^{2}+a \right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] x*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*a*b^4*
m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b^5*m*x
^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^5*x^4+
3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*b^3*m^
2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b^3*m*x
^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+2030*a
^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+20790*a^3*b
^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+9129*a^5*
m+10395*a^5)*(d*x)^m*((b*x^2+a)^2)^(5/2)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+
1)/(b*x^2+a)^5
```

maxima [A] time = 1.41, size = 243, normalized size = 0.78

$$\frac{\left(\left(m^{5}+25\,m^{4}+230\,m^{3}+950\,m^{2}+1689\,m+945\right)b^{5}d^{m}x^{11}+5\left(m^{5}+27\,m^{4}+262\,m^{3}+1122\,m^{2}+2041\,m+1155\right)ab^{4}d^{m}x^{9}+10\left(m^{5}+29\,m^{4}+302\,m^{3}+1366\,m^{2}+2577\,m+1485\right)a^{2}b^{3}d^{m}x^{7}+10\left(m^{5}+31\,m^{4}+350\,m^{3}+1730\,m^{2}+3489\,m+2079\right)a^{3}b^{2}d^{m}x^{5}+5\left(m^{5}+33\,m^{4}+406\,m^{3}+2262\,m^{2}+5353\,m+3465\right)a^{4}bd^{m}x^{3}+\left(m^{5}+35\,m^{4}+470\,m^{3}+3010\,m^{2}+9129\,m+10395\right)a^{5}d^{m}x\right)x^{m}}{m^{6}+36\,m^{5}+505\,m^{4}+3480\,m^{3}+12139\,m^{2}+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*d^m*x^11 + 5*(m^5 +
27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*d^m*x^9 + 10*(m^5 + 29*m
^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*d^m*x^7 + 10*(m^5 + 31*m^4
 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*d^m*x^5 + 5*(m^5 + 33*m^4 +
406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*d^m*x^3 + (m^5 + 35*m^4 + 470*m^3
 + 3010*m^2 + 9129*m + 10395)*a^5*d^m*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480
*m^3 + 12139*m^2 + 19524*m + 10395)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{m} \left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**(5/2), x)
```

3.611 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{a^3\sqrt{a^2+2\cdots}}{d(m\cdots}$$

Rubi [A] time = 0.08, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 270}

$$\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+1}}{d(m+1)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (3*a^2*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (3*a*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(5
 + m)*(a + b*x^2)) + (b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d
^7*(7 + m)*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3(dx)^m+\frac{3a^2b^4(dx)^{2+m}}{d^2}+\frac{3ab^5(dx)^{4+m}}{d^4}+\frac{b^6(dx)^{6+m}}{d^6}\right) d}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)}+\frac{3a^2b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)}+\frac{3ab^2}{}$$

Mathematica [A] time = 0.07, size = 131, normalized size = 0.64

$$\frac{x\sqrt{\left(a+bx^2\right)^2}(dx)^m\left(a^3\left(m^3+15m^2+71m+105\right)+3a^2b\left(m^3+13m^2+47m+35\right)x^2+3ab^2\left(m^3+11m^2+31m+21\right)x^4+b^3\left(m^3+9m^2+23m+15\right)x^6\right)}{(m+1)(m+3)(m+5)(m+7)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2*b*(
35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4 + b^
3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a + b*x
^2))
```

IntegrateAlgebraic [F] time = 1.17, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]
```

fricas [A] time = 0.99, size = 159, normalized size = 0.78

$$\frac{\left(\left(b^3m^3+9\,b^3m^2+23\,b^3m+15\,b^3\right)x^7+3\left(ab^2m^3+11\,ab^2m^2+31\,ab^2m+21\,ab^2\right)x^5+3\left(a^2bm^3+13\,a^2bm^2+47\,a^2bm+35\,a^2b\right)x^3+\left(a^3m^3+15\,a^3m^2+71\,a^3m+105\,a^3\right)x\right)(dx)^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m^
2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m +
```

```
 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*(d*x)^m/(m^
4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

giac [B] time = 0.21, size = 384, normalized size = 1.87

$$\frac{(dx)^m b^3m^3x^7\mathrm{sgn}\left(bx^2+a\right)+9\,(dx)^m b^3m^2x^7\mathrm{sgn}\left(bx^2+a\right)+3\,(dx)^m ab^2m^3x^5\mathrm{sgn}\left(bx^2+a\right)+23\,(dx)^m b^3mx^7\mathrm{sgn}\left(bx^2+a\right)+33\,(dx)^m ab^2m^2x^5\mathrm{sgn}\left(bx^2+a\right)+15\,(dx)^m b^3x^7\mathrm{sgn}\left(bx^2+a\right)+3\,(dx)^m a^2bm^3x^3\mathrm{sgn}\left(bx^2+a\right)+93\,(dx)^m ab^2mx^5\mathrm{sgn}\left(bx^2+a\right)+39\,(dx)^m a^2bm^2x^3\mathrm{sgn}\left(bx^2+a\right)+63\,(dx)^m ab^2x^5\mathrm{sgn}\left(bx^2+a\right)+(dx)^m a^3m^3x\mathrm{sgn}\left(bx^2+a\right)+141\,(dx)^m a^2bmx^3\mathrm{sgn}\left(bx^2+a\right)+15\,(dx)^m a^3m^2x\mathrm{sgn}\left(bx^2+a\right)+105\,(dx)^m a^2bx^3\mathrm{sgn}\left(bx^2+a\right)+71\,(dx)^m a^3mx\mathrm{sgn}\left(bx^2+a\right)+105\,(dx)^m a^3x\mathrm{sgn}\left(bx^2+a\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^3*m^3*x^7*sgn(b*x^2 + a) + 9*(d*x)^m*b^3*m^2*x^7*sgn(b*x^2 + a)
+ 3*(d*x)^m*a*b^2*m^3*x^5*sgn(b*x^2 + a) + 23*(d*x)^m*b^3*m*x^7*sgn(b*x^2 +
 a) + 33*(d*x)^m*a*b^2*m^2*x^5*sgn(b*x^2 + a) + 15*(d*x)^m*b^3*x^7*sgn(b*x^
2 + a) + 3*(d*x)^m*a^2*b*m^3*x^3*sgn(b*x^2 + a) + 93*(d*x)^m*a*b^2*m*x^5*sg
n(b*x^2 + a) + 39*(d*x)^m*a^2*b*m^2*x^3*sgn(b*x^2 + a) + 63*(d*x)^m*a*b^2*x
^5*sgn(b*x^2 + a) + (d*x)^m*a^3*m^3*x*sgn(b*x^2 + a) + 141*(d*x)^m*a^2*b*m*
x^3*sgn(b*x^2 + a) + 15*(d*x)^m*a^3*m^2*x*sgn(b*x^2 + a) + 105*(d*x)^m*a^2*
b*x^3*sgn(b*x^2 + a) + 71*(d*x)^m*a^3*m*x*sgn(b*x^2 + a) + 105*(d*x)^m*a^3*
x*sgn(b*x^2 + a))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

maple [A] time = 0.01, size = 199, normalized size = 0.97

$$\frac{\left(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23b^3m\,x^6+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93a\,b^2m\,x^4+39a^2b\,m^2x^2+63a\,b^2x^4+a^3m^3+141a^2bm\,x^2+15a^3m^2+105a^2b\,x^2+71a^3m+105a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x\,(dx)^m}{(m+7)\,(m+5)\,(m+3)\,(m+1)\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] x*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^2*x^4+
15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x^4+a^3
*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(d*x)^m*((b
*x^2+a)^2)^(3/2)/(m+7)/(m+5)/(m+3)/(m+1)/(b*x^2+a)^3
```

maxima [A] time = 1.44, size = 119, normalized size = 0.58

$$\frac{\left(\left(m^3+9\,m^2+23\,m+15\right)b^3d^mx^7+3\left(m^3+11\,m^2+31\,m+21\right)ab^2d^mx^5+3\left(m^3+13\,m^2+47\,m+35\right)a^2bd^mx^3+\left(m^3+15\,m^2+71\,m+105\right)a^3d^mx\right)x^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*d^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^2
*d^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*d^m*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*a^3*d^m*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx\right)^{m} \left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral((d*x)**m*((a + b*x**2)**2)**(3/2), x)
```

3.612 $\int (dx)^m\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=97

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+1}}{d(m+1)\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 14}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+1}}{d(m+1)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2)) +
 (b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a + b*x^2)
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^m \sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^m+\frac{b^2(dx)^{2+m}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)} + \frac{b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.55

$$\frac{x\sqrt{\left(a+bx^2\right)^2}\,(dx)^m\left(a(m+3)+b(m+1)x^2\right)}{(m+1)(m+3)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a*(3 + m) + b*(1 + m)*x^2))/((1 + m)*(3 + m
)*(a + b*x^2))
```

IntegrateAlgebraic [F] time = 0.75, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a^2+2abx^2+b^2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]
```

fricas [A] time = 2.03, size = 35, normalized size = 0.36

$$\frac{\left((bm+b)x^3+(am+3\,a)x\right)(dx)^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^3 + (a*m + 3*a)*x)*(d*x)^m/(m^2 + 4*m + 3)
```

giac [A] time = 0.16, size = 83, normalized size = 0.86

$$\frac{(dx)^m bmx^3\mathrm{sgn}\left(bx^2+a\right)+(dx)^m bx^3\mathrm{sgn}\left(bx^2+a\right)+(dx)^m amx\mathrm{sgn}\left(bx^2+a\right)+3\,(dx)^m ax\mathrm{sgn}\left(bx^2+a\right)}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b*m*x^3*sgn(b*x^2 + a) + (d*x)^m*b*x^3*sgn(b*x^2 + a) + (d*x)^m*a*
m*x*sgn(b*x^2 + a) + 3*(d*x)^m*a*x*sgn(b*x^2 + a))/(m^2 + 4*m + 3)
```

maple [A] time = 0.00, size = 56, normalized size = 0.58

$$\frac{\left(bm\,x^2+b\,x^2+am+3a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x\,(dx)^m}{(m+3)\,(m+1)\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] x*(b*m*x^2+b*x^2+a*m+3*a)*(d*x)^m*((b*x^2+a)^2)^(1/2)/(m+3)/(m+1)/(b*x^2+a)
```

maxima [A] time = 1.40, size = 35, normalized size = 0.36

$$\frac{\left(bd^m(m+1)x^3+ad^m(m+3)x\right)x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*d^m*(m + 1)*x^3 + a*d^m*(m + 3)*x)*x^m/(m^2 + 4*m + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{\left(a+bx^2\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**2)**2), x)
```

3.613 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=174

$$\frac{\left(a+bx^2\right)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+2)}-\frac{3a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+3)}+\frac{3a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+1)}-\frac{a^3\left(a\right.}{}$$

Rubi [A] time = 0.11, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 43}

$$\frac{\left(a+bx^2\right)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+2)}-\frac{3a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+3)}+\frac{3a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+1)}-\frac{a^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(1 + 2*p)) + (3*a^2
*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(1 + p)) - (3*a*(a + b
*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^p\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^7\left(1+\frac{bx^2}{a}\right)^{2p}dx$$

$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^2\right)$$

$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{2p}}{b^3}+\frac{3a^3\left(1+\frac{b}{a}\right.}{b^3}\right.\right.$$

$$= -\frac{a^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(1+2p)}+\frac{3a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(1+p)}-\frac{3}{}$$

Mathematica [A] time = 0.06, size = 110, normalized size = 0.63

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\left(-3a^3+3a^2b(2p+1)x^2-3ab^2\left(2p^2+3p+1\right)x^4+b^3\left(4p^3+12p^2+11p+3\right)x^6\right)}{4b^4(p+1)(p+2)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^2 - 3*a*b^2*(1
 + 3*p + 2*p^2)*x^4 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^6))/(4*b^4*(1 + p)*
(2 + p)*(1 + 2*p)*(3 + 2*p))
```

IntegrateAlgebraic [F] time = 0.48, size = 0, normalized size = 0.00

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]
```

fricas [A] time = 1.03, size = 163, normalized size = 0.94

$$\frac{\left(\left(4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4\right)x^8+6\,a^3bpx^2+2\left(2\,ab^3p^3+3\,ab^3p^2+ab^3p\right)x^6-3\left(2\,a^2b^2p^2+a^2b^2p\right)x^4-3\,a^4\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^8 + 6*a^3*b*p*x^2 + 2*(2
*a*b^3*p^3 + 3*a*b^3*p^2 + a*b^3*p)*x^6 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x^4
 - 3*a^4)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4*p^
2 + 25*b^4*p + 6*b^4)
```

giac [B] time = 0.19, size = 375, normalized size = 2.16

$$\frac{4\left(b^2x^4+2\,abx^2+a^2\right)^p b^4p^3x^8+12\left(b^2x^4+2\,abx^2+a^2\right)^p b^4p^2x^8+4\left(b^2x^4+2\,abx^2+a^2\right)^p ab^3p^3x^6+11\left(b^2x^4+2\,abx^2+a^2\right)^p b^4px^8+6\left(b^2x^4+2\,abx^2+a^2\right)^p ab^3p^2x^6+3\left(b^2x^4+2\,abx^2+a^2\right)^p b^4x^8+2\left(b^2x^4+2\,abx^2+a^2\right)^p ab^3px^6-6\left(b^2x^4+2\,abx^2+a^2\right)^p a^2b^2p^2x^4-3\left(b^2x^4+2\,abx^2+a^2\right)^p a^2b^2px^4+6\left(b^2x^4+2\,abx^2+a^2\right)^p a^3bpx^2-3\left(b^2x^4+2\,abx^2+a^2\right)^p a^4}{4\left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^3*x^8 + 12*(b^2*x^4 + 2*a*b*x^2
+ a^2)^p*b^4*p^2*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^3*x^6 + 11*(
b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p*x^8 + 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*
b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 2*(b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a*b^3*p*x^6 - 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4
- 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 + 6*(b^2*x^4 + 2*a*b*x^2 +
a^2)^p*a^3*b*p*x^2 - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4)/(4*b^4*p^4 + 20*b
^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

maple [A] time = 0.01, size = 150, normalized size = 0.86

$$-\frac{\left(-4b^3p^3x^6-12b^3p^2x^6-11b^3p\,x^6+6a\,b^2p^2x^4-3b^3x^6+9a\,b^2p\,x^4+3a\,b^2x^4-6a^2bp\,x^2-3a^2b\,x^2+3a^3\right)\left(b\,x^2+a\right)\left(b^2x^4+2ab\,x^2+a^2\right)^p}{4\left(4p^4+20p^3+35p^2+25p+6\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-4*b^3*p^3*x^6-12*b^3*p^2*x^6-11*b^3*p*x^6+
6*a*b^2*p^2*x^4-3*b^3*x^6+9*a*b^2*p*x^4+3*a*b^2*x^4-6*a^2*b*p*x^2-3*a^2*b*x
^2+3*a^3)*(b*x^2+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

maxima [A] time = 1.46, size = 115, normalized size = 0.66

$$\frac{\left(\left(4\,p^3+12\,p^2+11\,p+3\right)b^4x^8+2\left(2\,p^3+3\,p^2+p\right)ab^3x^6-3\left(2\,p^2+p\right)a^2b^2x^4+6\,a^3bpx^2-3\,a^4\right)\left(bx^2+a\right)^{2\,p}}{4\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6
- 3*(2*p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*
p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

mupad [B] time = 4.40, size = 206, normalized size = 1.18

$$\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,\left(\frac{x^8\,\left(p^3+3\,p^2+\frac{11\,p}{4}+\frac{3}{4}\right)}{4\,p^4+20\,p^3+35\,p^2+25\,p+6}-\frac{3\,a^4}{4\,b^4\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{3\,a^3\,p\,x^2}{2\,b^3\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{a\,p\,x^6\,\left(2\,p^2+3\,p+1\right)}{2\,b\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}-\frac{3\,a^2\,p\,x^4\,\left(2\,p+1\right)}{4\,b^2\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^8*((11*p)/4 + 3*p^2 + p^3 + 3/4))/(25*p +
 35*p^2 + 20*p^3 + 4*p^4 + 6) - (3*a^4)/(4*b^4*(25*p + 35*p^2 + 20*p^3 + 4*
p^4 + 6)) + (3*a^3*p*x^2)/(2*b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) + (a
*p*x^6*(3*p + 2*p^2 + 1))/(2*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - (3*a
^2*p*x^4*(2*p + 1))/(4*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**8*(a**2)**p/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) +
x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a
**3*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b
**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a
*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(
12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2
*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36
*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*
x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-I*sqrt(a)*sqrt(
1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6
) + 18*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36
*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(-I*sqrt(
a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b
**7*x**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**
2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -2)), (Integral(x**7/((
a + b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b*
*4 + 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*
sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(
a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b*
```

```
*5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -1)), (Integral(x**7/s
qrt((a + b*x**2)**2), x), Eq(p, -1/2)), (-3*a**4*(a**2 + 2*a*b*x**2 + b**2*
x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**
4) + 6*a**3*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*
b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) - 6*a**2*b**2*p**2*x**4*(
a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p
**2 + 100*b**4*p + 24*b**4) - 3*a**2*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*
x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**
4) + 4*a*b**3*p**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 +
80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 6*a*b**3*p**2*x**6*(
a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p
**2 + 100*b**4*p + 24*b**4) + 2*a*b**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x**
4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4)
+ 4*b**4*p**3*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b*
*4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 12*b**4*p**2*x**8*(a**2 +
 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 +
100*b**4*p + 24*b**4) + 11*b**4*p*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(
16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 3*b**
4*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 14
0*b**4*p**2 + 100*b**4*p + 24*b**4), True))
```

3.614 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=130

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(p+1)}+\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+1)}$$

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 43}

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(p+1)}+\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + 2*p)) - (a*(a +
 b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + p)) + ((a + b*x^2)^3*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(3 + 2*p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^5\left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$
$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{bx}{a}\right)^{2p}}{b^2}-\frac{2a^2\left(1+\frac{bx}{a}\right)^{1}}{b^2}\right.\right.$$
$$= \frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+2p)}-\frac{a\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+p)}+\frac{\left(a+b\right.}{}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\left(a^2-ab(2p+1)x^2+b^2\left(2p^2+3p+1\right)x^4\right)}{2b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(a^2 - a*b*(1 + 2*p)*x^2 + b^2*(1 + 3*p + 2*
p^2)*x^4))/(2*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

IntegrateAlgebraic [F] time = 0.42, size = 0, normalized size = 0.00

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]
```

fricas [A] time = 1.74, size = 108, normalized size = 0.83

$$\frac{\left(\left(2\,b^3p^2+3\,b^3p+b^3\right)x^6-2\,a^2bpx^2+\left(2\,ab^2p^2+ab^2p\right)x^4+a^3\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{2\left(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((2*b^3*p^2 + 3*b^3*p + b^3)*x^6 - 2*a^2*b*p*x^2 + (2*a*b^2*p^2 + a*b^2
*p)*x^4 + a^3)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b
^3*p + 3*b^3)
```

giac [A] time = 0.19, size = 235, normalized size = 1.81

$$\frac{2\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}b^{3}p^{2}x^{6}+3\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}b^{3}px^{6}+2\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}ab^{2}p^{2}x^{4}+\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}b^{3}x^{6}+\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}ab^{2}px^{4}-2\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}a^{2}bpx^{2}+\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}a^{3}}{2\left(4\,b^{3}p^{3}+12\,b^{3}p^{2}+11\,b^{3}p+3\,b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 +
 a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p^2*x^4 + (b^2*x^
4 + 2*a*b*x^2 + a^2)^p*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4
- 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)
^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

maple [A] time = 0.01, size = 96, normalized size = 0.74

$$\frac{\left(b\,x^{2}+a\right)\left(2b^{2}p^{2}x^{4}+3b^{2}p\,x^{4}+b^{2}x^{4}-2abp\,x^{2}-ab\,x^{2}+a^{2}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}}{2\left(4p^{3}+12p^{2}+11p+3\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/2*(b*x^2+a)*(2*b^2*p^2*x^4+3*b^2*p*x^4+b^2*x^4-2*a*b*p*x^2-a*b*x^2+a^2)*(
b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

maxima [A] time = 1.44, size = 79, normalized size = 0.61

$$\frac{\left(\left(2\,p^{2}+3\,p+1\right)b^{3}x^{6}+\left(2\,p^{2}+p\right)ab^{2}x^{4}-2\,a^{2}bpx^{2}+a^{3}\right)\left(bx^{2}+a\right)^{2\,p}}{2\left(4\,p^{3}+12\,p^{2}+11\,p+3\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^
3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

mupad [B] time = 4.27, size = 137, normalized size = 1.05

$${\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p\,\left(\frac{x^6\,\left(p^2+\frac{3\,p}{2}+\frac{1}{2}\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{a^3}{2\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{a^2\,p\,x^2}{b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}+\frac{a\,p\,x^4\,\left(2\,p+1\right)}{2\,b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^6*((3*p)/2 + p^2 + 1/2))/(11*p + 12*p^2 +
 4*p^3 + 3) + a^3/(2*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (a^2*p*x^2)/(b^2*(1
1*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^4*(2*p + 1))/(2*b*(11*p + 12*p^2 + 4*p^
3 + 3)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^{6}\left(a^{2}\right)^{p}}{6} & \text{for}\: b = 0 \\ \int \frac{x^{5}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx & \text{for}\: p = -\frac{3}{2} \\ -\frac{2a^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{3}+2b^{4}x^{2}} - \frac{2a^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{3}+2b^{4}x^{2}} - \frac{2a^{2}}{2ab^{3}+2b^{4}x^{2}} - \frac{2abx^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{3}+2b^{4}x^{2}} - \frac{2abx^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{3}+2b^{4}x^{2}} + \frac{b^{2}x^{4}}{2ab^{3}+2b^{4}x^{2}} & \text{for}\: p = -1 \\ \int \frac{x^{5}}{\sqrt{\left(a+bx^{2}\right)^{2}}}\, dx & \text{for}\: p = -\frac{1}{2} \\ \frac{a^{3}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+24b^{3}p^{2}+22b^{3}p+6b^{3}} - \frac{2a^{2}bpx^{2}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+24b^{3}p^{2}+22b^{3}p+6b^{3}} + \frac{2ab^{2}p^{2}x^{4}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+24b^{3}p^{2}+22b^{3}p+6b^{3}} + \frac{ab^{2}px^{4}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+24b^{3}p^{2}+22b^{3}p+6b^{3}} + \frac{2b^{3}p^{2}x^{6}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+24b^{3}p^{2}+22b^{3}p+6b^{3}} + \frac{3b^{3}px^{6}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+24b^{3}p^{2}+22b^{3}p+6b^{3}} + \frac{b^{3}x^{6}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{p}}{8b^{3}p^{3}+24b^{3}p^{2}+22b^{3}p+6b^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (Integral(x**5/((a + b*x**2)**2)**(
3/2), x), Eq(p, -3/2)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 +
2*b**4*x**2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2)
 - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) +
x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*
b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -1)), (Inte
gral(x**5/sqrt((a + b*x**2)**2), x), Eq(p, -1/2)), (a**3*(a**2 + 2*a*b*x**2
 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) - 2*a**2
*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 +
22*b**3*p + 6*b**3) + 2*a*b**2*p**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p
/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) + a*b**2*p*x**4*(a**2 +
2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3
) + 2*b**3*p**2*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b
**3*p**2 + 22*b**3*p + 6*b**3) + 3*b**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x*
*4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) + b**3*x**6*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b*
*3), True))
```

3.615 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+1)}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 43}

$$\frac{\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(1 + 2*p)) + ((a + b*
x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p\right) \int x^3 \left(1+\frac{bx^2}{a}\right)^{2p} dx$$

$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p\right) \text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p} dx, x, x^2\right)$$

$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p\right) \text{Subst}\left(\int \left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b} + \frac{a\left(1+\frac{bx}{a}\right)^{1+2}}{b}\right.\right.$$

$$= -\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(1+2p)} + \frac{\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p \left(b(2p+1)x^2-a\right)}{4b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-a + b*(1 + 2*p)*x^2))/(4*b^2*(1 + p)*(1 +
2*p))
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] Defer[IntegrateAlgebraic][x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]
```

fricas [A] time = 4.01, size = 70, normalized size = 0.83

$$\frac{\left(2\,abpx^2+\left(2\,b^2p+b^2\right)x^4-a^2\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*p*x^2 + (2*b^2*p + b^2)*x^4 - a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p
/(2*b^2*p^2 + 3*b^2*p + b^2)
```

giac [A] time = 0.22, size = 132, normalized size = 1.57

$$\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^2px^4+\left(b^2x^4+2\,abx^2+a^2\right)^pb^2x^4+2\left(b^2x^4+2\,abx^2+a^2\right)^pabpx^2-\left(b^2x^4+2\,abx^2+a^2\right)^pa^2}{4\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^2*p*x^4 + (b^2*x^4 + 2*a*b*x^2 + a^2
)^p*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b*p*x^2 - (b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

maple [A] time = 0.01, size = 60, normalized size = 0.71

$$-\frac{\left(-2x^2pb-b\,x^2+a\right)\left(b\,x^2+a\right)\left(b^2x^4+2ab\,x^2+a^2\right)^p}{4\left(2p^2+3p+1\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-b*x^2+a)*(b*x^2+a)/b^2/(2*p^2+3*
p+1)
```

maxima [A] time = 1.43, size = 54, normalized size = 0.64

$$\frac{\left(b^2\left(2\,p+1\right)x^4+2\,abpx^2-a^2\right)\left(bx^2+a\right)^{2\,p}}{4\left(2\,p^2+3\,p+1\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p
 + 1)*b^2)
```

mupad [B] time = 4.26, size = 85, normalized size = 1.01

$$\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,\left(\frac{x^4\,\left(2\,p+1\right)}{4\,\left(2\,p^2+3\,p+1\right)}-\frac{a^2}{4\,b^2\,\left(2\,p^2+3\,p+1\right)}+\frac{a\,p\,x^2}{2\,b\,\left(2\,p^2+3\,p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^4*(2*p + 1))/(4*(3*p + 2*p^2 + 1)) - a^2/
(4*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^2)/(2*b*(3*p + 2*p^2 + 1)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^4 (a^2)^p}{4} & \text{for } b = 0 \\ \frac{a \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2ab^2 + 2b^3x^2} + \frac{a \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2ab^2 + 2b^3x^2} + \frac{a}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2ab^2 + 2b^3x^2} & \text{for } p = -1 \\ \int \frac{x^3}{\sqrt{(a + bx^2)^2}}\, dx & \text{for } p = -\frac{1}{2} \\ -\frac{a^2 (a^2 + 2abx^2 + b^2x^4)^p}{8b^2p^2 + 12b^2p + 4b^2} + \frac{2abpx^2 (a^2 + 2abx^2 + b^2x^4)^p}{8b^2p^2 + 12b^2p + 4b^2} + \frac{2b^2px^4 (a^2 + 2abx^2 + b^2x^4)^p}{8b^2p^2 + 12b^2p + 4b^2} + \frac{b^2x^4 (a^2 + 2abx^2 + b^2x^4)^p}{8b^2p^2 + 12b^2p + 4b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**4*(a**2)**p/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2
*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*
x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(
2*a*b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2
*b**3*x**2), Eq(p, -1)), (Integral(x**3/sqrt((a + b*x**2)**2), x), Eq(p, -1
/2)), (-a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 12*b**2*p +
4*b**2) + 2*a*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 12
*b**2*p + 4*b**2) + 2*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**
2*p**2 + 12*b**2*p + 4*b**2) + b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p
/(8*b**2*p**2 + 12*b**2*p + 4*b**2), True))
```

3.616 $\int x \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=41

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(2p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b*(1 + 2*p))
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x \left(a^2+2abx^2+b^2x^4\right)^p \, dx &= \frac{1}{2}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^p \, dx, x, x^2\right)\\ &= \frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 29, normalized size = 0.71

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p}{4bp+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p)/(2*b + 4*b*p)
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] Defer[IntegrateAlgebraic][x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]
```

fricas [A] time = 0.88, size = 37, normalized size = 0.90

$$\frac{\left(bx^2+a\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{2\left(2\,bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] 1/2*(b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b*p + b)
```

giac [A] time = 0.26, size = 58, normalized size = 1.41

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p bx^2+\left(b^2x^4+2\,abx^2+a^2\right)^p a}{2\left(2\,bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
[Out] 1/2*((b^2*x^4 + 2*a*b*x^2 + a^2)^p*b*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a)
/(2*b*p + b)
```

maple [A] time = 0.00, size = 40, normalized size = 0.98

$$\frac{\left(b\,x^2+a\right)\left(b^2x^4+2ab\,x^2+a^2\right)^p}{2\left(2p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b/(1+2*p)
```

maxima [A] time = 1.32, size = 30, normalized size = 0.73

$$\frac{\left(bx^2+a\right)\left(bx^2+a\right)^{2p}}{2b\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)/(b*(2*p + 1))
```

mupad [B] time = 4.67, size = 46, normalized size = 1.12

$$\left(\frac{x^2}{2\,\left(2\,p+1\right)}+\frac{a}{2\,b\,\left(2\,p+1\right)}\right)\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (x^2/(2*(2*p + 1)) + a/(2*b*(2*p + 1)))*(a^2 + b^2*x^4 + 2*a*b*x^2)^p
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2}{2\sqrt{a^2}} & \text{for}\: b = 0 \wedge p = -\frac{1}{2} \\ \frac{x^2\left(a^2\right)^p}{2} & \text{for}\: b = 0 \\ \int \frac{x}{\sqrt{\left(a+bx^2\right)^2}}\, dx & \text{for}\: p = -\frac{1}{2} \\ \frac{a\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+2b} + \frac{bx^2\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**2/(2*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**2*(a**2)**p/2,
 Eq(b, 0)), (Integral(x/sqrt((a + b*x**2)**2), x), Eq(p, -1/2)), (a*(a**2 +
 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 2*b) + b*x**2*(a**2 + 2*a*b*x**2 + b**
2*x**4)**p/(4*b*p + 2*b), True))
```

3.617 $\int x^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2+cx^4\right) dx &= \int \left(ax^2+bx^4+cx^6\right) dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2 + cx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x^2*(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 1.05, size = 19, normalized size = 0.76

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/7*x^7*c + 1/5*x^5*b + 1/3*x^3*a
```

giac [A] time = 0.15, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{7}c\,x^7 + \frac{1}{5}b\,x^5 + \frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a),x)
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

maxima [A] time = 1.33, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^7}{7}+\frac{b\,x^5}{5}+\frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

sympy [A] time = 0.07, size = 19, normalized size = 0.76

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

3.618 $\int x\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)\,dx&=\int\left(ax+bx^3+cx^5\right)\,dx\\&=\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2+cx^4\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x*(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 1.05, size = 19, normalized size = 0.76

$$\frac{1}{6}x^6c+\frac{1}{4}x^4b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/6*x^6*c + 1/4*x^4*b + 1/2*x^2*a
```

giac [A] time = 0.15, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{6}c\,x^6+\frac{1}{4}b\,x^4+\frac{1}{2}a\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a),x)
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

maxima [A] time = 1.39, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^6}{6}+\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4),x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

sympy [A] time = 0.07, size = 19, normalized size = 0.76

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a),x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

3.619 $\int \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=20

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^2 + c*x^4,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right) dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^2 + c*x^4,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2+cx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + b*x^2 + c*x^4,x]
```

```
[Out] IntegrateAlgebraic[a + b*x^2 + c*x^4, x]
```

fricas [A] time = 2.47, size = 16, normalized size = 0.80

$$\frac{1}{5}x^5c + \frac{1}{3}x^3b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="fricas")
[Out] 1/5*x^5*c + 1/3*x^3*b + x*a
```

giac [A] time = 0.15, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="giac")
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{1}{5}c\,x^5 + \frac{1}{3}b\,x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^2+a,x)
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

maxima [A] time = 1.35, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="maxima")
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

mupad [B] time = 0.02, size = 16, normalized size = 0.80

$$\frac{c\,x^5}{5} + \frac{b\,x^3}{3} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^2 + c*x^4,x)
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

sympy [A] time = 0.06, size = 15, normalized size = 0.75

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**2+a,x)
```

```
[Out] a*x + b*x**3/3 + c*x**5/5
```

3.620 $\int \frac{a+bx^2+cx^4}{x} \, dx$

Optimal. Leaf size=21

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x} \, dx &= \int \left(\frac{a}{x}+bx+cx^3\right) \, dx\\ &= \frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x, x]
```

fricas [A] time = 2.11, size = 17, normalized size = 0.81

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="fricas")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

giac [A] time = 0.15, size = 20, normalized size = 0.95

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="giac")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{c\,x^4}{4}+\frac{b\,x^2}{2}+a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x,x)
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

maxima [A] time = 1.36, size = 20, normalized size = 0.95

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="maxima")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

mupad [B] time = 0.02, size = 17, normalized size = 0.81

$$\frac{b\,x^2}{2}+\frac{c\,x^4}{4}+a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x,x)
[Out] (b*x^2)/2 + (c*x^4)/4 + a*log(x)
```

sympy [A] time = 0.10, size = 17, normalized size = 0.81

$$a\log{(x)}+\frac{bx^{2}}{2}+\frac{cx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x,x)
[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

3.621 $\int \frac{a+bx^2+cx^4}{x^2} dx$

Optimal. Leaf size=18

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^2,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{x^2} \, dx &= \int \left(b+\frac{a}{x^2}+cx^2\right) dx \\ &= -\frac{a}{x}+bx+\frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^2,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^2,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^2, x]
```

fricas [A] time = 2.46, size = 20, normalized size = 1.11

$$\frac{cx^4 + 3\,bx^2 - 3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

giac [A] time = 0.15, size = 16, normalized size = 0.89

$$\frac{1}{3}\,cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
[Out] 1/3*c*x^3 + b*x - a/x
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{c\,x^3}{3} + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2,x)
[Out] -a/x+b*x+1/3*c*x^3
```

maxima [A] time = 1.39, size = 16, normalized size = 0.89

$$\frac{1}{3}\,cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

mupad [B] time = 0.03, size = 16, normalized size = 0.89

$$b\,x-\frac{a}{x}+\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^2,x)
```

```
[Out] b*x - a/x + (c*x^3)/3
```

sympy [A] time = 0.10, size = 12, normalized size = 0.67

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a/x + b*x + c*x**3/3
```

3.622 $\int \frac{a+bx^2+cx^4}{x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^3,x]
```

```
[Out] -a/(2*x^2) + (c*x^2)/2 + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^3}\,dx &= \int \left(\frac{a}{x^3}+\frac{b}{x}+cx\right)dx\\ &= -\frac{a}{2x^2}+\frac{cx^2}{2}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^3,x]
```

```
[Out] -1/2*a/x^2 + (c*x^2)/2 + b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^3,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^3, x]
```

fricas [A] time = 1.02, size = 22, normalized size = 1.05

$$\frac{cx^4+2\,bx^2\log(x)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + 2*b*x^2*log(x) - a)/x^2
```

giac [A] time = 0.15, size = 26, normalized size = 1.24

$$\frac{1}{2}\,cx^2+\frac{1}{2}\,b\log\left(x^2\right)-\frac{bx^2+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
[Out] 1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{c\,x^2}{2}+b\ln(x)-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^3,x)
[Out] -1/2*a/x^2+1/2*c*x^2+b*ln(x)
```

maxima [A] time = 1.34, size = 20, normalized size = 0.95

$$\frac{1}{2}\,cx^2+\frac{1}{2}\,b\log\left(x^2\right)-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*a/x^2
```

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{c\,x^2}{2}-\frac{a}{2\,x^2}+b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^3,x)
```

```
[Out] (c*x^2)/2 - a/(2*x^2) + b*log(x)
```

sympy [A] time = 0.13, size = 17, normalized size = 0.81

$$-\frac{a}{2x^2}+b\log{(x)}+\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x) + c*x**2/2
```

3.623 $\int \frac{a+bx^2+cx^4}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{a}{3x^3}-\frac{b}{x}+cx$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{b}{x}+cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/x + c*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^4}\,dx &= \int \left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)dx\\ &= -\frac{a}{3x^3}-\frac{b}{x}+cx\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b}{x}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/x + c*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^4,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^4, x]
```

fricas [A] time = 1.07, size = 21, normalized size = 1.17

$$\frac{3\,cx^4-3\,bx^2-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*c*x^4 - 3*b*x^2 - a)/x^3
```

giac [A] time = 0.15, size = 17, normalized size = 0.94

$$cx-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

maple [A] time = 0.01, size = 17, normalized size = 0.94

$$cx-\frac{b}{x}-\frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4,x)
[Out] -1/3*a/x^3-b/x+c*x
```

maxima [A] time = 1.37, size = 17, normalized size = 0.94

$$cx-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

mupad [B] time = 0.02, size = 18, normalized size = 1.00

$$c\,x-\frac{b\,x^2+\frac{a}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^4,x)
```

```
[Out] c*x - (a/3 + b*x^2)/x^3
```

sympy [A] time = 0.13, size = 17, normalized size = 0.94

$$cx+\frac{-a-3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] c*x + (-a - 3*b*x**2)/(3*x**3)
```

3.624 $\int \frac{a+bx^2+cx^4}{x^5} dx$

Optimal. Leaf size=21

$$-\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^5,x]
[Out] -a/(4*x^4) - b/(2*x^2) + c*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^5} dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^3}+\frac{c}{x}\right) dx\\ &= -\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^5,x]
[Out] -1/4*a/x^4 - b/(2*x^2) + c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^5,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^5, x]
```

fricas [A] time = 0.81, size = 23, normalized size = 1.10

$$\frac{4 \, c x^4 \log(x) - 2 \, b x^2 - a}{4 \, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
[Out] 1/4*(4*c*x^4*log(x) - 2*b*x^2 - a)/x^4
```

giac [A] time = 0.16, size = 27, normalized size = 1.29

$$\frac{1}{2} \, c \log\left(x^2\right) - \frac{3 \, c x^4 + 2 \, b x^2 + a}{4 \, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
[Out] 1/2*c*log(x^2) - 1/4*(3*c*x^4 + 2*b*x^2 + a)/x^4
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$c \ln(x) - \frac{b}{2x^2} - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^5,x)
[Out] -1/4*a/x^4-1/2*b/x^2+c*ln(x)
```

maxima [A] time = 1.31, size = 21, normalized size = 1.00

$$\frac{1}{2} \, c \log\left(x^2\right) - \frac{2 \, b x^2 + a}{4 \, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2) - 1/4*(2*b*x^2 + a)/x^4
```

mupad [B] time = 0.04, size = 20, normalized size = 0.95

$$c\,\ln(x)-\frac{\frac{b\,x^2}{2}+\frac{a}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^5,x)
```

```
[Out] c*log(x) - (a/4 + (b*x^2)/2)/x^4
```

sympy [A] time = 0.24, size = 19, normalized size = 0.90

$$c\log(x)+\frac{-a-2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] c*log(x) + (-a - 2*b*x**2)/(4*x**4)
```

3.625 $\int \frac{a+bx^2+cx^4}{x^6} dx$

Optimal. Leaf size=23

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(3*x^3) - c/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^6} dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^4}+\frac{c}{x^2}\right) dx\\ &= -\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/(3*x^3) - c/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^6} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^6,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^6, x]
```

fricas [A] time = 1.92, size = 21, normalized size = 0.91

$$-\frac{15 \, cx^4 + 5 \, bx^2 + 3 \, a}{15 \, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

giac [A] time = 0.15, size = 21, normalized size = 0.91

$$-\frac{15 \, cx^4 + 5 \, bx^2 + 3 \, a}{15 \, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$-\frac{c}{x} - \frac{b}{3x^3} - \frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6,x)
[Out] -1/5*a/x^5-1/3*b/x^3-c/x
```

maxima [A] time = 1.36, size = 21, normalized size = 0.91

$$-\frac{15 \, cx^4 + 5 \, bx^2 + 3 \, a}{15 \, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

mupad [B] time = 0.03, size = 20, normalized size = 0.87

$$-\frac{c\,x^4+\frac{b\,x^2}{3}+\frac{a}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^6,x)
```

```
[Out] -(a/5 + (b*x^2)/3 + c*x^4)/x^5
```

sympy [A] time = 0.26, size = 22, normalized size = 0.96

$$\frac{-3a-5bx^2-15cx^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] (-3*a - 5*b*x**2 - 15*c*x**4)/(15*x**5)
```

3.626 $\int \frac{a+bx^2+cx^4}{x^7}\,dx$

Optimal. Leaf size=25

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(4*x^4) - c/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^7}\,dx &= \int \left(\frac{a}{x^7}+\frac{b}{x^5}+\frac{c}{x^3}\right)dx\\ &= -\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -1/6*a/x^6 - b/(4*x^4) - c/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^7} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^7,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^7, x]
```

fricas [A] time = 1.77, size = 21, normalized size = 0.84

$$-\frac{6\,cx^4 + 3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

giac [A] time = 0.15, size = 21, normalized size = 0.84

$$-\frac{6\,cx^4 + 3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{c}{2x^2} - \frac{b}{4x^4} - \frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^7,x)
[Out] -1/6*a/x^6-1/4*b/x^4-1/2*c/x^2
```

maxima [A] time = 1.30, size = 21, normalized size = 0.84

$$-\frac{6\,cx^4 + 3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$-\frac{\frac{c\,x^4}{2}+\frac{b\,x^2}{4}+\frac{a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^7,x)
```

```
[Out] -(a/6 + (b*x^2)/4 + (c*x^4)/2)/x^6
```

sympy [A] time = 0.34, size = 22, normalized size = 0.88

$$\frac{-2a-3bx^2-6cx^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] (-2*a - 3*b*x**2 - 6*c*x**4)/(12*x**6)
```

3.627 $\int \frac{a+bx^2+cx^4}{x^8}\,dx$

Optimal. Leaf size=25

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^8,x]
[Out] -a/(7*x^7) - b/(5*x^5) - c/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^8}\,dx &= \int \left(\frac{a}{x^8}+\frac{b}{x^6}+\frac{c}{x^4}\right)dx\\ &= -\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^8,x]
[Out] -1/7*a/x^7 - b/(5*x^5) - c/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^8,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^8, x]
```

fricas [A] time = 0.67, size = 21, normalized size = 0.84

$$-\frac{35 \, cx^4 + 21 \, bx^2 + 15 \, a}{105 \, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="fricas")
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

giac [A] time = 0.17, size = 21, normalized size = 0.84

$$-\frac{35 \, cx^4 + 21 \, bx^2 + 15 \, a}{105 \, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="giac")
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{c}{3x^3} - \frac{b}{5x^5} - \frac{a}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8,x)
[Out] -1/7*a/x^7-1/5*b/x^5-1/3*c/x^3
```

maxima [A] time = 1.34, size = 21, normalized size = 0.84

$$-\frac{35 \, cx^4 + 21 \, bx^2 + 15 \, a}{105 \, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$-\frac{\frac{c\,x^4}{3}+\frac{b\,x^2}{5}+\frac{a}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^8,x)
```

```
[Out] -(a/7 + (b*x^2)/5 + (c*x^4)/3)/x^7
```

sympy [A] time = 0.32, size = 22, normalized size = 0.88

$$\frac{-15a-21bx^2-35cx^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8,x)
```

```
[Out] (-15*a - 21*b*x**2 - 35*c*x**4)/(105*x**7)
```

3.628 $\int x^2 \left(a + bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7 \left(2ac + b^2\right) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7 \left(2ac + b^2\right) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a + bx^2 + cx^4\right)^2 \, dx &= \int \left(a^2x^2 + 2abx^4 + \left(b^2 + 2ac\right)x^6 + 2bcx^8 + c^2x^{10}\right) dx \\ &= \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{1}{7}\left(b^2 + 2ac\right)x^7 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7 \left(2ac + b^2\right) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^2,x]
```

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*x^11)/11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^2+cx^4\right)^2\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^2*(a + b*x^2 + c*x^4)^2, x]

fricas [A] time = 1.03, size = 46, normalized size = 0.85

$$\frac{1}{11}x^{11}c^2+\frac{2}{9}x^9cb+\frac{1}{7}x^7b^2+\frac{2}{7}x^7ca+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/11*x^11*c^2 + 2/9*x^9*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 2/5*x^5*b*a + 1/3*x^3*a^2

giac [A] time = 0.15, size = 46, normalized size = 0.85

$$\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,b^2x^7+\frac{2}{7}\,acx^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{11}}{11}+\frac{2bc\,x^9}{9}+\frac{2ab\,x^5}{5}+\frac{\left(2ac+b^2\right)x^7}{7}+\frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^4+b*x^2+a)^2,x)

[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11

maxima [A] time = 1.31, size = 44, normalized size = 0.81

$$\frac{1}{11}c^2x^{11} + \frac{2}{9}bcx^9 + \frac{1}{7}\left(b^2 + 2ac\right)x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^2
*x^3
```

mupad [B] time = 0.03, size = 45, normalized size = 0.83

$$x^7\left(\frac{b^2}{7} + \frac{2\,a\,c}{7}\right) + \frac{a^2\,x^3}{3} + \frac{c^2\,x^{11}}{11} + \frac{2\,a\,b\,x^5}{5} + \frac{2\,b\,c\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^7*((2*a*c)/7 + b^2/7) + (a^2*x^3)/3 + (c^2*x^11)/11 + (2*a*b*x^5)/5 + (2*
b*c*x^9)/9
```

sympy [A] time = 0.08, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7 +
 b**2/7)
```

3.629 $\int x \left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 611}

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)^2\,dx&=\frac{1}{2}\text{Subst}\left(\int\left(a+bx+cx^2\right)^2\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^2+2abx+b^2\left(1+\frac{2ac}{b^2}\right)x^2+2bcx^3+c^2x^4\right)\,dx,x,x^2\right)\\&=\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2\left(30a^2+10x^4\left(2ac+b^2\right)+30abx^2+15bcx^6+6c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))
/60
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2+cx^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 2.30, size = 46, normalized size = 0.85

$$\frac{1}{10}x^{10}c^2+\frac{1}{4}x^8cb+\frac{1}{6}x^6b^2+\frac{1}{3}x^6ca+\frac{1}{2}x^4ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^2 + 1/4*x^8*c*b + 1/6*x^6*b^2 + 1/3*x^6*c*a + 1/2*x^4*b*a + 1/2
*x^2*a^2
```

giac [A] time = 0.16, size = 46, normalized size = 0.85

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,b^2x^6+\frac{1}{3}\,acx^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2
*a^2*x^2
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{10}}{10}+\frac{bc\,x^8}{4}+\frac{ab\,x^4}{2}+\frac{\left(2ac+b^2\right)x^6}{6}+\frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

maxima [A] time = 1.38, size = 44, normalized size = 0.81

$$\frac{1}{10} c^2 x^{10} + \frac{1}{4} bcx^8 + \frac{1}{6} \left(b^2 + 2 ac\right) x^6 + \frac{1}{2} abx^4 + \frac{1}{2} a^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^6 \left(\frac{b^2}{6} + \frac{a\,c}{3}\right) + \frac{a^2\,x^2}{2} + \frac{c^2\,x^{10}}{10} + \frac{a\,b\,x^4}{2} + \frac{b\,c\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^2)/2 + (c^2*x^10)/10 + (a*b*x^4)/2 + (b*c*x^
8)/4
```

sympy [A] time = 0.08, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{bcx^8}{4} + \frac{c^2x^{10}}{10} + x^6 \left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/
6)
```

3.630 $\int \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1090}

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right) dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 49, normalized size = 1.00

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2+cx^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2, x]
```

fricas [A] time = 1.14, size = 43, normalized size = 0.88

$$\frac{1}{9}x^9c^2+\frac{2}{7}x^7cb+\frac{1}{5}x^5b^2+\frac{2}{5}x^5ca+\frac{2}{3}x^3ba+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2 + 2/5*x^5*c*a + 2/3*x^3*b*a + x*a^2
```

giac [A] time = 0.15, size = 43, normalized size = 0.88

$$\frac{1}{9}c^2x^9+\frac{2}{7}bcx^7+\frac{1}{5}b^2x^5+\frac{2}{5}acx^5+\frac{2}{3}abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.00, size = 42, normalized size = 0.86

$$\frac{c^2x^9}{9}+\frac{2bc\,x^7}{7}+\frac{2ab\,x^3}{3}+\frac{\left(2ac+b^2\right)x^5}{5}+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2,x)
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

maxima [A] time = 1.33, size = 45, normalized size = 0.92

$$\frac{1}{9}c^2x^9+\frac{2}{7}bcx^7+\frac{1}{5}b^2x^5+a^2x+\frac{2}{15}\left(3\,cx^5+5\,bx^3\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)*
a
```

mupad [B] time = 0.02, size = 42, normalized size = 0.86

$$a^2\,x+x^5\,\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+\frac{c^2\,x^9}{9}+\frac{2\,a\,b\,x^3}{3}+\frac{2\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2,x)
```

```
[Out] a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7)
/7
```

sympy [A] time = 0.08, size = 48, normalized size = 0.98

$$a^2x+\frac{2abx^3}{3}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}+x^5\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

3.631 $\int \frac{(a+bx^2+cx^4)^2}{x} dx$

Optimal. Leaf size=47

$$a^2\log(x)+\frac{1}{4}x^4\left(2ac+b^2\right)+abx^2+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$a^2\log(x)+\frac{1}{4}x^4\left(2ac+b^2\right)+abx^2+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x,x]
```

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2+cx^4\right)^2}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+\left(b^2+2ac\right)x+2bcx^2+c^2x^3\right)dx,x,x^2\right)\\
&= abx^2+\frac{1}{4}\left(b^2+2ac\right)x^4+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}+a^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.00

$$a^2\log(x)+\frac{1}{4}x^4\left(2ac+b^2\right)+abx^2+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x,x]
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2+cx^4\right)^2}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x, x]
```

fricas [A] time = 0.57, size = 41, normalized size = 0.87

$$\frac{1}{8}c^2x^8+\frac{1}{3}bcx^6+\frac{1}{4}\left(b^2+2\,ac\right)x^4+abx^2+a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + a^2*log(x)
```

giac [A] time = 0.15, size = 46, normalized size = 0.98

$$\frac{1}{8}c^2x^8+\frac{1}{3}bcx^6+\frac{1}{4}b^2x^4+\frac{1}{2}acx^4+abx^2+\frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4 + 1/2*a*c*x^4 + a*b*x^2 + 1/2*a^2*l
og(x^2)
```

maple [A] time = 0.00, size = 44, normalized size = 0.94

$$\frac{c^2x^8}{8}+\frac{bc\,x^6}{3}+\frac{ac\,x^4}{2}+\frac{b^2x^4}{4}+ab\,x^2+a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x,x)
```

```
[Out] 1/8*c^2*x^8+1/3*b*c*x^6+1/2*x^4*a*c+1/4*b^2*x^4+a*b*x^2+a^2*ln(x)
```

maxima [A] time = 1.34, size = 44, normalized size = 0.94

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}\left(b^2 + 2\,ac\right)x^4 + abx^2 + \frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + 1/2*a^2*log(x
^2)
```

mupad [B] time = 0.02, size = 42, normalized size = 0.89

$$a^2\,\ln(x) + x^4\left(\frac{b^2}{4} + \frac{a\,c}{2}\right) + \frac{c^2\,x^8}{8} + a\,b\,x^2 + \frac{b\,c\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x,x)
```

```
[Out] a^2*log(x) + x^4*((a*c)/2 + b^2/4) + (c^2*x^8)/8 + a*b*x^2 + (b*c*x^6)/3
```

sympy [A] time = 0.14, size = 42, normalized size = 0.89

$$a^2\log{(x)} + abx^2 + \frac{bcx^6}{3} + \frac{c^2x^8}{8} + x^4\left(\frac{ac}{2} + \frac{b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b*c*x**6/3 + c**2*x**8/8 + x**4*(a*c/2 + b**2/4)
```

3.632 $\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^2}\,dx &= \int \left(2ab+\frac{a^2}{x^2}+\left(b^2+2ac\right)x^2+2bcx^4+c^2x^6\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{1}{3}\left(b^2+2ac\right)x^3+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^2}{x^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^2,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^2, x]
```

fricas [A] time = 0.72, size = 46, normalized size = 0.96

$$\frac{15\,c^2x^8 + 42\,bcx^6 + 35\,(b^2 + 2\,ac)x^4 + 210\,abx^2 - 105\,a^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
[Out] 1/105*(15*c^2*x^8 + 42*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 210*a*b*x^2 - 105*a
^2)/x
```

giac [A] time = 0.18, size = 44, normalized size = 0.92

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3 + \frac{2}{3}\,acx^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + 2*a*b*x - a^2/x
```

maple [A] time = 0.00, size = 45, normalized size = 0.94

$$\frac{c^2x^7}{7} + \frac{2bc\,x^5}{5} + \frac{2ac\,x^3}{3} + \frac{b^2x^3}{3} + 2abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^2,x)
[Out] 1/7*c^2*x^7+2/5*b*c*x^5+2/3*x^3*a*c+1/3*b^2*x^3+2*a*b*x-a^2/x
```

maxima [A] time = 1.22, size = 42, normalized size = 0.88

$$\frac{1}{7}\,c^2x^7+\frac{2}{5}\,bcx^5+\frac{1}{3}\,\left(b^2+2\,ac\right)x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*(b^2 + 2*a*c)*x^3 + 2*a*b*x - a^2/x
```

mupad [B] time = 0.02, size = 43, normalized size = 0.90

$$x^3\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)-\frac{a^2}{x}+\frac{c^2\,x^7}{7}+2\,a\,b\,x+\frac{2\,b\,c\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^2,x)
[Out] x^3*((2*a*c)/3 + b^2/3) - a^2/x + (c^2*x^7)/7 + 2*a*b*x + (2*b*c*x^5)/5
```

sympy [A] time = 0.14, size = 44, normalized size = 0.92

$$-\frac{a^2}{x}+2abx+\frac{2bcx^5}{5}+\frac{c^2x^7}{7}+x^3\left(\frac{2ac}{3}+\frac{b^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**2,x)
[Out] -a**2/x + 2*a*b*x + 2*b*c*x**5/5 + c**2*x**7/7 + x**3*(2*a*c/3 + b**2/3)
```

3.633 $\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2}{2x^2}+\frac{1}{2}x^2\left(2ac+b^2\right)+2ab\log(x)+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{2x^2}+\frac{1}{2}x^2\left(2ac+b^2\right)+2ab\log(x)+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + ((b^2 + 2*a*c)*x^2)/2 + (b*c*x^4)/2 + (c^2*x^6)/6 + 2*a*b*Lo
g[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(b^2\left(1+\frac{2ac}{b^2}\right)+\frac{a^2}{x^2}+\frac{2ab}{x}+2bcx+c^2x^2\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{2x^2}+\frac{1}{2}\left(b^2+2ac\right)x^2+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}+2ab\log(x)$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.90

$$\frac{1}{6}\left(-\frac{3a^2}{x^2}+3x^2\left(2ac+b^2\right)+12ab\log(x)+3bcx^4+c^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] ((-3*a^2)/x^2 + 3*(b^2 + 2*a*c)*x^2 + 3*b*c*x^4 + c^2*x^6 + 12*a*b*Log[x])/
6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^3, x]
```

fricas [A] time = 1.74, size = 47, normalized size = 0.92

$$\frac{c^2x^8+3\,bcx^6+3\left(b^2+2\,ac\right)x^4+12\,abx^2\log(x)-3\,a^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(c^2*x^8 + 3*b*c*x^6 + 3*(b^2 + 2*a*c)*x^4 + 12*a*b*x^2*log(x) - 3*a^2)
/x^2
```

giac [A] time = 0.15, size = 53, normalized size = 1.04

$$\frac{1}{6}c^2x^6 + \frac{1}{2}bcx^4 + \frac{1}{2}b^2x^2 + acx^2 + ab\log\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2 + a*c*x^2 + a*b*log(x^2) - 1/2*(2*a
*b*x^2 + a^2)/x^2
```

maple [A] time = 0.01, size = 45, normalized size = 0.88

$$\frac{c^2x^6}{6} + \frac{bc\,x^4}{2} + ac\,x^2 + \frac{b^2x^2}{2} + 2ab\ln(x) - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^3,x)
```

```
[Out] 1/6*c^2*x^6+1/2*b*c*x^4+x^2*a*c+1/2*b^2*x^2-1/2*a^2/x^2+2*a*b*ln(x)
```

maxima [A] time = 1.37, size = 44, normalized size = 0.86

$$\frac{1}{6}c^2x^6 + \frac{1}{2}bcx^4 + \frac{1}{2}\left(b^2 + 2\,ac\right)x^2 + ab\log\left(x^2\right) - \frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*(b^2 + 2*a*c)*x^2 + a*b*log(x^2) - 1/2*a^2/
x^2
```

mupad [B] time = 0.03, size = 43, normalized size = 0.84

$$x^2\left(\frac{b^2}{2} + a\,c\right) - \frac{a^2}{2\,x^2} + \frac{c^2\,x^6}{6} + 2\,a\,b\,\ln(x) + \frac{b\,c\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^3,x)
```

```
[Out] x^2*(a*c + b^2/2) - a^2/(2*x^2) + (c^2*x^6)/6 + 2*a*b*log(x) + (b*c*x^4)/2
```

sympy [A] time = 0.17, size = 44, normalized size = 0.86

$$-\frac{a^2}{2x^2} + 2ab\log\left(x\right) + \frac{bcx^4}{2} + \frac{c^2x^6}{6} + x^2\left(ac + \frac{b^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b*c*x**4/2 + c**2*x**6/6 + x**2*(a*c + b**2
/2)
```

3.634 $\int \frac{(a+bx^2+cx^4)^2}{x^4} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^4} dx &= \int \left(b^2\left(1+\frac{2ac}{b^2}\right)+\frac{a^2}{x^4}+\frac{2ab}{x^2}+2bcx^2+c^2x^4\right) dx\\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+\left(b^2+2ac\right)x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^4,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^4, x]
```

fricas [A] time = 1.94, size = 46, normalized size = 0.98

$$\frac{3\,c^2x^8+10\,bcx^6+15\,(b^2+2\,ac)x^4-30\,abx^2-5\,a^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
[Out] 1/15*(3*c^2*x^8 + 10*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 - 30*a*b*x^2 - 5*a^2)/x
^3
```

giac [A] time = 0.15, size = 42, normalized size = 0.89

$$\frac{1}{5}\,c^2x^5+\frac{2}{3}\,bcx^3+b^2x+2\,acx-\frac{6\,abx^2+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x + 2*a*c*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

maple [A] time = 0.01, size = 42, normalized size = 0.89

$$\frac{c^2x^5}{5}+\frac{2bc\,x^3}{3}+2acx+b^2x-\frac{2ab}{x}-\frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^4,x)
[Out] 1/5*c^2*x^5+2/3*b*c*x^3+2*a*c*x+b^2*x-2*a*b/x-1/3*a^2/x^3
```

maxima [A] time = 1.37, size = 42, normalized size = 0.89

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + \left(b^2 + 2\,ac\right)x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + (b^2 + 2*a*c)*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

mupad [B] time = 0.04, size = 44, normalized size = 0.94

$$x\,\left(b^2 + 2\,a\,c\right) - \frac{\frac{a^2}{3} + 2\,b\,a\,x^2}{x^3} + \frac{c^2\,x^5}{5} + \frac{2\,b\,c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^4,x)
```

```
[Out] x*(2*a*c + b^2) - (a^2/3 + 2*a*b*x^2)/x^3 + (c^2*x^5)/5 + (2*b*c*x^3)/3
```

sympy [A] time = 0.18, size = 46, normalized size = 0.98

$$\frac{2bcx^3}{3} + \frac{c^2x^5}{5} + x\left(2ac + b^2\right) + \frac{-a^2 - 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*b*c*x**3/3 + c**2*x**5/5 + x*(2*a*c + b**2) + (-a**2 - 6*a*b*x**2)/(3*x**
3)
```

3.635 $\int \frac{(a+bx^2+cx^4)^2}{x^5}\,dx$

Optimal. Leaf size=45

$$-\frac{a^2}{4x^4}+\log(x)\left(2ac+b^2\right)-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}$$

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{4x^4}+\log(x)\left(2ac+b^2\right)-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b*c*x^2 + (c^2*x^4)/4 + (b^2 + 2*a*c)*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2+cx^4)^2}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(2bc+\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2+2ac}{x}+c^2x\right)dx,x,x^2\right)\\
&= -\frac{a^2}{4x^4}-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}+\left(b^2+2ac\right)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.91

$$\log(x)\left(2ac+b^2\right)+\frac{\left(cx^4-a\right)\left(a+4bx^2+cx^4\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] ((-a + c*x^4)*(a + 4*b*x^2 + c*x^4))/(4*x^4) + (b^2 + 2*a*c)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2+cx^4\right)^2}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^5, x]
```

fricas [A] time = 1.01, size = 47, normalized size = 1.04

$$\frac{c^2x^8+4\,bcx^6+4\left(b^2+2\,ac\right)x^4\log(x)-4\,abx^2-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*x^8 + 4*b*c*x^6 + 4*(b^2 + 2*a*c)*x^4*log(x) - 4*a*b*x^2 - a^2)/x^
4
```

giac [A] time = 0.18, size = 60, normalized size = 1.33

$$\frac{1}{4}\,c^2x^4+bcx^2+\frac{1}{2}\left(b^2+2\,ac\right)\log\left(x^2\right)-\frac{3\,b^2x^4+6\,acx^4+4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(3*b^2*x^4 + 6*a*c
*x^4 + 4*a*b*x^2 + a^2)/x^4
```

maple [A] time = 0.01, size = 43, normalized size = 0.96

$$\frac{c^2x^4}{4}+bc\,x^2+2ac\ln(x)+b^2\ln(x)-\frac{ab}{x^2}-\frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^5,x)
```

```
[Out] 1/4*c^2*x^4+b*c*x^2-a*b/x^2-1/4*a^2/x^4+2*ln(x)*a*c+b^2*ln(x)
```

maxima [A] time = 1.34, size = 45, normalized size = 1.00

$$\frac{1}{4}\,c^2x^4+bcx^2+\frac{1}{2}\left(b^2+2\,ac\right)\log\left(x^2\right)-\frac{4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/
x^4
```

mupad [B] time = 0.04, size = 43, normalized size = 0.96

$$\ln(x)\,\left(b^2+2\,a\,c\right)-\frac{\frac{a^2}{4}+b\,a\,x^2}{x^4}+\frac{c^2\,x^4}{4}+b\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^5,x)
```

```
[Out] log(x)*(2*a*c + b^2) - (a^2/4 + a*b*x^2)/x^4 + (c^2*x^4)/4 + b*c*x^2
```

sympy [A] time = 0.37, size = 44, normalized size = 0.98

$$bcx^2+\frac{c^2x^4}{4}+\left(2ac+b^2\right)\log\left(x\right)+\frac{-a^2-4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] b*c*x**2 + c**2*x**4/4 + (2*a*c + b**2)*log(x) + (-a**2 - 4*a*b*x**2)/(4*x*
*4)
```

3.636 $\int \frac{(a+bx^2+cx^4)^2}{x^6} dx$

Optimal. Leaf size=48

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - (b^2 + 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^6} dx &= \int \left(2bc+\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2+2ac}{x^2}+c^2x^2\right) dx\\ &= -\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2+2ac}{x}+2bcx+\frac{c^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.02

$$-\frac{a^2}{5x^5}+\frac{-2ac-b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) + (-b^2 - 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^2}{x^6} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^6, x]
```

fricas [A] time = 1.68, size = 46, normalized size = 0.96

$$\frac{5\,c^2x^8 + 30\,bcx^6 - 15\,(b^2 + 2\,ac)x^4 - 10\,abx^2 - 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(5*c^2*x^8 + 30*b*c*x^6 - 15*(b^2 + 2*a*c)*x^4 - 10*a*b*x^2 - 3*a^2)/x
^5
```

giac [A] time = 0.15, size = 47, normalized size = 0.98

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{15\,b^2x^4 + 30\,acx^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*b^2*x^4 + 30*a*c*x^4 + 10*a*b*x^2 + 3*a^2)
/x^5
```

maple [A] time = 0.01, size = 43, normalized size = 0.90

$$\frac{c^2x^3}{3} + 2bcx - \frac{2ab}{3x^3} - \frac{2ac + b^2}{x} - \frac{a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^6,x)
```

```
[Out] 1/3*c^2*x^3+2*b*c*x-1/5*a^2/x^5-(2*a*c+b^2)/x-2/3*a*b/x^3
```

maxima [A] time = 1.32, size = 45, normalized size = 0.94

$$\frac{1}{3}\,c^2x^3+2\,bcx-\frac{15\left(b^2+2\,ac\right)x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*(b^2 + 2*a*c)*x^4 + 10*a*b*x^2 + 3*a^2)/x^
5
```

mupad [B] time = 0.04, size = 44, normalized size = 0.92

$$\frac{c^2\,x^3}{3}-\frac{x^4\,\left(b^2+2\,a\,c\right)+\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}}{x^5}+2\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^6,x)
```

```
[Out] (c^2*x^3)/3 - (x^4*(2*a*c + b^2) + a^2/5 + (2*a*b*x^2)/3)/x^5 + 2*b*c*x
```

sympy [A] time = 0.43, size = 48, normalized size = 1.00

$$2bcx+\frac{c^2x^3}{3}+\frac{-3a^2-10abx^2+x^4\left(-30ac-15b^2\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] 2*b*c*x + c**2*x**3/3 + (-3*a**2 - 10*a*b*x**2 + x**4*(-30*a*c - 15*b**2))/
(15*x**5)
```

3.637 $\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{2x^2}-\frac{ab}{2x^4}+2bc\log(x)+\frac{c^2x^2}{2}$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{2x^2}-\frac{ab}{2x^4}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - (b^2 + 2*a*c)/(2*x^2) + (c^2*x^2)/2 + 2*b*c*
Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(c^2+\frac{a^2}{x^4}+\frac{2ab}{x^3}+\frac{b^2+2ac}{x^2}+\frac{2bc}{x}\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2+2ac}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.98

$$-\frac{a^2+3abx^2+6acx^4+3b^2x^4-12bcx^6\log(x)-3c^2x^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -1/6*(a^2 + 3*a*b*x^2 + 3*b^2*x^4 + 6*a*c*x^4 - 3*c^2*x^8 - 12*b*c*x^6*Log[
x])/x^6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^7, x]
```

fricas [A] time = 0.56, size = 48, normalized size = 0.94

$$\frac{3\,c^2x^8+12\,bcx^6\log(x)-3\left(b^2+2\,ac\right)x^4-3\,abx^2-a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*c^2*x^8 + 12*b*c*x^6*log(x) - 3*(b^2 + 2*a*c)*x^4 - 3*a*b*x^2 - a^2)
/x^6
```

giac [A] time = 0.16, size = 54, normalized size = 1.06

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{11\,bcx^6 + 3\,b^2x^4 + 6\,acx^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(11*b*c*x^6 + 3*b^2*x^4 + 6*a*c*x^4 + 3*a*
b*x^2 + a^2)/x^6
```

maple [A] time = 0.01, size = 46, normalized size = 0.90

$$\frac{c^2x^2}{2} + 2bc\ln(x) - \frac{ac}{x^2} - \frac{b^2}{2x^2} - \frac{ab}{2x^4} - \frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^7,x)
```

```
[Out] 1/2*c^2*x^2-1/x^2*a*c-1/2*b^2/x^2-1/6*a^2/x^6-1/2*a*b/x^4+2*b*c*ln(x)
```

maxima [A] time = 1.34, size = 45, normalized size = 0.88

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{3\left(b^2 + 2\,ac\right)x^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(3*(b^2 + 2*a*c)*x^4 + 3*a*b*x^2 + a^2)/x^
6
```

mupad [B] time = 4.14, size = 46, normalized size = 0.90

$$\frac{c^2\,x^2}{2} - \frac{\frac{a^2}{6} + x^4\,\left(\frac{b^2}{2} + a\,c\right) + \frac{a\,b\,x^2}{2}}{x^6} + 2\,b\,c\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^7,x)
```

```
[Out] (c^2*x^2)/2 - (a^2/6 + x^4*(a*c + b^2/2) + (a*b*x^2)/2)/x^6 + 2*b*c*log(x)
```

sympy [A] time = 0.78, size = 48, normalized size = 0.94

$$2bc\log(x) + \frac{c^2x^2}{2} + \frac{-a^2 - 3abx^2 + x^4\left(-6ac - 3b^2\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*b*c*log(x) + c**2*x**2/2 + (-a**2 - 3*a*b*x**2 + x**4*(-6*a*c - 3*b**2))/
(6*x**6)
```

3.638 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^8} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^8,x]
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - (b^2 + 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^8} dx &= \int \left(c^2+\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2+2ac}{x^4}+\frac{2bc}{x^2}\right) dx\\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2+2ac}{3x^3}-\frac{2bc}{x}+c^2x\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.04

$$-\frac{a^2}{7x^7}+\frac{-2ac-b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^8,x]
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) + (-b^2 - 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^2}{x^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^8, x]
```

fricas [A] time = 1.53, size = 46, normalized size = 0.98

$$\frac{105\,c^2x^8 - 210\,bcx^6 - 35\left(b^2 + 2\,ac\right)x^4 - 42\,abx^2 - 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*c^2*x^8 - 210*b*c*x^6 - 35*(b^2 + 2*a*c)*x^4 - 42*a*b*x^2 - 15*a
^2)/x^7
```

giac [A] time = 0.17, size = 46, normalized size = 0.98

$$c^2x - \frac{210\,bcx^6 + 35\,b^2x^4 + 70\,acx^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="giac")
```

```
[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*b^2*x^4 + 70*a*c*x^4 + 42*a*b*x^2 + 15*a^2)
/x^7
```

maple [A] time = 0.01, size = 42, normalized size = 0.89

$$c^2x - \frac{2bc}{x} - \frac{2ab}{5x^5} - \frac{2ac + b^2}{3x^3} - \frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^8,x)
```

```
[Out] c^2*x-1/7*a^2/x^7-2/5*a*b/x^5-2*b*c/x-1/3*(2*a*c+b^2)/x^3
```

maxima [A] time = 1.34, size = 44, normalized size = 0.94

$$c^2x - \frac{210\,bcx^6 + 35\left(b^2 + 2\,ac\right)x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="maxima")
```

```
[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 42*a*b*x^2 + 15*a^2)/x^
7
```

mupad [B] time = 4.17, size = 45, normalized size = 0.96

$$c^2\,x - \frac{\frac{a^2}{7} + x^4\,\left(\frac{b^2}{3} + \frac{2\,a\,c}{3}\right) + \frac{2\,a\,b\,x^2}{5} + 2\,b\,c\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^8,x)
```

```
[Out] c^2*x - (a^2/7 + x^4*((2*a*c)/3 + b^2/3) + (2*a*b*x^2)/5 + 2*b*c*x^6)/x^7
```

sympy [A] time = 0.76, size = 46, normalized size = 0.98

$$c^2x + \frac{-15a^2 - 42abx^2 - 210bcx^6 + x^4\left(-70ac - 35b^2\right)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**8,x)
```

```
[Out] c**2*x + (-15*a**2 - 42*a*b*x**2 - 210*b*c*x**6 + x**4*(-70*a*c - 35*b**2))
/(105*x**7)
```

3.639 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^9} dx$

Optimal. Leaf size=48

$$-\frac{a^2}{8x^8}-\frac{2ac+b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{8x^8}-\frac{2ac+b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - (b^2 + 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Log[
x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^5}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2+2ac}{x^3}+\frac{2bc}{x^2}+\frac{c^2}{x}\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2+2ac}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.04

$$-\frac{a^2}{8x^8}+\frac{-2ac-b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^9,x]
[Out] -1/8*a^2/x^8 - (a*b)/(3*x^6) + (-b^2 - 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Log
[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^9,x]
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^9, x]
```

fricas [A] time = 1.23, size = 48, normalized size = 1.00

$$\frac{24\,c^2x^8\log(x)-24\,bcx^6-6\,(b^2+2\,ac)x^4-8\,abx^2-3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="fricas")
[Out] 1/24*(24*c^2*x^8*log(x) - 24*b*c*x^6 - 6*(b^2 + 2*a*c)*x^4 - 8*a*b*x^2 - 3*
a^2)/x^8
```

giac [A] time = 0.15, size = 58, normalized size = 1.21

$$\frac{1}{2}\,c^{2}\log\left(x^{2}\right)-\frac{25\,c^{2}x^{8}+24\,bcx^{6}+6\,b^{2}x^{4}+12\,acx^{4}+8\,abx^{2}+3\,a^{2}}{24\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2) - 1/24*(25*c^2*x^8 + 24*b*c*x^6 + 6*b^2*x^4 + 12*a*c*x^4 +
 8*a*b*x^2 + 3*a^2)/x^8
```

maple [A] time = 0.01, size = 45, normalized size = 0.94

$$c^{2}\ln(x)-\frac{bc}{x^{2}}-\frac{ac}{2x^{4}}-\frac{b^{2}}{4x^{4}}-\frac{ab}{3x^{6}}-\frac{a^{2}}{8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^9,x)
```

```
[Out] -b*c/x^2-1/8*a^2/x^8-1/3*a*b/x^6-1/2/x^4*a*c-1/4*b^2/x^4+c^2*ln(x)
```

maxima [A] time = 1.36, size = 48, normalized size = 1.00

$$\frac{1}{2}\,c^{2}\log\left(x^{2}\right)-\frac{24\,bcx^{6}+6\left(b^{2}+2\,ac\right)x^{4}+8\,abx^{2}+3\,a^{2}}{24\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2) - 1/24*(24*b*c*x^6 + 6*(b^2 + 2*a*c)*x^4 + 8*a*b*x^2 + 3*a
^2)/x^8
```

mupad [B] time = 4.18, size = 45, normalized size = 0.94

$$c^{2}\,\ln(x)-\frac{\frac{a^{2}}{8}+x^{4}\left(\frac{b^{2}}{4}+\frac{a\,c}{2}\right)+\frac{a\,b\,x^{2}}{3}+b\,c\,x^{6}}{x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^9,x)
```

```
[Out] c^2*log(x) - (a^2/8 + x^4*((a*c)/2 + b^2/4) + (a*b*x^2)/3 + b*c*x^6)/x^8
```

sympy [A] time = 1.31, size = 48, normalized size = 1.00

$$c^2\log(x) + \frac{-3a^2 - 8abx^2 - 24bcx^6 + x^4\left(-12ac - 6b^2\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**9,x)
```

```
[Out] c**2*log(x) + (-3*a**2 - 8*a*b*x**2 - 24*b*c*x**6 + x**4*(-12*a*c - 6*b**2)
)/(24*x**8)
```

3.640 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=52

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - (b^2 + 2*a*c)/(5*x^5) - (2*b*c)/(3*x^3) -
c^2/x
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^8}+\frac{b^2+2ac}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2+2ac}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.96

$$-\frac{35a^2+90abx^2+126acx^4+63b^2x^4+210bcx^6+315c^2x^8}{315x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -1/315*(35*a^2 + 90*a*b*x^2 + 63*b^2*x^4 + 126*a*c*x^4 + 210*b*c*x^6 + 315*
c^2*x^8)/x^9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^10, x]
```

fricas [A] time = 2.10, size = 46, normalized size = 0.88

$$-\frac{315\,c^2x^8+210\,bcx^6+63\,(b^2+2\,ac)x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35*
a^2)/x^9
```

giac [A] time = 0.15, size = 48, normalized size = 0.92

$$-\frac{315\,c^2x^8+210\,bcx^6+63\,b^2x^4+126\,acx^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*b^2*x^4 + 126*a*c*x^4 + 90*a*b*x^2 +
 35*a^2)/x^9
```

maple [A] time = 0.00, size = 45, normalized size = 0.87

$$-\frac{c^2}{x}-\frac{2bc}{3x^3}-\frac{2ab}{7x^7}-\frac{2ac+b^2}{5x^5}-\frac{a^2}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^10,x)
```

```
[Out] -2/7*a*b/x^7-1/9*a^2/x^9-1/5*(2*a*c+b^2)/x^5-c^2/x-2/3*b*c/x^3
```

maxima [A] time = 1.33, size = 46, normalized size = 0.88

$$-\frac{315\,c^{2}x^{8}+210\,bcx^{6}+63\,\left(b^{2}+2\,ac\right)x^{4}+90\,abx^{2}+35\,a^{2}}{315\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35*
a^2)/x^9
```

mupad [B] time = 0.03, size = 46, normalized size = 0.88

$$-\frac{\frac{a^{2}}{9}+x^{4}\,\left(\frac{b^{2}}{5}+\frac{2\,a\,c}{5}\right)+c^{2}\,x^{8}+\frac{2\,a\,b\,x^{2}}{7}+\frac{2\,b\,c\,x^{6}}{3}}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^10,x)
```

```
[Out] -(a^2/9 + x^4*((2*a*c)/5 + b^2/5) + c^2*x^8 + (2*a*b*x^2)/7 + (2*b*c*x^6)/3
)/x^9
```

sympy [A] time = 1.47, size = 49, normalized size = 0.94

$$\frac{-35a^{2}-90abx^{2}-210bcx^{6}-315c^{2}x^{8}+x^{4}\left(-126ac-63b^{2}\right)}{315x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**10,x)
```

```
[Out] (-35*a**2 - 90*a*b*x**2 - 210*b*c*x**6 - 315*c**2*x**8 + x**4*(-126*a*c - 6
3*b**2))/(315*x**9)
```

3.641 $\int \frac{(a+bx^2+cx^4)^2}{x^{11}} dx$

Optimal. Leaf size=54

$$-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -a^2/(10*x^10) - (a*b)/(4*x^8) - (b^2 + 2*a*c)/(6*x^6) - (b*c)/(2*x^4) - c^
2/(2*x^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2+2ac}{x^4}+\frac{2bc}{x^3}+\frac{c^2}{x^2}\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{10x^{10}}-\frac{ab}{4x^8}-\frac{b^2+2ac}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.98

$$-\frac{6a^2+5a(3bx^2+4cx^4)+10x^4(b^2+3bcx^2+3c^2x^4)}{60x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -1/60*(6*a^2 + 5*a*(3*b*x^2 + 4*c*x^4) + 10*x^4*(b^2 + 3*b*c*x^2 + 3*c^2*x^
4))/x^10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^11, x]
```

fricas [A] time = 1.77, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+30\,bcx^6+10\,(b^2+2\,ac)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2)
/x^10
```

giac [A] time = 0.22, size = 48, normalized size = 0.89

$$-\frac{30\,c^{2}x^{8}+30\,bcx^{6}+10\,b^{2}x^{4}+20\,acx^{4}+15\,abx^{2}+6\,a^{2}}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*b^2*x^4 + 20*a*c*x^4 + 15*a*b*x^2 + 6*a
^2)/x^10
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$-\frac{c^{2}}{2x^{2}}-\frac{bc}{2x^{4}}-\frac{ab}{4x^{8}}-\frac{2ac+b^{2}}{6x^{6}}-\frac{a^{2}}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^11,x)
```

```
[Out] -1/2*c^2/x^2-1/6*(2*a*c+b^2)/x^6-1/10*a^2/x^10-1/2*b*c/x^4-1/4*a*b/x^8
```

maxima [A] time = 1.28, size = 46, normalized size = 0.85

$$-\frac{30\,c^{2}x^{8}+30\,bcx^{6}+10\left(b^{2}+2\,ac\right)x^{4}+15\,abx^{2}+6\,a^{2}}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2)
/x^10
```

mupad [B] time = 4.12, size = 47, normalized size = 0.87

$$-\frac{\frac{a^{2}}{10}+x^{4}\left(\frac{b^{2}}{6}+\frac{a\,c}{3}\right)+\frac{c^{2}\,x^{8}}{2}+\frac{a\,b\,x^{2}}{4}+\frac{b\,c\,x^{6}}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^11,x)
```

```
[Out] -(a^2/10 + x^4*((a*c)/3 + b^2/6) + (c^2*x^8)/2 + (a*b*x^2)/4 + (b*c*x^6)/2)
/x^10
```

sympy [A] time = 2.08, size = 49, normalized size = 0.91

$$\frac{-6a^2 - 15abx^2 - 30bcx^6 - 30c^2x^8 + x^4(-20ac - 10b^2)}{60x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**11,x)
```

```
[Out] (-6*a**2 - 15*a*b*x**2 - 30*b*c*x**6 - 30*c**2*x**8 + x**4*(-20*a*c - 10*b*
*2))/(60*x**10)
```

3.642 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{12}} \, dx$

Optimal. Leaf size=54

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -a^2/(11*x^11) - (2*a*b)/(9*x^9) - (b^2 + 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
- c^2/(3*x^3)
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{12}} \, dx &= \int \left(\frac{a^2}{x^{12}}+\frac{2ab}{x^{10}}+\frac{b^2+2ac}{x^8}+\frac{2bc}{x^6}+\frac{c^2}{x^4}\right) dx\\ &= -\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}-\frac{b^2+2ac}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.04

$$-\frac{a^2}{11x^{11}}+\frac{-2ac-b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -1/11*a^2/x^11 - (2*a*b)/(9*x^9) + (-b^2 - 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
 - c^2/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^12, x]
```

fricas [A] time = 0.94, size = 46, normalized size = 0.85

$$-\frac{1155\,c^2x^8+1386\,bcx^6+495\left(b^2+2\,ac\right)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
+ 315*a^2)/x^11
```

giac [A] time = 0.19, size = 48, normalized size = 0.89

$$-\frac{1155\,c^2x^8+1386\,bcx^6+495\,b^2x^4+990\,acx^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*b^2*x^4 + 990*a*c*x^4 + 770*a*b*
x^2 + 315*a^2)/x^11
```

maple [A] time = 0.01, size = 45, normalized size = 0.83

$$-\frac{c^2}{3x^3}-\frac{2bc}{5x^5}-\frac{2ab}{9x^9}-\frac{2ac+b^2}{7x^7}-\frac{a^2}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^12,x)
```

```
[Out] -1/7*(2*a*c+b^2)/x^7-1/11*a^2/x^11-2/5*b*c/x^5-1/3*c^2/x^3-2/9*a*b/x^9
```

maxima [A] time = 1.38, size = 46, normalized size = 0.85

$$-\frac{1155\,c^2x^8+1386\,bcx^6+495\,\left(b^2+2\,ac\right)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
+ 315*a^2)/x^11
```

mupad [B] time = 4.16, size = 47, normalized size = 0.87

$$-\frac{\frac{a^2}{11}+x^4\,\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{c^2\,x^8}{3}+\frac{2\,a\,b\,x^2}{9}+\frac{2\,b\,c\,x^6}{5}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^12,x)
```

```
[Out] -(a^2/11 + x^4*((2*a*c)/7 + b^2/7) + (c^2*x^8)/3 + (2*a*b*x^2)/9 + (2*b*c*x
^6)/5)/x^11
```

sympy [A] time = 1.96, size = 49, normalized size = 0.91

$$\frac{-315a^2-770abx^2-1386bcx^6-1155c^2x^8+x^4\left(-990ac-495b^2\right)}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**12,x)
```

```
[Out] (-315*a**2 - 770*a*b*x**2 - 1386*b*c*x**6 - 1155*c**2*x**8 + x**4*(-990*a*c
 - 495*b**2))/(3465*x**11)
```

3.643 $\int \frac{(a+bx^2+cx^4)^2}{x^{13}} dx$

Optimal. Leaf size=54

$$-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -a^2/(12*x^12) - (a*b)/(5*x^10) - (b^2 + 2*a*c)/(8*x^8) - (b*c)/(3*x^6) - c
^2/(4*x^4)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^7}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2+2ac}{x^5}+\frac{2bc}{x^4}+\frac{c^2}{x^3}\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{12x^{12}}-\frac{ab}{5x^{10}}-\frac{b^2+2ac}{8x^8}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.93

$$-\frac{10a^2+24abx^2+30acx^4+15b^2x^4+40bcx^6+30c^2x^8}{120x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -1/120*(10*a^2 + 24*a*b*x^2 + 15*b^2*x^4 + 30*a*c*x^4 + 40*b*c*x^6 + 30*c^2
*x^8)/x^12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^13, x]
```

fricas [A] time = 0.81, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+40\,bcx^6+15\,(b^2+2\,ac)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a^
2)/x^12
```

giac [A] time = 0.15, size = 48, normalized size = 0.89

$$-\frac{30\,c^{2}x^{8}+40\,bcx^{6}+15\,b^{2}x^{4}+30\,acx^{4}+24\,abx^{2}+10\,a^{2}}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*b^2*x^4 + 30*a*c*x^4 + 24*a*b*x^2 + 10
*a^2)/x^12
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$-\frac{c^{2}}{4x^{4}}-\frac{bc}{3x^{6}}-\frac{ab}{5x^{10}}-\frac{2ac+b^{2}}{8x^{8}}-\frac{a^{2}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^13,x)
```

```
[Out] -1/3*b*c/x^6-1/12*a^2/x^12-1/5*a*b/x^10-1/8*(2*a*c+b^2)/x^8-1/4*c^2/x^4
```

maxima [A] time = 1.34, size = 46, normalized size = 0.85

$$-\frac{30\,c^{2}x^{8}+40\,bcx^{6}+15\,\left(b^{2}+2\,ac\right)x^{4}+24\,abx^{2}+10\,a^{2}}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="maxima")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a^
2)/x^12
```

mupad [B] time = 4.16, size = 47, normalized size = 0.87

$$-\frac{\frac{a^{2}}{12}+x^{4}\left(\frac{b^{2}}{8}+\frac{a\,c}{4}\right)+\frac{c^{2}\,x^{8}}{4}+\frac{a\,b\,x^{2}}{5}+\frac{b\,c\,x^{6}}{3}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^13,x)
```

```
[Out] -(a^2/12 + x^4*((a*c)/4 + b^2/8) + (c^2*x^8)/4 + (a*b*x^2)/5 + (b*c*x^6)/3)
/x^12
```

sympy [A] time = 2.70, size = 49, normalized size = 0.91

$$\frac{-10a^2 - 24abx^2 - 40bcx^6 - 30c^2x^8 + x^4\left(-30ac - 15b^2\right)}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**13,x)
```

```
[Out] (-10*a**2 - 24*a*b*x**2 - 40*b*c*x**6 - 30*c**2*x**8 + x**4*(-30*a*c - 15*b
**2))/(120*x**12)
```

3.644 $\int x^2 \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=89

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)*
x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3x^2+3a^2bx^4+3a\left(b^2+ac\right)x^6+b\left(b^2+6ac\right)x^8+3c\left(b^2+ac\right)x^{10}+3bc^2x^{12}+\right.\\ &= \frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}a\left(b^2+ac\right)x^7+\frac{1}{9}b\left(b^2+6ac\right)x^9+\frac{3}{11}c\left(b^2+ac\right)x^{11}+\frac{3}{13}bc^2x^{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 89, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)*
x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2 + cx^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 1.26, size = 87, normalized size = 0.98

$$\frac{1}{15}x^{15}c^3 + \frac{3}{13}x^{13}c^2b + \frac{3}{11}x^{11}cb^2 + \frac{3}{11}x^{11}c^2a + \frac{1}{9}x^9b^3 + \frac{2}{3}x^9cba + \frac{3}{7}x^7b^2a + \frac{3}{7}x^7ca^2 + \frac{3}{5}x^5ba^2 + \frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*c^3 + 3/13*x^13*c^2*b + 3/11*x^11*c*b^2 + 3/11*x^11*c^2*a + 1/9*x
^9*b^3 + 2/3*x^9*c*b*a + 3/7*x^7*b^2*a + 3/7*x^7*c*a^2 + 3/5*x^5*b*a^2 + 1/
3*x^3*a^3
```

giac [A] time = 0.19, size = 87, normalized size = 0.98

$$\frac{1}{15}\,c^3x^{15} + \frac{3}{13}\,bc^2x^{13} + \frac{3}{11}\,b^2cx^{11} + \frac{3}{11}\,ac^2x^{11} + \frac{1}{9}\,b^3x^9 + \frac{2}{3}\,abcx^9 + \frac{3}{7}\,ab^2x^7 + \frac{3}{7}\,a^2cx^7 + \frac{3}{5}\,a^2bx^5 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*b^2*c*x^11 + 3/11*a*c^2*x^11 + 1/9*b
^3*x^9 + 2/3*a*b*c*x^9 + 3/7*a*b^2*x^7 + 3/7*a^2*c*x^7 + 3/5*a^2*b*x^5 + 1/
3*a^3*x^3
```

maple [A] time = 0.00, size = 111, normalized size = 1.25

$$\frac{c^3x^{15}}{15} + \frac{3b\,c^2x^{13}}{13} + \frac{\left(a\,c^2 + 2b^2c + \left(2ac + b^2\right)c\right)x^{11}}{11} + \frac{\left(4abc + \left(2ac + b^2\right)b\right)x^9}{9} + \frac{3a^2b\,x^5}{5} + \frac{\left(a^2c + 2a\,b^2 + \left(2ac + b^2\right)a\right)x^7}{7} + \frac{a^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/15*c^3*x^15+3/13*b*c^2*x^13+1/11*(a*c^2+2*b^2*c+(2*a*c+b^2)*c)*x^11+1/9*(
4*a*b*c+(2*a*c+b^2)*b)*x^9+1/7*(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*x^7+3/5*a^2*b*
x^5+1/3*a^3*x^3
```

maxima [A] time = 1.39, size = 81, normalized size = 0.91

$$\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\left(b^2c+ac^2\right)x^{11}+\frac{1}{9}\left(b^3+6\,abc\right)x^9+\frac{3}{5}\,a^2bx^5+\frac{3}{7}\left(ab^2+a^2c\right)x^7+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*(b^2*c + a*c^2)*x^11 + 1/9*(b^3 + 6*
a*b*c)*x^9 + 3/5*a^2*b*x^5 + 3/7*(a*b^2 + a^2*c)*x^7 + 1/3*a^3*x^3
```

mupad [B] time = 0.03, size = 76, normalized size = 0.85

$$x^9\left(\frac{b^3}{9}+\frac{2\,a\,c\,b}{3}\right)+\frac{a^3\,x^3}{3}+\frac{c^3\,x^{15}}{15}+\frac{3\,a^2\,b\,x^5}{5}+\frac{3\,b\,c^2\,x^{13}}{13}+\frac{3\,a\,x^7\,\left(b^2+a\,c\right)}{7}+\frac{3\,c\,x^{11}\,\left(b^2+a\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^9*(b^3/9 + (2*a*b*c)/3) + (a^3*x^3)/3 + (c^3*x^15)/15 + (3*a^2*b*x^5)/5 +
 (3*b*c^2*x^13)/13 + (3*a*x^7*(a*c + b^2))/7 + (3*c*x^11*(a*c + b^2))/11
```

sympy [A] time = 0.09, size = 97, normalized size = 1.09

$$\frac{a^3x^3}{3}+\frac{3a^2bx^5}{5}+\frac{3bc^2x^{13}}{13}+\frac{c^3x^{15}}{15}+x^{11}\left(\frac{3ac^2}{11}+\frac{3b^2c}{11}\right)+x^9\left(\frac{2abc}{3}+\frac{b^3}{9}\right)+x^7\left(\frac{3a^2c}{7}+\frac{3ab^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*b*c**2*x**13/13 + c**3*x**15/15 + x**11*(
3*a*c**2/11 + 3*b**2*c/11) + x**9*(2*a*b*c/3 + b**3/9) + x**7*(3*a**2*c/7 +
 3*a*b**2/7)
```

3.645 $\int x\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=89

$$\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{3}{10}cx^{10}\left(ac+b^2\right)+\frac{1}{8}bx^8\left(6ac+b^2\right)+\frac{1}{2}ax^6\left(ac+b^2\right)+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 611}

$$\frac{3}{4}a^2bx^4+\frac{a^3x^2}{2}+\frac{3}{10}cx^{10}\left(ac+b^2\right)+\frac{1}{8}bx^8\left(6ac+b^2\right)+\frac{1}{2}ax^6\left(ac+b^2\right)+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^4)/4 + (a*(b^2 + a*c)*x^6)/2 + (b*(b^2 + 6*a*c)*x^
8)/8 + (3*c*(b^2 + a*c)*x^10)/10 + (b*c^2*x^12)/4 + (c^3*x^14)/14
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)^3\,dx &= \frac{1}{2}\text{Subst}\left(\int\left(a+bx+cx^2\right)^3\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(a^3+3a^2bx+3ab^2\left(1+\frac{ac}{b^2}\right)x^2+b^3\left(1+\frac{6ac}{b^2}\right)x^3+3b^2c\left(1+\frac{ac}{b^2}\right)x^4+\right.\right.\\ &= \frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{1}{2}a\left(b^2+ac\right)x^6+\frac{1}{8}b\left(b^2+6ac\right)x^8+\frac{3}{10}c\left(b^2+ac\right)x^{10}+\frac{1}{4}bc^2x^{12}\end{aligned}$$

Mathematica [A] time = 0.02, size = 79, normalized size = 0.89

$$\frac{1}{280}x^2\left(140a^3+210a^2bx^2+84cx^8\left(ac+b^2\right)+35bx^6\left(6ac+b^2\right)+140ax^4\left(ac+b^2\right)+70bc^2x^{10}+20c^3x^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(140*a^3 + 210*a^2*b*x^2 + 140*a*(b^2 + a*c)*x^4 + 35*b*(b^2 + 6*a*c)*
x^6 + 84*c*(b^2 + a*c)*x^8 + 70*b*c^2*x^10 + 20*c^3*x^12))/280
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2+cx^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.96, size = 87, normalized size = 0.98

$$\frac{1}{14}x^{14}c^3+\frac{1}{4}x^{12}c^2b+\frac{3}{10}x^{10}cb^2+\frac{3}{10}x^{10}c^2a+\frac{1}{8}x^8b^3+\frac{3}{4}x^8cba+\frac{1}{2}x^6b^2a+\frac{1}{2}x^6ca^2+\frac{3}{4}x^4ba^2+\frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*c^3 + 1/4*x^12*c^2*b + 3/10*x^10*c*b^2 + 3/10*x^10*c^2*a + 1/8*x^
8*b^3 + 3/4*x^8*c*b*a + 1/2*x^6*b^2*a + 1/2*x^6*c*a^2 + 3/4*x^4*b*a^2 + 1/2
*x^2*a^3
```

giac [A] time = 0.17, size = 87, normalized size = 0.98

$$\frac{1}{14}c^3x^{14}+\frac{1}{4}bc^2x^{12}+\frac{3}{10}b^2cx^{10}+\frac{3}{10}ac^2x^{10}+\frac{1}{8}b^3x^8+\frac{3}{4}abcx^8+\frac{1}{2}ab^2x^6+\frac{1}{2}a^2cx^6+\frac{3}{4}a^2bx^4+\frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*b^2*c*x^10 + 3/10*a*c^2*x^10 + 1/8*b^
3*x^8 + 3/4*a*b*c*x^8 + 1/2*a*b^2*x^6 + 1/2*a^2*c*x^6 + 3/4*a^2*b*x^4 + 1/2
*a^3*x^2
```

maple [A] time = 0.00, size = 111, normalized size = 1.25

$$\frac{c^3x^{14}}{14}+\frac{b\,c^2x^{12}}{4}+\frac{\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)x^{10}}{10}+\frac{\left(4abc+\left(2ac+b^2\right)b\right)x^8}{8}+\frac{3a^2b\,x^4}{4}+\frac{\left(a^2c+2a\,b^2+\left(2ac+b^2\right)a\right)x^6}{6}+\frac{a^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/14*c^3*x^14+1/4*b*c^2*x^12+1/10*(a*c^2+2*b^2*c+(2*a*c+b^2)*c)*x^10+1/8*(4
*a*b*c+(2*a*c+b^2)*b)*x^8+1/6*(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*x^6+3/4*a^2*b*x
^4+1/2*a^3*x^2
```

maxima [A] time = 1.37, size = 81, normalized size = 0.91

$$\frac{1}{14}\,c^3x^{14}+\frac{1}{4}\,bc^2x^{12}+\frac{3}{10}\left(b^2c+ac^2\right)x^{10}+\frac{1}{8}\left(b^3+6\,abc\right)x^8+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\left(ab^2+a^2c\right)x^6+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*(b^2*c + a*c^2)*x^10 + 1/8*(b^3 + 6*a
*b*c)*x^8 + 3/4*a^2*b*x^4 + 1/2*(a*b^2 + a^2*c)*x^6 + 1/2*a^3*x^2
```

mupad [B] time = 0.03, size = 76, normalized size = 0.85

$$x^8\left(\frac{b^3}{8}+\frac{3\,a\,c\,b}{4}\right)+\frac{a^3\,x^2}{2}+\frac{c^3\,x^{14}}{14}+\frac{3\,a^2\,b\,x^4}{4}+\frac{b\,c^2\,x^{12}}{4}+\frac{a\,x^6\,\left(b^2+a\,c\right)}{2}+\frac{3\,c\,x^{10}\,\left(b^2+a\,c\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^8*(b^3/8 + (3*a*b*c)/4) + (a^3*x^2)/2 + (c^3*x^14)/14 + (3*a^2*b*x^4)/4 +
 (b*c^2*x^12)/4 + (a*x^6*(a*c + b^2))/2 + (3*c*x^10*(a*c + b^2))/10
```

sympy [A] time = 0.10, size = 92, normalized size = 1.03

$$\frac{a^3x^2}{2}+\frac{3a^2bx^4}{4}+\frac{bc^2x^{12}}{4}+\frac{c^3x^{14}}{14}+x^{10}\left(\frac{3ac^2}{10}+\frac{3b^2c}{10}\right)+x^8\left(\frac{3abc}{4}+\frac{b^3}{8}\right)+x^6\left(\frac{a^2c}{2}+\frac{ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**2/2 + 3*a**2*b*x**4/4 + b*c**2*x**12/4 + c**3*x**14/14 + x**10*(3*a
*c**2/10 + 3*b**2*c/10) + x**8*(3*a*b*c/4 + b**3/8) + x**6*(a**2*c/2 + a*b*
*2/2)
```

3.646 $\int \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=81

$$a^3x+a^2bx^3+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1090}

$$a^2bx^3+a^3x+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c*
(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^3 \, dx = \int \left(a^3+3a^2bx^2+3ab^2\left(1+\frac{ac}{b^2}\right)x^4+b^3\left(1+\frac{6ac}{b^2}\right)x^6+3b^2c\left(1+\frac{ac}{b^2}\right)x^8+3bc^2x^{10}+c^3\right.$$

$$=a^3x+a^2bx^3+\frac{3}{5}a\left(b^2+ac\right)x^5+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{1}{3}c\left(b^2+ac\right)x^9+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Mathematica [A] time = 0.01, size = 81, normalized size = 1.00

$$a^3x+a^2bx^3+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c*
(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^2 + cx^4\right)^3 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3, x]
```

fricas [A] time = 0.75, size = 83, normalized size = 1.02

$$\frac{1}{13}x^{13}c^3 + \frac{3}{11}x^{11}c^2b + \frac{1}{3}x^9cb^2 + \frac{1}{3}x^9c^2a + \frac{1}{7}x^7b^3 + \frac{6}{7}x^7cba + \frac{3}{5}x^5b^2a + \frac{3}{5}x^5ca^2 + x^3ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^3 + 3/11*x^11*c^2*b + 1/3*x^9*c*b^2 + 1/3*x^9*c^2*a + 1/7*x^7*b
^3 + 6/7*x^7*c*b*a + 3/5*x^5*b^2*a + 3/5*x^5*c*a^2 + x^3*b*a^2 + x*a^3
```

giac [A] time = 0.15, size = 83, normalized size = 1.02

$$\frac{1}{13}c^3x^{13} + \frac{3}{11}bc^2x^{11} + \frac{1}{3}b^2cx^9 + \frac{1}{3}ac^2x^9 + \frac{1}{7}b^3x^7 + \frac{6}{7}abcx^7 + \frac{3}{5}ab^2x^5 + \frac{3}{5}a^2cx^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/3*a*c^2*x^9 + 1/7*b^3*x
^7 + 6/7*a*b*c*x^7 + 3/5*a*b^2*x^5 + 3/5*a^2*c*x^5 + a^2*b*x^3 + a^3*x
```

maple [A] time = 0.00, size = 107, normalized size = 1.32

$$\frac{c^3x^{13}}{13} + \frac{3b\,c^2x^{11}}{11} + \frac{\left(a\,c^2 + 2b^2c + \left(2ac + b^2\right)c\right)x^9}{9} + \frac{\left(4abc + \left(2ac + b^2\right)b\right)x^7}{7} + a^2b\,x^3 + \frac{\left(a^2c + 2a\,b^2 + \left(2ac + b^2\right)a\right)x^5}{5} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/13*c^3*x^13+3/11*b*c^2*x^11+1/9*(a*c^2+2*b^2*c+(2*a*c+b^2)*c)*x^9+1/7*(4*
a*b*c+(2*a*c+b^2)*b)*x^7+1/5*(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*x^5+a^2*b*x^3+a^
3*x
```

maxima [A] time = 1.35, size = 85, normalized size = 1.05

$$\frac{1}{13}\,c^3x^{13}+\frac{3}{11}\,bc^2x^{11}+\frac{1}{3}\,b^2cx^9+\frac{1}{7}\,b^3x^7+a^3x+\frac{1}{5}\left(3\,cx^5+5\,bx^3\right)a^2+\frac{1}{105}\left(35\,c^2x^9+90\,bcx^7+63\,b^2x^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/7*b^3*x^7 + a^3*x + 1/5
*(3*c*x^5 + 5*b*x^3)*a^2 + 1/105*(35*c^2*x^9 + 90*b*c*x^7 + 63*b^2*x^5)*a
```

mupad [B] time = 0.03, size = 72, normalized size = 0.89

$$x^7\left(\frac{b^3}{7}+\frac{6\,a\,c\,b}{7}\right)+a^3\,x+\frac{c^3\,x^{13}}{13}+a^2\,b\,x^3+\frac{3\,b\,c^2\,x^{11}}{11}+\frac{3\,a\,x^5\,\left(b^2+a\,c\right)}{5}+\frac{c\,x^9\,\left(b^2+a\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^7*(b^3/7 + (6*a*b*c)/7) + a^3*x + (c^3*x^13)/13 + a^2*b*x^3 + (3*b*c^2*x^
11)/11 + (3*a*x^5*(a*c + b^2))/5 + (c*x^9*(a*c + b^2))/3
```

sympy [A] time = 0.09, size = 87, normalized size = 1.07

$$a^3x+a^2bx^3+\frac{3bc^2x^{11}}{11}+\frac{c^3x^{13}}{13}+x^9\left(\frac{ac^2}{3}+\frac{b^2c}{3}\right)+x^7\left(\frac{6abc}{7}+\frac{b^3}{7}\right)+x^5\left(\frac{3a^2c}{5}+\frac{3ab^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*b*x**3 + 3*b*c**2*x**11/11 + c**3*x**13/13 + x**9*(a*c**2/3 +
 b**2*c/3) + x**7*(6*a*b*c/7 + b**3/7) + x**5*(3*a**2*c/5 + 3*a*b**2/5)
```

3.647 $\int \frac{(a+bx^2+cx^4)^3}{x}\,dx$

Optimal. Leaf size=85

$$a^3\log(x)+\frac{3}{2}a^2bx^2+\frac{3}{8}cx^8\left(ac+b^2\right)+\frac{1}{6}bx^6\left(6ac+b^2\right)+\frac{3}{4}ax^4\left(ac+b^2\right)+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}$$

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$\frac{3}{2}a^2bx^2+a^3\log(x)+\frac{3}{8}cx^8\left(ac+b^2\right)+\frac{1}{6}bx^6\left(6ac+b^2\right)+\frac{3}{4}ax^4\left(ac+b^2\right)+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c*
(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^3}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3a\left(b^2+ac\right)x+b\left(b^2+6ac\right)x^2+3c\left(b^2+ac\right)x^3+3bc^2x^4+\right.\right.$$
$$= \frac{3}{2}a^2bx^2+\frac{3}{4}a\left(b^2+ac\right)x^4+\frac{1}{6}b\left(b^2+6ac\right)x^6+\frac{3}{8}c\left(b^2+ac\right)x^8+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}+a^3$$

Mathematica [A] time = 0.02, size = 85, normalized size = 1.00

$$a^3\log(x)+\frac{3}{2}a^2bx^2+\frac{3}{8}cx^8\left(ac+b^2\right)+\frac{1}{6}bx^6\left(6ac+b^2\right)+\frac{3}{4}ax^4\left(ac+b^2\right)+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c*
(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^3}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x, x]
```

fricas [A] time = 1.93, size = 79, normalized size = 0.93

$$\frac{1}{12}c^3x^{12}+\frac{3}{10}bc^2x^{10}+\frac{3}{8}\left(b^2c+ac^2\right)x^8+\frac{1}{6}\left(b^3+6\,abc\right)x^6+\frac{3}{2}a^2bx^2+\frac{3}{4}\left(ab^2+a^2c\right)x^4+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a*
b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + a^3*log(x)
```

giac [A] time = 0.16, size = 87, normalized size = 1.02

$$\frac{1}{12}\,c^3x^{12}+\frac{3}{10}\,bc^2x^{10}+\frac{3}{8}\,b^2cx^8+\frac{3}{8}\,ac^2x^8+\frac{1}{6}\,b^3x^6+abcx^6+\frac{3}{4}\,ab^2x^4+\frac{3}{4}\,a^2cx^4+\frac{3}{2}\,a^2bx^2+\frac{1}{2}\,a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*b^2*c*x^8 + 3/8*a*c^2*x^8 + 1/6*b^3*x
^6 + a*b*c*x^6 + 3/4*a*b^2*x^4 + 3/4*a^2*c*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*lo
g(x^2)
```

maple [A] time = 0.00, size = 85, normalized size = 1.00

$$\frac{c^3x^{12}}{12}+\frac{3b\,c^2x^{10}}{10}+\frac{3a\,c^2x^8}{8}+\frac{3b^2c\,x^8}{8}+abc\,x^6+\frac{b^3x^6}{6}+\frac{3a^2c\,x^4}{4}+\frac{3a\,b^2x^4}{4}+\frac{3a^2b\,x^2}{2}+a^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x,x)
```

```
[Out] 1/12*c^3*x^12+3/10*b*c^2*x^10+3/8*x^8*a*c^2+3/8*x^8*b^2*c+x^6*a*b*c+1/6*b^3
*x^6+3/4*x^4*a^2*c+3/4*a*b^2*x^4+3/2*a^2*b*x^2+a^3*ln(x)
```

maxima [A] time = 1.39, size = 82, normalized size = 0.96

$$\frac{1}{12}\,c^3x^{12}+\frac{3}{10}\,bc^2x^{10}+\frac{3}{8}\left(b^2c+ac^2\right)x^8+\frac{1}{6}\left(b^3+6\,abc\right)x^6+\frac{3}{2}\,a^2bx^2+\frac{3}{4}\left(ab^2+a^2c\right)x^4+\frac{1}{2}\,a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a*
b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + 1/2*a^3*log(x^2)
```

mupad [B] time = 0.03, size = 73, normalized size = 0.86

$$a^3\,\ln(x)+x^6\,\left(\frac{b^3}{6}+a\,c\,b\right)+\frac{c^3\,x^{12}}{12}+\frac{3\,a^2\,b\,x^2}{2}+\frac{3\,b\,c^2\,x^{10}}{10}+\frac{3\,a\,x^4\,\left(b^2+a\,c\right)}{4}+\frac{3\,c\,x^8\,\left(b^2+a\,c\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x,x)
```

```
[Out] a^3*log(x) + x^6*(b^3/6 + a*b*c) + (c^3*x^12)/12 + (3*a^2*b*x^2)/2 + (3*b*c
^2*x^10)/10 + (3*a*x^4*(a*c + b^2))/4 + (3*c*x^8*(a*c + b^2))/8
```

sympy [A] time = 0.22, size = 92, normalized size = 1.08

$$a^3\log(x) + \frac{3a^2bx^2}{2} + \frac{3bc^2x^{10}}{10} + \frac{c^3x^{12}}{12} + x^8\left(\frac{3ac^2}{8} + \frac{3b^2c}{8}\right) + x^6\left(abc + \frac{b^3}{6}\right) + x^4\left(\frac{3a^2c}{4} + \frac{3ab^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x**2/2 + 3*b*c**2*x**10/10 + c**3*x**12/12 + x**8*(3
*a*c**2/8 + 3*b**2*c/8) + x**6*(a*b*c + b**3/6) + x**4*(3*a**2*c/4 + 3*a*b*
*2/4)
```

3.648 $\int \frac{(a+bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3}{x}+3a^2bx+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$3a^2bx-\frac{a^3}{x}+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*(
b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^2}\,dx &= \int \left(3a^2b+\frac{a^3}{x^2}+3a\left(b^2+ac\right)x^2+b\left(b^2+6ac\right)x^4+3c\left(b^2+ac\right)x^6+3bc^2x^8+c^3x^{10}\right)dx\\ &= -\frac{a^3}{x}+3a^2bx+a\left(b^2+ac\right)x^3+\frac{1}{5}b\left(b^2+6ac\right)x^5+\frac{3}{7}c\left(b^2+ac\right)x^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.02, size = 80, normalized size = 1.00

$$-\frac{a^3}{x}+3a^2bx+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*(
b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x^2, x]
```

fricas [A] time = 0.84, size = 83, normalized size = 1.04

$$\frac{105\,c^3x^{12}+385\,bc^2x^{10}+495\left(b^2c+ac^2\right)x^8+231\left(b^3+6\,abc\right)x^6+3465\,a^2bx^2+1155\left(ab^2+a^2c\right)x^4-1155\,a^3}{1155\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/1155*(105*c^3*x^12 + 385*b*c^2*x^10 + 495*(b^2*c + a*c^2)*x^8 + 231*(b^3
+ 6*a*b*c)*x^6 + 3465*a^2*b*x^2 + 1155*(a*b^2 + a^2*c)*x^4 - 1155*a^3)/x
```

giac [A] time = 0.15, size = 83, normalized size = 1.04

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{3}{7}\,ac^2x^7+\frac{1}{5}\,b^3x^5+\frac{6}{5}\,abcx^5+ab^2x^3+a^2cx^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 3/7*a*c^2*x^7 + 1/5*b^3*x^5
 + 6/5*a*b*c*x^5 + a*b^2*x^3 + a^2*c*x^3 + 3*a^2*b*x - a^3/x
```

maple [A] time = 0.00, size = 84, normalized size = 1.05

$$\frac{c^3x^{11}}{11}+\frac{b\,c^2x^9}{3}+\frac{3a\,c^2x^7}{7}+\frac{3b^2c\,x^7}{7}+\frac{6abc\,x^5}{5}+\frac{b^3x^5}{5}+a^2c\,x^3+a\,b^2x^3+3a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^2,x)
```

```
[Out] 1/11*c^3*x^11+1/3*b*c^2*x^9+3/7*x^7*a*c^2+3/7*b^2*c*x^7+6/5*x^5*a*b*c+1/5*b
^3*x^5+x^3*a^2*c+a*b^2*x^3+3*a^2*b*x-a^3/x
```

maxima [A] time = 1.36, size = 78, normalized size = 0.98

$$\frac{1}{11}c^3x^{11} + \frac{1}{3}bc^2x^9 + \frac{3}{7}\left(b^2c + ac^2\right)x^7 + \frac{1}{5}\left(b^3 + 6\,abc\right)x^5 + 3\,a^2bx + \left(ab^2 + a^2c\right)x^3 - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*(b^2*c + a*c^2)*x^7 + 1/5*(b^3 + 6*a*b*
c)*x^5 + 3*a^2*b*x + (a*b^2 + a^2*c)*x^3 - a^3/x
```

mupad [B] time = 0.03, size = 73, normalized size = 0.91

$$x^5\left(\frac{b^3}{5} + \frac{6\,a\,c\,b}{5}\right) - \frac{a^3}{x} + \frac{c^3\,x^{11}}{11} + \frac{b\,c^2\,x^9}{3} + a\,x^3\,\left(b^2 + a\,c\right) + \frac{3\,c\,x^7\,\left(b^2 + a\,c\right)}{7} + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^2,x)
```

```
[Out] x^5*(b^3/5 + (6*a*b*c)/5) - a^3/x + (c^3*x^11)/11 + (b*c^2*x^9)/3 + a*x^3*(
a*c + b^2) + (3*c*x^7*(a*c + b^2))/7 + 3*a^2*b*x
```

sympy [A] time = 0.22, size = 82, normalized size = 1.02

$$-\frac{a^3}{x} + 3a^2bx + \frac{bc^2x^9}{3} + \frac{c^3x^{11}}{11} + x^7\left(\frac{3ac^2}{7} + \frac{3b^2c}{7}\right) + x^5\left(\frac{6abc}{5} + \frac{b^3}{5}\right) + x^3\left(a^2c + ab^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*x + b*c**2*x**9/3 + c**3*x**11/11 + x**7*(3*a*c**2/7 + 3
*b**2*c/7) + x**5*(6*a*b*c/5 + b**3/5) + x**3*(a**2*c + a*b**2)
```

3.649 $\int \frac{(a+bx^2+cx^4)^3}{x^3} dx$

Optimal. Leaf size=86

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$3a^2b\log(x)-\frac{a^3}{2x^2}+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*(b^2 + a*c)*x^2)/2 + (b*(b^2 + 6*a*c)*x^4)/4 + (c*(b^2
+ a*c)*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10 + 3*a^2*b*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^3}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(3a\left(b^2+ac\right)+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b\left(b^2+6ac\right)x+3c\left(b^2+ac\right)x^2+3bc^2x^3+c\right.\right.$$
$$= -\frac{a^3}{2x^2}+\frac{3}{2}a\left(b^2+ac\right)x^2+\frac{1}{4}b\left(b^2+6ac\right)x^4+\frac{1}{2}c\left(b^2+ac\right)x^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}+3a^2b$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.91

$$\frac{1}{40}\left(-\frac{20a^3}{x^2}+120a^2b\log(x)+20cx^6\left(ac+b^2\right)+10bx^4\left(6ac+b^2\right)+60ax^2\left(ac+b^2\right)+15bc^2x^8+4c^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] ((-20*a^3)/x^2 + 60*a*(b^2 + a*c)*x^2 + 10*b*(b^2 + 6*a*c)*x^4 + 20*c*(b^2
+ a*c)*x^6 + 15*b*c^2*x^8 + 4*c^3*x^10 + 120*a^2*b*Log[x])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x^3, x]
```

fricas [A] time = 0.56, size = 85, normalized size = 0.99

$$\frac{4\,c^3x^{12}+15\,bc^2x^{10}+20\left(b^2c+ac^2\right)x^8+10\left(b^3+6\,abc\right)x^6+120\,a^2bx^2\log(x)+60\left(ab^2+a^2c\right)x^4-20\,a^3}{40\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/40*(4*c^3*x^12 + 15*b*c^2*x^10 + 20*(b^2*c + a*c^2)*x^8 + 10*(b^3 + 6*a*b
*c)*x^6 + 120*a^2*b*x^2*log(x) + 60*(a*b^2 + a^2*c)*x^4 - 20*a^3)/x^2
```

giac [A] time = 0.16, size = 98, normalized size = 1.14

$$\frac{1}{10}c^3x^{10} + \frac{3}{8}bc^2x^8 + \frac{1}{2}b^2cx^6 + \frac{1}{2}ac^2x^6 + \frac{1}{4}b^3x^4 + \frac{3}{2}abcx^4 + \frac{3}{2}ab^2x^2 + \frac{3}{2}a^2cx^2 + \frac{3}{2}a^2b\log\left(x^2\right) - \frac{3a^2bx^2 + a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/2*a*c^2*x^6 + 1/4*b^3*x^4
 + 3/2*a*b*c*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*c*x^2 + 3/2*a^2*b*log(x^2) - 1/2
*(3*a^2*b*x^2 + a^3)/x^2
```

maple [A] time = 0.01, size = 87, normalized size = 1.01

$$\frac{c^3x^{10}}{10} + \frac{3bc^2x^8}{8} + \frac{ac^2x^6}{2} + \frac{b^2cx^6}{2} + \frac{3abcx^4}{2} + \frac{b^3x^4}{4} + \frac{3a^2cx^2}{2} + \frac{3ab^2x^2}{2} + 3a^2b\ln(x) - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^3,x)
```

```
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*x^6*a*c^2+1/2*x^6*b^2*c+3/2*x^4*a*b*c+1/4*b
^3*x^4+3/2*x^2*a^2*c+3/2*a*b^2*x^2-1/2*a^3/x^2+3*a^2*b*ln(x)
```

maxima [A] time = 1.37, size = 82, normalized size = 0.95

$$\frac{1}{10}c^3x^{10} + \frac{3}{8}bc^2x^8 + \frac{1}{2}\left(b^2c + ac^2\right)x^6 + \frac{1}{4}\left(b^3 + 6abc\right)x^4 + \frac{3}{2}a^2b\log\left(x^2\right) + \frac{3}{2}\left(ab^2 + a^2c\right)x^2 - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*(b^2*c + a*c^2)*x^6 + 1/4*(b^3 + 6*a*b*
c)*x^4 + 3/2*a^2*b*log(x^2) + 3/2*(a*b^2 + a^2*c)*x^2 - 1/2*a^3/x^2
```

mupad [B] time = 0.04, size = 75, normalized size = 0.87

$$x^4\left(\frac{b^3}{4} + \frac{3acb}{2}\right) - \frac{a^3}{2x^2} + \frac{c^3x^{10}}{10} + \frac{3bc^2x^8}{8} + 3a^2b\ln(x) + \frac{3ax^2\left(b^2 + ac\right)}{2} + \frac{cx^6\left(b^2 + ac\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^3,x)
```

```
[Out] x^4*(b^3/4 + (3*a*b*c)/2) - a^3/(2*x^2) + (c^3*x^10)/10 + (3*b*c^2*x^8)/8 +
 3*a^2*b*log(x) + (3*a*x^2*(a*c + b^2))/2 + (c*x^6*(a*c + b^2))/2
```

sympy [A] time = 0.27, size = 92, normalized size = 1.07

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3bc^2x^8}{8}+\frac{c^3x^{10}}{10}+x^6\left(\frac{ac^2}{2}+\frac{b^2c}{2}\right)+x^4\left(\frac{3abc}{2}+\frac{b^3}{4}\right)+x^2\left(\frac{3a^2c}{2}+\frac{3ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*b*c**2*x**8/8 + c**3*x**10/10 + x**6*(
a*c**2/2 + b**2*c/2) + x**4*(3*a*b*c/2 + b**3/4) + x**2*(3*a**2*c/2 + 3*a*b
**2/2)
```

3.650 $\int \frac{(a+bx^2+cx^4)^3}{x^4} dx$

Optimal. Leaf size=83

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{3a^2b}{x}-\frac{a^3}{3x^3}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
(3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^4}\,dx &= \int \left(3a\left(b^2+ac\right)+\frac{a^3}{x^4}+\frac{3a^2b}{x^2}+b\left(b^2+6ac\right)x^2+3c\left(b^2+ac\right)x^4+3bc^2x^6+c^3x^8\right)dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3a\left(b^2+ac\right)x+\frac{1}{3}b\left(b^2+6ac\right)x^3+\frac{3}{5}c\left(b^2+ac\right)x^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
(3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x^4, x]
```

fricas [A] time = 1.27, size = 83, normalized size = 1.00

$$\frac{35\,c^3x^{12}+135\,bc^2x^{10}+189\left(b^2c+ac^2\right)x^8+105\left(b^3+6\,abc\right)x^6-945\,a^2bx^2+945\left(ab^2+a^2c\right)x^4-105\,a^3}{315\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^3*x^12 + 135*b*c^2*x^10 + 189*(b^2*c + a*c^2)*x^8 + 105*(b^3 +
6*a*b*c)*x^6 - 945*a^2*b*x^2 + 945*(a*b^2 + a^2*c)*x^4 - 105*a^3)/x^3
```

giac [A] time = 0.18, size = 84, normalized size = 1.01

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{3}{5}\,ac^2x^5+\frac{1}{3}\,b^3x^3+2\,abcx^3+3\,ab^2x+3\,a^2cx-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 1/3*b^3*x^3 +
 2*a*b*c*x^3 + 3*a*b^2*x + 3*a^2*c*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

maple [A] time = 0.01, size = 84, normalized size = 1.01

$$\frac{c^3x^9}{9}+\frac{3b\,c^2x^7}{7}+\frac{3a\,c^2x^5}{5}+\frac{3b^2c\,x^5}{5}+2abc\,x^3+\frac{b^3x^3}{3}+3a^2cx+3a\,b^2x-\frac{3a^2b}{x}-\frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^4,x)
```

```
[Out] 1/9*c^3*x^9+3/7*b*c^2*x^7+3/5*x^5*a*c^2+3/5*b^2*c*x^5+2*x^3*a*b*c+1/3*b^3*x
^3+3*a^2*c*x+3*a*b^2*x-3*a^2*b/x-1/3*a^3/x^3
```

maxima [A] time = 1.35, size = 80, normalized size = 0.96

$$\frac{1}{9}c^3x^9 + \frac{3}{7}bc^2x^7 + \frac{3}{5}\left(b^2c + ac^2\right)x^5 + \frac{1}{3}\left(b^3 + 6abc\right)x^3 + 3\left(ab^2 + a^2c\right)x - \frac{9a^2bx^2 + a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*(b^2*c + a*c^2)*x^5 + 1/3*(b^3 + 6*a*b*c)
*x^3 + 3*(a*b^2 + a^2*c)*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

mupad [B] time = 0.03, size = 77, normalized size = 0.93

$$x^3\left(\frac{b^3}{3} + 2acb\right) - \frac{\frac{a^3}{3} + 3ba^2x^2}{x^3} + \frac{c^3x^9}{9} + \frac{3bc^2x^7}{7} + 3ax\left(b^2 + ac\right) + \frac{3cx^5\left(b^2 + ac\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^4,x)
```

```
[Out] x^3*(b^3/3 + 2*a*b*c) - (a^3/3 + 3*a^2*b*x^2)/x^3 + (c^3*x^9)/9 + (3*b*c^2*
x^7)/7 + 3*a*x*(a*c + b^2) + (3*c*x^5*(a*c + b^2))/5
```

sympy [A] time = 0.24, size = 90, normalized size = 1.08

$$\frac{3bc^2x^7}{7} + \frac{c^3x^9}{9} + x^5\left(\frac{3ac^2}{5} + \frac{3b^2c}{5}\right) + x^3\left(2abc + \frac{b^3}{3}\right) + x\left(3a^2c + 3ab^2\right) + \frac{-a^3 - 9a^2bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**4,x)
```

```
[Out] 3*b*c**2*x**7/7 + c**3*x**9/9 + x**5*(3*a*c**2/5 + 3*b**2*c/5) + x**3*(2*a*
b*c + b**3/3) + x*(3*a**2*c + 3*a*b**2) + (-a**3 - 9*a**2*b*x**2)/(3*x**3)
```

3.651 $\int \frac{x^7}{a+bx^2+cx^4} dx$

Optimal. Leaf size=100

$$\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1114, 701, 634, 618, 206, 628}

$$\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\text{Subst}\left(\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}-\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}+\frac{\left(b^2-ac\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,\right.}{4c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\right.}{2c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 0.93

$$\frac{-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)+cx^2\left(cx^2-2b\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{a + bx^2 + cx^4}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 2.86, size = 313, normalized size = 3.13

$$\left[\frac{(b^2c^2 - 4\,ac^3)x^4 - 2\,(b^3c - 4\,abc^2)x^2 - (b^3 - 3\,abc)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^4 + 2\,bcx^2 + b^2 - 2\,ac - (2\,cx^2 + b)\sqrt{b^2 - 4\,ac}}{cx^4 + bx^2 + a}\right) + (b^4 - 5\,ab^2c + 4\,a^2c^2)\log\left(cx^4 + bx^2 + a\right)}{4\,(b^2c^3 - 4\,ac^4)}, \frac{(b^2c^2 - 4\,ac^3)x^4 - 2\,(b^3c - 4\,abc^2)x^2 + 2\,(b^3 - 3\,abc)\sqrt{-b^2 + 4\,ac}\arctan\left(-\frac{(2\,cx^2 + b)\sqrt{-b^2 + 4\,ac}}{b^2 - 4\,ac}\right) + (b^4 - 5\,ab^2c + 4\,a^2c^2)\log\left(cx^4 + bx^2 + a\right)}{4\,(b^2c^3 - 4\,ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c)
*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2
*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2
)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 0.56, size = 92, normalized size = 0.92

$$\frac{cx^4 - 2\,bx^2}{4\,c^2} + \frac{(b^2 - ac)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{(b^3 - 3\,abc)\arctan\left(\frac{2\,cx^2 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1/
2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

maple [A] time = 0.01, size = 142, normalized size = 1.42

$$\frac{x^{4}}{4c}+\frac{3ab\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}\;c^{2}}-\frac{b^{3}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}\;c^{3}}-\frac{b\,x^{2}}{2c^{2}}-\frac{a\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4c^{2}}+\frac{b^{2}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/c*x^4-1/2*b/c^2*x^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a+1/4/c^3*ln(c*x^4+b*x^2+
a)*b^2+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b-
1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.40, size = 842, normalized size = 8.42

$$\frac{x^{4}}{4c}-\frac{\ln\left(c\,x^{4}+b\,x^{2}+a\right)\left(8a^{2}c^{2}-10ab^{2}c+2b^{4}\right)}{2\left(16ac^{4}-4b^{2}c^{3}\right)}-\frac{b\,x^{2}}{2c^{2}}+\frac{b\,\mathrm{atan}\left(\frac{2c^{4}\left(4ac-b^{2}\right)\left(\frac{\frac{b\left(3ac-b^{2}\right)\left(\frac{8a^{2}c^{4}-8ab^{2}c^{3}}{c^{4}}-\frac{8ac^{2}\left(8a^{2}c^{2}-10ab^{2}c+2b^{4}\right)}{16ac^{4}-4b^{2}c^{3}}\right)}{8c^{3}\sqrt{4ac-b^{2}}}}{a}-\frac{ab\left(3ac-b^{2}\right)\left(8a^{2}c^{2}-10ab^{2}c+2b^{4}\right)}{c\sqrt{4ac-b^{2}}\left(16ac^{4}-4b^{2}c^{3}\right)}\right)-x^{2}\left(\frac{\frac{b\left(\frac{6b^{3}c^{3}-10abc^{4}}{c^{4}}+\frac{4bc^{2}\left(8a^{2}c^{2}-10ab^{2}c+2b^{4}\right)}{16ac^{4}-4b^{2}c^{3}}\right)\left(3ac-b^{2}\right)}{8c^{3}\sqrt{4ac-b^{2}}}}{a}+\frac{b^{2}\left(3ac-b^{2}\right)\left(8a^{2}c^{2}-10ab^{2}c+2b^{4}\right)}{2c\sqrt{4ac-b^{2}}\left(16ac^{4}-4b^{2}c^{3}\right)}+\frac{b\left(\frac{2a^{2}bc^{2}-3ab^{3}c+b^{5}}{c^{4}}+\frac{\left(\frac{6b^{3}c^{3}-10abc^{4}}{c^{4}}+\frac{4bc^{2}\left(8a^{2}c^{2}-10ab^{2}c+2b^{4}\right)}{16ac^{4}-4b^{2}c^{3}}\right)\left(8a^{2}c^{2}-10ab^{2}c+2b^{4}\right)}{2\left(16ac^{4}-4b^{2}c^{3}\right)}-\frac{b^{3}\left(3ac-b^{2}\right)^{2}}{2c^{4}\left(4ac-b^{2}\right)}\right)}{2a\sqrt{4ac-b^{2}}}\right)+\frac{b\left(\frac{\left(\frac{8a^{2}c^{4}-8ab^{2}c^{3}}{c^{4}}-\frac{8ac^{2}\left(8a^{2}c^{2}-10ab^{2}c+2b^{4}\right)}{16ac^{4}-4b^{2}c^{3}}\right)\left(8a^{2}c^{2}-10ab^{2}c+2b^{4}\right)}{2\left(16ac^{4}-4b^{2}c^{3}\right)}-\frac{a^{3}c^{2}-2a^{2}b^{2}c+ab^{4}}{c^{4}}+\frac{ab^{2}\left(3ac-b^{2}\right)^{2}}{c^{4}\left(4ac-b^{2}\right)}\right)}{2a\sqrt{4ac-b^{2}}}\right)}{9a^{2}b^{2}c^{2}-6ab^{4}c+b^{6}}\right)\left(3ac-b^{2}\right)}{2c^{3}\sqrt{4ac-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^4/(4*c) - (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(1
6*a*c^4 - 4*b^2*c^3)) - (b*x^2)/(2*c^2) + (b*atan((2*c^4*(4*a*c - b^2)*(((b
*(3*a*c - b^2)*((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2
 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3)))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*b
*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(c*(4*a*c - b^2)^(1/2)*(16
*a*c^4 - 4*b^2*c^3)))/a - x^2*(((b*((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2
*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(3*a*c - b^2))/(
8*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b
^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((b^5 + 2*a
```

```
^2*b*c^2 - 3*a*b^3*c)/c^4 + (((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2*(2*b^
4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 1
0*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^3*(3*a*c - b^2)^2)/(2*c^4*(4*a*
c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*a^2*c^4 - 8*a*b^2*c^3)/c^
4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b
^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*b^4 + a^3*c^2
 - 2*a^2*b^2*c)/c^4 + (a*b^2*(3*a*c - b^2)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4
*a*c - b^2)^(1/2))))/(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c))*(3*a*c - b^2))/(2*c
^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.91, size = 391, normalized size = 3.91

$$-\frac{bx^2}{2c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right)+\left(\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right)+\frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)
/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*
a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c +
 b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(
x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))
/(3*a*b*c - b**3)) + x**4/(4*c)
```

3.652 $\int \frac{x^5}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1114, 703, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2c} + \frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}\\
&= \frac{x^2}{2c} - \frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} + \frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}\\
&= \frac{x^2}{2c} - \frac{b\log\left(a+bx^2+cx^4\right)}{4c^2} - \frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c} - \frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.96

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - b\log\left(a+bx^2+cx^4\right) + 2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a + bx^2 + cx^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 1.01, size = 254, normalized size = 3.14

$$\left[\frac{2\left(b^{2}c - 4\,ac^{2}\right)x^{2} - \left(b^{2} - 2\,ac\right)\sqrt{b^{2} - 4\,ac}\,\log\left(\frac{2\,c^{2}x^{4} + 2\,bcx^{2} + b^{2} - 2\,ac + \left(2\,cx^{2} + b\right)\sqrt{b^{2} - 4\,ac}}{cx^{4} + bx^{2} + a}\right) - \left(b^{3} - 4\,abc\right)\log\left(cx^{4} + bx^{2} + a\right)}{4\left(b^{2}c^{2} - 4\,ac^{3}\right)}, \frac{2\left(b^{2}c - 4\,ac^{2}\right)x^{2} - 2\left(b^{2} - 2\,ac\right)\sqrt{-b^{2} + 4\,ac}\,\arctan\left(-\frac{\left(2\,cx^{2} + b\right)\sqrt{-b^{2} + 4\,ac}}{b^{2} - 4\,ac}\right) - \left(b^{3} - 4\,abc\right)\log\left(cx^{4} + bx^{2} + a\right)}{4\left(b^{2}c^{2} - 4\,ac^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b
*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.62, size = 75, normalized size = 0.93

$$\frac{x^{2}}{2\,c} - \frac{b\log\left(cx^{4} + bx^{2} + a\right)}{4\,c^{2}} + \frac{\left(b^{2} - 2\,ac\right)\arctan\left(\frac{2\,cx^{2} + b}{\sqrt{-b^{2} + 4\,ac}}\right)}{2\,\sqrt{-b^{2} + 4\,ac}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.00, size = 111, normalized size = 1.37

$$-\frac{a\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}\;c} + \frac{b^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}\;c^{2}} + \frac{x^{2}}{2c} - \frac{b\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/c*x^2-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.75, size = 655, normalized size = 8.09

$$\frac{x^2}{2c}+\frac{\ln\left(c x^4+b x^2+a\right)\left(2b^3-8abc\right)}{2\left(16ac^3-4b^2c^2\right)}+\frac{\operatorname{atan}\left(\frac{2c^2\left(4ac-b^2\right)\left(\frac{\frac{\left(8ab+\frac{8ac^2\left(2b^3-8abc\right)}{16ac^3-4b^2c^2}\right)\left(2ac-b^2\right)}{8c^2\sqrt{4ac-b^2}}+\frac{a\left(2b^3-8abc\right)\left(2ac-b^2\right)}{\sqrt{4ac-b^2}\left(16ac^3-4b^2c^2\right)}}{a}-x^2\left(\frac{\frac{\left(2ac-b^2\right)\left(\frac{4ac^3-6b^2c^2}{c^2}-\frac{4bc^2\left(2b^3-8abc\right)}{16ac^3-4b^2c^2}\right)}{8c^2\sqrt{4ac-b^2}}-\frac{b\left(2b^3-8abc\right)\left(2ac-b^2\right)}{2\sqrt{4ac-b^2}\left(16ac^3-4b^2c^2\right)}}{a}+\frac{b\left(\frac{\left(2b^3-8abc\right)\left(\frac{4ac^3-6b^2c^2}{c^2}-\frac{4bc^2\left(2b^3-8abc\right)}{16ac^3-4b^2c^2}\right)}{2\left(16ac^3-4b^2c^2\right)}-\frac{b^3-abc}{c^2}+\frac{b\left(2ac-b^2\right)^2}{2c^2\left(4ac-b^2\right)}\right)}{2a\sqrt{4ac-b^2}}\right)+\frac{b\left(\frac{ab^2}{c^2}+\frac{\left(2b^3-8abc\right)\left(8ab+\frac{8ac^2\left(2b^3-8abc\right)}{16ac^3-4b^2c^2}\right)}{2\left(16ac^3-4b^2c^2\right)}-\frac{a\left(2ac-b^2\right)^2}{c^2\left(4ac-b^2\right)}\right)}{2a\sqrt{4ac-b^2}}\right)}{4a^2c^2-4ab^2c+b^4}\right)\left(2ac-b^2\right)}{2c^2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^2
*c^2)) + (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))
/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2))
)/a - x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8
*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2*b^3 -
 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a
+ (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a
*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c)/
c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))
) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*a*c - b^2)^2
)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a^2*c^2 - 4*a*
b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.14, size = 316, normalized size = 3.90

$$\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)+2b^2c\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)}{2ac-b^2}\right)+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*
log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2) + sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b
- 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
 - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

3.653 $\int \frac{x^3}{a+bx^2+cx^4} dx$

Optimal. Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c}\\ &= \frac{\log\left(a+bx^2+cx^4\right)}{4c} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c}\\ &= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^2+cx^4\right)}{4c}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^2+cx^4\right) - \frac{2b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^2 + c*x^4])/(4*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 0.98, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)+(b^2-4ac)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4ac^2\right)}, \frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+(b^2-4ac)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2
+ a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 0.57, size = 59, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}\,c}+\frac{\log\left(cx^4+bx^2+a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
4*log(c*x^4 + b*x^2 + a)/c
```

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c}+\frac{\ln\left(cx^4+bx^2+a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.26, size = 118, normalized size = 1.87

$$\frac{4\,a\,c\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b^2\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4),x)
```

```
[Out] (4*a*c*log(a + b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a + b*x^2 +
c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^2)/(4
*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.03, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)+2a+2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-b*
sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c +
b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/
(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

3.654 $\int \frac{x}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= -\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)\\
&= -\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.08

$$
\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4),x]
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x}{a+bx^2+cx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x/(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 2.33, size = 129, normalized size = 3.58

$$
\left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}},\,-\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.57, size = 35, normalized size = 0.97

$$\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 36, normalized size = 1.00

$$\frac{\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.27, size = 41, normalized size = 1.14

$$\frac{\mathrm{atan}\left(\frac{2\,a\,c\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan((a*b + 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

sympy [B] time = 0.59, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2
```

3.655 $\int \frac{1}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} + \frac{\operatorname{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a}\\
&= \frac{\log(x)}{a} - \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a}\\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 113, normalized size = 1.64

$$
\frac{-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2 + c*x^4)), x]
```

fricas [A] time = 1.31, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-\left(b^2-4ac\right)\log\left(cx^4+bx^2+a\right)+4\left(b^2-4ac\right)\log(x)}{4\left(ab^2-4a^2c\right)},\ \frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left(b^2-4ac\right)\log\left(cx^4+bx^2+a\right)+4\left(b^2-4ac\right)\log(x)}{4\left(ab^2-4a^2c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 0.57, size = 68, normalized size = 0.99

$$-\frac{b\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}\,a}-\frac{\log\left(cx^4+bx^2+a\right)}{4a}+\frac{\log\left(x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

maple [A] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a}+\frac{\ln(x)}{a}-\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*ln(c*x^4+b*x^2+a)/a-1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))+ln(x)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.94, size = 1014, normalized size = 14.70

$$\frac{\ln(x)}{a}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)\left(8ac-2b^2\right)}{2\left(4ab^2-16a^2c\right)}+\frac{b\,\mathrm{atan}\left(\text{[illegible]}\right)}{2a\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log(x)/a + (log(a + b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)
) + (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3 -
 ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*
(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*
c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^
3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(
(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c
^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b
^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a
```

```
*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^
2)^(3/2))/(b^2*c^2) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*(((8*a*c - 2
*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(
4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))
/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/
(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*c - 6*b^2)) - (2*
(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c -
 b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c
 - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^
2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))
/(b^2*c^4*(25*a*c - 6*b^2))))/(2*a*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 4.67, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)+2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) +
 (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqr
t(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + l
og(x)/a
```

3.656 $\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx, x, x^2\right)}{2a}$$

$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} + \frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, \right.}{4a^2}$$

$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log\left(a+bx^2+cx^4\right)}{4a^2} - \frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2\right.}{2a^2}$$

$$= -\frac{1}{2ax^2} - \frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] time = 0.13, size = 135, normalized size = 1.52

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} - \frac{2a}{x^2} - 4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2 + c*x^4)), x]
```

fricas [A] time = 0.68, size = 293, normalized size = 3.29

$$\left[-\frac{\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}\,x^{2}\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+\left(2\,cx^{2}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-\left(b^{3}-4\,abc\right)x^{2}\log\left(cx^{4}+bx^{2}+a\right)+4\left(b^{3}-4\,abc\right)x^{2}\log(x)+2\,ab^{2}-8\,a^{2}c}{4\left(a^{2}b^{2}-4\,a^{3}c\right)x^{2}},\ -\frac{2\left(b^{2}-2\,ac\right)\sqrt{-b^{2}+4\,ac}\,x^{2}\arctan\left(-\frac{\left(2\,cx^{2}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-\left(b^{3}-4\,abc\right)x^{2}\log\left(cx^{4}+bx^{2}+a\right)+4\left(b^{3}-4\,abc\right)x^{2}\log(x)+2\,ab^{2}-8\,a^{2}c}{4\left(a^{2}b^{2}-4\,a^{3}c\right)x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

giac [A] time = 0.58, size = 94, normalized size = 1.06

$$\frac{b\log\left(cx^{4}+bx^{2}+a\right)}{4\,a^{2}}-\frac{b\log\left(x^{2}\right)}{2\,a^{2}}+\frac{\left(b^{2}-2\,ac\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\sqrt{-b^{2}+4\,ac}\,a^{2}}+\frac{bx^{2}-a}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x
^2 - a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{c\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}\;a}+\frac{b^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}\;a^{2}}-\frac{b\ln(x)}{a^{2}}+\frac{b\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4a^{2}}-\frac{1}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*b^2-1/2/a/x^2-b*ln(x)/a^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.89, size = 2033, normalized size = 22.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(c^5/a^3 + ((2*b^3 - 8*a*
b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((((2
*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*
b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b
^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^
2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a
^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2
)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(8*a^3*c
^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3 - 8*a*b*c)*(((2*a*c - b^2)*(
(20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^
3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) +
((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4
*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((
40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)/(64*a^9*(4*a*c - b^2)^(3/2)
) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)
^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2
)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3) - (2*a^3*(4*a*c - b^2)*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)
*(((2*b^3 - 8*a*b*c)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2
*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2
)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^(1/
2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((
```

```
a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c
^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a
^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*a*c - b^2)^3
)/(16*a^5*(4*a*c - b^2)^(3/2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2
*c^4 + b^4*c^2 - 4*a*b^2*c^3)) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^
2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c)*((a^2*c^4 - 4*a*b^2*c^3)/a
^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(
2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*
(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a
^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))
/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2
*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)
) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(16*a^
3*c - 4*a^2*b^2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3)))*(2*a*c - b^2))/(2*a^2*(4*a*c - b^2)^(1/2)) - (b*log(x))/
a^2 - (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
 - 1/(2*a*x^2)
```

sympy [B] time = 137.80, size = 345, normalized size = 3.88

$$\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{-8a^3c\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2(4ac-b^2)}\right) + 2a^2b^2\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2(4ac-b^2)}\right) + 3abc - b^3}{2ac^2 - b^2c}\right) + \left(\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{-8a^3c\left(\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2(4ac-b^2)}\right) + 2a^2b^2\left(\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2(4ac-b^2)}\right) + 3abc - b^3}{2ac^2 - b^2c}\right) - \frac{1}{2ax^2} - \frac{b\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] (b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*l
og(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a*
*2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) + (
b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*lo
g(x**2 + (-8*a**3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**
2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c -
 b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) - 1/
(2*a*x**2) - b*log(x)/a**2
```

3.657 $\int \frac{1}{x^5\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=114

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}-\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Rubi [A] time = 0.20, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2
 - a*c)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)\,dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{\left(b^2-ac\right)\log(x)}{a^3}+\frac{\text{Subst}\left(\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^3}$$
$$= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^3}-\frac{\left(b^2\right.}{}$$
$$= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{S}}{}$$
$$= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a\right.}{4a}$$

Mathematica [A] time = 0.23, size = 188, normalized size = 1.65

$$\frac{-\frac{a^2}{x^4}+4\log(x)\left(b^2-ac\right)-\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2\sqrt{b^2-4ac}+ac\sqrt{b^2-4ac}-3abc+b^3\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{2ab}{x^2}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(a^2/x^4) + (2*a*b)/x^2 + 4*(b^2 - a*c)*Log[x] - ((b^3 - 3*a*b*c + b^2*Sq
rt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c] + ((b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a
^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^2 + c*x^4)), x]
```

fricas [A] time = 2.11, size = 374, normalized size = 3.28

$$\left[-\frac{(b^3-3abc)\sqrt{b^2-4ac}\,x^4\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)+(b^4-5ab^2c+4a^2c^2)x^4\log(cx^4+bx^2+a)-4(b^4-5ab^2c+4a^2c^2)x^4\log(x)+a^2b^2-4a^3c-2(ab^3-4a^2bc)x^2}{4(a^3b^2-4a^4c)x^4}, \frac{2(b^3-3abc)\sqrt{-b^2+4ac}\,x^4\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-(b^4-5ab^2c+4a^2c^2)x^4\log(cx^4+bx^2+a)+4(b^4-5ab^2c+4a^2c^2)x^4\log(x)-a^2b^2+4a^3c+2(ab^3-4a^2bc)x^2}{4(a^3b^2-4a^4c)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b
^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 -
 5*a*b^2*c + 4*a^2*c^2)*x^4*log(c*x^4 + b*x^2 + a) - 4*(b^4 - 5*a*b^2*c + 4
*a^2*c^2)*x^4*log(x) + a^2*b^2 - 4*a^3*c - 2*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3
*b^2 - 4*a^4*c)*x^4), 1/4*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^4*arctan(
-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*x^4*log(c*x^4 + b*x^2 + a) + 4*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^4*log(
x) - a^2*b^2 + 4*a^3*c + 2*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2 - 4*a^4*c)*x^
4)]
```

giac [A] time = 0.55, size = 126, normalized size = 1.11

$$-\frac{(b^2-ac)\log\left(cx^4+bx^2+a\right)}{4\,a^3}+\frac{(b^2-ac)\log\left(x^2\right)}{2\,a^3}-\frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^3}-\frac{3\,b^2x^4-3\,acx^4-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2 - a*c)*log(x^2)/a^3
- 1/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
 4*a*c)*a^3) - 1/4*(3*b^2*x^4 - 3*a*c*x^4 - 2*a*b*x^2 + a^2)/(a^3*x^4)
```

maple [A] time = 0.01, size = 159, normalized size = 1.39

$$\frac{3bc\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2}-\frac{b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^3}-\frac{c\ln(x)}{a^2}+\frac{c\ln\left(c\,x^4+b\,x^2+a\right)}{4a^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(c\,x^4+b\,x^2+a\right)}{4a^3}+\frac{b}{2a^2x^2}-\frac{1}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/a^2*c*ln(c*x^4+b*x^2+a)-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2+3/2/a^2/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c-1/2/a^3/(4*a*c-b^2)^(1/2
```

```
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3-1/4/a/x^4-1/a^2*ln(x)*c+1/a^3*l
n(x)*b^2+1/2*b/a^2/x^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.37, size = 2451, normalized size = 21.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*
a^3*b^2)) - (1/(4*a) - (b*x^2)/(2*a^2))/x^4 - (log(x)*(a*c - b^2))/a^3 + (b
*atan((2*a^6*(4*a*c - b^2)*((((b*(3*a*c - b^2)*((4*a^4*b^4*c^2 - 8*a^5*b^2*
c^3)/a^6 - (2*a*b^2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3
*b^2)))/(4*a^3*(4*a*c - b^2)^(1/2)) - (b^3*c^2*(3*a*c - b^2)*(2*b^4 + 8*a^2
*c^2 - 10*a*b^2*c))/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(2*
b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) + (b^5*c^2*(3*a*c
 - b^2)^3)/(16*a^8*(4*a*c - b^2)^(3/2)) + (b*(3*a*c - b^2)*((4*a^2*b^4*c^3
- 5*a^3*b^2*c^4)/a^6 + (((4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*a*b^2*c^2
*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3*b^2))*(2*b^4 + 8*a^2*c
^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2))))/(4*a^3*(4*a*c - b^2)^(1/2)))
*(3*b^6 - 10*a^3*c^3 + 27*a^2*b^2*c^2 - 18*a*b^4*c))/(c^2*(b^6*c^2 - 6*a*b^
4*c^3 + 9*a^2*b^2*c^4)*(6*b^6 - 25*a^3*c^3 + 54*a^2*b^2*c^2 - 36*a*b^4*c))
- (16*a^9*x^2*((3*b*(b^4 + 3*a^2*c^2 - 4*a*b^2*c)*((((5*a^3*b*c^5 - 6*a^2*b
^3*c^4)/a^6 - (((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^7*b*c^3 - 12*a^
6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4*c - 4*a^3*b^2))
)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)))*(2*b^4 + 8*
a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) - (b^3*c^5)/a^6 + (b*(3*a
*c - b^2)*((b*((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^7*b*c^3 - 12*a^6
*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))
*(3*a*c - b^2))/(4*a^3*(4*a*c - b^2)^(1/2)) + (b*(40*a^7*b*c^3 - 12*a^6*b^3
*c^2)*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(8*a^9*(4*a*c - b^2)^
(1/2)*(16*a^4*c - 4*a^3*b^2))))/(4*a^3*(4*a*c - b^2)^(1/2)) + (b^2*(40*a^7*
b*c^3 - 12*a^6*b^3*c^2)*(3*a*c - b^2)^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(
```

```
32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2))))/(8*a^3*c^2*(6*b^6 - 25*a^3*
c^3 + 54*a^2*b^2*c^2 - 36*a*b^4*c)) + (((b^3*(40*a^7*b*c^3 - 12*a^6*b^3*c^2
)*(3*a*c - b^2)^3)/(64*a^15*(4*a*c - b^2)^(3/2)) - (((b*((10*a^5*b*c^4 + 2*
a^4*b^3*c^3)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10
*a*b^2*c))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(3*a*c - b^2))/(4*a^3*(4*a*c - b
^2)^(1/2)) + (b*(40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(3*a*c - b^2)*(2*b^4 + 8*a^
2*c^2 - 10*a*b^2*c))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(2
*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) + (b*((5*a^3*b*c
^5 - 6*a^2*b^3*c^4)/a^6 - (((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^7*b
*c^3 - 12*a^6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4*c -
 4*a^3*b^2)))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)))
*(3*a*c - b^2))/(4*a^3*(4*a*c - b^2)^(1/2)))*(3*b^6 - 10*a^3*c^3 + 27*a^2*b
^2*c^2 - 18*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(6*b^6 - 25*a^3*c^3 +
54*a^2*b^2*c^2 - 36*a*b^4*c)))*(4*a*c - b^2)^(3/2))/(b^6*c^2 - 6*a*b^4*c^3
+ 9*a^2*b^2*c^4) + (6*a^6*b*(4*a*c - b^2)^(3/2)*(b^4 + 3*a^2*c^2 - 4*a*b^2*
c)*((b^4*c^4 - a*b^2*c^5)/a^6 + (((4*a^2*b^4*c^3 - 5*a^3*b^2*c^4)/a^6 + (((
4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*a*b^2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a
*b^2*c))/(16*a^4*c - 4*a^3*b^2))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a
^4*c - 4*a^3*b^2)))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*
b^2)) - (b*((b*(3*a*c - b^2)*((4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*a*b^
2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3*b^2)))/(4*a^3*(4*
a*c - b^2)^(1/2)) - (b^3*c^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c)
)/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(3*a*c - b^2))/(4*a^3
*(4*a*c - b^2)^(1/2)) + (b^4*c^2*(3*a*c - b^2)^2*(2*b^4 + 8*a^2*c^2 - 10*a*
b^2*c))/(8*a^5*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2))))/(c^2*(b^6*c^2 - 6*a*
b^4*c^3 + 9*a^2*b^2*c^4)*(6*b^6 - 25*a^3*c^3 + 54*a^2*b^2*c^2 - 36*a*b^4*c)
))*(3*a*c - b^2))/(2*a^3*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.658 $\int \frac{x^6}{a+bx^2+cx^4} dx$

Optimal. Leaf size=203

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Rubi [A] time = 0.67, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1122, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{a+bx^2+cx^4}\,dx = \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^2)}{a+bx^2+cx^4}\,dx}{3c}$$

$$= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\int \frac{3ab+3(b^2-ac)x^2}{a+bx^2+cx^4}\,dx}{3c^2}$$

$$= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4a}}}{2c^2}$$

$$= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b+}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.15, size = 250, normalized size = 1.23

$$\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{bx}{c^2} + \frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 3
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{a + bx^2 + cx^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x^2 + c*x^4), x]
```

fricas [B] time = 1.31, size = 1564, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c
^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(
1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sq
rt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^
2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
```

```
^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*
a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

giac [B] time = 1.01, size = 2457, normalized size = 12.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 -
 (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 -
 4*a*c)*a^2*c^4)*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2
 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3
*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 -
 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan(2*sq
rt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*
```

```
b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c
^2) + 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*
c^5 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(
b^2 - 4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^
2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*
c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan
(2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8
*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c
^7)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

maple [B] time = 0.05, size = 467, normalized size = 2.30

$$-\frac{3\sqrt{2}\, ab \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\, c}-\frac{3\sqrt{2}\, ab \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\, c}+\frac{\sqrt{2}\, b^{3} \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\, c^{2}}+\frac{\sqrt{2}\, b^{3} \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\, c^{2}}+\frac{x^{3}}{3c}+\frac{\sqrt{2}\, a \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\, c}-\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\, c}-\frac{\sqrt{2}\, b^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\, c^{2}}+\frac{\sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\, c^{2}}-\frac{bx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3/c*x^3-b/c^2*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x
*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c^2*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-
```

```
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a+1/2/c^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx^3 - 3\,bx}{3\,c^2} - \frac{-\int \frac{(b^2-ac)x^2+ab}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^2
 + a), x)/c^2
```

mupad [B] time = 5.01, size = 4127, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3/(3*c) - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16
*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7
 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*
a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c -
3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2)*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b
*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b
^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*
b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*
```

b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*1i)/((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*
c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
+ 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
+ (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^
7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6
*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(
a^4*c - a^3*b^2))/c^3))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^
3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2
c(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*2i - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*
c^6)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a
*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2
*c^2 - 6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
- 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((
b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 -
6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((
((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(

```
4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(
1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c
^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4
*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*
c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*
c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*
((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(a^4*c - a^3*b^2))
/c^3))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - (b*x)/c^2
```

sympy [A] time = 5.16, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2} + \operatorname{RootSum}\left(t^4\left(256a^2c^7 - 128ab^2c^6 + 16b^4c^5\right) + t^2\left(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7\right) + a^5, \left(t \mapsto t\log\left(x + \frac{-64t^3a^2c^7 + 48t^3ab^2c^6 - 8t^3b^4c^5 + 14ta^3bc^3 - 28ta^2b^3c^2 + 14tab^5c - 2tb^7}{a^4c^2 - 3a^3b^2c + a^2b^4}\right)\right)\right) + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
+ _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**
5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t
**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c -
 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

3.659 $\int \frac{x^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Rubi [A] time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{a+bx^2+cx^4}\,dx = \frac{x}{c} - \frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.11, size = 202, normalized size = 1.13

$$-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2 + c*x^4),x]
```

[Out] IntegrateAlgebraic[x^4/(a + b*x^2 + c*x^4), x]

fricas [B] time = 1.80, size = 1059, normalized size = 5.92

$$-\frac{\sqrt{\frac{1}{2}}\, c \sqrt{-\frac{b^{3}-3\,abc+\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}}{b^{2}c^{3}-4\,ac^{4}}}\log\left(-2\left(ab^{2}-a^{2}c\right)x+\sqrt{\frac{1}{2}}\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}-\left(b^{3}c^{3}-4\,abc^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}\right)\sqrt{-\frac{b^{3}-3\,abc+\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}}{b^{2}c^{3}-4\,ac^{4}}}\right)-\sqrt{\frac{1}{2}}\, c \sqrt{-\frac{b^{3}-3\,abc+\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}}{b^{2}c^{3}-4\,ac^{4}}}\log\left(-2\left(ab^{2}-a^{2}c\right)x-\sqrt{\frac{1}{2}}\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}-\left(b^{3}c^{3}-4\,abc^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}\right)\sqrt{-\frac{b^{3}-3\,abc+\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}}{b^{2}c^{3}-4\,ac^{4}}}\right)+\sqrt{\frac{1}{2}}\, c \sqrt{-\frac{b^{3}-3\,abc-\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}}{b^{2}c^{3}-4\,ac^{4}}}\log\left(-2\left(ab^{2}-a^{2}c\right)x+\sqrt{\frac{1}{2}}\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}+\left(b^{3}c^{3}-4\,abc^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}\right)\sqrt{-\frac{b^{3}-3\,abc-\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}}{b^{2}c^{3}-4\,ac^{4}}}\right)-\sqrt{\frac{1}{2}}\, c \sqrt{-\frac{b^{3}-3\,abc-\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}}{b^{2}c^{3}-4\,ac^{4}}}\log\left(-2\left(ab^{2}-a^{2}c\right)x-\sqrt{\frac{1}{2}}\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}+\left(b^{3}c^{3}-4\,abc^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}\right)\sqrt{-\frac{b^{3}-3\,abc-\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}}{b^{2}c^{3}-4\,ac^{4}}}\right)-2\,x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x)/c

giac [B] time = 0.97, size = 2109, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -

4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*
b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a
*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*
c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)
)/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*
b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^
2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3
- 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*
c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)

maple [B] time = 0.03, size = 343, normalized size = 1.92

$$\frac{\sqrt{2}\, a \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, b^2 \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}+\frac{\sqrt{2}\, b \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/2/c*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 0.65, size = 3026, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4),x)
```

```
[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
```

```
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2
*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4
*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4
)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2
*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*
b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
```

```
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [A] time = 5.51, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3b^3c^3-4ta^2c^2+8tab^2c-2tb^4}{a^2c-ab^2}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

3.660 $\int \frac{x^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}}$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.08, size = 165, normalized size = 1.10

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x^2 + c*x^4), x]
```

fricas [B] time = 1.55, size = 559, normalized size = 3.73

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/
2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2
*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4
*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(
b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*s
qrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a
*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

giac [B] time = 1.05, size = 503, normalized size = 3.35

$$-\frac{\left(2\,b^{2}c^{2}-8\,ac^{3}-\sqrt{2}\sqrt{b^{2}-4\,ac}\sqrt{bc+\sqrt{b^{2}-4\,ac}\,c}\,b^{2}+4\,\sqrt{2}\sqrt{b^{2}-4\,ac}\sqrt{bc+\sqrt{b^{2}-4\,ac}\,c}\,ac+2\,\sqrt{2}\sqrt{b^{2}-4\,ac}\sqrt{bc+\sqrt{b^{2}-4\,ac}\,c}\,bc-\sqrt{2}\sqrt{b^{2}-4\,ac}\sqrt{bc+\sqrt{b^{2}-4\,ac}\,c}\,c^{2}-2\left(b^{2}-4\,ac\right)c^{2}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{b+\sqrt{b^{2}-4\,ac}}{c}}}\right)}{2\left(b^{4}-8\,ab^{2}c-2\,b^{3}c+16\,a^{2}c^{2}+8\,abc^{2}+b^{2}c^{2}-4\,ac^{3}\right)|c|}+\frac{\left(2\,b^{2}c^{2}-8\,ac^{3}-\sqrt{2}\sqrt{b^{2}-4\,ac}\sqrt{bc-\sqrt{b^{2}-4\,ac}\,c}\,b^{2}+4\,\sqrt{2}\sqrt{b^{2}-4\,ac}\sqrt{bc-\sqrt{b^{2}-4\,ac}\,c}\,ac+2\,\sqrt{2}\sqrt{b^{2}-4\,ac}\sqrt{bc-\sqrt{b^{2}-4\,ac}\,c}\,bc-\sqrt{2}\sqrt{b^{2}-4\,ac}\sqrt{bc-\sqrt{b^{2}-4\,ac}\,c}\,c^{2}-2\left(b^{2}-4\,ac\right)c^{2}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{b-\sqrt{b^{2}-4\,ac}}{c}}}\right)}{2\left(b^{4}-8\,ab^{2}c-2\,b^{3}c+16\,a^{2}c^{2}+8\,abc^{2}+b^{2}c^{2}-4\,ac^{3}\right)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a
*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*
b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)) + 1/2
*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*
c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*b^2*
c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
```

maple [A] time = 0.02, size = 208, normalized size = 1.39

$$\frac{\sqrt{2}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
```

```
^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1
/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 4.46, size = 416, normalized size = 2.77

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,\left(4\,a\,c^2-2\,b^2\,c\right)+\frac{x\,\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\,\left(b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{-\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\,\sqrt{-\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}-2\,\mathrm{atanh}\left(\frac{\left(x\,\left(4\,a\,c^2-2\,b^2\,c\right)-\frac{x\,\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\,\left(\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{\frac{\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\,\sqrt{\frac{\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-
(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2
*c^2)))^(1/2))/(a*c))*(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*
c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) - (
x*(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*
b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(((-(4*a*c - b^2
)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

sympy [A] time = 2.62, size = 75, normalized size = 0.50

$$\operatorname{RootSum}\left(t^4\left(256a^2c^3 - 128ab^2c^2 + 16b^4c\right) + t^2\left(-16abc + 4b^3\right) + a, \left(t \mapsto t\log\left(64t^3ac^2 - 16t^3b^2c - 2tb + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a*
b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 2*
_t*b + x)))
```

3.661 $\int \frac{1}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1093, 205}

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.08, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(-1), x]
```

fricas [B] time = 0.75, size = 613, normalized size = 4.09

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

giac [B] time = 0.58, size = 1024, normalized size = 6.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^
2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs
(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
```

```
 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

maple [A] time = 0.02, size = 116, normalized size = 0.77

$$-\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}-\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^{4}+bx^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 4.61, size = 763, normalized size = 5.09

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x\,16\mathrm{i}-a\,b^2\,c\,x\,8\mathrm{i}}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x\,16\mathrm{i}-a\,b^2\,c\,x\,8\mathrm{i}}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*
a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
- 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2
) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(b^3 + (
```

```
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 +
 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i
)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

sympy [A] time = 2.84, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

3.662 $\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=174

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Rubi [A] time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx &= -\frac{1}{ax}+\frac{\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,d\\ &= -\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.38, size = 191, normalized size = 1.10

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2 + c*x^4)), x]
```

fricas [B] time = 0.81, size = 1116, normalized size = 6.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2
*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a
^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))
)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 -
 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^
3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
- 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*s
qrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqr
t(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*
c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

giac [B] time = 0.97, size = 1839, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
```

```
a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^2*c
^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*
(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^
2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*
x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*
a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(
c)) + 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*
b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2
 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 -
2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(
1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)
*abs(c)) - 1/(a*x)
```

maple [A] time = 0.02, size = 232, normalized size = 1.33

$$\frac{\sqrt{2}\, bc \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, c \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}-\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b-1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b-1/a/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

mupad [B] time = 4.85, size = 2997, normalized size = 17.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^
3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b
*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
```

```
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2
*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - a
tan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*
b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c
^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a
^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5
 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^
3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^
3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*
(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(a
*x)
```

sympy [A] time = 4.92, size = 148, normalized size = 0.85

$$\text{RootSum}\left(t^4\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2+48t^3a^4b^2c-8t^3a^3b^4-10ta^2bc^2+10tab^3c-2tb^5}{ac^3-b^2c^2}\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**
3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2
+ 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

3.663 $\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=196

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Rubi [A] time = 0.42, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1123, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2+cx^4\right)}\,dx = -\frac{1}{3ax^3}+\frac{\int \frac{-3b-3cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{3a}$$
$$= -\frac{1}{3ax^3}+\frac{b}{a^2x}-\frac{\int \frac{-3\left(b^2-ac\right)-3bcx^2}{a+bx^2+cx^4}\,dx}{3a^2}$$
$$= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2}+\frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-}}$$
$$= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{}{\sqrt{b-}}\right)}{\sqrt{2}\,a^2\sqrt{b+\sqrt{b^2-4a}}}$$

Mathematica [A] time = 0.14, size = 216, normalized size = 1.10

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{x^3}+\frac{6b}{x}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^3 + (6*b)/x + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-b^2 + 2*a*c + b*Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2 + c*x^4)), x]
```

fricas [B] time = 0.85, size = 1622, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 -
 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4
+ a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3
 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-
(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^
2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 +
 (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3
*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*
a^3*b^2*c^3 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*
c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c
)))/(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(
b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 + (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^
```

```
2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2
 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*
c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 + (a^5*b^5 - 7*a^6*b^3*c + 12
*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2
- 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*b*x^2 + 2*a)/(a^2*x^3)
```

giac [B] time = 1.16, size = 1640, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c
 - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*c
^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*
a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
- 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 -
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)
*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*
a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b +
sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c +
16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/4*(sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c +
24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 5*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^2*c^3 + 14*a*b^3
```

```
*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 - 24*
a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 -
 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 12*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 10*(
b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3
- 6*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b - sqrt(a^4*b^2
- 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 +
8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/3*(3*b*x^2 - a)/(a^2*x^3
)
```

maple [B] time = 0.02, size = 368, normalized size = 1.88

$$\frac{\sqrt{2}\, c^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}\, a}+\frac{\sqrt{2}\, c^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}\, a}-\frac{\sqrt{2}\, b^{2} c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}\, a^{2}}-\frac{\sqrt{2}\, b^{2} c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}\, a^{2}}-\frac{\sqrt{2}\, b c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}\, a^{2}}+\frac{\sqrt{2}\, b c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}\, a^{2}}+\frac{b}{a^{2} x}-\frac{1}{3 a x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+1/2/a^2*c*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*b+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/a^2*c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/3/a/x^3+b/a^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((b*c*x^2 + b^2 - a*c)/(c*x^4 + b*x^2 + a), x)/a^2 + 1/3*(3*b*x^2
- a)/(a^2*x^3)
```

mupad [B] time = 0.79, size = 4160, normalized size = 21.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - (1/(3*a) - (b*x^2)/a^2)/x^3 - atan(((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6
*b^2*c)))^(1/2)*(16*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3
) - x*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7
*c^2 - 8*a^6*b^2*c)))^(1/2)*1i - (((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20
*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c
- 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*
c)))^(1/2)*(16*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^
4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3) + x
*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)^(1/2
) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2
- 8*a^6*b^2*c)))^(1/2)*1i)/((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*
b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a
*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^
(1/2)*(16*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3) - x*(4*a
^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b
^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a
*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a
^6*b^2*c)))^(1/2) + (((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 -
25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(1
6*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(4*a*c - b^2)^3)^(1
/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2
) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^
2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3) + x*(4*a^8*c^5 +
 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*
a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c -
 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c
)))^(1/2) - 2*a^6*b*c^5))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c
```

^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/
2)*2i - atan((((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^
2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a
c - b^2)^3)^(1/2))/(8(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^1
0*c^4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)
(-(b^7 + b^4(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^
2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) - x*(4*a^8*c^5 + 2*a
^6*b^4*c^3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3
*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))
^(1/2)*1i - ((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*
b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*
c^4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(
-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) + x*(4*a^8*c^5 + 2*a^6
*b^4*c^3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(
1/2)*1i)/(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^
3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*c^
4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(
b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) - x*(4*a^8*c^5 + 2*a^6*b
^4*c^3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/
2) + ((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*c^4 + 4
*a^8*b^4*c^2 - 20*a^9*b^2*c^3 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7 +
 b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) + x*(4*a^8*c^5 + 2*a^6*b^4*c^
3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 +
25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) -
2*a^6*b*c^5))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2
*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*2i

sympy [A] time = 16.67, size = 211, normalized size = 1.08

$$\operatorname{RootSum}\left(t^4\left(256a^7c^2-128a^6b^2c+16a^5b^4\right)+t^2\left(-80a^3bc^3+100a^2b^3c^2-36ab^5c+4b^7\right)+c^5,\left(t\mapsto t\log\left(x+\frac{-96t^3a^7bc^2+56t^3a^6b^3c-8t^3a^5b^5-4ta^4c^4+32ta^3b^2c^3-40ta^2b^4c^2+16tab^6c-2tb^8}{a^2c^5-3ab^2c^4+b^4c^3}\right)\right)\right)+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**7*c**2 - 128*a**6*b**2*c + 16*a**5*b**4) + _t**2*(-80
*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + c**5, Lambda(_t
, _t*log(x + (-96*_t**3*a**7*b*c**2 + 56*_t**3*a**6*b**3*c - 8*_t**3*a**5*b
**5 - 4*_t*a**4*c**4 + 32*_t*a**3*b**2*c**3 - 40*_t*a**2*b**4*c**2 + 16*_t*
a*b**6*c - 2*_t*b**8)/(a**2*c**5 - 3*a*b**2*c**4 + b**4*c**3)))) + (-a + 3*
b*x**2)/(3*a**2*x**3)
```

3.664 $\int \frac{x^7}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=132

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Rubi [A] time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 738, 773, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\operatorname{Subst}\left(\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\operatorname{Subst}\left(\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c\left(b^2-4ac\right)}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} - \frac{\left(b\left(b^2\right.\right.}{}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{\left(b\left(b^2-6ac\right)\right)}{}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}} + \frac{\log}{}$$

Mathematica [A] time = 0.17, size = 121, normalized size = 0.92

$$\frac{\frac{2\left(-2a^2c+ab\left(b-3cx^2\right)+b^3x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2
 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.97, size = 663, normalized size = 5.02

$$\left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + 2\left(b^5 - 7\,ab^3c + 12\,a^2bc^2\right)x^2 + \left(\left(b^3c - 6\,abc^2\right)x^4 + ab^3 - 6\,a^2bc + \left(b^4 - 6\,ab^2c\right)x^2\right)\sqrt{b^2 - 4\,ac}\,\log\left(\frac{2\,c^2x^4 + 2\,bcx^2 + b^2 - 2\,ac + \left(2\,cx^2 + b\right)\sqrt{b^2 - 4\,ac}}{cx^4 + bx^2 + a}\right) + \left(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^4 + \left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)x^2\right)\log\left(cx^4 + bx^2 + a\right)}{4\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + \left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)x^4 + \left(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4\right)x^2\right)}, \frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + 2\left(b^5 - 7\,ab^3c + 12\,a^2bc^2\right)x^2 + 2\left(\left(b^3c - 6\,abc^2\right)x^4 + ab^3 - 6\,a^2bc + \left(b^4 - 6\,ab^2c\right)x^2\right)\sqrt{-b^2 + 4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2 + b\right)\sqrt{-b^2 + 4\,ac}}{b^2 - 4\,ac}\right) + \left(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^4 + \left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)x^2\right)\log\left(cx^4 + bx^2 + a\right)}{4\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + \left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)x^4 + \left(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c
^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*
x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3
*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (
b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 +
 b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

giac [A] time = 0.60, size = 152, normalized size = 1.15

$$-\frac{\left(b^3 - 6\,abc\right)\arctan\left(\frac{2\,cx^2 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\left(b^2c^2 - 4\,ac^3\right)\sqrt{-b^2 + 4\,ac}} - \frac{b^2cx^4 - 4\,ac^2x^4 - b^3x^2 + 2\,abcx^2 - ab^2}{4\left(cx^4 + bx^2 + a\right)\left(b^2c^2 - 4\,ac^3\right)} + \frac{\log\left(cx^4 + bx^2 + a\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4
*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*
b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4
+ b*x^2 + a)/c^2
```

maple [A] time = 0.02, size = 222, normalized size = 1.68

$$-\frac{3ab\arctan\left(\frac{2c\,x^2 + b}{\sqrt{4ac - b^2}}\right)}{\left(4ac - b^2\right)^{\frac{3}{2}} c} + \frac{b^3\arctan\left(\frac{2c\,x^2 + b}{\sqrt{4ac - b^2}}\right)}{2\left(4ac - b^2\right)^{\frac{3}{2}} c^2} + \frac{a\ln\left(c\,x^4 + b\,x^2 + a\right)}{\left(4ac - b^2\right)c} - \frac{b^2\ln\left(c\,x^4 + b\,x^2 + a\right)}{4\left(4ac - b^2\right)c^2} + \frac{\frac{\left(3ac - b^2\right)b\,x^2}{\left(4ac - b^2\right)c^2} + \frac{\left(2ac - b^2\right)a}{\left(4ac - b^2\right)c^2}}{2c\,x^4 + 2b\,x^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^
4+b*x^2+a)+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a-1/4/c^2/(4*a*c-b^2)*ln(c*x^4
+b*x^2+a)*b^2-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a
*b+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.10, size = 1336, normalized size = 10.12

$$\frac{\frac{a\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}+\frac{bx^2\left(3ac-b^2\right)}{2c^2\left(4ac-b^2\right)}}{cx^4+bx^2+a}-\frac{\ln\left(cx^4+bx^2+a\right)\left(-128a^3c^3+96a^2b^2c^2-24ab^4c+2b^6\right)}{2\left(256a^3c^5-192a^2b^2c^4+48ab^4c^3-4b^6c^2\right)}+\frac{b\,\mathrm{atan}\left(\frac{[illegible]}{36a^2b^2c^2-12ab^4c+b^6}\right)\left(6ac-b^2\right)}{2c^2\left(4ac-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)) + (b*x^2*(3*a*c - b^2))/(2*c^2*(4*
a*c - b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3
*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4
*c^3 - 192*a^2*b^2*c^4)) + (b*atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4
*a*c - b^2)^3)*(x^2*(((b*((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((
8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)
)/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*
b^2*c^4)))*(6*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(3/2)) + (b*(8*b^3*c^4 - 32*
a*b*c^5)*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))
/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 +
 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - (b*((b^3 - 5*a*b*c)/
(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((8*
b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/
(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^
2*c^4)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c
```

```
^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (b^2*((b^3*c^4)/2 - 2*a
*b*c^5)*(6*a*c - b^2)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(
4*a*c - b^2)^(3/2))) - ((b*(6*a*c - b^2)*(8*a + (8*a*c^2*(2*b^6 - 128*a^3*c
^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
- 192*a^2*b^2*c^4)))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*b*(6*a*c - b^2)*(2*b^
6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/((4*a*c - b^2)^(3/2)*(256*a
^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) +
(b*(a/c^2 + ((8*a + (8*a*c^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b
^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*b^6 -
 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 +
48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (a*b^2*(6*a*c - b^2)^2)/(c^2*(4*a*c - b^
2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(b^6 + 36*a^2*b^2*c^2 - 12*a*b^4*c))*(6
*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 40.65, size = 745, normalized size = 5.64

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)+8a^2c+16ab^2c^2\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)-ab^2-2b^4c\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)}{6abc-b^3}\right)+\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)+8a^2c+16ab^2c^2\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)-ab^2-2b^4c\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)}{6abc-b^3}\right)+\frac{2a^2c-ab^2+x^2\left(3abc-b^3\right)}{8a^2c^3-2ab^2c^2+x^4\left(8ac^4-2b^2c^3\right)+x^2\left(8abc^3-2b^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(
-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c
**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x**2*(3*a
*b*c - b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3)
+ x**2*(8*a*b*c**3 - 2*b**3*c**2))
```

3.665 $\int \frac{x^5}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 722, 618, 206}

$$\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
```

```
0] && LtQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 1.19

$$\frac{2a\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + \frac{a(b-2cx^2)+b^2x^2}{2c(4ac-b^2)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.99, size = 407, normalized size = 5.22

$$\left[-\frac{ab^3-4a^2bc+\left(b^4-6ab^2c+8a^2c^2\right)x^2+2\left(ac^2x^4+abcx^2+a^2c\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4c-8a^2b^2c^2+16a^3c^3+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^4+\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^2\right)}, -\frac{ab^3-4a^2bc+\left(b^4-6ab^2c+8a^2c^2\right)x^2-4\left(ac^2x^4+abcx^2+a^2c\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{2\left(ab^4c-8a^2b^2c^2+16a^3c^3+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^4+\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4
 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8
*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a
*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a
^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

giac [A] time = 0.91, size = 96, normalized size = 1.23

$$-\frac{2\,a\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{b^2x^2-2\,acx^2+ab}{2\left(cx^4+bx^2+a\right)\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c
^2))
```

maple [A] time = 0.01, size = 104, normalized size = 1.33

$$\frac{2a\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{\frac{ab}{\left(4ac-b^2\right)c}-\frac{\left(2ac-b^2\right)x^2}{\left(4ac-b^2\right)c}}{2c\,x^4+2b\,x^2+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a/
(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.18, size = 187, normalized size = 2.40

$$-\frac{\frac{x^2\left(2\,a\,c-b^2\right)}{2\,c\left(4\,a\,c-b^2\right)}-\frac{a\,b}{2\,c\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}-\frac{2\,a\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{x^2\left(\frac{4\,a\,c^2}{{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{4\,a\left(b^3\,c^2-4\,a\,b\,c^3\right)\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{13/2}}\right){\left(4\,a\,c-b^2\right)}^4}{8\,a^2\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b)/(2*c*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) - (2*a*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*((4
*a*c^2)/(4*a*c - b^2)^(7/2) + (4*a*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(13/2))*(4*a*c - b^2)^4)/(8*a^2*c^2)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 3.90, size = 282, normalized size = 3.62

$$-a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+ab}{2ac}\right)+a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{16a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+ab}{2ac}\right)+\frac{ab+x^2\left(-2ac+b^2\right)}{8a^2c^2-2ab^2c+x^4\left(8ac^3-2b^2c^2\right)+x^2\left(8abc^2-2b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*c
- b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a
```

```
**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)*
*3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + (a*b + x**2*(-2*a
*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**
2*(8*a*b*c**2 - 2*b**3*c))
```

3.666 $\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=75

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 638, 618, 206}

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}\\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac}\\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 1.05

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x^
2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.86, size = 360, normalized size = 4.80

$$\left[\frac{2\,ab^{2}-8\,a^{2}c+\left(b^{3}-4\,abc\right)x^{2}-\left(bcx^{4}+b^{2}x^{2}+ab\right)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+\left(2\,cx^{2}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)}{2\left(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+\left(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3}\right)x^{4}+\left(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2}\right)x^{2}\right)},\ \frac{2\,ab^{2}-8\,a^{2}c+\left(b^{3}-4\,abc\right)x^{2}-2\left(bcx^{4}+b^{2}x^{2}+ab\right)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^{2}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{2\left(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+\left(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3}\right)x^{4}+\left(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*
x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*x^2 +
 a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

giac [A] time = 0.62, size = 82, normalized size = 1.09

$$\frac{b\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{2}-4\,ac\right)\sqrt{-b^{2}+4\,ac}}+\frac{bx^{2}+2\,a}{2\left(cx^{4}+bx^{2}+a\right)\left(b^{2}-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.01, size = 77, normalized size = 1.03

$$-\frac{b\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}+\frac{-b\,x^{2}-2a}{2\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 4.57, size = 178, normalized size = 2.37

$$\frac{b\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{x^2\,{\left(4\,a\,c-b^2\right)}^4\,\left(\frac{b^2\,c^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{b^2\,\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,{\left(4\,a\,c-b^2\right)}^{13/2}}\right)}{2\,b^2\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{\frac{a}{4\,a\,c-b^2}+\frac{b\,x^2}{2\,\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (b*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((b^2*c^
2)/(a*(4*a*c - b^2)^(7/2)) + (b^2*(2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c))/
(2*a*(4*a*c - b^2)^(13/2))))/(2*b^2*c^2)))/(4*a*c - b^2)^(3/2) - (a/(4*a*c
- b^2) + (b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [**B**] time = 1.88, size = 269, normalized size = 3.59

$$\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)}{2}-\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)}{2}+\frac{-2a-bx^2}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b*
*2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a*
*2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3
) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 + (-2*a - b*x**2)/(8
*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

3.667 $\int \frac{x}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=74

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1107, 614, 618, 206}

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)\\ &= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{c\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}\\ &= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(2c)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\ &= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 1.07

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx^2}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 1.73, size = 361, normalized size = 4.88

$$\left[-\frac{b^{3} - 4\,abc + 2\,{\left(b^{2}c - 4\,ac^{2}\right)}x^{2} + 2\,{\left(c^{2}x^{4} + bcx^{2} + ac\right)}\sqrt{b^{2} - 4\,ac}\log\left(\frac{2\,c^{2}x^{4} + 2\,bcx^{2} + b^{2} - 2\,ac - {\left(2\,cx^{2} + b\right)}\sqrt{b^{2} - 4\,ac}}{cx^{4} + bx^{2} + a}\right)}{2\,{\left(ab^{4} - 8\,a^{2}b^{2}c + 16\,a^{3}c^{2} + {\left(b^{4}c - 8\,ab^{2}c^{2} + 16\,a^{2}c^{3}\right)}x^{4} + {\left(b^{5} - 8\,ab^{3}c + 16\,a^{2}bc^{2}\right)}x^{2}\right)}}, -\frac{b^{3} - 4\,abc + 2\,{\left(b^{2}c - 4\,ac^{2}\right)}x^{2} - 4\,{\left(c^{2}x^{4} + bcx^{2} + ac\right)}\sqrt{-b^{2} + 4\,ac}\arctan\left(-\frac{{\left(2\,cx^{2} + b\right)}\sqrt{-b^{2} + 4\,ac}}{b^{2} - 4\,ac}\right)}{2\,{\left(ab^{4} - 8\,a^{2}b^{2}c + 16\,a^{3}c^{2} + {\left(b^{4}c - 8\,ab^{2}c^{2} + 16\,a^{2}c^{3}\right)}x^{4} + {\left(b^{5} - 8\,ab^{3}c + 16\,a^{2}bc^{2}\right)}x^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x
^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

giac [A] time = 0.58, size = 82, normalized size = 1.11

$$-\frac{2\,c\arctan\left(\frac{2\,cx^{2} + b}{\sqrt{-b^{2} + 4\,ac}}\right)}{{\left(b^{2} - 4\,ac\right)}\sqrt{-b^{2} + 4\,ac}} - \frac{2\,cx^{2} + b}{2\,{\left(cx^{4} + bx^{2} + a\right)}{\left(b^{2} - 4\,ac\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.01, size = 75, normalized size = 1.01

$$\frac{2c\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}+\frac{2c\,x^{2}+b}{2\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.31, size = 172, normalized size = 2.32

$$\frac{\frac{b}{2\left(4\,a\,c-b^2\right)}+\frac{c\,x^2}{4\,a\,c-b^2}}{c\,x^4+b\,x^2+a}-\frac{2\,c\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{x^2\,{\left(4\,a\,c-b^2\right)}^4\,\left(\frac{4\,c^4}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{4\,c^2\,\left(b^3\,c^2-4\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{a\,{\left(4\,a\,c-b^2\right)}^{13/2}}\right)}{8\,c^4}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (b/(2*(4*a*c - b^2)) + (c*x^2)/(4*a*c - b^2))/(a + b*x^2 + c*x^4) - (2*c*at
an((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((4*c^4)/(a*(
4*a*c - b^2)^(7/2)) + (4*c^2*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(a*(4*a
*c - b^2)^(13/2))))/(8*c^4)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 2.78, size = 267, normalized size = 3.61

$$-c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc}{2c^2}\right)+c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc}{2c^2}\right)+\frac{b+2cx^2}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*c
- b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*
a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/
(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)
)
```

3.668 $\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=122

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Rubi [A] time = 0.20, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax} + \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} - \frac{\left(b\left(b^2-6ac\right)\right.}{}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}}{2a^2}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx\right.}{4a^2}$$

Mathematica [A] time = 0.33, size = 207, normalized size = 1.70

$$\frac{\frac{2a\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + 4\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[
x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2 + c*x^4)^2), x]
```

fricas [B] time = 0.94, size = 813, normalized size = 6.66

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\left(ab^3c-4\,a^2bc^2\right)x^2+\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)\log\left(cx^4+bx^2+a\right)+4\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)\log\left(x\right)}{4\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2+\left(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3\right)x^4+\left(a^2b^5-8\,a^3b^3c+16\,a^4bc^2\right)x^2\right)},\ \frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\left(ab^3c-4\,a^2bc^2\right)x^2+2\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)\log\left(cx^4+bx^2+a\right)+4\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)\log\left(x\right)}{4\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2+\left(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3\right)x^4+\left(a^2b^5-8\,a^3b^3c+16\,a^4bc^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 +
 ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^
2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12
*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b
*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*
a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
]
```

giac [A] time = 0.56, size = 166, normalized size = 1.36

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}+\frac{b^2cx^4-4\,ac^2x^4+b^3x^2-2\,abcx^2+3\,ab^2-8\,a^2c}{4\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4
*a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a*
b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/
4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

maple [B] time = 0.02, size = 253, normalized size = 2.07

$$-\frac{bc\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}-\frac{3bc\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a}+\frac{b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(4ac-b^2\right)^{\frac{3}{2}}a^2}-\frac{b^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}-\frac{c\ln\left(c\,x^4+b\,x^2+a\right)}{\left(4ac-b^2\right)a}+\frac{b^2\ln\left(c\,x^4+b\,x^2+a\right)}{4\left(4ac-b^2\right)a^2}+\frac{c}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)}+\frac{\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x^2+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c-
1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)+1
/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*b*c+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*b^3+1/a^2*ln(x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.29, size = 5048, normalized size = 41.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(2*a*(4*a*c - b^2)) - (b*c*x^2)/(2*a*(4*a*c - b
^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3 +
96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c +
192*a^4*b^2*c^2)) + (b*atan((x^2*((((((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 +
36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*
a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*
b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*
```

b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2
*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056
*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*
a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*
b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
c))/(2(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b*((
6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^
4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
 - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 1
2*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*
(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5
*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2
c^2)))(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(
2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688
*a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c)
)/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 7
2*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((6*a*b^5*c^4 + 80*a^3*
b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/
(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*
c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a
^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c +
 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2
*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*
a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^
3*b^4*c + 192*a^4*b^2*c^2)) - (b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3
+ 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(256
0*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^
6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a
^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^
2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 10
56*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^
3*b^4*c + 192*a^4*b^2*c^2))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b
^2)^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 +
 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(
32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^

2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))))/(8*a^3*c^2
*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16
*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b
^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2)))/(b^6*c^2 -
12*a*b^4*c^3 + 36*a^2*b^2*c^4) + (((b*((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a
^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5
*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
c))/(2(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6
*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) - (((b*((4*a^2*b^6*c^2 - 36*a^3*b^
4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*
c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^
2 - 24*a*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a
^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b
^2)^(3/2)) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*
c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b
^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 4
8*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24
*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) +
(b^3*(6*a*c - b^2)^3*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4))/(6
4*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^
6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4
*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 40*a^3*c^
3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(b^6*c^2 -
12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72
*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(16*a^6*b^6*(4*a*c - b^2)^
(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2
) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*((((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 +
80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32
*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 -
48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 -
24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*
a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (b^2*c^4)/(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c) + (b*(6*a*c - b^2)*((b*((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 +
80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32
*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 -
48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2
)) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*
b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b^2)^(3/2

```
)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^
4*c + 192*a^4*b^2*c^2)))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^
2*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 +
96*a^2*b^2*c^2 - 24*a*b^4*c))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
))/(8*a^3*c^2*(4*a*c - b^2)^3*(b^6*c^2 - 12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6*
b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(6*a*c - b^2))/(2*a^2*(
4*a*c - b^2)^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.669 $\int \frac{1}{x^3(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+b}{2ax^2\left(b^2-4ac\right)(a+}$$

Rubi [A] time = 0.25, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}+\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bcx^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int\left(\frac{2\left(-b^2+3ac\right)}{ax^2} - \frac{2b\left(-b^2+4ac\right)}{a^2x} + \frac{2\left(-b^4+5ab^2c-3a^2\cdots\right.}{a^2\left(a+b\cdots\right.}\right.\right.}{2a\left(b^2-4ac\right)\cdots}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} - \frac{\text{Subst}\left(\int \frac{-b^4+5ab\cdots}{\cdots}\right.}{a\cdots}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\,\text{Subst}\left(\int \frac{b+2c\cdots}{a+bx+\cdots}\right.}{2a^3}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx^2+\cdots\right.}{2a^3}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\cdots\right.}{a^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.27, size = 248, normalized size = 1.53

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{a\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{a}{x^2} - 4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 -
4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
b^2 - 4*a*c)^(3/2))/(2*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2 + c*x^4)^2), x]
```

fricas [B] time = 1.81, size = 1007, normalized size = 6.22

$$\left[-\frac{a^2b^4-8a^3b^2c+16a^4c^2+2\left(ab^4c-7a^2b^2c^2+12a^3c^3\right)x^4+\left(2ab^5-15a^2b^3c+28a^3bc^2\right)x^2+\left(\left(b^4c-6ab^2c^2+6a^2c^3\right)x^6+\left(b^5-6ab^3c+6a^2bc^2\right)x^4+\left(ab^4-6a^2b^2c+6a^3c^2\right)x^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-\left(\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^6+\left(b^6-8ab^4c+16a^2b^2c^2\right)x^4+\left(ab^5-8a^2b^3c+16a^3bc^2\right)x^2\right)\log\left(cx^4+bx^2+a\right)+4\left(\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^6+\left(b^6-8ab^4c+16a^2b^2c^2\right)x^4+\left(ab^5-8a^2b^3c+16a^3bc^2\right)x^2\right)\log(x)}{2\left(\left(a^3b^4c-8a^4b^2c^2+16a^5c^3\right)x^6+\left(a^3b^5-8a^4b^3c+16a^5bc^2\right)x^4+\left(a^4b^4-8a^5b^2c+16a^6c^2\right)x^2\right)}, -\frac{a^2b^4-8a^3b^2c+16a^4c^2+2\left(ab^4c-7a^2b^2c^2+12a^3c^3\right)x^4+\left(2ab^5-15a^2b^3c+28a^3bc^2\right)x^2+2\left(\left(b^4c-6ab^2c^2+6a^2c^3\right)x^6+\left(b^5-6ab^3c+6a^2bc^2\right)x^4+\left(ab^4-6a^2b^2c+6a^3c^2\right)x^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left(\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^6+\left(b^6-8ab^4c+16a^2b^2c^2\right)x^4+\left(ab^5-8a^2b^3c+16a^3bc^2\right)x^2\right)\log\left(cx^4+bx^2+a\right)+4\left(\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^6+\left(b^6-8ab^4c+16a^2b^2c^2\right)x^4+\left(ab^5-8a^2b^3c+16a^3bc^2\right)x^2\right)\log(x)}{2\left(\left(a^3b^4c-8a^4b^2c^2+16a^5c^3\right)x^6+\left(a^3b^5-8a^4b^3c+16a^5bc^2\right)x^4+\left(a^4b^4-8a^5b^2c+16a^6c^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a
*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 -
6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)
*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 +
 (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^
2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2
+ a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*
b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b
*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

giac [A] time = 0.59, size = 182, normalized size = 1.12

$$\frac{\left(b^4-6ab^2c+6a^2c^2\right)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\left(a^3b^2-4a^4c\right)\sqrt{-b^2+4ac}}-\frac{2b^2cx^4-6ac^2x^4+2b^3x^2-7abcx^2+ab^2-4a^2c}{2\left(cx^6+bx^4+ax^2\right)\left(a^2b^2-4a^3c\right)}+\frac{b\log\left(cx^4+bx^2+a\right)}{2a^3}-\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2*b
^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 -
 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

maple [B] time = 0.02, size = 352, normalized size = 2.17

$$-\frac{c^{2}x^{2}}{\left(c x^{4}+b x^{2}+a\right)\left(4ac-b^{2}\right)a}+\frac{b^{2}c x^{2}}{2\left(c x^{4}+b x^{2}+a\right)\left(4ac-b^{2}\right)a^{2}}-\frac{6c^{2}\arctan\left(\frac{2c x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}a}+\frac{6b^{2}c\arctan\left(\frac{2c x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}a^{2}}-\frac{b^{4}\arctan\left(\frac{2c x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}a^{3}}-\frac{3bc}{2\left(c x^{4}+b x^{2}+a\right)\left(4ac-b^{2}\right)a}+\frac{b^{3}}{2\left(c x^{4}+b x^{2}+a\right)\left(4ac-b^{2}\right)a^{2}}+\frac{2bc\ln\left(c x^{4}+b x^{2}+a\right)}{\left(4ac-b^{2}\right)a^{2}}-\frac{b^{3}\ln\left(c x^{4}+b x^{2}+a\right)}{2\left(4ac-b^{2}\right)a^{3}}-\frac{2b\ln(x)}{a^{3}}-\frac{1}{2a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b
^2)*x^2*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*c+1/2/a^2/(c*x^4+b*x^2+a)*b
^3/(4*a*c-b^2)+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*b-1/2/a^3/(4*a*c-b^2)*
ln(c*x^4+b*x^2+a)*b^3-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2))*c^2+6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^
2*c-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4-1/2/a
^2/x^2-2/a^3*b*ln(x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.81, size = 5491, normalized size = 33.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (log(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))
/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (1/(2*a) - (x
^2*(2*b^3 - 7*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(3*a*c - b^2))/(a^2*(4
```

*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) - (2*b*log(x))/a^3 + (atan(((2*a^9*b^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c)*((4*(2*b^5*c^4 - 12*a*b^3*c^5 + 18*a^2*b*c^6))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3 + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) + (((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*a^6*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)) - (x^2*((((4*(54*a^3*c^8 - 2*b^6*c^5 + 18*a*b^4*c^6 - 54*a^2*b^2*c^7))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*a^3*b^5*c^5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (((((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2

- 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*
(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*
b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(a^3*(4*a*c
- b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^
6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^
2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7 -
64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^
2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(2*a^6*(4*a*c - b^
2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a
^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c
^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b
^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((4*(480*a^8*c^7 - a^4*b^8
*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9
*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3
*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a
^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 +
6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 -
6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c
^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(
a^3*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*
b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4
*c + 48*a^5*b^2*c^2)) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*a^3*b^5*c^
5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*
a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(
b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b
^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)
)/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*
c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2
c)^3(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 -
672*a^9*b^3*c^5))/(2*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*
c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 3
82*a^3*b^2*c^3 + 72*a*b^6*c)))*(2*a^9*b^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^
3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2
*(4*a*c - b^2)^(9/2)))/(36*a^4*c^6 + b^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^
4 - 72*a^3*b^2*c^5) + (b*((((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5
*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*
c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2
- 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3
- 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4

```
*a*c - b^2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 +
 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))
/(a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b
^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)) - (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3 + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))
/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2
+ 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) -
(2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*
a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 -
 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2
*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((a^7
*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3)
/(2*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))*(2*a^9*b
^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(
4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 49*a^3*c
^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^
4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b
^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)))*(b^4 + 6*a^2*c^2
 - 6*a*b^2*c))/(a^3*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.670 $\int \frac{x^8}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+$$

Rubi [A] time = 0.84, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1279, 1166, 205}

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3b^2-10ac\right)}{2c^2\left(b^2-4ac\right)}+\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^3}{2c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^4\left(10a+3bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{x^2\left(9ab+3\left(3b^2-10ac\right)x^2\right)}{a+bx^2+cx^4}\,dx}{6c\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \text{[cut off: } \int \frac{3a\left(3b^2-10ac\right)+3b\left(3b^2-1\ldots\right.}{a+bx^2+cx^4} \text{ over } 6c^2\left(b^2-4ac\ldots\right]$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \text{[cut off: } \left(3b^3-13abc-\frac{3b^4-19\ldots}{\ldots}\ldots\right. \text{ over } 4\ldots\text{]}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \text{[cut off: } \left(3b^3-13abc-\frac{3b^4-19\ldots}{\ldots}\ldots\right. \text{ over } 2\sqrt{2}\,c^{5/2}\left(b^2\ldots\right. \text{]}$$

Mathematica [A] time = 0.66, size = 327, normalized size = 0.99

$$\frac{-\frac{2\sqrt{c}\,x\left(2a^2c-ab\left(b-3cx^2\right)+b^3\left(-x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{2}\left(-20a^2c^2+19ab^2c-13abc\sqrt{b^2-4ac}+3b^3\sqrt{b^2-4ac}-3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(20a^2c^2-19ab^2c-13abc\sqrt{b^2-4ac}+3b^3\sqrt{b^2-4ac}+3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+4\sqrt{c}\,x}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2
+ 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 -
 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c
^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 2.06, size = 2856, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2*
c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt
(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*
b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5
*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^
3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 +
336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^
6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385
*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
```

)/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b
^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*s
qrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*
a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818
*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6
+ 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c +
385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(
a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2
)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 -
12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 25
00*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8
818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c
+ 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2
)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*
x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2
- 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 -
918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10
- 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^
5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a

```
^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(3*
a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^
3*c^2 - 4*a*b*c^3)*x^2)
```

giac [B] time = 1.17, size = 3339, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/2*(b^3*x^3 - 3*a*b*c*x^3 + a*b^2*x - 2*a^2*c*x)/((c*x^4 + b*x^2 + a)*(b^2
*c^2 - 4*a*c^3)) + x/c^2 + 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8
 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 19
2*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50
*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 +
13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b
^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 - 2*(
3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7
- 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 -
```

80*(b^2 - 4*a*c)*a^3*c^6)*abs(-b^2*c^2 + 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c)) - 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 320*a^4*c^7 + 6*(b^2 - 4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c)*a^2*b^2*c^5 + 80*(b^2 - 4*a*c)*a^3*c^6)*abs(-b^2*c^2 + 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 - sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c))

maple [B] time = 0.04, size = 844, normalized size = 2.55

$$\frac{3abx^3}{2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)c}-\frac{b^3x^3}{2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)c^2}+\frac{5\sqrt{2}\,a^2\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{5\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{19\sqrt{2}\,ab^2\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{19\sqrt{2}\,ab^2\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c}+\frac{3\sqrt{2}\,b^4\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c^2}+\frac{3\sqrt{2}\,b^4\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c^2}+\frac{a^2x}{\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)c}-\frac{ab^2x}{2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)c^2}+\frac{13\sqrt{2}\,ab\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{13\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{3\sqrt{2}\,b^3\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c^2}+\frac{3\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c^2}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/c^2*x+3/2/c/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^3*a-1/2/c^2/(c*x^4+b*x^2+a)*b
^3/(4*a*c-b^2)*x^3+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x-1/2/c^2/(c*x^4+b*x
^2+a)*a/(4*a*c-b^2)*x*b^2+13/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-3/4/
c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*a^2-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*b^2+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*b^4-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b+3/4/c^2/(4*a*c-b^2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2-19
/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2+3/4/c^2/(4*a*c
-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3-3\,abc\right)x^3+\left(ab^2-2\,a^2c\right)x}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^4+\left(b^3c^2-4\,abc^3\right)x^2\right)}+\frac{-\int\frac{3\,ab^2-10\,a^2c+\left(3\,b^3-13\,abc\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(b^2c^2-4\,ac^3\right)}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b
^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*b^
2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4
*a*c^3) + x/c^2
```

mupad [B] time = 1.57, size = 7599, normalized size = 22.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((b*x^3*(3*a*c - b^2))/(2*(4*a*c - b^2)) + (a*x*(2*a*c - b^2))/(2*(4*a*c -
b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7 + 48*a*b^8*c^
3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11
+ b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*
b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3
*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3
*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*
a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 268
80*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4
4800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*
a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10
*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2
*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 422
4*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
- 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7)
)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024
0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 2
13*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2
 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b
^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2
*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 -
1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10
```

240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4
*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(
-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*
c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128
0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 2
40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))
^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*
a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a
^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 -
736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b
^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*
c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13
+ 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10
656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^
6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4
*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2) + (63*a^3*b^5 - 573*a^4*b^3*c + 1300*a^5*b*c^2)/(4*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*(-(9*b^13 + 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4

*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i - atan(((((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/(((((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(

```
1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128
0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 2
40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))
^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*
a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a
^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 -
 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b
^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 - 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*
c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13
 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10
656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^
6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4
*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2) + (63*a^3*b^5 - 573*a^4*b^3*c + 1300*a^5*b*c^2)/(4*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*(-(9*b^13 - 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i + x/c^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.671 $\int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=271

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+c\right.}$$

Rubi [A] time = 0.57, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx}{2c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^2\left(6a+bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{ab+\left(b^2-6ac\right)x^2}{a+bx^2+cx^4}\,dx}{2c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+c}}{4c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2-}}}\right.}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4a}}}$$

Mathematica [A] time = 0.50, size = 282, normalized size = 1.04

$$\frac{-\frac{2\sqrt{c}\,x\left(a\left(b-2cx^2\right)+b^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}-8abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(4*c^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 1.44, size = 2257, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60
*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*
a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 8
8*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) +
sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c
^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x
- 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c
```

```
^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((
b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c
^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a
*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^
4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a
*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88
*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^
6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + s
qrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^
2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x -
 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^
3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a
^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

giac [B] time = 1.06, size = 2736, normalized size = 10.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^3 - 2*a*c*x^3 + a*b*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
- 1/16*(2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4
```

*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24
*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a
*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
- 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*
c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2 - 2*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*
c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a
^2*b*c^4)*abs(b^2*c - 4*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^
2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3
)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a
^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2
*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) + 1/16*(2*b^8*c^4 - 32*
a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^
5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*
a*c^3)*(b^2*c - 4*a*c^2)^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b

```
^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a
^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b
*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(b^2*c - 4
*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b
*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)
))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3
*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*
abs(b^2*c - 4*a*c^2)*abs(c))
```

maple [B] time = 0.03, size = 602, normalized size = 2.22

$$\frac{2\sqrt{2}\,ab\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{2\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,b^3\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{3\sqrt{2}\,a\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{3\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,b^2\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c}+\frac{\frac{abx}{2\left(4ac-b^2\right)c}-\frac{\left(2ac-b^2\right)x^3}{2\left(4ac-b^2\right)c}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2/(4*a*c-b^2)*a*b/c*x)/(c*x^4+b*x^2+a
)-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^2+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-1/4/(
4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+3/2/(4*a*c-b^2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+2/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^2-2\,ac\right)x^3+abx}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}-\frac{-\int \frac{\left(b^2-6\,ac\right)x^2+ab}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b)/
(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

mupad [B] time = 6.00, size = 6293, normalized size = 23.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^3*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x)/(2*c*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*
c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2
*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b
^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(1
6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a
^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9
*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))^(1/2)*1i)/(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5
```

*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 66*a^3*b^2*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a

^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9
 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^
2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4
*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3
+ 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^
4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*
b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 38
40*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a
*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*
a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a
^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*
b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c -
b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*
a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^
4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c
^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(
4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3
 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i)/((((16*a*b^7*c^2 - 1024*a
^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3
840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*
a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24
*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2
*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c -
 b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840
*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c
^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a
^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*
c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-
(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^
3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(40
96*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4
*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 384
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*
b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a
*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^

```
5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b
^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b
^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a
^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9
 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^
2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4
*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3
+ 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 6
6*a^3*b^2*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*(-(
b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280
*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i
```

sympy [A] time = 51.73, size = 379, normalized size = 1.40

$$\frac{abx + x^3(-2ac + b^2)}{8a^2c^2 - 2ab^2c + x^4(8ac^3 - 2b^2c^2) + x^2(8abc^2 - 2b^3c)} + \operatorname{RootSum}\left(t^4(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a^4b^4c^7 - 327680a^3b^6c^6 + 61440a^2b^8c^5 - 6144ab^{10}c^4 + 256b^{12}c^3) + t^2(-61440a^5bc^5 + 61440a^4b^3c^4 - 24064a^3b^5c^3 + 4608a^2b^7c^2 - 432ab^9c + 16b^{11}) + 1296a^5c^2 - 360a^4b^2c + 25a^3b^4, \left(t \mapsto t\log\left(x + \frac{49152t^3a^4c^7 - 40960t^3a^3b^2c^6 + 12288t^3a^2b^4c^5 - 1536t^3ab^6c^4 + 64t^3b^8c^3 - 1728ta^3bc^3 + 656ta^2b^3c^2 - 88tab^5c + 4tb^7}{324a^3c^2 - 81a^2b^2c + 5ab^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (a*b*x + x**3*(-2*a*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 -
 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + RootSum(_t**4*(1048576*a**6
*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4*b**4*c**7 - 327680*a**3*b**6*c
**6 + 61440*a**2*b**8*c**5 - 6144*a*b**10*c**4 + 256*b**12*c**3) + _t**2*(-
61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2
*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**2*c +
25*a**3*b**4, Lambda(_t, _t*log(x + (49152*_t**3*a**4*c**7 - 40960*_t**3*a*
*3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b**6*c**4 + 64*_t*
*3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*b**3*c**2 - 88*_t*a*b**5*c
 + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*b**2*c + 5*a*b**4))))
```

3.672 $\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=237

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.41, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1120, 1166, 205}

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\left(b^2-4ac\right)^{3/2}} + \frac{\left(b^2+\right.}{}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2+\right.}{2\sqrt{}}$$

Mathematica [A] time = 0.40, size = 235, normalized size = 0.99

$$\frac{1}{4}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 1.11, size = 1668, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*
x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b
*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b
^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*
c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2
+ 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^
2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a
*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3
+ 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*
```

```
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [B] time = 1.06, size = 2132, normalized size = 9.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*b^7*c^2 -
 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*(b^2 - 4*a*c)^2 + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2
 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arctan(2*s
qrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*
c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a
*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^
3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*b^
7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 -
```

```
(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 -
 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2
*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 -
 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arc
tan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 -
 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^
2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4
- 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c))
```

maple [B] time = 0.03, size = 452, normalized size = 1.91

$$-\frac{\sqrt{2}\, ac \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, ac \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b^2 \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, b \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{-\frac{b x^3}{2\left(4ac-b^2\right)}-\frac{ax}{4ac-b^2}}{c x^4+b x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/4/(4*a
*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx^3+2ax}{2\left(\left(b^2c-4ac^2\right)x^4+ab^2-4a^2c+\left(b^3-4abc\right)x^2\right)}+ \text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c
)
```

mupad [B] time = 5.91, size = 4973, normalized size = 20.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4
)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^
2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(
b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3
- 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((
(-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b
^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8
*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i - (((2048*a^4*c^5 - 32*a
*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b
^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*
b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3
*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6
*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^
5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b
*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*
b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5
*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1
536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3
*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 12
80*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 -
 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*
c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2
*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
- (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^
3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1
280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*a^2*b*c
```

^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((2
048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2)
- b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 409
6*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b
^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b
*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2
*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2)
 - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 76
8*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7
 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))^(1/2)*2i - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^
2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5
*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^
2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^
4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^1
0*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^
2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240
*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1
/2)*1i - (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^
4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^
3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*
(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^
3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1
280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c
+ 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-
(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/
(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6
*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 -
32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 2

```
4*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144
*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^
2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)
^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))
/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 76
8*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7
- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))^(1/2) + (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3
 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
 (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 51
2*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*
c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^
2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^
(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 +
 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*
c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*
a^2*b*c^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512
*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
- 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*2i - ((a*
x)/(4*a*c - b^2) + (b*x^3)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [A] time = 9.01, size = 296, normalized size = 1.25

$$\frac{-2ax-bx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\operatorname{RootSum}\left(t^4\left(1048576a^6c^7-1572864a^5b^2c^6+983040a^4b^4c^5-327680a^3b^6c^4+61440a^2b^8c^3-6144ab^{10}c^2+256b^{12}c\right)+t^2\left(-12288a^4bc^4+8192a^3b^3c^3-1536a^2b^5c^2+16b^9\right)+16a^3c^2+24a^2b^2c+9ab^4,\left(t\mapsto t\log\left(x+\frac{16384t^3a^3bc^4-12288t^3a^2b^3c^3+3072t^3ab^5c^2-256t^3b^7c+64ta^2c^2-128tab^2c-4tb^4}{4ac+3b^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-2*a*x - b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**2*
c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c**3
 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3
*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a
**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2*c**2
 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

3.673 $\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.26, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1119, 1166, 205}

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b -
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
+ Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(1+\frac{2b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(b^2-4ac\right)} + \frac{\left(c\left(2b-\sqrt{b^2\right.\right.}{}$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2b\right.}{\sqrt{2}}$$

Mathematica [A] time = 0.43, size = 222, normalized size = 1.00

$$\frac{-bx-2cx^3}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b +
 Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
 + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 1.57, size = 1680, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*
c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6
*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3
*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
+ 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256
*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2
- 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2
)*x - 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c
+ 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
```

```
2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + 2*b*x)/((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [B] time = 0.98, size = 1970, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/8*(4*b^6*c^2 -
 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*
c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)
^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c
^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3
- 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/
((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^
4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4
*a*c)*abs(c)) - 1/8*(4*b^6*c^2 - 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*
c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
```

```
c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*
(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*
c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/
sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c -
 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3
*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4*a*c)*abs(c))
```

maple [A] time = 0.08, size = 342, normalized size = 1.55

$$\frac{\sqrt{2}\, bc \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{x}{2\left(4ac-b^2\right)\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}+\frac{x}{2\left(4ac-b^2\right)\left(x^2+\frac{b}{2c}+\frac{\sqrt{-4ac+b^2}}{2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))+c/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-1/2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)+1/2/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+c/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*
c)
```

mupad [B] time = 1.35, size = 4854, normalized size = 21.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^2,x)

[Out] atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1
/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 +
 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a
^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*
b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)
^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*
b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2
*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*
a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^
3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(
((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 128
0*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4
- 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) -
b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*
a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a
^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^
2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b
^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8
*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*
b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240
*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1
/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(
32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*
c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 +
96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*
c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^
5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b
```

^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
)*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a
^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 -
1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(((-(4*a*
c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)
/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^
6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + atan(((((8*b^7*c^
2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*
a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 -
24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 614
4*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*
b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2)
 - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7
*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^
4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b
^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*
a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/
2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b
^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^1
2 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 384
0*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a
^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^
3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3)
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 2
4*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144
*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 +
384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*
(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^
3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1
280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 -
 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b
^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 +
 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*
c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*
(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c -

```
b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^
12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 38
40*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^5*c^3 -
512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240
*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1
/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16
*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 -
96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*
c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^
5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b
^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^
3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^
3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + ((b*x)/(2*(4*a*c - b^
2)) + (c*x^3)/(4*a*c - b^2))/(a + b*x^2 + c*x^4)
```

sympy [A] time = 20.75, size = 298, normalized size = 1.35

$$\frac{bx+2cx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\operatorname{RootSum}\left(t^4\left(1048576a^7c^6-1572864a^6b^2c^5+983040a^5b^4c^4-327680a^4b^6c^3+61440a^3b^8c^2-6144a^2b^{10}c+256ab^{12}\right)+t^2\left(-12288a^4bc^4+8192a^3b^3c^3-1536a^2b^5c^2+16b^9\right)+16a^2c^3+24ab^2c^2+9b^4c,\left(t\mapsto t\log\left(x+\frac{16384t^3a^5c^4-8192t^3a^4b^2c^3+512t^3a^2b^6c-64t^3ab^8-128ta^2bc^2-16tab^3c-4tb^5}{4ac^2+3b^2c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(
8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 - 1572864*a**6*b**2*c
**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**6*c**3 + 61440*a**3*b**8*c**2
- 6144*a**2*b**10*c + 256*a*b**12) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3*
b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*c**2
+ 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3*a**4*
b**2*c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a*b**8 - 128*_t*a**2*b*c**2 -
16*_t*a*b**3*c - 4*_t*b**5)/(4*a*c**2 + 3*b**2*c))))
```

3.674 $\int \frac{1}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2 \cdots\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{\cdots}}$$

Rubi [A] time = 0.51, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1092, 1166, 205}

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}} + $$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - $$

Mathematica [A] time = 0.42, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(-2), x]
```

fricas [B] time = 1.00, size = 2309, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
```

```
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

giac [B] time = 0.86, size = 2682, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1
/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^
3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c
)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
```

```
*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b
^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c
)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sq
rt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2
 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*
a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 6
4*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)
*abs(c)) - 1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5
*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7
+ 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b
*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*
(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))
*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2
- 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a
^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3
*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^
2 - 4*a^2*c)*abs(c))
```

maple [B] time = 0.06, size = 733, normalized size = 2.91

$$\frac{\sqrt{2}\, b^{2} c \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4 \sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a}+\frac{\sqrt{2}\, b^{2} c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4 \sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{3 \sqrt{2}\, c^{2} \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}-\frac{3 \sqrt{2}\, c^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}-\frac{b^{2} x}{4 \sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \left(x^{2}+\frac{b}{2 c}-\frac{\sqrt{-4 a c+b^{2}}}{2 c}\right) a}+\frac{b^{2} x}{4 \sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \left(x^{2}+\frac{b}{2 c}+\frac{\sqrt{-4 a c+b^{2}}}{2 c}\right) a}+\frac{\sqrt{2}\, b c \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4 \left(4 a c-b^{2}\right) \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{\sqrt{2}\, b c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4 \left(4 a c-b^{2}\right) \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a}+\frac{c x}{\sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \left(x^{2}+\frac{b}{2 c}-\frac{\sqrt{-4 a c+b^{2}}}{2 c}\right)}-\frac{c x}{\sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \left(x^{2}+\frac{b}{2 c}+\frac{\sqrt{-4 a c+b^{2}}}{2 c}\right)}-\frac{b x}{4 \left(4 a c-b^{2}\right) \left(x^{2}+\frac{b}{2 c}-\frac{\sqrt{-4 a c+b^{2}}}{2 c}\right) a}-\frac{b x}{4 \left(4 a c-b^{2}\right) \left(x^{2}+\frac{b}{2 c}+\frac{\sqrt{-4 a c+b^{2}}}{2 c}\right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b+c/(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)-1/4/(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b^2+1/4*c/(
4*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*b-c/(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)+1/4/
(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*b
^2-1/4*c/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c
-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

mupad [B] time = 6.00, size = 6404, normalized size = 25.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 +
 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
```

2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 38
40*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*
b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
- b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 384
0*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2
 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 409
6*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b
^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a
^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-
(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1
504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c
^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^
5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^
5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16
*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 24
0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(
1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4
- 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^

2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c -

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b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*
c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*
(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7
*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c
^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((
((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a
^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x
*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2
- 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^
9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 -
1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b
*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 1
6*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5
*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c
 + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5)))^(1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3
*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4)
)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))
/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)
^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3
*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^
9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.675 $\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{3b^2-10ac}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4\right.}{2\sqrt{2}\,a^2\left(b^2-\right.}$$

Rubi [A] time = 1.44, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b
^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 -
10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3b^2+10ac-3bcx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-b\left(3b^2-13ac\right)-c\left(3b^2-10ac\right)x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\cdots\right.\right.}{4a^2\left(b^2\cdots\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\cdots\right.}{2\sqrt{2}\,a^2\left(b^2-4ac\right)\cdots}$$

Mathematica [A] time = 0.60, size = 302, normalized size = 0.98

$$\frac{-\frac{2x\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4}{x}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4
*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^2 + cx^4\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2 + c*x^4)^2), x]
```

fricas [B] time = 1.19, size = 2912, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c
)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
+ (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
+ 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^
5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*
a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^
5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7
*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
```

*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sq
rt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 144
08*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*
c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b
^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*
c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10
- 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1
280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 -
4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 -
1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
- 5200*a^5*b*c^5 + (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*
b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3
*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2
)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)

giac [B] time = 1.34, size = 3087, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*x^4 - 10*a*c^2*x^4 + 3*b^3*x^2 - 11*a*b*c*x^2 + 2*a*b^2 - 8*a
^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*(6*a^4*b^8*c^2 - 8
0*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 5
6*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + (6*b^4*c^2 -
44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*
a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2 + 2*(3*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c
- 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 5
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 -
304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 41
6*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 50*(b^2 - 4*a*c)*a^3*b^3*c^2 + 10
4*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a^3*c))*arctan(2*sqrt(1/2)*x/sqr
t((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*
c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b
^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8
*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*ab
s(c)) + 1/16*(6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^
2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8
```

```
 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c +
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 17
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 5
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 25
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 1
28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 +
28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 -
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 -
6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a
*c)*a^6*b^2*c^4 + (6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*
b^2 - 4*a^3*c)^2 - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 3
7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*
b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50
*(b^2 - 4*a*c)*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a
^3*c))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a
^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*
a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6
*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7
*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c))
```

maple [B] time = 0.04, size = 712, normalized size = 2.31

$$-\frac{c^{2} x^{3}}{\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a}+\frac{b^{2} c x^{3}}{2\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a^{2}}+\frac{4 \sqrt{2}\, b c^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\left(4 a c-b^{2}\right) \sqrt{-4 a c+b^{2}}\, \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a}+\frac{4 \sqrt{2}\, b c^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\left(4 a c-b^{2}\right) \sqrt{-4 a c+b^{2}}\, \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{3 \sqrt{2}\, b^{3} c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4\left(4 a c-b^{2}\right) \sqrt{-4 a c+b^{2}}\, \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a^{2}}-\frac{3 \sqrt{2}\, b^{3} c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4\left(4 a c-b^{2}\right) \sqrt{-4 a c+b^{2}}\, \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a^{2}}-\frac{3 b c x}{2\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a}+\frac{5 \sqrt{2}\, c^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2\left(4 a c-b^{2}\right) \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{5 \sqrt{2}\, c^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2\left(4 a c-b^{2}\right) \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a}+\frac{b^{3} x}{2\left(c x^{4}+b x^{2}+a\right)\left(4 a c-b^{2}\right) a^{2}}-\frac{3 \sqrt{2}\, b^{2} c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4\left(4 a c-b^{2}\right) \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a^{2}}+\frac{3 \sqrt{2}\, b^{2} c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4\left(4 a c-b^{2}\right) \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a^{2}}-\frac{1}{a^{2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b
^2)*x^3*b^2-3/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x+1/2/a^2/(c*x^4+b*x^2+a)
*b^3/(4*a*c-b^2)*x+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-3/4/a^2*c/(4
```

```
*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*b^3-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+3/4/a^2*c/(4*a*c-b
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3
-1/a^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3 b^2 c-10 a c^2\right) x^4+2 a b^2-8 a^2 c+\left(3 b^3-11 a b c\right) x^2}{2\left(\left(a^2 b^2 c-4 a^3 c^2\right) x^5+\left(a^2 b^3-4 a^3 b c\right) x^3+\left(a^3 b^2-4 a^4 c\right) x\right)}+\frac{-\int \frac{3 b^3-13 a b c+\left(3 b^2 c-10 a c^2\right) x^2}{c x^4+b x^2+a}\, dx}{2\left(a^2 b^2-4 a^3 c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^2
)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a
^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/(c*
x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

mupad [B] time = 6.72, size = 7555, normalized size = 24.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan((((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(85
1968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4
 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(
9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^
2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/
```

2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*
a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b
*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680
*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^
12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*
a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4
*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*
b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b
^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 +
3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i - ((-(9*b^13 - 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 +
4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a
^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c
^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*
a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9
)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^
4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a
*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^
6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 -
6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*
a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*
c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264
*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^
7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2688
0*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44
800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a
*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^1
0*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2
*c^5)))^(1/2)*1i)/(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*
b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^
5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5))
)^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a
^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*
c^7 + x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*
c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(104
8576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c
^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x

*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^
5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b
^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + ((-(9*b^13 - 9*b
^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^
8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 +
 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c
- b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
 - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096
*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b
^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2
 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a
^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^
3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^1
0*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2
) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*
c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*
c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24
*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*
a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9 + 504*a^6*b^8*c^5 - 8112*a^7*b^6*c^
6 + 48704*a^8*b^4*c^7 - 129280*a^9*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^
11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*
c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - atan((((-(9*b^13 + 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^
11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*
c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 46
72*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^
5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2)
 + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c
^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c
 - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*
a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a

^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i - ((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^

```
4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) +
26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4
- 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c -
51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6
*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10
*b^2*c^5)))^(1/2) + ((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6
*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a
^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*
c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)
))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*
a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3
*c^7 - x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 +
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(10
48576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*
c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) -
x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c
^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*
b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2
- 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-
(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^
8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9
 + 504*a^6*b^8*c^5 - 8112*a^7*b^6*c^6 + 48704*a^8*b^4*c^7 - 129280*a^9*b^2*
c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a
^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25
*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - (
1/a + (b*x^2*(11*a*c - 3*b^2))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(10*a*c - 3*b
^2))/(2*a^2*(4*a*c - b^2)))/(a*x + b*x^3 + c*x^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.676 $\int \frac{x^{11}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=209

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx^2\left(b^2-7ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^4\left(bx^2\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^8\,(2}{4\left(b^2-4ac\right)}$$

Rubi [A] time = 0.40, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1114, 738, 818, 773, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx^2\left(b^2-7ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^4\left(bx^2\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*(b^2 - 7*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^8*(2*a + b*x^2))/(4*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^4*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*
c)*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c +
 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)
^(5/2)) + Log[a + b*x^2 + c*x^4]/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^5}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^3(8a+bx)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x(2a(\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\cdots\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\cdots\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\cdots\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\cdots\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 0.32, size = 244, normalized size = 1.17

$$\frac{-\frac{2bc\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{2a^3c^2+a^2bc\left(5cx^2-4b\right)+ab^3\left(b-5cx^2\right)+b^5x^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{32a^3c^3-39a^2b^2c^2+50a^2bc^3x^2+11ab^4c-30ab^3c^2x^2-b^6+4b^5cx^2}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + c\log\left(a+bx^2+cx^4\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x^2 - 30*a*b^3*
c^2*x^2 + 50*a^2*b*c^3*x^2)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (2*a^3*
c^2 + b^5*x^2 + a*b^3*(b - 5*c*x^2) + a^2*b*c*(-4*b + 5*c*x^2))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[
```

```
(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + b*x^2 +
 c*x^4])/(4*c^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^11/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 2.36, size = 1631, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
- 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^6 + (3*b^8 - 31*a*b^6*c
+ 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(3*a*b^7 - 34*a^2*
b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3 + 3
0*a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*
b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*
b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*
c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b
^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*
a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3
*c^5 - 64*a^4*b*c^6)*x^2), 1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2
- 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*
x^6 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*
x^4 + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x^2 + 2*
((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*
a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*
c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b
```

```
^2 - 4*a*c)) + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8
+ a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3
*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3
*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b
^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*
a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6
- 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^
4*b*c^6)*x^2)]
```

giac [A] time = 1.84, size = 306, normalized size = 1.46

$$-\frac{\left(b^{5} - 10\,ab^{3}c + 30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(b^{4}c^{3} - 8\,ab^{2}c^{4} + 16\,a^{2}c^{5}\right)\sqrt{-b^{2} + 4\,ac}} - \frac{3\,b^{4}c^{2}x^{8} - 24\,ab^{2}c^{3}x^{8} + 48\,a^{2}c^{4}x^{8} - 2\,b^{5}cx^{6} + 12\,ab^{3}c^{2}x^{6} - 4\,a^{2}bc^{3}x^{6} - 3\,b^{6}x^{4} + 20\,ab^{4}cx^{4} - 22\,a^{2}b^{2}c^{2}x^{4} + 32\,a^{3}c^{3}x^{4} - 6\,ab^{5}x^{2} + 40\,a^{2}b^{3}cx^{2} - 28\,a^{3}bc^{2}x^{2} - 3\,a^{2}b^{4} + 18\,a^{3}b^{2}c}{8\,\left(b^{4}c^{3} - 8\,ab^{2}c^{4} + 16\,a^{2}c^{5}\right)\left(cx^{4} + bx^{2} + a\right)^{2}} + \frac{\log\left(cx^{4} + bx^{2} + a\right)}{4\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) - 1/8*(3*b^4
*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 - 2*b^5*c*x^6 + 12*a*b^3*c^2*x
^6 - 4*a^2*b*c^3*x^6 - 3*b^6*x^4 + 20*a*b^4*c*x^4 - 22*a^2*b^2*c^2*x^4 + 32
*a^3*c^3*x^4 - 6*a*b^5*x^2 + 40*a^2*b^3*c*x^2 - 28*a^3*b*c^2*x^2 - 3*a^2*b^
4 + 18*a^3*b^2*c)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)
^2) + 1/4*log(c*x^4 + b*x^2 + a)/c^3
```

maple [B] time = 0.02, size = 547, normalized size = 2.62

$$-\frac{15a^{2}b\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\;c}+\frac{5a\,b^{3}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\;c^{2}}-\frac{b^{5}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\;c^{3}}+\frac{4a^{2}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c}-\frac{2a\,b^{2}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}+\frac{b^{4}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}+\frac{\frac{\left(25a^{2}c^{2}-15a\,b^{2}c+2b^{4}\right)b\,x^{6}}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}+\frac{\left(31a^{2}c^{2}-22a\,b^{2}c+3b^{4}\right)ab\,x^{2}}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}+\frac{\left(32a^{3}c^{3}+11a^{2}b^{2}c^{2}-19a\,b^{4}c+3b^{6}\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}+\frac{3\left(8a^{2}c^{2}-7a\,b^{2}c+b^{4}\right)a^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1
/2*(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^4+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^
2+3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+
b*x^2+a)^2+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a^2-2/c^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a*b^2+1/4/c^3/(16*a^2*c^2-8*a*b^2*c
+b^4)*ln(c*x^4+b*x^2+a)*b^4-15/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b+5/c^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/2/c^3/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 7.30, size = 2588, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^4*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(4*c^3*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2))/(2*c^
3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*(a*b^4 + 8*a^3*c^2 - 7*a^2*b^2*c))
/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^6*(2*b^4 + 25*a^2*c^2 - 15*a
*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 +
 c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log((a/c^4 + ((c^3*(-(b^2*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) - 1)*((8*a)/c + (2*(c^3
*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) - 1
)*(2*a + b*x^2))/c + (2*b*x^2*(3*b^4 + 62*a^2*c^2 - 26*a*b^2*c))/(c*(4*a*c
- b^2)^2)))/(4*c^3) + (x^2*(b^5 + 23*a^2*b*c^2 - 9*a*b^3*c))/(c^4*(4*a*c -
b^2)^2))*(a/c^4 - ((c^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a
*c - b^2)^5))^(1/2) + 1)*((8*a)/c - (2*(c^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a
*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) + 1)*(2*a + b*x^2))/c + (2*b*x^2*(3
*b^4 + 62*a^2*c^2 - 26*a*b^2*c))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (x^2*(b^5
+ 23*a^2*b*c^2 - 9*a*b^3*c))/(c^4*(4*a*c - b^2)^2)))*(2*b^10 - 2048*a^5*c^5
 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*
(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*
c^6 - 5120*a^4*b^2*c^7)) - (b*atan(((x^2*(((b*((6*b^5*c^3 - 52*a*b^3*c^4 +
124*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + ((8*b^5*c^6 - 64*a*b^
3*c^7 + 128*a^2*b*c^8)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*
b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^
2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a
^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(8*c^3*(4
*a*c - b^2)^(5/2)) + (b*(8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(b^4 + 3
0*a^2*c^2 - 10*a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3
*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(16*c^3*(4*a*c - b^2)^(5/2)*(16*
```

```
a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4
- 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))/(a*(4*a*c - b^2)
^2) - (b*((b^5 + 23*a^2*b*c^2 - 9*a*b^3*c)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*
c^5) + (((6*b^5*c^3 - 52*a*b^3*c^4 + 124*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 -
 8*a*b^2*c^5) + ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(2*b^10 - 2048*
a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*
c))/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80
*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(2*b^
10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 -
 40*a*b^8*c))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^
5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (b^2*((b^5*c^6)/2 - 4*a*b^3*c^7
 + 8*a^2*b*c^8)*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5*(16
*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5))))/(2*a*(4*a*c - b^2)^(5/2))) + ((b*((8*a
)/c + (8*a*c^2*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3
+ 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4
 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))*(b^4 + 30*a^2*c^
2 - 10*a*b^2*c))/(8*c^3*(4*a*c - b^2)^(5/2)) + (a*b*(b^4 + 30*a^2*c^2 - 10*
a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560
*a^4*b^2*c^4 - 40*a*b^8*c))/(c*(4*a*c - b^2)^(5/2)*(4096*a^5*c^8 - 4*b^10*c
^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))
)/(a*(4*a*c - b^2)^2) - (b*(a/c^4 + (((8*a)/c + (8*a*c^2*(2*b^10 - 2048*a^5
*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))
/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4
*c^6 - 5120*a^4*b^2*c^7))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a
^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4096*a^5*c^8 - 4*b^10*c^3
+ 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) -
(a*b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^4*(4*a*c - b^2)^5)))/(2*a*(4*a
*c - b^2)^(5/2)))*(32*a^2*c^6*(4*a*c - b^2)^5 + 2*b^4*c^4*(4*a*c - b^2)^5 -
 16*a*b^2*c^5*(4*a*c - b^2)^5))/(b^10 + 160*a^2*b^6*c^2 - 600*a^3*b^4*c^3 +
 900*a^4*b^2*c^4 - 20*a*b^8*c))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*c^3*(4*
a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.677 $\int \frac{x^9}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=121

$$-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 722, 618, 206}

$$-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^6*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*a*x^2*(2*a
 + b*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*a^2*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
```

```
0] && LtQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{(3a)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{a+bx+c\cdots}\right.}{\left(b^2-4ac\right)\cdots}$$

$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(6a^2\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac\cdots}\right.}{\left(b^2-4\cdots\right.}$$

$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.17, size = 194, normalized size = 1.60

$$\frac{1}{4}\left(\frac{24a^2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{a^2c\left(2cx^2-3b\right)+ab^2\left(b-4cx^2\right)+b^4x^2}{c^3\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2} + \frac{22a^2bc^2-20a^2c^3x^2-8ab^3c+16ab^2c^2x^2+b^5-2b^4cx^2}{c^3\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x^2 + 16*a*b^2*c^2*x^2 - 20*a^2*
c^3*x^2)/(c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^4*x^2 + a*b^2*(b -
4*c*x^2) + a^2*c*(-3*b + 2*c*x^2))/(c^3*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^
```

```
2) + (24*a^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.72, size = 973, normalized size = 8.04

$$\left[-\frac{a^2b^5-14\,a^3b^3c+40\,a^4bc^2+2\left(b^6c-12\,ab^4c^2+42\,a^2b^2c^3-40\,a^3c^4\right)x^6+\left(b^7-12\,ab^5c+30\,a^2b^3c^2+8\,a^3bc^3\right)x^4+2\left(ab^6-14\,a^2b^4c+46\,a^3b^2c^2-24\,a^4c^3\right)x^2-12\left(a^2c^4x^8+2\,a^2bc^3x^6+2\,a^3bc^2x^2+a^4c^2+\left(a^2b^2c^2+2\,a^3c^3\right)x^4\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{4\left(a^2b^6c^2-12\,a^3b^4c^3+48\,a^4b^2c^4-64\,a^5c^5+\left(b^6c^4-12\,ab^4c^5+48\,a^2b^2c^6-64\,a^3c^7\right)x^8+2\left(b^7c^3-12\,ab^5c^4+48\,a^2b^3c^5-64\,a^3bc^6\right)x^6+\left(b^8c^2-10\,ab^6c^3+24\,a^2b^4c^4+32\,a^3b^2c^5-128\,a^4c^6\right)x^4+2\left(ab^7c^2-12\,a^2b^5c^3+48\,a^3b^3c^4-64\,a^4bc^5\right)x^2\right)},\ -\frac{a^2b^5-14\,a^3b^3c+40\,a^4bc^2+2\left(b^6c-12\,ab^4c^2+42\,a^2b^2c^3-40\,a^3c^4\right)x^6+\left(b^7-12\,ab^5c+30\,a^2b^3c^2+8\,a^3bc^3\right)x^4+2\left(ab^6-14\,a^2b^4c+46\,a^3b^2c^2-24\,a^4c^3\right)x^2+24\left(a^2c^4x^8+2\,a^2bc^3x^6+2\,a^3bc^2x^2+a^4c^2+\left(a^2b^2c^2+2\,a^3c^3\right)x^4\right)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{4\left(a^2b^6c^2-12\,a^3b^4c^3+48\,a^4b^2c^4-64\,a^5c^5+\left(b^6c^4-12\,ab^4c^5+48\,a^2b^2c^6-64\,a^3c^7\right)x^8+2\left(b^7c^3-12\,ab^5c^4+48\,a^2b^3c^5-64\,a^3bc^6\right)x^6+\left(b^8c^2-10\,ab^6c^3+24\,a^2b^4c^4+32\,a^3b^2c^5-128\,a^4c^6\right)x^4+2\left(ab^7c^2-12\,a^2b^5c^3+48\,a^3b^3c^4-64\,a^4bc^5\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42
*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3
*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x^2 -
12*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 + a^4*c^2 + (a^2*b^2*c^
2 + 2*a^3*c^3)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*
a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a^2*b^6*c^2 -
 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^
5 - 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b
^2*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4
- 64*a^4*b*c^5)*x^2), -1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*
c - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12*a*b^5*c + 3
0*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2
 - 24*a^4*c^3)*x^2 + 24*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 +
a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 4
8*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*
a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x
^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^
6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2
)]
```

giac [A] time = 1.87, size = 212, normalized size = 1.75

$$\frac{6\,a^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^4cx^6-16\,ab^2c^2x^6+20\,a^2c^3x^6+b^5x^4-8\,ab^3cx^4-2\,a^2bc^2x^4+2\,ab^4x^2-20\,a^2b^2cx^2+12\,a^3c^2x^2+a^2b^3-10\,a^3bc}{4\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)\left(cx^4+bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 6*a^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*b^4*c*x^6 - 16*a*b^2*c^2*x^6 + 20*a^2*c^3*
x^6 + b^5*x^4 - 8*a*b^3*c*x^4 - 2*a^2*b*c^2*x^4 + 2*a*b^4*x^2 - 20*a^2*b^2*
c*x^2 + 12*a^3*c^2*x^2 + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.02, size = 267, normalized size = 2.21

$$\frac{6a^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^6}{\left(16a^2c^2-8a\,b^2c+b^4\right)c}+\frac{\left(2a^2c^2+8a\,b^2c-b^4\right)b\,x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{\left(10ac-b^2\right)a^2b}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}-\frac{\left(6a^2c^2-10a\,b^2c+b^4\right)a\,x^2}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}}{2\left(c\,x^4+b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*b*(
2*a^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-a*(6*a^2*c^2-10
*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2+1/2*a^2*b*(10*a*c-b^2)/c^2
/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+6*a^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.53, size = 444, normalized size = 3.67

$$\frac{6\,a^2\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{36\,a^3\,c^2}{{\left(4\,a\,c-b^2\right)}^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{36\,a^3\,b\,\left(16\,a^2\,b\,c^4-8\,a\,b^3\,c^3+b^5\,c^2\right)}{{\left(4\,a\,c-b^2\right)}^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{72\,a^4\,b\,c^2}{{\left(4\,a\,c-b^2\right)}^{15/2}}\right)\left(b^4\,{\left(4\,a\,c-b^2\right)}^5+16\,a^2\,c^2\,{\left(4\,a\,c-b^2\right)}^5-8\,a\,b^2\,c\,{\left(4\,a\,c-b^2\right)}^5\right)}{72\,a^4\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{\frac{x^6\,\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^3-10\,a\,b\,c\right)}{4\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^4\,\left(2\,a^2\,b\,c^2+8\,a\,b^3\,c-b^5\right)}{4\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x^2\,\left(6\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (6*a^2*atan(((x^2*((36*a^3*c^2)/((4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (36*a^3*b*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/((4*a*c - b^2
)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (72*a^4*b*c^2)/(4*a*c - b^2)^(1
5/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c
- b^2)^5))/(72*a^4*c^2)))/(4*a*c - b^2)^(5/2) - ((x^6*(b^4 + 10*a^2*c^2 - 8
*a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 - 10*a*b*c))/(4
*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^4*(2*a^2*b*c^2 - b^5 + 8*a*b^3*c)
)/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x^2*(b^4 + 6*a^2*c^2 - 10*a*b
^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c
^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [B] time = 4.64, size = 554, normalized size = 4.58

$$-3a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a^2b}{6a^2c}\right)+3a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-3a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a^2b}{6a^2c}\right)+\frac{10a^3bc-a^2b^3+x^6(-20a^2c^3+16ab^2c^2-2b^4c)+x^4(2a^2bc^2+8ab^3c-b^5)+x^2(-12a^3c^2+20a^2b^2c-2ab^4)}{64a^4c^4-32a^3b^2c^3+4a^2b^4c^2+x^8(64a^2c^6-32ab^2c^5+4b^4c^4)+x^6(128a^2bc^5-64ab^3c^4+8b^5c^3)+x^4(128a^3c^5-24ab^4c^3+4b^6c^2)+x^2(128a^3bc^4-64a^2b^3c^3+8ab^5c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**5*c**3*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**3*b**
4*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 3
*a**2*b)/(6*a**2*c)) + 3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a*
*5*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5) + 36*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a**2*b**6*sqrt(-1/(
4*a*c - b**2)**5) + 3*a**2*b)/(6*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x**6
*(-20*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**4*(2*a**2*b*c**2 + 8*a*b*
*3*c - b**5) + x**2*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(64*a**4*c
**4 - 32*a**3*b**2*c**3 + 4*a**2*b**4*c**2 + x**8*(64*a**2*c**6 - 32*a*b**2
*c**5 + 4*b**4*c**4) + x**6*(128*a**2*b*c**5 - 64*a*b**3*c**4 + 8*b**5*c**3
) + x**4*(128*a**3*c**5 - 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*(128*a**3*b*
c**4 - 64*a**2*b**3*c**3 + 8*a*b**5*c**2))
```

3.678 $\int \frac{x^7}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=119

$$\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 728, 722, 618, 206}

$$-\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*b*x^2*(2*
a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*a*b*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
```

```
0] && LtQ[p, -1]
```

Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{(3ab)\,\text{Subst}\left(\int \frac{1}{a+bx+c}\right.}{2\left(b^2-4ac\right.}$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(3ab)\,\text{Subst}\left(\int \frac{1}{b^2-4ac}\right.}{\left(b^2-4\right.}$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.19, size = 137, normalized size = 1.15

$$-\frac{8a^3c+a^2\left(b^2+10bcx^2+16c^2x^4\right)+abx^2\left(2b^2+bcx^2+6c^2x^4\right)+b^4x^4}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}-\frac{3ab\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(8*a^3*c + b^4*x^4 + a*b*x^2*(2*b^2 + b*c*x^2 + 6*c^2*x^4) + a^2*(b^2
+ 10*b*c*x^2 + 16*c^2*x^4))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) - (3*
a*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.96, size = 892, normalized size = 7.50

$$\left[-\frac{6\left(ab^3c^2-4a^2bc^3\right)x^6+a^2b^4+4a^3b^2c-32a^4c^2+\left(b^6-3ab^4c+12a^2b^2c^2-64a^3c^3\right)x^4+2\left(ab^5+a^2b^3c-20a^3bc^2\right)x^2-6\left(abc^3x^8+2ab^2c^2x^6+2a^2b^2cx^2+a^3bc+\left(ab^3c+2a^2bc^2\right)x^4\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{4\left(a^2b^6c-12a^3b^4c^2+48a^4b^2c^3-64a^5c^4+\left(b^6c^3-12ab^4c^4+48a^2b^2c^5-64a^3c^6\right)x^8+2\left(b^7c^2-12ab^5c^3+48a^2b^3c^4-64a^3bc^5\right)x^6+\left(b^8c-10ab^6c^2+24a^2b^4c^3+32a^3b^2c^4-128a^4c^5\right)x^4+2\left(ab^7c-12a^2b^5c^2+48a^3b^3c^3-64a^4bc^4\right)x^2\right)},\ -\frac{6\left(ab^3c^2-4a^2bc^3\right)x^6+a^2b^4+4a^3b^2c-32a^4c^2+\left(b^6-3ab^4c+12a^2b^2c^2-64a^3c^3\right)x^4+2\left(ab^5+a^2b^3c-20a^3bc^2\right)x^2-12\left(abc^3x^8+2ab^2c^2x^6+2a^2b^2cx^2+a^3bc+\left(ab^3c+2a^2bc^2\right)x^4\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{4\left(a^2b^6c-12a^3b^4c^2+48a^4b^2c^3-64a^5c^4+\left(b^6c^3-12ab^4c^4+48a^2b^2c^5-64a^3c^6\right)x^8+2\left(b^7c^2-12ab^5c^3+48a^2b^3c^4-64a^3bc^5\right)x^6+\left(b^8c-10ab^6c^2+24a^2b^4c^3+32a^3b^2c^4-128a^4c^5\right)x^4+2\left(ab^7c-12a^2b^5c^2+48a^3b^3c^3-64a^4bc^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^6 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2
 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b^3
*c - 20*a^3*b*c^2)*x^2 - 6*(a*b*c^3*x^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2
 + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
+ 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^
3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5
*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*
b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 +
 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2), -1/4*(6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^6
 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 -
 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x^2 - 12*(a*b*c^3*x
^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2 + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*
```

```
x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3
- 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^
4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 4
8*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

giac [A] time = 1.77, size = 171, normalized size = 1.44

$$-\frac{3\,ab\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,abc^2x^6+b^4x^4+ab^2cx^4+16\,a^2c^2x^4+2\,ab^3x^2+10\,a^2bcx^2+a^2b^2+8\,a^3c}{4\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\left(cx^4+bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*a*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*a*b*c^2*x^6 + b^4*x^4 + a*b^2*c*x^4 + 16*
a^2*c^2*x^4 + 2*a*b^3*x^2 + 10*a^2*b*c*x^2 + a^2*b^2 + 8*a^3*c)/((b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.02, size = 230, normalized size = 1.93

$$-\frac{3ab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{-\frac{3abc\,x^6}{16a^2c^2-8a\,b^2c+b^4}-\frac{\left(5ac+b^2\right)ab\,x^2}{\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(16a^2c^2+a\,b^2c+b^4\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(8ac+b^2\right)a^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}}{2\left(c\,x^4+b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-3
*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.44, size = 423, normalized size = 3.55

$$-\frac{\frac{x^2\left(5ca^2b+ab^3\right)}{2c\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{x^4\left(16a^2c^2+ab^2c+b^4\right)}{4c\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{a\left(8ca^2+ab^2\right)}{4c\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{3abcx^6}{2\left(16a^2c^2-8ab^2c+b^4\right)}}{x^4\left(b^2+2ac\right)+a^2+c^2x^8+2abx^2+2bcx^6}-\frac{3ab\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{9ab^2c^2}{\left(4ac-b^2\right)^{9/2}\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{9ab^3\left(32a^2bc^4-16ab^3c^3+2b^5c^2\right)}{2\left(4ac-b^2\right)^{15/2}\left(16a^2c^2-8ab^2c+b^4\right)}\right)+\frac{18a^2b^3c^2}{\left(4ac-b^2\right)^{15/2}}\right)\left(b^4\left(4ac-b^2\right)^5+16a^2c^2\left(4ac-b^2\right)^5-8ab^2c\left(4ac-b^2\right)^5\right)}{18a^2b^2c^2}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^2*(a*b^3 + 5*a^2*b*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(b
^4 + 16*a^2*c^2 + a*b^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(a*b^
2 + 8*a^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^6)/(2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2
+ 2*b*c*x^6) - (3*a*b*atan(((x^2*((9*a*b^2*c^2)/((4*a*c - b^2)^(9/2)*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) + (9*a*b^3*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c
^4))/(2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (18*a^2*b^3
*c^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)
^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*a^2*b^2*c^2)))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 3.81, size = 524, normalized size = 4.40

$$\frac{3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^2}{6abc}\right)}{2}-\frac{3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-3ab^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^2}{6abc}\right)}{2}+\frac{-8a^3c-a^2b^2-6abc^2x^6+x^4\left(-16a^2c^2-ab^2c-b^4\right)+x^2\left(-10a^2bc-2ab^3\right)}{64a^4c^3-32a^3b^2c^2+4a^2b^4c+x^8\left(64a^2c^5-32ab^2c^4+4b^4c^3\right)+x^6\left(128a^2bc^4-64ab^3c^3+8b^5c^2\right)+x^4\left(128a^3c^4-24ab^4c^2+4b^6c\right)+x^2\left(128a^3bc^3-64a^2b^3c^2+8ab^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**4*b*c**3*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**
5*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*
b**2)/(6*a*b*c))/2 - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**4*
b*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**
2)**5) + 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*
c - b**2)**5) + 3*a*b**2)/(6*a*b*c))/2 + (-8*a**3*c - a**2*b**2 - 6*a*b*c**
2*x**6 + x**4*(-16*a**2*c**2 - a*b**2*c - b**4) + x**2*(-10*a**2*b*c - 2*a*
b**3))/(64*a**4*c**3 - 32*a**3*b**2*c**2 + 4*a**2*b**4*c + x**8*(64*a**2*c*
*5 - 32*a*b**2*c**4 + 4*b**4*c**3) + x**6*(128*a**2*b*c**4 - 64*a*b**3*c**3
 + 8*b**5*c**2) + x**4*(128*a**3*c**4 - 24*a*b**4*c**2 + 4*b**6*c) + x**2*(
128*a**3*b*c**3 - 64*a**2*b**3*c**2 + 8*a*b**5*c))
```

3.679 $\int \frac{x^5}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=130

$$-\frac{\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^2\left(2ac+b^2\right)+3ab}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 738, 638, 618, 206}

$$\frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^2\left(2ac+b^2\right)+3ab}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*a*b + (b^2
 + 2*a*c)*x^2)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2 + 2*a*c)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-2bx}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ab+\left(b^2+2ac\right)x^2}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(b^2+2ac\right)\text{Subst}\left(\int \ldots\right.}{2\left(b^2-4\ldots\right.}$$

$$= \frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ab+\left(b^2+2ac\right)x^2}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+2ac\right)\text{Subst}\left(\int \ldots\right.}{\left(b^2-\ldots\right.}$$

$$= \frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ab+\left(b^2+2ac\right)x^2}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+2ac\right)\tanh^{-1}\left(\ldots\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.13, size = 145, normalized size = 1.12

$$\frac{1}{4}\left(\frac{4\left(2ac+b^2\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{\left(2ac+b^2\right)\left(b+2cx^2\right)}{c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{a\left(b-2cx^2\right)+b^2x^2}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 + 2*a*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b
^2*x^2 + a*(b - 2*c*x^2))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(b^
2 + 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/
4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.60, size = 907, normalized size = 6.98

$$\left[\frac{2\left(b^4c-2ab^2c^2-8a^2c^3\right)x^6+6a^2b^3-24a^3bc+3\left(b^5-2ab^3c-8a^2bc^2\right)x^4+2\left(5ab^4-22a^2b^2c+8a^3c^2\right)x^2+2\left(\left(b^2c^2+2ac^3\right)x^8+2\left(b^3c+2abc^2\right)x^6+\left(b^4+4ab^2c+4a^2c^2\right)x^4+a^2b^2+2a^3c+2\left(ab^3+2a^2bc\right)x^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{4\left(\left(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5\right)x^8+a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+2\left(b^7c-12ab^5c^2+48a^2b^3c^3-64a^3bc^4\right)x^6+\left(b^8-10ab^6c+24a^2b^4c^2+32a^3b^2c^3-128a^4c^4\right)x^4+2\left(ab^7-12a^2b^5c+48a^3b^3c^2-64a^4bc^3\right)x^2\right)},\frac{2\left(b^4c-2ab^2c^2-8a^2c^3\right)x^6+6a^2b^3-24a^3bc+3\left(b^5-2ab^3c-8a^2bc^2\right)x^4+2\left(5ab^4-22a^2b^2c+8a^3c^2\right)x^2-4\left(\left(b^2c^2+2ac^3\right)x^8+2\left(b^3c+2abc^2\right)x^6+\left(b^4+4ab^2c+4a^2c^2\right)x^4+a^2b^2+2a^3c+2\left(ab^3+2a^2bc\right)x^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{4\left(\left(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5\right)x^8+a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+2\left(b^7c-12ab^5c^2+48a^2b^3c^3-64a^3bc^4\right)x^6+\left(b^8-10ab^6c+24a^2b^4c^2+32a^3b^2c^3-128a^4c^4\right)x^4+2\left(ab^7-12a^2b^5c+48a^3b^3c^2-64a^4bc^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3*
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2
)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 + 4*a
*b^2*c + 4*a^2*c^2)*x^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sq
rt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7
 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(2*(b^4*c - 2*a*
b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3*(b^5 - 2*a*b^3*c - 8*
a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x^2 - 4*((b^2*c^2 +
```

```
2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x
^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4
)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*
x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 1.81, size = 161, normalized size = 1.24

$$\frac{\left(b^{2}+2\,ac\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{2\,b^{2}cx^{6}+4\,ac^{2}x^{6}+3\,b^{3}x^{4}+6\,abcx^{4}+10\,ab^{2}x^{2}-4\,a^{2}cx^{2}+6\,a^{2}b}{4\left(cx^{4}+bx^{2}+a\right)^{2}\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (b^2 + 2*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^2*c*x^6 + 4*a*c^2*x^6 + 3*b^3*x^
4 + 6*a*b*c*x^4 + 10*a*b^2*x^2 - 4*a^2*c*x^2 + 6*a^2*b)/((c*x^4 + b*x^2 + a
)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

maple [B] time = 0.02, size = 270, normalized size = 2.08

$$\frac{2ac\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{b^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{\frac{\left(2ac+b^{2}\right)c\,x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3\left(2ac+b^{2}\right)b\,x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{3a^{2}b}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{\left(2ac-5b^{2}\right)a\,x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*((2*a*c+b^2)*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3/2*b*(2*a*c+b^2)/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^4-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+3*a^
2*b/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+2/(16*a^2*c^2-8*a*b^2*c+b
^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*c+1/(16*a^2*c
^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b
^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.46, size = 460, normalized size = 3.54

$$\frac{\frac{3a^2b}{2(16a^2c^2-8ab^2c+b^4)}+\frac{x^2(5ab^2-2a^2c)}{2(16a^2c^2-8ab^2c+b^4)}+\frac{3bx^4(b^2+2ac)}{4(16a^2c^2-8ab^2c+b^4)}+\frac{cx^6(b^2+2ac)}{2(16a^2c^2-8ab^2c+b^4)}}{x^4(b^2+2ac)+a^2+c^2x^8+2abx^2+2bcx^6}+\frac{\operatorname{atan}\left(\frac{\left(x^2\left(\frac{(b^2+2ac)(b^2c^2+2ac^3)}{a(4ac-b^2)^{9/2}(16a^2c^2-8ab^2c+b^4)}+\frac{b(b^2+2ac)^2(32a^2bc^4-16ab^3c^3+2b^5c^2)}{2a(4ac-b^2)^{15/2}(16a^2c^2-8ab^2c+b^4)}\right)+\frac{2bc^2(b^2+2ac)^2}{(4ac-b^2)^{15/2}}\right)\left(b^4(4ac-b^2)^5+16a^2c^2(4ac-b^2)^5-8ab^2c(4ac-b^2)^5\right)}{8a^2c^4+8ab^2c^3+2b^4c^2}\right)(b^2+2ac)}{(4ac-b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*a^2*b)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(5*a*b^2 - 2*a^2*c))/(
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*x^4*(2*a*c + b^2))/(4*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) + (c*x^6*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + (atan(((
x^2*(((2*a*c + b^2)*(2*a*c^3 + b^2*c^2))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) + (b*(2*a*c + b^2)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^
2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b
*c^2*(2*a*c + b^2)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c
^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(8*a^2*c^4 + 2*b^4*c^2 + 8
*a*b^2*c^3))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 5.41, size = 580, normalized size = 4.46

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)\log\left(x^2+\frac{-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)+2abc+b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)+b^3}{4ac^2+2b^2c}\right)}{2}+\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)\log\left(x^2+\frac{64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)+2abc-b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}(2ac+b^2)+b^3}{4ac^2+2b^2c}\right)}{2}+\frac{6a^2b+x^6(4ac^2+2b^2c)+x^4(6abc+3b^3)+x^2(-4a^2c+10ab^2)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6(128a^2bc^3-64ab^3c^2+8b^5c)+x^4(128a^3c^3-24ab^4c+4b^6)+x^2(128a^3bc^2-64a^2b^3c+8ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 + (-64*a**3*c**3*sqrt(-
1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b*
*2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a
*c**2 + 2*b**2*c))/2 + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 +
 (64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**
2*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c -
b**2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c
 + b**2) + b**3)/(4*a*c**2 + 2*b**2*c))/2 + (6*a**2*b + x**6*(4*a*c**2 + 2*
b**2*c) + x**4*(6*a*b*c + 3*b**3) + x**2*(-4*a**2*c + 10*a*b**2))/(64*a**4*
c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 +
 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(
128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**
3*c + 8*a*b**5))
```

3.680 $\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=113

$$\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 638, 614, 618, 206}

$$\frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a + b*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*b*(b + 2*c*x^2))
/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*b*c*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{(3bc)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{2\left(b^2-4ac\right)^2}$$

$$= \frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(3bc)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^{}}\right.}{\left(b^2-4ac\right.}$$

$$= \frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.10, size = 114, normalized size = 1.01

$$\frac{-\frac{12bc\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{\left(b^2-4ac\right)\left(2a+bx^2\right)}{\left(a+bx^2+cx^4\right)^2} - \frac{3b\left(b+2cx^2\right)}{a+bx^2+cx^4}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 - (3*b*(b + 2*c*x^2))/
(a + b*x^2 + c*x^4) - (12*b*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqr
t[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a + bx^2 + cx^4\right)^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 2.67, size = 808, normalized size = 7.15

$$\left[-\frac{6\left(b^3c^2 - 4abc^3\right)x^6 + ab^4 + 4a^2b^2c - 32a^3c^2 + 9\left(b^4c - 4ab^2c^2\right)x^4 + 2\left(b^5 + ab^3c - 20a^2bc^2\right)x^2 - 6\left(bc^3x^8 + 2b^2c^2x^6 + 2ab^2cx^2 + \left(b^3c + 2abc^2\right)x^4 + a^2bc\right)\sqrt{b^2 - 4ac}\log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + \left(2cx^2 + b\right)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right)}{4\left(\left(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5\right)x^8 + a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3 + 2\left(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3bc^4\right)x^6 + \left(b^8 - 10ab^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4\right)x^4 + 2\left(ab^7 - 12a^2b^5c + 48a^3b^3c^2 - 64a^4bc^3\right)x^2\right)}, -\frac{6\left(b^3c^2 - 4abc^3\right)x^6 + ab^4 + 4a^2b^2c - 32a^3c^2 + 9\left(b^4c - 4ab^2c^2\right)x^4 + 2\left(b^5 + ab^3c - 20a^2bc^2\right)x^2 - 12\left(bc^3x^8 + 2b^2c^2x^6 + 2ab^2cx^2 + \left(b^3c + 2abc^2\right)x^4 + a^2bc\right)\sqrt{-b^2 + 4ac}\arctan\left(-\frac{\left(2cx^2 + b\right)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right)}{4\left(\left(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5\right)x^8 + a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3 + 2\left(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3bc^4\right)x^6 + \left(b^8 - 10ab^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4\right)x^4 + 2\left(ab^7 - 12a^2b^5c + 48a^3b^3c^2 - 64a^4bc^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(
b^4*c - 4*a*b^2*c^2)*x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 6*(b*c^3*
x^8 + 2*b^2*c^2*x^6 + 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sq
rt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7
 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*(6*(b^3*c^2 - 4
*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*
x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 12*(b*c^3*x^8 + 2*b^2*c^2*x^6
+ 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*sqrt(-b^2 + 4*a*c)*arc
tan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)
*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 1.79, size = 143, normalized size = 1.27

$$-\frac{3\,bc\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2x^6+9\,b^2cx^4+2\,b^3x^2+10\,abcx^2+ab^2+8\,a^2c}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*b*c^2*x^6 + 9*b^2*c*x^4 + 2*b^3*x^2 + 10*
a*b*c*x^2 + a*b^2 + 8*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a
^2*c^2))
```

maple [A] time = 0.01, size = 142, normalized size = 1.26

$$-\frac{3bc\,x^2}{2\left(4ac-b^2\right)^2\left(c\,x^4+b\,x^2+a\right)}-\frac{3bc\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}}-\frac{3b^2}{4\left(4ac-b^2\right)^2\left(c\,x^4+b\,x^2+a\right)}+\frac{-b\,x^2-2a}{4\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2-3/2*b/(4*a*c-b^2)^2/(c*x^4+b
*x^2+a)*x^2*c-3/4*b^2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)-3*b/(4*a*c-b^2)^(5/2)*c
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.39, size = 400, normalized size = 3.54

$$-\frac{\frac{8\,c\,a^2+a\,b^2}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(b^3+5\,a\,c\,b\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b^2\,c\,x^4}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,b\,c^2\,x^6}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}-\frac{3\,b\,c\,\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{9\,b^2\,c^4}{a\,{\left(4\,a\,c-b^2\right)}^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b^3\,c^2\,\left(144\,a^2\,b\,c^4-72\,a\,b^3\,c^3+9\,b^5\,c^2\right)}{a\,{\left(4\,a\,c-b^2\right)}^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{18\,b^3\,c^4}{{\left(4\,a\,c-b^2\right)}^{15/2}}\right)\,\left(b^4\,{\left(4\,a\,c-b^2\right)}^5+16\,a^2\,c^2\,{\left(4\,a\,c-b^2\right)}^5-8\,a\,b^2\,c\,{\left(4\,a\,c-b^2\right)}^5\right)}{18\,b^2\,c^4}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((a*b^2 + 8*a^2*c)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3 + 5*a*b
*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^4)/(4*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) + (3*b*c^2*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2
*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (3*b*c*atan(((x^2*((
9*b^2*c^4)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^3*c^
2*(9*b^5*c^2 - 72*a*b^3*c^3 + 144*a^2*b*c^4))/(a*(4*a*c - b^2)^(15/2)*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))) + (18*b^3*c^4)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*
c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*b
^2*c^4)))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 3.04, size = 491, normalized size = 4.35

$$\frac{3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2c}{6bc^2}\right)}{2}-\frac{3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2c}{6bc^2}\right)}{2}+\frac{-8a^2c-ab^2-9b^2cx^4-6bc^2x^6+x^2(-10abc-2b^3)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6(128a^2bc^3-64ab^3c^2+8b^5c)+x^4(128a^3c^3-24ab^4c+4b^6)+x^2(128a^3bc^2-64a^2b^3c+8ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*b*c**4*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c
**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b*
*2*c)/(6*b*c**2))/2 - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**3
*b*c**4*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b*
*2)**5) + 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a
*c - b**2)**5) + 3*b**2*c)/(6*b*c**2))/2 + (-8*a**2*c - a*b**2 - 9*b**2*c*x
**4 - 6*b*c**2*x**6 + x**2*(-10*a*b*c - 2*b**3))/(64*a**4*c**2 - 32*a**3*b*
*2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x
**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 2
4*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

3.681 $\int \frac{x}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=113

$$-\frac{6c^2 \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1107, 614, 618, 206}

$$-\frac{6c^2 \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b + 2*c*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*(b + 2*c*x^2)
)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*c^2*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{(3c)\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c^2\right)\operatorname{Subst}\left(\int \frac{1}{a+bx\cdots}\right.}{\left(b^2-4ac\cdots\right.}$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(6c^2\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4a\cdots}\right.}{\left(b^2-\cdots\right.}$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.10, size = 106, normalized size = 0.94

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{\left(b+2cx^2\right)\left(-2c\left(5a+3cx^4\right)+b^2-6bcx^2\right)}{\left(a+bx^2+cx^4\right)^2}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-(((b + 2*c*x^2)*(b^2 - 6*b*c*x^2 - 2*c*(5*a + 3*c*x^4)))/(a + b*x^2 + c*x
^4)^2) + (24*c^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c])/(4*(b^2 - 4*a*c)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 1.06, size = 809, normalized size = 7.16

$$\left[\frac{12\left(b^2c^3-4ac^4\right)x^6-b^5+14ab^3c-40a^2bc^2+18\left(b^3c^2-4abc^3\right)x^4+4\left(b^4c+ab^2c^2-20a^2c^3\right)x^2+12\left(c^4x^8+2bc^3x^6+2abc^2x^2+\left(b^2c^2+2ac^3\right)x^4+a^2c^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{4\left(\left(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5\right)x^8+a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+2\left(b^7c-12ab^5c^2+48a^2b^3c^3-64a^3bc^4\right)x^6+\left(b^8-10ab^6c+24a^2b^4c^2+32a^3b^2c^3-128a^4c^4\right)x^4+2\left(ab^7-12a^2b^5c+48a^3b^3c^2-64a^4bc^3\right)x^2\right)},\ \frac{12\left(b^2c^3-4ac^4\right)x^6-b^5+14ab^3c-40a^2bc^2+18\left(b^3c^2-4abc^3\right)x^4+4\left(b^4c+ab^2c^2-20a^2c^3\right)x^2-24\left(c^4x^8+2bc^3x^6+2abc^2x^2+\left(b^2c^2+2ac^3\right)x^4+a^2c^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{4\left(\left(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5\right)x^8+a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+2\left(b^7c-12ab^5c^2+48a^2b^3c^3-64a^3bc^4\right)x^6+\left(b^8-10ab^6c+24a^2b^4c^2+32a^3b^2c^3-128a^4c^4\right)x^4+2\left(ab^7-12a^2b^5c+48a^3b^3c^2-64a^4bc^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^
3*c^2 - 4*a*b*c^3)*x^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 + 12*(c^4*x
^8 + 2*b*c^3*x^6 + 2*a*b*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(12*(b^2*c^3 - 4*a*
c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*x^4 +
 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 - 24*(c^4*x^8 + 2*b*c^3*x^6 + 2*a*b
*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6
+ (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 1.77, size = 144, normalized size = 1.27

$$\frac{6\,c^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2+20\,ac^2x^2-b^3+10\,abc}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 + 20
*a*c^2*x^2 - b^3 + 10*a*b*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a
^2*c^2))
```

maple [A] time = 0.01, size = 141, normalized size = 1.25

$$\frac{3c^2x^2}{\left(4ac-b^2\right)^2\left(c\,x^4+b\,x^2+a\right)}+\frac{6c^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}}+\frac{3bc}{2\left(4ac-b^2\right)^2\left(c\,x^4+b\,x^2+a\right)}+\frac{2c\,x^2+b}{4\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3*c^2/(4*a*c-b^2)^2/(c*x^4+b*
x^2+a)*x^2+3/2*c/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b+6*c^2/(4*a*c-b^2)^(5/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.34, size = 386, normalized size = 3.42

$$\frac{\frac{3\,c^3\,x^6}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{4\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\left(b^2\,c+5\,a\,c^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b\,c^2\,x^4}{2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}+\frac{6\,c^2\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{36\,c^6}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{36\,b\,c^4\left(16\,a^2\,b\,c^4-8\,a\,b^3\,c^3+b^5\,c^2\right)}{a\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{72\,b\,c^6}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\left(4\,a\,c-b^2\right)^5\right)}{72\,c^6}\right)}{\left(4\,a\,c-b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*c^3*x^6)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(4*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (x^2*(5*a*c^2 + b^2*c))/(b^4 + 16*a^2*c^2 - 8*a*b^
2*c) + (9*b*c^2*x^4)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2)
 + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + (6*c^2*atan(((x^2*((36*c^6)/(a*
(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c^2 -
```

```
8*a*b^3*c^3 + 16*a^2*b*c^4))/(a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c))) + (72*b*c^6)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2
*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(72*c^6)))/(4*a*c - b^2)
^(5/2)
```

sympy [B] time = 2.91, size = 481, normalized size = 4.26

$$-3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^3c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^2b^2c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36ab^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3bc^2}{6c^3}\right)+3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{192a^3c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-144a^2b^2c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+36ab^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-3b^6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3bc^2}{6c^3}\right)+\frac{10abc-b^3+18bc^2x^4+12c^3x^6+x^2\left(20ac^2+4b^2c\right)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\left(64a^2c^4-32ab^2c^3+4b^4c^2\right)+x^6\left(128a^2bc^3-64ab^3c^2+8b^5c\right)+x^4\left(128a^3c^3-24ab^4c+4b^6\right)+x^2\left(128a^3bc^2-64a^2b^3c+8ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*c**5*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**4*c
**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3
*b*c**2)/(6*c**3)) + 3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**3
*c**5*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**5) + 36*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*
a*c - b**2)**5) + 3*b*c**2)/(6*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**4 +
 12*c**3*x**6 + x**2*(20*a*c**2 + 4*b**2*c))/(64*a**4*c**2 - 32*a**3*b**2*c
 + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*
(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*
b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

3.682 $\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=200

$$-\frac{\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+}{\sqrt{b}}\right.}{2a^3\left(b^2-4ac\right)^{5/2}}$$

Rubi [A] time = 0.30, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}+\frac{-2ac+b^2+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*b^4
- 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2
*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a
+ b*x^2 + c*x^4]/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-4ac\right)-3bcx}{x\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subs}}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subs}}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(x}{a^3}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(x}{a^3}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(x}{a^3}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{b\left(b^4\right.}{}$$

Mathematica [A] time = 0.52, size = 342, normalized size = 1.71

$$\frac{\frac{a^2\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{a\left(16a^2c^2-15ab^2c-14abc^2x^2+2b^4+2b^3cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}+b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{\left(-16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c+8ab^2c\sqrt{b^2-4ac}-b^4\sqrt{b^2-4ac}+b^5\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}+4\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(
2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2))/((b^2 - 4*
a*c)^2*(a + b*x^2 + c*x^4)) + 4*Log[x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2
+ b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2
 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + ((b^
5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2
- 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2 + c*x^4)^3), x]
```

fricas [B] time = 1.89, size = 2017, normalized size = 10.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*
a^3*b^2*c^3 - 64*a^4*c^4)*x^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 +
4*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 -
 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^
6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a
^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b
^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(x))/(a^5*b^6 -
12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3
+ 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5
```

```
*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 3
2*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c
^2 - 64*a^7*b*c^3)*x^2), 1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 -
96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6 + (4*a*b^6*c
 - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^4 + 2*(a*b^7 - 10*a^2*b
^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x^2 + 2*((b^5*c^2 - 10*a*b^3*c^3 + 30*
a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^
4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*
c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^
4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*
log(c*x^4 + b*x^2 + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(x))/(a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c
^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*
a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3
- 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*
c^3)*x^2)]
```

giac [A] time = 1.88, size = 323, normalized size = 1.62

$$-\frac{\left(b^{5} - 10\,ab^{3}c + 30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(a^{3}b^{4} - 8\,a^{4}b^{2}c + 16\,a^{5}c^{2}\right)\sqrt{-b^{2}+4\,ac}} + \frac{3\,b^{4}c^{2}x^{8} - 24\,ab^{2}c^{3}x^{8} + 48\,a^{2}c^{4}x^{8} + 6\,b^{5}cx^{6} - 44\,ab^{3}c^{2}x^{6} + 68\,a^{2}bc^{3}x^{6} + 3\,b^{6}x^{4} - 10\,ab^{4}cx^{4} - 58\,a^{2}b^{2}c^{2}x^{4} + 128\,a^{3}c^{3}x^{4} + 10\,ab^{5}x^{2} - 72\,a^{2}b^{3}cx^{2} + 92\,a^{3}bc^{2}x^{2} + 9\,a^{2}b^{4} - 66\,a^{3}b^{2}c + 96\,a^{4}c^{2}}{8\left(a^{3}b^{4} - 8\,a^{4}b^{2}c + 16\,a^{5}c^{2}\right)\left(cx^{4} + bx^{2} + a\right)^{2}} - \frac{\log\left(cx^{4} + bx^{2} + a\right)}{4\,a^{3}} + \frac{\log\left(x^{2}\right)}{2\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*b^4
*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 + 6*b^5*c*x^6 - 44*a*b^3*c^2*x
^6 + 68*a^2*b*c^3*x^6 + 3*b^6*x^4 - 10*a*b^4*c*x^4 - 58*a^2*b^2*c^2*x^4 + 1
28*a^3*c^3*x^4 + 10*a*b^5*x^2 - 72*a^2*b^3*c*x^2 + 92*a^3*b*c^2*x^2 + 9*a^2
*b^4 - 66*a^3*b^2*c + 96*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*
x^4 + b*x^2 + a)^2) - 1/4*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*log(x^2)/a^3
```

maple [B] time = 0.03, size = 822, normalized size = 4.11

$$-\frac{7bc^{3}x^{6}}{2\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a}+\frac{b^{3}c^{2}x^{6}}{2\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a^{2}}-\frac{29b^{2}c^{2}x^{4}}{4\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a}+\frac{b^{4}cx^{4}}{\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a^{2}}+\frac{4c^{3}x^{4}}{\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)}-\frac{3b^{3}cx^{2}}{\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a}+\frac{b^{5}x^{2}}{2\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a^{2}}-\frac{bc^{2}x^{2}}{2\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)}+\frac{6ac^{2}}{\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)}+\frac{3b^{4}}{4\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a}-\frac{15bc^{2}\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\,a}+\frac{5b^{3}c\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\,a^{2}}-\frac{b^{5}\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\,a^{3}}-\frac{21b^{2}c}{4\left(cx^{4}+bx^{2}+a\right)^{2}\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)}-\frac{4c^{2}\ln\left(cx^{4}+bx^{2}+a\right)}{\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a}+\frac{2b^{2}c\ln\left(cx^{4}+bx^{2}+a\right)}{\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a^{2}}-\frac{b^{4}\ln\left(cx^{4}+bx^{2}+a\right)}{4\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)a^{3}}+\frac{\ln\left(x\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -7/2/a/(c*x^4+b*x^2+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/a^2/(c*x^
4+b*x^2+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+4/(c*x^4+b*x^2+a)^2*c^3
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-29/4/a/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4*b^2+1/a^2/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^4*b^4-1/2/(c*x^4+b*x^2+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^2-3/a/(c*x^
4+b*x^2+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c+1/2/a^2/(c*x^4+b*x^2+a)^2
*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+6*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*c^2-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+3/4/a
/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4-4/a/(16*a^2*c^2-8*a*b^2*c
+b^4)*c^2*ln(c*x^4+b*x^2+a)+2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*x
^2+a)*b^2-1/4/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*b^4-15/a/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*b*c^2+5/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*b^3*c-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5+ln(x)/a^3
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 10.95, size = 9339, normalized size = 46.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(4*a*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (x^4*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(4*a^2*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (b*x^2*(a^2*c^2 - b^4 + 6*a*b^2*c))/(2*a^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^6*(7*a*c - b^2))/(2*a^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
- (log((((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^
5))^(1/2) + 1)*((b^2*c^3*(4*b^6 - 497*a^3*c^3 + 302*a^2*b^2*c^2 - 61*a*b^4*
c))/(a^4*(4*a*c - b^2)^4) - ((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)
/(a^6*(4*a*c - b^2)^5))^(1/2) + 1)*((4*b^2*c^2*(b^4 + 23*a^2*c^2 - 9*a*b^2*
```

c))/(a^2*(4*a*c - b^2)^2) + (b*c^2*(a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2
c)^2)/(a^6(4*a*c - b^2)^5))^(1/2) + 1)*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^
3 + (2*b*c^3*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4
*a^3) + (b*c^4*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a
^4*(4*a*c - b^2)^4)))/(4*a^3) - (b^2*c^4*(7*a*c - b^2)^2)/(a^6*(4*a*c - b^2
)^4) + (b^3*c^5*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6))*(((a^3*(-(b^2*(
b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*
(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) - 1)
*((4*b^2*c^2*(b^4 + 23*a^2*c^2 - 9*a*b^2*c))/(a^2*(4*a*c - b^2)^2) - (b*c^2
(a^3(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2
) - 1)*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*b*c^3*x^2*(b^4 + 10*a^2*c^2
 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3) + (b^2*c^3*(4*b^6 - 497*a^3*
c^3 + 302*a^2*b^2*c^2 - 61*a*b^4*c))/(a^4*(4*a*c - b^2)^4) + (b*c^4*x^2*(6*
b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/
(4*a^3) + (b^2*c^4*(7*a*c - b^2)^2)/(a^6*(4*a*c - b^2)^4) - (b^3*c^5*x^2*(7
*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6
*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 -
4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7
*b^2*c^4)) + (b*atan((((b*((4*a^2*b^8*c^3 - 61*a^3*b^6*c^4 + 302*a^4*b^4*c^
5 - 497*a^5*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^
2 - 256*a^9*b^2*c^3) - (((4*a^4*b^10*c^2 - 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4
 - 1312*a^7*b^4*c^5 + 1472*a^8*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^
6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) + ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3
 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*
a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*
c))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^
2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a
^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 -
 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a
^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c
^4)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) - (((b*(
(4*a^4*b^10*c^2 - 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4 - 1312*a^7*b^4*c^5 + 147
2*a^8*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 25
6*a^9*b^2*c^3) + ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024
*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2
 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(a^6*b^8 + 256*a^1
0*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
c^4)))(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) + (b*(
b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b
^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 32
0*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(8*a^3*(
4*a*c - b^2)^(5/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2
 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*
c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a

^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b
^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 51
20*a^7*b^2*c^4)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(4*a^7*b^10*c^2 -
 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6))
/(64*a^9*(4*a*c - b^2)^(15/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a
^8*b^4*c^2 - 256*a^9*b^2*c^3)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 32
5*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*
c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11
*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 6
1440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(
15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*
b^6*c^4 - 600*a^3*b^4*c^5 + 900*a^4*b^2*c^6)*(6*b^10 - 6400*a^5*c^5 + 960*a
^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) - (x^2*(((
(((b*((5120*a^10*b*c^9 + 2*a^4*b^13*c^3 - 36*a^5*b^11*c^4 + 276*a^6*b^9*c^5
 - 1216*a^7*b^7*c^6 + 3456*a^8*b^5*c^7 - 6144*a^9*b^3*c^8)/(a^6*b^12 + 4096
*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*
b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 -
1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a
^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97
280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^
10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 512
0*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2
*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 256
0*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*
b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227
328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*
b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5
120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^
2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10 - 2
048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*
b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 25
60*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) + (b*((8960*a^7*b*c^9 - 6*a^2*b^11*c^4
+ 137*a^3*b^9*c^5 - 1217*a^4*b^7*c^6 + 5256*a^5*b^5*c^7 - 11024*a^6*b^3*c^8
)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^
6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((5120*a^10*b*c^9 + 2*a^4
*b^13*c^3 - 36*a^5*b^11*c^4 + 276*a^6*b^9*c^5 - 1216*a^7*b^7*c^6 + 3456*a^8
*b^5*c^7 - 6144*a^9*b^3*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 24
0*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) -
 ((2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^
2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3
 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11
*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8
*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 409

6*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10
*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 -
1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^
8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^
4)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) + (b^3*(b
^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*
a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 +
 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(64*a^9*(4*a*c - b^2)^(15/2)*(
a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c
^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*b^8 + 160*a^4*c^4 + 180*a^
2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*
(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*
c^4 - 120*a*b^8*c)) + (3*b*((b^9*c^5 - 21*a*b^7*c^6 + 147*a^2*b^5*c^7 - 343
*a^3*b^3*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 -
 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((8960*a^7*b*
c^9 - 6*a^2*b^11*c^4 + 137*a^3*b^9*c^5 - 1217*a^4*b^7*c^6 + 5256*a^5*b^5*c^
7 - 11024*a^6*b^3*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*
b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((51
20*a^10*b*c^9 + 2*a^4*b^13*c^3 - 36*a^5*b^11*c^4 + 276*a^6*b^9*c^5 - 1216*a
^7*b^7*c^6 + 3456*a^8*b^5*c^7 - 6144*a^9*b^3*c^8)/(a^6*b^12 + 4096*a^12*c^6
 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 -
 6144*a^11*b^2*c^5) - ((2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*
b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c
^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*
b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
c^4)(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^
9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10 - 2048*a^5*c^5
 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*
(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*
c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a
^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5
- 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) -
(b*((b*((5120*a^10*b*c^9 + 2*a^4*b^13*c^3 - 36*a^5*b^11*c^4 + 276*a^6*b^9*c
^5 - 1216*a^7*b^7*c^6 + 3456*a^8*b^5*c^7 - 6144*a^9*b^3*c^8)/(a^6*b^12 + 40
96*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^1
0*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2
- 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12
*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 -
97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*
b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5
120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^
2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a
^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 -
10*a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2

560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(32*a^6*(4*a*c - b^2)^5*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 600*a^3*b^4*c^5 + 900*a^4*b^2*c^6) + (3*b*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c)*((((4*a^2*b^8*c^3 - 61*a^3*b^6*c^4 + 302*a^4*b^4*c^5 - 497*a^5*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - (((4*a^4*b^10*c^2 - 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4 - 1312*a^7*b^4*c^5 + 1472*a^8*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) + ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (b^6*c^4 - 14*a*b^4*c^5 + 49*a^2*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) + (b*((b*(((4*a^4*b^10*c^2 - 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4 - 1312*a^7*b^4*c^5 + 1472*a^8*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) + ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(

```
b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*
a^2*c^2 - 10*a*b^2*c)*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 -
1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6
*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(8*a^3*(4*a*c - b
^2)^(5/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9
*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 256
0*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3
*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(4*a^7*b^10*
c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*
c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4
*b^2*c^4 - 40*a*b^8*c))/(32*a^6*(4*a*c - b^2)^5*(a^6*b^8 + 256*a^10*c^4 - 1
6*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5
- 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(
16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 38
4*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2)
 - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^
2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b
^2)^6*(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 600*a^3*b^4*c^5 + 900*a^
4*b^2*c^6)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 77
75*a^4*b^2*c^4 - 120*a*b^8*c)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*a^3*(4*
a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.683 $\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=255

$$\frac{3b\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{3b\log(x)}{a^4}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(-20a^3\right.}{}$$

Rubi [A] time = 0.39, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{3b\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{3b\log(x)}{a^4}+\frac{-2ac+b^2+bcx^2}{4ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*x^2) + (b^2 - 2*a*c
 + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^2) + (3*b^4 - 20*a*b
^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2*x^2*(a
+ b*x^2 + c*x^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*L
og[x])/a^4 + (3*b*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^3}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$=\frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}-\frac{\text{Subst}\left(\int \frac{-3b^2+10ac-4bcx}{x^2\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{4a\left(b^2-4ac\right)}$$

$$=\frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)}+\text{Su}$$

$$=\frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)}+\text{Su}$$

$$=-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2}+\frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}+\frac{3b^4-20ab^2c+20a^2c}{4a^2\left(b^2-4ac\right)^2x}$$

$$=-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2}+\frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}+\frac{3b^4-20ab^2c+20a^2c}{4a^2\left(b^2-4ac\right)^2x}$$

$$=-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2}+\frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}+\frac{3b^4-20ab^2c+20a^2c}{4a^2\left(b^2-4ac\right)^2x}$$

$$=-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2}+\frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}+\frac{3b^4-20ab^2c+20a^2c}{4a^2\left(b^2-4ac\right)^2x}$$

Mathematica [A] time = 0.62, size = 402, normalized size = 1.58

$$\frac{\frac{a^2\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}-\frac{a\left(46a^2bc^2+28a^2c^3x^2-29ab^3c-26ab^2c^2x^2+4b^5+4b^4cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\left(-20a^3c^3+30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}-10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+b^6\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3\left(20a^3c^3-30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}+10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}-b^6\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{2a}{x^2}-12b\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a)/x^2 + (a^2*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c
*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - 12*b*Log[x] + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 +
b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2
 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + (3*(
-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8
*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^2 + cx^4\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2 + c*x^4)^3), x]
```

fricas [B] time = 2.67, size = 2312, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c
^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^8 + 3*(4*a*b^7*c - 45*
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^6 + 2*(3*a*b^8 - 30*a^2*b^
6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^4 + (9*a^2*b^7 - 104
*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x^2 + 3*((b^6*c^2 - 10*a*b^4*
c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*
b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^
2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4
*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x^2)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^
2*b^3*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 -
 64*a^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3
- 128*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^
2*c^3)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*
log(c*x^4 + b*x^2 + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*
a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4
```

```
)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4
)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^4 + (a
^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(x))/((a^4*b
^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c
- 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b
^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12
*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)*x^2), -1/4*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5
*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 4
0*a^4*c^5)*x^8 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*
b*c^4)*x^6 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 -
200*a^5*c^4)*x^4 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b
*c^3)*x^2 + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10
+ 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a
*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*
a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c +
30*a^4*b^2*c^2 - 20*a^5*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3
*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128
*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3
)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(c
*x^4 + b*x^2 + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^8
 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^6
 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^4 + (a^2*b^
7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(x))/((a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*
a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*
b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)*x^2)]
```

giac [**A**] time = 1.80, size = 382, normalized size = 1.50

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{9\,b^{5}c^{2}x^{8}-72\,ab^{3}c^{3}x^{8}+144\,a^{2}bc^{4}x^{8}+18\,b^{6}cx^{6}-136\,ab^{4}c^{2}x^{6}+236\,a^{2}b^{2}c^{3}x^{6}+56\,a^{3}c^{4}x^{6}+9\,b^{7}x^{4}-38\,ab^{5}cx^{4}-110\,a^{2}b^{3}c^{2}x^{4}+436\,a^{3}bc^{3}x^{4}+26\,ab^{6}x^{2}-192\,a^{2}b^{4}cx^{2}+316\,a^{3}b^{2}c^{2}x^{2}+72\,a^{4}c^{3}x^{2}+19\,a^{2}b^{5}-144\,a^{3}b^{3}c+260\,a^{4}bc^{2}}{8\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}+\frac{3\,b\log\left(cx^{4}+bx^{2}+a\right)}{4\,a^{4}}-\frac{3\,b\log\left(x^{2}\right)}{2\,a^{4}}+\frac{3\,bx^{2}-a}{2\,a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x^2 + b)/s
qrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)
) - 1/8*(9*b^5*c^2*x^8 - 72*a*b^3*c^3*x^8 + 144*a^2*b*c^4*x^8 + 18*b^6*c*x^
6 - 136*a*b^4*c^2*x^6 + 236*a^2*b^2*c^3*x^6 + 56*a^3*c^4*x^6 + 9*b^7*x^4 -
38*a*b^5*c*x^4 - 110*a^2*b^3*c^2*x^4 + 436*a^3*b*c^3*x^4 + 26*a*b^6*x^2 - 1
```

```
92*a^2*b^4*c*x^2 + 316*a^3*b^2*c^2*x^2 + 72*a^4*c^3*x^2 + 19*a^2*b^5 - 144*
a^3*b^3*c + 260*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(c*x^4 + b
*x^2 + a)^2) + 3/4*b*log(c*x^4 + b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^4 + 1/2*
(3*b*x^2 - a)/(a^4*x^2)
```

maple [B] time = 0.03, size = 1002, normalized size = 3.93

$$-\frac{7c^4x^6}{(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a}+\frac{13b^2c^3x^6}{2(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a^2}-\frac{b^4c^2x^6}{(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a^3}-\frac{37bc^3x^4}{2(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a}+\frac{55b^3c^2x^4}{4(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a^2}-\frac{2b^5cx^4}{(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a^3}-\frac{9c^3x^2}{(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)}-\frac{7b^2c^2x^2}{2(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a}+\frac{6b^4cx^2}{(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a^2}-\frac{b^6x^2}{(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a^3}-\frac{29bc^2}{2(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)}+\frac{9b^3c}{(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a}-\frac{5b^5}{4(cx^4+bx^2+a)^2(16a^2c^2-8ab^2c+b^4)a^2}+\frac{12bc^2\ln(cx^4+bx^2+a)}{(16a^2c^2-8ab^2c+b^4)a^2}-\frac{6b^3c\ln(cx^4+bx^2+a)}{(16a^2c^2-8ab^2c+b^4)a^3}+\frac{3b^5\ln(cx^4+bx^2+a)}{4(16a^2c^2-8ab^2c+b^4)a^4}-\frac{30c^3\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8ab^2c+b^4)\sqrt{4ac-b^2}\,a}+\frac{45b^2c^2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8ab^2c+b^4)\sqrt{4ac-b^2}\,a^2}-\frac{15b^4c\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8ab^2c+b^4)\sqrt{4ac-b^2}\,a^3}+\frac{3b^6\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(16a^2c^2-8ab^2c+b^4)\sqrt{4ac-b^2}\,a^4}-\frac{3b\ln(x)}{a^4}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -7/a/(c*x^4+b*x^2+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+13/2/a^2/(c*x^4+b
*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b^2-1/a^3/(c*x^4+b*x^2+a)^2*c^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b^4-37/2/a/(c*x^4+b*x^2+a)^2*b*c^3/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^4+55/4/a^2/(c*x^4+b*x^2+a)^2*b^3*c^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^4-2/a^3/(c*x^4+b*x^2+a)^2*b^5*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4
-9/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3-7/2/a/(c*x^4+b*x^2+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c^2+6/a^2/(c*x^4+b*x^2+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2*b^4*c-1/a^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^2*b^6-29/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2+9/a/(
c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c-5/4/a^2/(c*x^4+b*x^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*b^5+12/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*
x^4+b*x^2+a)*b-6/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*x^2+a)*b^3+3/4
/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*b^5-30/a/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^3+45/a
^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))*b^2*c^2-15/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*c+3/2/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^6-1/2/a^3/x^2-3*b
*ln(x)/a^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 11.76, size = 10074, normalized size = 39.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x^2 + c*x^4)^3),x)

[Out] (log(((27*c^5*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((9*c^3*(4*b^10 - 100*a^5*c^5 + 342*a^2*b^6*c^2 - 837*a^3*b^4*c^3 + 780*a^4*b^2*c^4 - 61*a*b^8*c))/(a^6*(4*a*c - b^2)^4) - ((3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((6*c^3*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*(3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^4 + (12*b*c^2*(b^6 - 10*a^3*c^3 + 23*a^2*b^2*c^2 - 9*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4) + (9*b*c^4*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (27*b*c^4*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2)/(a^9*(4*a*c - b^2)^4))*((27*c^5*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((9*c^3*(4*b^10 - 100*a^5*c^5 + 342*a^2*b^6*c^2 - 837*a^3*b^4*c^3 + 780*a^4*b^2*c^4 - 61*a*b^8*c))/(a^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((6*c^3*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*(3*b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^4 + (12*b*c^2*(b^6 - 10*a^3*c^3 + 23*a^2*b^2*c^2 - 9*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4) + (9*b*c^4*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (27*b*c^4*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2)/(a^9*(4*a*c - b^2)^4)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*b*log(x))/a^4 - (1/(2*a) + (x^4*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^6*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3))/(4*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^8*(b^4 + 10*a^2*c^2 - 7*a*b^2*c))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^6*(2*a*c + b^2) + a^2*x^2 + c^2*x^10 + 2*a*b*x^4 + 2*b*c*x^8) - (3*atan((x^2*((((27000*a^6*c^11 + 27*b^12*c^5 - 567*a*b^10*c^6 + 4779*a^2*b^8*c^7 - 20601*a^3*b^6*c^8 + 47790*a^4*b^4*c^9 - 56700*a^5*b^2*c^10)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((129600*a^9*b*c^10 + 54*a^3*b^13*c^4 - 1233*a^4*b^11*c^5 + 11583*a^5*b^9*c^6 - 57204*a^6*b^7*c^7 + 156276*a^7*b^5*c^8 - 223200*a^8*b^3*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c

^10 + 6*a^6*b^14*c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^5 + 792*a^9*b^8*c^
6 - 22272*a^10*b^6*c^7 + 100608*a^11*b^4*c^8 - 199680*a^12*b^2*c^9)/(a^9*b^
12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3
+ 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 6144*a^5*b*c^5 + 960*
a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^
16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960
*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*
c^8))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560
*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c
 + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2
c^5)))(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 768
0*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c
+ 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*b^11 - 6144*a
^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^
9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560
*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*((3*((153600*a^13*c^10 + 6*a^6*b^14*
c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^5 + 792*a^9*b^8*c^6 - 22272*a^10*b^
6*c^7 + 100608*a^11*b^4*c^8 - 199680*a^12*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^
6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c
^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 384
0*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9
*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 9
7280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(2*(4*a^4*b
^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 51
20*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c
^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20
*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) - (3*(
b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11 - 6144*a^5*b*c^5 +
960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(16384
0*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 2
4960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*
b^3*c^8))/(8*a^4*(4*a*c - b^2)^(5/2)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^
8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 40
96*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*
a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10
*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*
c^2 - 10*a*b^4*c)^2*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b
^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c
^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^
13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(32*a^8*(4*a*c - b
^2)^5*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^
7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c +
240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^
5)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))
/(8*a^3*c^2*(4*a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*

a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((153600*a^13*c^10 + 6*a^6*b^14*c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^ 5 + 792*a^9*b^8*c^6 - 22272*a^10*b^6*c^7 + 100608*a^11*b^4*c^8 - 199680*a^1 2*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 614 4*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a *b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^1 1*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640* a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^ 6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c ^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)) /(4*a^4*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a *b^4*c)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 768 0*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^1 0*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(8*a^4*(4*a*c - b^2)^(5/2)*(4* a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11* b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b ^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^ 4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^ 6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*((129600*a^9*b*c^10 + 54 *a^3*b^13*c^4 - 1233*a^4*b^11*c^5 + 11583*a^5*b^9*c^6 - 57204*a^6*b^7*c^7 + 156276*a^7*b^5*c^8 - 223200*a^8*b^3*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^ 10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144 *a^14*b^2*c^5) - (((153600*a^13*c^10 + 6*a^6*b^14*c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^5 + 792*a^9*b^8*c^6 - 22272*a^10*b^6*c^7 + 100608*a^11*b^4*c ^8 - 199680*a^12*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240* a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4* b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13 *c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 22732 8*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a ^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2 *b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^ 10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 512 0*a^8*b^2*c^4)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) + (27*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^ 3*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11 *c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 2949 12*a^15*b^3*c^8))/(64*a^12*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4

- 6144*a^14*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2
*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 -
960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 +
120*a*b^10*c)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c
- b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(
4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*
b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(1
0800*a^6*c^8 + 27*b^12*c^2 - 540*a*b^10*c^3 + 4320*a^2*b^8*c^4 - 17280*a^3*
b^6*c^5 + 35100*a^4*b^4*c^6 - 32400*a^5*b^2*c^7) + (((27*b^9*c^4 - 378*a*b^
7*c^5 + 2700*a^4*b*c^8 + 1863*a^2*b^5*c^6 - 3780*a^3*b^3*c^7)/(a^9*b^8 + 25
6*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) + (((900*a
^8*c^8 - 36*a^3*b^10*c^3 + 549*a^4*b^8*c^4 - 3078*a^5*b^6*c^5 + 7533*a^6*b^
4*c^6 - 7020*a^7*b^2*c^7)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11
*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920*a^11*b*c^7 - 12*a^6*b^11*c^2 + 204*a
^7*b^9*c^3 - 1332*a^8*b^7*c^4 + 4056*a^9*b^5*c^5 - 5376*a^10*b^3*c^6)/(a^9*
b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) -
((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5
+ 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*
b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*
a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5
- 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*
(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^
3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^
6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*b^11 - 6144*a^5*b*c^5
+ 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2
*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4
*c^3 + 5120*a^8*b^2*c^4)) + (3*((3*((1920*a^11*b*c^7 - 12*a^6*b^11*c^2 + 20
4*a^7*b^9*c^3 - 1332*a^8*b^7*c^4 + 4056*a^9*b^5*c^5 - 5376*a^10*b^3*c^6)/(a
^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)
- ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c
^5 + 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a
^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(a^9*b^8 + 256*a^13*c^4 -
16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*
c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)
))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(
5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(4*a^10*b^10*c^
2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*
c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a
^4*b^3*c^4 - 120*a*b^9*c))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c
^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096
*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2
c^4)))(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b
^2)^(5/2)) - (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(4*a^10*
b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^
14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 +

7680*a^4*b^3*c^4 - 120*a*b^9*c))/(32*a^8*(4*a*c - b^2)^5*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)*(10800*a^6*c^8 + 27*b^12*c^2 - 540*a*b^10*c^3 + 4320*a^2*b^8*c^4 - 17280*a^3*b^6*c^5 + 35100*a^4*b^4*c^6 - 32400*a^5*b^2*c^7)) - (b*((((3*((1920*a^11*b*c^7 - 12*a^6*b^11*c^2 + 204*a^7*b^9*c^3 - 1332*a^8*b^7*c^4 + 4056*a^9*b^5*c^5 - 5376*a^10*b^3*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*((900*a^8*c^8 - 36*a^3*b^10*c^3 + 549*a^4*b^8*c^4 - 3078*a^5*b^6*c^5 + 7533*a^6*b^4*c^6 - 7020*a^7*b^2*c^7)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920*a^11*b*c^7 - 12*a^6*b^11*c^2 + 204*a^7*b^9*c^3 - 1332*a^8*b^7*c^4 + 4056*a^9*b^5*c^5 - 5376*a^10*b^3*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) + (27*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6))/(64*a^12*(4*a*c -

```
 b^2)^(15/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 25
6*a^12*b^2*c^3)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3
- 39*a*b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c -
b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a
*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4
*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^
3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a
^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)*(10800*a^6
*c^8 + 27*b^12*c^2 - 540*a*b^10*c^3 + 4320*a^2*b^8*c^4 - 17280*a^3*b^6*c^5
+ 35100*a^4*b^4*c^6 - 32400*a^5*b^2*c^7)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c
^2 - 10*a*b^4*c))/(2*a^4*(4*a*c - b^2)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.684 $\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=400

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+}}$$

Rubi [A] time = 1.73, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1120, 1275, 1279, 1166, 205}

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3bx\left(b^2-8ac\right)}{8c^2\left(b^2-4ac\right)^2}+\frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^5\left(12ab-x^2\left(b^2-28ac\right)\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^3\left(b^2-28ac\right)}{8c\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*b*(b^2 - 8*a*c)*x)/(8*c^2*(b^2 - 4*a*c)^2) + ((b^2 - 28*a*c)*x^3)/(8*c*
(b^2 - 4*a*c)^2) + (x^7*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) + (x^5*(12*a*b - (b^2 - 28*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*(b^4 - 9*a*b^2*c + 28*a^2*c^2 - (b^5 - 11*a*b^3*c + 44*a^2*b*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b
^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}}{\left(a+bx^2+cx^4\right)^3}\,dx &= \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{x^6\left(14a+bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)} \\
&= \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-28ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^4\left(60ab-3\left(b^2-28ac\right)x^2\right)}{a+bx^2+cx^4}\,dx}{8\left(b^2-4ac\right)^2} \\
&= \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-28ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{x^2(\cdots}{\cdots}}{\cdots} \\
&= -\frac{3b\left(b^2-8ac\right)x}{8c^2\left(b^2-4ac\right)^2} + \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-\cdots\right.\right.}{8\left(b^2-4ac\right)^2\left(a+\cdots\right.} \\
&= -\frac{3b\left(b^2-8ac\right)x}{8c^2\left(b^2-4ac\right)^2} + \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-\cdots\right.\right.}{8\left(b^2-4ac\right)^2\left(a+\cdots\right.} \\
&= -\frac{3b\left(b^2-8ac\right)x}{8c^2\left(b^2-4ac\right)^2} + \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-\cdots\right.\right.}{8\left(b^2-4ac\right)^2\left(a+\cdots\right.}
\end{aligned}
$$

Mathematica [A] time = 1.17, size = 455, normalized size = 1.14

$$
\frac{-\frac{4\left(a^2cx\left(2cx^2-3b\right)+ab^2x\left(b-4cx^2\right)+b^4x^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2\sqrt{b^2-4ac}-44a^2bc^2+11ab^3c-9ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}-b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2\sqrt{b^2-4ac}+44a^2bc^2-11ab^3c-9ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}+b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{2x\left(48a^2bc^2-44a^2c^3x^2-17ab^3c+37ab^2c^2x^2+2b^5-5b^4cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}}{16c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5 - 17*a*b^3*c + 48*a^2*b*c^2 - 5*b^4*c*x^2 + 37*a*b^2*c^2*x^2 -
 44*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*(b^4*x^3 + a*b
^2*x*(b - 4*c*x^2) + a^2*c*x*(-3*b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^5 + 11*a*b^3*c - 44*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^5 - 11*a*b^3*c +
44*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2
```

```
*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^10/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 2.01, size = 4279, normalized size = 10.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + 2*(3*b^5 - 20*a*b^3*c
- 4*a^2*b*c^2)*x^5 + 2*(6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x^3 + 3*sqrt(1
/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6
*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)
*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^
4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 12
80*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 -
1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c
^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 2
0*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a
^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5
*b^2*c^3 + 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*
b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*
a^6*b*c^6 - (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8
 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*
c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^
4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1
280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c
^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*
b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 2
2*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 -
 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 -
1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c
^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) - 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*
```

c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c
^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)
x^4 + 2(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*
b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20
*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^
5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*
a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 +
1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^
6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2
*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4
)*x - 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c
^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 - (b^14*c^5 -
30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 4
9664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b
^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a
*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*
a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1
680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^
8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*
c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2
*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c
^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 -
1024*a^5*c^10))) + 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 +
a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^
2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*
a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2
- 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6
*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a
*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20
*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 102
4*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
+ 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4
189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^
13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 -
27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*
b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 8192
0*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 -
1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*
c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 -
21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^
5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1
024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 +
2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*
c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*
a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) - 3*sqr

```
t(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*
c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2
- 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c
^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680
*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 +
 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2
 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^
6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5
- 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 102
4*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*
a^5*b^2*c^3 + 38416*a^6*c^4)*x - 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a
^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 250
88*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*
c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a
^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401
*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13
+ 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^
5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a
^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8
- 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^1
0 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14
 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^
4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 6*(a^2*b^3 - 8*a^3*b*c)*x)/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*
a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4
*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)
```

giac [B] time = 3.63, size = 2430, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6
*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 24*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^5*c^2 + 32*a*b^5*c^2 - 2*b^6*c^2 - 128*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 160
*a^2*b^3*c^3 + 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^4 + 256*a^3*b*c^4 - 192*a^2*b^2*c^4 + 448*a^3*c^5 + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
```

(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt (b^2 - 4*a*c)*c)*b^4*c^2 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^ 2 - 4*a*c)*c)*a^3*c^3 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4 *a*c)*c)*a*b^2*c^3 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a *c)*c)*a^2*c^4 + 2*(b^2 - 4*a*c)*b^5*c - 24*(b^2 - 4*a*c)*a*b^3*c^2 + 2*(b^ 2 - 4*a*c)*b^4*c^2 + 64*(b^2 - 4*a*c)*a^2*b*c^3 - 20*(b^2 - 4*a*c)*a*b^2*c^ 3 + 112*(b^2 - 4*a*c)*a^2*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c^2 - 8*a*b^3 *c^3 + 16*a^2*b*c^4 + sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)^2 - 4*(a* b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b^6*c^3 - 2*b^7*c ^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2*c^5 - 96*a^2*b^3 *c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2*b^2*c^6 - 64*a^3 *c^7)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt (2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 2*b^7*c + 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a ^2*b^3*c^2 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2) *sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 32*a*b^5*c^2 + 2*b^6*c^2 - 128*s qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c - sq rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) *a*b^3*c^3 + 160*a^2*b^3*c^3 - 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^ 2 - 4*a*c)*c)*a^2*b*c^4 - 256*a^3*b*c^4 + 192*a^2*b^2*c^4 - 448*a^3*c^5 - s qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 14*sqrt(2)*s qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b ^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 96*sqrt(2)*sqrt(b^2 - 4 *a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4 *a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c) *sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sq rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b* c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^5*c + 24*(b^2 - 4*a*c)*a *b^3*c^2 - 2*(b^2 - 4*a*c)*b^4*c^2 - 64*(b^2 - 4*a*c)*a^2*b*c^3 + 20*(b^2 - 4*a*c)*a*b^2*c^3 - 112*(b^2 - 4*a*c)*a^2*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b ^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 - sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2* b*c^4)^2 - 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^ 4 + 16*a^2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b ^6*c^3 - 2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2* c^5 - 96*a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2* b^2*c^6 - 64*a^3*c^7)*abs(c)) - 1/8*(5*b^4*c*x^7 - 37*a*b^2*c^2*x^7 + 44*a^ 2*c^3*x^7 + 3*b^5*x^5 - 20*a*b^3*c*x^5 - 4*a^2*b*c^2*x^5 + 6*a*b^4*x^3 - 49 *a^2*b^2*c*x^3 + 28*a^3*c^2*x^3 + 3*a^2*b^3*x - 24*a^3*b*c*x)/((b^4*c^2 - 8 *a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)

maple [B] time = 0.05, size = 1141, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*(44*a^2*c^2-37*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^7+1/8*b*
(4*a^2*c^2+20*a*b^2*c-3*b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(
28*a^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/8*a^2*b*(8*a*
c-b^2)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-21/4/(16*a^2*c^2
-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2+27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*a*b^2-3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^4+33/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*a^2*b-33/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*a*b^3+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5+21/4/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*a^2-27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2+3
/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4+33/4/(16*a^2*c^2-
8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*b-33/16/c/(16*a^2*
c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^3+3/16/c^2/(16
*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + (3*b^5 - 20*a*b^3*c - 4*a
^2*b*c^2)*x^5 + (6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x^3 + 3*(a^2*b^3 - 8*
a^3*b*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3
*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6
```

```
*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^
3*b*c^4)*x^2) + 3/8*integrate((a*b^3 - 8*a^2*b*c + (b^4 - 9*a*b^2*c + 28*a^
2*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

mupad [B] time = 9.04, size = 10912, normalized size = 27.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^3*(6*a*b^4 + 28*a^3*c^2 - 49*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (x^7*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)) - (b*x^5*(4*a^2*c^2 - 3*b^4 + 20*a*b^2*c))/(8*c^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) - (3*a^2*b*x*(8*a*c - b^2))/(8*c^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
- atan(((((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 8192
0*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*
c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 128
0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*((9*(b^4*(-(4*a*
c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3
*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 +
 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576
*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^
8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 19660
80*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256
*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840
*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c
^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2)
 - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*
a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c
^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17
*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*
c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12
*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 +
2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*b^10 - 14112*a
^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^
8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*
b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 -
769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*
c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2
)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*
b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 +
```

860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440
*a^9*b^2*c^14)))^(1/2)*1i - (((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168
*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^
7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 2
40*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) +
 (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2
*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1
069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(
1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^
7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*
a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^
2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*
a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 +
b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*
a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a
^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6
 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(10485
76*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*
c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196
6080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) +
(x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^
4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^
2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 +
1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c
^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 30105
60*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*
b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a
*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258
048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^
8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i)/((((3*(256*a*b^13*c^3 + 2097
152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6
+ 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3
 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 -
6144*a^5*b^2*c^8)) - (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320
*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 31
6864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*
b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(
-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c
^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5
*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c
^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144
*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))
/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c

^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a
^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 -
 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c
^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)
^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*
c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 86016
0*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*
b^2*c^14)))^(1/2) - (x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^
3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 -
16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)
^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^
3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936
*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^
15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 5376
0*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b
^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (3*(189*a^
3*b^8 + 197568*a^7*c^4 - 3645*a^4*b^6*c + 29844*a^5*b^4*c^2 - 117936*a^6*b^
2*c^3))/(256*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1
280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (((3*(256*a*b^13*
c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^
4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9
+ b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*
b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19
 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^1
1*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 30
10560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11
*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 4
0*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 -
258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120
*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^
6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4
*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256
*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c
^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5
*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8
+ 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c
- b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720
*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^
10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 26
21440*a^9*b^2*c^14)))^(1/2) + (x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2
 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 +
b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*
a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a
^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6

+ 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(10485
76*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*
c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196
6080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)))*
((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^1
5*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 10698
24*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 -
7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*
b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^
14)))^(1/2)*2i - atan(((((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*
b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2
883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^
2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(
-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^1
5*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 10698
24*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 -
7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*
b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^
14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*
b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(
-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*
b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 -
2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^
(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*
a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8
+ 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196608
0*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*
(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b
^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4
- 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560
*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b
^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25804
8*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*
b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i - (((3*(256*a*b^13*c^3 + 209715
2*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 +
1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3 -

24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)) + (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*
a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316
864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b
^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^
6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*
b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^
13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*
a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/
(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^
6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a
^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 +
1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c
^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)
^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*
c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 86016
0*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*
b^2*c^14)))^(1/2) + (x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^
3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 -
16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*
c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c
^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 234393
6*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
- 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c
^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 537
60*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*
b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i)/((((3*(
256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b^9*c^5
- 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c^8))/(512*(40
96*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
+ 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 6
3440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*
b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a
*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 +
b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4
*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^
12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5
120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8
+ 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4
- 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560
*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^

2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b
^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25804
8*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*
b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*b^10 - 14112*a^5*c^5 + 188
1*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^8*c))/(32*(2
56*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(
-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^1
5*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 10698
24*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 -
7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*
b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^
14)))^(1/2) - (3*(189*a^3*b^8 + 197568*a^7*c^4 - 3645*a^4*b^6*c + 29844*a^5
*b^4*c^2 - 117936*a^6*b^2*c^3))/(256*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c
^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c
^8)) + (((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920
*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c
^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280
*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*(b^19 + b^4*
(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3
*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 -
 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576
*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^
8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 19660
80*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256
*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840
*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c
^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^1
5)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440
*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*
c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^1
7*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20
*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^1
2*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 +
 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*b^10 - 14112*
a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b
^8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9
 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^
9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 4
9*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^
2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10

```
+ 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 26214
40*a^9*b^2*c^14)))^(1/2)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 172
0320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4
- 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*
a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2
*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^
18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048
*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b
^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.685 $\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=348

$$\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{}{4(}$$

Rubi [A] time = 0.89, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1120, 1275, 1279, 1166, 205}

$$\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^3\left(x^2\left(20ac+b^2\right)+12ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x\left(20ac+b^2\right)}{8c\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b^2 + 20*a*c)*x)/(8*c*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x^2))/(4*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) + (x^3*(12*a*b + (b^2 + 20*a*c)*x^2))/(8*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3 - 16*a*b*c - (b^4 - 18*a*b^2*c - 40
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((b^3 - 16*a*b*c + (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b
^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{x^4\left(10a-bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^2\left(36ab+\left(b^2+20ac\right)x^2\right)}{a+bx^2+cx^4}\,d}{8\left(b^2-4ac\right)^2}$$

$$= -\frac{\left(b^2+20ac\right)x}{8c\left(b^2-4ac\right)^2} + \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \int$$

$$= -\frac{\left(b^2+20ac\right)x}{8c\left(b^2-4ac\right)^2} + \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(b}{}$$

$$= -\frac{\left(b^2+20ac\right)x}{8c\left(b^2-4ac\right)^2} + \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(b}{}$$

Mathematica [A] time = 0.96, size = 381, normalized size = 1.09

$$\frac{\frac{4\left(-2a^2cx+abx\left(b-3cx^2\right)+b^3x^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2x\left(-36a^2c^2+11ab^2c-16abc^2x^2-2b^4+b^3cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(40a^2c^2+18ab^2c-16abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-40a^2c^2-18ab^2c-16abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4 + 11*a*b^2*c - 36*a^2*c^2 + b^3*c*x^2 - 16*a*b*c^2*x^2))/((b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (4*(-2*a^2*c*x + b^3*x^3 + a*b*x*(b - 3
*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-b^4 +
18*a*b^2*c + 40*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b^4 - 18*a*b^2*c - 4
0*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a + bx^2 + cx^4\right)^3}\, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[x^8/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 2.29, size = 3725, normalized size = 10.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/16*(2*(b^3*c - 16*a*b*c^2)*x^7 - 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^5 - 4
*(a*b^3 + 14*a^2*b*c)*x^3 + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8
*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^
4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c
^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4
*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 6
40*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2
*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*
b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*
c^5 - (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 345
60*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*
c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c
^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2
*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 64
0*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 6
25*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 +
1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6
*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
c^3 + 2(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2
+ 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(
b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*
c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^

6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 -
20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*
a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^
3)*x - 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^
3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 - (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^
2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 5324
8*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7
 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))
*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 2
0*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a
^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 16
0*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^1
0*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7
 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 +
a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*
b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^
3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680
*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 +
 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^
10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^
10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*
b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c
+ 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c -
 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 + (
b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*
b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625
*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 12
80*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^
2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b
^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*
c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^
4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 -
640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 +
2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^
3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 3
5*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 1
60*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b
^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 -
 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^
8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8
))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x -
1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 236
80*a^4*b^2*c^4 + 32000*a^5*c^5 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c
^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b

```
*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*
a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-
(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8
*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)
*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b
^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 -
 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024
*a^5*c^8))) - 2*(a^2*b^2 + 20*a^3*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3
+ 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c -
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)
```

giac [B] time = 2.46, size = 4558, normalized size = 13.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/64*(2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b^9*c^6 - 2688*a^3*b^7*c^7 + 20
48*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^6*b*c^10 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^13*c^2 + 34*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^11*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 - 344*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 5632*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 448*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 10240*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 5120*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 768*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 2560*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 2*(b^2 - 4*a*c)*b^11*c
^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 - 448*(b^2 - 4*a*c)*a^2*b^7*c^6 + 896*(b^2
- 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 - 4*a*c)*a^4*b^3*c^8 - 5120*(b^2 - 4*a*c)*
a^5*b*c^9 - (2*b^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
```

*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^8*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 2*a*b^8*c^3 - 192*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^5*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*
c^4 - 16*a^2*b^6*c^4 + 896*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*
c^5 + 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 384*a^3*b^4*c^5 - 1280*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b*c^6 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c^6 - 1792*a^4*b^2*c^6 + 320*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
c^7 + 2560*a^5*c^7 + 2*(b^2 - 4*a*c)*a*b^6*c^3 + 24*(b^2 - 4*a*c)*a^2*b^4*c
^4 - 288*(b^2 - 4*a*c)*a^3*b^2*c^5 + 640*(b^2 - 4*a*c)*a^4*c^6)*abs(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3))*arctan(2*sqrt(1/2)*x/sqrt((b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3 + sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)^2 - 4*(a*b^4*c -
 8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/(b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((a*b^10*c^3 - 20*a^2*b^8*c^4 - 2*a*b^9*c^4
 + 160*a^3*b^6*c^5 + 32*a^2*b^7*c^5 + a*b^8*c^5 - 640*a^4*b^4*c^6 - 192*a^3
*b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a^5*b^2*c^7 + 512*a^4*b^3*c^7 + 96*a^3*b^4
*c^7 - 1024*a^6*c^8 - 512*a^5*b*c^8 - 256*a^4*b^2*c^8 + 256*a^5*c^9)*abs(b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*abs(c)) + 1/64*(2*b^13*c^4 - 68*a*b^11*c^5
+ 688*a^2*b^9*c^6 - 2688*a^3*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9
 - 20480*a^6*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^13*c^2 + 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^11*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^12*c^3 - 344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^9*c^4 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^10*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^11*c
^4 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7
*c^5 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
8*c^5 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*
c^5 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
5*c^6 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^6*c^6 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^7*c^6 - 5632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^3*c^7 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^4*c^7 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b^5*c^7 + 10240*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b*c^8 + 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c

```
)*c)*a^4*b^3*c^8 - 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^5*b*c^9 - 2*(b^2 - 4*a*c)*b^11*c^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 -
448*(b^2 - 4*a*c)*a^2*b^7*c^6 + 896*(b^2 - 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 -
 4*a*c)*a^4*b^3*c^8 - 5120*(b^2 - 4*a*c)*a^5*b*c^9 - (2*b^5*c^2 - 40*a*b^3*
c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c -
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2 +
 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 + 8*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^7*c^3 + 2*a*b^8*c^3 - 192*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 + sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 16*a^2*b^6*c^4 + 896*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 288*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^4*c^5 - 384*a^3*b^4*c^5 - 1280*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^5*c^6 - 640*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 144*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 1792*a^4*b^2*c^6 + 320*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 2560*a^5*c^7 - 2*(b^2 - 4*a*
c)*a*b^6*c^3 - 24*(b^2 - 4*a*c)*a^2*b^4*c^4 + 288*(b^2 - 4*a*c)*a^3*b^2*c^5
 - 640*(b^2 - 4*a*c)*a^4*c^6)*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*arctan
(2*sqrt(1/2)*x/sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 - sqrt((b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)^2 - 4*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((
a*b^10*c^3 - 20*a^2*b^8*c^4 - 2*a*b^9*c^4 + 160*a^3*b^6*c^5 + 32*a^2*b^7*c^
5 + a*b^8*c^5 - 640*a^4*b^4*c^6 - 192*a^3*b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a
^5*b^2*c^7 + 512*a^4*b^3*c^7 + 96*a^3*b^4*c^7 - 1024*a^6*c^8 - 512*a^5*b*c^
8 - 256*a^4*b^2*c^8 + 256*a^5*c^9)*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*ab
s(c)) + 1/8*(b^3*c*x^7 - 16*a*b*c^2*x^7 - b^4*x^5 - 5*a*b^2*c*x^5 - 36*a^2*
c^2*x^5 - 2*a*b^3*x^3 - 28*a^2*b*c*x^3 - a^2*b^2*x - 20*a^3*c*x)/((b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.04, size = 953, normalized size = 2.74

$$-\frac{5\sqrt{2}\,a^{2}c\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{5\sqrt{2}\,a^{2}c\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{9\sqrt{2}\,ab^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{9\sqrt{2}\,ab^{2}\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\sqrt{2}\,b^{4}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{16\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\,c}+\frac{\sqrt{2}\,b^{4}\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{16\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\,c}+\frac{\sqrt{2}\,ab\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\sqrt{2}\,b^{3}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{16\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\,c}+\frac{\sqrt{2}\,b^{3}\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{16\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\,c}+\frac{-\frac{\left(16ac-b^{2}\right)bx^{7}}{8\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)}-\frac{\left(14ac+b^{2}\right)abx^{3}}{4\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)c}-\frac{\left(36a^{2}c^{2}+5ab^{2}c+b^{4}\right)x^{5}}{8\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)c}-\frac{\left(20ac+b^{2}\right)a^{2}x}{8\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)c}}{\left(cx^{4}+bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*b*(16*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-1/8*(36*a^2*c^2+5*a*b^2
*c+b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*a/c*b*(14*a*c+b^2)/(16*a^2*c^2
```

```
-8*a*b^2*c+b^4)*x^3-1/8*a^2*(20*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c
*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-1/16
/(16*a^2*c^2-8*a*b^2*c+b^4)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3-5/2/(16*a^2*c^2-8*a
*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2-9/8/(16*a^2*c^2-
8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2+1/16/(16*a^2*
c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4-1/(16*a^2
*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/
c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*b^3-5/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a*b^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3c-16\,abc^2\right)x^7-\left(b^4+5\,ab^2c+36\,a^2c^2\right)x^5-2\left(ab^3+14\,a^2bc\right)x^3-\left(a^2b^2+20\,a^3c\right)x}{8\left(\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)x^8+a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3+2\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4\right)x^6+\left(b^6c-6\,ab^4c^2+32\,a^3c^4\right)x^4+2\left(ab^5c-8\,a^2b^3c^2+16\,a^3bc^3\right)x^2\right)}-\frac{-\int\frac{ab^2+20\,a^2c+\left(b^3-16\,abc\right)x^2}{cx^4+bx^2+a}\,dx}{8\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^3*c - 16*a*b*c^2)*x^7 - (b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^5 - 2*(a*b
^3 + 14*a^2*b*c)*x^3 - (a^2*b^2 + 20*a^3*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b
^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2) - 1/8*integrate(-(a*b^2 + 20*a^2
*c + (b^3 - 16*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)
```

mupad [B] time = 8.54, size = 9575, normalized size = 27.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*
b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^
```

6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c
^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 -
40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 -
258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*
a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4
- 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b
^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3
*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1
140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c
^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a
c - b^2)^15)^(1/2))/(512(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 7
20*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*
c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26
21440*a^9*b^2*c^12)))^(1/2) - (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208
*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^
4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^1
5*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3
- 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7
- 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 294912
0*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i - (((5242880*a^7*c^8 - 25
6*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6
- 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a
^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*
(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13
*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*
a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15
)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*
c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160
*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^
2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a
^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4
*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(
4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*
b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 186
3680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(10
48576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^
14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 19
66080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) +
(x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(
32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))

)*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i)/((((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(

b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^
2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6
*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(
1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5
 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^
6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c
^12)))^(1/2) + (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c^3 -
36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8
 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b
^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-
(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4
 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b
^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11
 - 2621440*a^9*b^2*c^12)))^(1/2) - (35*a^2*b^7 - 1176*a^3*b^5*c + 6400*a^5*b
*c^3 + 9456*a^4*b^3*c^2)/(256*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*
a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6))))*(-
(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c
^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^
6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^
5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a
^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*
c^12)))^(1/2)*2i + atan(((((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^
8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(51
2*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c
^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 - b^2*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 +
 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b
^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10
*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5
3760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*
b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c
^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5
*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^
4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^1
5*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3
 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7
 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 294912
0*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(b^8 + 800*a^4*c^4 + 31
4*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 1
6*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c -
b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3

+ 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7
*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^
10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 +
53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^
7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i - (((5
242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 +
2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24
*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*
a^5*b^2*c^6)) + (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*
c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^
5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c
*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*
c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^
5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^
11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*
a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(
32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^
13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 68096
0*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^1
6*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 8601
60*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*
b^2*c^12)))^(1/2) + (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c
^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
- 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*
b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*
a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a
c(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^1
8*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*
a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*
c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i)/((((5242880*a^7*c^8 - 256*a*b^12*c
^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456
*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
- 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 -
b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 101
60*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^
6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(
512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680
*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c
^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^
(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5
- 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*
a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 +

34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b
^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10
*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5
3760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*
b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(b^8 +
800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c
+ 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17
- b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 1
0160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*
c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))
/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 76
80*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8
*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12))
)^(1/2) + (((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*
a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 409
6*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b
^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) -
1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*
c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b
^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^
3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c
^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949
120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9
*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a
^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8
+ 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b
^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-
(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4
+ 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b
^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11
- 2621440*a^9*b^2*c^12)))^(1/2) + (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 +
208*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2)
- 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^
9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a
*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*
c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12
*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 29
49120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (35*a^2*b^7 - 1176*a^3
*b^5*c + 6400*a^5*b*c^3 + 9456*a^4*b^3*c^2)/(256*(b^12*c + 4096*a^6*c^7 - 2
4*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144
*a^5*b^2*c^6))))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^
8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*
b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(

```
-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^
4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*
b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11
 - 2621440*a^9*b^2*c^12)))^(1/2)*2i - ((x^3*(a*b^3 + 14*a^2*b*c))/(4*c*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^7*(b^3 - 16*a*b*c))/(8*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)) + (x^5*(b^4 + 36*a^2*c^2 + 5*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)) + (a^2*x*(20*a*c + b^2))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
)/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.686 $\int \frac{x^6}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=298

$$\frac{3x\left(x^2\left(4ac+b^2\right)+4ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\left(-\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\right.}{}$$

Rubi [A] time = 0.68, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1275, 1166, 205}

$$\frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(x^2\left(4ac+b^2\right)+4ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\left(-\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^3*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x*(4*a*b +
 (b^2 + 4*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2 + 4*
a*c - (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (3*(b^2 + 4*a*c + (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(
b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{x^2\left(6a-3bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4ab+\left(b^2+4ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{12ab-3\left(b^2+4ac\right)x^2}{a+bx^2+cx^4}\,dx}{8\left(b^2-4ac\right)^2}$$

$$= \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4ab+\left(b^2+4ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3\left(b^2+4ac-\frac{b\left(b^2+12\cdots\right.}{\sqrt{b^2-4\cdots}}\right.\right.}{16\left(b\cdots\right.}$$

$$= \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4ab+\left(b^2+4ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\left(b^2+4ac-\frac{b\left(b^2+12a\cdots\right.}{\sqrt{b^2-4ac\cdots}}\right.}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\cdots\right.}$$

Mathematica [A] time = 0.85, size = 343, normalized size = 1.15

$$\frac{-\frac{4\left(ax\left(b-2cx^2\right)+b^2x^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{8abcx+24ac^2x^3+4b^3x+6b^2cx^3}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-12abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}+12abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}}{16c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^3*x + 8*a*b*c*x + 6*b^2*c*x^3 + 24*a*c^2*x^3)/((b^2 - 4*a*c)^2*(a + b
*x^2 + c*x^4)) - (4*(b^2*x^3 + a*x*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^3 - 12*a*b*c + b^2*Sqrt[b^2 - 4*a*c]
 + 4*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sq
rt[c]*(b^3 + 12*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 1.66, size = 3128, normalized size = 10.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^2*c + 4*a*c^2)*x^7 + 2*(5*b^3 + 16*a*b*c)*x^5 + 24*a^2*b*x + 2*(
19*a*b^2 - 4*a^2*c)*x^3 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c +
 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a
*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*
c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 128
0*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 6
40*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*
c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*
a^3*b*c^3 + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*
b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c
^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*
a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*
c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 -
 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4
```

+ 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (
b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^
5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*
b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^
2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^
4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b
^3*c^2 - 128*a^3*b*c^3 + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^1
0*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6
 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b
^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^
6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*
a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640
*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) + 3*sqrt(1/2)*((b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6
 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*
a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 12
80*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c
^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8
*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)
)*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 - 24*a*b^5
*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b
^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*
c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 128
0*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^
10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5
- 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^
4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*
b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2
)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*
a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*
a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3
*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 +
160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10
*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 -
1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b
^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^10*c^
2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^
6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*
b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^

```
2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280
*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4
 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c
^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)))
)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*
a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)
```

giac [B] time = 2.82, size = 1750, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/16*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 16*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^4*c - 4*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 16
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^3*c^2 + 32*a*b^3*c^2 + 6*b^4*c^2 - 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 64*a^2*b*c^3 - 16*a*b^2*c^3 - 32*a^2*c^4
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 4*(b^2 - 4*a*c)*b^3*c - 16*(b^2 - 4*a*c)
*a*b*c^2 - 6*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1
/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sqrt((b^5 - 8*a*b^3*c + 16*a^2
*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((b^8 - 16*a*b^6*c - 2*b^7*
c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*c^3 - 96*a^2*b^3*
c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*abs(c)) - 3/16*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 16*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 4*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^4*c + 4*b^5*c + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 32*a*b^3*c^2 - 6*b^4*c^2 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 64*a^2*b*c^3 + 16*a*b^2*c^3
 + 32*a^2*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
- 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 3*sqrt(2)*sqrt
```

```
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 4*(b^2 - 4*a*c)*b^3*c + 16
*(b^2 - 4*a*c)*a*b*c^2 + 6*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a
rctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt((b^5 - 8*a*
b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((b^8 - 16*a*
b^6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*c^3
 - 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*abs(c)) + 1/8*(3*b^2*c*x^7 + 12*a*c^2*x^7 + 5*b^3*x^5 +
16*a*b*c*x^5 + 19*a*b^2*x^3 - 4*a^2*c*x^3 + 12*a^2*b*x)/((c*x^4 + b*x^2 + a
)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

maple [B] time = 0.04, size = 753, normalized size = 2.53

$$\frac{9\sqrt{2}\,abc\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{9\sqrt{2}\,abc\,\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{3\sqrt{2}\,b^3\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{16\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{3\sqrt{2}\,b^3\,\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{16\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{3\sqrt{2}\,ac\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{3\sqrt{2}\,ac\,\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{3\sqrt{2}\,b^2\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{16\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{3\sqrt{2}\,b^2\,\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{16\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\frac{3\left(4ac+b^2\right)c\,x^7}{8\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{\left(16ac+5b^2\right)b\,x^5}{128a^2c^2-64ab^2c+8b^4}+\frac{3a^2bx}{2\left(16a^2c^2-8ab^2c+b^4\right)}-\frac{\left(4ac-19b^2\right)a\,x^3}{8\left(16a^2c^2-8ab^2c+b^4\right)}}{\left(cx^4+bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (3/8*c*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*b*(16*a*c+5*b^2)/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(4*a*c-19*b^2)*a/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3+3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*b*x)/(c*x^4+b*x^2+a)^2-3/4/(16*a^2*c
^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-3/16/(16*a^2*c^2-8*a*b^2*c+b^4)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^2+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*a*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*a+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+9/4/(16*a
^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b+3/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^2*c + 4*a*c^2)*x^7 + (5*b^3 + 16*a*b*c)*x^5 + 12*a^2*b*x + (19*a*
b^2 - 4*a^2*c)*x^3)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2
) + 3/8*integrate(((b^2 + 4*a*c)*x^2 - 4*a*b)/(c*x^4 + b*x^2 + a), x)/(b^4
- 8*a*b^2*c + 16*a^2*c^2)
```

mupad [B] time = 8.18, size = 8521, normalized size = 28.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 16384
0*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 409
6*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^
5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 819
20*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 10
24*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a
^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4
*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8
+ 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*
a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327
680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 -
 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5
 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*
a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 25
8048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*
b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) - (x*(9*b^6*c - 288*a^3*c^4 + 126*a
*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*
a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*
a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*
a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10
*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^
12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2
949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*1i - (((3*(1024*a*b^11*c
^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^
4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c
^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
 + (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^
11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^
6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2
+ 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^
```

10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2
621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*
c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^
8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^
15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*
a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20
*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16
*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 86016
0*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2
c^10)))^(1/2) + (x(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^
3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)
))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*
c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b
^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 7
20*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*
c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621
440*a^9*b^2*c^10)))^(1/2)*1i)/((3*(576*a^4*c^4 + 540*a^2*b^4*c^2 + 1584*a^3
*b^2*c^3 + 45*a*b^6*c))/(256*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (((3*(1
024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4
 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2
4*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7
 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^
5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40
*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 2
58048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8
*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 2
62144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c
^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c
)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11
*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*
b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 +
720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10
*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 262
1440*a^9*b^2*c^10)))^(1/2) - (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 57
6*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 5
60*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 +
 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*
b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 2580
48*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^
4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (((3*(1024*a*b^11*c^2 - 1048576*a^6
*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310
720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^

6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*2i + ((x^3*(19*a*b^2 - 4*a^2*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c*x^7*(4*a*c + b^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a^2*b*x)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^5*(16*a*c + 5*b^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^

3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7
- 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)
- (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a
*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*
b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b
^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3
- 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^
6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^1
0)))^(1/2)*1i - (((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c
^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(
b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4
- 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b
^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7
*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c +
1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 +
53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7
*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^
2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*
c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^
3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7
*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5
*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^
11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*
c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949
120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (x*(9*b^6*c - 288*a^3*c^4
+ 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^
2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 +
81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4
+ 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 10485
76*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760
*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*
c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*1i)/((((3*(1024*a
*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 65
5360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^
2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b
^10*c)) - (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*
a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61
440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^1
8*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*
a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c
^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*
a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(
32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((
9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 -

4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) - (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (3*(576*a^4*c^4 + 540*a^2*b^4*c^2 + 1584*a^3*b^2*c^3 + 45*a*b^6*c))/(256*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*2i

sympy [B] time = 23.39, size = 627, normalized size = 2.10

$$\frac{12a^2bx + x^7\left(12ac^2 + 3b^2c\right) + x^5\left(16abc + 5b^3\right) + x^3\left(-4a^2c + 19ab^2\right)}{128a^4c^2 - 64a^3b^2c + 8a^2b^4 + x^8\left(128a^2c^4 - 64ab^2c^3 + 8b^4c^2\right) + x^6\left(256a^2bc^3 - 128ab^3c^2 + 16b^5c\right) + x^4\left(256a^3c^3 - 48ab^4c + 8b^6\right) + x^2\left(256a^3bc^2 - 128a^2b^3c + 16ab^5\right)} + \operatorname{RootSum}\left(t^4\left(68719476736a^{10}c^{11} - 171798691840a^9b^2c^{10} + 193273528320a^8b^4c^9 - 128849018880a^7b^6c^8 + 56371445760a^6b^8c^7 - 16911433728a^5b^{10}c^6 + 3523215360a^4b^{12}c^5 - 503316480a^3b^{14}c^4 + 47185920a^2b^{16}c^3 - 2621440ab^{18}c^2 + 65536b^{20}c\right) + t^2\left(-188743680a^7bc^7 + 141557760a^6b^3c^6 - 2359296a^5b^5c^5 - 26542080a^4b^7c^4 + 9584640a^3b^9c^3 - 1290240a^2b^{11}c^2 + 46080ab^{13}c + 2304b^{15}\right) + 20736a^5c^4 + 103680a^4b^2c^3 + 142560a^3b^4c^2 + 32400a^2b^6c + 2025ab^8, \left(t \mapsto t\log\left(x + \frac{33554432t^3a^6c^7 - 16777216t^3a^5b^2c^6 - 10485760t^3a^4b^4c^5 + 10485760t^3a^3b^6c^4 - 3276800t^3a^2b^8c^3 + 458752t^3ab^{10}c^2 - 24576t^3b^{12}c - 64512ta^3bc^3 - 43776ta^2b^3c^2 - 21312tab^5c - 144tb^7}{432a^2c^2 + 1080ab^2c + 135b^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (12*a**2*b*x + x**7*(12*a*c**2 + 3*b**2*c) + x**5*(16*a*b*c + 5*b**3) + x**
3*(-4*a**2*c + 19*a*b**2))/(128*a**4*c**2 - 64*a**3*b**2*c + 8*a**2*b**4 +
x**8*(128*a**2*c**4 - 64*a*b**2*c**3 + 8*b**4*c**2) + x**6*(256*a**2*b*c**3
 - 128*a*b**3*c**2 + 16*b**5*c) + x**4*(256*a**3*c**3 - 48*a*b**4*c + 8*b**
6) + x**2*(256*a**3*b*c**2 - 128*a**2*b**3*c + 16*a*b**5)) + RootSum(_t**4*
(68719476736*a**10*c**11 - 171798691840*a**9*b**2*c**10 + 193273528320*a**8
*b**4*c**9 - 128849018880*a**7*b**6*c**8 + 56371445760*a**6*b**8*c**7 - 169
11433728*a**5*b**10*c**6 + 3523215360*a**4*b**12*c**5 - 503316480*a**3*b**1
4*c**4 + 47185920*a**2*b**16*c**3 - 2621440*a*b**18*c**2 + 65536*b**20*c) +
 _t**2*(-188743680*a**7*b*c**7 + 141557760*a**6*b**3*c**6 - 2359296*a**5*b*
*5*c**5 - 26542080*a**4*b**7*c**4 + 9584640*a**3*b**9*c**3 - 1290240*a**2*b
**11*c**2 + 46080*a*b**13*c + 2304*b**15) + 20736*a**5*c**4 + 103680*a**4*b
**2*c**3 + 142560*a**3*b**4*c**2 + 32400*a**2*b**6*c + 2025*a*b**8, Lambda(
_t, _t*log(x + (33554432*_t**3*a**6*c**7 - 16777216*_t**3*a**5*b**2*c**6 -
10485760*_t**3*a**4*b**4*c**5 + 10485760*_t**3*a**3*b**6*c**4 - 3276800*_t*
*3*a**2*b**8*c**3 + 458752*_t**3*a*b**10*c**2 - 24576*_t**3*b**12*c - 64512
*_t*a**3*b*c**3 - 43776*_t*a**2*b**3*c**2 - 21312*_t*a*b**5*c - 144*_t*b**7
)/(432*a**2*c**2 + 1080*a*b**2*c + 135*b**4)))
```

3.687 $\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=289

$$\frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-4ac+7b^2+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 0.71, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1178, 1166, 205}

$$\frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-4ac+7b^2+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x*(7*b^2 - 4*a
*c + 12*b*c*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(3*b
^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{3a\left(b^2+4ac\right)-12abcx^2}{a+bx^2+cx^4}\,dx}{8a\left(b^2-4ac\right)^2}$$

$$= \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(3b^2+4ac-2b\sqrt{}\right.\right.}{8\left(\right.}$$

$$= \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{c}\left(3b^2+4ac-2b\right.}{4\sqrt{2}\left(b^2-4a\right.}$$

Mathematica [A] time = 0.71, size = 285, normalized size = 0.99

$$\frac{1}{8}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{4acx-7b^2x-12bcx^3}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (-7*b^2*x + 4*
a*c*x - 12*b*c*x^3)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt
[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a + bx^2 + cx^4\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 1.10, size = 3128, normalized size = 10.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*b*c^2*x^7 + 2*(19*b^2*c - 4*a*c^2)*x^5 + 2*(5*b^3 + 16*a*b*c)*x^3
 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*
a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqr
t(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*
c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20
*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^
7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a
^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x +
 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4 - (a*b^13 -
 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 1433
6*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6
*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*
a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4
*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c +
160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^
10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2
```

```
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c
 + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqr
t(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^
2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b
^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 +
16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*
c^4 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5
*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*
c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*s
qrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^
6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 -
20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*
a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280
*a^5*b^2*c^4 - 1024*a^6*c^5))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*
b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10
 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 10
24*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^
3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*
c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c +
40*a*b^2*c^2 + 16*a^2*c^3)*x + 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2
*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7
*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^1
0 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1
024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8
*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/s
qrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*
b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4
*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a
^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*
b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a
^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 -
 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c
 + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*lo
g(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*
c + 128*a^3*b^2*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2
 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^
6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*
a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^
10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
```

```
1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*
c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^
6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) + 6*(a*b^2 + 4
*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^
4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)
```

giac [B] time = 2.64, size = 1861, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2 + 64*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2 - 4*a*c)*b^3*
c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/
2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sqrt((b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((a*b^8 - 16*a^2*b^6*c - 2*a
*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*c^2 - 256*a^4*b^2*c^3 - 96
*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128*a^4*b*c^4 + 48*a^3*b^2*c^
4 - 64*a^4*c^5)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6
 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*a
*b^4*c^2 - 2*b^5*c^2 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 +
 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^2*b^2*c^3 - 16
*a*b^3*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 128*a^3*c
^4 + 96*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*
c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 48*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 24*sq
```

rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 12*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^
4*c + 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a^2*c^3 + 24*(b^2 - 4*a*c)
*a*b*c^3)*arctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt(
(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((
a*b^8 - 16*a^2*b^6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*
c^2 - 256*a^4*b^2*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128
*a^4*b*c^4 + 48*a^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) - 1/8*(12*b*c^2*x^7 + 19*
b^2*c*x^5 - 4*a*c^2*x^5 + 5*b^3*x^3 + 16*a*b*c*x^3 + 3*a*b^2*x + 12*a^2*c*x
)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))

maple [B] time = 0.04, size = 617, normalized size = 2.13

$$-\frac{3\sqrt{2}\, a c^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\left(16a^{2}c^{2}-8a b^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{3\sqrt{2}\, a c^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\left(16a^{2}c^{2}-8a b^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{9\sqrt{2}\, b^{2} c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\left(16a^{2}c^{2}-8a b^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{9\sqrt{2}\, b^{2} c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\left(16a^{2}c^{2}-8a b^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{3\sqrt{2}\, b c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{4\left(16a^{2}c^{2}-8a b^{2}c+b^{4}\right)\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{3\sqrt{2}\, b c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{4\left(16a^{2}c^{2}-8a b^{2}c+b^{4}\right)\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{-\frac{3b c^{2}x^{7}}{2\left(16a^{2}c^{2}-8a b^{2}c+b^{4}\right)}+\frac{\left(4ac-19b^{2}\right)c x^{5}}{128a^{2}c^{2}-64a b^{2}c+8b^{4}}-\frac{\left(16ac+5b^{2}\right)b x^{3}}{8\left(16a^{2}c^{2}-8a b^{2}c+b^{4}\right)}-\frac{3\left(4ac+b^{2}\right)a x}{8\left(16a^{2}c^{2}-8a b^{2}c+b^{4}\right)}}{\left(c x^{4}+b x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^4+b*x^2+a)^3,x)

[Out] (-3/2*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*c*(4*a*c-19*b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^5-1/8*b*(16*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8
a(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/4/(16*a^2*
c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)
*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-9/8/(16*a^2*c^2-8*a*b^2*c+b
^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-3/4/(16*a^2*c^2-8*a*b^2*
c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*a-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*b^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] -1/8*(12*b*c^2*x^7 + (19*b^2*c - 4*a*c^2)*x^5 + (5*b^3 + 16*a*b*c)*x^3 + 3*
(a*b^2 + 4*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -

```
 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^
6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^
2) - 3/8*integrate((4*b*c*x^2 - b^2 - 4*a*c)/(c*x^4 + b*x^2 + a), x)/(b^4 -
 8*a*b^2*c + 16*a^2*c^2)
```

mupad [B] time = 7.59, size = 8397, normalized size = 29.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c
^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2
4*a*b^10*c)) - (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 +
 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5
 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*
a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 25
8048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*
b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 13
1072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6
))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))
)*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^
2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3
*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720
*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^
5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 262144
0*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(1
6*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9
*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4
160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6
- 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*
b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 8
60160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^1
0*b^2*c^9)))^(1/2)*1i - (((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^
3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12
+ 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 61
44*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 +
 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4
+ 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 10485
76*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760
*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*
c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2
560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 +
```

163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*
c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7
 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c
^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 4
0*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 -
258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^
9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) + (x*(144*a^2*c^5 + 117*b^4*c^3 +
 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 56
0*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 -
61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2
*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 25804
8*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4
*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*1i)/((((3*(262144*a^6*c^8 - 64*b^12*c^
2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2
c^7))/(128(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 384
0*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*((9*((-(4*a*c - b^2)^
15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 +
11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(5
12*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^
4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6
 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)
*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81
920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*
c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15
 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^
4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 104
8576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 537
60*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^
6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c
^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81
920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1
024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*
a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^
5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7
 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (3*(45*b^5*c^3 + 3
60*a*b^3*c^4 + 144*a^2*b*c^5))/(64*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (
((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81
920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*
b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^1
0*c)) + (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^
2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 6144
0*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^1

8*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) + (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*2i - ((x^3*(5*b^3 + 16*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^5*(4*a*c^2 - 19*b^2*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^7)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*x*(4*a*c + b^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2

) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c
^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 10
48576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53
760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b
^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*1i - (((3*(262
144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*
b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2
- 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) +
(x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*
c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b
^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 7
20*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*
c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621
440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7
 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 +
 (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*
b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b
^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2
 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^
7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^
9)))^(1/2) + (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*
a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(
4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*
c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*
c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7
680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^
8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))
^(1/2)*1i)/((((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2
*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c
^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c
^5 - 24*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*
b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*
b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^1
0 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c
^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 29491
20*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c
^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*
b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*
b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2
*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440
*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18
*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6
*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8
- 2621440*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*

```
c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*
c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^1
1*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6
*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c +
 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^1
0*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 26
21440*a^10*b^2*c^9)))^(1/2) - (3*(45*b^5*c^3 + 360*a*b^3*c^4 + 144*a^2*b*c^
5))/(64*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^
4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (((3*(262144*a^6*c^8 - 64*b^
12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^
5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3
+ 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(b^15 + (-(
4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*
c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*
c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7
680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^
8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))
^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^
4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^
2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15
)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4
*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^2
0 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^
3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080
*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) + (x*(14
4*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^
4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1
/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7
*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 +
1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 +
53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8
*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2))*(-(9*(b^15
 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^
3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a
*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c
^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*
a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*
c^9)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**3,x)
```

[Out] Timed out

3.688 $\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=311

$$-\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(cx^2\left(20ac+b^2\right)+b\left(8ac+b^2\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 0.70, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1119, 1178, 1166, 205}

$$-\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(cx^2\left(20ac+b^2\right)+b\left(8ac+b^2\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b*(b^2 + 8
*a*c) + c*(b^2 + 20*a*c)*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
(Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{b-10cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)x^2\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-b\left(b^2-16ac\right)-c\left(\cdots\right.}{a+bx^2+\cdots}}{8a\left(b^2-\cdots\right.}$$

$$= -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)x^2\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(c\left(b^2+20ac\cdots\right.\right.}{\cdots}$$

$$= -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)x^2\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b^2+20ac\cdots\right.}{8\sqrt{2}\,a\left(b\cdots\right.}$$

Mathematica [A] time = 0.85, size = 334, normalized size = 1.07

$$\frac{1}{16}\left(-\frac{4x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(8abc+20ac^2x^2+b^3+b^2cx^2\right)}{a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(b^3 + 8
*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
 + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b
^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^3 + 5
2*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 1.68, size = 3777, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*x^7 + 4*(b^3*c + 14*a*b*c^2)*x^5 + 2*(b^4 + 5*
a*b^2*c + 36*a^2*c^2)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 -
 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^
4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c
^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^1
0*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 6
40*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a
*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x + 1/2*sqrt(1/2)*(b^11 - 53*
a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^
5*b*c^5 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 +
 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^
6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 -
 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c +
```

160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt(
(b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2
 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^
4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c
^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 -
 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6
+ (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 +
(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2
*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a
^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^1
1*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 128
0*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2
*b^2*c^4 + 10000*a^3*c^5)*x - 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^
7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 - (a^3*b^14
- 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 +
1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2
*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*
c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^
2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 6
40*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c +
625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 +
 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^
6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) + sqrt(1/2)*((
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4
*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-
(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b
^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)
*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b
^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 -
 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024
*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3
*c^5)*x + 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5
*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 + (a^3*b^14 - 38*a^4*b^12*c + 48
0*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 3
2768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a
^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*
c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3
*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 128
0*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^
10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 -
 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*
c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^
2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c -

```
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4
 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c +
280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c
^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^
2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4
*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*
a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b
^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x - 1/2*sqrt(1
/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3
*c^4 - 25600*a^5*b*c^5 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 272
0*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 4
0960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8
*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)
))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 -
 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024
*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c +
160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a
^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c
^4 - 1024*a^8*c^5))) - 2*(a*b^3 - 16*a^2*b*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^
3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a
^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2
*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
```

giac [B] time = 2.45, size = 4270, normalized size = 13.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*(2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*
c^5 + 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4
```

*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 5120*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6 + (2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c - 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 56*a^2*b^7*c^2 - 832*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 1664*a^4*b^3*c^4 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2048*a^5*b*c^5 + 2*(b^2 - 4*a*c)*a*b^7*c - 48*(b^2 - 4*a*c)*a^2*b^5*c^2 + 288*(b^2 - 4*a*c)*a^3*b^3*c^3 - 512*(b^2 - 4*a*c)*a^4*b*c^4)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))*arctan(2*sqrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2 + a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*a^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c^5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*abs(c)) - 1/64*(2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 + 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b

```
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2
- 4*a*c)*a^2*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*
a^4*b^6*c^4 + 5120*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c
^6 + (2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^
3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^8*c + 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 - 56*a^2*b^7*c^2 - 832
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^5*c^3 + 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4
 + 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 1664*a^4*b^3*c
^4 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 2048*a^5*b*c^5
 - 2*(b^2 - 4*a*c)*a*b^7*c + 48*(b^2 - 4*a*c)*a^2*b^5*c^2 - 288*(b^2 - 4*a*
c)*a^3*b^3*c^3 + 512*(b^2 - 4*a*c)*a^4*b*c^4)*abs(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2))*arctan(2*sqrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 - s
qrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*
a^4*c^2)*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2
+ 16*a^3*c^3)))/((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 +
 32*a^4*b^7*c^2 + a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*
b^6*c^3 + 1280*a^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^
5 - 512*a^7*b*c^5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)*abs(c)) + 1/8*(b^2*c^2*x^7 + 20*a*c^3*x^7 + 2*b^3*c*x^5 + 28*
a*b*c^2*x^5 + b^4*x^3 + 5*a*b^2*c*x^3 + 36*a^2*c^2*x^3 - a*b^3*x + 16*a^2*b
*c*x)/((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.16, size = 2958, normalized size = 9.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^3,x)

[Out] 52*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+9*c^3/(
-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-9*c^3/(-4*a*c+b^2)^
2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-27*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b
^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b^3-1/16*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*b^7+1/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*b^6-1/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^6+52*c
^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-1/16*c/(-4*a*c
+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^7-27*c^3/(-4*a*c+b^2)^(
5/2)/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+1/16/(-4*a*c+b^2)^2/(4*a*c-b
^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^6+1/16/(-4*a*c+b^2)^
2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^6+20*c^3/(
-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*x^
3+3/4*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2
)/c)^2*x^3*b^5-56*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4
*a*c+b^2)^(1/2)/c)^2*x*a^3+3/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+
1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^4+3/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2
+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^4+20*c^3/(-4*a*c+b^2)^2/(4*a*c-b
^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*x^3-3/4*c/(-4*a*c+b^2)^(
5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^5+56*c^3/
(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x
*a^3-1/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1
/2)/c)^2/a*x^3*b^7-9*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*
a*c+b^2)^(1/2)/c)^2*a*x^3*b^2+12*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*
b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a^2*b-6*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x
^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a*b^3+12*c^2/(-4*a*c+b^2)^2/(4*a*c
-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a^2*b-6*c/(-4*a*c+b^2)^2
/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a*b^3+20*c^4/(-4*
a*c+b^2)^2/(4*a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+3/4*c^2/(-4*a*c+b^2)^2/(4*a
*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4+4*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^
2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*x^3*b-3*c^2/(-4*a*c+b^2)^(5/2)/(4
*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b^3+42*c^2/(-4*a
```

```
*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a^2*
b^2-21/2*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(
1/2)/c)^2*x*a*b^4+15/4*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^5-4*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2
)^(1/2)/c)^2*a^2*x^3*b+3*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-
1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b^3-42*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)
^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a^2*b^2+21/2*c/(-4*a*c+b^2)^(
5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a*b^4+15/4*c^
2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5-20*c^4/(-4*a*c
+b^2)^2/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-3/4*c^2/(-4*a*c+b^2)^2/(4*a
*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4-9*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2
+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b^2+7/8/(-4*a*c+b^2)^(5/2)/(4*a*
c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^6+3/4/(-4*a*c+b^2)^2/
(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^5-7/8/(-4*a*c+b^
2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^6+1/16/
(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a
*x^3*b^7+3/4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/
2)/c)^2*x*b^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*c^2 + 20*a*c^3)*x^7 + 2*(b^3*c + 14*a*b*c^2)*x^5 + (b^4 + 5*a*b^2
*c + 36*a^2*c^2)*x^3 - (a*b^3 - 16*a^2*b*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3
+ 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + 1/8*integrate((b^3 - 16*a*b*c
+ (b^2*c + 20*a*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)
```

mupad [B] time = 8.37, size = 9731, normalized size = 31.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((b*x*(16*a*c - b^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(b^4 + 36*a
```

^2*c^2 + 5*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^5*(14*a*c^
2 + b^2*c))/(4*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*a*c^2 + b^2*c
))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8
 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9
216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^
5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c
+ 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)
) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a
^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 -
680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^
5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 +
 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440
*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*
c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^
2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-
(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c
^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^
6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^
2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a
^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2
c^9)))^(1/2) - (x(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5
))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2
c^3)))(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*
a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 -
 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c -
 b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a
^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5
+ 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 262144
0*a^12*b^2*c^9)))^(1/2)*1i - (((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a
^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6
 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240
*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (
x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^
13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 68096
0*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^1
6*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 8601
60*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12
*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 -
 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8
 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17
 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 -
10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5

*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2)
)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7
680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^
8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)
))^(1/2) + (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(3
2*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)
))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b
^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 6809
60*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)
^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^
16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860
160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^1
2*b^2*c^9)))^(1/2)*1i)/((((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^
11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 55
05024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*
b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(
b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^
2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6
*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(
1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2
 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^
9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*
c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 4096
0*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 25
6*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^
2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160
*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6
+ 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(51
2*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a
^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6
 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1
/2) - (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^
2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-
(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c
^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^
6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^
2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a
^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2
*c^9)))^(1/2) + (((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 +
 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^
6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
- 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(b^17 + b
^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 1016
0*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6

+ 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(5
12*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*
a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^
6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(
1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^
7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^
4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a
*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^1
1*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 186368
0*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b
^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*
c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 19660
80*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x
*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 +
256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 +
b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 101
60*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^
6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(
512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680
*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c
^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^
(1/2) - (8000*a^3*c^7 - 35*b^6*c^4 - 84*a*b^4*c^5 + 12720*a^2*b^2*c^6)/(256
*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*
c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(b^17 + b^2*(-(4*a*c - b^2)
^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3
4880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3
*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 104
8576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 537
60*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b
^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*2i + atan((((
(256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 + 122880*a^3*b^9*c^
4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a
^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3
 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(b^17 - b^2*(-(4*a*c - b^2)
^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3
4880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3
*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 104
8576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 537
60*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b
^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b
*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5
*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c +
 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2
) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*
b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55

*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13
*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^
12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 +
2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(800*a^3*c^6 - b^
6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a
^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2
)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 +
34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^
3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 10
48576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53
760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*
b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i - (((256*
a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 8
19200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^
12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 38
40*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^
(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*
a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7
- 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*
a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^
7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^
7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7
- 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*
c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a
^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1
720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c
^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^
15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10
 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^
4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 29491
20*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(800*a^3*c^6 - b^6*c^3
 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^
6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)
^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880
*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7
 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576
*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a
^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c
^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i)/((((256*a*b^1
3*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200
*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 +
4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^
6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2)
 - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b
^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*

a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*
c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^1
2*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2
949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256
*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 -
327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^
4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 172032
0*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 +
43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c
+ 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40
*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 2
58048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^
11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(800*a^3*c^6 - b^6*c^3 + 34
*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c +
96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2
) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*
b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55
*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13
*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^
12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 +
2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (((256*a*b^13*c^2 +
4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7
*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8
*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^
4 - 6144*a^7*b^2*c^5)) + (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 17203
20*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 +
43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c
+ 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 4
0*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 -
258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a
^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^1
1*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*
a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 -
256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*
c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^
5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c
*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^1
8*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^
8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c
^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c
^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 -

```
40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 -
 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*
a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (8000*a^3*c^7 - 35*b^6*c^4 -
 84*a*b^4*c^5 + 12720*a^2*b^2*c^6)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b
^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^
2*c^5))))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 114
0*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5
 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c
 - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720
*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^
5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621
440*a^12*b^2*c^9)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.689 $\int \frac{1}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=355

$$\frac{x\left(3bcx^2\left(b^2-8ac\right)+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\cdots}{\sqrt{b\cdots}}\right.}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 1.84, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1092, 1178, 1166, 205}

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3bcx^2\left(b^2-8ac\right)+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*
((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(8*a^2*(b^2 - 4*
a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (3*Sqrt[c]*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt
[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{b^2-2ac-4\left(b^2-4ac\right)-5bcx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc\left(b^2-8ac\right)x^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{3(b \cdots}{\cdots}}{\cdots}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc\left(b^2-8ac\right)x^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3c\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc\left(b^2-8ac\right)x^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{c}\cdots}{\cdots}$$

Mathematica [A] time = 1.02, size = 372, normalized size = 1.05

$$\frac{\frac{2x\left(28a^2c^2-25ab^2c-24abc^2x^2+3b^4+3b^3cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(56a^2c^2-10ab^2c+8abc\sqrt{b^2-4ac}-b^3\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{4ax\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] ((4*a*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2
*x*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2))/((b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56
*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b^
3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]))/(16*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(-3), x]
```

fricas [B] time = 1.44, size = 4323, normalized size = 12.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*x^7 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)
*x^5 + 2*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 - 3*sqrt(1/2)*((a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2
*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c +
32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9
- 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^
10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3
+ 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^
4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160
*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*
(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*
a^4*c^7)*x + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a
^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 8
7808*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*
c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a
^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 24
```

01*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^
3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*
b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160
*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^
8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b
^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c
^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*
b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 3*sqrt(1/2)*((a^2*b^4*c^2 -
8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(
a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32
*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 -
21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10
- 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 10
24*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 +
2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*
c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a
^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(2
1*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^
4*c^7)*x - 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3
*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 878
08*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^
3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^1
2*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401
*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^
5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a
^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8
- 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^1
0 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4
- 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^
4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8
*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^
2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a
^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21
*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 -
20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024
*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 24
01*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^
3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7
*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*
b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*
c^7)*x + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b
^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808
*a^7*c^7 + (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3
+ 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*

```
b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a
^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 +
1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*
c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7
*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 -
22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10
- 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 -
 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*
c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*
b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5
*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21*a
*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 2
0*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a
^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401
*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b
^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*b^
8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^
7)*x - 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8
*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a
^7*c^7 + (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 +
15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*
c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4
*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 12
80*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^
2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b
^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22
*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 -
20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1
024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^
3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 2*(5*a*b^4 - 37*a^2*b^2*c + 44*a^
3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a
^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2)*x^2)
```

giac [B] time = 1.43, size = 2705, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c + s
```

qrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7
*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 -
 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176*a^
2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^4*c
^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 8
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a*c)
*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2 -
4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*c^4
 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b
^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))
)/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a
^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3
- 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^8 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 34*a*b^6*c^2 - 2*
b^7*c^2 - 368*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^4 - 736*a^3*b^2*c^4 - 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*c^5 + 896*a^4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^6*c + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^3*c^2 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -

```
4*a*c)*c)*b^5*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^3 - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^3 - 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^3*c^3 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^6*c + 26*(b^2 - 4*a*c)*a*b^4*c^2 + 2*(b^2
- 4*a*c)*b^5*c^2 - 128*(b^2 - 4*a*c)*a^2*b^2*c^3 - 22*(b^2 - 4*a*c)*a*b^3*c
^3 + 224*(b^2 - 4*a*c)*a^3*c^4 + 88*(b^2 - 4*a*c)*a^2*b*c^4)*arctan(2*sqrt(
1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((a^2*b^5 - 8*a^3*b
^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c
- 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/
((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 +
a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c
^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/8*(3*b^3*c^2*
x^7 - 24*a*b*c^3*x^7 + 6*b^4*c*x^5 - 49*a*b^2*c^2*x^5 + 28*a^2*c^3*x^5 + 3*
b^5*x^3 - 20*a*b^3*c*x^3 - 4*a^2*b*c^2*x^3 + 5*a*b^4*x - 37*a^2*b^2*c*x + 4
4*a^3*c^2*x)/((a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.13, size = 3360, normalized size = 9.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -24*c^4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b
^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5-168*c^5/(-4*a*c+b^2)^(5/2)/(4*
a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+27/8*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(
x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^6-3*c/(-4*a*c+b^2)^2/(4*a*c
-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^5+27*c^2/(-4*a*c+b
^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x*b^2+27*c^2
/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x*
b^2+15*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3-24*c^3/(-4*a
*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b-57
/2*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4-15*c^3/(-4*a
*c+b^2)^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3-24*c^3/(-4*a*c+b^2)^2/(4*
a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b+20*c^3/(-4*a*c+
b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*b*x+6
6*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/
c)^2*a*b^2*x^3-66*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4
```

*a*c+b^2)^(1/2)/c)^2*a*b^2*x^3-20*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2
+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*b*x-57/2*c^3/(-4*a*c+b^2)^(5/2)/(4
*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4-15*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)
^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x*b^3-3*c/(-4*a*c+b^2)^2/(4*a
*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^5-168*c^5/(-4*a*
c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-27/8*c/(-4*a*c+b^2)^(5
/2))/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^6+15*c^2
/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*
a*x*b^3+27/8*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^6-
3/16*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^8+114*c^4/
(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+27/8*c^2/(-4*a
*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^6-3/16*c/(-4*a*c+b^2)
^(5/2)/(4*a*c-b^2)^2/a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^8+114*c^4/(-4*a*c+b^2)^(5/
2)/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+24*c^4/(-4*a*c+b^2)^2/(4*a*c-b^2)
^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b+3*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^5-3/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a^2*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^7+3/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^7-72*c
^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^
2*x^3*a^2+5/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^
2)^(1/2)/c)^2/a*x*b^7-3/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/
2*(-4*a*c+b^2)^(1/2)/c)^2/a^2*x^3*b^8+3/16/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^
2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a^2*x^3*b^7+3/16/(-4*a*c+b^2)^2/(4*a*
c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a^2*x^3*b^7+5/16/(-4*a*c+
b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x*b^6+5/16/
(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x*b
^6-5/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2
)/c)^2/a*x*b^7+3/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a
*c+b^2)^(1/2)/c)^2/a^2*x^3*b^8+72*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2
+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*a^2+45/2*c^2/(-4*a*c+b^2)^(5/2)/(4
*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^4-15/4*c/(-4*a*c
+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^5-44
*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*
a^2*x-44*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/

```
2)/c)^2*a^2*x+15*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+
b^2)^(1/2)/c)^2*x^3*b^3-21/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/
2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^4-45/2*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(
x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^4+15*c^2/(-4*a*c+b^2)^2/(4*a*
c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^3-21/4*c/(-4*a*c+b^
2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^4+15/4*c/(-
4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b
^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(b^{3}c^{2}-8\,abc^{3}\right)x^{7}+\left(6\,b^{4}c-49\,ab^{2}c^{2}+28\,a^{2}c^{3}\right)x^{5}+\left(3\,b^{5}-20\,ab^{3}c-4\,a^{2}bc^{2}\right)x^{3}+\left(5\,ab^{4}-37\,a^{2}b^{2}c+44\,a^{3}c^{2}\right)x}{8\left(\left(a^{2}b^{4}c^{2}-8\,a^{3}b^{2}c^{3}+16\,a^{4}c^{4}\right)x^{8}+a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}+2\left(a^{2}b^{5}c-8\,a^{3}b^{3}c^{2}+16\,a^{4}bc^{3}\right)x^{6}+\left(a^{2}b^{6}-6\,a^{3}b^{4}c+32\,a^{5}c^{3}\right)x^{4}+2\left(a^{3}b^{5}-8\,a^{4}b^{3}c+16\,a^{5}bc^{2}\right)x^{2}\right)}-\frac{-3\int\frac{b^{4}-9\,ab^{2}c+28\,a^{2}c^{2}+\left(b^{3}c-8\,abc^{2}\right)x^{2}}{cx^{4}+bx^{2}+a}\,dx}{8\left(a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*x^7 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*x^
5 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 + (5*a*b^4 - 37*a^2*b^2*c + 44*a
^3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^
5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (
a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5
*b*c^2)*x^2) - 3/8*integrate(-(b^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^3*c - 8*a*
b*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

mupad [B] time = 9.00, size = 10979, normalized size = 30.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (x^5*(6*b^4*c + 28*a^2*c^3 - 49*a*b^2*c^2))/(8*a^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) - (x^3*(4*a^2*b*c^2 - 3*b^5 + 20*a*b^3*c))/(8*a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) + (3*c*x^7*(b^3*c - 8*a*b*c^2))/(8*a^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
- atan(((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 9420
8*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4
*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*
c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^
5)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 7
69*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c
^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b
```

^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 8
60160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*
a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c
^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4
*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(
9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*
c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824
*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))
/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 76
80*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b
^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9
)))^(1/2) + (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^
5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b
^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4
 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560
*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6
*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 25804
8*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13
*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i - (((3*(7340032*a^9*c^9 - 256*a
^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 -
 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a
^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^
3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*(b^19 + b^4*(-(4*a*c -
b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 +
 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^
7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41
*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576
*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a
^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6
*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c
^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b
^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 9
6*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/
2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b
^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 +
3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c -
11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 -
 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4
- 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 29491
20*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*a^4*c^7 + 9*b^8
*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 +
 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^1

9 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 -
8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b
^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*
(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8
*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6
 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1
/2)*1i)/((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 942
08*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^
4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10
*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c
^5)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 +
769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*
c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2
)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*
b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 +
860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440
*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*
c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^
4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-
(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15
*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 106982
4*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))
)/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7
680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*
b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^
9)))^(1/2) + (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c
^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*
b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1
720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^
4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 301056
0*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b
^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^
6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 2580
48*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^1
3*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (((3*(7340032*a^9*c^9 - 256*a^2
*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2
949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4
*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3
+ 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*(b^19 + b^4*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 6
3440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*
b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a

*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a
^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9
*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c
^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7
 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5
*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*
a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2)
 - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^1
1*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 30
10560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11
*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 4
0*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 -
258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120
*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*a^4*c^7 + 9*b^8*c
^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 2
56*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19
+ b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 86
20*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7
*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^
2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a
^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b
^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 -
 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2
) + (3*(189*b^7*c^5 - 3456*a*b^5*c^6 - 56448*a^3*b*c^8 + 22608*a^2*b^3*c^7)
)/(256*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a
^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5))))*(-(9*(b^19 + b^4*(-(4*
a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13
*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343
936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2
) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1
048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 5
3760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^
12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i - atan
(((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*
b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 -
 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 24
0*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) -
(x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*
b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 10
69824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*
(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2
 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a
^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^
2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40

960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i - (((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/

2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i)/((3*(189*b^7*c^5 - 3456*a*b^5*c^6 - 56448*a^3*b*c^8 + 22608*a^2*b^3*c^7))/(256*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(

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1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^
2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*
a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b
^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 4
0960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 +
 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*
(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8
620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^
7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(
a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*
b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6
- 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/
2) - (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 619
2*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2
- 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^
9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 31686
4*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3
*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4
*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c
+ 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b
^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8
 - 2621440*a^14*b^2*c^9)))^(1/2)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^1
9 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^
11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3
010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 1
1*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 -
40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 -
 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 294912
0*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.690 $\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=425

$$-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3x\left(b^2-4ac\right)^2}+\frac{36a^2c^2+bcx^2\left(5b^2-32ac\right)-35ab^2c+5b^4}{8a^2x\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right.\right.}{8\sqrt{2}\,a^3\left(b^2-4ac\right)^2\sqrt{\ }}$$

Rubi [A] time = 0.96, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2+bcx^2\left(5b^2-32ac\right)-35ab^2c+5b^4}{8a^2x\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^3\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3x\left(b^2-4ac\right)^2}+\frac{-2ac+b^2+bcx^2}{4ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*
c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)^2) + (5*b^4 - 35*a*b^
2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x^2)/(8*a^2*(b^2 - 4*a*c)^2*x*(a +
b*x^2 + c*x^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 -
47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b
^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
```

```
tegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx = \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{-5b^2+18ac-7bcx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc\left(5b^2-32ac\right)x^2}{8a^2\left(b^2-4ac\right)^2x\left(a+bx^2+cx^4\right)} + \frac{\int 3}{}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a}{8a^2\left(b^2-4ac\right)}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a}{8a^2\left(b^2-4ac\right)}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a}{8a^2\left(b^2-4ac\right)}$$

Mathematica [A] time = 1.76, size = 454, normalized size = 1.07

$$-\frac{\frac{3\sqrt{2}\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2bc^2-47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}+5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}-124a^2bc^2+47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}-5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{2x\left(84a^2bc^2+52a^2c^3x^2-52ab^3c-47ab^2c^2x^2+7b^5+7b^4cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{4ax\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{16}{x}}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/16*(16/x + (4*a*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(7*b^5 - 52*a*b^3*c + 84*a^2*b*c^2 + 7*b^
4*c*x^2 - 47*a*b^2*c^2*x^2 + 52*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*S
qrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^5 + 47*a*b^
3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c
] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^2 + cx^4\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2 + c*x^4)^3), x]
```

fricas [B] time = 2.59, size = 4924, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + 2*(30*b^5*c - 227*a*
b^3*c^2 + 392*a^2*b*c^3)*x^6 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2
*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*x^4 + 2*(25*a*b^5 - 1
94*a^2*b^3*c + 364*a^3*b*c^2)*x^2 + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (
a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6
*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*
a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*
a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725
*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5
 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*
b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 1
60*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log
(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^
4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x + 27/2*sqrt(1/2)*(125*b^17
- 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^
9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1
324800*a^8*b*c^8 - (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960
*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4
*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10
*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a
^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 -
 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 4
95*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 184
80*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^
3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94
725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*
```

c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x - 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 - (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x + 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^

3*c^7 + 1324800*a^8*b*c^8 + (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x - 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 + (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)

giac [B] time = 2.62, size = 5273, normalized size = 12.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/64*(10*a^6*b^14*c^2 - 254*a^7*b^12*c^3 + 2712*a^8*b^10*c^4 - 15552*a^9*b
^8*c^5 + 50432*a^10*b^6*c^6 - 87552*a^11*b^4*c^7 + 63488*a^12*b^2*c^8 - 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^14 + 127*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^12*c + 10*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^13*c - 1356*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^2 - 214*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^11*c^2 - 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^12*c^2 + 777
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c^3 + 1
856*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^9*c^3 +
 107*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^10*c^3
 - 25216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^6
*c^4 - 8128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b
^7*c^4 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*
b^8*c^4 + 43776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^11*b^4*c^5 + 17920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^10*b^5*c^5 + 4064*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^9*b^6*c^5 - 31744*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^12*b^2*c^6 - 15872*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^11*b^3*c^6 - 8960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^10*b^4*c^6 + 7936*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^11*b^2*c^7 - 10*(b^2 - 4*a*c)*a^6*b^12*c^2 + 214*(b^2
- 4*a*c)*a^7*b^10*c^3 - 1856*(b^2 - 4*a*c)*a^8*b^8*c^4 + 8128*(b^2 - 4*a*c)
*a^9*b^6*c^5 - 17920*(b^2 - 4*a*c)*a^10*b^4*c^6 + 15872*(b^2 - 4*a*c)*a^11*
b^2*c^7 + (10*b^6*c^2 - 114*a*b^4*c^3 + 416*a^2*b^2*c^4 - 480*a^3*c^5 - 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 57*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 208*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 74*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 240*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 120*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 37*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 74*(b^2 - 4
*a*c)*a*b^2*c^3 - 120*(b^2 - 4*a*c)*a^2*c^4)*(a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)^2 + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^11 - 102*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 10*sqrt(2)*sqrt(b*c + sqrt(
```

b^2 - 4*a*c)*c)*a^3*b^10*c - 10*a^3*b^11*c + 836*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 + 164*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 + 204*a^4*b^9*c^2 - 3440*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^3 - 1016*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 - 82*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 - 1672*a^5*b^7*c^3 + 7104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 + 2816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 + 508*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 + 6880*a^6*b^5*c^4 - 5888*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^5 - 2944*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 - 1408*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 - 14208*a^7*b^3*c^5 + 1472*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 + 11776*a^8*b*c^6 + 10*(b^2 - 4*a*c)*a^3*b^9*c - 164*(b^2 - 4*a*c)*a^4*b^7*c^2 + 1016*(b^2 - 4*a*c)*a^5*b^5*c^3 - 2816*(b^2 - 4*a*c)*a^6*b^3*c^4 + 2944*(b^2 - 4*a*c)*a^7*b*c^5)*abs(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + sqrt((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)^2 - 4*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/((a^7*b^10 - 20*a^8*b^8*c - 2*a^7*b^9*c + 160*a^9*b^6*c^2 + 32*a^8*b^7*c^2 + a^7*b^8*c^2 - 640*a^10*b^4*c^3 - 192*a^9*b^5*c^3 - 16*a^8*b^6*c^3 + 1280*a^11*b^2*c^4 + 512*a^10*b^3*c^4 + 96*a^9*b^4*c^4 - 1024*a^12*c^5 - 512*a^11*b*c^5 - 256*a^10*b^2*c^5 + 256*a^11*c^6)*abs(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*abs(c)) + 3/64*(10*a^6*b^14*c^2 - 254*a^7*b^12*c^3 + 2712*a^8*b^10*c^4 - 15552*a^9*b^8*c^5 + 50432*a^10*b^6*c^6 - 87552*a^11*b^4*c^7 + 63488*a^12*b^2*c^8 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^14 + 127*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^12*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^13*c - 1356*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^2 - 214*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^11*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^12*c^2 + 7776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c^3 + 1856*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^9*c^3 + 107*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^10*c^3 - 25216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^6*c^4 - 8128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^7*c^4 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^8*c^4 + 43776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^4*c^5 + 17920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^5*c^5 + 4064*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^6*c^5 - 31744*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^2*c^6 - 15872*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^3*c^6 - 8960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^4*c^6 + 7936*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^2*c^7 - 10*(b^2 - 4*a*c)*a^6*b^12*c^2 + 214*(b^2 - 4*a*c)*a^7*b^10*c^3 - 1856*(b^2 - 4*a*c)*a^8*b^8*c^4 + 8128*(b^2 - 4*a*c)*a^

```
9*b^6*c^5 - 17920*(b^2 - 4*a*c)*a^10*b^4*c^6 + 15872*(b^2 - 4*a*c)*a^11*b^2
*c^7 + (10*b^6*c^2 - 114*a*b^4*c^3 + 416*a^2*b^2*c^4 - 480*a^3*c^5 - 5*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 57*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 208*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 74*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 240*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 37*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 74*(b^2 - 4*a*
c)*a*b^2*c^3 - 120*(b^2 - 4*a*c)*a^2*c^4)*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)^2 - 2*(5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^11 - 102*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b^10*c + 10*a^3*b^11*c + 836*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^5*b^7*c^2 + 164*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
8*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - 204*a^4*b^9
*c^2 - 3440*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^3 - 1016*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 - 82*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 1672*a^5*b^7*c^3 + 7104*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 + 2816*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^6*b^4*c^4 + 508*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4
- 6880*a^6*b^5*c^4 - 5888*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^5
 - 2944*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 - 1408*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 14208*a^7*b^3*c^5 + 1472*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 - 11776*a^8*b*c^6 - 10*(b^2 -
 4*a*c)*a^3*b^9*c + 164*(b^2 - 4*a*c)*a^4*b^7*c^2 - 1016*(b^2 - 4*a*c)*a^5*
b^5*c^3 + 2816*(b^2 - 4*a*c)*a^6*b^3*c^4 - 2944*(b^2 - 4*a*c)*a^7*b*c^5)*ab
s(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^5 -
 8*a^4*b^3*c + 16*a^5*b*c^2 - sqrt((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)^2
 - 4*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/((a^7*b^10 - 20*a^8*b^
8*c - 2*a^7*b^9*c + 160*a^9*b^6*c^2 + 32*a^8*b^7*c^2 + a^7*b^8*c^2 - 640*a^
10*b^4*c^3 - 192*a^9*b^5*c^3 - 16*a^8*b^6*c^3 + 1280*a^11*b^2*c^4 + 512*a^1
0*b^3*c^4 + 96*a^9*b^4*c^4 - 1024*a^12*c^5 - 512*a^11*b*c^5 - 256*a^10*b^2*
c^5 + 256*a^11*c^6)*abs(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*abs(c)) - 1/8*(
7*b^4*c^2*x^7 - 47*a*b^2*c^3*x^7 + 52*a^2*c^4*x^7 + 14*b^5*c*x^5 - 99*a*b^3
*c^2*x^5 + 136*a^2*b*c^3*x^5 + 7*b^6*x^3 - 43*a*b^4*c*x^3 + 25*a^2*b^2*c^2*
x^3 + 68*a^3*c^3*x^3 + 9*a*b^5*x - 66*a^2*b^3*c*x + 108*a^3*b*c^2*x)/((a^3*
b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/(a^3*x)
```

maple [B] time = 0.06, size = 1567, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -17/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*c^3+45/4/a/(16*a^2*c
^2-8*a*b^2*c+b^4)*c^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-45/4/a/(16*a^2*c^2-8*a*b^2*c+b^
4)*c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)+43/8/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^3*b^4*c+33/4/a/(c*x^4+b*x^2+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c-
25/8/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c^2+47/8/a^2/(c
*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7*b^2-17/a/(c*x^4+b*x^2+a)
^2*c^3*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-13/2/a/(c*x^4+b*x^2+a)^2*c^4/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^7-27/2/(c*x^4+b*x^2+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4
)*x*c^2-141/16/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^3+15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b^5+15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5-141/16/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+93/4/a/(16*a^2*c^2-8*a*b^2*c+b^
4)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+93/4/a/(16*a^2*c^2-8*a*b^
2*c+b^4)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-111/16/a^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+111/16/a^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b^4-15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*b^4-9/8/a^2/(c*x^4+b*x^2+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x-7/8
/a^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^6-1/a^3/x+99/8/a^2/
(c*x^4+b*x^2+a)^2*c^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-7/8/a^3/(c*x^4+b*x
^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7*b^4-7/4/a^3/(c*x^4+b*x^2+a)^2*c*
b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + (30*b^5*c - 227*a*b^3
*c^2 + 392*a^2*b*c^3)*x^6 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*b^
6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*x^4 + (25*a*b^5 - 194*a^2*b^
3*c + 364*a^3*b*c^2)*x^2)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 +
 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c
+ 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*x) - 3/8*integrate((5*b^5 - 42*a*b^3*c + 92*a^
2*b*c^2 + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*x^2)/(c*x^4 + b*x^2 + a), x
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
```

mupad [B] time = 9.37, size = 12130, normalized size = 28.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] - atan(((x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^2
0*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^
13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 8698
15812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b
^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a*c -
b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^1
5*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
- 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*
a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
+ 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3
 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 19660
80*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(245
760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^13 - 10911744*a^13*b^21*c^3 + 22
0397568*a^14*b^19*c^4 - 2673082368*a^15*b^17*c^5 + 21630025728*a^16*b^15*c^
6 - 122607894528*a^17*b^13*c^7 + 496773365760*a^18*b^11*c^8 - 1438679826432
*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10 - 3949222428672*a^21*b^5*c^11 +
 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)
^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 +
1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 439042
56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 104857
6*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760
*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*
b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(10995116277
76*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*
a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 1240
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17180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*1i + (x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1185410973696*a^23*b*c^13 - 245760*a^12*b^23*c^2 + 10911744*a^13*b^21*c^3 - 220397568*a^14*b^19*c^4 + 2673082368*a^15*b^17*c^5 - 21630025728*a^16*b^15*c^6 + 122607894528*a^17*b^13*c^7 - 496773365760*a^18*b^11*c^8 + 1438679826432*a^19*b^9*c^9 - 2918430277632*a^20*b^7*c^10 + 3949222428672*a^21*b^5*c^11 - 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a *b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*1i)/((x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1185410973696*a^23*b*c^13 - 245760*a^12*b^23*c^2 + 10911744*a^13*b^21*c^3 - 220397568*a^14*b^19*c^4 + 2673082368*a^15*b^17*c^5 - 21630025728*a^16*b^15*c^6 + 122607894528*a^17*b^13*c^7 - 496773365760*a^18*b^11*c^8 + 1438679826432*a^19*b^9*c^9 - 2918430277632*a^20*b^7*c^10 + 3949222428672*a^21*b^5*c^11 - 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2) - (x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^

13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 8698
15812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b
^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a*c -
b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^1
5*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
- 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*
a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
+ 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3
 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 19660
80*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(245
760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^13 - 10911744*a^13*b^21*c^3 + 22
0397568*a^14*b^19*c^4 - 2673082368*a^15*b^17*c^5 + 21630025728*a^16*b^15*c^
6 - 122607894528*a^17*b^13*c^7 + 496773365760*a^18*b^11*c^8 - 1438679826432
*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10 - 3949222428672*a^21*b^5*c^11 +
 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)
^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 +
1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 439042
56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 104857
6*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760
*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*
b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(10995116277
76*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*
a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 1240
17180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^
9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656
976384*a^25*b^3*c^12))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) +
18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a
^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^
7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
 + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^
10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^1
2*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 +
 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2) + 191102976000*a^17*c
^14 + 2851200*a^9*b^16*c^6 - 92568960*a^10*b^14*c^7 + 1312630272*a^11*b^12*
c^8 - 10611136512*a^12*b^10*c^9 + 53445353472*a^13*b^8*c^10 - 171591892992*
a^14*b^6*c^11 + 342580396032*a^15*b^4*c^12 - 388363714560*a^16*b^2*c^13))*(
-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17
794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5
*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*
c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*
b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c -

b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*2i - atan(((x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(245760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^13 - 10911744*a^13*b^21*c^3 + 220397568*a^14*b^19*c^4 - 2673082368*a^15*b^17*c^5 + 21630025728*a^16*b^15*c^6 - 122607894528*a^17*b^13*c^7 + 496773365760*a^18*b^11*c^8 - 1438679826432*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10 - 3949222428672*a^21*b^5*c^11 + 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*1i + (x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^

9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 174731349196
8*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 + 25*b^6*(-(
4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095
*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*
b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^
9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*
(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a
^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*
b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6
 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1
/2)*(1185410973696*a^23*b*c^13 - 245760*a^12*b^23*c^2 + 10911744*a^13*b^21*
c^3 - 220397568*a^14*b^19*c^4 + 2673082368*a^15*b^17*c^5 - 21630025728*a^16
*b^15*c^6 + 122607894528*a^17*b^13*c^7 - 496773365760*a^18*b^11*c^8 + 14386
79826432*a^19*b^9*c^9 - 2918430277632*a^20*b^7*c^10 + 3949222428672*a^21*b^
5*c^11 - 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 + 25*b^6*(-(4*a*c -
b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^1
5*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
- 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*
a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
+ 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3
 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 19660
80*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(109
9511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 23
0686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^
6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680
*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 -
 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^
(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1
299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 4390425
6*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(
-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576
*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*
a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b
^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*1i)/((x*(2717
90899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 19103846
4*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89
374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^
8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 110105
5131648*a^19*b^2*c^13) + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) +
 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*
a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b
^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2

) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c
^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^
12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7
+ 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1185410973696*a^23*
b*c^13 - 245760*a^12*b^23*c^2 + 10911744*a^13*b^21*c^3 - 220397568*a^14*b^1
9*c^4 + 2673082368*a^15*b^17*c^5 - 21630025728*a^16*b^15*c^6 + 122607894528
*a^17*b^13*c^7 - 496773365760*a^18*b^11*c^8 + 1438679826432*a^19*b^9*c^9 -
2918430277632*a^20*b^7*c^10 + 3949222428672*a^21*b^5*c^11 - 3208340570112*a
^22*b^3*c^12 + x*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 1892352
0*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13
*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 +
 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^
15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*
a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40
*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 -
 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 294912
0*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 -
 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 +
2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^
13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 28346784
15360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*
c^12)))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*
c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61
26640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160
*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(
-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18
*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a
^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^
4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2) - (x*(271790899200*a^20*c^14 - 230400
*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 220780339
2*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 3
33226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17
*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-
(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 177
94*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*
b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c
^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b
^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c -
b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a
^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c
^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 26
21440*a^16*b^2*c^9)))^(1/2)*(245760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^
13 - 10911744*a^13*b^21*c^3 + 220397568*a^14*b^19*c^4 - 2673082368*a^15*b^1
7*c^5 + 21630025728*a^16*b^15*c^6 - 122607894528*a^17*b^13*c^7 + 4967733657

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60*a^18*b^11*c^8 - 1438679826432*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10
 - 3949222428672*a^21*b^5*c^11 + 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b
^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b
^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5
 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520
39680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c +
694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)
^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*
c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860
160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^
16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 115
34336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 -
22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*
b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 37795
71220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12))*(-(9*(25*b^21 + 25*
b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 -
 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 199056
00*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9
*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/
(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 768
0*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*
b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^
9)))^(1/2) + 191102976000*a^17*c^14 + 2851200*a^9*b^16*c^6 - 92568960*a^10*
b^14*c^7 + 1312630272*a^11*b^12*c^8 - 10611136512*a^12*b^10*c^9 + 534453534
72*a^13*b^8*c^10 - 171591892992*a^14*b^6*c^11 + 342580396032*a^15*b^4*c^12
- 388363714560*a^16*b^2*c^13))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(
1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 12
99860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256
*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-
(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*
a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a
^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^
6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*2i - (1/a + (x
^4*(15*b^6 + 324*a^3*c^3 + 25*a^2*b^2*c^2 - 91*a*b^4*c))/(8*a^3*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) + (b*x^6*(30*b^4*c + 392*a^2*c^3 - 227*a*b^2*c^2))/(8*
a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c*x^8*(5*b^4*c + 60*a^2*c^3 - 37*a
*b^2*c^2))/(8*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^2*(25*b^4 + 364*a^
2*c^2 - 194*a*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^5*(2*a*c +
 b^2) + a^2*x + c^2*x^9 + 2*a*b*x^3 + 2*b*c*x^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.691 $\int \frac{x^5}{a-bx^2+cx^4} dx$

Optimal. Leaf size=82

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1114, 703, 634, 618, 206, 628}

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) + (b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{a-bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a-bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2c}+\frac{\text{Subst}\left(\int \frac{-a+bx}{a-bx+cx^2}\,dx,x,x^2\right)}{2c}\\
&= \frac{x^2}{2c}+\frac{b\,\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4c^2}\\
&= \frac{x^2}{2c}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c}+\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.98

$$
\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log\left(a-bx^2+cx^4\right)+2cx^2}{4c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a-bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^5/(a - b*x^2 + c*x^4), x]
```

fricas [A] time = 1.06, size = 259, normalized size = 3.16

$$\left[\frac{2\left(b^2c-4\,ac^2\right)x^2-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac+\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+\left(b^3-4\,abc\right)\log\left(cx^4-bx^2+a\right)}{4\left(b^2c^2-4\,ac^3\right)},\ \frac{2\left(b^2c-4\,ac^2\right)x^2-2\left(b^2-2\,ac\right)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2-b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(b^3-4\,abc\right)\log\left(cx^4-bx^2+a\right)}{4\left(b^2c^2-4\,ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 - 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b
*x^2 + a)) + (b^3 - 4*a*b*c)*log(c*x^4 - b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*log(c*x^4 -
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.53, size = 78, normalized size = 0.95

$$\frac{x^2}{2\,c}+\frac{b\log\left(cx^4-bx^2+a\right)}{4\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c + 1/4*b*log(c*x^4 - b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.00, size = 116, normalized size = 1.41

$$-\frac{a\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c}+\frac{b^2\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2}+\frac{x^2}{2c}+\frac{b\ln\left(c\,x^4-b\,x^2+a\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/2/c*x^2+1/4*b*ln(c*x^4-b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
-b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*
c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.74, size = 656, normalized size = 8.00

$$\frac{x^2}{2c}-\frac{\ln\left(cx^4-bx^2+a\right)\left(2b^3-8abc\right)}{2\left(16ac^3-4b^2c^2\right)}-\frac{\mathrm{atan}\left(\frac{2c^2\left(4ac-b^2\right)\left(\frac{\frac{\left(8ab+\frac{8ac^2\left(2b^3-8abc\right)}{16ac^3-4b^2c^2}\right)\left(2ac-b^2\right)}{8c^2\sqrt{4ac-b^2}}+\frac{a\left(2b^3-8abc\right)\left(2ac-b^2\right)}{\sqrt{4ac-b^2}\left(16ac^3-4b^2c^2\right)}}{a}+x^2\left(\frac{\frac{\left(2ac-b^2\right)\left(\frac{4ac^3-6b^2c^2}{c^2}-\frac{4bc^2\left(2b^3-8abc\right)}{16ac^3-4b^2c^2}\right)}{8c^2\sqrt{4ac-b^2}}-\frac{b\left(2b^3-8abc\right)\left(2ac-b^2\right)}{2\sqrt{4ac-b^2}\left(16ac^3-4b^2c^2\right)}}{a}+\frac{b\left(\frac{\left(2b^3-8abc\right)\left(\frac{4ac^3-6b^2c^2}{c^2}-\frac{4bc^2\left(2b^3-8abc\right)}{16ac^3-4b^2c^2}\right)}{2\left(16ac^3-4b^2c^2\right)}-\frac{b^3-abc}{c^2}+\frac{b\left(2ac-b^2\right)^2}{2c^2\left(4ac-b^2\right)}\right)}{2a\sqrt{4ac-b^2}}\right)+\frac{b\left(\frac{ab^2}{c^2}+\frac{\left(2b^3-8abc\right)\left(8ab+\frac{8ac^2\left(2b^3-8abc\right)}{16ac^3-4b^2c^2}\right)}{2\left(16ac^3-4b^2c^2\right)}-\frac{a\left(2ac-b^2\right)^2}{c^2\left(4ac-b^2\right)}\right)}{2a\sqrt{4ac-b^2}}\right)}{4a^2c^2-4ab^2c+b^4}\right)\left(2ac-b^2\right)}{2c^2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b*x^2 + c*x^4),x)
```

```
[Out] x^2/(2*c) - (log(a - b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^2
*c^2)) - (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))
/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2))
)/a + x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8
*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2*b^3 -
 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a
+ (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a
*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c)/
c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))
) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*a*c - b^2)^2
)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a^2*c^2 - 4*a*
b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.76, size = 311, normalized size = 3.79

$$\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4c^2\,(4ac-b^2)}\right)\log\left(x^2 + \frac{ab - 8ac^2\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4c^2(4ac-b^2)}\right) + 2b^2c\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4c^2(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4c^2\,(4ac-b^2)}\right)\log\left(x^2 + \frac{ab - 8ac^2\left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4c^2(4ac-b^2)}\right) + 2b^2c\left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4c^2(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4-b*x**2+a),x)
```

```
[Out] (b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*l
og(x**2 + (a*b - 8*a*c**2*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*
c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (b/(4*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (a*b - 8*a*
c**2*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2
))) + 2*b**2*c*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*
a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

3.692 $\int \frac{x^3}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=64

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1114, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a - b*x^2 + c*x^4]/(4*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a-bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a-bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx, x, x^2\right)}{4c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx, x, x^2\right)}{4c}\\ &= \frac{\log\left(a-bx^2+cx^4\right)}{4c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, -b+2cx^2\right)}{2c}\\ &= \frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log\left(a-bx^2+cx^4\right)}{4c}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.02

$$\frac{\frac{2b\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] ((2*b*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 - b*x^2 + c*x^4])/(4*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a-bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^3/(a - b*x^2 + c*x^4), x]
```

fricas [A] time = 0.69, size = 206, normalized size = 3.22

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^4-bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)},-\frac{2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{\left(2\,cx^2-b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(b^2-4\,ac\right)\log\left(cx^4-bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x
^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 -
 b*x^2 + a))/(b^2*c - 4*a*c^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x
^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^4 - b*x^2
 + a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 0.57, size = 62, normalized size = 0.97

$$\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c}+\frac{\log\left(cx^4-bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/4
*log(c*x^4 - b*x^2 + a)/c
```

maple [A] time = 0.00, size = 63, normalized size = 0.98

$$\frac{b\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c}+\frac{\ln\left(c\,x^4-b\,x^2+a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/4*ln(c*x^4-b*x^2+a)/c+1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.40, size = 120, normalized size = 1.88

$$\frac{4\,a\,c\,\ln\left(c\,x^4-b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b^2\,\ln\left(c\,x^4-b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}-\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2 + c*x^4),x)
```

```
[Out] (4*a*c*log(a - b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a - b*x^2 +
c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) - (2*c*x^2)/(4
*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.45, size = 223, normalized size = 3.48

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)-2a-2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)-2a-2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*(
-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*(b*sqrt(-4*a*c + b**
2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(b*sqrt(-4*a*c + b**2)/(4
*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

3.693 $\int \frac{x}{a-bx^2+cx^4} dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1107, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2 + c*x^4),x]
```

```
[Out] ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a-bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, -b+2cx^2\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{-b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.17

$$\frac{\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2 + c*x^4),x]
[Out] ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a-bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a - b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[x/(a - b*x^2 + c*x^4), x]
```

fricas [A] time = 0.87, size = 134, normalized size = 3.83

$$\left[\frac{\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2-b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 - b)*sqrt(b^2 - 4*
a*c))/(c*x^4 - b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.57, size = 37, normalized size = 1.06

$$\frac{\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 38, normalized size = 1.09

$$\frac{\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.30, size = 42, normalized size = 1.20

$$-\frac{\mathrm{atan}\left(\frac{a\,b-2\,a\,c\,x^2}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2 + c*x^4),x)
```

```
[Out] -atan((a*b - 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

sympy [B] time = 0.72, size = 131, normalized size = 3.74

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4-b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2
```

3.694 $\int \frac{1}{x\left(a-bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=70

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a - b*x^2 + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a-bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a-bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}+\frac{\text{Subst}\left(\int \frac{b-cx}{a-bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4a}+\frac{b\,\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}-\frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2a}\\
&= \frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 117, normalized size = 1.67

$$
\frac{\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}-b+2cx^2\right)-\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}-b+2cx^2\right)+4\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] + (b - Sqrt[b^2 - 4*a*c])*Log[-b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2] - (b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a-bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a - b*x^2 + c*x^4)), x]
```

fricas [A] time = 0.88, size = 230, normalized size = 3.29

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^4-bx^2+a\right)+4\left(b^2-4\,ac\right)\log(x)}{4\left(ab^2-4\,a^2c\right)},\ -\frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{\left(2\,cx^2-b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(b^2-4\,ac\right)\log\left(cx^4-bx^2+a\right)-4\left(b^2-4\,ac\right)\log(x)}{4\left(ab^2-4\,a^2c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x
^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 -
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), -1/4*(2*sqrt(-b^2
+ 4*a*c)*b*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 -
 4*a*c)*log(c*x^4 - b*x^2 + a) - 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 0.57, size = 71, normalized size = 1.01

$$\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\sqrt{-b^2+4\,ac}\,a}-\frac{\log\left(cx^4-bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/4
*log(c*x^4 - b*x^2 + a)/a + 1/2*log(x^2)/a
```

maple [A] time = 0.01, size = 69, normalized size = 0.99

$$\frac{b \arctan\left(\frac{2c\,x^{2}-b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}\;a}+\frac{\ln(x)}{a}-\frac{\ln\left(c\,x^{4}-b\,x^{2}+a\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/a*ln(x)-1/4*ln(c*x^4-b*x^2+a)/a+1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
-b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.89, size = 1015, normalized size = 14.50

$$\frac{\ln(x)}{a}+\frac{\ln\left(c\,x^{4}-b\,x^{2}+a\right)\left(8\,a\,c-2\,b^{2}\right)}{2\left(4\,a\,b^{2}-16\,a^{2}\,c\right)}-\frac{b\,\mathrm{atan}\left(\frac{16\,a^{3}\,x^{2}\left(\frac{\left(3\,b^{3}-8\,a\,b\,c\right)\left(\frac{\left(8\,a\,c-2\,b^{2}\right)^{2}\left(10\,b\,c^{3}-\frac{\left(12\,b^{3}\,c^{2}-40\,a\,b\,c^{3}\right)\left(8\,a\,c-2\,b^{2}\right)}{2\left(4\,a\,b^{2}-16\,a^{2}\,c\right)}\right)}{4\left(4\,a\,b^{2}-16\,a^{2}\,c\right)^{2}}-\frac{b^{2}\left(10\,b\,c^{3}-\frac{\left(12\,b^{3}\,c^{2}-40\,a\,b\,c^{3}\right)\left(8\,a\,c-2\,b^{2}\right)}{2\left(4\,a\,b^{2}-16\,a^{2}\,c\right)}\right)}{16\,a^{2}\left(4\,a\,c-b^{2}\right)}+\frac{b^{2}\left(12\,b^{3}\,c^{2}-40\,a\,b\,c^{3}\right)\left(8\,a\,c-2\,b^{2}\right)}{16\,a^{2}\left(4\,a\,b^{2}-16\,a^{2}\,c\right)\left(4\,a\,c-b^{2}\right)}\right)}{8\,a^{3}\,c^{2}\left(25\,a\,c-6\,b^{2}\right)}-\frac{\left(10\,a^{2}\,c^{2}-14\,a\,b^{2}\,c+3\,b^{4}\right)\left(\frac{b^{3}\left(12\,b^{3}\,c^{2}-40\,a\,b\,c^{3}\right)}{64\,a^{3}\left(4\,a\,c-b^{2}\right)^{3/2}}-\frac{b\left(12\,b^{3}\,c^{2}-40\,a\,b\,c^{3}\right)\left(8\,a\,c-2\,b^{2}\right)^{2}}{16\,a\left(4\,a\,b^{2}-16\,a^{2}\,c\right)^{2}\sqrt{4\,a\,c-b^{2}}}+\frac{b\left(8\,a\,c-2\,b^{2}\right)\left(10\,b\,c^{3}-\frac{\left(12\,b^{3}\,c^{2}-40\,a\,b\,c^{3}\right)\left(8\,a\,c-2\,b^{2}\right)}{2\left(4\,a\,b^{2}-16\,a^{2}\,c\right)}\right)}{4\,a\left(4\,a\,b^{2}-16\,a^{2}\,c\right)\sqrt{4\,a\,c-b^{2}}}\right)}{8\,a^{3}\,c^{2}\sqrt{4\,a\,c-b^{2}}\left(25\,a\,c-6\,b^{2}\right)}\right)\left(4\,a\,c-b^{2}\right)^{3/2}}{b^{2}\,c^{2}}-\frac{2\left(3\,b^{3}-8\,a\,b\,c\right)\left(4\,a\,c-b^{2}\right)^{3/2}\left(\frac{\left(8\,a\,c-2\,b^{2}\right)^{2}\left(4\,b^{2}\,c^{2}-\frac{2\,a\,b^{2}\,c^{2}\left(8\,a\,c-2\,b^{2}\right)}{4\,a\,b^{2}-16\,a^{2}\,c}\right)}{4\left(4\,a\,b^{2}-16\,a^{2}\,c\right)^{2}}-\frac{b^{2}\left(4\,b^{2}\,c^{2}-\frac{2\,a\,b^{2}\,c^{2}\left(8\,a\,c-2\,b^{2}\right)}{4\,a\,b^{2}-16\,a^{2}\,c}\right)}{16\,a^{2}\left(4\,a\,c-b^{2}\right)}+\frac{b^{4}\,c^{2}\left(8\,a\,c-2\,b^{2}\right)}{4\,a\left(4\,a\,b^{2}-16\,a^{2}\,c\right)\left(4\,a\,c-b^{2}\right)}\right)}{b^{2}\,c^{4}\left(25\,a\,c-6\,b^{2}\right)}+\frac{2\left(4\,a\,c-b^{2}\right)\left(10\,a^{2}\,c^{2}-14\,a\,b^{2}\,c+3\,b^{4}\right)\left(\frac{b^{5}\,c^{2}}{16\,a^{2}\left(4\,a\,c-b^{2}\right)^{3/2}}-\frac{b^{3}\,c^{2}\left(8\,a\,c-2\,b^{2}\right)^{2}}{4\left(4\,a\,b^{2}-16\,a^{2}\,c\right)^{2}\sqrt{4\,a\,c-b^{2}}}+\frac{b\left(8\,a\,c-2\,b^{2}\right)\left(4\,b^{2}\,c^{2}-\frac{2\,a\,b^{2}\,c^{2}\left(8\,a\,c-2\,b^{2}\right)}{4\,a\,b^{2}-16\,a^{2}\,c}\right)}{4\,a\left(4\,a\,b^{2}-16\,a^{2}\,c\right)\sqrt{4\,a\,c-b^{2}}}\right)}{b^{2}\,c^{4}\left(25\,a\,c-6\,b^{2}\right)}\right)}{2\,a\sqrt{4\,a\,c-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2 + c*x^4)),x)
```

```
[Out] log(x)/a + (log(a - b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)
) - (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3 -
 ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*
(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*
c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^
3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(
(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c
^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b
^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a
```

```
*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^
2)^(3/2))/(b^2*c^2) - (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*(((8*a*c - 2
*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(
4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))
/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/
(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*c - 6*b^2)) + (2*
(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c -
 b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c
 - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^
2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))
/(b^2*c^4*(25*a*c - 6*b^2))))/(2*a*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 5.74, size = 253, normalized size = 3.61

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)-2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)-2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right)+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a**2*
c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(-b*sq
rt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c)) +
(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a**2*c
*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(b*sqrt(
-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c)) + log
(x)/a
```

3.695 $\int \frac{1}{x^3(a-bx^2+cx^4)}\,dx$

Optimal. Leaf size=89

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) + (b*Log[x])/a^2 - (b*Log[a - b*x^2 + c*x^4])/(4*a^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3(a-bx^2+cx^4)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a-bx+cx^2)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \frac{b-cx}{x(a-bx+cx^2)}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \left(\frac{b}{ax} - \frac{-b^2+ac+bcx}{a(a-bx+cx^2)}\right) dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} + \frac{b\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{-b^2+ac+bcx}{a-bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$
$$= -\frac{1}{2ax^2} + \frac{b\log(x)}{a^2} - \frac{b\,\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx, x, x^2\right)}{4a^2} + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx\right.}{4a^2}$$
$$= -\frac{1}{2ax^2} + \frac{b\log(x)}{a^2} - \frac{b\log(a-bx^2+cx^4)}{4a^2} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, -b+\right.}{2a^2}$$
$$= -\frac{1}{2ax^2} + \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} + \frac{b\log(x)}{a^2} - \frac{b\log(a-bx^2+cx^4)}{4a^2}$$

Mathematica [A] time = 0.14, size = 139, normalized size = 1.56

$$\frac{\frac{(-b\sqrt{b^2-4ac}-2ac+b^2)\log(-\sqrt{b^2-4ac}-b+2cx^2)}{\sqrt{b^2-4ac}} - \frac{(b\sqrt{b^2-4ac}-2ac+b^2)\log(\sqrt{b^2-4ac}-b+2cx^2)}{\sqrt{b^2-4ac}} - \frac{2a}{x^2} + 4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 + 4*b*Log[x] + ((b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*Log[-b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((b^2 - 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a-bx^2+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a - b*x^2 + c*x^4)), x]
```

fricas [A] time = 1.63, size = 298, normalized size = 3.35

$$\left[-\frac{(b^2-2ac)\sqrt{b^2-4ac}\,x^2\log\left(\frac{2c^2x^4-2bcx^2+b^2-2ac+(2cx^2-b)\sqrt{b^2-4ac}}{cx^4-bx^2+a}\right)+(b^3-4abc)x^2\log\left(cx^4-bx^2+a\right)-4\left(b^3-4abc\right)x^2\log(x)+2ab^2-8a^2c}{4\left(a^2b^2-4a^3c\right)x^2}, -\frac{2\left(b^2-2ac\right)\sqrt{-b^2+4ac}\,x^2\arctan\left(-\frac{(2cx^2-b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+(b^3-4abc)x^2\log\left(cx^4-bx^2+a\right)-4\left(b^3-4abc\right)x^2\log(x)+2ab^2-8a^2c}{4\left(a^2b^2-4a^3c\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 - 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^3 - 4
*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4
*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

giac [A] time = 0.59, size = 95, normalized size = 1.07

$$-\frac{b\log\left(cx^4-bx^2+a\right)}{4\,a^2}+\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}-\frac{bx^2+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*b*log(c*x^4 - b*x^2 + a)/a^2 + 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*
arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(b*
x^2 + a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 123, normalized size = 1.38

$$-\frac{c\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a}+\frac{b^2\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2}+\frac{b\ln(x)}{a^2}-\frac{b\ln\left(c\,x^4-b\,x^2+a\right)}{4a^2}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/2/a/x^2+1/a^2*b*ln(x)-1/4*b*ln(c*x^4-b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^2-b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.84, size = 2032, normalized size = 22.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2 + c*x^4)),x)
```

```
[Out] (b*log(x))/a^2 - 1/(2*a*x^2) + (log(a - b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(
2*(16*a^3*c - 4*a^2*b^2)) + (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*
c)*(c^5/a^3 + ((2*b^3 - 8*a*b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a
^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3
*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16
*a^3*c - 4*a^2*b^2)) - ((((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 +
 ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a
^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 -
 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^
2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(4
0*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^
3*c - 4*a^2*b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3
- 8*a*b*c)*(((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*
a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4
*a^2*(4*a*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c
^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*
(16*a^3*c - 4*a^2*b^2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)
/(64*a^9*(4*a*c - b^2)^(3/2)) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a
^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3
*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c
- b^2))/(4*a^2*(4*a*c - b^2)^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(
8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2
)^(3/2))/(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3) + (2*a^3*(4*a*c - b^2)*(3*b^5
+ 13*a^2*b*c^2 - 15*a*b^3*c)*(((2*b^3 - 8*a*b*c)*((((4*a^3*b*c^3 - 4*a^2*b^
3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c
 - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c -
```

```
b^2))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))))/(2*(16*a^3*c - 4*a
^2*b^2)) + ((2*a*c - b^2)*((a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)
*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a
^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)
) - (b^2*c^2*(2*a*c - b^2)^3)/(16*a^5*(4*a*c - b^2)^(3/2))))/(c^2*(a^2*c^2
- 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3)) - (2*a^3*(4*a*c
- b^2)^(3/2)*(3*b^4 + a^2*c^2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c
)*((a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b
^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(
16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((((4*
a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c
- 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3
- 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))
)/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)
/(8*a^3*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*
a*b^2*c)*(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3)))*(2*a*c - b^2))/(2*a^2*(4*a*c
 - b^2)^(1/2))
```

sympy [B] time = 142.97, size = 350, normalized size = 3.93

$$\left(-\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-8a^3c\left(-\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2(4ac-b^2)}\right)+2a^2b^2\left(-\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2(4ac-b^2)}\right)-3abc+b^3}{2ac^2-b^2c}\right)+\left(-\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-8a^3c\left(-\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2(4ac-b^2)}\right)+2a^2b^2\left(-\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2(4ac-b^2)}\right)-3abc+b^3}{2ac^2-b^2c}\right)-\frac{1}{2ax^2}+\frac{b\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*
log(x**2 + (-8*a**3*c*(-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a
*c - b**2)/(4*a**2*(4*a*c - b**2))) - 3*a*b*c + b**3)/(2*a*c**2 - b**2*c))
+ (-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))
)*log(x**2 + (-8*a**3*c*(-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(4*a**2*(4*a*c - b**2))) - 3*a*b*c + b**3)/(2*a*c**2 - b**2*c)
) - 1/(2*a*x**2) + b*log(x)/a**2
```

3.696 $\int \frac{x^4}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Rubi [A] time = 0.36, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1122, 1166, 208}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a-bx^2+cx^4}\,dx &= \frac{x}{c}-\frac{\int \frac{a-bx^2}{a-bx^2+cx^4}\,dx}{c}\\ &= \frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 208, normalized size = 1.16

$$\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sqrt
[b^2 - 4*a*c]]) + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*S
qrt[-b + Sqrt[b^2 - 4*a*c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a-bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^4/(a - b*x^2 + c*x^4), x]
```

fricas [B] time = 1.16, size = 1051, normalized size = 5.87

$$-\frac{\sqrt{\frac{1}{2}}\,c\sqrt{\frac{b^3-3abc+(b^2c^3-4ac^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\log\left(-2(ab^2-a^2c)x+\sqrt{\frac{1}{2}}\left(b^4-5ab^2c+4a^2c^2-(b^3c^3-4abc^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}\right)\sqrt{\frac{b^3-3abc+(b^2c^3-4ac^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\right)-\sqrt{\frac{1}{2}}\,c\sqrt{\frac{b^3-3abc+(b^2c^3-4ac^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\log\left(-2(ab^2-a^2c)x-\sqrt{\frac{1}{2}}\left(b^4-5ab^2c+4a^2c^2-(b^3c^3-4abc^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}\right)\sqrt{\frac{b^3-3abc+(b^2c^3-4ac^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\right)+\sqrt{\frac{1}{2}}\,c\sqrt{\frac{b^3-3abc-(b^2c^3-4ac^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\log\left(-2(ab^2-a^2c)x+\sqrt{\frac{1}{2}}\left(b^4-5ab^2c+4a^2c^2+(b^3c^3-4abc^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}\right)\sqrt{\frac{b^3-3abc-(b^2c^3-4ac^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\right)-\sqrt{\frac{1}{2}}\,c\sqrt{\frac{b^3-3abc-(b^2c^3-4ac^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\log\left(-2(ab^2-a^2c)x-\sqrt{\frac{1}{2}}\left(b^4-5ab^2c+4a^2c^2+(b^3c^3-4abc^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}\right)\sqrt{\frac{b^3-3abc-(b^2c^3-4ac^4)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\right)-2x}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
- a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c
 + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)
))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a
*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 -
(b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))
)*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt((b^3 - 3*a
*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*
c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a
*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1
/2)*c*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x -
 sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3
- 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4))) - 2*x)/c
```

giac [B] time = 0.98, size = 2153, normalized size = 12.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt
```

(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 2*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt(-(b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 + 2*a*b^3*c^4 + 16*a^3*c^5 - 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 2*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt(-(b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 + 2*a*b^3*c^4 + 16*a^3*c^5 - 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)

maple [B] time = 0.03, size = 343, normalized size = 1.92

$$\frac{\sqrt{2}\, a \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\sqrt{-4 a c+b^{2}}\, \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}+\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{\sqrt{-4 a c+b^{2}}\, \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}-\frac{\sqrt{2}\, b^{2} \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \sqrt{-4 a c+b^{2}}\, \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, c}-\frac{\sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \sqrt{-4 a c+b^{2}}\, \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, c}-\frac{\sqrt{2}\, b \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, c}+\frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/c*x-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
a-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+1/2/c*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] x/c + integrate((b*x^2 - a)/(c*x^4 - b*x^2 + a), x)/c
```

mupad [B] time = 0.67, size = 3000, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^2 + c*x^4),x)
```

```
[Out] x/c + atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((
b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(
b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^
3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
```

c^4)))^(1/2))/c)((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a
*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2
*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5
 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((
b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*
a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 -
4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2) - (2*a^2*b)/c))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*
a*b^2*c^4)))^(1/2)*2i + atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*
c^3 - 16*a*b*c^4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a
*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))^(1/2))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b
^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 - b^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c
^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^
5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16
*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 - b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 -
 4*a*b^2*c))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b
^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*

```
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((
b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(
b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2) - (2*a^2*b)/c))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [A] time = 2.78, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^{4}\left(256a^{2}c^{5}-128ab^{2}c^{4}+16b^{4}c^{3}\right)+t^{2}\left(-48a^{2}bc^{2}+28ab^{3}c-4b^{5}\right)+a^{3},\left(t\mapsto t\log\left(x+\frac{-32t^{3}abc^{4}+8t^{3}b^{3}c^{3}-4ta^{2}c^{2}+8tab^{2}c-2tb^{4}}{a^{2}c-ab^{2}}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(-48
*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + a**3, Lambda(_t, _t*log(x + (-32*_t*
*3*a*b*c**4 + 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**
4)/(a**2*c - a*b**2)))) + x/c
```

3.697 $\int \frac{x^2}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1130, 208}

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - (Sqrt[b + Sqrt[b^2 - 4*a*
c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx = -\left(\frac{1}{2}\left(-1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{-\frac{b}{2}+\frac{1}{2}\sqrt{}}$$

$$= \frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.11, size = 137, normalized size = 0.91

$$\frac{\sqrt{\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)-\sqrt{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[-b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b
^2 - 4*a*c]]]) + Sqrt[-b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a - b*x^2 + c*x^4), x]
```

fricas [B] time = 0.82, size = 551, normalized size = 3.67

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/sqr
```

```
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2
*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*
a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*sq
rt(1/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c
^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c
^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2*sqrt(1
/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))
*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2
- 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

giac [B] time = 1.06, size = 513, normalized size = 3.42

$$\frac{\left(2 b^2c^2 - 8 ac^3 - \sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-\sqrt{b^2-4ac}\,c}\,b^2 + 4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-\sqrt{b^2-4ac}\,c}\,ac - 2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-\sqrt{b^2-4ac}\,c}\,bc - \sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-\sqrt{b^2-4ac}\,c}\,c^2 - 2\left(b^2-4ac\right)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{-\frac{b+\sqrt{b^2-4ac}}{c}}}\right)}{2\left(b^4 - 8ab^2c + 2b^3c + 16a^2c^2 - 8abc^2 + b^2c^2 - 4ac^3\right)|c|} - \frac{\left(2 b^2c^2 - 8 ac^3 - \sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc+\sqrt{b^2-4ac}\,c}\,b^2 + 4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc+\sqrt{b^2-4ac}\,c}\,ac - 2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc+\sqrt{b^2-4ac}\,c}\,bc - \sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc+\sqrt{b^2-4ac}\,c}\,c^2 - 2\left(b^2-4ac\right)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{-\frac{b-\sqrt{b^2-4ac}}{c}}}\right)}{2\left(b^4 - 8ab^2c + 2b^3c + 16a^2c^2 - 8abc^2 + b^2c^2 - 4ac^3\right)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 -
 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*
c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 -
4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b + sqrt(b^2 - 4*a*c))/c))/((b^4 -
8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)) -
 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2
- 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)
*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b*c
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 -
 4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b - sqrt(b^2 - 4*a*c))/c))/((b^4 -
 8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
```

maple [A] time = 0.01, size = 208, normalized size = 1.39

$$-\frac{\sqrt{2}\,b\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} + \frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)-1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/2*2
```

```
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)-1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^4 - bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 - b*x^2 + a), x)
```

mupad [B] time = 4.54, size = 416, normalized size = 2.77

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,\left(4\,a\,c^2-2\,b^2\,c\right)+\frac{x\,\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\,\left(b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\,\sqrt{\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}-2\,\mathrm{atanh}\left(\frac{\left(x\,\left(4\,a\,c^2-2\,b^2\,c\right)-\frac{x\,\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\,\left(\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{-\frac{\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\,\sqrt{-\frac{\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((
b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*
c^2)))^(1/2))/(a*c))*((b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c
+ 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) - (x*
(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(b^
4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b
*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(-((-(4*a*c - b^2
)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

sympy [A] time = 1.25, size = 75, normalized size = 0.50

$$\mathrm{RootSum}\left(t^4\left(256a^2c^3 - 128ab^2c^2 + 16b^4c\right) + t^2\left(16abc - 4b^3\right) + a, \left(t \mapsto t\log\left(64t^3ac^2 - 16t^3b^2c + 2tb + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(16*a*b
*c - 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c + 2*_
t*b + x)))
```

3.698 $\int \frac{1}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1093, 208}

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTanh[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a-bx^2+cx^4}\,dx = \frac{c\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{-\sqrt{b^2-4ac}-b}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]]/
Sqrt[-b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^
2 - 4*a*c]]]/Sqrt[-b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a-bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a - b*x^2 + c*x^4)^(-1), x]
```

fricas [B] time = 0.82, size = 605, normalized size = 4.03

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
- 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(a
^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 -
 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt((b - (a*b^2 - 4*a^2
*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2
- 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b - (a*b^2 - 4
*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((
b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x
 - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sq
rt((b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)))
```

giac [B] time = 0.57, size = 1050, normalized size = 7.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(-b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^
3*c + 2*b^4*c + 16*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqr
t(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(-b*c - sqrt(b^
2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
 sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^
2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^
2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b + sqrt(b^2 - 4*a*c))/c))/((
a*b^4 - 8*a^2*b^2*c + 2*a*b^3*c + 16*a^3*c^2 - 8*a^2*b*c^2 + a*b^2*c^2 - 4*
a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sq
rt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(-b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c^2 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*
sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqr
t(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4
*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b - sqrt(b
```

```
^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c + 2*a*b^3*c + 16*a^3*c^2 - 8*a^2*b*c
^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

maple [A] time = 0.01, size = 116, normalized size = 0.77

$$-\frac{\sqrt{2}\,c\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4-b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^4-bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 - b*x^2 + a), x)
```

mupad [B] time = 0.49, size = 763, normalized size = 5.09

$$-\operatorname{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x\,16\mathrm{i}-a\,b^2\,c\,x\,8\mathrm{i}}{4\,a\,b^4\,\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2\mathrm{i}-\operatorname{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x\,16\mathrm{i}-a\,b^2\,c\,x\,8\mathrm{i}}{4\,a\,b^4\,\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2 + c*x^4),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*((b^3 + (b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a
^2*b^2*c))^(1/2) + 64*a^3*c^2*((b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)
- 32*a^2*b^2*c*((b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*((b^3 + (b^6
```

```
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128
*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4
*a*b^4*(-((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*
a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-((b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4
+ 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(-((b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
- 64*a^2*b^2*c))^(1/2)))*(-((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

sympy [A] time = 1.25, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(16abc-4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{-32t^3a^2bc+8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(16*a*b
*c - 4*b**3) + c, Lambda(_t, _t*log(x + (-32*_t**3*a**2*b*c + 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

3.699 $\int \frac{1}{x^2(a-bx^2+cx^4)} dx$

Optimal. Leaf size=172

$$\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Rubi [A] time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1123, 1166, 208}

$$\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a-bx^2+cx^4\right)}\,dx &= -\frac{1}{ax}+\frac{\int \frac{b-cx^2}{a-bx^2+cx^4}\,dx}{a} \\
&= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a} \\
&= -\frac{1}{ax}+\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.40, size = 199, normalized size = 1.16

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sqrt[b^2
- 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b + Sqrt[b^2
- 4*a*c]]))/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a-bx^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a - b*x^2 + c*x^4)), x]
```

fricas [B] time = 0.84, size = 1108, normalized size = 6.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c
^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4
*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*
sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^3 - 3*a
*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5 - 5
*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4
*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c))) + sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2
*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4
- 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^
3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(
b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*
b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*
b^2 - 4*a^4*c))) - 2)/(a*x)
```

giac [B] time = 1.01, size = 1877, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 -
```

```
4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^
2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c
)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c -
 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^
2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt
(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(
-b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*
a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*ar
ctan(2*sqrt(1/2)*x/sqrt(-(a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4
- 8*a^4*b^2*c + 2*a^3*b^3*c + 16*a^5*c^2 - 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^
4*c^3)*abs(a)*abs(c)) - 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^
2 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*
a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c
)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*
c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 - 2*(sqrt(2)*sqr
t(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*
b^5*c + 16*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*s
qrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(-b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*
c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt(-(a*b - sqrt(a^2*b^2 - 4*a^
3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c + 2*a^3*b^3*c + 16*a^5*c^2 - 8*a^4*b*
c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(c)) - 1/(a*x)
```

maple [A] time = 0.02, size = 232, normalized size = 1.35

$$-\frac{\sqrt{2}\, b c \,\operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \sqrt{-4 a c+b^{2}}\, \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{\sqrt{2}\, b c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \sqrt{-4 a c+b^{2}}\, \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a}+\frac{\sqrt{2}\, c \,\operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{2 \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{1}{a x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/a/x+1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b-1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 - b)/(c*x^4 - b*x^2 + a), x)/a - 1/(a*x)
```

mupad [B] time = 4.93, size = 2979, normalized size = 17.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^
6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^
2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 -
4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
```

^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b
^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - (
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
c - b^2)^3)^(1/2))/(8(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*
b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*((b^5 + b^2*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan(((x*(4*a^
4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*
b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*
(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1
i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*
c^3 - 8*a^5*b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*
b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*
c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^2 +
x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5
*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16
*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*((b^5 - b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(a*x)

sympy [A] time = 3.83, size = 148, normalized size = 0.86

$$\operatorname{RootSum}\left(t^{4}\left(256a^{5}c^{2}-128a^{4}b^{2}c+16a^{3}b^{4}\right)+t^{2}\left(-48a^{2}bc^{2}+28ab^{3}c-4b^{5}\right)+c^{3},\left(t\mapsto t\log\left(x+\frac{-64t^{3}a^{5}c^{2}+48t^{3}a^{4}b^{2}c-8t^{3}a^{3}b^{4}+10ta^{2}bc^{2}-10tab^{3}c+2tb^{5}}{ac^{3}-b^{2}c^{2}}\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(-48
*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t*
*3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 + 10*_t*a**2*b*c**2
 - 10*_t*a*b**3*c + 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

3.700 $\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=69

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}+\frac{x^2}{2a}$$

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1114, 703, 634, 618, 206, 628}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) - ((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[
b]) - Log[a - b + 2*a*x^2 + a*x^4]/(2*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a-b+2ax+ax^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2a}+\frac{\text{Subst}\left(\int \frac{-a+b-2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2a}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx,x,2a\left(1+x^2\right)\right)}{a}\\
&= \frac{x^2}{2a}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.90

$$\frac{x^2-\log\left(a\left(x^2+1\right)^2-b\right)}{2a}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -1/2*((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(a^(3/2)*Sqrt[b]) + (x^
2 - Log[-b + a*(1 + x^2)^2])/(2*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^5/(a - b + 2*a*x^2 + a*x^4), x]
```

fricas [A] time = 1.05, size = 156, normalized size = 2.26

$$\left[\frac{2\,abx^2-2\,ab\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{ab}\,(a+b)\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}\,\left(x^2+1\right)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,a^2b},\ \frac{abx^2-ab\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{-ab}\,(a+b)\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{2\,a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*(a + b)*lo
g((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a -
b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(-a*b)*
(a + b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(a^2*b)]
```

giac [A] time = 0.26, size = 60, normalized size = 0.87

$$\frac{x^2}{2\,a}+\frac{(a+b)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}\,a}-\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a + b)*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2
*log(a*x^4 + 2*a*x^2 + a - b)/a
```

maple [A] time = 0.00, size = 86, normalized size = 1.25

$$-\frac{b\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}\;a}+\frac{x^2}{2a}-\frac{\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{\ln\left(a\,x^4+2a\,x^2+a-b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a-b)/a-1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+
2*a)/(a*b)^(1/2))-1/2/a/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*
b
```

maxima [A] time = 3.00, size = 74, normalized size = 1.07

$$\frac{x^2}{2\,a}+\frac{(a+b)\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a}-\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a + 1/4*(a + b)*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b))
)/(sqrt(a*b)*a) - 1/2*log(a*x^4 + 2*a*x^2 + a - b)/a
```

mupad [B] time = 0.39, size = 166, normalized size = 2.41

$$\frac{x^2}{2\,a}-\ln\left(a\,\sqrt{a^3\,b}-b\,\sqrt{a^3\,b}-a^2\,b\,x^2+a\,x^2\,\sqrt{a^3\,b}\right)\left(\frac{\frac{a^2}{2}+\frac{\sqrt{a^3\,b}}{4}}{a^3}+\frac{\sqrt{a^3\,b}}{4\,a^2\,b}\right)-\ln\left(a\,\sqrt{a^3\,b}-b\,\sqrt{a^3\,b}+a^2\,b\,x^2+a\,x^2\,\sqrt{a^3\,b}\right)\left(\frac{\frac{a^2}{2}-\frac{\sqrt{a^3\,b}}{4}}{a^3}-\frac{\sqrt{a^3\,b}}{4\,a^2\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x^2/(2*a) - log(a*(a^3*b)^(1/2) - b*(a^3*b)^(1/2) - a^2*b*x^2 + a*x^2*(a^3*
b)^(1/2))*((a^2/2 + (a^3*b)^(1/2)/4)/a^3 + (a^3*b)^(1/2)/(4*a^2*b)) - log(a
*(a^3*b)^(1/2) - b*(a^3*b)^(1/2) + a^2*b*x^2 + a*x^2*(a^3*b)^(1/2))*((a^2/2
 - (a^3*b)^(1/2)/4)/a^3 - (a^3*b)^(1/2)/(4*a^2*b))
```

sympy [B] time = 1.79, size = 138, normalized size = 2.00

$$\left(-\frac{1}{2a}-\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}-\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)+a-b}{a+b}\right)+\left(-\frac{1}{2a}+\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}+\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)+a-b}{a+b}\right)+\frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(2*a) - sqrt(a**3*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a) -
 sqrt(a**3*b)*(a + b)/(4*a**3*b)) + a - b)/(a + b)) + (-1/(2*a) + sqrt(a**3
*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a) + sqrt(a**3*b)*(a + b)
/(4*a**3*b)) + a - b)/(a + b)) + x**2/(2*a)
```

3.701 $\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1114, 634, 618, 206, 628}

$$\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a - b + 2*a*
x^2 + a*x^4]/(4*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x}{a-b+2ax+ax^2}\,dx, x, x^2\right)\\ &= -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)\right)+\frac{\mathrm{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{4a}\\ &= \frac{\log\left(a-b+2ax^2+ax^4\right)}{4a}+\mathrm{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{\log\left(a-b+2ax^2+ax^4\right)}{4a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.91

$$\frac{\log\left(a\left(x^2+1\right)^2-b\right)+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((2*Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[-b + a*(1 +
 x^2)^2])/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^3/(a - b + 2*a*x^2 + a*x^4), x]
```

fricas [A] time = 0.96, size = 134, normalized size = 2.39

$$\left[\frac{b \log\left(ax^4 + 2\,ax^2 + a - b\right) + \sqrt{ab}\,\log\left(\frac{ax^4 + 2\,ax^2 + 2\,\sqrt{ab}\left(x^2+1\right) + a + b}{ax^4 + 2\,ax^2 + a - b}\right)}{4\,ab}, \frac{b \log\left(ax^4 + 2\,ax^2 + a - b\right) - 2\,\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{4\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*log((a*x^4 + 2*a*x^2 + 2*s
qrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a - b)))/(a*b), 1/4*(b*log(a
*x^4 + 2*a*x^2 + a - b) - 2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(a*b
)]
```

giac [A] time = 0.26, size = 46, normalized size = 0.82

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}} + \frac{\log\left(ax^4 + 2\,ax^2 + a - b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b) + 1/4*log(a*x^4 + 2*a*x^2 +
a - b)/a
```

maple [A] time = 0.00, size = 49, normalized size = 0.88

$$\frac{\operatorname{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}} + \frac{\ln\left(a\,x^4 + 2a\,x^2 + a - b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/4/a*ln(a*x^4+2*a*x^2+a-b)+1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)
^(1/2))
```

maxima [A] time = 2.97, size = 60, normalized size = 1.07

$$-\frac{\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\,\sqrt{ab}} + \frac{\log\left(ax^4 + 2\,ax^2 + a - b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] -1/4*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/sqrt(a*b) + 1/4*l
og(a*x^4 + 2*a*x^2 + a - b)/a
```

mupad [B] time = 0.17, size = 153, normalized size = 2.73

$$\frac{\ln\left(x^2\sqrt{a^3 b}+a b-a^2-a^2 x^2\right)}{4 a}+\frac{\ln\left(x^2\sqrt{a^3 b}-a b+a^2+a^2 x^2\right)}{4 a}-\frac{\ln\left(x^2\sqrt{a^3 b}-a b+a^2+a^2 x^2\right)\sqrt{a^3 b}}{4 a^2 b}+\frac{\ln\left(x^2\sqrt{a^3 b}+a b-a^2-a^2 x^2\right)\sqrt{a^3 b}}{4 a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] log(x^2*(a^3*b)^(1/2) + a*b - a^2 - a^2*x^2)/(4*a) + log(x^2*(a^3*b)^(1/2)
- a*b + a^2 + a^2*x^2)/(4*a) - (log(x^2*(a^3*b)^(1/2) - a*b + a^2 + a^2*x^2
)*(a^3*b)^(1/2))/(4*a^2*b) + (log(x^2*(a^3*b)^(1/2) + a*b - a^2 - a^2*x^2)*
(a^3*b)^(1/2))/(4*a^2*b)
```

sympy [B] time = 0.84, size = 110, normalized size = 1.96

$$\left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right)+\left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (1/(4*a) - sqrt(a**3*b)/(4*a**2*b))*log(x**2 + (4*a*b*(1/(4*a) - sqrt(a**3*
b)/(4*a**2*b)) + a - b)/a) + (1/(4*a) + sqrt(a**3*b)/(4*a**2*b))*log(x**2 +
 (4*a*b*(1/(4*a) + sqrt(a**3*b)/(4*a**2*b)) + a - b)/a)
```

3.702 $\int \frac{x}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b + 2*a*x^2 + a*x^4),x]
[Out] -1/2*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a - b + 2*a*x^2 + a*x^4),x]
[Out] IntegrateAlgebraic[x/(a - b + 2*a*x^2 + a*x^4), x]
```

fricas [A] time = 1.11, size = 91, normalized size = 2.94

$$\left[\frac{\sqrt{ab}\,\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}\left(x^2+1\right)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,ab}, \frac{\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4
 + 2*a*x^2 + a - b))/(a*b), 1/2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a))/(
a*b)]
```

giac [A] time = 0.26, size = 23, normalized size = 0.74

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b)
```

maple [A] time = 0.00, size = 26, normalized size = 0.84

$$-\frac{\operatorname{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

maxima [A] time = 3.02, size = 37, normalized size = 1.19

$$\frac{\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] 1/4*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/sqrt(a*b)
```

mupad [B] time = 4.34, size = 31, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sqrt{b}\,x^2}{a\,x^2+a-b}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] atanh((a^(1/2)*b^(1/2)*x^2)/(a - b + a*x^2))/(2*a^(1/2)*b^(1/2))
```

sympy [A] time = 0.34, size = 53, normalized size = 1.71

$$\frac{\sqrt{\frac{1}{ab}}\log\left(-b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4}-\frac{\sqrt{\frac{1}{ab}}\log\left(b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] sqrt(1/(a*b))*log(-b*sqrt(1/(a*b)) + x**2 + 1)/4 - sqrt(1/(a*b))*log(b*sqrt
(1/(a*b)) + x**2 + 1)/4
```

3.703 $\int \frac{1}{x\left(a-b+2ax^2+ax^4\right)}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\log(x)}{a-b}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1114, 705, 29, 634, 618, 206, 628}

$$-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)}+\frac{\log(x)}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*(a - b)*Sqrt[b]) + Log[x]
/(a - b) - Log[a - b + 2*a*x^2 + a*x^4]/(4*(a - b))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a-b+2ax^2+ax^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a-b+2ax+ax^2\right)}\,dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2(a-b)}+\frac{\operatorname{Subst}\left(\int \frac{-2a-ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2(a-b)}\\
&= \frac{\log(x)}{a-b}-\frac{\operatorname{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{4(a-b)}-\frac{a\operatorname{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2(a-b)}\\
&= \frac{\log(x)}{a-b}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)}+\frac{a\operatorname{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)}{a-b}\\
&= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2(a-b)\sqrt{b}}+\frac{\log(x)}{a-b}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 1.17

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{a}\left(x^2+1\right)-\sqrt{b}\right)+\left(\sqrt{b}-\sqrt{a}\right)\log\left(\sqrt{a}\left(x^2+1\right)+\sqrt{b}\right)-4\sqrt{b}\log(x)}{4\sqrt{b}(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-4*Sqrt[b]*Log[x] + (Sqrt[a] + Sqrt[b])*Log[-Sqrt[b] + Sqrt[a]*(1 + x^2)]
+ (-Sqrt[a] + Sqrt[b])*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*Sqrt[b]*(-a + b
))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a-b+2ax^2+ax^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a - b + 2*a*x^2 + a*x^4)), x]
```

fricas [A] time = 0.97, size = 151, normalized size = 1.96

$$\left[-\frac{\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2ax^2-2\left(bx^2+b\right)\sqrt{\frac{a}{b}}+a+b}{ax^4+2ax^2+a-b}\right)+\log\left(ax^4+2ax^2+a-b\right)-4\log(x)}{4(a-b)},-\frac{2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{-\frac{a}{b}}}{ax^2+a}\right)+\log\left(ax^4+2ax^2+a-b\right)-4\log(x)}{4(a-b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b) + a + b)/(a
*x^4 + 2*a*x^2 + a - b)) + log(a*x^4 + 2*a*x^2 + a - b) - 4*log(x))/(a - b)
, -1/4*(2*sqrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x^2 + a)) + log(a*x^4 + 2*a*x^2
 + a - b) - 4*log(x))/(a - b)]
```

giac [A] time = 0.33, size = 71, normalized size = 0.92

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}(a-b)}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\log\left(x^2\right)}{2(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*(a - b)) - 1/4*log(a*x^4
+ 2*a*x^2 + a - b)/(a - b) + 1/2*log(x^2)/(a - b)
```

maple [A] time = 0.01, size = 71, normalized size = 0.92

$$\frac{a\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\,(a-b)\,\sqrt{ab}}+\frac{\ln(x)}{a-b}-\frac{\ln\left(a\,x^4+2a\,x^2+a-b\right)}{4\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] ln(x)/(a-b)-1/4*ln(a*x^4+2*a*x^2+a-b)/(a-b)+1/2*a/(a-b)/(a*b)^(1/2)*arctanh
(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

maxima [A] time = 3.13, size = 85, normalized size = 1.10

$$-\frac{a\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\sqrt{ab}\,(a-b)}-\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,(a-b)}+\frac{\log\left(x^2\right)}{2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] -1/4*a*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/(sqrt(a*b)*(a -
 b)) - 1/4*log(a*x^4 + 2*a*x^2 + a - b)/(a - b) + 1/2*log(x^2)/(a - b)
```

mupad [B] time = 4.56, size = 183, normalized size = 2.38

$$\frac{\ln(x)}{a-b}-\frac{\ln\left(16\,a^4+20\,a^4\,x^2+\frac{\left(b-\sqrt{a\,b}\right)\left(x^2\left(16\,a^5+80\,b\,a^4\right)-16\,a^4\,b+16\,a^5\right)}{4\left(a\,b-b^2\right)}\right)\left(b-\sqrt{a\,b}\right)}{4\,\left(a\,b-b^2\right)}-\frac{\ln\left(16\,a^4+20\,a^4\,x^2+\frac{\left(b+\sqrt{a\,b}\right)\left(x^2\left(16\,a^5+80\,b\,a^4\right)-16\,a^4\,b+16\,a^5\right)}{4\left(a\,b-b^2\right)}\right)\left(b+\sqrt{a\,b}\right)}{4\,\left(a\,b-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] log(x)/(a - b) - (log(16*a^4 + 20*a^4*x^2 + ((b - (a*b)^(1/2))*(x^2*(80*a^4
*b + 16*a^5) - 16*a^4*b + 16*a^5))/(4*(a*b - b^2)))*(b - (a*b)^(1/2)))/(4*(
a*b - b^2)) - (log(16*a^4 + 20*a^4*x^2 + ((b + (a*b)^(1/2))*(x^2*(80*a^4*b
+ 16*a^5) - 16*a^4*b + 16*a^5))/(4*(a*b - b^2)))*(b + (a*b)^(1/2)))/(4*(a*b
 - b^2))
```

sympy [B] time = 5.31, size = 184, normalized size = 2.39

$$\left(-\frac{1}{4\,(a-b)}-\frac{\sqrt{ab}}{4b\,(a-b)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right)+\left(-\frac{1}{4\,(a-b)}+\frac{\sqrt{ab}}{4b\,(a-b)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}+\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}+\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right)+\frac{\log(x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(4*(a - b)) - sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (4*a*b*(-1/(4*(a - b)
) - sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b)) - sqrt(a*b)/(4*b*
(a - b))) + b)/a) + (-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (
4*a*b*(-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b
)) + sqrt(a*b)/(4*b*(a - b))) + b)/a) + log(x)/(a - b)
```

3.704 $\int \frac{1}{x^3(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{2x^2(a-b)}-\frac{\sqrt{a}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)^2}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{2a\log(x)}{(a-b)^2}$$

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$-\frac{1}{2x^2(a-b)}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{\sqrt{a}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)^2}-\frac{2a\log(x)}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a - b)*x^2) - (Sqrt[a]*(a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])
/(2*(a - b)^2*Sqrt[b]) - (2*a*Log[x])/(a - b)^2 + (a*Log[a - b + 2*a*x^2 +
a*x^4])/(2*(a - b)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a-b+2ax^2+ax^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+2ax+ax^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2(a-b)x^2} + \frac{\text{Subst}\left(\int \frac{-2a-ax}{x\left(a-b+2ax+ax^2\right)}\,dx, x, x^2\right)}{2(a-b)}$$
$$= -\frac{1}{2(a-b)x^2} + \frac{\text{Subst}\left(\int \left(-\frac{2a}{(a-b)x} + \frac{a(3a+b+2ax)}{(a-b)\left(a-b+2ax+ax^2\right)}\right)dx, x, x^2\right)}{2(a-b)}$$
$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\,\text{Subst}\left(\int \frac{3a+b+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2(a-b)^2}$$
$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\,\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2(a-b)^2} + \frac{(a(a+b))\,\text{Subst}}{}$$
$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\log\left(a-b+2ax^2+ax^4\right)}{2(a-b)^2} - \frac{(a(a+b))\,\text{Subst}\left(\int \frac{}{4a}\right.}{(a-}$$
$$= -\frac{1}{2(a-b)x^2} - \frac{\sqrt{a}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2(a-b)^2\sqrt{b}} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\log\left(a-b+2ax^2\right.}{2(a-b)^2}$$

Mathematica [A] time = 0.09, size = 146, normalized size = 1.51

$$\frac{-8a\sqrt{b}\,x^2\log(x)+\sqrt{a}\,x^2\left(\sqrt{a}+\sqrt{b}\right)^2\log\left(\sqrt{a}\left(x^2+1\right)-\sqrt{b}\right)-\left(\sqrt{a}-\sqrt{b}\right)\left(\left(ax^2-\sqrt{a}\sqrt{b}\,x^2\right)\log\left(\sqrt{a}\left(x^2+1\right)+\sqrt{b}\right)+2\left(\sqrt{a}\sqrt{b}+b\right)\right)}{4\sqrt{b}\,x^2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-8*a*Sqrt[b]*x^2*Log[x] + Sqrt[a]*(Sqrt[a] + Sqrt[b])^2*x^2*Log[-Sqrt[b] +
 Sqrt[a]*(1 + x^2)] - (Sqrt[a] - Sqrt[b])*(2*(Sqrt[a]*Sqrt[b] + b) + (a*x^2
 - Sqrt[a]*Sqrt[b]*x^2)*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)]))/(4*(a - b)^2*Sqr
t[b]*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a-b+2ax^2+ax^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a - b + 2*a*x^2 + a*x^4)), x]
```

fricas [A] time = 0.78, size = 209, normalized size = 2.15

$$\left[\frac{(a+b)x^2\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{\frac{a}{b}}+a+b}{ax^4+2\,ax^2+a-b}\right)+2\,ax^2\log\left(ax^4+2\,ax^2+a-b\right)-8\,ax^2\log(x)-2\,a+2\,b}{4\left(a^2-2\,ab+b^2\right)x^2},\ \frac{(a+b)x^2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{-\frac{a}{b}}}{ax^2+a}\right)+ax^2\log\left(ax^4+2\,ax^2+a-b\right)-4\,ax^2\log(x)-a+b}{2\left(a^2-2\,ab+b^2\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*x^2*sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b)
+ a + b)/(a*x^4 + 2*a*x^2 + a - b)) + 2*a*x^2*log(a*x^4 + 2*a*x^2 + a - b)
- 8*a*x^2*log(x) - 2*a + 2*b)/((a^2 - 2*a*b + b^2)*x^2), 1/2*((a + b)*x^2*s
qrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x^2 + a)) + a*x^2*log(a*x^4 + 2*a*x^2 + a
- b) - 4*a*x^2*log(x) - a + b)/((a^2 - 2*a*b + b^2)*x^2)]
```

giac [A] time = 0.28, size = 126, normalized size = 1.30

$$\frac{a\log\left(ax^4+2\,ax^2+a-b\right)}{2\left(a^2-2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2-2\,ab+b^2}+\frac{\left(a^2+ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\left(a^2-2\,ab+b^2\right)\sqrt{-ab}}+\frac{2\,ax^2-a+b}{2\left(a^2-2\,ab+b^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*log(x^2)/(a^2 -
2*a*b + b^2) + 1/2*(a^2 + a*b)*arctan((a*x^2 + a)/sqrt(-a*b))/((a^2 - 2*a*b
 + b^2)*sqrt(-a*b)) + 1/2*(2*a*x^2 - a + b)/((a^2 - 2*a*b + b^2)*x^2)
```

maple [A] time = 0.01, size = 122, normalized size = 1.26

$$-\frac{a^2\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\,(a-b)^2\,\sqrt{ab}}-\frac{ab\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\,(a-b)^2\,\sqrt{ab}}-\frac{2a\ln(x)}{(a-b)^2}+\frac{a\ln\left(a\,x^4+2a\,x^2+a-b\right)}{2\,(a-b)^2}-\frac{1}{2\,(a-b)\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a-b)/x^2-2*a*ln(x)/(a-b)^2+1/2*a*ln(a*x^4+2*a*x^2+a-b)/(a-b)^2-1/2/(a
-b)^2*a^2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))-1/2/(a-b)^2*a/
(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*b
```

maxima [A] time = 2.97, size = 123, normalized size = 1.27

$$\frac{a \log\left(ax^{4} + 2\,ax^{2} + a - b\right)}{2\left(a^{2} - 2\,ab + b^{2}\right)} - \frac{a \log\left(x^{2}\right)}{a^{2} - 2\,ab + b^{2}} + \frac{\left(a^{2} + ab\right) \log\left(\frac{ax^{2} + a - \sqrt{ab}}{ax^{2} + a + \sqrt{ab}}\right)}{4\left(a^{2} - 2\,ab + b^{2}\right)\sqrt{ab}} - \frac{1}{2\,(a - b)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*log(x^2)/(a^2 -
2*a*b + b^2) + 1/4*(a^2 + a*b)*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqr
t(a*b)))/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - 1/2/((a - b)*x^2)
```

mupad [B] time = 4.87, size = 389, normalized size = 4.01

$$\frac{\ln\left(100\,a\,(a\,b)^{7/2} - 198\,b\,(a\,b)^{7/2} - a^3\,(a\,b)^{5/2} + 100\,b^3\,(a\,b)^{5/2} - b^5\,(a\,b)^{3/2} + a^2\,b^6 - 100\,a^3\,b^5 + 198\,a^4\,b^4 - 100\,a^5\,b^3 + a^6\,b^2 + a^2\,b^6\,x^2 - 100\,a^3\,b^5\,x^2 + 198\,a^4\,b^4\,x^2 - 100\,a^5\,b^3\,x^2 + a^6\,b^2\,x^2\right)\left(\frac{a\,\sqrt{a\,b}}{4} + b\left(\frac{a}{2} + \frac{\sqrt{a\,b}}{4}\right)\right)}{a^2\,b - 2\,a\,b^2 + b^3} - \frac{2\,a\,\ln(x)}{a^2 - 2\,a\,b + b^2} - \frac{\ln\left(198\,b\,(a\,b)^{7/2} - 100\,a\,(a\,b)^{7/2} + a^3\,(a\,b)^{5/2} - 100\,b^3\,(a\,b)^{5/2} + b^5\,(a\,b)^{3/2} + a^2\,b^6 - 100\,a^3\,b^5 + 198\,a^4\,b^4 - 100\,a^5\,b^3 + a^6\,b^2 + a^2\,b^6\,x^2 - 100\,a^3\,b^5\,x^2 + 198\,a^4\,b^4\,x^2 - 100\,a^5\,b^3\,x^2 + a^6\,b^2\,x^2\right)\left(\frac{a\,\sqrt{a\,b}}{4} - b\left(\frac{a}{2} - \frac{\sqrt{a\,b}}{4}\right)\right)}{a^2\,b - 2\,a\,b^2 + b^3} - \frac{1}{2\,x^2\,(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] (log(100*a*(a*b)^(7/2) - 198*b*(a*b)^(7/2) - a^3*(a*b)^(5/2) + 100*b^3*(a*b
)^(5/2) - b^5*(a*b)^(3/2) + a^2*b^6 - 100*a^3*b^5 + 198*a^4*b^4 - 100*a^5*b
^3 + a^6*b^2 + a^2*b^6*x^2 - 100*a^3*b^5*x^2 + 198*a^4*b^4*x^2 - 100*a^5*b^
3*x^2 + a^6*b^2*x^2)*((a*(a*b)^(1/2))/4 + b*(a/2 + (a*b)^(1/2)/4)))/(a^2*b
- 2*a*b^2 + b^3) - (2*a*log(x))/(a^2 - 2*a*b + b^2) - (log(198*b*(a*b)^(7/2
) - 100*a*(a*b)^(7/2) + a^3*(a*b)^(5/2) - 100*b^3*(a*b)^(5/2) + b^5*(a*b)^(
3/2) + a^2*b^6 - 100*a^3*b^5 + 198*a^4*b^4 - 100*a^5*b^3 + a^6*b^2 + a^2*b^
6*x^2 - 100*a^3*b^5*x^2 + 198*a^4*b^4*x^2 - 100*a^5*b^3*x^2 + a^6*b^2*x^2)*
((a*(a*b)^(1/2))/4 - b*(a/2 - (a*b)^(1/2)/4)))/(a^2*b - 2*a*b^2 + b^3) - 1/
(2*x^2*(a - b))
```

sympy [B] time = 32.93, size = 372, normalized size = 3.84

$$-\frac{2a\log(x)}{(a-b)^2} + \left(\frac{a}{2\,(a-b)^2} - \frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right)\log\left(x^2 + \frac{-4a^2b\left(\frac{a}{2(a-b)^2} - \frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right) + a^2 + 8ab^2\left(\frac{a}{2(a-b)^2} - \frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right) + 3ab - 4b^3\left(\frac{a}{2(a-b)^2} - \frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right)}{a^2+ab}\right) + \left(\frac{a}{2\,(a-b)^2} + \frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right)\log\left(x^2 + \frac{-4a^2b\left(\frac{a}{2(a-b)^2} + \frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right) + a^2 + 8ab^2\left(\frac{a}{2(a-b)^2} + \frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right) + 3ab - 4b^3\left(\frac{a}{2(a-b)^2} + \frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right)}{a^2+ab}\right) - \frac{1}{x^2\,(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] -2*a*log(x)/(a - b)**2 + (a/(2*(a - b)**2) - sqrt(a*b)*(a + b)/(4*b*(a**2 -
 2*a*b + b**2)))*log(x**2 + (-4*a**2*b*(a/(2*(a - b)**2) - sqrt(a*b)*(a + b
)/(4*b*(a**2 - 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a - b)**2) - sqrt(a
*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2)
 - sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))))/(a**2 + a*b)) + (a/(2*(a
```

```
 - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2)))*log(x**2 + (-4*a
**2*b*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) +
a**2 + 8*a*b**2*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b +
b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 -
 2*a*b + b**2))))/(a**2 + a*b)) - 1/(x**2*(2*a - 2*b))
```

3.705 $\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=114

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}+\frac{x}{a}$$

Rubi [A] time = 0.17, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1122, 1166, 205}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]
])/(2*a^(5/4)*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*a^(5/4)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx = \frac{x}{a} - \frac{\int \frac{a-b+2ax^2}{a-b+2ax^2+ax^4}\,dx}{a}$$
$$= \frac{x}{a} - \frac{1}{2}\left(2 - \frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx - \frac{1}{2}\left(2 + \frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,d$$
$$= \frac{x}{a} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}$$

Mathematica [A] time = 0.09, size = 144, normalized size = 1.26

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2a\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]])
/(2*a*Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^2*ArcTan[(S
qrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(2*a*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b
])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^4/(a - b + 2*a*x^2 + a*x^4), x]
```

fricas [B] time = 0.85, size = 603, normalized size = 5.29

$$\frac{a\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\log\left(-(3a^2-2ab-b^2)x + \left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}} - 3a^2b - ab^2\right)\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\right) - a\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\log\left(-(3a^2-2ab-b^2)x - \left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}} - 3a^2b - ab^2\right)\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\right) - a\sqrt{\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-a-3b}{a^2b}}\log\left(-(3a^2-2ab-b^2)x + \left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}} + 3a^2b + ab^2\right)\sqrt{\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-a-3b}{a^2b}}\right) + a\sqrt{\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-a-3b}{a^2b}}\log\left(-(3a^2-2ab-b^2)x - \left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}} + 3a^2b + ab^2\right)\sqrt{\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-a-3b}{a^2b}}\right) + 4x}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*b)/(a^2*b))
*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b))
- 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3
*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*
b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((9*a^2 + 6*a*b + b^2
)/(a^5*b)) - 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*
b)) + a + 3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)
) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6
*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b
^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2
)/(a^5*b)) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((
9*a^2 + 6*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 +
6*a*b + b^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + 4*x)/a
```

giac [B] time = 0.36, size = 511, normalized size = 4.48

$$-\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^4-\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^3b-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^2b^2-2\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,b^2\right)a^2+\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^3b-7\sqrt{a^2+\sqrt{ab}\,a}\,a^2b^2+4\sqrt{a^2+\sqrt{ab}\,a}\,ab^3\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^2+\sqrt{a^4-(a^2-ab)a^2}}{a^2}}}\right)}{2\left(3a^6b-7a^5b^2+4a^4b^3\right)}+\frac{\left(3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a^4-\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a^3b-4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a^2b^2-2\left(3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,ab-4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,b^2\right)a^2-\left(3\sqrt{a^2-\sqrt{ab}\,a}\,a^3b-7\sqrt{a^2-\sqrt{ab}\,a}\,a^2b^2+4\sqrt{a^2-\sqrt{ab}\,a}\,ab^3\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^2-\sqrt{a^4-(a^2-ab)a^2}}{a^2}}}\right)}{2\left(3a^6b-7a^5b^2+4a^4b^3\right)}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^4 - sqrt(a^2 + sqrt(a*b)*a)*sqr
t(a*b)*a^3*b - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 2*(3*sqrt(a^2
+ sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*b^2)*a^2
 + (3*sqrt(a^2 + sqrt(a*b)*a)*a^3*b - 7*sqrt(a^2 + sqrt(a*b)*a)*a^2*b^2 + 4
*sqrt(a^2 + sqrt(a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 + sqrt(a^4 - (a^
2 - a*b)*a^2))/a^2))/(3*a^6*b - 7*a^5*b^2 + 4*a^4*b^3) + 1/2*(3*sqrt(a^2 -
sqrt(a*b)*a)*sqrt(a*b)*a^4 - sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^3*b - 4*sq
rt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 2*(3*sqrt(a^2 - sqrt(a*b)*a)*sqrt
(a*b)*a*b - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b^2)*a^2 - (3*sqrt(a^2 - sq
rt(a*b)*a)*a^3*b - 7*sqrt(a^2 - sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(a*
b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - sqrt(a^4 - (a^2 - a*b)*a^2))/a^2)
)/(3*a^6*b - 7*a^5*b^2 + 4*a^4*b^3) + x/a
```

maple [B] time = 0.03, size = 210, normalized size = 1.84

$$-\frac{a\,\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(-a+\sqrt{ab}\right)a}}-\frac{a\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(a+\sqrt{ab}\right)a}}-\frac{b\,\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(-a+\sqrt{ab}\right)a}}-\frac{b\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(a+\sqrt{ab}\right)a}}+\frac{\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{\sqrt{\left(-a+\sqrt{ab}\right)a}}-\frac{\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{\sqrt{\left(a+\sqrt{ab}\right)a}}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] x/a+1/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2))-1/2/
(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2)
)*a-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*
a)^(1/2))*b-1/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)^(1/2
))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)
^(1/2))*a-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2
)+a)*a)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{a}-\frac{\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^2b-4\sqrt{a^2+\sqrt{ab}\,a}\,ab^2+3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^2-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab\right)a\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)}+\frac{\left(3\sqrt{a^2-\sqrt{ab}\,a}\,a^2b-4\sqrt{a^2-\sqrt{ab}\,a}\,ab^2-3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a^2+4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,ab\right)a\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] x/a - integrate((2*a*x^2 + a - b)/(a*x^4 + 2*a*x^2 + a - b), x)/a
```

mupad [B] time = 4.79, size = 1097, normalized size = 9.62

$$\frac{x}{a}-2\,\mathrm{atanh}\left(\frac{24x\sqrt{a^5b^3}\sqrt{-\frac{3}{16a^2}-\frac{1}{16ab}-\frac{3\sqrt{a^5b^3}}{16a^4b^2}-\frac{\sqrt{a^5b^3}}{16a^5b}}}{4ab^2-\frac{6\sqrt{a^5b^3}}{a}-6a^2b+2b^3+\frac{2b^2\sqrt{a^5b^3}}{a^3}+\frac{4b\sqrt{a^5b^3}}{a^2}}+\frac{8x\sqrt{a^5b^3}\sqrt{-\frac{3}{16a^2}-\frac{1}{16ab}-\frac{3\sqrt{a^5b^3}}{16a^4b^2}-\frac{\sqrt{a^5b^3}}{16a^5b}}}{\frac{4\sqrt{a^5b^3}}{a}-\frac{6\sqrt{a^5b^3}}{b}+2ab^2+4a^2b-6a^3+\frac{2b\sqrt{a^5b^3}}{a^2}}-\frac{8ab^2x\sqrt{-\frac{3}{16a^2}-\frac{1}{16ab}-\frac{3\sqrt{a^5b^3}}{16a^4b^2}-\frac{\sqrt{a^5b^3}}{16a^5b}}}{4ab+\frac{4\sqrt{a^5b^3}}{a^2}-6a^2+2b^2-\frac{6\sqrt{a^5b^3}}{ab}+\frac{2b\sqrt{a^5b^3}}{a^3}}-\frac{24a^2bx\sqrt{-\frac{3}{16a^2}-\frac{1}{16ab}-\frac{3\sqrt{a^5b^3}}{16a^4b^2}-\frac{\sqrt{a^5b^3}}{16a^5b}}}{4ab+\frac{4\sqrt{a^5b^3}}{a^2}-6a^2+2b^2-\frac{6\sqrt{a^5b^3}}{ab}+\frac{2b\sqrt{a^5b^3}}{a^3}}\right)\sqrt{-\frac{3a\sqrt{a^5b^3}+b\sqrt{a^5b^3}+a^4b+3a^3b^2}{16a^5b^2}}+2\,\mathrm{atanh}\left(\frac{24x\sqrt{a^5b^3}\sqrt{\frac{3\sqrt{a^5b^3}}{16a^4b^2}-\frac{1}{16ab}-\frac{3}{16a^2}+\frac{\sqrt{a^5b^3}}{16a^5b}}}{\frac{6\sqrt{a^5b^3}}{a}+4ab^2-6a^2b+2b^3-\frac{2b^2\sqrt{a^5b^3}}{a^3}-\frac{4b\sqrt{a^5b^3}}{a^2}}-\frac{8x\sqrt{a^5b^3}\sqrt{\frac{3\sqrt{a^5b^3}}{16a^4b^2}-\frac{1}{16ab}-\frac{3}{16a^2}+\frac{\sqrt{a^5b^3}}{16a^5b}}}{\frac{4\sqrt{a^5b^3}}{a}-\frac{6\sqrt{a^5b^3}}{b}-2ab^2-4a^2b+6a^3+\frac{2b\sqrt{a^5b^3}}{a^2}}+\frac{8ab^2x\sqrt{\frac{3\sqrt{a^5b^3}}{16a^4b^2}-\frac{1}{16ab}-\frac{3}{16a^2}+\frac{\sqrt{a^5b^3}}{16a^5b}}}{4ab-\frac{4\sqrt{a^5b^3}}{a^2}-6a^2+2b^2+\frac{6\sqrt{a^5b^3}}{ab}-\frac{2b\sqrt{a^5b^3}}{a^3}}+\frac{24a^2bx\sqrt{\frac{3\sqrt{a^5b^3}}{16a^4b^2}-\frac{1}{16ab}-\frac{3}{16a^2}+\frac{\sqrt{a^5b^3}}{16a^5b}}}{4ab-\frac{4\sqrt{a^5b^3}}{a^2}-6a^2+2b^2+\frac{6\sqrt{a^5b^3}}{ab}-\frac{2b\sqrt{a^5b^3}}{a^3}}\right)\sqrt{\frac{3a\sqrt{a^5b^3}+b\sqrt{a^5b^3}-a^4b-3a^3b^2}{16a^5b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x/a - 2*atanh((24*x*(a^5*b^3)^(1/2)*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b^
3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b^2 - (6*(
a^5*b^3)^(1/2))/a - 6*a^2*b + 2*b^3 + (2*b^2*(a^5*b^3)^(1/2))/a^3 + (4*b*(a
^5*b^3)^(1/2))/a^2) + (8*x*(a^5*b^3)^(1/2)*(- 3/(16*a^2) - 1/(16*a*b) - (3*
(a^5*b^3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((4*(a^5
*b^3)^(1/2))/a - (6*(a^5*b^3)^(1/2))/b + 2*a*b^2 + 4*a^2*b - 6*a^3 + (2*b*(
a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b^3)
^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*(a^5*
b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 - (6*(a^5*b^3)^(1/2))/(a*b) + (2*b*(a^5*b^3
)^(1/2))/a^3) - (24*a^2*b*x*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b^3)^(1/2)
)/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*(a^5*b^3)^(
1/2))/a^2 - 6*a^2 + 2*b^2 - (6*(a^5*b^3)^(1/2))/(a*b) + (2*b*(a^5*b^3)^(1/2
))/a^3))*(-(3*a*(a^5*b^3)^(1/2) + b*(a^5*b^3)^(1/2) + a^4*b + 3*a^3*b^2)/(1
6*a^5*b^2))^(1/2) + 2*atanh((24*x*(a^5*b^3)^(1/2)*((3*(a^5*b^3)^(1/2))/(16*
a^4*b^2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((6
*(a^5*b^3)^(1/2))/a + 4*a*b^2 - 6*a^2*b + 2*b^3 - (2*b^2*(a^5*b^3)^(1/2))/a
^3 - (4*b*(a^5*b^3)^(1/2))/a^2) - (8*x*(a^5*b^3)^(1/2)*((3*(a^5*b^3)^(1/2))
```

```
/(16*a^4*b^2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2)
)/((4*(a^5*b^3)^(1/2))/a - (6*(a^5*b^3)^(1/2))/b - 2*a*b^2 - 4*a^2*b + 6*a^
3 + (2*b*(a^5*b^3)^(1/2))/a^2) + (8*a*b^2*x*((3*(a^5*b^3)^(1/2))/(16*a^4*b^
2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b -
(4*(a^5*b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 + (6*(a^5*b^3)^(1/2))/(a*b) - (2*b*
(a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*((3*(a^5*b^3)^(1/2))/(16*a^4*b^2) - 1/(
16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b - (4*(a^5*
b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 + (6*(a^5*b^3)^(1/2))/(a*b) - (2*b*(a^5*b^3
)^(1/2))/a^3))*((3*a*(a^5*b^3)^(1/2) + b*(a^5*b^3)^(1/2) - a^4*b - 3*a^3*b^
2)/(16*a^5*b^2))^(1/2)
```

sympy [A] time = 2.02, size = 105, normalized size = 0.92

$$\operatorname{RootSum}\left(256t^4a^5b^2 + t^2\left(32a^4b + 96a^3b^2\right) + a^3 - 3a^2b + 3ab^2 - b^3, \left(t \mapsto t\log\left(x + \frac{64t^3a^4b + 4ta^3 + 24ta^2b + 4tab^2}{3a^2 - 2ab - b^2}\right)\right)\right) + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(32*a**4*b + 96*a**3*b**2) + a**3 - 3*a
**2*b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (64*_t**3*a**4*b + 4*_t*a**3
 + 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 - 2*a*b - b**2)))) + x/a
```

3.706 $\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1130, 205}

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a
^(3/4)*Sqrt[b]) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a]
+ Sqrt[b]]])/(2*a^(3/4)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx = -\left(\frac{1}{2}\left(-1+\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx\right)+\frac{1}{2}\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx$$

$$= -\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [A] time = 0.10, size = 128, normalized size = 1.17

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (-(((Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]])/Sqrt
[a - Sqrt[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + S
qrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a - b + 2*a*x^2 + a*x^4), x]
```

fricas [B] time = 1.76, size = 267, normalized size = 2.45

$$\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)+\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sqrt(1/(a^3
*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) + x) - 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)
```

```
/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) +
 x) - 1/4*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(1/
(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b)) + x) + 1/4*sqrt((a*b*sqrt(1/(a^3*b)) -
 1)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b))
 + x)
```

giac [B] time = 0.36, size = 199, normalized size = 1.83

$$\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)}-\frac{\left(3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a-4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt
(a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a - b)*a + 4*a^2)
)/a))/(3*a^4*b - 4*a^3*b^2) - 1/2*(3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a -
4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*
a - sqrt(-4*(a - b)*a + 4*a^2))/a))/(3*a^4*b - 4*a^3*b^2)
```

maple [A] time = 0.01, size = 134, normalized size = 1.23

$$\frac{a\,\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(-a+\sqrt{ab}\right)a}}+\frac{a\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(a+\sqrt{ab}\right)a}}-\frac{\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{\left(-a+\sqrt{ab}\right)a}}+\frac{\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{\left(a+\sqrt{ab}\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/((-a+(a*b)^(1/2))*a)^(1/2)*arctanh(1/((-a+(a*b)^(1/2))*a)^(1/2)*a*x)+1
/2/(a*b)^(1/2)/((-a+(a*b)^(1/2))*a)^(1/2)*arctanh(1/((-a+(a*b)^(1/2))*a)^(1
/2)*a*x)*a+1/2/((a+(a*b)^(1/2))*a)^(1/2)*arctan(1/((a+(a*b)^(1/2))*a)^(1/2)
*a*x)+1/2/(a*b)^(1/2)/((a+(a*b)^(1/2))*a)^(1/2)*a*arctan(1/((a+(a*b)^(1/2))
*a)^(1/2)*a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ax^4+2\,ax^2+a-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a - b), x)
```

mupad [B] time = 0.30, size = 216, normalized size = 1.98

$$-2\,\mathrm{atanh}\left(\frac{2\left(x\left(4a^3+4ba^2\right)-\frac{4ax\left(\sqrt{a^3b^3}+a^2b\right)}{b}\right)\sqrt{-\frac{\sqrt{a^3b^3}+a^2b}{16a^3b^2}}}{2ab-2a^2}\right)\sqrt{-\frac{\sqrt{a^3b^3}+a^2b}{16a^3b^2}}-2\,\mathrm{atanh}\left(\frac{2\left(x\left(4a^3+4ba^2\right)+\frac{4ax\left(\sqrt{a^3b^3}-a^2b\right)}{b}\right)\sqrt{\frac{\sqrt{a^3b^3}-a^2b}{16a^3b^2}}}{2ab-2a^2}\right)\sqrt{\frac{\sqrt{a^3b^3}-a^2b}{16a^3b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] - 2*atanh((2*(x*(4*a^2*b + 4*a^3) - (4*a*x*((a^3*b^3)^(1/2) + a^2*b))/b)*(-
((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2))/(2*a*b - 2*a^2))*(-((a^3*b^3
)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - 2*atanh((2*(x*(4*a^2*b + 4*a^3) + (4
*a*x*((a^3*b^3)^(1/2) - a^2*b))/b)*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))
^(1/2))/(2*a*b - 2*a^2))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)
```

sympy [A] time = 0.60, size = 44, normalized size = 0.40

$$\mathrm{RootSum}\left(256t^4a^3b^2+32t^2a^2b+a-b,\left(t\mapsto t\log\left(-64t^3a^2b-4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b + a - b, Lambda(_t, _t*log(-6
4*_t**3*a**2*b - 4*_t*a + x)))
```

3.707 $\int \frac{1}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}+\sqrt{b}}}$$

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1093, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[
b]]*Sqrt[b]) - ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]]/(2*a^(1/4)*Sqrt[
Sqrt[a] + Sqrt[b]]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx = \frac{\sqrt{a}\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\sqrt{b}} - \frac{\sqrt{a}\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\sqrt{b}}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\,\sqrt{b}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\,\sqrt{b}}$$

Mathematica [A] time = 0.06, size = 105, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a - Sqrt[a]*Sqrt[b]]*
Sqrt[b]) - ArcTan[(Sqrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a + Sqrt[a
]*Sqrt[b]]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a - b + 2*a*x^2 + a*x^4)^(-1), x]
```

fricas [B] time = 0.78, size = 553, normalized size = 5.07

$$-\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)+\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(-\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)-\frac{1}{4}\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}\log\left(\left(b+\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}+x\right)+\frac{1}{4}\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}\log\left(-\left(b+\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*l
og((b - (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^2)
```

```
/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) + 1/4*sqrt(-((a*b -
 b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*log(-(b - (a^2*b -
a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a
^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) - 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b -
 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2))*log((b + (a^2*b - a*b^2)/sqrt(a^3*b -
 2*a^2*b^2 + a*b^3))*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) - 1)
/(a*b - b^2)) + x) + 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
- 1)/(a*b - b^2))*log(-(b + (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
)*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2)) + x)
```

giac [B] time = 0.25, size = 299, normalized size = 2.74

$$\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^2b-4\sqrt{a^2+\sqrt{ab}\,a}\,ab^2-3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^2+4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^5b-7a^4b^2+4a^3b^3\right)}+\frac{\left(3\sqrt{a^2-\sqrt{ab}\,a}\,a^2b-4\sqrt{a^2-\sqrt{ab}\,a}\,ab^2+3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a^2-4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,ab\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^5b-7a^4b^2+4a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^2*b - 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^2 - 3*
sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2 + 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)
*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a - b)*a + 4*a^2))/a
))/(3*a^5*b - 7*a^4*b^2 + 4*a^3*b^3) + 1/2*(3*sqrt(a^2 - sqrt(a*b)*a)*a^2*b
 - 4*sqrt(a^2 - sqrt(a*b)*a)*a*b^2 + 3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^
2 - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sq
rt((2*a - sqrt(-4*(a - b)*a + 4*a^2))/a))/(3*a^5*b - 7*a^4*b^2 + 4*a^3*b^3)
```

maple [A] time = 0.01, size = 74, normalized size = 0.68

$$-\frac{a\,\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(-a+\sqrt{ab}\right)a}}-\frac{a\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(a+\sqrt{ab}\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a*b)^(1/2)/((-a+(a*b)^(1/2))*a)^(1/2)*arctanh(1/((-a+(a*b)^(1/2))*a)^
(1/2)*a*x)*a-1/2/(a*b)^(1/2)/((a+(a*b)^(1/2))*a)^(1/2)*a*arctan(1/((a+(a*b)
^(1/2))*a)^(1/2)*a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^4+2\,ax^2+a-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] integrate(1/(a*x^4 + 2*a*x^2 + a - b), x)
```

mupad [B] time = 5.78, size = 322, normalized size = 2.95

$$\frac{\ln\left(4\,a^3\,b\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}-4\,a^3\,x+\frac{4\,a^4\,b\,x}{a\,b+\sqrt{a\,b^3}}\right)\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}}{4}+\frac{\ln\left(4\,a^3\,x-4\,a^3\,b\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}-\frac{4\,a^4\,b\,x}{a\,b-\sqrt{a\,b^3}}\right)\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}}{4}-\ln\left(4\,a^3\,x+4\,a^3\,b\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}-\frac{4\,a^4\,b\,x}{a\,b+\sqrt{a\,b^3}}\right)\,\sqrt{\frac{a\,b-\sqrt{a\,b^3}}{16\,\left(a\,b^3-a^2\,b^2\right)}}-\ln\left(4\,a^3\,x+16\,a^3\,b\,\sqrt{-\frac{1}{16\,a\,b-16\,\sqrt{a\,b^3}}}-\frac{4\,a^4\,b\,x}{a\,b-\sqrt{a\,b^3}}\right)\,\sqrt{\frac{a\,b+\sqrt{a\,b^3}}{16\,\left(a\,b^3-a^2\,b^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] (log(4*a^3*b*(-1/(a*b + (a*b^3)^(1/2)))^(1/2) - 4*a^3*x + (4*a^4*b*x)/(a*b
+ (a*b^3)^(1/2)))*(-1/(a*b + (a*b^3)^(1/2)))^(1/2))/4 + (log(4*a^3*x - 4*a^
3*b*(-1/(a*b - (a*b^3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b - (a*b^3)^(1/2)))*(
-1/(a*b - (a*b^3)^(1/2)))^(1/2))/4 - log(4*a^3*x + 4*a^3*b*(-1/(a*b + (a*b^
3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b + (a*b^3)^(1/2)))*((a*b - (a*b^3)^(1/2)
)/(16*(a*b^3 - a^2*b^2)))^(1/2) - log(4*a^3*x + 16*a^3*b*(-1/(16*a*b - 16*(
a*b^3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b - (a*b^3)^(1/2)))*((a*b + (a*b^3)^(
1/2))/(16*(a*b^3 - a^2*b^2)))^(1/2)
```

sympy [A] time = 0.95, size = 63, normalized size = 0.58

$$\operatorname{RootSum}\left(t^{4}\left(256a^{2}b^{2}-256ab^{3}\right)+32t^{2}ab+1,\left(t\mapsto t\log\left(-64t^{3}a^{2}b+64t^{3}ab^{2}-4ta-4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 - 256*a*b**3) + 32*_t**2*a*b + 1, Lambda(_t, _
t*log(-64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a - 4*_t*b + x)))
```

3.708 $\int \frac{1}{x^2(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=121

$$-\frac{1}{x(a-b)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1123, 1166, 205}

$$-\frac{1}{x(a-b)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(1/((a - b)*x)) - (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2
*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]) + (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[b]]])/(2*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-b+2ax^2+ax^4\right)}\,dx = -\frac{1}{(a-b)x}-\frac{\int \frac{-2a-ax^2}{a-b+2ax^2+ax^4}\,dx}{-a+b}$$

$$= -\frac{1}{(a-b)x}-\frac{a\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}+\frac{a\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}}$$

$$= -\frac{1}{(a-b)x}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.15, size = 143, normalized size = 1.18

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}+a\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}}-\frac{\left(a-\sqrt{a}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{2}{x}}{2(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (2/x + ((a + Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]
)/(Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt
[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]))/(2*
(-a + b))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a-b+2ax^2+ax^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a - b + 2*a*x^2 + a*x^4)), x]
```

fricas [B] time = 1.51, size = 1612, normalized size = 13.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*((a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sqr
t((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 +
15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log((3*a
^2 + a*b)*x + (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*sqrt
((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 1
5*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*
b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 -
 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 -
b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)
*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^
4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log(
(3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*
sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4
 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b
^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4))) + (a - b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^3 -
b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^
3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*
log((3*a^2 + a*b)*x + (6*a^2*b + 2*a*b^2 + (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b
^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3
*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^
2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a
^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 2
0*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^
4))*log((3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 + (a^4*b - 2*a^3*b^2 + 2*a*b^4
 - b^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20
*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 +
15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2
+ 3*a*b^3 - b^4))) - 4)/((a - b)*x)
```

giac [B] time = 0.38, size = 698, normalized size = 5.77

$$\frac{\left(\left(3\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}ab-4\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}b^2\right)(a-b)^2|a|-2\left(3\sqrt{a^2+\sqrt{ab}a}a^3b-7\sqrt{a^2+\sqrt{ab}a}a^2b^2+4\sqrt{a^2+\sqrt{ab}a}ab^3\right)|a-b||a|+\left(3\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a^4-10\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a^3b+11\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a^2b^2-4\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}ab^3\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^2-ab+\sqrt{(a^2-ab)^2-(a^2-ab)(a^2-2ab+b^2)}}{a^2-ab}}}\right)}{2\left(3a^6b-13a^5b^2+21a^4b^3-15a^3b^4+4a^2b^5\right)|a-b|}-\frac{\left(\left(3\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}ab-4\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}b^2\right)(a-b)^2|a|+2\left(3\sqrt{a^2-\sqrt{ab}a}a^3b-7\sqrt{a^2-\sqrt{ab}a}a^2b^2+4\sqrt{a^2-\sqrt{ab}a}ab^3\right)|a-b||a|+\left(3\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a^4-10\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a^3b+11\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a^2b^2-4\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}ab^3\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^2-ab-\sqrt{(a^2-ab)^2-(a^2-ab)(a^2-2ab+b^2)}}{a^2-ab}}}\right)}{2\left(3a^6b-13a^5b^2+21a^4b^3-15a^3b^4+4a^2b^5\right)|a-b|}-\frac{1}{(a-b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 + sqrt(a*b)*a)*s
qrt(a*b)*b^2)*(a - b)^2*abs(a) - 2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^3*b - 7*sqr
t(a^2 + sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^3)*abs(a - b)*
abs(a) + (3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^4 - 10*sqrt(a^2 + sqrt(a*b)
*a)*sqrt(a*b)*a^3*b + 11*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 4*sqrt
(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - a*b + sqr
t((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - a*b)))/((3*a^6*b
 - 13*a^5*b^2 + 21*a^4*b^3 - 15*a^3*b^4 + 4*a^2*b^5)*abs(a - b)) - 1/2*((3*
sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)
*b^2)*(a - b)^2*abs(a) + 2*(3*sqrt(a^2 - sqrt(a*b)*a)*a^3*b - 7*sqrt(a^2 -
sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(a*b)*a)*a*b^3)*abs(a - b)*abs(a) +
 (3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^4 - 10*sqrt(a^2 - sqrt(a*b)*a)*sqrt
(a*b)*a^3*b + 11*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 4*sqrt(a^2 - s
qrt(a*b)*a)*sqrt(a*b)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - a*b - sqrt((a^2 -
 a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - a*b)))/((3*a^6*b - 13*a^
5*b^2 + 21*a^4*b^3 - 15*a^3*b^4 + 4*a^2*b^5)*abs(a - b)) - 1/((a - b)*x)
```

maple [B] time = 0.01, size = 180, normalized size = 1.49

$$\frac{a^{2} \operatorname{arctanh}\left(\frac{a x}{\sqrt{\left(-a+\sqrt{a b}\right) a}}\right)}{2 \left(a-b\right) \sqrt{a b}\, \sqrt{\left(-a+\sqrt{a b}\right) a}}+\frac{a^{2} \arctan\left(\frac{a x}{\sqrt{\left(a+\sqrt{a b}\right) a}}\right)}{2 \left(a-b\right) \sqrt{a b}\, \sqrt{\left(a+\sqrt{a b}\right) a}}+\frac{a \operatorname{arctanh}\left(\frac{a x}{\sqrt{\left(-a+\sqrt{a b}\right) a}}\right)}{2 \left(a-b\right) \sqrt{\left(-a+\sqrt{a b}\right) a}}-\frac{a \arctan\left(\frac{a x}{\sqrt{\left(a+\sqrt{a b}\right) a}}\right)}{2 \left(a-b\right) \sqrt{\left(a+\sqrt{a b}\right) a}}-\frac{1}{\left(a-b\right) x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/(a-b)/x+1/2*a/(a-b)/((-a+(a*b)^(1/2))*a)^(1/2)*arctanh(1/((-a+(a*b)^(1/2
))*a)^(1/2)*a*x)+1/2*a^2/(a-b)/(a*b)^(1/2)/((-a+(a*b)^(1/2))*a)^(1/2)*arcta
nh(1/((-a+(a*b)^(1/2))*a)^(1/2)*a*x)-1/2*a/(a-b)/((a+(a*b)^(1/2))*a)^(1/2)*
arctan(1/((a+(a*b)^(1/2))*a)^(1/2)*a*x)+1/2*a^2/(a-b)/(a*b)^(1/2)/((a+(a*b)
^(1/2))*a)^(1/2)*arctan(1/((a+(a*b)^(1/2))*a)^(1/2)*a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{2}\left(\frac{\left(6 \sqrt{a^{2}+\sqrt{ab}\, a}\, a^{2} b-8 \sqrt{a^{2}+\sqrt{ab}\, a}\, a b^{2}-3 \sqrt{a^{2}+\sqrt{ab}\, a}\, \sqrt{ab}\, a^{2}+\sqrt{a^{2}+\sqrt{ab}\, a}\, \sqrt{ab}\, a b+4 \sqrt{a^{2}+\sqrt{ab}\, a}\, \sqrt{ab}\, b^{2}\right) a \arctan\left(\frac{2 \sqrt{\frac{1}{2}}\, x}{\sqrt{\frac{2 a+\sqrt{-4 (a-b) a+4 a^{2}}}{a}}}\right)}{3 a^{5} b-7 a^{4} b^{2}+4 a^{3} b^{3}}+\frac{\left(6 \sqrt{a^{2}-\sqrt{ab}\, a}\, a^{2} b-8 \sqrt{a^{2}-\sqrt{ab}\, a}\, a b^{2}+3 \sqrt{a^{2}-\sqrt{ab}\, a}\, \sqrt{ab}\, a^{2}-\sqrt{a^{2}-\sqrt{ab}\, a}\, \sqrt{ab}\, a b-4 \sqrt{a^{2}-\sqrt{ab}\, a}\, \sqrt{ab}\, b^{2}\right) a \arctan\left(\frac{2 \sqrt{\frac{1}{2}}\, x}{\sqrt{\frac{2 a-\sqrt{-4 (a-b) a+4 a^{2}}}{a}}}\right)}{3 a^{5} b-7 a^{4} b^{2}+4 a^{3} b^{3}}\right) a}{a-b}-\frac{1}{(a-b) x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] -a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a - b), x)/(a - b) - 1/((a - b)*x
)
```

mupad [B] time = 5.12, size = 2774, normalized size = 22.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] atan(((x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) + (-(3*a*b^2 + a^2*b + 3
*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^
2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2
 - x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4
 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5
- 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b + 3*a*(a*b^
3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/
2)*1i + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) - (-(3*a*b^2 + a^2*b +
 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*
b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b
^2 + x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b
^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^
5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b + 3*a*(a*
b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(
1/2)*1i)/(6*a^6*b - 2*a^7 + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) +
(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^
5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192
*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^
3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*
a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b
^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^
2*b^3 + a^3*b^2)))^(1/2) - (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) - (
-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5
 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*
a^6*b^3 - 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3
)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a
^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^
2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2
*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 - 6*a^5*b^2))*(-(3*a*b^2 + a^2*b + 3*a*
(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2))
)^(1/2)*2i - 1/(x*(a - b)) + atan(((x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*
b^3) + (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b
^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^
4 + 192*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) -
b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*
```

```
b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(
-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5
 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^
5*b^3) - (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a
*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*
b^4 + 192*a^6*b^3 - 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2)
- b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^
9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))
*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b
^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i)/(6*a^6*b - 2*a^7 + (x*(8*a^7*b - 4*a^
8 + 4*a^4*b^4 - 8*a^5*b^3) + (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*
b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 3
2*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b
- 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3
*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*
b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/
2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2) - (x*(8*a^7*b - 4*a^8
 + 4*a^4*b^4 - 8*a^5*b^3) - (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b
^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32
*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b -
 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*
b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b
^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2
))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 - 6*a^5*b^
2))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4
- b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*2i
```

sympy [A] time = 6.14, size = 134, normalized size = 1.11

$$\operatorname{RootSum}\left(t^4\left(256a^3b^2-768a^2b^3+768ab^4-256b^5\right)+t^2\left(32a^2b+96ab^2\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^3a^4b-128t^3a^3b^2+128t^3ab^4-64t^3b^5+4ta^3+40ta^2b+20tab^2}{3a^2+ab}\right)\right)\right)-\frac{1}{x(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 - 768*a**2*b**3 + 768*a*b**4 - 256*b**5) + _t*
*2*(32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (64*_t**3*a**4*b - 12
8*_t**3*a**3*b**2 + 128*_t**3*a*b**4 - 64*_t**3*b**5 + 4*_t*a**3 + 40*_t*a*
*2*b + 20*_t*a*b**2)/(3*a**2 + a*b)))) - 1/(x*(a - b))
```

3.709 $\int \frac{x^5}{a+b+2ax^2+ax^4} dx$

Optimal. Leaf size=69

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} - \frac{\log\left(ax^4+2ax^2+a+b\right)}{2a} + \frac{x^2}{2a}$$

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 703, 634, 618, 204, 628}

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} - \frac{\log\left(ax^4+2ax^2+a+b\right)}{2a} + \frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) + ((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[b
]) - Log[a + b + 2*a*x^2 + a*x^4]/(2*a)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{a+b+2ax+ax^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2a} + \frac{\text{Subst}\left(\int \frac{-a-b-2ax}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^2}{2a} - \frac{\text{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{2a} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^2}{2a} - \frac{\log\left(a+b+2ax^2+ax^4\right)}{2a} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)}{a}\\
&= \frac{x^2}{2a} + \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} - \frac{\log\left(a+b+2ax^2+ax^4\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.90

$$
\frac{\sqrt{a}\left(x^2-\log\left(a\left(x^2+1\right)^2+b\right)\right)+\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{2a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Sqrt[a]*(x^2 - Log
[b + a*(1 + x^2)^2]))/(2*a^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b + 2*a*x^2 + a*x^4), x]
```

fricas [A] time = 0.82, size = 157, normalized size = 2.28

$$\left[\frac{2\,abx^2-2\,ab\log\left(ax^4+2\,ax^2+a+b\right)+\sqrt{-ab}\,(a-b)\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}\,(x^2+1)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,a^2b},\ \frac{abx^2-ab\log\left(ax^4+2\,ax^2+a+b\right)-\sqrt{ab}\,(a-b)\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{2\,a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a + b) + sqrt(-a*b)*(a - b)*l
og((a*x^4 + 2*a*x^2 + 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 + a
+ b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(a*b)
*(a - b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(a^2*b)]
```

giac [A] time = 0.25, size = 58, normalized size = 0.84

$$\frac{x^2}{2\,a}+\frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*l
og(a*x^4 + 2*a*x^2 + a + b)/a
```

maple [A] time = 0.01, size = 84, normalized size = 1.22

$$-\frac{b\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}\;a}+\frac{x^2}{2a}+\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{\ln\left(a\,x^4+2a\,x^2+a+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2/a*x^2-1/2*ln(a*x^4+2*a*x^2+a+b)/a+1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2
*a)/(a*b)^(1/2))-1/2/a/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*b
```

maxima [A] time = 3.03, size = 58, normalized size = 0.84

$$\frac{x^2}{2a} + \frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a} - \frac{\log\left(ax^4 + 2\,ax^2 + a + b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*l
og(a*x^4 + 2*a*x^2 + a + b)/a
```

mupad [B] time = 0.18, size = 302, normalized size = 4.38

$$\frac{x^2}{2a} - \frac{\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{2\,a} - \frac{\mathrm{atan}\left(\frac{a\,b\left(x^2\left(\frac{\frac{\sqrt{a}\,(2\,a-2\,b)}{\sqrt{b}}+\frac{(a-b)\left(4\,a\,b-12\,a^2\right)}{4\,a^{3/2}\,\sqrt{b}}}{a+b}+\frac{\sqrt{a}\left(6\,a-2\,b-\frac{(a-b)^2}{b}+\frac{2\,a\,b-6\,a^2}{a}\right)}{\sqrt{b}\,(a+b)}\right)-\frac{\frac{(a-b)\left(16\,a\,b-\frac{8\,a^3+8\,b\,a^2}{a}+16\,a^2\right)}{4\,a^{3/2}\,\sqrt{b}}-\frac{\left(16\,a^3+16\,b\,a^2\right)(a-b)}{8\,a^{5/2}\,\sqrt{b}}}{a+b}+\frac{\sqrt{a}\left(4\,a+4\,b-\frac{8\,a\,b-\frac{8\,a^3+8\,b\,a^2}{2\,a}+8\,a^2}{a}-\frac{(a-b)^2\left(a^3+b\,a^2\right)}{a^3\,b}\right)}{\sqrt{b}\,(a+b)}\right)}{a^2-2\,a\,b+b^2}\right)(a-b)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x^2/(2*a) - log(a + b + 2*a*x^2 + a*x^4)/(2*a) - (atan((a*b*(x^2*(((a^(1/2)
*(2*a - 2*b))/b^(1/2) + ((a - b)*(4*a*b - 12*a^2))/(4*a^(3/2)*b^(1/2)))/(a
+ b) + (a^(1/2)*(6*a - 2*b - (a - b)^2/b + (2*a*b - 6*a^2)/a))/(b^(1/2)*(a
+ b))) - (((a - b)*(16*a*b - (8*a^2*b + 8*a^3)/a + 16*a^2))/(4*a^(3/2)*b^(1
/2)) - ((16*a^2*b + 16*a^3)*(a - b))/(8*a^(5/2)*b^(1/2)))/(a + b) + (a^(1/2
)*(4*a + 4*b - (8*a*b - (8*a^2*b + 8*a^3)/(2*a) + 8*a^2)/a - ((a - b)^2*(a^
2*b + a^3))/(a^3*b)))/(b^(1/2)*(a + b))))/(a^2 - 2*a*b + b^2))*(a - b))/(2*
a^(3/2)*b^(1/2))
```

sympy [B] time = 1.60, size = 144, normalized size = 2.09

$$\left(-\frac{1}{2a} - \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2 + \frac{4ab\left(-\frac{1}{2a} - \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right) + a + b}{a-b}\right) + \left(-\frac{1}{2a} + \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2 + \frac{4ab\left(-\frac{1}{2a} + \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right) + a + b}{a-b}\right) + \frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(2*a) - sqrt(-a**3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a) -
 sqrt(-a**3*b)*(a - b)/(4*a**3*b)) + a + b)/(a - b)) + (-1/(2*a) + sqrt(-a*
*3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a) + sqrt(-a**3*b)*(a -
b)/(4*a**3*b)) + a + b)/(a - b)) + x**2/(2*a)
```

3.710 $\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=54

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 634, 618, 204, 628}

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a + b + 2*a*
x^2 + a*x^4]/(4*a)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a+b+2ax+ax^2}\,dx, x, x^2\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)\right)+\frac{\operatorname{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{4a}\\ &= \frac{\log\left(a+b+2ax^2+ax^4\right)}{4a}+\operatorname{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\log\left(a+b+2ax^2+ax^4\right)}{4a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.91

$$\frac{\log\left(a\left(x^2+1\right)^2+b\right)-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((-2*Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[b + a*(1 +
x^2)^2])/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b + 2*a*x^2 + a*x^4), x]
```

fricas [A] time = 0.73, size = 131, normalized size = 2.43

$$\left[\frac{b\log\left(ax^4+2\,ax^2+a+b\right)-\sqrt{-ab}\,\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}\left(x^2+1\right)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab},\ \frac{b\log\left(ax^4+2\,ax^2+a+b\right)+2\,\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{4\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(-a*b)*log((a*x^4 + 2*a*x^2 + 2*
sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 + a + b)))/(a*b), 1/4*(b*log
(a*x^4 + 2*a*x^2 + a + b) + 2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(a*b
)]
```

giac [A] time = 0.23, size = 42, normalized size = 0.78

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}}+\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*log(a*x^4 + 2*a*x^2 + a
+ b)/a
```

maple [A] time = 0.00, size = 47, normalized size = 0.87

$$-\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{\ln\left(a\,x^4+2a\,x^2+a+b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/4/a*ln(a*x^4+2*a*x^2+a+b)-1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^
(1/2))
```

maxima [A] time = 2.87, size = 42, normalized size = 0.78

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}}+\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*log(a*x^4 + 2*a*x^2 + a
+ b)/a
```

mupad [B] time = 0.09, size = 85, normalized size = 1.57

$$\frac{\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{4\,a}-\frac{\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}}{a+b}+\frac{a^{3/2}}{\sqrt{b}\,\left(a+b\right)}+\frac{\sqrt{a}\,\sqrt{b}\,x^2}{a+b}+\frac{a^{3/2}\,x^2}{\sqrt{b}\,\left(a+b\right)}\right)}{2\,\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] log(a + b + 2*a*x^2 + a*x^4)/(4*a) - atan((a^(1/2)*b^(1/2))/(a + b) + a^(3/
2)/(b^(1/2)*(a + b)) + (a^(1/2)*b^(1/2)*x^2)/(a + b) + (a^(3/2)*x^2)/(b^(1/
2)*(a + b)))/(2*a^(1/2)*b^(1/2))
```

sympy [B] time = 0.60, size = 117, normalized size = 2.17

$$\left(\frac{1}{4a}-\frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2+\frac{-4ab\left(\frac{1}{4a}-\frac{\sqrt{-a^3b}}{4a^2b}\right)+a+b}{a}\right)+\left(\frac{1}{4a}+\frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2+\frac{-4ab\left(\frac{1}{4a}+\frac{\sqrt{-a^3b}}{4a^2b}\right)+a+b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (1/(4*a) - sqrt(-a**3*b)/(4*a**2*b))*log(x**2 + (-4*a*b*(1/(4*a) - sqrt(-a*
*3*b)/(4*a**2*b)) + a + b)/a) + (1/(4*a) + sqrt(-a**3*b)/(4*a**2*b))*log(x*
*2 + (-4*a*b*(1/(4*a) + sqrt(-a**3*b)/(4*a**2*b)) + a + b)/a)
```

3.711 $\int \frac{x}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b + 2*a*x^2 + a*x^4),x]
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b + 2*a*x^2 + a*x^4),x]
[Out] IntegrateAlgebraic[x/(a + b + 2*a*x^2 + a*x^4), x]
```

fricas [A] time = 0.88, size = 91, normalized size = 2.94

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{-ab}\left(x^2+1\right)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab}, -\frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*
x^4 + 2*a*x^2 + a + b))/(a*b), -1/2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a))
/(a*b)]
```

giac [A] time = 0.24, size = 21, normalized size = 0.68

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.00, size = 26, normalized size = 0.84

$$\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

maxima [A] time = 2.94, size = 21, normalized size = 0.68

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 0.05, size = 24, normalized size = 0.77

$$\frac{\mathrm{atan}\left(\frac{\sqrt{a}+\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] atan((a^(1/2) + a^(1/2)*x^2)/b^(1/2))/(2*a^(1/2)*b^(1/2))
```

sympy [B] time = 0.46, size = 60, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}+\frac{\sqrt{-\frac{1}{ab}}\log\left(b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-b*sqrt(-1/(a*b)) + x**2 + 1)/4 + sqrt(-1/(a*b))*log(b*
sqrt(-1/(a*b)) + x**2 + 1)/4
```

3.712 $\int \frac{1}{x(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4(a+b)}+\frac{\log(x)}{a+b}$$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 705, 29, 634, 618, 204, 628}

$$-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4(a+b)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)}+\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*Sqrt[b]*(a + b)) + Log[x]
/(a + b) - Log[a + b + 2*a*x^2 + a*x^4]/(4*(a + b))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+b+2ax^2+ax^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a+b+2ax+ax^2\right)}\,dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2(a+b)} + \frac{\operatorname{Subst}\left(\int \frac{-2a-ax}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{2(a+b)}\\
&= \frac{\log(x)}{a+b} - \frac{\operatorname{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{4(a+b)} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{2(a+b)}\\
&= \frac{\log(x)}{a+b} - \frac{\log\left(a+b+2ax^2+ax^4\right)}{4(a+b)} + \frac{a\operatorname{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)}{a+b}\\
&= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)} + \frac{\log(x)}{a+b} - \frac{\log\left(a+b+2ax^2+ax^4\right)}{4(a+b)}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 105, normalized size = 1.52

$$\frac{i\left(\sqrt{a}+i\sqrt{b}\right)\log\left(\sqrt{a}\left(x^2+1\right)-i\sqrt{b}\right)+\left(-\sqrt{b}-i\sqrt{a}\right)\log\left(\sqrt{a}\left(x^2+1\right)+i\sqrt{b}\right)+4\sqrt{b}\log(x)}{4\sqrt{b}\,(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (4*Sqrt[b]*Log[x] + I*(Sqrt[a] + I*Sqrt[b])*Log[(-I)*Sqrt[b] + Sqrt[a]*(1 +
 x^2)] + ((-I)*Sqrt[a] - Sqrt[b])*Log[I*Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*Sq
rt[b]*(a + b))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b+2ax^2+ax^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b + 2*a*x^2 + a*x^4)), x]
```

fricas [A] time = 0.58, size = 147, normalized size = 2.13

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-\log\left(ax^4+2\,ax^2+a+b\right)+4\,\log(x)}{4\,(a+b)},\ \frac{2\,\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{ax^2+a}\right)-\log\left(ax^4+2\,ax^2+a+b\right)+4\,\log(x)}{4\,(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/b) + a - b)/(
a*x^4 + 2*a*x^2 + a + b)) - log(a*x^4 + 2*a*x^2 + a + b) + 4*log(x))/(a + b
), 1/4*(2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*x^2 + a)) - log(a*x^4 + 2*a*x^2 +
 a + b) + 4*log(x))/(a + b)]
```

giac [A] time = 0.23, size = 61, normalized size = 0.88

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,(a+b)}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,(a+b)}+\frac{\log\left(x^2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*log(a*x^4 +
2*a*x^2 + a + b)/(a + b) + 1/2*log(x^2)/(a + b)
```

maple [A] time = 0.01, size = 63, normalized size = 0.91

$$-\frac{a \arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\,(a+b)\sqrt{ab}}+\frac{\ln(x)}{a+b}-\frac{\ln\left(a\,x^4+2a\,x^2+a+b\right)}{4\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/4*ln(a*x^4+2*a*x^2+a+b)/(a+b)-1/2*a/(a+b)/(a*b)^(1/2)*arctan(1/2*(2*a*x^
2+2*a)/(a*b)^(1/2))+ln(x)/(a+b)
```

maxima [A] time = 3.01, size = 61, normalized size = 0.88

$$-\frac{a \arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(a+b)}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,(a+b)}+\frac{\log\left(x^2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*log(a*x^4 +
2*a*x^2 + a + b)/(a + b) + 1/2*log(x^2)/(a + b)
```

mupad [B] time = 4.64, size = 71, normalized size = 1.03

$$\frac{\ln(x)}{a+b}-\frac{4\,b\,\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{16\,b^2+16\,a\,b}-\frac{\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\,\sqrt{b}\,\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] log(x)/(a + b) - (4*b*log(a + b + 2*a*x^2 + a*x^4))/(16*a*b + 16*b^2) - (a^
(1/2)*atan(a^(1/2)/b^(1/2) + (a^(1/2)*x^2)/b^(1/2)))/(2*b^(1/2)*(a + b))
```

sympy [B] time = 5.96, size = 194, normalized size = 2.81

$$\left(-\frac{1}{4\,(a+b)}-\frac{\sqrt{-ab}}{4b\,(a+b)}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)+a-4b^2\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)-b}{a}\right)+\left(-\frac{1}{4\,(a+b)}+\frac{\sqrt{-ab}}{4b\,(a+b)}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{4(a+b)}+\frac{\sqrt{-ab}}{4b(a+b)}\right)+a-4b^2\left(-\frac{1}{4(a+b)}+\frac{\sqrt{-ab}}{4b(a+b)}\right)-b}{a}\right)+\frac{\log(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(4*(a + b)) - sqrt(-a*b)/(4*b*(a + b)))*log(x**2 + (-4*a*b*(-1/(4*(a +
b)) - sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4*(a + b)) - sqrt(-a*b)/(
4*b*(a + b))) - b)/a) + (-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b)))*log(x**
2 + (-4*a*b*(-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4
*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) - b)/a) + log(x)/(a + b)
```

3.713 $\int \frac{1}{x^3(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2(a+b)}+\frac{\sqrt{a}\,(a-b)\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)^2}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}$$

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{2x^2(a+b)}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2}+\frac{\sqrt{a}\,(a-b)\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a + b)*x^2) + (Sqrt[a]*(a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/
(2*Sqrt[b]*(a + b)^2) - (2*a*Log[x])/(a + b)^2 + (a*Log[a + b + 2*a*x^2 + a
*x^4])/(2*(a + b)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+b+2ax^2+ax^4\right)} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^2 \left(a+b+2ax+ax^2\right)} \, dx, x, x^2\right)$$

$$= -\frac{1}{2(a+b)x^2} + \frac{\text{Subst}\left(\int \frac{-2a-ax}{x\left(a+b+2ax+ax^2\right)} \, dx, x, x^2\right)}{2(a+b)}$$

$$= -\frac{1}{2(a+b)x^2} + \frac{\text{Subst}\left(\int \left(-\frac{2a}{(a+b)x} + \frac{a(3a-b+2ax)}{(a+b)\left(a+b+2ax+ax^2\right)}\right) dx, x, x^2\right)}{2(a+b)}$$

$$= -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\,\text{Subst}\left(\int \frac{3a-b+2ax}{a+b+2ax+ax^2} \, dx, x, x^2\right)}{2(a+b)^2}$$

$$= -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\,\text{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2} \, dx, x, x^2\right)}{2(a+b)^2} + \frac{(a(a-b))\,\text{Subst}\left(\right.}{2}$$

$$= -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\log\left(a+b+2ax^2+ax^4\right)}{2(a+b)^2} - \frac{(a(a-b))\,\text{Subst}\left(\int \frac{}{-4a}\right.}{(a+}$$

$$= -\frac{1}{2(a+b)x^2} + \frac{\sqrt{a}\,(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\log\left(a+b+2ax^2+\right.}{2(a+b)^2}$$

Mathematica [C] time = 0.10, size = 163, normalized size = 1.83

$$\frac{\left(2a^{3/2}\sqrt{b}-ia^2+iab\right)\log\left(\sqrt{a}\,x^2+\sqrt{a}-i\sqrt{b}\right)}{4\sqrt{a}\,\sqrt{b}\,(a+b)^2} + \frac{\left(2a^{3/2}\sqrt{b}+ia^2-iab\right)\log\left(\sqrt{a}\,x^2+\sqrt{a}+i\sqrt{b}\right)}{4\sqrt{a}\,\sqrt{b}\,(a+b)^2} - \frac{1}{2x^2(a+b)} - \frac{2a\log(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/2*1/((a + b)*x^2) - (2*a*Log[x])/(a + b)^2 + (((-I)*a^2 + 2*a^(3/2)*Sqrt
[b] + I*a*b)*Log[Sqrt[a] - I*Sqrt[b] + Sqrt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a
+ b)^2) + ((I*a^2 + 2*a^(3/2)*Sqrt[b] - I*a*b)*Log[Sqrt[a] + I*Sqrt[b] + Sq
rt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a + b)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a+b+2ax^2+ax^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b + 2*a*x^2 + a*x^4)), x]
```

fricas [A] time = 1.07, size = 208, normalized size = 2.34

$$\left[-\frac{(a-b)x^2\sqrt{-\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-2\,ax^2\log\left(ax^4+2\,ax^2+a+b\right)+8\,ax^2\log(x)+2\,a+2\,b}{4\left(a^2+2\,ab+b^2\right)x^2},-\frac{(a-b)x^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{ax^2+a}\right)-ax^2\log\left(ax^4+2\,ax^2+a+b\right)+4\,ax^2\log(x)+a+b}{2\left(a^2+2\,ab+b^2\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - b)*x^2*sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/
b) + a - b)/(a*x^4 + 2*a*x^2 + a + b)) - 2*a*x^2*log(a*x^4 + 2*a*x^2 + a +
b) + 8*a*x^2*log(x) + 2*a + 2*b)/((a^2 + 2*a*b + b^2)*x^2), -1/2*((a - b)*x
^2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*x^2 + a)) - a*x^2*log(a*x^4 + 2*a*x^2 +
a + b) + 4*a*x^2*log(x) + a + b)/((a^2 + 2*a*b + b^2)*x^2)]
```

giac [A] time = 0.28, size = 125, normalized size = 1.40

$$\frac{a\log\left(ax^4+2\,ax^2+a+b\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2+2\,ab+b^2}+\frac{\left(a^2-ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{ab}}+\frac{2\,ax^2-a-b}{2\left(a^2+2\,ab+b^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*log(x^2)/(a^2 +
2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*b))/((a^2 + 2*a*b
+ b^2)*sqrt(a*b)) + 1/2*(2*a*x^2 - a - b)/((a^2 + 2*a*b + b^2)*x^2)
```

maple [A] time = 0.01, size = 110, normalized size = 1.24

$$\frac{a^2\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\,(a+b)^2\sqrt{ab}}-\frac{ab\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\,(a+b)^2\sqrt{ab}}-\frac{2a\ln(x)}{(a+b)^2}+\frac{a\ln\left(a\,x^4+2a\,x^2+a+b\right)}{2\,(a+b)^2}-\frac{1}{2\,(a+b)\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2*a*ln(a*x^4+2*a*x^2+a+b)/(a+b)^2+1/2/(a+b)^2*a^2/(a*b)^(1/2)*arctan(1/2*
(2*a*x^2+2*a)/(a*b)^(1/2))-1/2/(a+b)^2*a/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*
a)/(a*b)^(1/2))*b-1/2/(a+b)/x^2-2*a*ln(x)/(a+b)^2
```

maxima [A] time = 2.90, size = 104, normalized size = 1.17

$$\frac{a\log\left(ax^4+2\,ax^2+a+b\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2+2\,ab+b^2}+\frac{\left(a^2-ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{ab}}-\frac{1}{2\,(a+b)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*log(x^2)/(a^2 +
2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*b))/((a^2 + 2*a*b
+ b^2)*sqrt(a*b)) - 1/2/((a + b)*x^2)
```

mupad [B] time = 7.39, size = 3313, normalized size = 37.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] (8*a*b*log(((2*a^5)/(a + b)^3 - (a/(2*(a + b)^2) - (-(a*(a - b)^2)/(b*(a +
b)^4))^(1/2)/4)*((12*a^5*x^2)/(a + b)^2 - (a/(2*(a + b)^2) - (-(a*(a - b)^2
)/(b*(a + b)^4))^(1/2)/4)*((8*a^4*(3*a - b))/(a + b) + 16*a^4*(a/(2*(a + b)
^2) - (-(a*(a - b)^2)/(b*(a + b)^4))^(1/2)/4)*(a + b + a*x^2 - 5*b*x^2) + (
4*a^4*x^2*(7*a + 5*b))/(a + b)) + (a^4*(15*a - b))/(a + b)^2) + (a^5*x^2)/(
a + b)^3)*((2*a^5)/(a + b)^3 - (a/(2*(a + b)^2) + (-(a*(a - b)^2)/(b*(a + b
)^4))^(1/2)/4)*((12*a^5*x^2)/(a + b)^2 - (a/(2*(a + b)^2) + (-(a*(a - b)^2)
/(b*(a + b)^4))^(1/2)/4)*((8*a^4*(3*a - b))/(a + b) + 16*a^4*(a/(2*(a + b)^
2) + (-(a*(a - b)^2)/(b*(a + b)^4))^(1/2)/4)*(a + b + a*x^2 - 5*b*x^2) + (4
*a^4*x^2*(7*a + 5*b))/(a + b)) + (a^4*(15*a - b))/(a + b)^2) + (a^5*x^2)/(a
 + b)^3)))/(32*a*b^2 + 16*a^2*b + 16*b^3) - (2*a*log(x))/(2*a*b + a^2 + b^2
) - 1/(2*x^2*(a + b)) + (a^(1/2)*atan(((13*a^2 - 34*a*b + b^2)*((8*a*b*((14
*a^5*b + 15*a^6 - a^4*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*((40*a^
6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) + (8*
a*b*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/((32*a*b^2
+ 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(32*a*b^2 + 16*a^2*
b + 16*b^3)))/(32*a*b^2 + 16*a^2*b + 16*b^3) - (2*a^5)/(3*a*b^2 + 3*a^2*b +
 a^3 + b^3) + (a^(1/2)*((a^(1/2)*(a - b)*((40*a^6*b + 24*a^7 - 8*a^4*b^3 +
8*a^5*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + 16
*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b
^2 + 3*a^2*b + a^3 + b^3))))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (2*a^(3/2)*b
^(1/2)*(a - b)*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/
((2*a*b + a^2 + b^2)*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^
3 + b^3)))*(a - b))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (a^2*(a - b)^2*(64*a^
7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/(2*(2*a*b + a^2 + b^2
```

)^2*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*(24*a*
b^(13/2) + 4*b^(15/2) + 4*a^6*b^(3/2) + 24*a^5*b^(5/2) + 60*a^4*b^(7/2) + 8
0*a^3*b^(9/2) + 60*a^2*b^(11/2)))/((a + b)^3*(98*a*b + a^2 + b^2)*(a^(13/2)
 - 2*a^(11/2)*b + a^(9/2)*b^2)) - (x^2*(((13*a^2 - 34*a*b + b^2)*(a^5/(3*a*
b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*((12*a^5*b + 12*a^6)/(3*a*b^2 + 3*a^2*b
 + a^3 + b^3) - (8*a*b*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*b^2)/(3*a*
b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a
^5*b^3 + 192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b +
 a^3 + b^3))))/(32*a*b^2 + 16*a^2*b + 16*b^3)))/(32*a*b^2 + 16*a^2*b + 16*b
^3) - (a^(1/2)*((a^(1/2)*(a - b)*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*
b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b
^4 + 224*a^5*b^3 + 192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 +
 3*a^2*b + a^3 + b^3))))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) - (2*a^(3/2)*b^(1/
2)*(a - b)*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 + 192*a^6*b^2))/((
2*a*b + a^2 + b^2)*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3
 + b^3)))*(a - b))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (a^2*(a - b)^2*(32*a^7*
b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 + 192*a^6*b^2))/(2*(2*a*b + a^2 + b^2
)^2*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/((a +
 b)^3*(98*a*b + a^2 + b^2)) + (a^(1/2)*(a^2 - 34*a*b + 13*b^2)*((8*a*b*((a^
(1/2)*(a - b)*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*b^2)/(3*a*b^2 + 3*a
^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 +
192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^
3))))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) - (2*a^(3/2)*b^(1/2)*(a - b)*(32*a^7*
b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 + 192*a^6*b^2))/((2*a*b + a^2 + b^2)*
(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(32*a*b^2
 + 16*a^2*b + 16*b^3) - (a^(1/2)*(a - b)*((12*a^5*b + 12*a^6)/(3*a*b^2 + 3*
a^2*b + a^3 + b^3) - (8*a*b*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*b^2)/
(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b^4 +
224*a^5*b^3 + 192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^
2*b + a^3 + b^3))))/(32*a*b^2 + 16*a^2*b + 16*b^3)))/(4*b^(1/2)*(2*a*b + a^
2 + b^2)) + (a^(3/2)*(a - b)^3*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^
3 + 192*a^6*b^2))/(64*b^(3/2)*(2*a*b + a^2 + b^2)^3*(3*a*b^2 + 3*a^2*b + a^
3 + b^3))))/(b^(1/2)*(a + b)^3*(98*a*b + a^2 + b^2)))*(24*a*b^(13/2) + 4*b^
(15/2) + 4*a^6*b^(3/2) + 24*a^5*b^(5/2) + 60*a^4*b^(7/2) + 80*a^3*b^(9/2) +
 60*a^2*b^(11/2)))/(a^(13/2) - 2*a^(11/2)*b + a^(9/2)*b^2) + (a^(1/2)*((a^(
1/2)*(a - b)*((14*a^5*b + 15*a^6 - a^4*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3)
 - (8*a*b*((40*a^6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^2*b +
 a^3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*
b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(3
2*a*b^2 + 16*a^2*b + 16*b^3)))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) - (8*a*b*((a
^(1/2)*(a - b)*((40*a^6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^
2*b + a^3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96
*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (2*a^(3/2)*b^(1/2)*(a - b)*(64*a^7*b +
 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/((2*a*b + a^2 + b^2)*(32*a

```
*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(32*a*b^2 + 16
*a^2*b + 16*b^3) + (a^(3/2)*(a - b)^3*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*
a^5*b^3 + 96*a^6*b^2))/(64*b^(3/2)*(2*a*b + a^2 + b^2)^3*(3*a*b^2 + 3*a^2*b
 + a^3 + b^3)))*(a^2 - 34*a*b + 13*b^2)*(24*a*b^(13/2) + 4*b^(15/2) + 4*a^6
*b^(3/2) + 24*a^5*b^(5/2) + 60*a^4*b^(7/2) + 80*a^3*b^(9/2) + 60*a^2*b^(11/
2)))/(b^(1/2)*(a + b)^3*(98*a*b + a^2 + b^2)*(a^(13/2) - 2*a^(11/2)*b + a^(
9/2)*b^2)))*(a - b))/(2*b^(1/2)*(2*a*b + a^2 + b^2))
```

sympy [B] time = 41.75, size = 386, normalized size = 4.34

$$-\frac{2a\log(x)}{(a+b)^2}+\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}\,(a-b)}{4b\left(a^2+2ab+b^2\right)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)-3ab+4b^3\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)}{a^2-ab}\right)+\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}\,(a-b)}{4b\left(a^2+2ab+b^2\right)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)-3ab+4b^3\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)}{a^2-ab}\right)-\frac{1}{x^2(2a+2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -2*a*log(x)/(a + b)**2 + (a/(2*(a + b)**2) - sqrt(-a*b)*(a - b)/(4*b*(a**2
+ 2*a*b + b**2)))*log(x**2 + (4*a**2*b*(a/(2*(a + b)**2) - sqrt(-a*b)*(a -
b)/(4*b*(a**2 + 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) - sqrt(
-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**
2) - sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))))/(a**2 - a*b)) + (a/(2
*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2)))*log(x**2 + (
4*a**2*b*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))
) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a
*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(
a**2 + 2*a*b + b**2))))/(a**2 - a*b)) - 1/(x**2*(2*a + 2*b))
```

3.714 $\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=432

$$\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(\sqrt{2}\sqrt[4]{\right.}{4\sqrt{2}a^{5/4}\sqrt{a+}}$$

Rubi [A] time = 0.89, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1122, 1169, 634, 618, 204, 628}

$$\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((a + b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]]
 - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt
[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - ((a + b + 2*Sqrt[a]*Sqrt[a + b])*Arc
Tan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[
a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ((a
 + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[
a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[-Sq
rt[a] + Sqrt[a + b]]) - ((a + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] +
Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a
^(5/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx = \frac{x}{a} - \frac{\int \frac{a+b+2ax^2}{a+b+2ax^2+ax^4}\,dx}{a}$$

$$= \frac{x}{a} - \frac{\int \frac{\frac{\sqrt{2}(a+b)\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}} - \left(a+b-2\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}} - \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}} + x^2}\,dx}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\int \frac{\frac{\sqrt{2}(a+b)\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}} + \left(a+b-2\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}} + \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}} + x^2}\,dx}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{x}{a} + \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}} + 2x}{\frac{\sqrt{a+b}}{\sqrt{a}} - \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}} + x^2}\,dx}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\int \cdots}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\cdots}$$

$$= \frac{x}{a} + \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\log\left(\sqrt{a+b} - \sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x + \sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{(a\cdots}{\cdots}$$

$$= \frac{x}{a} + \frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}} - \sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\tan\cdots}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\cdots}$$

Mathematica [C] time = 0.10, size = 164, normalized size = 0.38

$$-\frac{i\left(\sqrt{a}-i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}} + \frac{i\left(\sqrt{a}+i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}} + \frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a - ((I/2)*(Sqrt[a] - I*Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*
Sqrt[b]]])/(a*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*(Sqrt[a] + I*Sq
rt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a + I*Sqr
t[a]*Sqrt[b]]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b + 2*a*x^2 + a*x^4), x]
```

fricas [A] time = 2.05, size = 615, normalized size = 1.42

$$\frac{a\sqrt{\frac{a^{2}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}+a-3b}{a^{2}b}}\log\left(-\left(3a^{2}+2ab-b^{2}\right)x+\left(a^{4}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}+3a^{2}b-ab^{2}\right)\sqrt{\frac{a^{2}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}+a-3b}{a^{2}b}}\right)-a\sqrt{\frac{a^{2}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}+a-3b}{a^{2}b}}\log\left(-\left(3a^{2}+2ab-b^{2}\right)x-\left(a^{4}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}+3a^{2}b-ab^{2}\right)\sqrt{\frac{a^{2}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}+a-3b}{a^{2}b}}\right)-a\sqrt{-\frac{a^{2}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}-a+3b}{a^{2}b}}\log\left(-\left(3a^{2}+2ab-b^{2}\right)x+\left(a^{4}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}-3a^{2}b+ab^{2}\right)\sqrt{-\frac{a^{2}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}-a+3b}{a^{2}b}}\right)+a\sqrt{-\frac{a^{2}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}-a+3b}{a^{2}b}}\log\left(-\left(3a^{2}+2ab-b^{2}\right)x-\left(a^{4}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}-3a^{2}b+ab^{2}\right)\sqrt{-\frac{a^{2}b\sqrt{-\frac{9a^{2}-6ab+b^{2}}{a^{5}b}}-a+3b}{a^{2}b}}\right)+4x}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3*b)/(a^2*b))
*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b))
 + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a -
3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3
*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4*b*sqrt(-(9*a^2 - 6*a*b + b
^2)/(a^5*b)) + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^
5*b)) + a - 3*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^
5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^
2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*
a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*
a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4
*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqr
t(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + 4*x)/a
```

giac [A] time = 0.35, size = 533, normalized size = 1.23

$$\frac{\left(3\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,a^{4}+\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,a^{3}b-4\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,a^{2}b^{2}+2\left(3\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,ab+4\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,b^{2}\right)a^{2}-\left(3\sqrt{a^{2}+\sqrt{-ab}\,a}\,a^{3}b+7\sqrt{a^{2}+\sqrt{-ab}\,a}\,a^{2}b^{2}+4\sqrt{a^{2}+\sqrt{-ab}\,a}\,ab^{3}\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^{2}+\sqrt{a^{4}-(a^{2}+ab)a^{2}}}{a^{2}}}}\right)}{2\left(3a^{6}b+7a^{5}b^{2}+4a^{4}b^{3}\right)}-\frac{\left(3\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,a^{4}+\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,a^{3}b-4\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,a^{2}b^{2}+2\left(3\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,ab+4\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,b^{2}\right)a^{2}+\left(3\sqrt{a^{2}-\sqrt{-ab}\,a}\,a^{3}b+7\sqrt{a^{2}-\sqrt{-ab}\,a}\,a^{2}b^{2}+4\sqrt{a^{2}-\sqrt{-ab}\,a}\,ab^{3}\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^{2}-\sqrt{a^{4}-(a^{2}+ab)a^{2}}}{a^{2}}}}\right)}{2\left(3a^{6}b+7a^{5}b^{2}+4a^{4}b^{3}\right)}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + sqrt(a^2 + sqrt(-a*b)*a)*s
qrt(-a*b)*a^3*b - 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 2*(3*sqrt
(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)
*b^2)*a^2 - (3*sqrt(a^2 + sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 + sqrt(-a*b)*a)*
a^2*b^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 + sq
```

```
rt(a^4 - (a^2 + a*b)*a^2))/a^2))/(3*a^6*b + 7*a^5*b^2 + 4*a^4*b^3) - 1/2*(3
*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a
*b)*a^3*b - 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 2*(3*sqrt(a^2 -
 sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b^2)*
a^2 + (3*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^
2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - sqrt(a^4
 - (a^2 + a*b)*a^2))/a^2))/(3*a^6*b + 7*a^5*b^2 + 4*a^4*b^3) + x/a
```

maple [B] time = 0.14, size = 1658, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/a*x+1/8/a/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*
(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/a^2/b*ln(-x^2*a^(1/2)+x*(2*(a
*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(a^2+a*b)^(1/2)*(a+b)^(1/2)*(2*(a^2+a
*b)^(1/2)-2*a)^(1/2)-1/4/a^(3/2)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a
)^(1/2)-(a+b)^(1/2))*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^(1
/2)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+
a*b)^(1/2)-2*a)^(1/2)+1/a/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/
2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/
2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(a+b)^(1/2)-1/4/a/b/(4*a^(1/2)*(a+b)^(1/2)
-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^
(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1
/2)-2*a)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^2/b/(4*a^(1/
2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b
))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(
2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)
-2*a)^(1/2)+1/2/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/
2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/
2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1
/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/2/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a
*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2)
)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2
*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b
))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-
1/8/a^2/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(a^2+
a*b)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/4/a^(3/2)/b*ln(x^2*a
^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(a^2+a*b)^(1/2)*(2*(a^2
+a*b)^(1/2)-2*a)^(1/2)+1/4/a^(1/2)/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*
a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/a/(4*a^(1/2)*(a+b)^(1
/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a
)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(a+b)^(1/2)+1
```

```
/4/a/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2
)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)
+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2
*a)^(1/2)+1/4/a^2/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arc
tan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a
*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+
b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/2/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)
-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(
1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/
2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/2/a^(1/2)/b/(
4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(
a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1
/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{a} - \frac{\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,a^2 b + 4\sqrt{a^2+\sqrt{-ab}\,a}\,a b^2 - 3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2 - 4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab\right) a \arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^4 b + 4a^3 b^2\right)} + \frac{\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,a^2 b + 4\sqrt{a^2-\sqrt{-ab}\,a}\,a b^2 + 3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2 + 4\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab\right) a \arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^4 b + 4a^3 b^2\right)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] x/a - integrate((2*a*x^2 + a + b)/(a*x^4 + 2*a*x^2 + a + b), x)/a
```

mupad [B] time = 4.65, size = 1147, normalized size = 2.66

$$\frac{x}{a} + 2\,\mathrm{atanh}\left(\frac{24x\sqrt{-a^5b^3}\sqrt{\frac{1}{16ab}-\frac{3}{16a^2}+\frac{3\sqrt{-a^5b^3}}{16a^4b^2}-\frac{\sqrt{-a^5b^3}}{16a^5b}}}{\frac{6\sqrt{-a^5b^3}}{a}+4ab^2+6a^2b-2b^3-\frac{2b^2\sqrt{-a^5b^3}}{a^3}+\frac{4b\sqrt{-a^5b^3}}{a^2}} - \frac{8x\sqrt{-a^5b^3}\sqrt{\frac{1}{16ab}-\frac{3}{16a^2}+\frac{3\sqrt{-a^5b^3}}{16a^4b^2}-\frac{\sqrt{-a^5b^3}}{16a^5b}}}{\frac{4\sqrt{-a^5b^3}}{a}+\frac{6\sqrt{-a^5b^3}}{b}-2ab^2+4a^2b+6a^3-\frac{2b\sqrt{-a^5b^3}}{a^2}} - \frac{8ab^2x\sqrt{\frac{1}{16ab}-\frac{3}{16a^2}+\frac{3\sqrt{-a^5b^3}}{16a^4b^2}-\frac{\sqrt{-a^5b^3}}{16a^5b}}}{4ab+\frac{4\sqrt{-a^5b^3}}{a^2}+6a^2-2b^2+\frac{6\sqrt{-a^5b^3}}{ab}-\frac{2b\sqrt{-a^5b^3}}{a^3}} + \frac{24a^2bx\sqrt{\frac{1}{16ab}-\frac{3}{16a^2}+\frac{3\sqrt{-a^5b^3}}{16a^4b^2}-\frac{\sqrt{-a^5b^3}}{16a^5b}}}{4ab+\frac{4\sqrt{-a^5b^3}}{a^2}+6a^2-2b^2+\frac{6\sqrt{-a^5b^3}}{ab}-\frac{2b\sqrt{-a^5b^3}}{a^3}}\right)\sqrt{\frac{3a\sqrt{-a^5b^3}-b\sqrt{-a^5b^3}+a^4b-3a^3b^2}{16a^5b^2}} + 2\,\mathrm{atanh}\left(\frac{24x\sqrt{-a^5b^3}\sqrt{\frac{1}{16ab}-\frac{3}{16a^2}-\frac{3\sqrt{-a^5b^3}}{16a^4b^2}+\frac{\sqrt{-a^5b^3}}{16a^5b}}}{\frac{6\sqrt{-a^5b^3}}{a}-4ab^2-6a^2b+2b^3-\frac{2b^2\sqrt{-a^5b^3}}{a^3}+\frac{4b\sqrt{-a^5b^3}}{a^2}} - \frac{8x\sqrt{-a^5b^3}\sqrt{\frac{1}{16ab}-\frac{3}{16a^2}-\frac{3\sqrt{-a^5b^3}}{16a^4b^2}+\frac{\sqrt{-a^5b^3}}{16a^5b}}}{\frac{4\sqrt{-a^5b^3}}{a}+\frac{6\sqrt{-a^5b^3}}{b}+2ab^2-4a^2b-6a^3-\frac{2b\sqrt{-a^5b^3}}{a^2}} - \frac{8ab^2x\sqrt{\frac{1}{16ab}-\frac{3}{16a^2}-\frac{3\sqrt{-a^5b^3}}{16a^4b^2}+\frac{\sqrt{-a^5b^3}}{16a^5b}}}{4ab-\frac{4\sqrt{-a^5b^3}}{a^2}+6a^2-2b^2-\frac{6\sqrt{-a^5b^3}}{ab}+\frac{2b\sqrt{-a^5b^3}}{a^3}} + \frac{24a^2bx\sqrt{\frac{1}{16ab}-\frac{3}{16a^2}-\frac{3\sqrt{-a^5b^3}}{16a^4b^2}+\frac{\sqrt{-a^5b^3}}{16a^5b}}}{4ab-\frac{4\sqrt{-a^5b^3}}{a^2}+6a^2-2b^2-\frac{6\sqrt{-a^5b^3}}{ab}+\frac{2b\sqrt{-a^5b^3}}{a^3}}\right)\sqrt{-\frac{3a\sqrt{-a^5b^3}-b\sqrt{-a^5b^3}-a^4b+3a^3b^2}{16a^5b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x/a + 2*atanh((24*x*(-a^5*b^3)^(1/2)*(1/(16*a*b) - 3/(16*a^2) + (3*(-a^5*b^
3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((6*(-a^5*b^3)
^(1/2))/a + 4*a*b^2 + 6*a^2*b - 2*b^3 - (2*b^2*(-a^5*b^3)^(1/2))/a^3 + (4*b
*(-a^5*b^3)^(1/2))/a^2) - (8*x*(-a^5*b^3)^(1/2)*(1/(16*a*b) - 3/(16*a^2) +
(3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((4
*(-a^5*b^3)^(1/2))/a + (6*(-a^5*b^3)^(1/2))/b - 2*a*b^2 + 4*a^2*b + 6*a^3 -
 (2*b*(-a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(1/(16*a*b) - 3/(16*a^2) + (3*(-a
^5*b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b +
(4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 + (6*(-a^5*b^3)^(1/2))/(a*b) - (2*
b*(-a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*(1/(16*a*b) - 3/(16*a^2) + (3*(-a^5*
b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*
(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 + (6*(-a^5*b^3)^(1/2))/(a*b) - (2*b*(
-a^5*b^3)^(1/2))/a^3))*((3*a*(-a^5*b^3)^(1/2) - b*(-a^5*b^3)^(1/2) + a^4*b
```

```
- 3*a^3*b^2)/(16*a^5*b^2))^(1/2) + 2*atanh((24*x*(-a^5*b^3)^(1/2)*(1/(16*a*
b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)^(1/2)/(16*
a^5*b))^(1/2))/((6*(-a^5*b^3)^(1/2))/a - 4*a*b^2 - 6*a^2*b + 2*b^3 - (2*b^2
*(-a^5*b^3)^(1/2))/a^3 + (4*b*(-a^5*b^3)^(1/2))/a^2) - (8*x*(-a^5*b^3)^(1/2
)*(1/(16*a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)
^(1/2)/(16*a^5*b))^(1/2))/((4*(-a^5*b^3)^(1/2))/a + (6*(-a^5*b^3)^(1/2))/b
+ 2*a*b^2 - 4*a^2*b - 6*a^3 - (2*b*(-a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(1/(
16*a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)^(1/2)
/(16*a^5*b))^(1/2))/(4*a*b - (4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 - (6*
(-a^5*b^3)^(1/2))/(a*b) + (2*b*(-a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*(1/(16*
a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)^(1/2)/(1
6*a^5*b))^(1/2))/(4*a*b - (4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 - (6*(-a
^5*b^3)^(1/2))/(a*b) + (2*b*(-a^5*b^3)^(1/2))/a^3))*(-(3*a*(-a^5*b^3)^(1/2)
 - b*(-a^5*b^3)^(1/2) - a^4*b + 3*a^3*b^2)/(16*a^5*b^2))^(1/2)
```

sympy [A] time = 2.20, size = 105, normalized size = 0.24

$$\operatorname{RootSum}\left(256t^4a^5b^2 + t^2\left(-32a^4b + 96a^3b^2\right) + a^3 + 3a^2b + 3ab^2 + b^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^4b + 4ta^3 - 24ta^2b + 4tab^2}{3a^2 + 2ab - b^2}\right)\right)\right) + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(-32*a**4*b + 96*a**3*b**2) + a**3 + 3*
a**2*b + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b + 4*_t*a*
*3 - 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 + 2*a*b - b**2)))) + x/a
```

3.715 $\int \frac{x^2}{a+b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=331

$$\frac{\log \left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\log \left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\cdots\right.}{2\sqrt{2}a^{3/\cdots}}$$

Rubi [A] time = 0.26, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1129, 634, 618, 204, 628}

$$\frac{\log \left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\log \left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}+\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + S
qrt[a + b]]]/(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ArcTan[(Sqrt
[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]]/
(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt[a + b] - Sqrt[2]
*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*S
qrt[-Sqrt[a] + Sqrt[a + b]]) - Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt
[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] + Sqrt
[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1129

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = \frac{\int \frac{x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\int \frac{x}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4a} + \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4a} + \frac{\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+2}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}}}{4\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{}}}$$

$$= \frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\log\left(\sqrt{a+b}+\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}}\right.}{4\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\log\left(\sqrt{a+b}-\sqrt{2}\right.}{4\sqrt{2}\,a^{3}}$$

Mathematica [C] time = 0.12, size = 143, normalized size = 0.43

$$\frac{\frac{\left(\sqrt{b}+i\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{a}\,\sqrt{b}}}+\frac{\left(\sqrt{b}-i\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{a}\,\sqrt{b}}}}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((I*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/Sq
rt[a - I*Sqrt[a]*Sqrt[b]] + (((-I)*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sq
rt[a + I*Sqrt[a]*Sqrt[b]]])/Sqrt[a + I*Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b]
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b + 2*a*x^2 + a*x^4), x]
```

fricas [A] time = 1.65, size = 279, normalized size = 0.84

$$\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)+\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(-1/(a^3
*b)) + 1)/(a*b))*sqrt(-1/(a^3*b)) + x) - 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1
)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*sqrt(-1/(a^3*b))
 + x) - 1/4*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sq
rt(-1/(a^3*b)) - 1)/(a*b))*sqrt(-1/(a^3*b)) + x) + 1/4*sqrt(-(a*b*sqrt(-1/(
a^3*b)) - 1)/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*sqrt
(-1/(a^3*b)) + x)
```

giac [A] time = 0.34, size = 203, normalized size = 0.61

$$-\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a+4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^4b+4a^3b^2\right)}+\frac{\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a+4\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^4b+4a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a + 4*sqrt(a^2 + sqrt(-a*b)*a)*
sqrt(-a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a + b)*a + 4
*a^2))/a))/(3*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*
b)*a + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x
/sqrt((2*a - sqrt(-4*(a + b)*a + 4*a^2))/a))/(3*a^4*b + 4*a^3*b^2)
```

maple [B] time = 0.06, size = 724, normalized size = 2.19

$$-\frac{\sqrt{-2a+2\sqrt{(a+b)a}}\,\sqrt{-2a+2\sqrt{a^2+ab}}\,\arctan\left(\frac{-2\sqrt{a}\,x+\sqrt{-2a+2\sqrt{(a+b)a}}}{\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}}\right)}{4\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}\,\sqrt{a}\,b}+\frac{\sqrt{-2a+2\sqrt{(a+b)a}}\,\sqrt{-2a+2\sqrt{a^2+ab}}\,\arctan\left(\frac{2\sqrt{a}\,x+\sqrt{-2a+2\sqrt{(a+b)a}}}{\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}}\right)}{4\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}\,\sqrt{a}\,b}+\frac{\sqrt{-2a+2\sqrt{a^2+ab}}\,\ln\left(-\sqrt{a}\,x^2+\sqrt{-2a+2\sqrt{(a+b)a}}\,x-\sqrt{a+b}\right)}{8\sqrt{a}\,b}-\frac{\sqrt{-2a+2\sqrt{a^2+ab}}\,\ln\left(\sqrt{a}\,x^2+\sqrt{-2a+2\sqrt{(a+b)a}}\,x+\sqrt{a+b}\right)}{8\sqrt{a}\,b}-\frac{\sqrt{-2a+2\sqrt{(a+b)a}}\,\sqrt{a^2+ab}\,\sqrt{-2a+2\sqrt{a^2+ab}}\,\arctan\left(\frac{-2\sqrt{a}\,x+\sqrt{-2a+2\sqrt{(a+b)a}}}{\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}}\right)}{4\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}\,a^{\frac{3}{2}}b}+\frac{\sqrt{-2a+2\sqrt{(a+b)a}}\,\sqrt{a^2+ab}\,\sqrt{-2a+2\sqrt{a^2+ab}}\,\arctan\left(\frac{2\sqrt{a}\,x+\sqrt{-2a+2\sqrt{(a+b)a}}}{\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}}\right)}{4\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}\,a^{\frac{3}{2}}b}+\frac{\sqrt{a^2+ab}\,\sqrt{-2a+2\sqrt{a^2+ab}}\,\ln\left(-\sqrt{a}\,x^2+\sqrt{-2a+2\sqrt{(a+b)a}}\,x-\sqrt{a+b}\right)}{8a^{\frac{3}{2}}b}-\frac{\sqrt{a^2+ab}\,\sqrt{-2a+2\sqrt{a^2+ab}}\,\ln\left(\sqrt{a}\,x^2+\sqrt{-2a+2\sqrt{(a+b)a}}\,x+\sqrt{a+b}\right)}{8a^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/8*(a^2+a*b)^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(3/2)/b*ln(-a^(1/2)*x^
2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))-1/4/(2*a+4*(a+b)^(1/2)*a^(1
/2)-2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(a^2+a*b)^(1/2)
*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(3/2)/b*arctan((-2*a^(1/2)*x+(-2*a+2*((a+
b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/
8*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(1/2)/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a
)^(1/2))^(1/2)*x-(a+b)^(1/2))-1/4/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1
/2))^(1/2)*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^
(1/2)/b*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(
1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))-1/8*(a^2+a*b)^(1/2)*(-2*a+2*(a^2+a*b
)^(1/2))^(1/2)/a^(3/2)/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a
+b)^(1/2))+1/4/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*
((a+b)*a)^(1/2))^(1/2)*(a^2+a*b)^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(3/
2)/b*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)
*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))-1/8*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(1/
2)/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))+1/4/(2*a+
4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*((a+b)*a)^(1/2))^(1/
2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(1/2)/b*arctan((2*a^(1/2)*x+(-2*a+2*((a
+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ax^4 + 2\,ax^2 + a + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a + b), x)
```

mupad [B] time = 0.28, size = 222, normalized size = 0.67

$$-2\,\mathrm{atanh}\left(\frac{2\left(x\left(4\,a^2\,b-4\,a^3\right)+\frac{4\,a\,x\left(\sqrt{-a^3\,b^3}+a^2\,b\right)}{b}\right)\sqrt{\frac{\sqrt{-a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}}{2\,a^2+2\,b\,a}\right)\sqrt{\frac{\sqrt{-a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}-2\,\mathrm{atanh}\left(\frac{2\left(x\left(4\,a^2\,b-4\,a^3\right)-\frac{4\,a\,x\left(\sqrt{-a^3\,b^3}-a^2\,b\right)}{b}\right)\sqrt{-\frac{\sqrt{-a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}}{2\,a^2+2\,b\,a}\right)\sqrt{-\frac{\sqrt{-a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] - 2*atanh((2*(x*(4*a^2*b - 4*a^3) + (4*a*x*((-a^3*b^3)^(1/2) + a^2*b))/b)*(
((-a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2))/(2*a*b + 2*a^2))*(((-a^3*b^
3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - 2*atanh((2*(x*(4*a^2*b - 4*a^3) - (
4*a*x*((-a^3*b^3)^(1/2) - a^2*b))/b)*(-((-a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b
^2))^(1/2))/(2*a*b + 2*a^2))*(-((-a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/
2)
```

sympy [A] time = 0.83, size = 44, normalized size = 0.13

$$\mathrm{RootSum}\left(256t^4a^3b^2-32t^2a^2b+a+b,\left(t\mapsto t\log\left(64t^3a^2b-4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b + a + b, Lambda(_t, _t*log(64
*_t**3*a**2*b - 4*_t*a + x)))
```

3.716 $\int \frac{1}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=359

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan\cdots}{2\sqrt{2}\cdots}$$

Rubi [A] time = 0.26, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + S
qrt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) +
ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sq
rt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - L
og[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x
^2]/(4*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt
[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*
Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx = \frac{\int \frac{\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}-x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+x}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{a}\sqrt{a+b}} + \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{a}\sqrt{a+b}} - \frac{\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+\cdots}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+\cdots}}}{\sqrt[4]{a}}\cdots}}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}\cdots}}$$

$$= -\frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\log\left(\sqrt{a+b}+\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\cdots}\right.}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{\log\left(\sqrt{a+b}\cdots\right.}{4\sqrt{2}\cdots}$$

Mathematica [C] time = 0.07, size = 119, normalized size = 0.33

$$\frac{i\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\,\sqrt{b}}}\right)}{2\sqrt{b}\,\sqrt{a+i\sqrt{a}\,\sqrt{b}}}-\frac{i\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\,\sqrt{b}}}\right)}{2\sqrt{b}\,\sqrt{a-i\sqrt{a}\,\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ((-1/2*I)*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/(Sqrt[a - I*Sqrt
[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b
]]])/(Sqrt[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b + 2*a*x^2 + a*x^4)^(-1), x]
```

fricas [B] time = 1.12, size = 567, normalized size = 1.58

$$\frac{1}{4}\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(\left((a^2b+ab^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+b\right)\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}+x\right)-\frac{1}{4}\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(-\left((a^2b+ab^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+b\right)\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}+x\right)-\frac{1}{4}\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}\log\left(\left((a^2b+ab^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-b\right)\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}+x\right)+\frac{1}{4}\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}\log\left(-\left((a^2b+ab^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-b\right)\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)
)*log(((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*b
 + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4*s
qrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2))*log
(-((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*b + b
^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4*sqrt(
-((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))*log(((
a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*b + b^2)
*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x) + 1/4*sqrt(-((
a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))*log(-((a^
2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*b + b^2)*s
qrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x)
```

giac [A] time = 0.25, size = 307, normalized size = 0.86

$$\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,a^2b+4\sqrt{a^2+\sqrt{-ab}\,a}\,ab^2+3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2+4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^5b+7a^4b^2+4a^3b^3\right)}+\frac{\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,a^2b+4\sqrt{a^2-\sqrt{-ab}\,a}\,ab^2-3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2-4\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^5b+7a^4b^2+4a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*a^2*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b^2 +
3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt
(-a*b)*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a + b)*a + 4*a
^2))/a))/(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3) + 1/2*(3*sqrt(a^2 - sqrt(-a*b)*a
)*a^2*b + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^2 - 3*sqrt(a^2 - sqrt(-a*b)*a)*sqr
t(-a*b)*a^2 - 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b)*abs(a)*arctan(2*sq
rt(1/2)*x/sqrt((2*a - sqrt(-4*(a + b)*a + 4*a^2))/a))/(3*a^5*b + 7*a^4*b^2
+ 4*a^3*b^3)
```

maple [B] time = 0.08, size = 913, normalized size = 2.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/8/(a+b)^(1/2)/a/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)
^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/8/(a+b)^(1/2)/b*ln
(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*
b)^(1/2))^(1/2)-1/(a+b)^(1/2)/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))
^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1
/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/4/(a+b)^(1/2)/a/b/(2*a+4*(a+b)^(1/2
)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(
1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*(
(a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/4/(a
+b)^(1/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*
a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+
b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))
^(1/2)+1/8/(a+b)^(1/2)/a/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+
(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/8/(a+b)^(1/2)
/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^
2+a*b)^(1/2))^(1/2)+1/(a+b)^(1/2)/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1
/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)
^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))-1/4/(a+b)^(1/2)/a/b/(2*a+4*(a+b)^(
1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)
^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2
```

```
*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/4/
(a+b)^(1/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2
*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a
+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2)
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^4 + 2\,ax^2 + a + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] integrate(1/(a*x^4 + 2*a*x^2 + a + b), x)
```

mupad [B] time = 5.16, size = 986, normalized size = 2.75

$$2\,\mathrm{atanh}\left(\frac{8\,a^3\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+a\,b^3\right)}-\frac{\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}}}{\frac{2\,a^4\,b^2}{a^2\,b^2+a\,b^3}-\frac{2\,a^3\,b\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}-\frac{8\,a^5\,b^2\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+a\,b^3\right)}-\frac{\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\frac{2\,a^6\,b^4}{a^2\,b^2+a\,b^3}-\frac{2\,a^4\,b^4\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}-\frac{2\,a^5\,b^3\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}+\frac{8\,a^4\,b\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+a\,b^3\right)}-\frac{\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}}\,\sqrt{-a\,b^3}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\frac{2\,a^6\,b^4}{a^2\,b^2+a\,b^3}-\frac{2\,a^4\,b^4\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}-\frac{2\,a^5\,b^3\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}\right)\,\sqrt{\frac{a\,b-\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}}-2\,\mathrm{atanh}\left(\frac{8\,a^5\,b^2\,x\,\sqrt{\frac{\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}+\frac{a\,b}{16\,\left(a^2\,b^2+a\,b^3\right)}}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\frac{2\,a^6\,b^4}{a^2\,b^2+a\,b^3}+\frac{2\,a^4\,b^4\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}+\frac{2\,a^5\,b^3\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}-\frac{8\,a^3\,x\,\sqrt{\frac{\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}+\frac{a\,b}{16\,\left(a^2\,b^2+a\,b^3\right)}}}{\frac{2\,a^4\,b^2}{a^2\,b^2+a\,b^3}+\frac{2\,a^3\,b\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}+\frac{8\,a^4\,b\,x\,\sqrt{\frac{\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}+\frac{a\,b}{16\,\left(a^2\,b^2+a\,b^3\right)}}\,\sqrt{-a\,b^3}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\frac{2\,a^6\,b^4}{a^2\,b^2+a\,b^3}+\frac{2\,a^4\,b^4\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}+\frac{2\,a^5\,b^3\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}\right)\,\sqrt{\frac{a\,b+\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] 2*atanh((8*a^3*x*((a*b)/(16*(a*b^3 + a^2*b^2)) - (-a*b^3)^(1/2)/(16*(a*b^3
+ a^2*b^2)))^(1/2))/((2*a^4*b^2)/(a*b^3 + a^2*b^2) - (2*a^3*b*(-a*b^3)^(1/2
))/(a*b^3 + a^2*b^2)) - (8*a^5*b^2*x*((a*b)/(16*(a*b^3 + a^2*b^2)) - (-a*b^
3)^(1/2)/(16*(a*b^3 + a^2*b^2)))^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2
*a^6*b^4)/(a*b^3 + a^2*b^2) - (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)
- (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)) + (8*a^4*b*x*((a*b)/(16*(a*
b^3 + a^2*b^2)) - (-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)))^(1/2)*(-a*b^3)^(1/
2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2*b^2) - (2*a^4
*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) - (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3
+ a^2*b^2)))*((a*b - (-a*b^3)^(1/2))/(16*(a*b^3 + a^2*b^2)))^(1/2) - 2*atan
h((8*a^5*b^2*x*((-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)) + (a*b)/(16*(a*b^3 +
a^2*b^2)))^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2
*b^2) + (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) + (2*a^5*b^3*(-a*b^3)^
(1/2))/(a*b^3 + a^2*b^2)) - (8*a^3*x*((-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2))
 + (a*b)/(16*(a*b^3 + a^2*b^2)))^(1/2))/((2*a^4*b^2)/(a*b^3 + a^2*b^2) + (2
*a^3*b*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)) + (8*a^4*b*x*((-a*b^3)^(1/2)/(16*
(a*b^3 + a^2*b^2)) + (a*b)/(16*(a*b^3 + a^2*b^2)))^(1/2)*(-a*b^3)^(1/2))/((
2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2*b^2) + (2*a^4*b^4*(
-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) + (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3 + a^2*
b^2)))*((a*b + (-a*b^3)^(1/2))/(16*(a*b^3 + a^2*b^2)))^(1/2)
```

sympy [A] time = 1.24, size = 63, normalized size = 0.18

$$\mathrm{RootSum}\left(t^4\left(256a^2b^2+256ab^3\right)-32t^2ab+1,\left(t\mapsto t\log\left(64t^3a^2b+64t^3ab^2-4ta+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 + 256*a*b**3) - 32*_t**2*a*b + 1, Lambda(_t, _
t*log(64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a + 4*_t*b + x)))
```

3.717 $\int \frac{1}{x^2(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=433

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\cdots}\right.}{4\sqrt{2}(a+b)^{3/2}\sqrt{\cdots}}$$

Rubi [A] time = 0.52, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1123, 1169, 634, 618, 204, 628}

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{1}{x(a+b)}+\frac{\sqrt[4]{a}\left(\sqrt{a+b}+2\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{\sqrt[4]{a}\left(\sqrt{a+b}+2\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}+\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(1/((a + b)*x)) + (a^(1/4)*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a]
 + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[
2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] + Sqrt[
a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[a + b]]])/(2*Sqrt[2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]])
+ (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt
[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*(a + b)^(3/2)*Sqrt[-S
qrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b]
 + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2
]*(a + b)^(3/2)*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b+2ax^2+ax^4\right)}\,dx = -\frac{1}{(a+b)x} + \frac{\int \frac{-2a-ax^2}{a+b+2ax^2+ax^4}\,dx}{a+b}$$

$$= -\frac{1}{(a+b)x} + \frac{\int \frac{-2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}-\left(-2a+\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\int \frac{-2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}-}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+}}{\sqrt[4]{a}}}}{2\sqrt{2}\sqrt[4]{a}\,(a+b)^{3/2}\sqrt{}}$$

$$= -\frac{1}{(a+b)x} + \frac{\left(\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\right)\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+2x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\left(\sqrt[4]{a}\left(2\sqrt{a}\right.\right.}{4}$$

$$= -\frac{1}{(a+b)x} + \frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\right.}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= -\frac{1}{(a+b)x} + \frac{\sqrt[4]{a}\left(2\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,(a+b)^{3/2}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{\sqrt[4]{a}\left(2\sqrt{a}+\right.}{2\sqrt{}}$$

Mathematica [C] time = 0.15, size = 174, normalized size = 0.40

$$\frac{1}{x(-a-b)} + \frac{\left(-\sqrt{a}\sqrt{b}+ia\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}\,(a+b)} + \frac{\left(-\sqrt{a}\sqrt{b}-ia\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}\,(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] 1/((-a - b)*x) + ((I*a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqr
t[a]*Sqrt[b]]])/(2*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b)) + (((-I)*a
- Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(2*Sqrt
[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a+b+2ax^2+ax^4\right)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b + 2*a*x^2 + a*x^4)), x]
```

fricas [B] time = 0.76, size = 1582, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*((a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt
(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 +
15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*log(-(3*
a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*sqr
t(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 +
 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3
 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4
)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*
b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*lo
g(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^
5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3
*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2
 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a
^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4))) + (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 2
0*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4))*log(-(3*a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 - (a^4*b + 2*a^3*b^2 - 2*a*b^
4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 +
20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b - (a^3*b + 3*a
^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2
+ 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^
2 + 3*a*b^3 + b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3
*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b
^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4))*log(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 - (a^4*b + 2*a^3*b^2 -
```

```
2*a*b^4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*
b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b - (a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^
5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*
a^2*b^2 + 3*a*b^3 + b^4))) - 4)/((a + b)*x)
```

giac [B] time = 0.38, size = 742, normalized size = 1.71

$$\frac{\left(\left(3\sqrt{a^2-\sqrt{-ab}\,a}\sqrt{-ab}\,ab+4\sqrt{a^2-\sqrt{-ab}\,a}\sqrt{-ab}\,b^2\right)(a+b)^2|a|-2\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,a^3b+7\sqrt{a^2-\sqrt{-ab}\,a}\,a^2b^2+4\sqrt{a^2-\sqrt{-ab}\,a}\,ab^3\right)|a||-a-b|-\left(3\sqrt{a^2-\sqrt{-ab}\,a}\sqrt{-ab}\,a^4+10\sqrt{a^2-\sqrt{-ab}\,a}\sqrt{-ab}\,a^3b+11\sqrt{a^2-\sqrt{-ab}\,a}\sqrt{-ab}\,a^2b^2+4\sqrt{a^2-\sqrt{-ab}\,a}\sqrt{-ab}\,ab^3\right)|a|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{2a^2+2ab+\sqrt{-4(a^2+2ab+b^2)(a^2+ab)+4(a^2+ab)^2}}{a^2+ab}}}\right)}{2\left(3a^6b+13a^5b^2+21a^4b^3+15a^3b^4+4a^2b^5\right)|-a-b|}-\frac{\left(\left(3\sqrt{a^2+\sqrt{-ab}\,a}\sqrt{-ab}\,ab+4\sqrt{a^2+\sqrt{-ab}\,a}\sqrt{-ab}\,b^2\right)(a+b)^2|a|+2\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,a^3b+7\sqrt{a^2+\sqrt{-ab}\,a}\,a^2b^2+4\sqrt{a^2+\sqrt{-ab}\,a}\,ab^3\right)|a||-a-b|-\left(3\sqrt{a^2+\sqrt{-ab}\,a}\sqrt{-ab}\,a^4+10\sqrt{a^2+\sqrt{-ab}\,a}\sqrt{-ab}\,a^3b+11\sqrt{a^2+\sqrt{-ab}\,a}\sqrt{-ab}\,a^2b^2+4\sqrt{a^2+\sqrt{-ab}\,a}\sqrt{-ab}\,ab^3\right)|a|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{2a^2+2ab-\sqrt{-4(a^2+2ab+b^2)(a^2+ab)+4(a^2+ab)^2}}{a^2+ab}}}\right)}{2\left(3a^6b+13a^5b^2+21a^4b^3+15a^3b^4+4a^2b^5\right)|-a-b|}-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 - sqrt(-a*b)*a
)*sqrt(-a*b)*b^2)*(a + b)^2*abs(a) - 2*(3*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b +
7*sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3)*abs(
a)*abs(-a - b) - (3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + 10*sqrt(a^2 -
 sqrt(-a*b)*a)*sqrt(-a*b)*a^3*b + 11*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^
2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b^3)*abs(a))*arctan(2*sqrt(
1/2)*x/sqrt((2*a^2 + 2*a*b + sqrt(-4*(a^2 + 2*a*b + b^2)*(a^2 + a*b) + 4*(a
^2 + a*b)^2))/(a^2 + a*b)))/((3*a^6*b + 13*a^5*b^2 + 21*a^4*b^3 + 15*a^3*b^
4 + 4*a^2*b^5)*abs(-a - b)) - 1/2*((3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a
*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*b^2)*(a + b)^2*abs(a) + 2*(3*sqr
t(a^2 + sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 + sqrt(-a*b)*a)*a^2*b^2 + 4*sqrt(a
^2 + sqrt(-a*b)*a)*a*b^3)*abs(a)*abs(-a - b) - (3*sqrt(a^2 + sqrt(-a*b)*a)*
sqrt(-a*b)*a^4 + 10*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^3*b + 11*sqrt(a^2
 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)
*a*b^3)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt((2*a^2 + 2*a*b - sqrt(-4*(a^2 + 2
*a*b + b^2)*(a^2 + a*b) + 4*(a^2 + a*b)^2))/(a^2 + a*b)))/((3*a^6*b + 13*a^
5*b^2 + 21*a^4*b^3 + 15*a^3*b^4 + 4*a^2*b^5)*abs(-a - b)) - 1/((a + b)*x)
```

maple [B] time = 0.07, size = 3318, normalized size = 7.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/4*a^(3/2)/(a+b)^2/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*a
rctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1
/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a
*b)^(1/2))^(1/2)-1/2*a^2/(a+b)^(5/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*
a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4
*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/
```

2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/4/a^(1/2)/(a+b)^2/(2*a+4*(a+b)^(1/2)*a^
(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))
^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)
a)^(1/2))^(1/2)(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/2*a^2/(a+
b)^(5/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^
(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)
a)^(1/2))^(1/2))(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(
1/2)-1/4*a^(3/2)/(a+b)^2/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1
/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*
a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a
^2+a*b)^(1/2))^(1/2)+1/4/a^(1/2)/(a+b)^2/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b
)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a
+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(
1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/4/(a+b)^(5/2)*ln(a^(1
/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/
2))^(1/2)*(a^2+a*b)^(1/2)-1/8*a^(1/2)/(a+b)^2*ln(-a^(1/2)*x^2+(-2*a+2*((a+b
)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+1/8*a^(1/2)
/(a+b)^2*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a
+2*(a^2+a*b)^(1/2))^(1/2)-1/4*a/(a+b)^(5/2)*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a
)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-2*a^2/(a+b)^(5
/2)/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x
+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/
2))^(1/2))+1/4*a/(a+b)^(5/2)*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)
x-(a+b)^(1/2))(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+2*a^2/(a+b)^(5/2)/(2*a+4*(a+
b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+
b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/
4/(a+b)^(5/2)*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))
*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/(a+b)/x-1/2*a/(a+b)^(5/2)
/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x
+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/
2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a
^2+a*b)^(1/2)+1/2*a/(a+b)^(5/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1
/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)
^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2
a+2(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/4*a^(1/2)/(a+b)^2/b/(2*a+4*(a
+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+
b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-
2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)
+1/4*a^(1/2)/(a+b)^2/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*
arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(
1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+
a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-2*a/(a+b)^(5/2)*b/(2*a+4*(a+b)^(1/2)*a^(1
/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(
1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/8/a^(1/2)/(a+b
)^2*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a

```
^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/8/a^(1/2)/(a+b)^2*ln(-a^(1/2)*x^2+(-
2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*
(a^2+a*b)^(1/2)-1/8*a^(3/2)/(a+b)^2/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/
2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+1/4*a^2/(a+b)^(5/2)
/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a
^2+a*b)^(1/2))^(1/2)+2*a/(a+b)^(5/2)*b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*
a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4
*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/2*a/(a+b)^(5/2)/(2*a+4*(a+
b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b
)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2
*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/4*a^(1/2)/(a+b
)^2/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x
+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/
2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/
8*a^(1/2)/(a+b)^2/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^
(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/2/(a+b)^(5/2)/(2*a+
4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2
*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2
))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^
(1/2)+1/2/(a+b)^(5/2)/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*a
rctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/
2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*
b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/2*a/(a+b)^(5/2)/(2*a+4*(a+b)^(1/2)*a^(1/2
)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1
/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)
^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+1/4*a/(a+b)^(5/2)/b*ln(-a^(1/2
)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2)
)^(1/2)*(a^2+a*b)^(1/2)-1/4*a/(a+b)^(5/2)/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a
)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2
)+1/8*a^(1/2)/(a+b)^2/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+
b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/4*a^(1/2)/(a+b)^
2/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+
(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2
))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+1/8
*a^(3/2)/(a+b)^2/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1
/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/4*a^2/(a+b)^(5/2)/b*ln(a^(1/2)*x^2+(-
2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{2}\left(\frac{\left(6\sqrt{a^2+\sqrt{-ab}\,a}\,a^2b+8\sqrt{a^2+\sqrt{-ab}\,a}\,ab^2+3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2+\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab-4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,b^2\right)a\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{3a^5b+7a^4b^2+4a^3b^3}+\frac{\left(6\sqrt{a^2-\sqrt{-ab}\,a}\,a^2b+8\sqrt{a^2-\sqrt{-ab}\,a}\,ab^2-3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2-\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab+4\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,b^2\right)a\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{2a-\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{3a^5b+7a^4b^2+4a^3b^3}\right)a}{a+b}-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] -a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a + b), x)/(a + b) - 1/((a + b)*x
)
```

mupad [B] time = 5.27, size = 2848, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] - 1/(x*(a + b)) - atan((((-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^
3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*
a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b -
3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3
*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*
b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b
^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*
a^2*b^3 + a^3*b^2)))^(1/2)*1i - ((-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) +
b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8
*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 -
a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b
^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 +
640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*
(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 +
b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i)/(6*a^6*b + 2*a^7 + ((-(3*a*b^2 - a^2
*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3
+ a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128
*a^7*b^2 + x*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(1
6*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 32
0*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8
- 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b
^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2) + ((-(3*a*b^2
- a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2
*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3
+ 128*a^7*b^2 - x*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2
))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6
 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4
*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*
(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b
^3 + 6*a^5*b^2))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2)
)/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*2i - atan((((-(3*a*b^2
- a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2
*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3
+ 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2
```

```
))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6
 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4
*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*
(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i - ((-(
3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5
 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*
a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b
^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64
*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a
^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1
/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1
i)/(6*a^6*b + 2*a^7 + ((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)
^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^
4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b + 3*
a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b
^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^
3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2
 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^
2*b^3 + a^3*b^2)))^(1/2) + ((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a
*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b +
32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 - a^2*b
 + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 +
a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a
^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*
a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 +
 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 + 6*a^5*b^2))*(-(3*a*b^2 - a^2*b
+ 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a
^3*b^2)))^(1/2)*2i
```

sympy [A] time = 4.54, size = 134, normalized size = 0.31

$$\operatorname{RootSum}\left(t^4\left(256a^3b^2+768a^2b^3+768ab^4+256b^5\right)+t^2\left(-32a^2b+96ab^2\right)+a,\left(t\mapsto t\log\left(x+\frac{-64t^3a^4b-128t^3a^3b^2+128t^3ab^4+64t^3b^5+4ta^3-40ta^2b+20tab^2}{3a^2-ab}\right)\right)\right)-\frac{1}{x(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 + 768*a**2*b**3 + 768*a*b**4 + 256*b**5) + _t*
*2*(-32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b -
128*_t**3*a**3*b**2 + 128*_t**3*a*b**4 + 64*_t**3*b**5 + 4*_t*a**3 - 40*_t*
a**2*b + 20*_t*a*b**2)/(3*a**2 - a*b)))) - 1/(x*(a + b))
```

3.718 $\int \frac{x}{1+x^2+x^4} dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)$$
$$= \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^2 + x^4),x]
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1+x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 + x^2 + x^4),x]
[Out] IntegrateAlgebraic[x/(1 + x^2 + x^4), x]
```

fricas [A] time = 0.93, size = 18, normalized size = 0.90

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

giac [A] time = 0.15, size = 18, normalized size = 0.90

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x^2+1),x)
[Out] 1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

maxima [A] time = 2.84, size = 18, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

mupad [B] time = 0.06, size = 20, normalized size = 1.00

$$\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + x^4 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/3
```

sympy [A] time = 0.17, size = 26, normalized size = 1.30

$$\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x**2+1),x)
[Out] sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3
```

3.719 $\int \frac{x}{10+2x^2+x^4} \, dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1107, 618, 204}

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{10+2x+x^2}\,dx, x, x^2\right)$$
$$= -\operatorname{Subst}\left(\int \frac{1}{-36-x^2}\,dx, x, 2\left(1+x^2\right)\right)$$
$$= \frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(1+x^2\right)\right)$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(10 + 2*x^2 + x^4),x]
[Out] ArcTan[(1 + x^2)/3]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{10+2x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(10 + 2*x^2 + x^4),x]
[Out] IntegrateAlgebraic[x/(10 + 2*x^2 + x^4), x]
```

fricas [A] time = 0.82, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="fricas")
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

giac [A] time = 0.59, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="giac")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+10),x)
```

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

maxima [A] time = 2.92, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="maxima")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

mupad [B] time = 0.06, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2 + x^4 + 10),x)
```

```
[Out] atan(x^2/3 + 1/3)/6
```

sympy [A] time = 0.12, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+10),x)
```

```
[Out] atan(x**2/3 + 1/3)/6
```

3.720 $\int \frac{x^2}{20+9x^2+x^4}\,dx$

Optimal. Leaf size=23

$$\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1130, 203}

$$\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(20 + 9*x^2 + x^4),x]
```

```
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{20+9x^2+x^4}\,dx &= -\left(4\int \frac{1}{4+x^2}\,dx\right)+5\int \frac{1}{5+x^2}\,dx\\ &= -2\tan^{-1}\left(\frac{x}{2}\right)+\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(20 + 9*x^2 + x^4),x]
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{20+9x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(20 + 9*x^2 + x^4),x]
[Out] IntegrateAlgebraic[x^2/(20 + 9*x^2 + x^4), x]
```

fricas [A] time = 1.65, size = 18, normalized size = 0.78

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)-2\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="fricas")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

giac [A] time = 0.19, size = 18, normalized size = 0.78

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)-2\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="giac")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

maple [A] time = 0.01, size = 19, normalized size = 0.83

$$-2\arctan\left(\frac{x}{2}\right)+\sqrt{5}\,\arctan\left(\frac{\sqrt{5}\,x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+9*x^2+20),x)
```

```
[Out] -2*arctan(1/2*x)+arctan(1/5*x*5^(1/2))*5^(1/2)
```

maxima [A] time = 2.98, size = 18, normalized size = 0.78

$$\sqrt{5} \arctan\left(\frac{1}{5}\sqrt{5}x\right) - 2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="maxima")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

mupad [B] time = 4.37, size = 18, normalized size = 0.78

$$\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right) - 2\,\mathrm{atan}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(9*x^2 + x^4 + 20),x)
```

```
[Out] 5^(1/2)*atan((5^(1/2)*x)/5) - 2*atan(x/2)
```

sympy [A] time = 0.21, size = 20, normalized size = 0.87

$$-2\,\mathrm{atan}\left(\frac{x}{2}\right) + \sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+9*x**2+20),x)
```

```
[Out] -2*atan(x/2) + sqrt(5)*atan(sqrt(5)*x/5)
```

3.721 $\int \frac{x^2}{1-x^2+x^4}\,dx$

Optimal. Leaf size=74

$$\frac{\log \left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log \left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1127, 1161, 618, 204, 1164, 628}

$$\frac{\log \left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log \left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 + Log[1 - Sqrt[3]*x + x^
2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
```

```
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1-x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1-x^2+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}\,x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}\,x-x^2}\,dx}{4\sqrt{3}}\\
&= \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)-\frac{1}{2}\operatorname{Sub}\\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.14, size = 94, normalized size = 1.27

$$\frac{\sqrt{-1-i\sqrt{3}}\,\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+\sqrt{-1+i\sqrt{3}}\,\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(1 - x^2 + x^4),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2] + Sqrt[-1
 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(2*Sqrt[6])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1-x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 - x^2 + x^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(1 - x^2 + x^4), x]
```

fricas [B] time = 0.82, size = 159, normalized size = 2.15

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^2+2}+\sqrt{3}\right)-\frac{1}{24}\sqrt{6}\sqrt{2}\log\left(\sqrt{6}\sqrt{2}x+2x^2+2\right)+\frac{1}{24}\sqrt{6}\sqrt{2}\log\left(-\sqrt{6}\sqrt{2}x+2x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/6*sqrt(6)*
sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)
*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/24*sqrt(6)*sqrt(2)*log
(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/24*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2)
*x + 2*x^2 + 2)
```

giac [A] time = 0.17, size = 56, normalized size = 0.76

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{2}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\arctan\left(2x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(2*x - sqrt(3))
```

maple [A] time = 0.02, size = 57, normalized size = 0.77

$$\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-x^2+1),x)
```

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))+1/12*3^(1/2)*ln(x^2-3^(1/2)
*x+1)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^4 - x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 - x^2 + 1), x)
```

mupad [B] time = 0.08, size = 44, normalized size = 0.59

$$-\mathrm{atan}\left(\frac{x}{2} - \frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \mathrm{atan}\left(\frac{x}{2} + \frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 - x^2 + 1),x)
```

```
[Out] atan(x/2 + (3^(1/2)*x*1i)/2)*((3^(1/2)*1i)/6 + 1/2) - atan(x/2 - (3^(1/2)*x
*1i)/2)*((3^(1/2)*1i)/6 - 1/2)
```

sympy [A] time = 0.31, size = 63, normalized size = 0.85

$$\frac{\sqrt{3}\,\log\left(x^2 - \sqrt{3}\,x + 1\right)}{12} - \frac{\sqrt{3}\,\log\left(x^2 + \sqrt{3}\,x + 1\right)}{12} + \frac{\mathrm{atan}\left(2x - \sqrt{3}\right)}{2} + \frac{\mathrm{atan}\left(2x + \sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/12
 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

3.722 $\int \frac{x^2}{2-2x^2+x^4}\,dx$

Optimal. Leaf size=188

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right.$$

Rubi [A] time = 0.18, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1 + S
qrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/
Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*
Sqrt[2*(1 + Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx+\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+\cdots}{-\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\cdots}}{4\sqrt{2\left(1+\cdots\right.}}$$

$$= \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\text{Subst}\left(\int \frac{\cdots}{2\left(1-\cdots\right.}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^
(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2-2x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(2 - 2*x^2 + x^4), x]
```

fricas [A] time = 0.81, size = 247, normalized size = 1.31

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}\right)-\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}}\sqrt{2\sqrt{2}+4}-\sqrt{2}-1\right)-\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}}\sqrt{2\sqrt{2}+4}+\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2^(3/4)*x*sqrt(2*sqrt(2)
 + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)
*log(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(3/4)*sqrt
(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqr
t(2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2
*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2
*sqrt(2))*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

giac [A] time = 0.85, size = 147, normalized size = 0.78

$$\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)+\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2)
)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x - 2
^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

maple [B] time = 0.10, size = 308, normalized size = 1.64

$$\frac{\sqrt{2}\,\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2}\,\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(x^{2}-\sqrt{2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x^{2}-\sqrt{2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(x^{2}+\sqrt{2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x^{2}+\sqrt{2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-2*x^2+2),x)
```

```
[Out] -1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))+1/4*
2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2
))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^
(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)
-x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arct
an((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*
ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2
)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{x^{4}-2\,x^{2}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 - 2*x^2 + 2), x)
```

mupad [B] time = 4.37, size = 101, normalized size = 0.54

$$\mathrm{atanh}\left(32\,x\,\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^{3}\right)\left(2\,\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\,\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)+\mathrm{atanh}\left(32\,x\,\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^{3}\right)\left(2\,\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\,\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 - 2*x^2 + 2),x)
```

```
[Out] atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*
(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*((-
```

```
 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32
- 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

sympy [A] time = 0.83, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4 + 16t^2 + 1, \left(t \mapsto t \log\left(64t^3 + 4t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-2*x**2+2),x)
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

3.723 $\int x^7\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=171

$$\frac{b(7b^2-12ac)(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{9/2}}-\frac{b(7b^2-12ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{(-32ac+35b^2-4$$

Rubi [A] time = 0.16, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 742, 779, 612, 621, 206}

$$\frac{(-32ac+35b^2-42bcx^2)(a+bx^2+cx^4)^{3/2}}{480c^3}-\frac{b(7b^2-12ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{b(7b^2-12ac)(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{9/2}}+\frac{x^4(a+bx^2+cx^4)^{3/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^4) + (x^
4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + ((35*b^2 - 32*a*c - 42*b*c*x^2)*(a +
b*x^2 + c*x^4)^(3/2))/(480*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int x^7\sqrt{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\text{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{10c}\\
&= \frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(35b^2-32ac-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{480c^3}-\frac{\left(b\left(7b^2-12ac\right)\right)\text{S}}{}\\
&= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(35b^2-32ac-\right.}{}\\
&= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(35b^2-32ac-\right.}{}\\
&= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(35b^2-32ac-\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 164, normalized size = 0.96

$$
\frac{-\frac{\left(32ac-35b^2+42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{5\left(12abc-7b^3\right)\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{256c^{7/2}}+x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(3/2) - ((-35*b^2 + 32*a*c + 42*b*c*x^2)*(a + b*x^
2 + c*x^4)^(3/2))/(48*c^2) + (5*(-7*b^3 + 12*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^2
)*Sqrt[a + b*x^2 + c*x^4] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*
Sqrt[a + b*x^2 + c*x^4])]))/(256*c^(7/2)))/(10*c)
```

IntegrateAlgebraic [A] time = 0.46, size = 170, normalized size = 0.99

$$
\frac{\left(-48a^2bc^2+40ab^3c-7b^5\right)\log\left(-2c^{9/2}\sqrt{a+bx^2+cx^4}+bc^4+2c^5x^2\right)}{512c^{9/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-256a^2c^2+460ab^2c-232abc^2x^2+128ac^3x^4-105b^4+70b^3cx^2-56b^2c^2x^4+48bc^3x^6+384c^4x^8\right)}{3840c^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-105*b^4 + 460*a*b^2*c - 256*a^2*c^2 + 70*b^3*c*x
^2 - 232*a*b*c^2*x^2 - 56*b^2*c^2*x^4 + 128*a*c^3*x^4 + 48*b*c^3*x^6 + 384*
```

```
c^4*x^8))/(3840*c^4) + ((-7*b^5 + 40*a*b^3*c - 48*a^2*b*c^2)*Log[b*c^4 + 2*
c^5*x^2 - 2*c^(9/2)*Sqrt[a + b*x^2 + c*x^4]])/(512*c^(9/2))
```

fricas [A] time = 0.94, size = 367, normalized size = 2.15

$$\left[\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+4\left(384c^5x^8+48bc^4x^6-105b^4c+460ab^2c^2-256a^2c^3-8\left(7b^2c^3-16ac^4\right)x^4+2\left(35b^3c^2-116abc^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{15360c^5}, -\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-2\left(384c^5x^8+48bc^4x^6-105b^4c+460ab^2c^2-256a^2c^3-8\left(7b^2c^3-16ac^4\right)x^4+2\left(35b^3c^2-116abc^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{7680c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8
*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) +
 4*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 -
8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^2)*sqrt(c*x^4
 + b*x^2 + a))/c^5, -1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(-c
)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*
x^2 + a*c)) - 2*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 2
56*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^
2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

giac [A] time = 0.25, size = 172, normalized size = 1.01

$$\frac{1}{3840}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-460ab^2c+256a^2c^2}{c^4}\right)-\frac{\left(7b^5-40ab^3c+48a^2bc^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{512c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16
*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b^2
*c + 256*a^2*c^2)/c^4) - 1/512*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(
-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2)
```

maple [A] time = 0.02, size = 296, normalized size = 1.73

$$\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^4}{10c}+\frac{3\sqrt{cx^4+bx^2+a}\,abx^2}{32c^2}-\frac{7\sqrt{cx^4+bx^2+a}\,b^3x^2}{128c^3}+\frac{3a^2b\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{32c^{\frac{5}{2}}}-\frac{5ab^3\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{64c^{\frac{7}{2}}}+\frac{7b^5\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{512c^{\frac{9}{2}}}-\frac{7\left(cx^4+bx^2+a\right)^{\frac{3}{2}}bx^2}{80c^2}+\frac{3\sqrt{cx^4+bx^2+a}\,ab^2}{64c^3}-\frac{7\sqrt{cx^4+bx^2+a}\,b^4}{256c^4}-\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}a}{15c^2}+\frac{7\left(cx^4+bx^2+a\right)^{\frac{3}{2}}b^2}{96c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/10*x^4*(c*x^4+b*x^2+a)^(3/2)/c-7/80*b/c^2*x^2*(c*x^4+b*x^2+a)^(3/2)+7/96*
b^2/c^3*(c*x^4+b*x^2+a)^(3/2)-7/128*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)*x^2-7/256
*b^4/c^4*(c*x^4+b*x^2+a)^(1/2)-5/64*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))*a+7/512*b^5/c^(9/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*
```

```
x^2+a)^(1/2))+3/32*b/c^2*a*(c*x^4+b*x^2+a)^(1/2)*x^2+3/64*b^2/c^3*a*(c*x^4+
b*x^2+a)^(1/2)+3/32*b/c^(5/2)*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^
(1/2))-1/15*a/c^2*(c*x^4+b*x^2+a)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 5.31, size = 315, normalized size = 1.84

$$\frac{x^4\left(c\,x^4+b\,x^2+a\right)^{3/2}}{10\,c}+\frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2+a}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^2\left(c\,x^4+b\,x^2+a\right)^{3/2}}{4\,c}+\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)\sqrt{c\,x^4+b\,x^2+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^4+b\,x^2+a}+\frac{2\,c\,x^2+b}{\sqrt{c}}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{8\,c}\right)}{20\,c}-\frac{a\left(\frac{\left(8\,c\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)\sqrt{c\,x^4+b\,x^2+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^4+b\,x^2+a}+\frac{2\,c\,x^2+b}{\sqrt{c}}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + (7*b*((a*((b/(4*c) + x^2/2)*(a + b
*x^2 + c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^2*(a + b*x^2 + c*x^4)^(3/2))/(4*
c) + (5*b*(((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2)
)/(24*c^2) + (log(2*(a + b*x^2 + c*x^4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3
 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(20*c) - (a*(((8*c*(a + c*x^4) - 3*b^2
+ 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(24*c^2) + (log(2*(a + b*x^2 + c*x^
4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(5*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7*sqrt(a + b*x**2 + c*x**4), x)
```

3.724 $\int x^5\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=153

$$-\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{(5b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}$$

Rubi [A] time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 742, 640, 612, 621, 206}

$$\frac{(5b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(a+bx^2+cx^4)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^3) - (5*b*(a
 + b*x^2 + c*x^4)^(3/2))/(48*c^2) + (x^2*(a + b*x^2 + c*x^4)^(3/2))/(8*c) -
 ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(256*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,x^2\right)$$

$$=\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}+\frac{\text{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{8c}$$

$$=-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}+\frac{\left(5b^2-4ac\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\right.}{32c^2}$$

$$=\frac{\left(5b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}$$

$$=\frac{\left(5b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}$$

$$=\frac{\left(5b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}$$

Mathematica [A] time = 0.07, size = 136, normalized size = 0.89

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(b\left(8c^2x^4-52ac\right)+24c^2x^2\left(a+2cx^4\right)+15b^3-10b^2cx^2\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{768c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^3 - 10*b^2*c*x^2 + 24*c^2*x^2*(a +
 2*c*x^4) + b*(-52*a*c + 8*c^2*x^4)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(768*c^(7/2))
```

IntegrateAlgebraic [A] time = 0.35, size = 132, normalized size = 0.86

$$\frac{\left(16a^2c^2-24ab^2c+5b^4\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{256c^{7/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-52abc+24ac^2x^2+15b^3-10b^2cx^2+8bc^2x^4+48c^3x^6\right)}{384c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^2 + 24*a*c^2*x^2 +
 8*b*c^2*x^4 + 48*c^3*x^6))/(384*c^3) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(256*c^(7/2))
```

fricas [A] time = 1.74, size = 303, normalized size = 1.98

$$\left[\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+4\left(48c^4x^6+8bc^3x^4+15b^3c-52abc^2-2\left(5b^2c^2-12ac^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{1536c^4},\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)+2\left(48c^4x^6+8bc^3x^4+15b^3c-52abc^2-2\left(5b^2c^2-12ac^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{768c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c
*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(
48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)
*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c
^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2
*x^4 + b*c*x^2 + a*c)) + 2*(48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*a*b*c^
2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

giac [A] time = 0.23, size = 134, normalized size = 0.88

$$\frac{1}{384}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{\left(5b^4-24ab^2c+16a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{256c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*c^2
)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 1/256*(5*b^4 - 24*a*b^2*c + 16*a^2*
c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(7/
2)
```

maple [A] time = 0.02, size = 247, normalized size = 1.61

$$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;a\,x^{2}}{16c}+\frac{5\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{2}x^{2}}{64c^{2}}-\frac{a^{2}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{16c^{\frac{3}{2}}}+\frac{3a\,b^{2}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{5}{2}}}-\frac{5b^{4}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{256c^{\frac{7}{2}}}+\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}x^{2}}{8c}-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;ab}{32c^{2}}+\frac{5\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{3}}{128c^{3}}-\frac{5\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}b}{48c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*x^2*(c*x^4+b*x^2+a)^(3/2)/c-5/48*b*(c*x^4+b*x^2+a)^(3/2)/c^2+5/64*b^2/c
^2*(c*x^4+b*x^2+a)^(1/2)*x^2+5/128*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)+3/32*b^2/c
^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-5/256*b^4/c^(7/2)*
ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/16*a/c*(c*x^4+b*x^2+a)^(1
/2)*x^2-1/32*a/c^2*(c*x^4+b*x^2+a)^(1/2)*b-1/16*a^2/c^(3/2)*ln((c*x^2+1/2*b
)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.64, size = 193, normalized size = 1.26

$$\frac{x^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{3/2}}{8\,c}-\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^{2}}{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}+\frac{\ln\left(\sqrt{c\,x^{4}+b\,x^{2}+a}+\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^{2}}{4}\right)}{2\,c^{3/2}}\right)}{8\,c}-\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^{4}+a\right)-3\,b^{2}+2\,b\,c\,x^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{24\,c^{2}}+\frac{\ln\left(2\,\sqrt{c\,x^{4}+b\,x^{2}+a}+\frac{2\,c\,x^{2}+b}{\sqrt{c}}\right)\left(b^{3}-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (x^2*(a + b*x^2 + c*x^4)^(3/2))/(8*c) - (a*((b/(4*c) + x^2/2)*(a + b*x^2 +
c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2))))/(8*c) - (5*b*(((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^
2)*(a + b*x^2 + c*x^4)^(1/2))/(24*c^2) + (log(2*(a + b*x^2 + c*x^4)^(1/2) +
 (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(16*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4), x)
```

3.725 $\int x^3\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c}$$

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c^2) + (a + b*x^2 + c*x^4)^(
3/2)/(6*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(32*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} - \frac{b\,\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx, x, x^2\right)}{4c}$$
$$= -\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+}}\right.}{32c^2}$$
$$= -\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\right.}{16c^2}$$
$$= -\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2}}\right.}{32c^{5/2}}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.94

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(8c\left(a+cx^4\right)-3b^2+2bcx^2\right)+3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{96c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 2*b*c*x^2 + 8*c*(a + c*x^4)) +
 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]
)])/(96*c^(5/2))
```

IntegrateAlgebraic [A] time = 0.27, size = 107, normalized size = 0.99

$$\frac{(4abc-b^3)\log\left(-2c^{5/2}\sqrt{a+bx^2+cx^4}+bc^2+2c^3x^2\right)}{32c^{5/2}}+\frac{\sqrt{a+bx^2+cx^4}\,(8ac-3b^2+2bcx^2+8c^2x^4)}{48c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 8*a*c + 2*b*c*x^2 + 8*c^2*x^4))/(48*c^2)
 + ((-b^3 + 4*a*b*c)*Log[b*c^2 + 2*c^3*x^2 - 2*c^(5/2)*Sqrt[a + b*x^2 + c*x
^4]])/(32*c^(5/2))
```

fricas [A] time = 1.06, size = 237, normalized size = 2.19

$$\left[-\frac{3(b^3-4abc)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{c}-4ac\right)-4(8c^3x^4+2bc^2x^2-3b^2c+8ac^2)\sqrt{cx^4+bx^2+a}}{192c^3},\ -\frac{3(b^3-4abc)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{-c}}{2(c^2x^4+bcx^2+ac)}\right)-2(8c^3x^4+2bc^2x^2-3b^2c+8ac^2)\sqrt{cx^4+bx^2+a}}{96c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqr
t(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 + 2*b*c^
2*x^2 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/96*(3*(b^3 - 4*
a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(
c^2*x^4 + b*c*x^2 + a*c)) - 2*(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c + 8*a*c^2)
*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

giac [A] time = 0.22, size = 98, normalized size = 0.91

$$\frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{(b^3-4abc)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{32c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) -
1/32*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) - b))/c^(5/2)
```

maple [A] time = 0.01, size = 139, normalized size = 1.29

$$-\frac{\sqrt{cx^4+bx^2+a}\,bx^2}{8c}-\frac{ab\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{8c^{\frac{3}{2}}}+\frac{b^3\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{32c^{\frac{5}{2}}}-\frac{\sqrt{cx^4+bx^2+a}\,b^2}{16c^2}+\frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c-1/8*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16*b^2/c^2*
(c*x^4+b*x^2+a)^(1/2)-1/8*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))*a+1/32*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [**B**] time = 4.52, size = 87, normalized size = 0.81

$$\frac{\left(8\,c\,\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}{48\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^4+b\,x^2+a}+\frac{2\,c\,x^2+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{32\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(48*c^2)
+ (log(2*(a + b*x^2 + c*x^4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c)
)/(32*c^(5/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{a+bx^{2}+cx^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4), x)
```

3.726 $\int x\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=83

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1107, 612, 621, 206}

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8c}-\frac{\left(b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16c}\\ &= \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8c}-\frac{\left(b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c}\\ &= \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.00

$$\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2))
```

IntegrateAlgebraic [A] time = 0.21, size = 85, normalized size = 1.02

$$\frac{\left(b^2-4ac\right)\log\left(-2c^{3/2}\sqrt{a+bx^2+cx^4}+bc+2c^2x^2\right)}{16c^{3/2}}+\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) + ((b^2 - 4*a*c)*Log[b*c + 2*
c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(3/2))
```

fricas [A] time = 0.93, size = 197, normalized size = 2.37

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2+bc\right)}{32\,c^2},\ \frac{\left(b^2-4\,ac\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)+2\,\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2+bc\right)}{16\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*
(2*c^2*x^2 + b*c))/c^2, 1/16*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x
^4 + b*x^2 + a)*(2*c^2*x^2 + b*c))/c^2]
```

giac [A] time = 0.20, size = 76, normalized size = 0.92

$$\frac{1}{8}\,\sqrt{cx^4+bx^2+a}\left(2\,x^2+\frac{b}{c}\right)+\frac{\left(b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(-2*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)
```

maple [A] time = 0.01, size = 101, normalized size = 1.22

$$\frac{a\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4\sqrt{c}}-\frac{b^2\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{3}{2}}}+\frac{\left(2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2+a}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/4/c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.62, size = 72, normalized size = 0.87

$$\frac{\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2+a}}{2}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^2/2)*(a + b*x^2 + c*x^4)^(1/2))/2 + (log((a + b*x^2 + c*x^4)^
(1/2) + (b/2 + c*x^2)/c^(1/2))*(a*c - b^2/4))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4), x)
```

3.727 $\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 734, 843, 621, 206, 724}

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/2 - (Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/2 + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(4*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\sqrt{a+bx^2+cx^4} - \frac{1}{4}\,\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\sqrt{a+bx^2+cx^4} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) + \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+\cdots}}\right.\\
&= \frac{1}{2}\sqrt{a+bx^2+cx^4} - a\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right) + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{4c-x\cdots}\right.\\
&= \frac{1}{2}\sqrt{a+bx^2+cx^4} - \frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 106, normalized size = 0.97

$$\frac{1}{4}\left(2\sqrt{a+bx^2+cx^4}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^2 + c*x^4] - 2*Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])] + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2
 + c*x^4])])/Sqrt[c])/4
```

IntegrateAlgebraic [A] time = 0.25, size = 108, normalized size = 0.99

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{b\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{4\sqrt{c}}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/2 + Sqrt[a]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a
+ b*x^2 + c*x^4]/Sqrt[a]] - (b*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4]])/(4*Sqrt[c])
```

fricas [A] time = 0.69, size = 566, normalized size = 5.19

$$\left[\frac{b\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+2\sqrt{a}c\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)+4\sqrt{cx^4+bx^2+a}\,c}{8c}, -\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-\sqrt{a}c\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)-2\sqrt{cx^4+bx^2+a}\,c}{4c}, \frac{4\sqrt{-a}c\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+b\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+4\sqrt{cx^4+bx^2+a}\,c}{8c}, \frac{2\sqrt{-a}c\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)-b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)+2\sqrt{cx^4+bx^2+a}\,c}{4c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4
*sqrt(c*x^4 + b*x^2 + a)*c)/c, -1/4*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*c*log(-
((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(a) + 8*a^2)/x^4) - 2*sqrt(c*x^4 + b*x^2 + a)*c)/c, 1/8*(4*sqrt(-a)*c*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 +
 a^2)) + b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c)/c, 1/4*(
```

```
2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c
*x^4 + a*b*x^2 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c
*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c
)/c]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Value
```

maple [A] time = 0.01, size = 91, normalized size = 0.83

$$-\frac{\sqrt{a}\,\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{2}+\frac{b\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4\sqrt{c}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)+1/4*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))/c^(1/2)-1/2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.42, size = 88, normalized size = 0.81

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x^{2}}+\frac{\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2}+\frac{b\,\ln\left(\sqrt{c\,x^{4}+b\,x^{2}+a}+\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}\right)}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x,x)
```

```
[Out] (a + b*x^2 + c*x^4)^(1/2)/2 - (a^(1/2)*log(b/2 + a/x^2 + (a^(1/2)*(a + b*x^
2 + c*x^4)^(1/2))/x^2))/2 + (b*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2
)/c^(1/2)))/(4*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x, x)
```

3.728 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 732, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}+\frac{1}{4}b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)+\frac{1}{2}c\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+}}\right.\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)+c\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\right.\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 112, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/x^2 - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

IntegrateAlgebraic [A] time = 0.22, size = 114, normalized size = 1.02

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{1}{2}\sqrt{c}\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/x^2 + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[
a + b*x^2 + c*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (Sqrt[c]*Log[b + 2*c*x^2 - 2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4]])/2
```

fricas [A] time = 1.08, size = 601, normalized size = 5.37

$$\left[\frac{2a\sqrt{c}x^2\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+\sqrt{a}bx^2\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)-4\sqrt{cx^4+bx^2+a}a}{8ax^2},\ -\frac{4a\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-\sqrt{a}bx^2\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)+4\sqrt{cx^4+bx^2+a}a}{8ax^2},\ \frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+a\sqrt{c}x^2\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)-2\sqrt{cx^4+bx^2+a}a}{4ax^2},\ \frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)-2a\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-2\sqrt{cx^4+bx^2+a}a}{4ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*
x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/
x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), -1/8*(4*a*sqrt(-c)*x^2*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c
)) - sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x
^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 + b*x^2 + a)*a)/
(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*
a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + a*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b
*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 2
```

```
*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*
x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*a*sq
rt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x
^4 + b*c*x^2 + a*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2)]
```

giac [A] time = 0.29, size = 148, normalized size = 1.32

$$\frac{b\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}-\frac{1}{2}\sqrt{c}\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)+\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) -
1/2*sqrt(c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b)
) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/((sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)
```

maple [A] time = 0.01, size = 140, normalized size = 1.25

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^2}{2a}-\frac{b\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{4\sqrt{a}}+\frac{\sqrt{c}\;\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;b}{2a}-\frac{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2*b/a*(c*x^4+b*x^2+a)^(1/2)-1/4*b/a^(1/2
)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/2*c/a*(c*x^4+b*x^2+
a)^(1/2)*x^2+1/2*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.55, size = 91, normalized size = 0.81

$$\frac{\sqrt{c}\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,x^2}-\frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^2}+\frac{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^3,x)
```

```
[Out] (c^(1/2)*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/2 - (a + b
*x^2 + c*x^4)^(1/2)/(2*x^2) - (b*log(b/2 + a/x^2 + (a^(1/2)*(a + b*x^2 + c*
x^4)^(1/2))/x^2))/(4*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**3, x)
```

3.729 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^5} dx$

Optimal. Leaf size=88

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1114, 720, 724, 206}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a*x^4) + ((b^2 - 4*a*c)*ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16a}$$
$$= -\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4} + \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8a}$$
$$= -\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4} + \frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.00

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}} - \frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -1/8*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) + ((b^2 - 4*a*c)*ArcTa
nh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(3/2))
```

IntegrateAlgebraic [A] time = 0.33, size = 91, normalized size = 1.03

$$\frac{(4ac-b^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{3/2}} + \frac{(-2a-bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] ((-2*a - b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a*x^4) + ((-b^2 + 4*a*c)*ArcTan
h[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(3/2))
```

fricas [A] time = 1.17, size = 215, normalized size = 2.44

$$\left[-\frac{\left(b^{2}-4 \, a c\right) \sqrt{a} x^{4} \log\left(-\frac{\left(b^{2}+4 \, a c\right) x^{4}+8 \, a b x^{2}-4 \, \sqrt{c x^{4}+b x^{2}+a} \left(b x^{2}+2 \, a\right) \sqrt{a}+8 \, a^{2}}{x^{4}}\right)+4 \, \sqrt{c x^{4}+b x^{2}+a} \left(a b x^{2}+2 \, a^{2}\right)}{32 \, a^{2} x^{4}}, -\frac{\left(b^{2}-4 \, a c\right) \sqrt{-a} x^{4} \arctan\left(\frac{\sqrt{c x^{4}+b x^{2}+a} \left(b x^{2}+2 \, a\right) \sqrt{-a}}{2 \, \left(a c x^{4}+a b x^{2}+a^{2}\right)}\right)+2 \, \sqrt{c x^{4}+b x^{2}+a} \left(a b x^{2}+2 \, a^{2}\right)}{16 \, a^{2} x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*s
qrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 +
 b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4), -1/16*((b^2 - 4*a*c)*sqrt(-a)*x^4
*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x
^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4)]
```

giac [B] time = 0.22, size = 241, normalized size = 2.74

$$-\frac{\left(b^{2}-4 \, a c\right) \arctan\left(-\frac{\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}}{\sqrt{-a}}\right)}{8 \, \sqrt{-a} a}+\frac{\left(\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}\right)^{3} b^{2}+4 \, \left(\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}\right)^{3} a c+8 \, \left(\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}\right)^{2} a b \sqrt{c}+\left(\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}\right) a b^{2}+4 \, \left(\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}\right) a^{2} c}{8 \, {\left({\left(\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}\right)}^{2}-a\right)}^{2} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*(b^2 - 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a)
)/(sqrt(-a)*a) + 1/8*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 + 4*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))^2*a*b*sqrt(c) + (sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 +
 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c)/(((sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^2 - a)^2*a)
```

maple [B] time = 0.01, size = 193, normalized size = 2.19

$$-\frac{\sqrt{c x^{4}+b x^{2}+a}\, b c x^{2}}{8 a^{2}}-\frac{c \ln\left(\frac{b x^{2}+2 a+2 \sqrt{c x^{4}+b x^{2}+a}\, \sqrt{a}}{x^{2}}\right)}{4 \sqrt{a}}+\frac{b^{2} \ln\left(\frac{b x^{2}+2 a+2 \sqrt{c x^{4}+b x^{2}+a}\, \sqrt{a}}{x^{2}}\right)}{16 a^{\frac{3}{2}}}+\frac{\sqrt{c x^{4}+b x^{2}+a}\, c}{4 a}-\frac{\sqrt{c x^{4}+b x^{2}+a}\, b^{2}}{8 a^{2}}+\frac{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}} b}{8 a^{2} x^{2}}-\frac{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}}{4 a x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(c*x^4+b*x^2+a)^(3/2)+1/8*b/a^2/x^2*(c*x^4+b*x^2+a)^(3/2)-1/8*b^
2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*b^2/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a
```

```
)^(1/2)*a^(1/2))/x^2)-1/8*b/a^2*c*(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*c/a*(c*x^4+
b*x^2+a)^(1/2)-1/4*c/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))
/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^5,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**5, x)
```

3.730 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^7} \, dx$

Optimal. Leaf size=116

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}+\frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}$$

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 730, 720, 724, 206}

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}+\frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] (b*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a^2*x^4) - (a + b*x^2 + c*x^4
)^(3/2)/(6*a*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/(32*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{b\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right)}{4a}\\
&= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}+\frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx}}\right.}{32a^2}\\
&= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x\right.}{16a^2}\\
&= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx}}\right.}{32a^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 108, normalized size = 0.93

$$
-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\left(8a^2+2ax^2\left(b+4cx^2\right)-3b^2x^4\right)}{48a^2x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] -1/48*(Sqrt[a + b*x^2 + c*x^4]*(8*a^2 - 3*b^2*x^4 + 2*a*x^2*(b + 4*c*x^2)))
/(a^2*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x
^2 + c*x^4])])/(32*a^(5/2))
```

IntegrateAlgebraic [A] time = 0.57, size = 108, normalized size = 0.93

$$\frac{\left(b^3-4abc\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{\sqrt{a+bx^2+cx^4}\,\left(-8a^2-2abx^2-8acx^4+3b^2x^4\right)}{48a^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 - 2*a*b*x^2 + 3*b^2*x^4 - 8*a*c*x^4))/(48*
a^2*x^6) + ((b^3 - 4*a*b*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])
/Sqrt[a]])/(16*a^(5/2))
```

fricas [A] time = 0.99, size = 261, normalized size = 2.25

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{a}\,x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(2\,a^2bx^2-\left(3\,ab^2-8\,a^2c\right)x^4+8\,a^3\right)\sqrt{cx^4+bx^2+a}}{192\,a^3x^6},\ \frac{3\left(b^3-4\,abc\right)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)-2\left(2\,a^2bx^2-\left(3\,ab^2-8\,a^2c\right)x^4+8\,a^3\right)\sqrt{cx^4+bx^2+a}}{96\,a^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
+ 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(2*a^2*
b*x^2 - (3*a*b^2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^6)
, 1/96*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(2*a^2*b*x^2 - (3*a*b^
2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^6)]
```

giac [B] time = 0.30, size = 359, normalized size = 3.09

$$\frac{\left(b^3-4\,abc\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a^2}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5b^3-12\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5abc-48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^4a^2c^{\frac{3}{2}}-8\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3ab^3-48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3a^2bc-48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2a^2b^2\sqrt{c}-3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^2b^3-36\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^3bc-16\,a^4c^{\frac{3}{2}}}{48\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/16*(b^3 - 4*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-
a))/(sqrt(-a)*a^2) - 1/48*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*b^3
- 12*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c - 48*(sqrt(c)*x^2 - sq
```

```
rt(c*x^4 + b*x^2 + a))^4*a^2*c^(3/2) - 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^3*a*b^3 - 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^2*b*c - 48*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^2*b^2*sqrt(c) - 3*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^2*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
a))*a^3*b*c - 16*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 -
 a)^3*a^2)
```

maple [B] time = 0.01, size = 222, normalized size = 1.91

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{2} c\,x^{2}}{16 a^{3}}+\frac{b c \ln \left(\frac{b\,x^{2}+2 a+2 \sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{a}}{x^{2}}\right)}{8 a^{\frac{3}{2}}}-\frac{b^{3} \ln \left(\frac{b\,x^{2}+2 a+2 \sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{a}}{x^{2}}\right)}{32 a^{\frac{5}{2}}}-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;b c}{8 a^{2}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{3}}{16 a^{3}}-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}} b^{2}}{16 a^{3} x^{2}}+\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}} b}{8 a^{2} x^{4}}-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{6 a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(3/2)/a/x^6+1/8*b/a^2/x^4*(c*x^4+b*x^2+a)^(3/2)-1/16*b
^2/a^3/x^2*(c*x^4+b*x^2+a)^(3/2)+1/16*b^3/a^3*(c*x^4+b*x^2+a)^(1/2)-1/32*b^
3/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/16*b^2/a^3*
c*(c*x^4+b*x^2+a)^(1/2)*x^2-1/8*b/a^2*c*(c*x^4+b*x^2+a)^(1/2)+1/8*b/a^(3/2)
*c*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^7,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b x^{2} + c x^{4}}}{x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**7, x)
```

3.731 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=161

$$\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}-\frac{(5b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{5b(a+bx^2+cx^4)^{3/2}}{48a^2x^6}-$$

Rubi [A] time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 744, 806, 720, 724, 206}

$$-\frac{(5b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}+\frac{5b(a+bx^2+cx^4)^{3/2}}{48a^2x^6}-\frac{(a+bx^2+cx^4)^{3/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^3*x^4) - (a
 + b*x^2 + c*x^4)^(3/2)/(8*a*x^8) + (5*b*(a + b*x^2 + c*x^4)^(3/2))/(48*a^2
*x^6) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])])/(256*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^2\right)}{8a}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6} + \frac{\left(5b^2-4ac\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^2 \right.}{32a^2}$$

$$= -\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6}$$

$$= -\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6}$$

$$= -\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6}$$

Mathematica [A] time = 0.10, size = 141, normalized size = 0.88

$$\frac{3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) - \frac{2\sqrt{a}\sqrt{a+bx^2+cx^4}\left(48a^3+8a^2x^2\left(b+3cx^2\right)-2abx^4\left(5b+26cx^2\right)+15b^3x^6\right)}{x^8}}{768a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] ((-2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(48*a^3 + 15*b^3*x^6 + 8*a^2*x^2*(b +
3*c*x^2) - 2*a*b*x^4*(5*b + 26*c*x^2)))/x^8 + 3*(5*b^4 - 24*a*b^2*c + 16*a^
2*c^2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(768*a^(
7/2))
```

IntegrateAlgebraic [A] time = 0.74, size = 141, normalized size = 0.88

$$\frac{\left(-16a^2c^2+24ab^2c-5b^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{128a^{7/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(-48a^3-8a^2bx^2-24a^2cx^4+10ab^2x^4+52abcx^6-15b^3x^6\right)}{384a^3x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-48*a^3 - 8*a^2*b*x^2 + 10*a*b^2*x^4 - 24*a^2*c*x
^4 - 15*b^3*x^6 + 52*a*b*c*x^6))/(384*a^3*x^8) + ((-5*b^4 + 24*a*b^2*c - 16
*a^2*c^2)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(128*a^
(7/2))
```

fricas [A] time = 0.77, size = 325, normalized size = 2.02

$$\left[\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{a}x^8\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2+4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)-4\left(\left(15ab^3-52a^2bc\right)x^6+8a^3bx^2-2\left(5a^2b^2-12a^3c\right)x^4+48a^4\right)\sqrt{cx^4+bx^2+a}}{1536a^4x^8},\ -\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{-a}x^8\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(\left(15ab^3-52a^2bc\right)x^6+8a^3bx^2-2\left(5a^2b^2-12a^3c\right)x^4+48a^4\right)\sqrt{cx^4+bx^2+a}}{768a^4x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*c
)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2
)/x^4) - 4*((15*a*b^3 - 52*a^2*b*c)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 12*a
^3*c)*x^4 + 48*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^8), -1/768*(3*(5*b^4 -
24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3 - 52*a^2*b*c
)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 12*a^3*c)*x^4 + 48*a^4)*sqrt(c*x^4 + b
*x^2 + a))/(a^4*x^8)]
```

giac [B] time = 0.28, size = 617, normalized size = 3.83

$$-\frac{\left(5b^4-24ab^2c+16a^2c^2\right)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{128\sqrt{-a}a^3}+\frac{15\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^7b^4-72\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^7ab^2c+48\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^7a^2c^2-55\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5ab^4+264\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5a^2b^2c+336\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5a^3c^2+1152\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^4a^3bc^{\frac{3}{2}}+73\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3a^2b^4+648\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3a^3b^2c+336\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3a^4c^2+384\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2a^3b^3\sqrt{c}+256\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2a^4bc^{\frac{3}{2}}+15\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)a^3b^4+312\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)a^4b^2c+48\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)a^5c^2+128a^5bc^{\frac{3}{2}}}{384\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/128*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/384*(15*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))^7*b^4 - 72*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^2*
c + 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*c^2 - 55*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^5*a*b^4 + 264*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^5*a^2*b^2*c + 336*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*c^2 +
 1152*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^3*b*c^(3/2) + 73*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^2*b^4 + 648*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^3*a^3*b^2*c + 336*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^
4*c^2 + 384*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*b^3*sqrt(c) + 256
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^4*b*c^(3/2) + 15*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))*a^3*b^4 + 312*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*a^4*b^2*c + 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^5*c^2 + 128*
a^5*b*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^4*a^3)
```

maple [B] time = 0.02, size = 387, normalized size = 2.40

$$\frac{\sqrt{cx^4+bx^2+a}\,bc^2x^2}{32a^3}-\frac{5\sqrt{cx^4+bx^2+a}\,b^3cx^2}{128a^4}+\frac{c^2\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{16a^{\frac{3}{2}}}-\frac{3b^2c\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{5}{2}}}+\frac{5b^4\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{256a^{\frac{7}{2}}}-\frac{\sqrt{cx^4+bx^2+a}\,c^2}{16a^2}+\frac{7\sqrt{cx^4+bx^2+a}\,b^2c}{64a^3}-\frac{5\sqrt{cx^4+bx^2+a}\,b^4}{128a^4}-\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}bc}{32a^3x^2}+\frac{5\left(cx^4+bx^2+a\right)^{\frac{3}{2}}b^3}{128a^4x^2}+\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}c}{16a^2x^4}-\frac{5\left(cx^4+bx^2+a\right)^{\frac{3}{2}}b^2}{64a^3x^4}+\frac{5\left(cx^4+bx^2+a\right)^{\frac{3}{2}}b}{48a^2x^6}-\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{8ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^9,x)
```

```
[Out] -1/8*(c*x^4+b*x^2+a)^(3/2)/a/x^8+5/48*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^6-5/64*
b^2/a^3/x^4*(c*x^4+b*x^2+a)^(3/2)+5/128*b^3/a^4/x^2*(c*x^4+b*x^2+a)^(3/2)-5
/128*b^4/a^4*(c*x^4+b*x^2+a)^(1/2)+5/256*b^4/a^(7/2)*ln((b*x^2+2*a+2*(c*x^4
+b*x^2+a)^(1/2)*a^(1/2))/x^2)-5/128*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/2)*x^2+7/6
4*b^2/a^3*c*(c*x^4+b*x^2+a)^(1/2)-3/32*b^2/a^(5/2)*c*ln((b*x^2+2*a+2*(c*x^4
+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/16*c/a^2/x^4*(c*x^4+b*x^2+a)^(3/2)-1/32*c/a
^3*b/x^2*(c*x^4+b*x^2+a)^(3/2)+1/32*c^2/a^3*b*(c*x^4+b*x^2+a)^(1/2)*x^2-1/1
6*c^2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*c^2/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x
^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^9,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**9, x)
```

3.732 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=199

$$-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{9/2}}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{\left(35b^2-32ac\right)\left(a\right.}{480a}$$

Rubi [A] time = 0.23, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 744, 834, 806, 720, 724, 206}

$$-\frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{480a^3x^6}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{9/2}}+\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^4*x^4) -
(a + b*x^2 + c*x^4)^(3/2)/(10*a*x^10) + (7*b*(a + b*x^2 + c*x^4)^(3/2))/(80
*a^2*x^8) - ((35*b^2 - 32*a*c)*(a + b*x^2 + c*x^4)^(3/2))/(480*a^3*x^6) - (
b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
b*x^2 + c*x^4])])/(512*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^2\right)}{10a}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{4}\left(35b^2-32ac\right)+\frac{7bcx}{2}\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x\right.}{40a^2}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} - \frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{480a^3x^6} - \frac{\left(b\left(7b\right.\right.}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}$$

Mathematica [A] time = 0.12, size = 173, normalized size = 0.87

$$-\frac{b\left(48a^2c^2-40ab^2c+7b^4\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{9/2}} - \frac{\sqrt{a+bx^2+cx^4}\left(384a^4+16a^3\left(3bx^2+8cx^4\right)-8a^2\left(7b^2x^4+29bcx^6+32c^2x^8\right)+10ab^2x^6\left(7b+46cx^2\right)-105b^4x^8\right)}{3840a^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] -1/3840*(Sqrt[a + b*x^2 + c*x^4]*(384*a^4 - 105*b^4*x^8 + 10*a*b^2*x^6*(7*b
 + 46*c*x^2) + 16*a^3*(3*b*x^2 + 8*c*x^4) - 8*a^2*(7*b^2*x^4 + 29*b*c*x^6 +
 32*c^2*x^8)))/(a^4*x^10) - (b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2
*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(512*a^(9/2))
```

IntegrateAlgebraic [A] time = 1.01, size = 176, normalized size = 0.88

$$\frac{\left(48a^2bc^2-40ab^3c+7b^5\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{256a^{9/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(-384a^4-48a^3bx^2-128a^3cx^4+56a^2b^2x^4+232a^2bcx^6+256a^2c^2x^8-70ab^3x^6-460ab^2cx^8+105b^4x^8\right)}{3840a^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-384*a^4 - 48*a^3*b*x^2 + 56*a^2*b^2*x^4 - 128*a^
3*c*x^4 - 70*a*b^3*x^6 + 232*a^2*b*c*x^6 + 105*b^4*x^8 - 460*a*b^2*c*x^8 +
256*a^2*c^2*x^8))/(3840*a^4*x^10) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*Ar
cTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(256*a^(9/2))
```

fricas [A] time = 2.67, size = 389, normalized size = 1.95

$$\left[\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{a}\,x^{10}\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)+4\left(\left(105ab^4-460a^2b^2c+256a^3c^2\right)x^8-48a^4bx^2-2\left(35a^2b^3-116a^3bc\right)x^6-384a^5+8\left(7a^3b^2-16a^4c\right)x^4\right)\sqrt{cx^4+bx^2+a}}{15360a^5x^{10}},\ \frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{-a}\,x^{10}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(\left(105ab^4-460a^2b^2c+256a^3c^2\right)x^8-48a^4bx^2-2\left(35a^2b^3-116a^3bc\right)x^6-384a^5+8\left(7a^3b^2-16a^4c\right)x^4\right)\sqrt{cx^4+bx^2+a}}{7680a^5x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 +
4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) +
8*a^2)/x^4) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^8 - 48*a^4*b*x
^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 384*a^5 + 8*(7*a^3*b^2 - 16*a^4*c)*
x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10), 1/7680*(15*(7*b^5 - 40*a*b^3*c +
48*a^2*b*c^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 256
*a^3*c^2)*x^8 - 48*a^4*b*x^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 384*a^5 +
 8*(7*a^3*b^2 - 16*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10)]
```

giac [B] time = 0.36, size = 842, normalized size = 4.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/256*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/3840*(105*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^9*b^5 - 600*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*
b^3*c + 720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^2*b*c^2 - 490*(sqrt
(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^5 + 2800*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^7*a^2*b^3*c - 3360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7
*a^3*b*c^2 - 7680*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^4*c^(5/2) + 8
96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^5 - 5120*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))^5*a^3*b^3*c - 15360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^5*a^4*b*c^2 - 24320*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4
*b^2*c^(3/2) - 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*c^(5/2) -
 790*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^3*b^5 - 9200*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^3*a^4*b^3*c - 12000*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))^3*a^5*b*c^2 - 3840*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^
```

```
4*b^4*sqrt(c) - 5120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^5*b^2*c^(3
/2) - 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^6*c^(5/2) - 105*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*b^5 - 3240*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))*a^5*b^3*c - 720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^6
*b*c^2 - 1280*a^6*b^2*c^(3/2) + 512*a^7*c^(5/2))/(((sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))^2 - a)^5*a^4)
```

maple [B] time = 0.02, size = 442, normalized size = 2.22

$$-\frac{3\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{2}c^{2}x^{2}}{64a^{4}}+\frac{7\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{4}c\,x^{2}}{256a^{5}}-\frac{3b\,c^{2}\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{a}}{x^{2}}\right)}{32a^{\frac{5}{2}}}+\frac{5b^{3}c\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{a}}{x^{2}}\right)}{64a^{\frac{7}{2}}}-\frac{7b^{5}\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{a}}{x^{2}}\right)}{512a^{\frac{9}{2}}}+\frac{3\sqrt{c\,x^{4}+b\,x^{2}+a}\;b\,c^{2}}{32a^{3}}-\frac{13\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{3}c}{128a^{4}}+\frac{7\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{5}}{256a^{5}}+\frac{3\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}b^{2}c}{64a^{4}x^{2}}-\frac{7\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}b^{4}}{256a^{5}x^{2}}-\frac{3\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}bc}{32a^{3}x^{4}}+\frac{7\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}{128a^{4}x^{4}}+\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}c}{15a^{2}x^{6}}-\frac{7\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}b^{2}}{96a^{3}x^{6}}+\frac{7\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}b}{80a^{2}x^{8}}-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{10a\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^11,x)
```

```
[Out] -1/10*(c*x^4+b*x^2+a)^(3/2)/a/x^10+7/80*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-7/9
6*b^2/a^3/x^6*(c*x^4+b*x^2+a)^(3/2)+7/128*b^3/a^4/x^4*(c*x^4+b*x^2+a)^(3/2)
-7/256*b^4/a^5/x^2*(c*x^4+b*x^2+a)^(3/2)+7/256*b^5/a^5*(c*x^4+b*x^2+a)^(1/2
)-7/512*b^5/a^(9/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+7/2
56*b^4/a^5*c*(c*x^4+b*x^2+a)^(1/2)*x^2-13/128*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/
2)+5/64*b^3/a^(7/2)*c*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3
/32*b/a^3*c/x^4*(c*x^4+b*x^2+a)^(3/2)+3/64*b^2/a^4*c/x^2*(c*x^4+b*x^2+a)^(3
/2)-3/64*b^2/a^4*c^2*(c*x^4+b*x^2+a)^(1/2)*x^2+3/32*b/a^3*c^2*(c*x^4+b*x^2+
a)^(1/2)-3/32*b/a^(5/2)*c^2*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/
x^2)+1/15*c/a^2/x^6*(c*x^4+b*x^2+a)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^11,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**11, x)
```

3.733 $\int x^7 \left(a + bx^2 + cx^4\right)^{3/2} dx$

Optimal. Leaf size=223

$$-\frac{3b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4096c^{11/2}}+\frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b\left(3b\right.}{}$$

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 742, 779, 612, 621, 206}

$$\frac{\left(-16ac+21b^2-30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{560c^3}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{256c^4}+\frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{3b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4096c^{11/2}}+\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(
2048*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(25
6*c^4) + (x^4*(a + b*x^2 + c*x^4)^(5/2))/(14*c) + ((21*b^2 - 16*a*c - 30*b*
c*x^2)*(a + b*x^2 + c*x^4)^(5/2))/(560*c^3) - (3*b*(b^2 - 4*a*c)^2*(3*b^2 -
 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4096*c
^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^7 \left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int x^3\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$= \frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c} + \frac{\text{Subst}\left(\int x\left(-2a-\frac{9bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{14c}$$

$$= \frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c} + \frac{\left(21b^2-16ac-30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{560c^3} - \frac{\left(b\left(3b^2-4ac\right)\right)\cdots}{\cdots}$$

$$= -\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{256c^4} + \frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c} + \frac{\left(21b^2-16a\cdots\right.}{\cdots}$$

$$= \frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5} - \frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a\cdots\right.}{256c^4}$$

$$= \frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5} - \frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a\cdots\right.}{256c^4}$$

$$= \frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5} - \frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a\cdots\right.}{256c^4}$$

Mathematica [A] time = 0.25, size = 192, normalized size = 0.86

$$\frac{-\frac{\left(16ac-21b^2+30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{40c^2}+\frac{7\left(4abc-3b^3\right)\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{2048c^{9/2}}+x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(5/2) - ((-21*b^2 + 16*a*c + 30*b*c*x^2)*(a + b*x^
2 + c*x^4)^(5/2))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^2)
*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^
2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(
2048*c^(9/2)))/(14*c)
```

IntegrateAlgebraic [A] time = 0.98, size = 255, normalized size = 1.14

$$\frac{3\left(-64a^3bc^3+80a^2b^3c^2-28ab^5c+3b^7\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{4096c^{11/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-2048a^3c^3+5488a^2b^2c^2-2336a^2bc^3x^2+1024a^2c^4x^4-2520ab^4c+1456ab^3c^2x^2-992ab^2c^3x^4+704abc^4x^6+8192ac^5x^8+315b^6-210b^5cx^2+168b^4c^2x^4-144b^3c^3x^6+128b^2c^4x^8+6400bc^5x^{10}+5120c^6x^{12}\right)}{71680c^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*
a^3*c^3 - 210*b^5*c*x^2 + 1456*a*b^3*c^2*x^2 - 2336*a^2*b*c^3*x^2 + 168*b^4
*c^2*x^4 - 992*a*b^2*c^3*x^4 + 1024*a^2*c^4*x^4 - 144*b^3*c^3*x^6 + 704*a*b
*c^4*x^6 + 128*b^2*c^4*x^8 + 8192*a*c^5*x^8 + 6400*b*c^5*x^10 + 5120*c^6*x^
12))/(71680*c^5) + (3*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*
Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(4096*c^(11/2))
```

fricas [A] time = 1.30, size = 535, normalized size = 2.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/286720*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^12 + 6400*b*c^6*x^10 + 128*(b^2*c^5 + 64
*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4
- 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2
*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^2)*sqrt(c*x^
4 + b*x^2 + a))/c^6, 1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 6
4*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt
(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(5120*c^7*x^12 + 6400*b*c^6*x^10 + 128*
(b^2*c^5 + 64*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 -
2048*a^3*c^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 - 124*a*b^2*
c^4 + 128*a^2*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x
^2)*sqrt(c*x^4 + b*x^2 + a))/c^6]
```

giac [B] time = 0.40, size = 669, normalized size = 3.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 -
 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*
b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(
-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2))*a + 1/307
20*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*x^2 + b/c)*x^2 - (9*b^2*c^3 -
 20*a*c^4)/c^5)*x^2 + (21*b^3*c^2 - 68*a*b*c^3)/c^5)*x^2 - (105*b^4*c - 448
*a*b^2*c^2 + 240*a^2*c^3)/c^5)*x^2 + (315*b^5 - 1680*a*b^3*c + 1808*a^2*b*c
^2)/c^5) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*log(abs
(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(11/2))*b + 1/4
30080*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*(12*x^2 + b/c)*x^2 - (11*b
```

```
^2*c^4 - 24*a*c^5)/c^6)*x^2 + (99*b^3*c^3 - 316*a*b*c^4)/c^6)*x^2 - (231*b^
4*c^2 - 972*a*b^2*c^3 + 512*a^2*c^4)/c^6)*x^2 + (1155*b^5*c - 6048*a*b^3*c^
2 + 6352*a^2*b*c^3)/c^6)*x^2 - (3465*b^6 - 21840*a*b^4*c + 34608*a^2*b^2*c^
2 - 8192*a^3*c^3)/c^6) - 105*(33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*
a^3*b*c^3)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))
/c^(13/2))*c
```

maple [B] time = 0.04, size = 534, normalized size = 2.39

$$\frac{\sqrt{c x^4+b x^2+a}\, c x^{12}}{14}+\frac{5\sqrt{c x^4+b x^2+a}\, b x^{10}}{56}+\frac{4\sqrt{c x^4+b x^2+a}\, a x^8}{35}+\frac{\sqrt{c x^4+b x^2+a}\, b^2 x^8}{560 c}+\frac{11\sqrt{c x^4+b x^2+a}\, a b x^6}{1120 c}-\frac{9\sqrt{c x^4+b x^2+a}\, b^3 x^6}{4480 c^2}+\frac{\sqrt{c x^4+b x^2+a}\, a^2 x^4}{70 c}-\frac{31\sqrt{c x^4+b x^2+a}\, a b^2 x^4}{2240 c^2}+\frac{3\sqrt{c x^4+b x^2+a}\, b^4 x^4}{1280 c^3}-\frac{73\sqrt{c x^4+b x^2+a}\, a^2 b x^2}{2240 c^2}+\frac{13\sqrt{c x^4+b x^2+a}\, a b^3 x^2}{640 c^3}-\frac{3\sqrt{c x^4+b x^2+a}\, b^5 x^2}{1024 c^4}+\frac{3 a^3 b \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{64 c^{\frac{5}{2}}}-\frac{15 a^2 b^3 \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{256 c^{\frac{7}{2}}}+\frac{21 a b^5 \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{1024 c^{\frac{9}{2}}}-\frac{9 b^7 \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{4096 c^{\frac{11}{2}}}-\frac{\sqrt{c x^4+b x^2+a}\, a^3}{35 c^2}+\frac{49\sqrt{c x^4+b x^2+a}\, a^2 b^2}{640 c^3}-\frac{9\sqrt{c x^4+b x^2+a}\, a b^4}{256 c^4}+\frac{9\sqrt{c x^4+b x^2+a}\, b^6}{2048 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/560*b^2*x^8/c*(c*x^4+b*x^2+a)^(1/2)-9/4480*b^3/c^2*x^6*(c*x^4+b*x^2+a)^(1
/2)+3/1280*b^4/c^3*x^4*(c*x^4+b*x^2+a)^(1/2)-3/1024*b^5/c^4*x^2*(c*x^4+b*x^
2+a)^(1/2)+1/70*a^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)+49/640*a^2*b^2/c^3*(c*x^4+b
*x^2+a)^(1/2)+3/64*a^3*b/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))-9/256*a*b^4/c^4*(c*x^4+b*x^2+a)^(1/2)+21/1024*a*b^5/c^(9/2)*ln((c*x^2
+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-15/256*a^2*b^3/c^(7/2)*ln((c*x^2+1/2
*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+4/35*a*x^8*(c*x^4+b*x^2+a)^(1/2)+9/2048*
b^6/c^5*(c*x^4+b*x^2+a)^(1/2)+1/14*c*x^12*(c*x^4+b*x^2+a)^(1/2)+11/1120*a*b
*x^6/c*(c*x^4+b*x^2+a)^(1/2)-31/2240*a*b^2/c^2*x^4*(c*x^4+b*x^2+a)^(1/2)+13
/640*a*b^3/c^3*x^2*(c*x^4+b*x^2+a)^(1/2)-73/2240*a^2*b/c^2*x^2*(c*x^4+b*x^2
+a)^(1/2)+5/56*b*x^10*(c*x^4+b*x^2+a)^(1/2)-9/4096*b^7/c^(11/2)*ln((c*x^2+1
/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/35*a^3/c^2*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.734 $\int x^5 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=204

$$\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2048c^{9/2}}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{\left(7b^2-4ac\right)}{}$$

Rubi [A] time = 0.18, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 742, 640, 612, 621, 206}

$$\frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384c^3}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2048c^{9/2}}-\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(102
4*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*c^3
) - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(120*c^2) + (x^2*(a + b*x^2 + c*x^4)^(5
/2))/(12*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(2048*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5\left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int x^2\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$= \frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{12c} + \frac{\text{Subst}\left(\int\left(-a-\frac{7bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{12c}$$

$$= -\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120c^2} + \frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{12c} + \frac{\left(7b^2-4ac\right)\text{Subst}\left(\int\left(a+bx+cx\right.\right.}{48c^2}$$

$$= \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384c^3} - \frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120c^2} + \frac{x^2\left(a+bx^2+c\right.}{12c}$$

$$= -\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+b\right.}{384c^3}$$

$$= -\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+b\right.}{384c^3}$$

$$= -\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+b\right.}{384c^3}$$

Mathematica [A] time = 0.16, size = 175, normalized size = 0.86

$$\frac{\frac{\left(7b^2-4ac\right)\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{512c^{7/2}}+x^2\left(a+bx^2+cx^4\right)^{5/2}-\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{10c}}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((-7*b*(a + b*x^2 + c*x^4)^(5/2))/(10*c) + x^2*(a + b*x^2 + c*x^4)^(5/2) +
((7*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 +
 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(512*c^(7/2)))/(12*c)
```

IntegrateAlgebraic [A] time = 0.77, size = 209, normalized size = 1.02

$$\frac{\sqrt{a+bx^2+cx^4}\left(-1296a^2bc^2+480a^2c^3x^2+760ab^3c-432ab^2c^2x^2+288abc^3x^4+2240ac^4x^6-105b^5+70b^4cx^2-56b^3c^2x^4+48b^2c^3x^6+1664bc^4x^8+1280c^5x^{10}\right)}{15360c^4}+\frac{\left(64a^3c^3-144a^2b^2c^2+60ab^4c-7b^6\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{2048c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-105*b^5 + 760*a*b^3*c - 1296*a^2*b*c^2 + 70*b^4*
c*x^2 - 432*a*b^2*c^2*x^2 + 480*a^2*c^3*x^2 - 56*b^3*c^2*x^4 + 288*a*b*c^3*
x^4 + 48*b^2*c^3*x^6 + 2240*a*c^4*x^6 + 1664*b*c^4*x^8 + 1280*c^5*x^10))/(1
5360*c^4) + ((-7*b^6 + 60*a*b^4*c - 144*a^2*b^2*c^2 + 64*a^3*c^3)*Log[b + 2
*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(2048*c^(9/2))
```

fricas [A] time = 2.37, size = 451, normalized size = 2.21

$$\left[-\frac{15\left(7b^6-60ab^4c+144a^2b^2c^2-64a^3c^3\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)-4\left(1280c^6x^{10}+1664bc^5x^8+16\left(3b^2c^4+140ac^5\right)x^6-105b^5c+760ab^3c^2-1296a^2bc^3-8\left(7b^3c^3-36abc^4\right)x^4+2\left(35b^4c^2-216ab^2c^3+240a^2c^4\right)x^2\right)\sqrt{cx^4+bx^2+a}}{61440c^5}, -\frac{15\left(7b^6-60ab^4c+144a^2b^2c^2-64a^3c^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-2\left(1280c^6x^{10}+1664bc^5x^8+16\left(3b^2c^4+140ac^5\right)x^6-105b^5c+760ab^3c^2-1296a^2bc^3-8\left(7b^3c^3-36abc^4\right)x^4+2\left(35b^4c^2-216ab^2c^3+240a^2c^4\right)x^2\right)\sqrt{cx^4+bx^2+a}}{30720c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^10 + 1664*b*c^5*x^8 + 16*(3*b^2*c^4 + 140*a
*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 - 8*(7*b^3*c^3 - 36*
a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^2)*sqrt(c*x^4
 + b*x^2 + a))/c^5, -1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64
*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c
)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(1280*c^6*x^10 + 1664*b*c^5*x^8 + 16*(3*b^
2*c^4 + 140*a*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 - 8*(7*
b^3*c^3 - 36*a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^
2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

giac [B] time = 0.40, size = 535, normalized size = 2.62

$$\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{7}{2}}}\right)a+\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-460ab^2c+256a^2c^2}{c^4}\right)-\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{9}{2}}}\right)b+\frac{1}{30720}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\left(8\left(10x^2+\frac{b}{c}\right)x^2-\frac{9b^2c^3-20ac^4}{c^5}\right)x^2+\frac{21b^3c^2-68abc^3}{c^5}\right)x^2-\frac{105b^4c-448ab^2c^2+240a^2c^3}{c^5}\right)x^2+\frac{315b^5-1680ab^3c+1808a^2bc^2}{c^5}\right)+\frac{15\left(21b^6-140ab^4c+240a^2b^2c^2-64a^3c^3\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{11}{2}}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*
c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c
^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(7/2
))*a + 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^
2*c^2 - 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 -
 460*a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*l
og(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2))*b
+ 1/30720*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*x^2 + b/c)*x^2 - (9*b^
2*c^3 - 20*a*c^4)/c^5)*x^2 + (21*b^3*c^2 - 68*a*b*c^3)/c^5)*x^2 - (105*b^4*
c - 448*a*b^2*c^2 + 240*a^2*c^3)/c^5)*x^2 + (315*b^5 - 1680*a*b^3*c + 1808*
a^2*b*c^2)/c^5) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*
log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(11/2))*
c
```

maple [B] time = 0.02, size = 432, normalized size = 2.12

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;c\,x^{10}}{12}+\frac{13\sqrt{c\,x^{4}+b\,x^{2}+a}\;b\,x^{8}}{120}+\frac{7\sqrt{c\,x^{4}+b\,x^{2}+a}\;a\,x^{6}}{48}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{2}x^{6}}{320c}+\frac{3\sqrt{c\,x^{4}+b\,x^{2}+a}\;ab\,x^{4}}{160c}-\frac{7\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{3}x^{4}}{1920c^{2}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;a^{2}x^{2}}{32c}-\frac{9\sqrt{c\,x^{4}+b\,x^{2}+a}\;a\,b^{2}x^{2}}{320c^{2}}+\frac{7\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{4}x^{2}}{1536c^{3}}-\frac{a^{3}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{3}{2}}}+\frac{9a^{2}b^{2}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{128c^{\frac{5}{2}}}-\frac{15a\,b^{4}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{512c^{\frac{7}{2}}}+\frac{7b^{6}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2048c^{\frac{9}{2}}}-\frac{27\sqrt{c\,x^{4}+b\,x^{2}+a}\;a^{2}b}{320c^{2}}+\frac{19\sqrt{c\,x^{4}+b\,x^{2}+a}\;a\,b^{3}}{384c^{3}}-\frac{7\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{5}}{1024c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 7/1536*b^4/c^3*x^2*(c*x^4+b*x^2+a)^(1/2)+1/320*b^2*x^6/c*(c*x^4+b*x^2+a)^(1
/2)-7/1920*b^3/c^2*x^4*(c*x^4+b*x^2+a)^(1/2)+1/32*a^2*x^2/c*(c*x^4+b*x^2+a)
^(1/2)-15/512*a*b^4/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
+9/128*a^2*b^2/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-27/3
20*a^2*b/c^2*(c*x^4+b*x^2+a)^(1/2)+19/384*a*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)+1
3/120*b*x^8*(c*x^4+b*x^2+a)^(1/2)+3/160*a*b*x^4/c*(c*x^4+b*x^2+a)^(1/2)-9/3
20*a*b^2/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)-1/32*a^3/c^(3/2)*ln((c*x^2+1/2*b)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/12*c*x^10*(c*x^4+b*x^2+a)^(1/2)+7/48*a*x^6*(
c*x^4+b*x^2+a)^(1/2)-7/1024*b^5/c^4*(c*x^4+b*x^2+a)^(1/2)+7/2048*b^6/c^(9/2
)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^5*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5} \left(a + b x^{2} + c x^{4}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.735 $\int x^3 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=150

$$-\frac{3b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}+\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^3}{32c^2}$$

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 640, 612, 621, 206}

$$\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{3b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3) - (b*(b
 + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (a + b*x^2 + c*x^4)^(5/2)
/(10*c) - (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*
x^2 + c*x^4])])/(512*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$
$$=\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}-\frac{b\,\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{4c}$$
$$=-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}+\frac{\left(3b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\right.}{64}$$
$$=\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{(a+}{}$$
$$=\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{(a+}{}$$
$$=\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{(a+}{}$$

Mathematica [A] time = 0.14, size = 149, normalized size = 0.99

$$-\frac{3b\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)}{512c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/32*(b*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/c^2 + (a + b*x^2 + c*x^4)
^(5/2)/(10*c) - (3*b*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2
```

```
 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])]))/(512*c^(7/2))
```

IntegrateAlgebraic [A] time = 0.60, size = 162, normalized size = 1.08

$$\frac{3\left(16a^2bc^2-8ab^3c+b^5\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{512c^{7/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(128a^2c^2-100ab^2c+56abc^2x^2+256ac^3x^4+15b^4-10b^3cx^2+8b^2c^2x^4+176bc^3x^6+128c^4x^8\right)}{1280c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^4 - 100*a*b^2*c + 128*a^2*c^2 - 10*b^3*c*x^2
 + 56*a*b*c^2*x^2 + 8*b^2*c^2*x^4 + 256*a*c^3*x^4 + 176*b*c^3*x^6 + 128*c^4
*x^8))/(1280*c^3) + (3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*Log[b + 2*c*x^2 - 2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(512*c^(7/2))
```

fricas [A] time = 0.74, size = 361, normalized size = 2.41

$$\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+4\left(128c^5x^8+176bc^4x^6+15b^4c-100ab^2c^2+128a^2c^3+8\left(b^2c^3+32ac^4\right)x^4-2\left(5b^3c^2-28abc^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{5120c^4}, \frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)+2\left(128c^5x^8+176bc^4x^6+15b^4c-100ab^2c^2+128a^2c^3+8\left(b^2c^3+32ac^4\right)x^4-2\left(5b^3c^2-28abc^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{2560c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c
*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(
128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b
^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2
 + a))/c^4, 1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/2
*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))
+ 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 +
 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^4 +
b*x^2 + a))/c^4]
```

giac [B] time = 0.39, size = 414, normalized size = 2.76

$$\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3\left(b^3-4abc\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}\right)a+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{7}{2}}}\right)b+\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-460ab^2c+256a^2c^2}{c^4}\right)-\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{9}{2}}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2)
 - 3*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) - b))/c^(5/2))*a + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c
)*x^2 - (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
```

```
a))*sqrt(c) - b))/c^(7/2))*b + 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(
8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^
2)/c^4)*x^2 - (105*b^4 - 460*a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a
*b^3*c + 48*a^2*b*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*s
qrt(c) - b))/c^(9/2))*c
```

maple [B] time = 0.02, size = 316, normalized size = 2.11

$$\frac{\sqrt{c x^4+b x^2+a}\, c x^8}{10}+\frac{11\sqrt{c x^4+b x^2+a}\, b x^6}{80}+\frac{\sqrt{c x^4+b x^2+a}\, a x^4}{5}+\frac{\sqrt{c x^4+b x^2+a}\, b^2 x^4}{160c}+\frac{7\sqrt{c x^4+b x^2+a}\, a b x^2}{160c}-\frac{\sqrt{c x^4+b x^2+a}\, b^3 x^2}{128c^2}-\frac{3a^2 b \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{32c^{\frac{3}{2}}}+\frac{3a b^3 \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{64c^{\frac{5}{2}}}-\frac{3b^5 \ln\left(\frac{c x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{512c^{\frac{7}{2}}}+\frac{\sqrt{c x^4+b x^2+a}\, a^2}{10c}-\frac{5\sqrt{c x^4+b x^2+a}\, a b^2}{64c^2}+\frac{3\sqrt{c x^4+b x^2+a}\, b^4}{256c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/160*b^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)-1/128*b^3/c^2*x^2*(c*x^4+b*x^2+a)^(1/
2)+3/64*a*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/32*
a^2*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-5/64*a*b^2/c^
2*(c*x^4+b*x^2+a)^(1/2)+7/160*a*b*x^2/c*(c*x^4+b*x^2+a)^(1/2)+1/10*c*x^8*(c
*x^4+b*x^2+a)^(1/2)+1/5*a*x^4*(c*x^4+b*x^2+a)^(1/2)+3/256*b^4/c^3*(c*x^4+b*
x^2+a)^(1/2)-3/512*b^5/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/
2))+1/10*a^2/c*(c*x^4+b*x^2+a)^(1/2)+11/80*b*x^6*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.88, size = 223, normalized size = 1.49

$$\frac{\left(c x^4+b x^2+a\right)^{5/2}}{10 c}-\frac{b\left(\frac{3 a\left(\ln\left(\sqrt{c x^4+b x^2+a}+\frac{c x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\sqrt{c}}-\frac{b^2}{8 c^{3/2}}\right)+\frac{\left(2 c x^2+b\right)\sqrt{c x^4+b x^2+a}}{4 c}\right)}{4}+\frac{x^2\left(c x^4+b x^2+a\right)^{3/2}}{4}-\frac{3 b^2\left(\ln\left(\sqrt{c x^4+b x^2+a}+\frac{c x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\sqrt{c}}-\frac{b^2}{8 c^{3/2}}\right)+\frac{\left(2 c x^2+b\right)\sqrt{c x^4+b x^2+a}}{4 c}\right)}{16 c}+\frac{b\left(c x^4+b x^2+a\right)^{3/2}}{8 c}\right)}{4 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (a + b*x^2 + c*x^4)^(5/2)/(10*c) - (b*((3*a*(log((a + b*x^2 + c*x^4)^(1/2)
+ (b/2 + c*x^2)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^2)
```

```
*(a + b*x^2 + c*x^4)^(1/2))/(4*c)))/4 + (x^2*(a + b*x^2 + c*x^4)^(3/2))/4 -
 (3*b^2*(log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))*(a/(2*c^(1/
2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(4*c)))/
(16*c) + (b*(a + b*x^2 + c*x^4)^(3/2))/(8*c)))/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.736 $\int x\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}$$

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1107, 612, 621, 206}

$$-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^2) + ((b +
2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*c) + (3*(b^2 - 4*a*c)^2*ArcTanh[(b
+ 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int x\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}-\frac{\left(3\left(b^2-4ac\right)\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{32c}$$

$$=-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3\left(b^2-\right.\right.}{}$$

$$=-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3\left(b^2-\right.\right.}{}$$

$$=-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{3\left(b^2-\right.}{}$$

Mathematica [A] time = 0.09, size = 126, normalized size = 1.02

$$\frac{\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)}{8c^{3/2}}+2\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2) + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(
b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/
(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(8*c^(3/2)))/(32*c)
```

IntegrateAlgebraic [A] time = 0.47, size = 130, normalized size = 1.05

$$\frac{\sqrt{a+bx^2+cx^4}\left(20abc+40ac^2x^2-3b^3+2b^2cx^2+24bc^2x^4+16c^3x^6\right)}{128c^2}-\frac{3\left(16a^2c^2-8ab^2c+b^4\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{256c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-3*b^3 + 20*a*b*c + 2*b^2*c*x^2 + 40*a*c^2*x^2 +
24*b*c^2*x^4 + 16*c^3*x^6))/(128*c^2) - (3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*L
og[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(256*c^(5/2))
```

fricas [A] time = 0.85, size = 297, normalized size = 2.40

$$\left[\frac{3\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\left(2\,cx^{2}+b\right)\sqrt{c}-4\,ac\right)+4\left(16\,c^{4}x^{6}+24\,bc^{3}x^{4}-3\,b^{3}c+20\,abc^{2}+2\left(b^{2}c^{2}+20\,ac^{3}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}+a}}{512\,c^{3}},-\frac{3\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\left(2\,cx^{2}+b\right)\sqrt{-c}}{2\left(c^{2}x^{4}+bcx^{2}+ac\right)}\right)-2\left(16\,c^{4}x^{6}+24\,bc^{3}x^{4}-3\,b^{3}c+20\,abc^{2}+2\left(b^{2}c^{2}+20\,ac^{3}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}+a}}{256\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2
 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(16*c
^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*
sqrt(c*x^4 + b*x^2 + a))/c^3, -1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b
*c*x^2 + a*c)) - 2*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b
^2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

giac [B] time = 0.39, size = 317, normalized size = 2.56

$$\frac{1}{16}\left(2\,\sqrt{cx^{4}+bx^{2}+a}\left(2\,x^{2}+\frac{b}{c}\right)+\frac{\left(b^{2}-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}\right)a+\frac{1}{96}\left(2\,\sqrt{cx^{4}+bx^{2}+a}\left(2\left(4\,x^{2}+\frac{b}{c}\right)x^{2}-\frac{3\,b^{2}-8\,ac}{c^{2}}\right)-\frac{3\left(b^{3}-4\,abc\right)\log\left(\left|-2\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}\right)b+\frac{1}{768}\left(2\,\sqrt{cx^{4}+bx^{2}+a}\left(2\left(4\left(6\,x^{2}+\frac{b}{c}\right)x^{2}-\frac{5\,b^{2}c-12\,ac^{2}}{c^{3}}\right)x^{2}+\frac{15\,b^{3}-52\,abc}{c^{3}}\right)+\frac{3\left(5\,b^{4}-24\,ab^{2}c+16\,a^{2}c^{2}\right)\log\left(\left|-2\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{7}{2}}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(-2*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2))*a + 1/96*(2*sq
rt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3
- 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))
/c^(5/2))*b + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5
*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^
2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(7/2))*c
```

maple [B] time = 0.02, size = 242, normalized size = 1.95

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;c\,x^{6}}{8}+\frac{3\sqrt{c\,x^{4}+b\,x^{2}+a}\;b\,x^{4}}{16}+\frac{5\sqrt{c\,x^{4}+b\,x^{2}+a}\;a\,x^{2}}{16}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{2}x^{2}}{64c}+\frac{3a^{2}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{16\sqrt{c}}-\frac{3a\,b^{2}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{3}{2}}}+\frac{3b^{4}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{256c^{\frac{5}{2}}}+\frac{5\sqrt{c\,x^{4}+b\,x^{2}+a}\;ab}{32c}-\frac{3\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{3}}{128c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 5/16*a*x^2*(c*x^4+b*x^2+a)^(1/2)-3/128*b^3/c^2*(c*x^4+b*x^2+a)^(1/2)+3/256*
b^4/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/64*b^2*x^2/c*
```

```
(c*x^4+b*x^2+a)^(1/2)+5/32*a*b/c*(c*x^4+b*x^2+a)^(1/2)-3/32*a*b^2/c^(3/2)*l
n((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*a^2*ln((c*x^2+1/2*b)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/16*b*x^4*(c*x^4+b*x^2+a)^(1/2)+1/8*c
*x^6*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.96, size = 115, normalized size = 0.93

$$\frac{\left(c\,x^2+\frac{b}{2}\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{8\,c}+\frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\,\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2+a}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b/2 + c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(8*c) + ((3*a*c - (3*b^2)/4)*((b/
(4*c) + x^2/2)*(a + b*x^2 + c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) +
 (b/2 + c*x^2)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(8*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.737 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=155

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}+\frac{\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+c}}{16c}$$

Rubi [A] time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 734, 814, 843, 621, 206, 724}

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}+\frac{\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c}+\frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c) + (a + b*x^2 + c
*x^4)^(3/2)/6 - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
c*x^4])])/2 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(32*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - \frac{1}{4}\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12a\right.}{x\sqrt{a+bx+cx^2}}\right.}{16c}$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+b}}\right.$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - a^2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx\right.$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a}}\right.$$

Mathematica [A] time = 0.15, size = 143, normalized size = 0.92

$$\frac{1}{96}\left(-48a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) - \frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{c^{3/2}} + \frac{2\sqrt{a+bx^2+cx^4}\left(8c\left(4a+cx^4\right)+3b^2+14bcx^2\right)}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((2*Sqrt[a + b*x^2 + c*x^4]*(3*b^2 + 14*b*c*x^2 + 8*c*(4*a + c*x^4)))/c - 4
8*a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] - (3*b
*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])
/c^(3/2))/96
```

IntegrateAlgebraic [A] time = 0.62, size = 148, normalized size = 0.95

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right) + \frac{\left(b^3-12abc\right)\log\left(-2c^{3/2}\sqrt{a+bx^2+cx^4}+bc+2c^2x^2\right)}{32c^{3/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(32ac+3b^2+14bcx^2+8c^2x^4\right)}{48c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(3*b^2 + 32*a*c + 14*b*c*x^2 + 8*c^2*x^4))/(48*c)
+ a^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]]
+ ((b^3 - 12*a*b*c)*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]
])/(32*c^(3/2))
```

fricas [A] time = 1.46, size = 727, normalized size = 4.69

$$\left[\frac{48\,a^{\frac{3}{2}} c^{2} \log\left(-\frac{{\left(b^{2} + 4\,a c\right)} x^{4} + 8\,a b x^{2} - 4\,\sqrt{c x^{4} + b x^{2} + a} {\left(b x^{2} + 2\,a\right)} \sqrt{a} + 8\,a^{2}}{x^{4}}\right) - 3\,{\left(b^{3} - 12\,a b c\right)} \sqrt{c} \log\left(-8\,c^{2} x^{4} - 8\,b c x^{2} - b^{2} - 4\,\sqrt{c x^{4} + b x^{2} + a} {\left(2\,c x^{2} + b\right)} \sqrt{c} - 4\,a c\right) + 4\,{\left(8\,c^{3} x^{4} + 14\,b c^{2} x^{2} + 3\,b^{2} c + 32\,a c^{2}\right)} \sqrt{c x^{4} + b x^{2} + a}}{192\,c^{2}}, \frac{24\,a^{\frac{3}{2}} c^{2} \log\left(-\frac{{\left(b^{2} + 4\,a c\right)} x^{4} + 8\,a b x^{2} - 4\,\sqrt{c x^{4} + b x^{2} + a} {\left(b x^{2} + 2\,a\right)} \sqrt{a} + 8\,a^{2}}{x^{4}}\right) + 3\,{\left(b^{3} - 12\,a b c\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{4} + b x^{2} + a} {\left(2\,c x^{2} + b\right)} \sqrt{-c}}{2\,{\left(c^{2} x^{4} + b c x^{2} + a c\right)}}\right) + 2\,{\left(8\,c^{3} x^{4} + 14\,b c^{2} x^{2} + 3\,b^{2} c + 32\,a c^{2}\right)} \sqrt{c x^{4} + b x^{2} + a}}{96\,c^{2}}, \frac{96\,\sqrt{-a} a c^{2} \arctan\left(\frac{\sqrt{c x^{4} + b x^{2} + a} {\left(b x^{2} + 2\,a\right)} \sqrt{-a}}{2\,{\left(a c x^{4} + a b x^{2} + a^{2}\right)}}\right) - 3\,{\left(b^{3} - 12\,a b c\right)} \sqrt{c} \log\left(-8\,c^{2} x^{4} - 8\,b c x^{2} - b^{2} - 4\,\sqrt{c x^{4} + b x^{2} + a} {\left(2\,c x^{2} + b\right)} \sqrt{c} - 4\,a c\right) + 4\,{\left(8\,c^{3} x^{4} + 14\,b c^{2} x^{2} + 3\,b^{2} c + 32\,a c^{2}\right)} \sqrt{c x^{4} + b x^{2} + a}}{192\,c^{2}}, \frac{48\,\sqrt{-a} a c^{2} \arctan\left(\frac{\sqrt{c x^{4} + b x^{2} + a} {\left(b x^{2} + 2\,a\right)} \sqrt{-a}}{2\,{\left(a c x^{4} + a b x^{2} + a^{2}\right)}}\right) + 3\,{\left(b^{3} - 12\,a b c\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{4} + b x^{2} + a} {\left(2\,c x^{2} + b\right)} \sqrt{-c}}{2\,{\left(c^{2} x^{4} + b c x^{2} + a c\right)}}\right) + 2\,{\left(8\,c^{3} x^{4} + 14\,b c^{2} x^{2} + 3\,b^{2} c + 32\,a c^{2}\right)} \sqrt{c x^{4} + b x^{2} + a}}{96\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/192*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 3*(b^3 - 12*a*b*c)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt
(c*x^4 + b*x^2 + a))/c^2, 1/96*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 3
*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)
*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*
c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/192*(96*sqrt(-a)*a*c^2*arctan
(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^
2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^4 + 14*b*c^2
*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/96*(48*sqrt(-a)*
a*c^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 + 14
*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.index.cc index_m operator + Erro
r: Bad Argument Value
```

maple [A] time = 0.02, size = 192, normalized size = 1.24

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;c\,x^{4}}{6}+\frac{7\sqrt{c\,x^{4}+b\,x^{2}+a}\;b\,x^{2}}{24}-\frac{a^{\frac{3}{2}}\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{a}}{x^{2}}\right)}{2}+\frac{3ab\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{8\sqrt{c}}-\frac{b^{3}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{3}{2}}}+\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\;a}{3}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{2}}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/6*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/c*b^2
*(c*x^4+b*x^2+a)^(1/2)-1/32/c^(3/2)*b^3*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))+3/8*a*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2
)+2/3*a*(c*x^4+b*x^2+a)^(1/2)-1/2*a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(
1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x, x)
```

3.738 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=150

$$\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2}+\frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{2\cdots}{2\sqrt{a}\sqrt{a\cdots}}\right.$$

Rubi **[A]** time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 732, 814, 843, 621, 206, 724}

$$\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2}+\frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/8 - (a + b*x^2 + c*x^4)^(3/2)/(
2*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x
^4])])/4 + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2
 + c*x^4])])/(16*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} + \frac{3}{4}\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx,x,x^2\right)$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} - \frac{3\,\text{Subst}\left(\int \frac{-4abc-c\left(b^2+4ac\right)x}{x\sqrt{a+bx+cx^2}}\,dx,x,\right.}{16c}$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} + \frac{1}{4}(3ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+c}}\right.$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} - \frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\right.$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} - \frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+}}\right.$$

Mathematica [A] time = 0.12, size = 134, normalized size = 0.89

$$\frac{1}{16}\left(\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}} + \frac{2\sqrt{a+bx^2+cx^4}\left(-4a+5bx^2+2cx^4\right)}{x^2} - 12\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] ((2*Sqrt[a + b*x^2 + c*x^4]*(-4*a + 5*b*x^2 + 2*c*x^4))/x^2 - 12*Sqrt[a]*b*
ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] + (3*(b^2 + 4*a*
c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c])/16
```

IntegrateAlgebraic [A] time = 0.67, size = 138, normalized size = 0.92

$$-\frac{3\left(4ac+b^2\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{16\sqrt{c}} + \frac{\sqrt{a+bx^2+cx^4}\left(-4a+5bx^2+2cx^4\right)}{8x^2} + \frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-4*a + 5*b*x^2 + 2*c*x^4))/(8*x^2) + (3*Sqrt[a]*b
*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/2 - (3*(
b^2 + 4*a*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(16*Sqrt
[c])
```

fricas [A] time = 2.13, size = 713, normalized size = 4.75

$$\left[\frac{12\,\sqrt{a}bcx^{2}\log\left(-\frac{(b^{2}+4ac)x^{4}+8abx^{2}-4\sqrt{cx^{4}+bx^{2}+a}(bx^{2}+2a)\sqrt{a}+8a^{2}}{x^{4}}\right)+3\left(b^{2}+4ac\right)\sqrt{c}x^{2}\log\left(-8c^{2}x^{4}-8bcx^{2}-b^{2}-4\sqrt{cx^{4}+bx^{2}+a}\left(2cx^{2}+b\right)\sqrt{c}-4ac\right)+4\left(2c^{2}x^{4}+5bcx^{2}-4ac\right)\sqrt{cx^{4}+bx^{2}+a}}{32cx^{2}}, \frac{6\,\sqrt{a}bcx^{2}\log\left(-\frac{(b^{2}+4ac)x^{4}+8abx^{2}-4\sqrt{cx^{4}+bx^{2}+a}(bx^{2}+2a)\sqrt{a}+8a^{2}}{x^{4}}\right)-3\left(b^{2}+4ac\right)\sqrt{-c}x^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}(2cx^{2}+b)\sqrt{-c}}{2(c^{2}x^{4}+bcx^{2}+ac)}\right)+2\left(2c^{2}x^{4}+5bcx^{2}-4ac\right)\sqrt{cx^{4}+bx^{2}+a}}{16cx^{2}}, \frac{24\,\sqrt{-a}bcx^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}(bx^{2}+2a)\sqrt{-a}}{2(acx^{4}+abx^{2}+a^{2})}\right)+3\left(b^{2}+4ac\right)\sqrt{c}x^{2}\log\left(-8c^{2}x^{4}-8bcx^{2}-b^{2}-4\sqrt{cx^{4}+bx^{2}+a}\left(2cx^{2}+b\right)\sqrt{c}-4ac\right)+4\left(2c^{2}x^{4}+5bcx^{2}-4ac\right)\sqrt{cx^{4}+bx^{2}+a}}{32cx^{2}}, \frac{12\,\sqrt{-a}bcx^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}(bx^{2}+2a)\sqrt{-a}}{2(acx^{4}+abx^{2}+a^{2})}\right)-3\left(b^{2}+4ac\right)\sqrt{-c}x^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}(2cx^{2}+b)\sqrt{-c}}{2(c^{2}x^{4}+bcx^{2}+ac)}\right)+2\left(2c^{2}x^{4}+5bcx^{2}-4ac\right)\sqrt{cx^{4}+bx^{2}+a}}{16cx^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^
4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 3*(b^2 + 4*a*c)*sqrt(c
)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c*x^4 + b*x
^2 + a))/(c*x^2), 1/16*(6*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c
*x^4 + b*x^2 + a))/(c*x^2), 1/32*(24*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 3*(b^2 +
4*a*c)*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqr
t(c*x^4 + b*x^2 + a))/(c*x^2), 1/16*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*
x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c
*x^4 + b*x^2 + a))/(c*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{
-1,[1]%%%}]%%},[2,1,0]%%%}+%%%{%%{[1,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,0]%%%}
 / %%%{%%%{1,[1]%%%},[4,0,0]%%%}+%%%{%%%{-2,[1]%%%},[2,1,0]%%%}+%%%{%%%{1,[
1]%%%},[0,2,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.02, size = 170, normalized size = 1.13

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^2}{4}+\frac{3a\sqrt{c}\;\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4}-\frac{3\sqrt{a}\;b\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{4}+\frac{3b^2\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16\sqrt{c}}+\frac{5\sqrt{c\,x^4+b\,x^2+a}\;b}{8}-\frac{\sqrt{c\,x^4+b\,x^2+a}\;a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] 1/4*c*x^2*(c*x^4+b*x^2+a)^(1/2)+5/8*b*(c*x^4+b*x^2+a)^(1/2)+3/16*b^2*ln((c*
x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4*a*c^(1/2)*ln((c*x^2+1
/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2*a/x^2*(c*x^4+b*x^2+a)^(1/2)-3/4*a^
(1/2)*b*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^3,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**3, x)
```

3.739 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=151

$$-\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}-\frac{(a+bx^2+cx^4)^{3/2}}{4x^4}-\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2}+\frac{3}{4}b\sqrt{c}\tanh^{-1}\left(\frac{}{2\sqrt{c}\ \cdots}\right.$$

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 732, 812, 843, 621, 206, 724}

$$-\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}-\frac{(a+bx^2+cx^4)^{3/2}}{4x^4}-\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2}+\frac{3}{4}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (-3*(b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*x^2) - (a + b*x^2 + c*x^4)^(3
/2)/(4*x^4) - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*
x^2 + c*x^4])])/(16*Sqrt[a]) + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c
]*Sqrt[a + b*x^2 + c*x^4])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} + \frac{3}{8}\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} - \frac{3}{16}\,\text{Subst}\left(\int \frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} + \frac{1}{4}(3bc)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+c}}\right.$$

$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} + \frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x\right.$$

$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} - \frac{3\left(b^2+4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx}}\right.}{16\sqrt{a}}$$

Mathematica [A] time = 0.17, size = 134, normalized size = 0.89

$$\frac{1}{16}\left(-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}} - \frac{2\sqrt{a+bx^2+cx^4}\left(2a+5bx^2-4cx^4\right)}{x^4} + 12b\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] ((-2*(2*a + 5*b*x^2 - 4*c*x^4)*Sqrt[a + b*x^2 + c*x^4])/x^4 - (3*(b^2 + 4*a
*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a] + 1
2*b*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/16
```

IntegrateAlgebraic [A] time = 0.71, size = 134, normalized size = 0.89

$$\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8\sqrt{a}} + \frac{\sqrt{a+bx^2+cx^4}\left(-2a-5bx^2+4cx^4\right)}{8x^4} - \frac{3}{4}b\sqrt{c}\,\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-2*a - 5*b*x^2 + 4*c*x^4))/(8*x^4) + (3*(b^2 + 4*
a*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*Sqrt[a])
- (3*b*Sqrt[c]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/4
```

fricas [A] time = 2.08, size = 713, normalized size = 4.72

$$\left[\frac{12\,ab\sqrt{c}x^4\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+3\left(b^2+4\,ac\right)\sqrt{a}x^4\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(4\,acx^4-5\,abx^2-2\,a^2\right)\sqrt{cx^4+bx^2+a}}{32\,ax^4}, -\frac{24\,ab\sqrt{-c}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-3\left(b^2+4\,ac\right)\sqrt{a}x^4\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\left(4\,acx^4-5\,abx^2-2\,a^2\right)\sqrt{cx^4+bx^2+a}}{32\,ax^4}, \frac{6\,ab\sqrt{c}x^4\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+3\left(b^2+4\,ac\right)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(4\,acx^4-5\,abx^2-2\,a^2\right)\sqrt{cx^4+bx^2+a}}{16\,ax^4}, -\frac{12\,ab\sqrt{-c}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-3\left(b^2+4\,ac\right)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)-2\left(4\,acx^4-5\,abx^2-2\,a^2\right)\sqrt{cx^4+bx^2+a}}{16\,ax^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(12*a*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^4*lo
g(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(a) + 8*a^2)/x^4) + 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt(c*x^4 + b*x
^2 + a))/(a*x^4), -1/32*(24*a*b*sqrt(-c)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 3*(b^2 + 4*a*c)*sq
rt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt
(c*x^4 + b*x^2 + a))/(a*x^4), 1/16*(6*a*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*
c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*
(b^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sq
rt(c*x^4 + b*x^2 + a))/(a*x^4), -1/16*(12*a*b*sqrt(-c)*x^4*arctan(1/2*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 3*(b
^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*s
qrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt
(c*x^4 + b*x^2 + a))/(a*x^4)]
```

giac [B] time = 0.45, size = 302, normalized size = 2.00

$$-\frac{3}{4}\,b\sqrt{c}\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)+\frac{1}{2}\,\sqrt{cx^4+bx^2+a}\,c+\frac{3\left(b^2+4\,ac\right)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}}+\frac{5\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2+4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3ac+16\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2ab\sqrt{c}-3\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)ab^2+4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)a^2c-8\,a^2b\sqrt{c}}{8\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/4*b*sqrt(c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) +
b)) + 1/2*sqrt(c*x^4 + b*x^2 + a)*c + 3/8*(b^2 + 4*a*c)*arctan(-(sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) + 1/8*(5*(sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))^3*b^2 + 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a
*c + 16*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a*b*sqrt(c) - 3*(sqrt(c)*
x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 + 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*a^2*c - 8*a^2*b*sqrt(c))/((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 -
 a)^2
```

maple [A] time = 0.02, size = 174, normalized size = 1.15

$$-\frac{3\sqrt{a}\, c \ln\left(\frac{b x^{2}+2 a+2 \sqrt{c x^{4}+b x^{2}+a}\, \sqrt{a}}{x^{2}}\right)}{4}-\frac{3 b^{2} \ln\left(\frac{b x^{2}+2 a+2 \sqrt{c x^{4}+b x^{2}+a}\, \sqrt{a}}{x^{2}}\right)}{16 \sqrt{a}}+\frac{3 b \sqrt{c}\, \ln\left(\frac{c x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{4}+b x^{2}+a}\right)}{4}+\frac{\sqrt{c x^{4}+b x^{2}+a}\, c}{2}-\frac{5 \sqrt{c x^{4}+b x^{2}+a}\, b}{8 x^{2}}-\frac{\sqrt{c x^{4}+b x^{2}+a}\, a}{4 x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^5,x)
```

```
[Out] 1/2*c*(c*x^4+b*x^2+a)^(1/2)+3/4*b*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))-1/4*a/x^4*(c*x^4+b*x^2+a)^(1/2)-5/8*b/x^2*(c*x^4+b*x^2+a)^(1
/2)-3/16/a^(1/2)*b^2*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3/
4*a^(1/2)*c*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^5,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b x^{2} + c x^{4}\right)^{\frac{3}{2}}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**5,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**5, x)
```

3.740 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=163

$$\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}-\frac{(x^2(8ac+b^2)+2ab)\sqrt{a+bx^2+cx^4}}{16ax^4}+\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right.$$

Rubi [A] time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 732, 810, 843, 621, 206, 724}

$$\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}-\frac{(x^2(8ac+b^2)+2ab)\sqrt{a+bx^2+cx^4}}{16ax^4}+\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-\frac{(a+bx^2+cx^4)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a*x^4) - (a + b*
x^2 + c*x^4)^(3/2)/(6*x^6) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqr
t[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + \frac{1}{4}\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right)$$
$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b\left(b^2-12ac\right)-8a}{x\sqrt{a+bx+cx^2}}\right.}{16a}$$
$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + \frac{1}{2}c^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.$$
$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + c^2\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,d\right.$$
$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + \frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\right.}{32a^{3/2}}$$

Mathematica [A]   time = 0.22, size = 149, normalized size = 0.91

$$\frac{1}{96}\left(\frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}} - \frac{2\sqrt{a+bx^2+cx^4}\left(8a^2+14abx^2+32acx^4+3b^2x^4\right)}{ax^6} + 48c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] ((-2*Sqrt[a + b*x^2 + c*x^4]*(8*a^2 + 14*a*b*x^2 + 3*b^2*x^4 + 32*a*c*x^4))
/(a*x^6) + (3*b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*
x^2 + c*x^4])])/a^(3/2) + 48*c^(3/2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4])])/96
```

IntegrateAlgebraic [A]   time = 0.88, size = 148, normalized size = 0.91

$$\frac{\left(b^3-12abc\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{3/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(-8a^2-14abx^2-32acx^4-3b^2x^4\right)}{48ax^6} - \frac{1}{2}c^{3/2}\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 - 14*a*b*x^2 - 3*b^2*x^4 - 32*a*c*x^4))/(4
8*a*x^6) + ((b^3 - 12*a*b*c)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x
^4])/Sqrt[a]])/(16*a^(3/2)) - (c^(3/2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4]])/2
```

fricas [A] time = 1.56, size = 771, normalized size = 4.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(48*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^
6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8
*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6), -1/192*(96*a^2*sqrt(-c)*c*x^6*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 +
 a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(14*a^
2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x
^6), 1/96*(24*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-a
)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) - 2*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(
c*x^4 + b*x^2 + a))/(a^2*x^6), -1/96*(48*a^2*sqrt(-c)*c*x^6*arctan(1/2*sqrt
(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 3*(
b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*
a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^
2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6)]
```

giac [B] time = 0.68, size = 412, normalized size = 2.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*c^(3/2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b
)) - 1/16*(b^3 - 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/
sqrt(-a))/(sqrt(-a)*a) + 1/48*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*
b^3*sqrt(c) + 60*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c^(3/2) + 48
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a*b^2*c + 96*(sqrt(c)*x^2 - sqrt
```

```
(c*x^4 + b*x^2 + a))^4*a^2*c^2 + 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^
3*a*b^3*sqrt(c) - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*c^2 - 3*
(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*b^3*sqrt(c) + 36*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^3*b*c^(3/2) + 64*a^4*c^2)/(((sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^2 - a)^3*a*sqrt(c))
```

maple [A] time = 0.02, size = 202, normalized size = 1.24

$$-\frac{3bc\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{8\sqrt{a}}+\frac{b^3\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{3}{2}}}+\frac{c^{\frac{3}{2}}\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,b^2}{16a\,x^2}-\frac{2\sqrt{c\,x^4+b\,x^2+a}\,c}{3x^2}-\frac{7\sqrt{c\,x^4+b\,x^2+a}\,b}{24x^4}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^7,x)
```

```
[Out] 1/2*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-7/24*b/x^4*(c*x
^4+b*x^2+a)^(1/2)-1/16/a*b^2/x^2*(c*x^4+b*x^2+a)^(1/2)+1/32/a^(3/2)*b^3*ln(
(b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3/8*b*c/a^(1/2)*ln((b*x^2+
2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-2/3*c/x^2*(c*x^4+b*x^2+a)^(1/2)-1
/6*a/x^6*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^7,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**7, x)
```

3.741 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=133

$$-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}+\frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8}$$

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1114, 720, 724, 206}

$$\frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (3*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^2*x^4) - ((2
*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*a*x^8) - (3*(b^2 - 4*a*c)^2*ArcT
anh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(256*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^9}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{\left(3\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^2\right)}{32a}$$

$$= \frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} + \frac{\left(3\left(b^2-4\right.\right.}{}$$

$$= \frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{\left(3\left(b^2-4\right.\right.}{}$$

$$= \frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{3\left(b^2-4a\right.}{}$$

Mathematica [A] time = 0.17, size = 138, normalized size = 1.04

$$-\frac{\frac{3\left(b^2-4ac\right)\left(x^4\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{a}\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}\right)}{8a^{3/2}x^4}+\frac{2\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{x^8}}{32a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/32*((2*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/x^8 + (3*(b^2 - 4*a*c)*(
-2*Sqrt[a]*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*x^4*ArcTan
h[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]))/(8*a^(3/2)*x^4))/a
```

IntegrateAlgebraic [A] time = 1.00, size = 139, normalized size = 1.05

$$\frac{3\left(16a^2c^2-8ab^2c+b^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-16a^3-24a^2bx^2-40a^2cx^4-2ab^2x^4-20abcx^6+3b^3x^6\right)}{128a^2x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-16*a^3 - 24*a^2*b*x^2 - 2*a*b^2*x^4 - 40*a^2*c*x
^4 + 3*b^3*x^6 - 20*a*b*c*x^6))/(128*a^2*x^8) + (3*(b^4 - 8*a*b^2*c + 16*a^
2*c^2)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(128*a^(5/
2))
```

fricas [A] time = 1.17, size = 319, normalized size = 2.40

$$\left[\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{a}\,x^8\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)+4\left(\left(3ab^3-20a^2bc\right)x^6-24a^3bx^2-2\left(a^2b^2+20a^3c\right)x^4-16a^4\right)\sqrt{cx^4+bx^2+a}}{512a^3x^8},\ \frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{-a}\,x^8\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(\left(3ab^3-20a^2bc\right)x^6-24a^3bx^2-2\left(a^2b^2+20a^3c\right)x^4-16a^4\right)\sqrt{cx^4+bx^2+a}}{256a^3x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*c)*x^
4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^
4) + 4*((3*a*b^3 - 20*a^2*b*c)*x^6 - 24*a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*c)*
x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^8), 1/256*(3*(b^4 - 8*a*b^2*c
 + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^6 - 24*
a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*c)*x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2 + a))/(
a^3*x^8)]
```

giac [B] time = 0.49, size = 606, normalized size = 4.56

$$\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{128\sqrt{-a}\,a^2}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^7b^4-24\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^7ab^2c-80\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^7a^2c^2-256\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^6a^2bc^{\frac{3}{2}}-11\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5ab^4-168\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5a^2b^2c-48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5a^3c^2-128\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^4a^2b^3\sqrt{c}-11\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3a^2b^4-168\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3a^3b^2c-48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3a^4c^2-256\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2a^4bc^{\frac{3}{2}}+3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^3b^4-24\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^4b^2c-80\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^5c^2}{128\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 3/128*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/128*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b
*x^2 + a))^7*b^4 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^2*c - 8
0*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*c^2 - 256*(sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))^6*a^2*b*c^(3/2) - 11*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))^5*a*b^4 - 168*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^2*c
- 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*c^2 - 128*(sqrt(c)*x^2 -
```

```
 sqrt(c*x^4 + b*x^2 + a))^4*a^2*b^3*sqrt(c) - 11*(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 + a))^3*a^2*b^4 - 168*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^3
*b^2*c - 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^4*c^2 - 256*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^4*b*c^(3/2) + 3*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))*a^3*b^4 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*
b^2*c - 80*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^5*c^2)/(((sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^2 - a)^4*a^2)
```

maple [B] time = 0.02, size = 260, normalized size = 1.95

$$-\frac{3c^2\ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{16\sqrt{a}}+\frac{3b^2c\ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{3}{2}}}-\frac{3b^4\ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{256a^{\frac{5}{2}}}-\frac{5\sqrt{c x^4+b x^2+a}\,bc}{32a x^2}+\frac{3\sqrt{c x^4+b x^2+a}\,b^3}{128a^2x^2}-\frac{\sqrt{c x^4+b x^2+a}\,b^2}{64a x^4}-\frac{5\sqrt{c x^4+b x^2+a}\,c}{16x^4}-\frac{3\sqrt{c x^4+b x^2+a}\,b}{16x^6}-\frac{\sqrt{c x^4+b x^2+a}\,a}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^9,x)
```

```
[Out] -1/64*b^2/a/x^4*(c*x^4+b*x^2+a)^(1/2)+3/128*b^3/a^2/x^2*(c*x^4+b*x^2+a)^(1/
2)-3/16*c^2/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-5/1
6*c/x^4*(c*x^4+b*x^2+a)^(1/2)-3/256*b^4/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^
2+a)^(1/2)*a^(1/2))/x^2)-1/8*a/x^8*(c*x^4+b*x^2+a)^(1/2)-5/32/a*c*b/x^2*(c*
x^4+b*x^2+a)^(1/2)+3/32/a^(3/2)*c*b^2*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)
*a^(1/2))/x^2)-3/16*b/x^6*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^9,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**9,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**9, x)
```

3.742 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=162

$$\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}-\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8}$$

Rubi [A] time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 730, 720, 724, 206}

$$-\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}+\frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8}-\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] (-3*b*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^3*x^4) +
(b*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*a^2*x^8) - (a + b*x^2 + c*x
^4)^(5/2)/(10*a*x^10) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[
a]*Sqrt[a + b*x^2 + c*x^4])])/(512*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ax^{10}} - \frac{b\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^2\right)}{4a}$$

$$= \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ax^{10}} + \frac{\left(3b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\right.}{64a^2}$$

$$= -\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx\right.}{1}$$

$$= -\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx\right.}{1}$$

$$= -\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx\right.}{1}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 1.03

$$\frac{b\left(16a^{3/2}\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}-3x^4\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}-x^4\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)\right)\right)}{512a^{7/2}x^8}-\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -1/10*(a + b*x^2 + c*x^4)^(5/2)/(a*x^10) + (b*(16*a^(3/2)*(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(3/2) - 3*(b^2 - 4*a*c)*x^4*(2*Sqrt[a]*(2*a + b*x^2)*Sqrt[
a + b*x^2 + c*x^4] - (b^2 - 4*a*c)*x^4*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])])))/(512*a^(7/2)*x^8)
```

IntegrateAlgebraic [A] time = 1.36, size = 174, normalized size = 1.07

$$\frac{\sqrt{a+bx^2+cx^4}\left(-128a^4-176a^3bx^2-256a^3cx^4-8a^2b^2x^4-56a^2bcx^6-128a^2c^2x^8+10ab^3x^6+100ab^2cx^8-15b^4x^8\right)}{1280a^3x^{10}}-\frac{3\left(16a^2bc^2-8ab^3c+b^5\right)\tanh^{-1}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{256a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-128*a^4 - 176*a^3*b*x^2 - 8*a^2*b^2*x^4 - 256*a^
3*c*x^4 + 10*a*b^3*x^6 - 56*a^2*b*c*x^6 - 15*b^4*x^8 + 100*a*b^2*c*x^8 - 12
8*a^2*c^2*x^8))/(1280*a^3*x^10) - (3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*ArcTa
nh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(256*a^(7/2))
```

fricas [A] time = 1.56, size = 383, normalized size = 2.36

$$\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{a}x^{10}\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2+4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)-4\left(\left(15ab^4-100a^2b^2c+128a^3c^2\right)x^8+176a^4bx^2-2\left(5a^2b^3-28a^3bc\right)x^6+128a^5+8\left(a^3b^2+32a^4c\right)x^4\right)\sqrt{cx^4+bx^2+a}}{5120a^4x^{10}},\ -\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{-a}x^{10}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(\left(15ab^4-100a^2b^2c+128a^3c^2\right)x^8+176a^4bx^2-2\left(5a^2b^3-28a^3bc\right)x^6+128a^5+8\left(a^3b^2+32a^4c\right)x^4\right)\sqrt{cx^4+bx^2+a}}{2560a^4x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 + 4*a*
c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^
2)/x^4) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^8 + 176*a^4*b*x^2 -
 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(a^3*b^2 + 32*a^4*c)*x^4)*sqr
t(c*x^4 + b*x^2 + a))/(a^4*x^10), -1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)
/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x
^8 + 176*a^4*b*x^2 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(a^3*b^2
+ 32*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^10)]
```

giac [B] time = 0.53, size = 832, normalized size = 5.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -3/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/1280*(15*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))^9*b^5 - 120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*b^3
*c + 240*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^2*b*c^2 + 1280*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^8*a^3*c^(5/2) - 70*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^7*a*b^5 + 560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^
2*b^3*c + 2720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^3*b*c^2 + 5120*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^3*b^2*c^(3/2) + 128*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^5 + 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^5*a^3*b^3*c + 3840*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^4*
b*c^2 + 1280*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^3*b^4*sqrt(c) + 25
60*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4*b^2*c^(3/2) + 2560*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*c^(5/2) + 70*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^3*a^3*b^5 + 2000*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3
*a^4*b^3*c + 2400*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^5*b*c^2 + 256
0*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^5*b^2*c^(3/2) - 15*(sqrt(c)*x
^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*b^5 + 120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))*a^5*b^3*c + 1040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^6*b*c^2
 + 256*a^7*c^(5/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^5*a^3)
```

maple [B] time = 0.02, size = 337, normalized size = 2.08

$$\frac{3b\,c^2 \ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{3}{2}}}-\frac{3b^3 c \ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{64a^{\frac{5}{2}}}+\frac{3b^5 \ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{512a^{\frac{7}{2}}}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,c^2}{10a\,x^2}+\frac{5\sqrt{c\,x^4+b\,x^2+a}\,b^2 c}{64a^2x^2}-\frac{3\sqrt{c\,x^4+b\,x^2+a}\,b^4}{256a^3x^2}-\frac{7\sqrt{c\,x^4+b\,x^2+a}\,bc}{160a\,x^4}+\frac{\sqrt{c\,x^4+b\,x^2+a}\,b^3}{128a^2x^4}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,b^2}{160a\,x^6}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,c}{5x^6}-\frac{11\sqrt{c\,x^4+b\,x^2+a}\,b}{80x^8}-\frac{\sqrt{c\,x^4+b\,x^2+a}\,a}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^11,x)
```

```
[Out] -1/160/a*b^2/x^6*(c*x^4+b*x^2+a)^(1/2)+1/128/a^2*b^3/x^4*(c*x^4+b*x^2+a)^(1
/2)-3/256/a^3*b^4/x^2*(c*x^4+b*x^2+a)^(1/2)+3/512/a^(7/2)*b^5*ln((b*x^2+2*a
+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-1/10/a*c^2/x^2*(c*x^4+b*x^2+a)^(1/2)
+3/32*b*c^2/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3/6
4*b^3*c/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-7/160*b
*c/a/x^4*(c*x^4+b*x^2+a)^(1/2)+5/64*b^2*c/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-1/1
0*a/x^10*(c*x^4+b*x^2+a)^(1/2)-11/80*b/x^8*(c*x^4+b*x^2+a)^(1/2)-1/5*c/x^6*
(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^11,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**11,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**11, x)
```

3.743 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=216

$$-\frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2048a^{9/2}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{(7b^2-4ac}{}$$

Rubi [A] time = 0.22, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 744, 806, 720, 724, 206}

$$-\frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{384a^3x^8}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2048a^{9/2}}+\frac{7b(a+bx^2+cx^4)^{5/2}}{120a^2x^{10}}-\frac{(a+bx^2+cx^4)^{5/2}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(1024
*a^4*x^4) - ((7*b^2 - 4*a*c)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*
a^3*x^8) - (a + b*x^2 + c*x^4)^(5/2)/(12*a*x^12) + (7*b*(a + b*x^2 + c*x^4)
^(5/2))/(120*a^2*x^10) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2048*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{5/2}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+cx\right)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{5/2}}{12ax^{12}} + \frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120a^2x^{10}} + \frac{\left(7b^2-4ac\right)\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x,\right.}{48a^2}$$

$$= -\frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384a^3x^8} - \frac{\left(a+bx^2+cx^4\right)^{5/2}}{12ax^{12}} + \frac{7b\left(a+bx^2+cx^4\right)}{120a^2x^{10}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+\right.}{384a^3x^8}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+\right.}{384a^3x^8}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+\right.}{384a^3x^8}$$

Mathematica [A] time = 0.21, size = 206, normalized size = 0.95

$$-\frac{\frac{\left(\frac{7b^2}{2}-2ac\right)\left(16a^{3/2}\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}-3x^4\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}-x^4\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)\right)\right)}{256a^{7/2}x^8}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{x^{12}}-\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{10ax^{10}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -1/12*((a + b*x^2 + c*x^4)^(5/2)/x^12 - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(10
*a*x^10) + (((7*b^2)/2 - 2*a*c)*(16*a^(3/2)*(2*a + b*x^2)*(a + b*x^2 + c*x^
4)^(3/2) - 3*(b^2 - 4*a*c)*x^4*(2*Sqrt[a]*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*
x^4] - (b^2 - 4*a*c)*x^4*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
c*x^4])])))/(256*a^(7/2)*x^8))/a
```

IntegrateAlgebraic [A] time = 1.94, size = 221, normalized size = 1.02

$$\frac{\left(-64a^3c^3+144a^2b^2c^2-60ab^4c+7b^6\right)\tanh^{-1}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{1024a^{9/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-1280a^5-1664a^4bx^2-2240a^4cx^4-48a^3b^2x^4-288a^3bcx^6-480a^3c^2x^8+56a^2b^3x^6+432a^2b^2cx^8+1296a^2bc^2x^{10}-70ab^4x^8-760ab^3cx^{10}+105b^5x^{10}\right)}{15360a^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-1280*a^5 - 1664*a^4*b*x^2 - 48*a^3*b^2*x^4 - 224
0*a^4*c*x^4 + 56*a^2*b^3*x^6 - 288*a^3*b*c*x^6 - 70*a*b^4*x^8 + 432*a^2*b^2
*c*x^8 - 480*a^3*c^2*x^8 + 105*b^5*x^10 - 760*a*b^3*c*x^10 + 1296*a^2*b*c^2
*x^10))/(15360*a^4*x^12) + ((7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*
c^3)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(1024*a^(9/2
))
```

fricas [A] time = 1.51, size = 473, normalized size = 2.19

$$\left[-\frac{15\left(7 b^{6}-60 a b^{4} c+144 a^{2} b^{2} c^{2}-64 a^{3} c^{3}\right) \sqrt{a} x^{12} \log\left(-\frac{\left(b^{2}+4 a c\right) x^{4}+8 a b x^{2}+4 \sqrt{c x^{4}+b x^{2}+a}\left(b x^{2}+2 a\right) \sqrt{a}+8 a^{2}}{x^{4}}\right)-4\left(\left(105 a b^{5}-760 a^{2} b^{3} c+1296 a^{3} b c^{2}\right) x^{10}-2\left(35 a^{2} b^{4}-216 a^{3} b^{2} c+240 a^{4} c^{2}\right) x^{8}-1664 a^{5} b x^{2}+8\left(7 a^{3} b^{3}-36 a^{4} b c\right) x^{6}-1280 a^{6}-16\left(3 a^{4} b^{2}+140 a^{5} c\right) x^{4}\right) \sqrt{c x^{4}+b x^{2}+a}}{61440 a^{5} x^{12}}, \frac{15\left(7 b^{6}-60 a b^{4} c+144 a^{2} b^{2} c^{2}-64 a^{3} c^{3}\right) \sqrt{-a} x^{12} \arctan\left(\frac{\sqrt{c x^{4}+b x^{2}+a}\left(b x^{2}+2 a\right) \sqrt{-a}}{2\left(a c x^{4}+a b x^{2}+a^{2}\right)}\right)+2\left(\left(105 a b^{5}-760 a^{2} b^{3} c+1296 a^{3} b c^{2}\right) x^{10}-2\left(35 a^{2} b^{4}-216 a^{3} b^{2} c+240 a^{4} c^{2}\right) x^{8}-1664 a^{5} b x^{2}+8\left(7 a^{3} b^{3}-36 a^{4} b c\right) x^{6}-1280 a^{6}-16\left(3 a^{4} b^{2}+140 a^{5} c\right) x^{4}\right) \sqrt{c x^{4}+b x^{2}+a}}{30720 a^{5} x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x
^12*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c
^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^8 - 1664*a^5*b*x^
2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 1280*a^6 - 16*(3*a^4*b^2 + 140*a^5*c)*
x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^12), 1/30720*(15*(7*b^6 - 60*a*b^4*c +
 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*b^5 - 76
0*a^2*b^3*c + 1296*a^3*b*c^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^
4*c^2)*x^8 - 1664*a^5*b*x^2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 1280*a^6 - 1
6*(3*a^4*b^2 + 140*a^5*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^12)]
```

giac [B] time = 0.70, size = 1235, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/1024*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*arctan(-(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/15360*(105*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11*b^6 - 900*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^11*a*b^4*c + 2160*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11
*a^2*b^2*c^2 - 960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11*a^3*c^3 - 595
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*b^6 + 5100*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^9*a^2*b^4*c - 12240*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^9*a^3*b^2*c^2 - 15040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^4*c
^3 - 76800*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^8*a^4*b*c^(5/2) + 1386*(
```

```
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*b^6 - 11880*(sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))^7*a^3*b^4*c - 97440*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
 + a))^7*a^4*b^2*c^2 - 24960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^5*
c^3 - 112640*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^4*b^3*c^(3/2) - 61
440*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^5*b*c^(5/2) - 1686*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*b^6 - 42600*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^5*a^4*b^4*c - 128160*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^5*a^5*b^2*c^2 - 24960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^6*c^3 -
15360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4*b^5*sqrt(c) - 61440*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*b^3*c^(3/2) - 92160*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^4*a^6*b*c^(5/2) - 595*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^3*a^4*b^6 - 25620*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*
a^5*b^4*c - 58320*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^6*b^2*c^2 - 1
5040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^7*c^3 - 30720*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^2*a^6*b^3*c^(3/2) - 12288*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^2*a^7*b*c^(5/2) + 105*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*a^5*b^6 - 900*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^6*b^4*c - 132
00*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^7*b^2*c^2 - 960*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^8*c^3 - 3072*a^8*b*c^(5/2))/(((sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))^2 - a)^6*a^4)
```

maple [B] time = 0.03, size = 457, normalized size = 2.12

$$\frac{c^3 \ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{3}{2}}}-\frac{9b^2c^2 \ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{128a^{\frac{5}{2}}}+\frac{15b^4 c \ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{512a^{\frac{7}{2}}}-\frac{7b^6 \ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{2048a^{\frac{9}{2}}}+\frac{27\sqrt{c x^4+b x^2+a}\, b c^2}{320a^2x^2}-\frac{19\sqrt{c x^4+b x^2+a}\, b^3 c}{384a^3x^2}+\frac{7\sqrt{c x^4+b x^2+a}\, b^5}{1024a^4x^2}-\frac{\sqrt{c x^4+b x^2+a}\, c^2}{32a x^4}+\frac{9\sqrt{c x^4+b x^2+a}\, b^2 c}{320a^2x^4}-\frac{7\sqrt{c x^4+b x^2+a}\, b^4}{1536a^3x^4}-\frac{3\sqrt{c x^4+b x^2+a}\, b c}{160a x^6}+\frac{7\sqrt{c x^4+b x^2+a}\, b^3}{1920a^2x^6}-\frac{\sqrt{c x^4+b x^2+a}\, b^2}{320a x^8}-\frac{7\sqrt{c x^4+b x^2+a}\, c}{48x^8}-\frac{13\sqrt{c x^4+b x^2+a}\, b}{120x^{10}}-\frac{\sqrt{c x^4+b x^2+a}\, a}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^13,x)
```

```
[Out] -1/320/a*b^2/x^8*(c*x^4+b*x^2+a)^(1/2)+7/1920/a^2*b^3/x^6*(c*x^4+b*x^2+a)^(
1/2)-7/1536/a^3*b^4/x^4*(c*x^4+b*x^2+a)^(1/2)+7/1024/a^4*b^5/x^2*(c*x^4+b*x
^2+a)^(1/2)-7/2048/a^(9/2)*b^6*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2
))/x^2)-9/128*c^2*b^2/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2)
)/x^2)-1/32*c^2/a/x^4*(c*x^4+b*x^2+a)^(1/2)-1/12*a/x^12*(c*x^4+b*x^2+a)^(1/
2)-7/48*c/x^8*(c*x^4+b*x^2+a)^(1/2)-13/120*b/x^10*(c*x^4+b*x^2+a)^(1/2)+1/3
2*c^3/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+27/320*c^
2*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)+15/512/a^(7/2)*b^4*c*ln((b*x^2+2*a+2*(c*x
^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-19/384/a^3*b^3*c/x^2*(c*x^4+b*x^2+a)^(1/2)+
9/320/a^2*b^2*c/x^4*(c*x^4+b*x^2+a)^(1/2)-3/160/a*b*c/x^6*(c*x^4+b*x^2+a)^(
1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^13,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^13, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**13,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**13, x)
```

3.744 $\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{\left(-16ac+15b^2-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}$$

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 742, 779, 621, 206}

$$\frac{\left(-16ac+15b^2-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2 - 16*a*c - 10*b*c*x^2)*Sqrt[
a + b*x^2 + c*x^4])/(48*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
```

```
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2-16ac-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Subst}}{32}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2-16ac-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Subst}}{}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2-16ac-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{b\left(5b^2-12ac\right)\tanh^{-1}}{32c^{7/2}}$$

Mathematica [A] time = 0.06, size = 104, normalized size = 0.86

$$\frac{\left(36abc-15b^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(8c\left(cx^4-2a\right)+15b^2-10bcx^2\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^2 - 10*b*c*x^2 + 8*c*(-2*a + c*x^4
)) + (-15*b^3 + 36*a*b*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])])/(96*c^(7/2))
```

IntegrateAlgebraic [A] time = 0.29, size = 101, normalized size = 0.83

$$\frac{\left(5b^3-12abc\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{32c^{7/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-16ac+15b^2-10bcx^2+8c^2x^4\right)}{48c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^2 - 16*a*c - 10*b*c*x^2 + 8*c^2*x^4))/(48*c^
3) + ((5*b^3 - 12*a*b*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4
]])/(32*c^(7/2))
```

fricas [A] time = 2.96, size = 241, normalized size = 1.99

$$\left[-\frac{3\left(5b^3-12abc\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)-4\left(8c^3x^4-10bc^2x^2+15b^2c-16ac^2\right)\sqrt{cx^4+bx^2+a}}{192c^4},\ \frac{3\left(5b^3-12abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)+2\left(8c^3x^4-10bc^2x^2+15b^2c-16ac^2\right)\sqrt{cx^4+bx^2+a}}{96c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*
sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 - 10*
b*c^2*x^2 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*b
^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sq
rt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c
- 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

giac [A] time = 0.21, size = 103, normalized size = 0.85

$$\frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2x^2\left(\frac{4x^2}{c}-\frac{5b}{c^2}\right)+\frac{15b^2-16ac}{c^3}\right)+\frac{\left(5b^3-12abc\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{32c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c - 5*b/c^2) + (15*b^2 - 16*a*c)
/c^3) + 1/32*(5*b^3 - 12*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))*sqrt(c) - b))/c^(7/2)
```

maple [A] time = 0.02, size = 162, normalized size = 1.34

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;x^4}{6c}-\frac{5\sqrt{c\,x^4+b\,x^2+a}\;b\,x^2}{24c^2}+\frac{3ab\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{8c^{\frac{5}{2}}}-\frac{5b^3\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{32c^{\frac{7}{2}}}-\frac{\sqrt{c\,x^4+b\,x^2+a}\;a}{3c^2}+\frac{5\sqrt{c\,x^4+b\,x^2+a}\;b^2}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+5/16*b
^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*b^3/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)
^(1/2))-1/3*a/c^2*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^7/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 + c*x**4), x)
```

3.745 $\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}$$

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 742, 640, 621, 206}

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[a + b*x^2 + c*x^4])/(4*c
) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\operatorname{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}\\
&= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.85

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\left(2cx^2-3b\right)\sqrt{a+bx^2+cx^4}}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

[Out] (2*Sqrt[c]*(-3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (3*b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(5/2))

IntegrateAlgebraic [A] time = 0.25, size = 91, normalized size = 0.88

$$\frac{\left(4ac-3b^2\right)\log\left(-2c^{5/2}\sqrt{a+bx^2+cx^4}+bc^2+2c^3x^2\right)}{16c^{5/2}}+\frac{\left(2cx^2-3b\right)\sqrt{a+bx^2+cx^4}}{8c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/Sqrt[a + b*x^2 + c*x^4],x]

[Out] ((-3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((-3*b^2 + 4*a*c)*Log[
b*c^2 + 2*c^3*x^2 - 2*c^(5/2)*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(5/2))

fricas [A] time = 1.12, size = 203, normalized size = 1.95

$$\left[-\frac{\left(3b^2-4ac\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)-4\sqrt{cx^4+bx^2+a}\left(2c^2x^2-3bc\right)}{32c^3}, -\frac{\left(3b^2-4ac\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-2\sqrt{cx^4+bx^2+a}\left(2c^2x^2-3bc\right)}{16c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [-1/32*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c
*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c^2*x^2 - 3*b*c))/c^3, -1/16*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt
(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*s
qrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 - 3*b*c))/c^3]

giac [A] time = 0.24, size = 82, normalized size = 0.79

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(\frac{2x^2}{c}-\frac{3b}{c^2}\right)-\frac{\left(3b^2-4ac\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2/c - 3*b/c^2) - 1/16*(3*b^2 - 4*a*c)*log(
abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2)

maple [A] time = 0.01, size = 116, normalized size = 1.12

$$\frac{\sqrt{cx^4+bx^2+a}\,x^2}{4c}-\frac{a\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{4c^{\frac{3}{2}}}+\frac{3b^2\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{16c^{\frac{5}{2}}}-\frac{3\sqrt{cx^4+bx^2+a}\,b}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^2*(c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2/c^
(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*a/c^(3/2)*ln((c*x
^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 + c*x**4), x)
```

3.746 $\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1114, 640, 621, 206}

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2c}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 68, normalized size = 1.00

$$\frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 + c*x^4],x]
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(4*c^(3/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 70, normalized size = 1.03

$$\frac{b\log\left(-2c^{3/2}\sqrt{a+bx^2+cx^4}+bc+2c^2x^2\right)}{4c^{3/2}} + \frac{\sqrt{a+bx^2+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + b*x^2 + c*x^4],x]
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) + (b*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a +
 b*x^2 + c*x^4]])/(4*c^(3/2))
```

fricas [A] time = 0.86, size = 161, normalized size = 2.37

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^4 - 8\,bcx^2 - b^2 + 4\,\sqrt{cx^4 + bx^2 + a}\left(2\,cx^2 + b\right)\sqrt{c} - 4\,ac\right) + 4\,\sqrt{cx^4 + bx^2 + a}\,c}{8\,c^2}, \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right) + 2\,\sqrt{cx^4 + bx^2 + a}\,c}{4\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c)/c^2, 1/4*(b
*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^
4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c)/c^2]
```

giac [A] time = 0.21, size = 61, normalized size = 0.90

$$\frac{b\log\left(\left|-2\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{4\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3
/2) + 1/2*sqrt(c*x^4 + b*x^2 + a)/c
```

maple [A] time = 0.01, size = 56, normalized size = 0.82

$$-\frac{b\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}} + \sqrt{c\,x^4 + b\,x^2 + a}\right)}{4c^{\frac{3}{2}}} + \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.43, size = 55, normalized size = 0.81

$$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,c}-\frac{b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}\,+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (a + b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 +
 c*x^2)/c^(1/2)))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 + c*x**4), x)
```

3.747 $\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1107, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 + c*x^4],x]
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c])
```

IntegrateAlgebraic [A] time = 0.11, size = 41, normalized size = 0.95

$$-\frac{\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a + b*x^2 + c*x^4],x]
[Out] -1/2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]/Sqrt[c]
```

fricas [A] time = 0.91, size = 118, normalized size = 2.74

$$\left[\frac{\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)}{4\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
+ b)*sqrt(c) - 4*a*c)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))/c]
```

giac [A] time = 0.21, size = 40, normalized size = 0.93

$$-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/sqrt(
c)
```

maple [A] time = 0.01, size = 35, normalized size = 0.81

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.69, size = 34, normalized size = 0.79

$$\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))/(2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**2 + c*x**4), x)
```

3.748 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1114, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
[Out] -1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.11, size = 45, normalized size = 1.02

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]]/Sqrt[a]
```

fricas [A] time = 1.02, size = 124, normalized size = 2.82

$$\left[\frac{\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

giac [A] time = 0.25, size = 38, normalized size = 0.86

$$\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 39, normalized size = 0.89

$$-\frac{\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.44, size = 44, normalized size = 1.00

$$-\frac{\ln\left(\frac{1}{x^2}\right)}{2\sqrt{a}}-\frac{\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}\,+b\,x^2\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] - log(1/x^2)/(2*a^(1/2)) - log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) +
b*x^2)/(2*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

3.749 $\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1114, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.00

$$
\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}} - \frac{\sqrt{a+bx^2+cx^4}}{2ax^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

IntegrateAlgebraic [A] time = 0.20, size = 76, normalized size = 1.06

$$
-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{\sqrt{a+bx^2+cx^4}}{2ax^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(a*x^2) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - S
qrt[a + b*x^2 + c*x^4]/Sqrt[a]])/(2*a^(3/2))
```

fricas [A] time = 0.58, size = 179, normalized size = 2.49

$$\left[\frac{\sqrt{a}\, bx^2 \log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right) - 4\,\sqrt{cx^4+bx^2+a}\,a}{8\,a^2x^2}, -\frac{\sqrt{-a}\,bx^2 \arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right) + 2\,\sqrt{cx^4+bx^2+a}\,a}{4\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a)
/(a^2*x^2), -1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*a)/(
a^2*x^2)]
```

giac [A] time = 0.43, size = 114, normalized size = 1.58

$$-\frac{b \arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a} + \frac{\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)b + 2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^2 - a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*
a) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a)
```

maple [A] time = 0.01, size = 63, normalized size = 0.88

$$\frac{b \ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}} - \frac{\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*b/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2
+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.48, size = 56, normalized size = 0.78

$$\frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^2}{2}+a}{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{4\,a^{3/2}}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (b*atanh((a + (b*x^2)/2)/(a^(1/2)*(a + b*x^2 + c*x^4)^(1/2))))/(4*a^(3/2))
- (a + b*x^2 + c*x^4)^(1/2)/(2*a*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

3.750 $\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 744, 806, 724, 206}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x
^2) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*
x^4])])/(16*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
```

```
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4} - \frac{\operatorname{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16a^2}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8a^2}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.84

$$
\frac{\left(4ac-3b^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}} + \frac{\left(3bx^2-2a\right)\sqrt{a+bx^2+cx^4}}{8a^2x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*a + 3*b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((-3*b^2 + 4*a*c)*
ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

IntegrateAlgebraic [A] time = 0.31, size = 91, normalized size = 0.84

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{\left(3bx^2-2a\right)\sqrt{a+bx^2+cx^4}}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*a + 3*b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((3*b^2 - 4*a*c)*A
rcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(5/2))
```

fricas [A] time = 1.20, size = 221, normalized size = 2.05

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{a}\,x^4\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}\left(3\,abx^2-2\,a^2\right)}{32\,a^3x^4},\ \frac{\left(3\,b^2-4\,ac\right)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\,\sqrt{cx^4+bx^2+a}\left(3\,abx^2-2\,a^2\right)}{16\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4
*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4
 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*x^4), 1/16*((3*b^2 - 4*a*c)*sqrt(-a
)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*x^4)]
```

giac [B] time = 0.30, size = 221, normalized size = 2.05

$$\frac{\left(3\,b^2-4\,ac\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^2}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3ac-5\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)ab^2-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^2c-8\,a^2b\sqrt{c}}{8\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2 - 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a
))/(sqrt(-a)*a^2) - 1/8*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 -
4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*c - 5*(sqrt(c)*x^2 - sqrt(c*x
```

```
^4 + b*x^2 + a))*a*b^2 - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c -
8*a^2*b*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^2)
```

maple [A] time = 0.01, size = 127, normalized size = 1.18

$$\frac{c\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}-\frac{3b^{2}\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{16a^{\frac{5}{2}}}+\frac{3\sqrt{c\,x^{4}+b\,x^{2}+a}\,b}{8a^{2}x^{2}}-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^2-3/16*b
^2/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/4*c/a^(3/2
)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5}\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

3.751 $\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Rubi [A] time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 744, 834, 806, 724, 206}

$$-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[a + b*x^2 + c*x^4])/(24*a^2*
x^4) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^2 + c*x^4])/(48*a^3*x^2) + (b*(5*b^2
 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*
a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
```

```
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{6a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(15b^2-16ac\right)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{12a^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2} - \frac{\left(b\left(5b^2-1\right.\right.}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{\left(b\left(5b^2-1\right.\right.}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{b\left(5b^2-12\right.}{}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.77

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(-8a^2+2a\left(5bx^2+8cx^4\right)-15b^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 - 15*b^2*x^4 + 2*a*(5*b*x^2 + 8*c*x^4)))/(
48*a^3*x^6) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])])/(32*a^(7/2))
```

IntegrateAlgebraic [A] time = 0.47, size = 110, normalized size = 0.76

$$\frac{\left(12abc-5b^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(-8a^2+10abx^2+16acx^4-15b^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 + 10*a*b*x^2 - 15*b^2*x^4 + 16*a*c*x^4))/(
48*a^3*x^6) + ((-5*b^3 + 12*a*b*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 +
c*x^4])/Sqrt[a]])/(16*a^(7/2))
```

fricas [A] time = 1.45, size = 265, normalized size = 1.83

$$\left[-\frac{3\left(5b^3-12abc\right)\sqrt{a}x^6\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)-4\left(10a^2bx^2-\left(15ab^2-16a^2c\right)x^4-8a^3\right)\sqrt{cx^4+bx^2+a}}{192a^4x^6}, -\frac{3\left(5b^3-12abc\right)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)-2\left(10a^2bx^2-\left(15ab^2-16a^2c\right)x^4-8a^3\right)\sqrt{cx^4+bx^2+a}}{96a^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(10*
a^2*b*x^2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^
4*x^6), -1/96*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(10*a^2*b*x^
2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]
```

giac [B] time = 0.26, size = 335, normalized size = 2.31

$$-\frac{\left(5b^3-12abc\right)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\sqrt{-a}a^3}+\frac{15\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5b^3-36\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5abc-40\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3ab^3+96\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3a^2bc+96\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2a^3c^{\frac{3}{2}}+33\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)a^2b^3+36\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)a^3bc+48a^3b^2\sqrt{c}-32a^4c^{\frac{3}{2}}}{48\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*(5*b^3 - 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sq
rt(-a))/(sqrt(-a)*a^3) + 1/48*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5
*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c - 40*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^3*a*b^3 + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^3*a^2*b*c + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*c^(3/2)
+ 33*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*b^3 + 36*(sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))*a^3*b*c + 48*a^3*b^2*sqrt(c) - 32*a^4*c^(3/2))/(((sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^3*a^3)
```

maple [A] time = 0.02, size = 176, normalized size = 1.21

$$-\frac{3bc\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{8a^{\frac{5}{2}}}+\frac{5b^3\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{7}{2}}}+\frac{\sqrt{cx^4+bx^2+a}\,c}{3a^2x^2}-\frac{5\sqrt{cx^4+bx^2+a}\,b^2}{16a^3x^2}+\frac{5\sqrt{cx^4+bx^2+a}\,b}{24a^2x^4}-\frac{\sqrt{cx^4+bx^2+a}}{6ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-5/16*
b^2/a^3/x^2*(c*x^4+b*x^2+a)^(1/2)+5/32*b^3/a^(7/2)*ln((b*x^2+2*a+2*(c*x^4+b
```

```
*x^2+a)^(1/2)*a^(1/2))/x^2)-3/8*b/a^(5/2)*c*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)
^(1/2)*a^(1/2))/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7} \sqrt{a + bx^{2} + cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

3.752 $\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}-\frac{\left(16ac+15b^2+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}$$

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1114, 742, 779, 621, 204}

$$-\frac{\left(16ac+15b^2+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(x^4*Sqrt[a + b*x^2 - c*x^4])/(6*c) - ((15*b^2 + 16*a*c + 10*b*c*x^2)*Sqrt
[a + b*x^2 - c*x^4])/(48*c^3) - (b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(32*c^(7/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
```

```
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\left(15b^2+16ac+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3} + \frac{\left(b\left(5b^2+12ac\right)\right)\text{Subst}}{32}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\left(15b^2+16ac+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3} + \frac{\left(b\left(5b^2+12ac\right)\right)\text{Subst}}{}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\left(15b^2+16ac+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3} - \frac{b\left(5b^2+12ac\right)\tan^{-1}\left(\right.}{32c^{7/2}}$$

Mathematica [A] time = 0.08, size = 107, normalized size = 0.86

$$\frac{-2\sqrt{c}\sqrt{a+bx^2-cx^4}\left(8c\left(2a+cx^4\right)+15b^2+10bcx^2\right)-3b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]*(15*b^2 + 10*b*c*x^2 + 8*c*(2*a + c*x^4
)) - 3*b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 -
c*x^4])])/(96*c^(7/2))
```

IntegrateAlgebraic [B] time = 23.35, size = 3247, normalized size = 26.19

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 - c*x^4]*(-196*a*b^12*c^(5/2) - 2352*a^2*b^10*c^(7/2) - 111
36*a^3*b^8*c^(9/2) - 34304*a^4*b^6*c^(11/2) - 95232*a^5*b^4*c^(13/2) - 1597
44*a^6*b^2*c^(15/2) - 65536*a^7*c^(17/2) + 15*b^14*Sqrt[-c]*Sqrt[-c^2] + 18
0*a*b^12*Sqrt[-c]*c*Sqrt[-c^2] + 1632*a^2*b^10*Sqrt[-c]*c^2*Sqrt[-c^2] + 11
904*a^3*b^8*Sqrt[-c]*c^3*Sqrt[-c^2] + 43776*a^4*b^6*Sqrt[-c]*c^4*Sqrt[-c^2]
 + 58368*a^5*b^4*Sqrt[-c]*c^5*Sqrt[-c^2] - 190*b^13*c^(5/2)*x^2 - 4944*a*b^
11*c^(7/2)*x^2 - 36576*a^2*b^9*c^(9/2)*x^2 - 147968*a^3*b^7*c^(11/2)*x^2 -
477696*a^4*b^5*c^(13/2)*x^2 - 921600*a^5*b^3*c^(15/2)*x^2 - 434176*a^6*b*c^
(17/2)*x^2 + 180*b^13*Sqrt[-c]*c*Sqrt[-c^2]*x^2 + 2880*a*b^11*Sqrt[-c]*c^2*
Sqrt[-c^2]*x^2 + 31104*a^2*b^9*Sqrt[-c]*c^3*Sqrt[-c^2]*x^2 + 156672*a^3*b^7
*Sqrt[-c]*c^4*Sqrt[-c^2]*x^2 + 267264*a^4*b^5*Sqrt[-c]*c^5*Sqrt[-c^2]*x^2 -
 1880*b^12*c^(7/2)*x^4 - 38400*a*b^10*c^(9/2)*x^4 - 195456*a^2*b^8*c^(11/2)
*x^4 - 710656*a^3*b^6*c^(13/2)*x^4 - 1959936*a^4*b^4*c^(15/2)*x^4 - 1081344
*a^5*b^2*c^(17/2)*x^4 - 32768*a^6*c^(19/2)*x^4 + 1248*b^12*Sqrt[-c]*c^2*Sqr
t[-c^2]*x^4 + 15744*a*b^10*Sqrt[-c]*c^3*Sqrt[-c^2]*x^4 + 170496*a^2*b^8*Sqr
t[-c]*c^4*Sqrt[-c^2]*x^4 + 485376*a^3*b^6*Sqrt[-c]*c^5*Sqrt[-c^2]*x^4 - 983
04*a^4*b^4*Sqrt[-c]*c^6*Sqrt[-c^2]*x^4 - 11712*b^11*c^(9/2)*x^6 - 88832*a*b
^9*c^(11/2)*x^6 - 245760*a^2*b^7*c^(13/2)*x^6 - 1794048*a^3*b^5*c^(15/2)*x^
6 - 1261568*a^4*b^3*c^(17/2)*x^6 - 196608*a^5*b*c^(19/2)*x^6 - 1920*b^11*Sq
rt[-c]*c^3*Sqrt[-c^2]*x^6 + 46080*a*b^9*Sqrt[-c]*c^4*Sqrt[-c^2]*x^6 + 43622
4*a^2*b^7*Sqrt[-c]*c^5*Sqrt[-c^2]*x^6 - 294912*a^3*b^5*Sqrt[-c]*c^6*Sqrt[-c
^2]*x^6 - 9728*b^10*c^(11/2)*x^8 + 122880*a*b^8*c^(13/2)*x^8 - 688128*a^2*b
^6*c^(15/2)*x^8 - 720896*a^3*b^4*c^(17/2)*x^8 - 393216*a^4*b^2*c^(19/2)*x^8
 - 3072*b^10*Sqrt[-c]*c^4*Sqrt[-c^2]*x^8 + 208896*a*b^8*Sqrt[-c]*c^5*Sqrt[-
c^2]*x^8 - 294912*a^2*b^6*Sqrt[-c]*c^6*Sqrt[-c^2]*x^8 + 45056*b^9*c^(13/2)*
x^10 - 147456*a*b^7*c^(15/2)*x^10 - 196608*a^2*b^5*c^(17/2)*x^10 - 262144*a
^3*b^3*c^(19/2)*x^10 + 49152*b^9*Sqrt[-c]*c^5*Sqrt[-c^2]*x^10 - 98304*a*b^7
*Sqrt[-c]*c^6*Sqrt[-c^2]*x^10))/(48*(b^12*c^(9/2) + 24*a*b^10*c^(11/2) + 24
0*a^2*b^8*c^(13/2) + 1280*a^3*b^6*c^(15/2) + 3840*a^4*b^4*c^(17/2) + 6144*a
^5*b^2*c^(19/2) + 4096*a^6*c^(21/2) + 24*b^11*c^(11/2)*x^2 + 480*a*b^9*c^(1
3/2)*x^2 + 3840*a^2*b^7*c^(15/2)*x^2 + 15360*a^3*b^5*c^(17/2)*x^2 + 30720*a
^4*b^3*c^(19/2)*x^2 + 24576*a^5*b*c^(21/2)*x^2 + 192*b^10*c^(13/2)*x^4 + 30
```

```
72*a*b^8*c^(15/2)*x^4 + 18432*a^2*b^6*c^(17/2)*x^4 + 49152*a^3*b^4*c^(19/2)
*x^4 + 49152*a^4*b^2*c^(21/2)*x^4 + 512*b^9*c^(15/2)*x^6 + 6144*a*b^7*c^(17
/2)*x^6 + 24576*a^2*b^5*c^(19/2)*x^6 + 32768*a^3*b^3*c^(21/2)*x^6)) + (9280
*a^4*b^12*c^3*Sqrt[-c^2] + 222720*a^5*b^10*c^4*Sqrt[-c^2] + 2227200*a^6*b^8
*c^5*Sqrt[-c^2] + 11878400*a^7*b^6*c^6*Sqrt[-c^2] + 35635200*a^8*b^4*c^7*Sq
rt[-c^2] + 57016320*a^9*b^2*c^8*Sqrt[-c^2] + 38010880*a^10*c^9*Sqrt[-c^2] -
 5*b^19*Sqrt[-c]*Sqrt[c]*x^2 + 60*a*b^17*Sqrt[-c]*c^(3/2)*x^2 - 544*a^2*b^1
5*Sqrt[-c]*c^(5/2)*x^2 - 6272*a^3*b^13*Sqrt[-c]*c^(7/2)*x^2 + 220416*a^4*b^
11*Sqrt[-c]*c^(9/2)*x^2 + 3021824*a^5*b^9*Sqrt[-c]*c^(11/2)*x^2 + 15040512*
a^6*b^7*Sqrt[-c]*c^(13/2)*x^2 + 34013184*a^7*b^5*Sqrt[-c]*c^(15/2)*x^2 + 5*
b^19*Sqrt[-c^2]*x^2 - 60*a*b^17*c*Sqrt[-c^2]*x^2 + 544*a^2*b^15*c^2*Sqrt[-c
^2]*x^2 + 6272*a^3*b^13*c^3*Sqrt[-c^2]*x^2 + 2304*a^4*b^11*c^4*Sqrt[-c^2]*x
^2 + 1432576*a^5*b^9*c^5*Sqrt[-c^2]*x^2 + 20594688*a^6*b^7*c^6*Sqrt[-c^2]*x
^2 + 108527616*a^7*b^5*c^7*Sqrt[-c^2]*x^2 + 285081600*a^8*b^3*c^8*Sqrt[-c^2
]*x^2 + 228065280*a^9*b*c^9*Sqrt[-c^2]*x^2 + 60*b^18*Sqrt[-c]*c^(3/2)*x^4 -
 704*a*b^16*Sqrt[-c]*c^(5/2)*x^4 - 17280*a^2*b^14*Sqrt[-c]*c^(7/2)*x^4 - 15
360*a^3*b^12*Sqrt[-c]*c^(9/2)*x^4 + 1936384*a^4*b^10*Sqrt[-c]*c^(11/2)*x^4
- 60*b^18*c*Sqrt[-c^2]*x^4 + 704*a*b^16*c^2*Sqrt[-c^2]*x^4 + 17280*a^2*b^14
*c^3*Sqrt[-c^2]*x^4 + 15360*a^3*b^12*c^4*Sqrt[-c^2]*x^4 - 154624*a^4*b^10*c
^5*Sqrt[-c^2]*x^4 + 28508160*a^5*b^8*c^6*Sqrt[-c^2]*x^4 + 171048960*a^6*b^6
*c^7*Sqrt[-c^2]*x^4 + 456130560*a^7*b^4*c^8*Sqrt[-c^2]*x^4 + 456130560*a^8*
b^2*c^9*Sqrt[-c^2]*x^4 - 160*b^17*Sqrt[-c]*c^(5/2)*x^6 - 12672*a*b^15*Sqrt[
-c]*c^(7/2)*x^6 - 30208*a^2*b^13*Sqrt[-c]*c^(9/2)*x^6 + 260096*a^3*b^11*Sqr
t[-c]*c^(11/2)*x^6 + 4718592*a^4*b^9*Sqrt[-c]*c^(13/2)*x^6 + 160*b^17*c^2*S
qrt[-c^2]*x^6 + 12672*a*b^15*c^3*Sqrt[-c^2]*x^6 + 30208*a^2*b^13*c^4*Sqrt[-
c^2]*x^6 - 260096*a^3*b^11*c^5*Sqrt[-c^2]*x^6 + 32768*a^4*b^9*c^6*Sqrt[-c^2
]*x^6 + 57016320*a^5*b^7*c^7*Sqrt[-c^2]*x^6 + 228065280*a^6*b^5*c^8*Sqrt[-c
^2]*x^6 + 304087040*a^7*b^3*c^9*Sqrt[-c^2]*x^6 - 2176*b^16*Sqrt[-c]*c^(7/2)
*x^8 - 23552*a*b^14*Sqrt[-c]*c^(9/2)*x^8 + 210944*a^2*b^12*Sqrt[-c]*c^(11/2
)*x^8 - 98304*a^3*b^10*Sqrt[-c]*c^(13/2)*x^8 + 2176*b^16*c^3*Sqrt[-c^2]*x^8
 + 23552*a*b^14*c^4*Sqrt[-c^2]*x^8 - 210944*a^2*b^12*c^5*Sqrt[-c^2]*x^8 + 9
8304*a^3*b^10*c^6*Sqrt[-c^2]*x^8 - 9216*b^15*Sqrt[-c]*c^(9/2)*x^10 + 86016*
a*b^13*Sqrt[-c]*c^(11/2)*x^10 - 98304*a^2*b^11*Sqrt[-c]*c^(13/2)*x^10 + 921
6*b^15*c^4*Sqrt[-c^2]*x^10 - 86016*a*b^13*c^5*Sqrt[-c^2]*x^10 + 98304*a^2*b
^11*c^6*Sqrt[-c^2]*x^10 + 16384*b^14*Sqrt[-c]*c^(11/2)*x^12 - 32768*a*b^12*
Sqrt[-c]*c^(13/2)*x^12 - 16384*b^14*c^5*Sqrt[-c^2]*x^12 + 32768*a*b^12*c^6*
Sqrt[-c^2]*x^12)/(16*b^5*c^(7/2)*(b^2 + 4*a*c)^3*(b^2 + 4*a*c + 8*b*c*x^2)^
3) + ((5*b^3 + 12*a*b*c)*ArcTan[(-2*Sqrt[-c]*Sqrt[c]*x^2 + 2*Sqrt[c]*Sqrt[a
 + b*x^2 - c*x^4])/b])/(32*c^(7/2)) + (Sqrt[-c]*(5*b^3 + 12*a*b*c)*Log[b^2
+ 4*a*c + 4*b*c*x^2 - 8*c^2*x^4 - 8*Sqrt[-c]*c*x^2*Sqrt[a + b*x^2 - c*x^4]]
)/(64*c^4)
```

fricas [A] time = 1.21, size = 249, normalized size = 2.01

$$\left[-\frac{3\left(5b^3+12abc\right)\sqrt{-c}\log\left(8c^2x^4-8bcx^2+b^2-4\sqrt{-cx^4+bx^2+a}\left(2cx^2-b\right)\sqrt{-c}-4ac\right)+4\left(8c^3x^4+10bc^2x^2+15b^2c+16ac^2\right)\sqrt{-cx^4+bx^2+a}}{192c^4}, -\frac{3\left(5b^3+12abc\right)\sqrt{c}\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}\left(2cx^2-b\right)\sqrt{c}}{2\left(c^2x^4-bcx^2-ac\right)}\right)+2\left(8c^3x^4+10bc^2x^2+15b^2c+16ac^2\right)\sqrt{-cx^4+bx^2+a}}{96c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*
sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*(8*c^3*x^4 + 1
0*b*c^2*x^2 + 15*b^2*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4, -1/96*(3*
(5*b^3 + 12*a*b*c)*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b
)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c)) + 2*(8*c^3*x^4 + 10*b*c^2*x^2 + 15*b^2
*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4]
```

giac [A] time = 0.26, size = 112, normalized size = 0.90

$$-\frac{1}{48}\sqrt{-cx^4+bx^2+a}\left(2x^2\left(\frac{4x^2}{c}+\frac{5b}{c^2}\right)+\frac{15b^2+16ac}{c^3}\right)-\frac{\left(5b^3+12abc\right)\log\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{32\sqrt{-c}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c + 5*b/c^2) + (15*b^2 + 16*a*
c)/c^3) - 1/32*(5*b^3 + 12*a*b*c)*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b
*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*c^3)
```

maple [A] time = 0.02, size = 168, normalized size = 1.35

$$-\frac{\sqrt{-cx^4+bx^2+a}\,x^4}{6c}-\frac{5\sqrt{-cx^4+bx^2+a}\,bx^2}{24c^2}+\frac{3ab\arctan\left(\frac{\left(x^2-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-cx^4+bx^2+a}}\right)}{8c^{\frac{5}{2}}}+\frac{5b^3\arctan\left(\frac{\left(x^2-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-cx^4+bx^2+a}}\right)}{32c^{\frac{7}{2}}}-\frac{\sqrt{-cx^4+bx^2+a}\,a}{3c^2}-\frac{5\sqrt{-cx^4+bx^2+a}\,b^2}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*x^4*(-c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(-c*x^4+b*x^2+a)^(1/2)-5/1
6*b^2/c^3*(-c*x^4+b*x^2+a)^(1/2)+5/32*b^3/c^(7/2)*arctan(c^(1/2)*(x^2-1/2*b
/c)/(-c*x^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*arctan(c^(1/2)*(x^2-1/2*b/c)/(-
c*x^4+b*x^2+a)^(1/2))-1/3/c^2*a*(-c*x^4+b*x^2+a)^(1/2)
```

maxima [A] time = 2.43, size = 153, normalized size = 1.23

$$-\frac{\sqrt{-cx^4+bx^2+a}\,x^4}{6c}-\frac{5\sqrt{-cx^4+bx^2+a}\,bx^2}{24c^2}-\frac{5b^3\arcsin\left(-\frac{2cx^2-b}{\sqrt{b^2+4ac}}\right)}{32c^{\frac{7}{2}}}-\frac{3ab\arcsin\left(-\frac{2cx^2-b}{\sqrt{b^2+4ac}}\right)}{8c^{\frac{5}{2}}}-\frac{5\sqrt{-cx^4+bx^2+a}\,b^2}{16c^3}-\frac{\sqrt{-cx^4+bx^2+a}\,a}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-c*x^4 + b*x^2 + a)*x^4/c - 5/24*sqrt(-c*x^4 + b*x^2 + a)*b*x^2/c
^2 - 5/32*b^3*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(7/2) - 3/8*a*b*ar
csin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(5/2) - 5/16*sqrt(-c*x^4 + b*x^2 +
 a)*b^2/c^3 - 1/3*sqrt(-c*x^4 + b*x^2 + a)*a/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(x^7/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 - c*x**4), x)
```

3.753 $\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=107

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}$$

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1114, 742, 640, 621, 204}

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) - (x^2*Sqrt[a + b*x^2 - c*x^4])/(4*c
) - ((3*b^2 + 4*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4
])])/(16*c^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}-\frac{\text{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)}{4c}\\
&= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}+\frac{\left(3b^2+4ac\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}+\frac{\left(3b^2+4ac\right)\text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx,x,\frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}-\frac{\left(3b^2+4ac\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.83

$$-\frac{\left(4ac+3b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{\left(3b+2cx^2\right)\sqrt{a+bx^2-cx^4}}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/8*((3*b + 2*c*x^2)*Sqrt[a + b*x^2 - c*x^4])/c^2 - ((3*b^2 + 4*a*c)*ArcTa
n[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(16*c^(5/2))
```

IntegrateAlgebraic [C] time = 30.87, size = 91, normalized size = 0.85

$$\frac{\left(-3b-2cx^2\right)\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{i\left(4ac+3b^2\right)\log\left(2i\sqrt{c}\sqrt{a+bx^2-cx^4}+b-2cx^2\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-3*b - 2*c*x^2)*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) - ((I/16)*(3*b^2 + 4*a*c
)*Log[b - 2*c*x^2 + (2*I)*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]])/c^(5/2)
```

fricas [A] time = 0.65, size = 211, normalized size = 1.97

$$\left[-\frac{\left(3\,b^2+4\,ac\right)\sqrt{-c}\,\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c}-4\,ac\right)+4\,\sqrt{-cx^4+bx^2+a}\left(2\,c^2x^2+3\,bc\right)}{32\,c^3},\,-\frac{\left(3\,b^2+4\,ac\right)\sqrt{c}\,\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{c}}{2\left(c^2x^4-bcx^2-ac\right)}\right)+2\,\sqrt{-cx^4+bx^2+a}\left(2\,c^2x^2+3\,bc\right)}{16\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 + 4*a*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-
c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^2
+ a)*(2*c^2*x^2 + 3*b*c))/c^3, -1/16*((3*b^2 + 4*a*c)*sqrt(c)*arctan(1/2*sq
rt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c)) + 2
*sqrt(-c*x^4 + b*x^2 + a)*(2*c^2*x^2 + 3*b*c))/c^3]
```

giac [A] time = 0.21, size = 91, normalized size = 0.85

$$-\frac{1}{8}\,\sqrt{-cx^4+bx^2+a}\left(\frac{2\,x^2}{c}+\frac{3\,b}{c^2}\right)-\frac{\left(3\,b^2+4\,ac\right)\log\left(\left|2\left(\sqrt{-c}\,x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{16\,\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2/c + 3*b/c^2) - 1/16*(3*b^2 + 4*a*c)*lo
g(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*
c^2)
```

maple [A] time = 0.02, size = 120, normalized size = 1.12

$$-\frac{\sqrt{-c\,x^4+b\,x^2+a}\;x^2}{4c}+\frac{a\arctan\left(\frac{\left(x^2-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{4c^{\frac{3}{2}}}+\frac{3b^2\arctan\left(\frac{\left(x^2-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{16c^{\frac{5}{2}}}-\frac{3\sqrt{-c\,x^4+b\,x^2+a}\;b}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*x^2*(-c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(-c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2
/c^(5/2)*arctan((x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2)*c^(1/2))+1/4*a/c^(3/2)
*arctan((x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2)*c^(1/2))
```

maxima [A] time = 2.42, size = 105, normalized size = 0.98

$$-\frac{\sqrt{-cx^4+bx^2+a}\,x^2}{4\,c}-\frac{3\,b^2\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{16\,c^{\frac{5}{2}}}-\frac{a\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{4\,c^{\frac{3}{2}}}-\frac{3\,\sqrt{-cx^4+bx^2+a}\,b}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-c*x^4 + b*x^2 + a)*x^2/c - 3/16*b^2*arcsin(-(2*c*x^2 - b)/sqrt(b
^2 + 4*a*c))/c^(5/2) - 1/4*a*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(3/
2) - 3/8*sqrt(-c*x^4 + b*x^2 + a)*b/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 - c*x**4), x)
```

3.754 $\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1114, 640, 621, 204}

$$-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(2*c) - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(4*c^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2-cx^4}}{2c}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)}{4c}\\
&= -\frac{\sqrt{a+bx^2-cx^4}}{2c}+\frac{b\,\text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx,x,\frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)}{2c}\\
&= -\frac{\sqrt{a+bx^2-cx^4}}{2c}-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$
-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 - c*x^4]/c - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 - c*x^4])])/(4*c^(3/2))
```

IntegrateAlgebraic [C] time = 13.92, size = 77, normalized size = 1.10

$$
-\frac{\sqrt{a+bx^2-cx^4}}{2c}-\frac{ib\log\left(-2ic^{3/2}\sqrt{a+bx^2-cx^4}-bc+2c^2x^2\right)}{4c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 - c*x^4]/c - ((I/4)*b*Log[-(b*c) + 2*c^2*x^2 - (2*I)*c^
(3/2)*Sqrt[a + b*x^2 - c*x^4]])/c^(3/2)
```

fricas [A] time = 0.73, size = 169, normalized size = 2.41

$$\left[-\frac{b\sqrt{-c}\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c}-4\,ac\right)+4\,\sqrt{-cx^4+bx^2+a}\,c}{8\,c^2},-\frac{b\sqrt{c}\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{c}}{2\left(c^2x^4-bcx^2-ac\right)}\right)+2\,\sqrt{-cx^4+bx^2+a}\,c}{4\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(b*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2 +
 a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2, -1
/4*(b*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/(c^
2*x^4 - b*c*x^2 - a*c)) + 2*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2]
```

giac [A] time = 0.27, size = 70, normalized size = 1.00

$$-\frac{b\log\left(\left|2\left(\sqrt{-c}\,x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{4\,\sqrt{-c}\,c}-\frac{\sqrt{-cx^4+bx^2+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(
sqrt(-c)*c) - 1/2*sqrt(-c*x^4 + b*x^2 + a)/c
```

maple [A] time = 0.01, size = 58, normalized size = 0.83

$$\frac{b\arctan\left(\frac{\left(x^2-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{4c^{\frac{3}{2}}}-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(-c*x^4+b*x^2+a)^(1/2)/c+1/4*b/c^(3/2)*arctan((x^2-1/2*b/c)/(-c*x^4+b*
x^2+a)^(1/2)*c^(1/2))
```

maxima [A] time = 2.47, size = 50, normalized size = 0.71

$$-\frac{b\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{4\,c^{\frac{3}{2}}}-\frac{\sqrt{-cx^4+bx^2+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(3/2) - 1/2*sqrt(-c*x^4 +
 b*x^2 + a)/c
```

mupad [B] time = 4.59, size = 62, normalized size = 0.89

$$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{2\,c}-\frac{b\,\ln\left(\frac{\frac{b}{2}-c\,x^2}{\sqrt{-c}}+\sqrt{-c\,x^4+b\,x^2+a}\right)}{4\,{\left(-c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] - (a + b*x^2 - c*x^4)^(1/2)/(2*c) - (b*log((b/2 - c*x^2)/(-c)^(1/2) + (a +
b*x^2 - c*x^4)^(1/2)))/(4*(-c)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{a + bx^{2} - cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 - c*x**4), x)
```

3.755 $\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1107, 621, 204}

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/(2*Sqrt[c])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx, x, \frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/Sqrt[c]
```

IntegrateAlgebraic [B] time = 0.20, size = 127, normalized size = 2.89

$$\frac{\sqrt{-c}\,\log\left(-8\sqrt{-c}\,cx^2\sqrt{a+bx^2-cx^4}\,+4ac+b^2+4bcx^2-8c^2x^4\right)}{4c} - \frac{\tan^{-1}\left(\frac{2\sqrt{-c}\sqrt{c}\,x^2}{b}-\frac{2\sqrt{c}\sqrt{a+bx^2-cx^4}}{b}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*ArcTan[(2*Sqrt[-c]*Sqrt[c]*x^2)/b - (2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]
)/b]/Sqrt[c] + (Sqrt[-c]*Log[b^2 + 4*a*c + 4*b*c*x^2 - 8*c^2*x^4 - 8*Sqrt[-
c]*c*x^2*Sqrt[a + b*x^2 - c*x^4]])/(4*c)
```

fricas [A] time = 0.85, size = 124, normalized size = 2.82

$$\left[-\frac{\sqrt{-c}\,\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c}\,-4\,ac\right)}{4\,c}, -\frac{\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{c}}{2\left(c^2x^4-bcx^2-ac\right)}\right)}{2\,\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2 + a)
*(2*c*x^2 - b)*sqrt(-c) - 4*a*c)/c, -1/2*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a
)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c))/sqrt(c)]
```

giac [A] time = 0.21, size = 45, normalized size = 1.02

$$-\frac{\log\left(\left|2\left(\sqrt{-c}\,x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/sqr
t(-c)
```

maple [A] time = 0.01, size = 36, normalized size = 0.82

$$\frac{\arctan\left(\frac{\left(x^2-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/c^(1/2)*arctan((x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2)*c^(1/2))
```

maxima [A] time = 2.39, size = 28, normalized size = 0.64

$$-\frac{\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/sqrt(c)
```

mupad [B] time = 4.79, size = 40, normalized size = 0.91

$$\frac{\ln\left(\frac{\frac{b}{2}-c\,x^2}{\sqrt{-c}}+\sqrt{-c\,x^4+b\,x^2+a}\right)}{2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] log((b/2 - c*x^2)/(-c)^(1/2) + (a + b*x^2 - c*x^4)^(1/2))/(2*(-c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**2 - c*x**4), x)
```

3.756 $\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1114, 724, 204}

$$-\frac{\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx, x, \frac{-2a+bx^2}{\sqrt{-a+bx^2+cx^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{-2a+bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
[Out] ArcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

IntegrateAlgebraic [A] time = 0.11, size = 48, normalized size = 1.02

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}-\frac{\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
[Out] -(ArcTan[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[-a + b*x^2 + c*x^4]/Sqrt[a]]/Sqrt[a])
```

fricas [A] time = 1.05, size = 129, normalized size = 2.74

$$\left[-\frac{\sqrt{-a}\,\log\left(\frac{\left(b^2-4\,ac\right)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\,\left(bx^2-2\,a\right)\sqrt{-a}+8\,a^2}{x^4}\right)}{4\,a}, \frac{\arctan\left(\frac{\sqrt{cx^4+bx^2-a}\,\left(bx^2-2\,a\right)\sqrt{a}}{2\,\left(acx^4+abx^2-a^2\right)}\right)}{2\,\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] [-1/4*sqrt(-a)*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4)/a, 1/2*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2))/sqrt(a)]

giac [A] time = 0.21, size = 36, normalized size = 0.77

$$\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")

[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a))/sqrt(a)

maple [A] time = 0.02, size = 45, normalized size = 0.96

$$-\frac{\ln\left(\frac{b\,x^2-2a+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^4+b*x^2-a)^(1/2),x)

[Out] -1/2/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)

maxima [A] time = 2.33, size = 36, normalized size = 0.77

$$-\frac{\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")

[Out] -1/2*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/sqrt(a)

mupad [B] time = 4.52, size = 52, normalized size = 1.11

$$-\frac{\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{-a}}-\frac{\ln\left(2\,\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}-2\,a+b\,x^2\right)}{2\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] - log(1/x^2)/(2*(-a)^(1/2)) - log(2*(-a)^(1/2)*(b*x^2 - a + c*x^4)^(1/2) -
2*a + b*x^2)/(2*(-a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.757 $\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1114, 730, 724, 204}

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqr
t[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)}{4a}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx, x, \frac{-2a+bx^2}{\sqrt{-a+bx^2+cx^4}}\right)}{2a}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2} - \frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 76, normalized size = 0.99

$$\frac{b\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}} + \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 80, normalized size = 1.04

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2} - \frac{b\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}} - \frac{\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt
[-a + b*x^2 + c*x^4]/Sqrt[a]])/(2*a^(3/2))
```

fricas [A] time = 1.11, size = 188, normalized size = 2.44

$$\left[-\frac{\sqrt{-a}\,bx^2\log\left(\frac{\left(b^2-4\,ac\right)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{-a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2-a}\,a}{8\,a^2x^2},\ \frac{\sqrt{a}\,bx^2\arctan\left(\frac{\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{a}}{2\left(acx^4+abx^2-a^2\right)}\right)+2\,\sqrt{cx^4+bx^2-a}\,a}{4\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b*x^2*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 + b
*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 - a)*
a)/(a^2*x^2), 1/4*(sqrt(a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2
- 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*a)/(a
^2*x^2)]
```

giac [A] time = 0.22, size = 111, normalized size = 1.44

$$\frac{b\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)b-2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a))/a^(3/2) - 1/
2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*b - 2*a*sqrt(c))/(((sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 - a))^2 + a)*a)
```

maple [A] time = 0.01, size = 74, normalized size = 0.96

$$-\frac{b\ln\left(\frac{b\,x^2-2a+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{4\sqrt{-a}\,\,a}+\frac{\sqrt{c\,x^4+b\,x^2-a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2-a)^(1/2)/a/x^2-1/4*b/a/(-a)^(1/2)*ln((b*x^2-2*a+2*(-a)^(1/
2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

maxima [A] time = 2.41, size = 62, normalized size = 0.81

$$-\frac{b \arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}} + \frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{4\,a^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2 - a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(3/2) +
 1/2*sqrt(c*x^4 + b*x^2 - a)/(a*x^2)
```

mupad [B] time = 4.55, size = 64, normalized size = 0.83

$$\frac{\sqrt{c\,x^4 + b\,x^2 - a}}{2\,a\,x^2} - \frac{b\,\mathrm{atanh}\left(\frac{a-\frac{b\,x^2}{2}}{\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}\right)}{4\,(-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] (b*x^2 - a + c*x^4)^(1/2)/(2*a*x^2) - (b*atanh((a - (b*x^2)/2)/((-a)^(1/2)*
(b*x^2 - a + c*x^4)^(1/2))))/(4*(-a)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{-a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.758 $\int \frac{1}{x^5 \sqrt{-a+bx^2+cx^4}} \, dx$

Optimal. Leaf size=115

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1114, 744, 806, 724, 204}

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*
x^2) - ((3*b^2 + 4*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c
*x^4])])/(16*a^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
```

```
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\left(3b^2+4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx,x,x\right.}{16a^2}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}-\frac{\left(3b^2+4ac\right)\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx,x,\frac{-2a+}{\sqrt{-a+bx}}\right.}{8a^2}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}-\frac{\left(3b^2+4ac\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 95, normalized size = 0.83

$$\frac{\left(4ac+3b^2\right)\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{\left(2a+3bx^2\right)\sqrt{-a+bx^2+cx^4}}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((2*a + 3*b*x^2)*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((3*b^2 + 4*a*c)*A
rcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

IntegrateAlgebraic [A] time = 0.27, size = 95, normalized size = 0.83

$$\frac{\left(-4ac-3b^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{\left(2a+3bx^2\right)\sqrt{-a+bx^2+cx^4}}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((2*a + 3*b*x^2)*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((-3*b^2 - 4*a*c)*
ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(5/2))
```

fricas [A] time = 1.24, size = 230, normalized size = 2.00

$$\left[-\frac{\left(3\,b^2+4\,ac\right)\sqrt{-a}\,x^4\log\left(\frac{\left(b^2-4\,ac\right)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{-a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2-a}\left(3\,abx^2+2\,a^2\right)}{32\,a^3x^4},\ \frac{\left(3\,b^2+4\,ac\right)\sqrt{a}\,x^4\arctan\left(\frac{\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{a}}{2\left(acx^4+abx^2-a^2\right)}\right)+2\,\sqrt{cx^4+bx^2-a}\left(3\,abx^2+2\,a^2\right)}{16\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 + 4*a*c)*sqrt(-a)*x^4*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4
*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*x^
4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x^4), 1/16*((3*b^2 + 4*a*c)*sqrt(a
)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a
*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x^4)]
```

giac [B] time = 0.23, size = 224, normalized size = 1.95

$$\frac{\left(3\,b^2+4\,ac\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^3b^2+4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^3ac+5\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)ab^2-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)a^2c-8\,a^2b\sqrt{c}}{8\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^2+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2 + 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a)
)/a^(5/2) - 1/8*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*b^2 + 4*(sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*a*c + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
```

```
^2 - a))*a*b^2 - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*a^2*c - 8*a^2*b*
sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^2 + a)^2*a^2)
```

maple [A] time = 0.01, size = 149, normalized size = 1.30

$$-\frac{c \ln \left(\frac{b x^2-2a+2\sqrt{-a}\,\sqrt{c x^4+b x^2-a}}{x^2}\right)}{4\sqrt{-a}\,a}-\frac{3b^2 \ln \left(\frac{b x^2-2a+2\sqrt{-a}\,\sqrt{c x^4+b x^2-a}}{x^2}\right)}{16\sqrt{-a}\,a^2}+\frac{3\sqrt{c x^4+b x^2-a}\,b}{8a^2x^2}+\frac{\sqrt{c x^4+b x^2-a}}{4a x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/4*(c*x^4+b*x^2-a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^2-3/16*b^
2/a^2/(-a)^(1/2)*ln((b*x^2-2*a+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)-1/4
*c/a/(-a)^(1/2)*ln((b*x^2-2*a+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

maxima [A] time = 2.43, size = 126, normalized size = 1.10

$$-\frac{3\,b^2 \arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{16\,a^{\frac{5}{2}}}-\frac{c \arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{4\,a^{\frac{3}{2}}}+\frac{3\,\sqrt{cx^4+bx^2-a}\,b}{8\,a^2x^2}+\frac{\sqrt{cx^4+bx^2-a}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/16*b^2*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(5/2
) - 1/4*c*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(3/2
) + 3/8*sqrt(c*x^4 + b*x^2 - a)*b/(a^2*x^2) + 1/4*sqrt(c*x^4 + b*x^2 - a)/(
a*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^5*(b*x^2 - a + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2-a)**(1/2),x)
[Out] Integral(1/(x**5*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.759 $\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=154

$$-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{\left(16ac+15b^2\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}$$

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1114, 744, 834, 806, 724, 204}

$$\frac{\left(16ac+15b^2\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[-a + b*x^2 + c*x^4])/(24*a^2
*x^4) + ((15*b^2 + 16*a*c)*Sqrt[-a + b*x^2 + c*x^4])/(48*a^3*x^2) - (b*(5*b
^2 + 12*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(3
2*a^(7/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
```

```
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{\operatorname{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)}{6a}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{4}\left(15b^2+16ac\right)+\frac{5bcx}{2}}{x^2\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)}{12a^2}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\left(15b^2+16ac\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2} + \frac{\left(b\left(5b\right.\right.}{}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\left(15b^2+16ac\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2} - \frac{\left(b\left(5b\right.\right.}{}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\left(15b^2+16ac\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2} - \frac{b\left(5b^2\right.}{}$$

Mathematica [A] time = 0.08, size = 116, normalized size = 0.75

$$\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}} + \frac{\sqrt{-a+bx^2+cx^4}\left(8a^2+2a\left(5bx^2+8cx^4\right)+15b^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-a + b*x^2 + c*x^4]*(8*a^2 + 15*b^2*x^4 + 2*a*(5*b*x^2 + 8*c*x^4)))/(
48*a^3*x^6) + (b*(5*b^2 + 12*a*c)*ArcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sqrt[-a
+ b*x^2 + c*x^4])])/(32*a^(7/2))
```

IntegrateAlgebraic [A] time = 0.41, size = 114, normalized size = 0.74

$$\frac{\left(-12abc-5b^3\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}} + \frac{\sqrt{-a+bx^2+cx^4}\left(8a^2+10abx^2+16acx^4+15b^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-a + b*x^2 + c*x^4]*(8*a^2 + 10*a*b*x^2 + 15*b^2*x^4 + 16*a*c*x^4))/(
48*a^3*x^6) + ((-5*b^3 - 12*a*b*c)*ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*x^2 +
c*x^4])/Sqrt[a]])/(16*a^(7/2))
```

fricas [A] time = 4.01, size = 272, normalized size = 1.77

$$\left[-\frac{3\left(5\,b^{3}+12\,abc\right)\sqrt{-a}\,x^{6}\log\left(\frac{\left(b^{2}-4\,ac\right)x^{4}-8\,abx^{2}-4\,\sqrt{cx^{4}+bx^{2}-a}\left(bx^{2}-2\,a\right)\sqrt{-a}+8\,a^{2}}{x^{4}}\right)-4\left(10\,a^{2}bx^{2}+\left(15\,ab^{2}+16\,a^{2}c\right)x^{4}+8\,a^{3}\right)\sqrt{cx^{4}+bx^{2}-a}}{192\,a^{4}x^{6}},\ \frac{3\left(5\,b^{3}+12\,abc\right)\sqrt{a}\,x^{6}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}-a}\left(bx^{2}-2\,a\right)\sqrt{a}}{2\left(acx^{4}+abx^{2}-a^{2}\right)}\right)+2\left(10\,a^{2}bx^{2}+\left(15\,ab^{2}+16\,a^{2}c\right)x^{4}+8\,a^{3}\right)\sqrt{cx^{4}+bx^{2}-a}}{96\,a^{4}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-a)*x^6*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*(10
*a^2*b*x^2 + (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))/(a
^4*x^6), 1/96*(3*(5*b^3 + 12*a*b*c)*sqrt(a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x
^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*(10*a^2*b*x^2
+ (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))/(a^4*x^6)]
```

giac [B] time = 0.27, size = 344, normalized size = 2.23

$$\frac{\left(5\,b^{3}+12\,abc\right)\arctan\left(-\frac{\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}-a}}{\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}}-\frac{15\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}-a}\right)^{5}b^{3}+36\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}-a}\right)^{5}abc+40\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}-a}\right)^{3}ab^{3}+96\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}-a}\right)^{3}a^{2}bc-96\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}-a}\right)^{2}a^{3}c^{\frac{3}{2}}+33\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}-a}\right)a^{2}b^{3}-36\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}-a}\right)a^{3}bc-48\,a^{3}b^{2}\sqrt{c}-32\,a^{4}c^{\frac{3}{2}}}{48\left(\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}-a}\right)^{2}+a\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(5*b^3 + 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqr
t(a))/a^(7/2) - 1/48*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^5*b^3 + 36
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^5*a*b*c + 40*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 - a))^3*a*b^3 + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*a
^2*b*c - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^2*a^3*c^(3/2) + 33*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*a^2*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 - a))*a^3*b*c - 48*a^3*b^2*sqrt(c) - 32*a^4*c^(3/2))/(((sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 - a))^2 + a)^3*a^3)
```

maple [A] time = 0.02, size = 202, normalized size = 1.31

$$-\frac{3bc\ln\left(\frac{b\,x^{2}-2a+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{8\sqrt{-a}\,a^{2}}-\frac{5b^{3}\ln\left(\frac{b\,x^{2}-2a+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{32\sqrt{-a}\,a^{3}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}\,c}{3a^{2}x^{2}}+\frac{5\sqrt{c\,x^{4}+b\,x^{2}-a}\,b^{2}}{16a^{3}x^{2}}+\frac{5\sqrt{c\,x^{4}+b\,x^{2}-a}\,b}{24a^{2}x^{4}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{6a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/6*(c*x^4+b*x^2-a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^4+5/16*b
^2/a^3/x^2*(c*x^4+b*x^2-a)^(1/2)-5/32*b^3/a^3/(-a)^(1/2)*ln((b*x^2-2*a+2*(-
```

```
a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)-3/8*b/a^2*c/(-a)^(1/2)*ln((b*x^2-2*a+2
*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2-a)^(1/2)
```

maxima [A] time = 2.26, size = 179, normalized size = 1.16

$$-\frac{5 b^3 \arcsin\left(-\frac{b}{\sqrt{b^2+4ac}}+\frac{2a}{\sqrt{b^2+4ac}\,x^2}\right)}{32 a^{\frac{7}{2}}}-\frac{3 bc \arcsin\left(-\frac{b}{\sqrt{b^2+4ac}}+\frac{2a}{\sqrt{b^2+4ac}\,x^2}\right)}{8 a^{\frac{5}{2}}}+\frac{5\sqrt{cx^4+bx^2-a}\,b^2}{16a^3x^2}+\frac{\sqrt{cx^4+bx^2-a}\,c}{3a^2x^2}+\frac{5\sqrt{cx^4+bx^2-a}\,b}{24a^2x^4}+\frac{\sqrt{cx^4+bx^2-a}}{6ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/32*b^3*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(7/2
) - 3/8*b*c*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(5
/2) + 5/16*sqrt(c*x^4 + b*x^2 - a)*b^2/(a^3*x^2) + 1/3*sqrt(c*x^4 + b*x^2 -
 a)*c/(a^2*x^2) + 5/24*sqrt(c*x^4 + b*x^2 - a)*b/(a^2*x^4) + 1/6*sqrt(c*x^4
 + b*x^2 - a)/(a*x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(b*x^2 - a + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.760 $\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{\left(b(15b^2-52ac)-2cx^2(5b^2-12ac)\right)\sqrt{a+bx^2+cx^4}}{8c^3(b^2-4ac)}-\frac{bx^4\sqrt{a+bx^2+c}}{c(b^2-4ac)}$$

Rubi [A] time = 0.24, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 738, 832, 779, 621, 206}

$$-\frac{\left(b(15b^2-52ac)-2cx^2(5b^2-12ac)\right)\sqrt{a+bx^2+cx^4}}{8c^3(b^2-4ac)}+\frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{bx^4\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^6*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x^4*Sqrt[a
 + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*(5*b^2 -
 12*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^3*(b^2 - 4*a*c)) + (3*(5*b^2 -
4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
```

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\text{Subst}\left(\int \frac{x\left(-6ab-\frac{3}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx,\right.}{3c\left(b^2-4ac\right)}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12a\right.\right.}{8c^3\left(b^2-4a\right.}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12a\right.\right.}{8c^3\left(b^2-4a\right.}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12a\right.\right.}{8c^3\left(b^2-4a\right.}$$

Mathematica [A] time = 0.19, size = 181, normalized size = 0.95

$$\frac{\frac{2\sqrt{c}\left(4a^2c\left(6cx^2-13b\right)+a\left(15b^3-62b^2cx^2-20bc^2x^4+8c^3x^6\right)+b^2x^2\left(15b^2+5bcx^2-2c^2x^4\right)\right)}{\sqrt{a+bx^2+cx^4}}-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x^2) + b^2*x^2*(15*b^2 + 5*b*c*x^2 - 2*c^
2*x^4) + a*(15*b^3 - 62*b^2*c*x^2 - 20*b*c^2*x^4 + 8*c^3*x^6)))/Sqrt[a + b*
x^2 + c*x^4] - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7/2)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.76, size = 169, normalized size = 0.89

$$\frac{-52a^2bc+24a^2c^2x^2+15ab^3-62ab^2cx^2-20abc^2x^4+8ac^3x^6+15b^4x^2+5b^3cx^4-2b^2c^2x^6}{8c^3\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}} - \frac{3\left(5b^2-4ac\right)\log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{16c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (15*a*b^3 - 52*a^2*b*c + 15*b^4*x^2 - 62*a*b^2*c*x^2 + 24*a^2*c^2*x^2 + 5*b
^3*c*x^4 - 20*a*b*c^2*x^4 - 2*b^2*c^2*x^6 + 8*a*c^3*x^6)/(8*c^3*(-b^2 + 4*a
*c)*Sqrt[a + b*x^2 + c*x^4]) - (3*(5*b^2 - 4*a*c)*Log[b + 2*c*x^2 - 2*Sqrt[
c]*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(7/2))
```

fricas [A] time = 1.83, size = 591, normalized size = 3.11

$$\left[-\frac{3\left(5ab^4-24a^2b^2c+16a^3c^2+\left(5b^4c-24ab^2c^2+16a^2c^3\right)x^4+\left(5b^5-24ab^3c+16a^2bc^2\right)x^2\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)-4\left(2\left(b^2c^3-4ac^4\right)x^6-15ab^3c+52a^2bc^2-5\left(b^3c^2-4abc^3\right)x^4-\left(15b^4c-62ab^2c^2+24a^2c^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{32\left(ab^2c^4-4a^2c^5+\left(b^2c^5-4ac^6\right)x^4+\left(b^3c^4-4abc^5\right)x^2\right)}, -\frac{3\left(5ab^4-24a^2b^2c+16a^3c^2+\left(5b^4c-24ab^2c^2+16a^2c^3\right)x^4+\left(5b^5-24ab^3c+16a^2bc^2\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)-2\left(2\left(b^2c^3-4ac^4\right)x^6-15ab^3c+52a^2bc^2-5\left(b^3c^2-4abc^3\right)x^4-\left(15b^4c-62ab^2c^2+24a^2c^3\right)x^2\right)\sqrt{cx^4+bx^2+a}}{16\left(ab^2c^4-4a^2c^5+\left(b^2c^5-4ac^6\right)x^4+\left(b^3c^4-4abc^5\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^4*c - 24*a*b^2*c^2 +
16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2)*sqrt(c)*log(-8*c
^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c)
- 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - 5*(b^
3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^2)*sqrt(c
*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^4 + (b^3*
c^4 - 4*a*b*c^5)*x^2), -1/16*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b
^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)
*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c
^2*x^4 + b*c*x^2 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^3*c + 52*a
^2*b*c^2 - 5*(b^3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*
c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*
c^6)*x^4 + (b^3*c^4 - 4*a*b*c^5)*x^2)]
```

giac [A] time = 0.27, size = 215, normalized size = 1.13

$$\frac{\left(\left(\frac{2\left(b^2c^2-4ac^3\right)x^2}{b^2c^3-4ac^4}-\frac{5\left(b^3c-4abc^2\right)}{b^2c^3-4ac^4}\right)x^2-\frac{15b^4-62ab^2c+24a^2c^2}{b^2c^3-4ac^4}\right)x^2-\frac{15ab^3-52a^2bc}{b^2c^3-4ac^4}}{8\sqrt{cx^4+bx^2+a}}-\frac{3\left(5b^2-4ac\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(((2*(b^2*c^2 - 4*a*c^3)*x^2/(b^2*c^3 - 4*a*c^4) - 5*(b^3*c - 4*a*b*c^2
)/(b^2*c^3 - 4*a*c^4))*x^2 - (15*b^4 - 62*a*b^2*c + 24*a^2*c^2)/(b^2*c^3 -
4*a*c^4))*x^2 - (15*a*b^3 - 52*a^2*b*c)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^4 + b
*x^2 + a) - 3/16*(5*b^2 - 4*a*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))*sqrt(c) - b))/c^(7/2)
```

maple [**B**] time = 0.02, size = 354, normalized size = 1.86

$$\frac{x^{6}}{4\sqrt{c\,x^{4}+b\,x^{2}+a}\;c}-\frac{13a\,b^{2}x^{2}}{4\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}\;c^{2}}+\frac{15b^{4}x^{2}}{16\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}\;c^{3}}-\frac{5b\,x^{4}}{8\sqrt{c\,x^{4}+b\,x^{2}+a}\;c^{2}}-\frac{13a\,b^{3}}{8\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}\;c^{3}}+\frac{3a\,x^{2}}{4\sqrt{c\,x^{4}+b\,x^{2}+a}\;c^{2}}+\frac{15b^{5}}{32\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}\;c^{4}}-\frac{15b^{2}x^{2}}{16\sqrt{c\,x^{4}+b\,x^{2}+a}\;c^{3}}-\frac{3a\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{5}{2}}}+\frac{15b^{2}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{16c^{\frac{7}{2}}}-\frac{13ab}{8\sqrt{c\,x^{4}+b\,x^{2}+a}\;c^{3}}+\frac{15b^{3}}{32\sqrt{c\,x^{4}+b\,x^{2}+a}\;c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x^6/c/(c*x^4+b*x^2+a)^(1/2)-5/8*b/c^2*x^4/(c*x^4+b*x^2+a)^(1/2)-15/16*b
^2/c^3*x^2/(c*x^4+b*x^2+a)^(1/2)+15/32*b^3/c^4/(c*x^4+b*x^2+a)^(1/2)+15/16*
b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+15/32*b^5/c^4/(4*a*c-b^2)/(c*
x^4+b*x^2+a)^(1/2)+15/16*b^2/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))-13/8*b/c^3*a/(c*x^4+b*x^2+a)^(1/2)-13/4*b^2/c^2*a/(4*a*c-b^2)/(c*
x^4+b*x^2+a)^(1/2)*x^2-13/8*b^3/c^3*a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+3/4
/c^2*a*x^2/(c*x^4+b*x^2+a)^(1/2)-3/4/c^(5/2)*a*ln((c*x^2+1/2*b)/c^(1/2)+(c*
x^4+b*x^2+a)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^9/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{9}}{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**9/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.761 $\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\left(-8ac+3b^2-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 738, 779, 621, 206}

$$\frac{\left(-8ac+3b^2-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 - 8*a
*c - 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (3*b*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
```

```
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\operatorname{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}\\
&= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(3b^2-8ac-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{(3b)\operatorname{Subst}\left(\int \frac{}{\sqrt{a}} \right.}{4}\\
&= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(3b^2-8ac-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{4c-} \right.}{}\\
&= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(3b^2-8ac-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{3b\tanh^{-1}\left(\frac{b}{2\sqrt{c}\sqrt{}} \right.}{4c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 137, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}\left(8a^2c+a\left(-3b^2+10bcx^2+4c^2x^4\right)-b^2x^2\left(3b+cx^2\right)\right)}{\sqrt{a+bx^2+cx^4}}+3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(8*a^2*c - b^2*x^2*(3*b + c*x^2) + a*(-3*b^2 + 10*b*c*x^2 + 4*c
^2*x^4)))/Sqrt[a + b*x^2 + c*x^4] + 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(5/2)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.58, size = 131, normalized size = 0.98

$$\frac{8a^2c-3ab^2+10abcx^2+4ac^2x^4-3b^3x^2-b^2cx^4}{2c^2\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{3b\log\left(-2c^{5/2}\sqrt{a+bx^2+cx^4}+bc^2+2c^3x^2\right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*a*b^2 + 8*a^2*c - 3*b^3*x^2 + 10*a*b*c*x^2 - b^2*c*x^4 + 4*a*c^2*x^4)/(
2*c^2*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (3*b*Log[b*c^2 + 2*c^3*x^2
- 2*c^(5/2)*Sqrt[a + b*x^2 + c*x^4]])/(4*c^(5/2))
```

fricas [A] time = 1.58, size = 459, normalized size = 3.43

$$\left[\frac{3\left(\left(b^3c-4abc^2\right)x^4+ab^3-4a^2bc+\left(b^4-4ab^2c\right)x^2\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+4\left(\left(b^2c^2-4ac^3\right)x^4+3ab^2c-8a^2c^2+\left(3b^3c-10abc^2\right)x^2\right)\sqrt{cx^4+bx^2+a}}{8\left(ab^2c^3-4a^2c^4+\left(b^2c^4-4ac^5\right)x^4+\left(b^3c^3-4abc^4\right)x^2\right)}, \frac{3\left(\left(b^3c-4abc^2\right)x^4+ab^3-4a^2bc+\left(b^4-4ab^2c\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)+2\left(\left(b^2c^2-4ac^3\right)x^4+3ab^2c-8a^2c^2+\left(3b^3c-10abc^2\right)x^2\right)\sqrt{cx^4+bx^2+a}}{4\left(ab^2c^3-4a^2c^4+\left(b^2c^4-4ac^5\right)x^4+\left(b^3c^3-4abc^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^
2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8*a^
2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3 - 4
*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2), 1/4*(3*((b
^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^2)*sqrt(-c)
*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x
^2 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c
- 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c
^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)]
```

giac [A] time = 0.27, size = 154, normalized size = 1.15

$$\frac{\left(\frac{(b^2c-4\,ac^2)x^2}{b^2c^2-4\,ac^3}+\frac{3\,b^3-10\,abc}{b^2c^2-4\,ac^3}\right)x^2+\frac{3\,ab^2-8\,a^2c}{b^2c^2-4\,ac^3}}{2\,\sqrt{cx^4+bx^2+a}}+\frac{3\,b\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}\,-b\right|\right)}{4\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((b^2*c - 4*a*c^2)*x^2/(b^2*c^2 - 4*a*c^3) + (3*b^3 - 10*a*b*c)/(b^2*c
^2 - 4*a*c^3))*x^2 + (3*a*b^2 - 8*a^2*c)/(b^2*c^2 - 4*a*c^3))/sqrt(c*x^4 +
b*x^2 + a) + 3/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(
c) - b))/c^(5/2)
```

maple [B] time = 0.02, size = 264, normalized size = 1.97

$$\frac{2ab\,x^2}{\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c}-\frac{3b^3x^2}{4\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c^2}+\frac{x^4}{2\sqrt{c\,x^4+b\,x^2+a}\;c}+\frac{a\,b^2}{\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c^2}-\frac{3b^4}{8\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c^3}+\frac{3b\,x^2}{4\sqrt{c\,x^4+b\,x^2+a}\;c^2}-\frac{3b\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{5}{2}}}+\frac{a}{\sqrt{c\,x^4+b\,x^2+a}\;c^2}-\frac{3b^2}{8\sqrt{c\,x^4+b\,x^2+a}\;c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*x^4/c/(c*x^4+b*x^2+a)^(1/2)+3/4*b/c^2*x^2/(c*x^4+b*x^2+a)^(1/2)-3/8*b^2
/c^3/(c*x^4+b*x^2+a)^(1/2)-3/4*b^3/c^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^
2-3/8*b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-3/4*b/c^(5/2)*ln((c*x^2+1/2
*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/c^2*a/(c*x^4+b*x^2+a)^(1/2)+2/c*a*b/(4
*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/c^2*a*b^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.762 $\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 738, 640, 621, 206}

$$\frac{x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*Sqrt[a + b
*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])]/(2*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\operatorname{Subst}\left(\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{b^2-4ac}\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2c}\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{c}\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 107, normalized size = 0.93

$$\frac{\frac{2\sqrt{c}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{\sqrt{a+bx^2+cx^4}}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*x^2 + a*(b - 2*c*x^2)))/Sqrt[a + b*x^2 + c*x^4] - (b^2 - 4
*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2
)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.46, size = 96, normalized size = 0.83

$$\frac{ab-2acx^2+b^2x^2}{c\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{\log\left(-2c^{3/2}\sqrt{a+bx^2+cx^4}+bc+2c^2x^2\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*b + b^2*x^2 - 2*a*c*x^2)/(c*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - Lo
g[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]]/(2*c^(3/2))
```

fricas [A] time = 1.62, size = 387, normalized size = 3.37

$$\left[\frac{\left(\left(b^2c-4ac^2\right)x^4+ab^2-4a^2c+\left(b^3-4abc\right)x^2\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)-4\sqrt{cx^4+bx^2+a}\left(abc+\left(b^2c-2ac^2\right)x^2\right)}{4\left(ab^2c^2-4a^2c^3+\left(b^2c^3-4ac^4\right)x^4+\left(b^3c^2-4abc^3\right)x^2\right)}, -\frac{\left(\left(b^2c-4ac^2\right)x^4+ab^2-4a^2c+\left(b^3-4abc\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)+2\sqrt{cx^4+bx^2+a}\left(abc+\left(b^2c-2ac^2\right)x^2\right)}{2\left(ab^2c^2-4a^2c^3+\left(b^2c^3-4ac^4\right)x^4+\left(b^3c^2-4abc^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(
c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c - 2*a*c^2)*
x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c
^3)*x^2), -1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*
x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^
2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c - 2*a*c
^2)*x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a
*b*c^3)*x^2)]
```

giac [A] time = 0.26, size = 101, normalized size = 0.88

$$-\frac{\frac{\left(b^2-2\,ac\right)x^2}{b^2c-4\,ac^2}+\frac{ab}{b^2c-4\,ac^2}}{\sqrt{cx^4+bx^2+a}}-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((b^2 - 2*a*c)*x^2/(b^2*c - 4*a*c^2) + a*b/(b^2*c - 4*a*c^2))/sqrt(c*x^4 +
 b*x^2 + a) - 1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(3/2)
```

maple [A] time = 0.02, size = 149, normalized size = 1.30

$$\frac{b^2x^2}{2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c}+\frac{b^3}{4\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c^2}-\frac{x^2}{2\sqrt{c\,x^4+b\,x^2+a}\;c}+\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2c^{\frac{3}{2}}}+\frac{b}{4\sqrt{c\,x^4+b\,x^2+a}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4*b/c^2/(c*x^4+b*x^2+a)^(1/2)+1/2*b^2/c/
(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^4+b*x^2+
a)^(1/2)+1/2/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.76, size = 84, normalized size = 0.73

$$\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,c^{3/2}}+\frac{\frac{a\,b}{2}-x^2\,\left(a\,c-\frac{b^2}{2}\right)}{2\,c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))/(2*c^(3/2)) + ((a*b)
/2 - x^2*(a*c - b^2/2))/(2*c*(a*c - b^2/4)*(a + b*x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.763 $\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1114, 636}

$$\frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 36, normalized size = 1.00

$$\frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

IntegrateAlgebraic [A] time = 0.37, size = 38, normalized size = 1.06

$$-\frac{-2a-bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
[Out] -((-2*a - b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

fricas [A] time = 1.75, size = 67, normalized size = 1.86

$$\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^
2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [A] time = 0.28, size = 44, normalized size = 1.22

$$\frac{\frac{bx^2}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] (b*x^2/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

maple [**A**] time = 0.01, size = 38, normalized size = 1.06

$$-\frac{b\,x^{2}+2a}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(4ac-b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -(b*x^2+2*a)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [**B**] time = 4.47, size = 37, normalized size = 1.03

$$-\frac{b\,x^{2}+2\,a}{\left(4\,a\,c-b^{2}\right)\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -(2*a + b*x^2)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.764 $\int \frac{x}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{b+2cx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1107, 613}

$$-\frac{b+2cx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b + 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{b+2cx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.03

$$\frac{b+2cx^2}{(4ac-b^2)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^(3/2),x]
[Out] (b + 2*c*x^2)/((-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

IntegrateAlgebraic [A] time = 0.31, size = 37, normalized size = 1.03

$$\frac{-b-2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x^2 + c*x^4)^(3/2),x]
[Out] (-b - 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [A] time = 1.08, size = 67, normalized size = 1.86

$$-\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)}{\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [A] time = 0.20, size = 45, normalized size = 1.25

$$-\frac{\frac{2\,cx^2}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] -(2*c*x^2/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 1.00

$$\frac{2c\,x^2+b}{\sqrt{c\,x^4+b\,x^2+a}\,\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (2*c*x^2+b)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.36, size = 35, normalized size = 0.97

$$\frac{2\,c\,x^2+b}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (b + 2*c*x^2)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.765 $\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 740, 12, 724, 206}

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{a}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 89, normalized size = 1.00

$$
\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2))
```

IntegrateAlgebraic [A] time = 0.47, size = 95, normalized size = 1.07

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{2ac-b^2-bcx^2}{a\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-b^2 + 2*a*c - b*c*x^2)/(a*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ArcTa
nh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]]/a^(3/2)
```

fricas [B] time = 3.18, size = 389, normalized size = 4.37

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)\sqrt{a}\,\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(abcx^2+ab^2-2\,a^2c\right)\sqrt{cx^4+bx^2+a}}{4\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)},\ \frac{\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(abcx^2+ab^2-2\,a^2c\right)\sqrt{cx^4+bx^2+a}}{2\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(c*x^4 +
b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4
*a^3*b*c)*x^2), 1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a
)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(c*x^4 +
 b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 -
4*a^3*b*c)*x^2)]
```

giac [A] time = 0.20, size = 110, normalized size = 1.24

$$\frac{\frac{abcx^2}{a^2b^2-4\,a^3c}+\frac{ab^2-2\,a^2c}{a^2b^2-4\,a^3c}}{\sqrt{cx^4+bx^2+a}}+\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (a*b*c*x^2/(a^2*b^2 - 4*a^3*c) + (a*b^2 - 2*a^2*c)/(a^2*b^2 - 4*a^3*c))/sqr
t(c*x^4 + b*x^2 + a) + arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt
(-a))/(sqrt(-a)*a)
```

maple [A] time = 0.01, size = 99, normalized size = 1.11

$$-\frac{\left(2c\,x^{2}+b\right)b}{2\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}\;a}-\frac{\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}+\frac{1}{2\sqrt{c\,x^{4}+b\,x^{2}+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2/a/(c*x^4+b*x^2+a)^(1/2)-1/2*b/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)-1/2/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(c\,x^{4}+b\,x^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.766 $\int \frac{1}{x^3(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2x^2(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{ax^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 740, 806, 724, 206}

$$-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2x^2(b^2-4ac)}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}+\frac{-2ac+b^2+bcx^2}{ax^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]) - ((3
*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^2) + (3*b*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
```

```
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} - \frac{(3b)\,\text{Subst}\left(\int \frac{}{x\sqrt{\cdots}} \cdots\right)}{4\cdots}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{4a-\cdots} \cdots\right)}{\cdots}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} + \frac{3b\tanh^{-1}\left(\frac{2\cdots}{2\sqrt{a}\sqrt{\cdots}}\right)}{4a^{5/2}}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 0.99

$$\frac{\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx^2-8c^2x^4\right)+3b^2x^2\left(b+cx^2\right)\right)}{x^2\sqrt{a+bx^2+cx^4}}-3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2
*x^4)))/(x^2*Sqrt[a + b*x^2 + c*x^4]) - 3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.58, size = 134, normalized size = 0.96

$$\frac{-4a^2c+ab^2-10abcx^2-8ac^2x^4+3b^3x^2+3b^2cx^4}{2a^2x^2\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}-\frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*b^2 - 4*a^2*c + 3*b^3*x^2 - 10*a*b*c*x^2 + 3*b^2*c*x^4 - 8*a*c^2*x^4)/(2
*a^2*(-b^2 + 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]) - (3*b*ArcTanh[(Sqrt[c]*x^
2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/(2*a^(5/2))
```

fricas [A] time = 1.45, size = 485, normalized size = 3.49

$$\left[\frac{3\left(\left(b^3c-4abc^2\right)x^6+\left(b^4-4ab^2c\right)x^4+\left(ab^3-4a^2bc\right)x^2\right)\sqrt{a}\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2+4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)-4\left(\left(3ab^2c-8a^2c^2\right)x^4+a^2b^2-4a^3c+\left(3ab^3-10a^2bc\right)x^2\right)\sqrt{cx^4+bx^2+a}}{8\left(\left(a^3b^2c-4a^4c^2\right)x^6+\left(a^3b^3-4a^4bc\right)x^4+\left(a^4b^2-4a^5c\right)x^2\right)}, -\frac{3\left(\left(b^3c-4abc^2\right)x^6+\left(b^4-4ab^2c\right)x^4+\left(ab^3-4a^2bc\right)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(\left(3ab^2c-8a^2c^2\right)x^4+a^2b^2-4a^3c+\left(3ab^3-10a^2bc\right)x^2\right)\sqrt{cx^4+bx^2+a}}{4\left(\left(a^3b^2c-4a^4c^2\right)x^6+\left(a^3b^3-4a^4bc\right)x^4+\left(a^4b^2-4a^5c\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2
 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^4
+ a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5
*c)*x^2), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3
 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^
2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^
```

```
3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*
x^2)]
```

giac [A] time = 0.28, size = 200, normalized size = 1.44

$$-\frac{\frac{\left(a^{2} b^{2} c-2 \, a^{3} c^{2}\right) x^{2}}{a^{4} b^{2}-4 \, a^{5} c}+\frac{a^{2} b^{3}-3 \, a^{3} b c}{a^{4} b^{2}-4 \, a^{5} c}}{\sqrt{c x^{4}+b x^{2}+a}}-\frac{3 \, b \arctan\left(-\frac{\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}}{\sqrt{-a}}\right)}{2 \, \sqrt{-a} a^{2}}+\frac{\left(\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}\right) b+2 \, a \sqrt{c}}{2 \, {\left({\left(\sqrt{c} x^{2}-\sqrt{c x^{4}+b x^{2}+a}\right)}^{2}-a\right)} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b^2*c - 2*a^3*c^2)*x^2/(a^4*b^2 - 4*a^5*c) + (a^2*b^3 - 3*a^3*b*c)/(
a^4*b^2 - 4*a^5*c))/sqrt(c*x^4 + b*x^2 + a) - 3/2*b*arctan(-(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/2*((sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^2 - a)*a^2)
```

maple [A] time = 0.02, size = 195, normalized size = 1.40

$$\frac{3 b^{2} c \,x^{2}}{2 \left(4 a c -b^{2}\right) \sqrt{c \,x^{4}+b \,x^{2}+a}\, a^{2}}+\frac{3 b^{3}}{4 \left(4 a c -b^{2}\right) \sqrt{c \,x^{4}+b \,x^{2}+a}\, a^{2}}-\frac{2 \left(2 c \,x^{2}+b \right) c}{\left(4 a c -b^{2}\right) \sqrt{c \,x^{4}+b \,x^{2}+a}\, a}+\frac{3 b \ln \left(\frac{b \,x^{2}+2 a +2 \sqrt{c \,x^{4}+b \,x^{2}+a}\, \sqrt{a}}{x^{2}}\right)}{4 a^{\frac{5}{2}}}-\frac{3 b}{4 \sqrt{c \,x^{4}+b \,x^{2}+a}\, a^{2}}-\frac{1}{2 \sqrt{c \,x^{4}+b \,x^{2}+a}\, a \,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/a/x^2/(c*x^4+b*x^2+a)^(1/2)-3/4*b/a^2/(c*x^4+b*x^2+a)^(1/2)+3/2*b^2/a^
2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2*c+3/4*b^3/a^2/(4*a*c-b^2)/(c*x^4+b*
x^2+a)^(1/2)+3/4*b/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x
^2)-2*c/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a + b x^{2} + c x^{4}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.767 $\int \frac{1}{x^5\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}+\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2+cx^4}}{8a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2x^4\left(b^2-4ac\right)}+\frac{\quad}{ax^4\left(b\right.}$$

Rubi [A] time = 0.21, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 740, 834, 806, 724, 206}

$$\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2+cx^4}}{8a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2x^4\left(b^2-4ac\right)}-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}+\frac{-2ac+b^2+bcx^2}{ax^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^4*Sqrt[a + b*x^2 + c*x^4]) - ((5
*b^2 - 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(15*
b^2 - 52*a*c)*Sqrt[a + b*x^2 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^2) - (3*(5*b^
2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*
a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+12ac\right)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}b(15}{}\right.}{}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(15b^2-52ac\right)}{8a^3\left(b^2-\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(15b^2-52ac\right)}{8a^3\left(b^2-\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(15b^2-52ac\right)}{8a^3\left(b^2-\right.}$$

Mathematica [A] time = 0.13, size = 179, normalized size = 0.92

$$\frac{3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{2\sqrt{a}\left(-8a^3c+2a^2\left(b^2+10bcx^2-12c^2x^4\right)+abx^2\left(-5b^2+62bcx^2+52c^2x^4\right)-15b^3x^4\left(b+cx^2\right)\right)}{x^4\sqrt{a+bx^2+cx^4}}}{16a^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-8*a^3*c - 15*b^3*x^4*(b + c*x^2) + 2*a^2*(b^2 + 10*b*c*x^2 -
12*c^2*x^4) + a*b*x^2*(-5*b^2 + 62*b*c*x^2 + 52*c^2*x^4)))/(x^4*Sqrt[a + b*
x^2 + c*x^4]) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(7/2)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.87, size = 175, normalized size = 0.90

$$\frac{-8a^3c+2a^2b^2+20a^2bcx^2-24a^2c^2x^4-5ab^3x^2+62ab^2cx^4+52abc^2x^6-15b^4x^4-15b^3cx^6}{8a^3x^4\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}} - \frac{3\left(4ac-5b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (2*a^2*b^2 - 8*a^3*c - 5*a*b^3*x^2 + 20*a^2*b*c*x^2 - 15*b^4*x^4 + 62*a*b^2
*c*x^4 - 24*a^2*c^2*x^4 - 15*b^3*c*x^6 + 52*a*b*c^2*x^6)/(8*a^3*(-b^2 + 4*a
*c)*x^4*Sqrt[a + b*x^2 + c*x^4]) - (3*(-5*b^2 + 4*a*c)*ArcTanh[(Sqrt[c]*x^2
 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(7/2))
```

fricas [A] time = 2.74, size = 615, normalized size = 3.15

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^8+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^6+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x^4\right)\sqrt{a}\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\left(\left(15\,ab^3c-52\,a^2bc^2\right)x^6-2\,a^3b^2+8\,a^4c+\left(15\,ab^4-62\,a^2b^2c+24\,a^3c^2\right)x^4+5\left(a^2b^3-4\,a^3bc\right)x^2\right)\sqrt{cx^4+bx^2+a}}{32\left(\left(a^4b^2c-4\,a^5c^2\right)x^8+\left(a^4b^3-4\,a^5bc\right)x^6+\left(a^5b^2-4\,a^6c\right)x^4\right)},\ \frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^8+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^6+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(\left(15\,ab^3c-52\,a^2bc^2\right)x^6-2\,a^3b^2+8\,a^4c+\left(15\,ab^4-62\,a^2b^2c+24\,a^3c^2\right)x^4+5\left(a^2b^3-4\,a^3bc\right)x^2\right)\sqrt{cx^4+bx^2+a}}{16\left(\left(a^4b^2c-4\,a^5c^2\right)x^8+\left(a^4b^3-4\,a^5bc\right)x^6+\left(a^5b^2-4\,a^6c\right)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(a)*lo
g(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(a) + 8*a^2)/x^4) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^6 - 2*a^3*b^2 + 8
*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^4 + 5*(a^2*b^3 - 4*a^3*b*
c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^8 + (a^4*b^3 -
4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4), 1/16*(3*((5*b^4*c - 24*a*b^2*c^2
 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 2
4*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*
(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3*c - 52*a^2
*b*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*
x^4 + 5*(a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^4*b^2*c - 4
*a^5*c^2)*x^8 + (a^4*b^3 - 4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4)]
```

giac [A] time = 0.38, size = 350, normalized size = 1.79

$$\frac{\frac{\left(a^3b^3c-3\,a^4bc^2\right)x^2}{a^6b^2-4\,a^7c}+\frac{a^3b^4-4\,a^4b^2c+2\,a^5c^2}{a^6b^2-4\,a^7c}}{\sqrt{cx^4+bx^2+a}}+\frac{3\left(5\,b^2-4\,ac\right)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^3}-\frac{7\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2-4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3ac+8\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2ab\sqrt{c}-9\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)ab^2-4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)a^2c-16\,a^2b\sqrt{c}}{8\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^3*b^3*c - 3*a^4*b*c^2)*x^2/(a^6*b^2 - 4*a^7*c) + (a^3*b^4 - 4*a^4*b^2*c
 + 2*a^5*c^2)/(a^6*b^2 - 4*a^7*c))/sqrt(c*x^4 + b*x^2 + a) + 3/8*(5*b^2 - 4
*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a
^3) - 1/8*(7*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 - 4*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^3*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a
))^2*a*b*sqrt(c) - 9*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 - 4*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c - 16*a^2*b*sqrt(c))/(((sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^3)
```

maple [A] time = 0.02, size = 314, normalized size = 1.61

$$\frac{13b\,c^2x^2}{2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a^2}-\frac{15b^3c\,x^2}{8\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a^3}+\frac{13b^2c}{4\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a^2}-\frac{15b^4}{16\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a^3}+\frac{3c\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{4a^{\frac{5}{2}}}-\frac{15b^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{16a^{\frac{7}{2}}}-\frac{3c}{4\sqrt{c\,x^4+b\,x^2+a}\;a^2}+\frac{15b^2}{16\sqrt{c\,x^4+b\,x^2+a}\;a^3}+\frac{5b}{8\sqrt{c\,x^4+b\,x^2+a}\;a^2x^2}-\frac{1}{4\sqrt{c\,x^4+b\,x^2+a}\;a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/4/a/x^4/(c*x^4+b*x^2+a)^(1/2)+5/8*b/a^2/x^2/(c*x^4+b*x^2+a)^(1/2)+15/16*
b^2/a^3/(c*x^4+b*x^2+a)^(1/2)-15/8*b^3/a^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2
)*x^2*c-15/16*b^4/a^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-15/16*b^2/a^(7/2)*l
n((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+13/2*b/a^2*c^2/(4*a*c-b^
2)/(c*x^4+b*x^2+a)^(1/2)*x^2+13/4*b^2/a^2*c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/
2)-3/4*c/a^2/(c*x^4+b*x^2+a)^(1/2)+3/4*c/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x
^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5} \left(a + b x^{2} + c x^{4}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(1/(x**5*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.768 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=50

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3, 2016, 1588}

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{x\sqrt{bx^2+cx^4}}{3c} - \frac{(2b)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}$$
$$= -\frac{2b\sqrt{bx^2+cx^4}}{3c^2x} + \frac{x\sqrt{bx^2+cx^4}}{3c}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.68

$$\frac{(cx^2-2b)\sqrt{x^2(b+cx^2)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

IntegrateAlgebraic [A] time = 0.04, size = 34, normalized size = 0.68

$$\frac{(cx^2-2b)\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
[Out] ((-2*b + c*x^2)*Sqrt[b*x^2 + c*x^4])/(3*c^2*x)
```

fricas [A] time = 0.97, size = 30, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\,(cx^2-2\,b)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.00, size = 37, normalized size = 0.74

$$-\frac{\left(c\,x^2 + b\right)\left(-c\,x^2 + 2b\right)x}{3\sqrt{c\,x^4 + b\,x^2}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(1/2),x)
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.26, size = 34, normalized size = 0.68

$$\frac{c^2x^4 - bcx^2 - 2\,b^2}{3\,\sqrt{cx^2 + b}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

mupad [B] time = 4.60, size = 33, normalized size = 0.66

$$-\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(\frac{2\,b}{3\,c^2} - \frac{x^2}{3\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^(1/2),x)
[Out] -((b*x^2 + c*x^4)^(1/2)*((2*b)/(3*c^2) - x^2/(3*c)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

3.769 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3, 2018, 640, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right) \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c} \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c} \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.26

$$\frac{x\left(\sqrt{c}\,x\left(b+cx^2\right)-b\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) - b*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b +
c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.20, size = 68, normalized size = 1.17

$$\frac{b\log\left(-2c^{3/2}\sqrt{bx^2+cx^4}+bc+2c^2x^2\right)}{4c^{3/2}}+\frac{\sqrt{bx^2+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) + (b*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[b*x^2 +
 c*x^4]])/(4*c^(3/2))
```

fricas [A] time = 1.02, size = 114, normalized size = 1.97

$$\left[\frac{b\sqrt{c}\log\left(-2\,cx^2 - b + 2\,\sqrt{cx^4 + bx^2}\,\sqrt{c}\right) + 2\,\sqrt{cx^4 + bx^2}\,c}{4\,c^2}, \frac{b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right) + \sqrt{cx^4 + bx^2}\,c}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/
(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

giac [A] time = 0.20, size = 59, normalized size = 1.02

$$\frac{b\log\left(\left|-2\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} - b\right|\right)}{4\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(3/2)
+ 1/2*sqrt(c*x^4 + b*x^2)/c
```

maple [A] time = 0.01, size = 64, normalized size = 1.10

$$\frac{\sqrt{c\,x^2 + b}\,\left(-bc\ln\left(\sqrt{c}\,x + \sqrt{c\,x^2 + b}\right) + \sqrt{c\,x^2 + b}\,c^{\frac{3}{2}}x\right)x}{2\sqrt{c\,x^4 + b\,x^2}\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(3/2)-b*ln(c^(1/2)*x+(c*x^2+b)^(
1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

maxima [A] time = 1.16, size = 52, normalized size = 0.90

$$-\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 1/2*sqrt(
c*x^4 + b*x^2)/c
```

mupad [B] time = 4.61, size = 53, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{2\,c}-\frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4
)^(1/2)))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

3.770 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3, 1588}

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{\sqrt{bx^2+cx^4}}{cx}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\sqrt{x^2\left(b+cx^2\right)}}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

IntegrateAlgebraic [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

fricas [A] time = 0.95, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^4+bx^2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)/(c*x)
```

giac [A] time = 0.18, size = 31, normalized size = 1.41

$$-\frac{2\sqrt{b}}{\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(b)/((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)
```

maple [A] time = 0.00, size = 26, normalized size = 1.18

$$\frac{\left(c\,x^{2}+b\right)x}{\sqrt{c\,x^{4}+b\,x^{2}}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(1/2),x)
[Out] (c*x^2+b)/(c*x^4+b*x^2)^(1/2)/c*x
```

maxima [A] time = 1.22, size = 13, normalized size = 0.59

$$\frac{\sqrt{cx^{2}+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(c*x^2 + b)/c
```

mupad [B] time = 4.37, size = 20, normalized size = 0.91

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^(1/2),x)
[Out] (b*x^2 + c*x^4)^(1/2)/(c*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

3.771 $\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}} \, dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3, 2013, 620, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{x}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= \operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2*
(b + c*x^2)])
```

IntegrateAlgebraic [A] time = 0.14, size = 40, normalized size = 1.29

$$-\frac{\log\left(-2\sqrt{c}\sqrt{bx^2+cx^4}+b+2cx^2\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] -1/2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

fricas [A] time = 0.92, size = 74, normalized size = 2.39

$$\left[\frac{\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

giac [A] time = 0.19, size = 39, normalized size = 1.26

$$-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/sqrt(c)
```

maple [A] time = 0.00, size = 44, normalized size = 1.42

$$\frac{\sqrt{c\,x^2+b}\;x\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)}{\sqrt{c\,x^4+b\,x^2}\;\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))/c^(1/
2)
```

maxima [A] time = 1.06, size = 32, normalized size = 1.03

$$\frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c)
```

mupad [B] time = 4.56, size = 33, normalized size = 1.06

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

3.772 $\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3, 2008, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.73

$$-\frac{x\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
[Out] -((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(b
 + c*x^2)]))
```

IntegrateAlgebraic [A] time = 0.04, size = 30, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

fricas [A] time = 0.99, size = 80, normalized size = 2.67

$$\left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sqr
t(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))/b]
```

giac [A] time = 0.19, size = 46, normalized size = 1.53

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b)
)/(sqrt(-b)*sgn(x))
```

maple [B] time = 0.00, size = 50, normalized size = 1.67

$$-\frac{\sqrt{c\,x^2+b}\;x\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\;\sqrt{b}}{x}\right)}{\sqrt{c\,x^4+b\,x^2}\;\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^
(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(1/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

3.773 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3, 2014}

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{bx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

IntegrateAlgebraic [A] time = 0.13, size = 23, normalized size = 1.00

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

fricas [A] time = 0.84, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

giac [A] time = 0.18, size = 25, normalized size = 1.09

$$\frac{1}{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))
```

maple [A] time = 0.00, size = 26, normalized size = 1.13

$$-\frac{c\,x^2+b}{\sqrt{c\,x^4+b\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.02, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

mupad [B] time = 4.31, size = 21, normalized size = 0.91

$$-\frac{\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^(1/2)),x)
[Out] -(b*x^2 + c*x^4)^(1/2)/(b*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

3.774 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3, 2025, 2008, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(2*b^(3/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{\sqrt{bx^2+cx^4}}{2bx^3} - \frac{c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{2bx^3} + \frac{c\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{2bx^3} + \frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 1.15

$$\frac{c\sqrt{x^2\left(b+cx^2\right)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{2\sqrt{\frac{cx^2}{b}+1}}-\frac{b}{2cx^2}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] (c*Sqrt[x^2*(b + c*x^2)]*(-1/2*b/(c*x^2) + ArcTanh[Sqrt[1 + (c*x^2)/b]]/(2*
Sqrt[1 + (c*x^2)/b])))/(b^2*x)
```

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 1.00

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^
4]])/(2*b^(3/2))
```

fricas [A] time = 1.07, size = 133, normalized size = 2.25

$$\left[\frac{\sqrt{b}\,cx^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\,b}{4\,b^2x^3},-\frac{\sqrt{-b}\,cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,b}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]-1/2/b/x*sqrt(b*(1/x)^2+c)-2*c/4/b/sqrt(
b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 73, normalized size = 1.24

$$-\frac{\sqrt{c\,x^2+b}\,\left(-bc\,x^2\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)+\sqrt{c\,x^2+b}\,b^{\frac{3}{2}}\right)}{2\sqrt{c\,x^4+b\,x^2}\,b^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2*b+(c*x^2
+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{cx^4+bx^2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

mupad [B] time = 4.64, size = 76, normalized size = 1.29

$$-\frac{\left(\frac{\sqrt{c}\,x^2\,\sqrt{c+\frac{b}{x^2}}}{2\,b}+\frac{c^{3/2}\,x^3\,\mathrm{asin}\left(\frac{\sqrt{b}\,1{}\mathrm{i}}{\sqrt{c}\,x}\right)\,1{}\mathrm{i}}{2\,b^{3/2}}\right)\,\sqrt{\frac{b}{c\,x^2}+1}}{x\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((((c^(1/2)*x^2*(c + b/x^2)^(1/2))/(2*b) + (c^(3/2)*x^3*asin((b^(1/2)*1i)/(
c^(1/2)*x))*1i)/(2*b^(3/2)))*(b/(c*x^2) + 1)^(1/2))/(x*(b*x^2 + c*x^4)^(1/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{x^{2}\left(b + cx^{2}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

3.775 $\int \frac{1}{x^3 \sqrt{2+2a-2(1+a)+bx^2+cx^4}} \, dx$

Optimal. Leaf size=52

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2} - \frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.107, Rules used = {3, 2016, 2014}

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2} - \frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(2c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} + \frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(2cx^2-b\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

IntegrateAlgebraic [A] time = 0.15, size = 35, normalized size = 0.67

$$\frac{\left(2cx^2-b\right)\sqrt{bx^2+cx^4}}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
[Out] ((-b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^4)
```

fricas [A] time = 0.82, size = 31, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\left(2\,cx^2-b\right)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

giac [A] time = 0.19, size = 57, normalized size = 1.10

$$\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}\,+b}{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) + b)/(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2))^3
```

maple [A] time = 0.00, size = 37, normalized size = 0.71

$$-\frac{\left(c\,x^2+b\right)\left(-2c\,x^2+b\right)}{3\sqrt{c\,x^4+b\,x^2}\;b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.09, size = 44, normalized size = 0.85

$$\frac{2\sqrt{cx^4+bx^2}\,c}{3\,b^2x^2}-\frac{\sqrt{cx^4+bx^2}}{3\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/(b*x^4)
```

mupad [B] time = 4.47, size = 29, normalized size = 0.56

$$-\frac{\left(b-2\,c\,x^2\right)\,\sqrt{c\,x^4+b\,x^2}}{3\,b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b - 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b^2*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

3.776 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3, 2025, 2008, 206}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) - (3
*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(3c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}+\frac{\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 0.51

$$-\frac{c^2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{cx^2}{b}+1\right)}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
[Out] -((c^2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (c*x^2)/b])
/(b^3*x))
```

IntegrateAlgebraic [A] time = 0.07, size = 71, normalized size = 0.82

$$\frac{\left(3cx^2-2b\right)\sqrt{bx^2+cx^4}}{8b^2x^5}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*b + 3*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^5) - (3*c^2*ArcTanh[(Sqrt[b
]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

fricas [A] time = 1.00, size = 163, normalized size = 1.87

$$\left[\frac{3\sqrt{b}c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(3\,bcx^2-2\,b^2\right)}{16\,b^3x^5},\ \frac{3\,\sqrt{-b}\,c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\left(3\,bcx^2-2\,b^2\right)}{8\,b^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sqrt
(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^
4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]2*(-2*b^2/16/b^3/x/x+3*b*c/16/b^3)/x*sqr
t(b*(1/x)^2+c)+6*c^2/16/b^2/sqrt(b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 94, normalized size = 1.08

$$-\frac{\sqrt{c\,x^2+b}\,\left(3b\,c^2x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^2+b}\,b^{\frac{3}{2}}c\,x^2+2\sqrt{c\,x^2+b}\,b^{\frac{5}{2}}\right)}{8\sqrt{c\,x^4+b\,x^2}\,b^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b)^(1/2)*(3*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*b*c^2-3*(c*
x^2+b)^(1/2)*b^(3/2)*x^2*c+2*(c*x^2+b)^(1/2)*b^(5/2))/x^3/(c*x^4+b*x^2)^(1/
2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\, \sqrt{c\, x^4 + b\, x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

3.777 $\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{a+cx^4}}{2c}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4, 261}

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + c*x^4]/(2*c)
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{x^3}{\sqrt{a+cx^4}}\,dx\\ &= \frac{\sqrt{a+cx^4}}{2c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
[Out] Sqrt[a + c*x^4]/(2*c)
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{\sqrt{a + cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
[Out] Sqrt[a + c*x^4]/(2*c)
```

fricas [A] time = 1.20, size = 14, normalized size = 0.78

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(c*x^4 + a)/c
```

giac [A] time = 0.19, size = 14, normalized size = 0.78

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(c*x^4 + a)/c
```

maple [A] time = 0.01, size = 15, normalized size = 0.83

$$\frac{\sqrt{c\,x^4 + a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+a)^(1/2)/c
```

maxima [A] time = 0.97, size = 14, normalized size = 0.78

$$\frac{\sqrt{cx^4+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(c*x^4 + a)/c
```

mupad [B] time = 4.66, size = 14, normalized size = 0.78

$$\frac{\sqrt{c\,x^4+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + c*x^4)^(1/2),x)
[Out] (a + c*x^4)^(1/2)/(2*c)
```

sympy [A] time = 0.89, size = 22, normalized size = 1.22

$$\begin{cases} \frac{\sqrt{a+cx^4}}{2c} & \text{for}\ c \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+a)**(1/2),x)
[Out] Piecewise((sqrt(a + c*x**4)/(2*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True))
```

3.778 $\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4, 275, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{x}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{a+cx^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

IntegrateAlgebraic [A] time = 0.04, size = 32, normalized size = 1.07

$$-\frac{\log\left(\sqrt{a+cx^4}-\sqrt{c}\,x^2\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
[Out] -1/2*Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4]]/Sqrt[c]
```

fricas [A] time = 2.24, size = 63, normalized size = 2.10

$$\left[\frac{\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\,\sqrt{c}\,x^2-a\right)}{4\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2}{\sqrt{cx^4+a}}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a)/sqrt(c), -1/2*sqrt(-
c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + a))/c]
```

giac [A] time = 0.16, size = 25, normalized size = 0.83

$$-\frac{\log\left(\left|-\sqrt{c}\,x^2+\sqrt{cx^4+a}\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/sqrt(c)
```

maple [A] time = 0.01, size = 24, normalized size = 0.80

$$\frac{\ln\left(\sqrt{c}\;x^2+\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

maxima [B] time = 2.43, size = 45, normalized size = 1.50

$$-\frac{\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x^2))/
sqrt(c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + c*x^4)^(1/2),x)
```

```
[Out] int(x/(a + c*x^4)^(1/2), x)
```

sympy [A] time = 1.10, size = 20, normalized size = 0.67

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+a)**(1/2),x)
```

```
[Out] asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

3.779 $\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=27

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{1}{x\sqrt{a+cx^4}}\,dx$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^4\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^4}\right)}{2c}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
[Out] -1/2*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.04, size = 27, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
[Out] -1/2*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/Sqrt[a]
```

fricas [A] time = 1.14, size = 63, normalized size = 2.33

$$\left[\frac{\log\left(\frac{cx^4-2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-a}}{a}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4)/sqrt(a), 1/2*sqrt(-
a)*arctan(sqrt(c*x^4 + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.15, size = 23, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 29, normalized size = 1.07

$$-\frac{\ln\left(\frac{2a+2\sqrt{c\,x^4+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^4+a)^(1/2))/x^2)
```

maxima [A] time = 2.29, size = 37, normalized size = 1.37

$$\frac{\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 4.55, size = 19, normalized size = 0.70

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)^(1/2)),x)
```

```
[Out] -atanh((a + c*x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

sympy [A] time = 1.25, size = 22, normalized size = 0.81

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}\,x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(c)*x**2))/(2*sqrt(a))
```

3.780 $\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4, 264}

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + c*x^4]/(2*a*x^2)
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{1}{x^3\sqrt{a+cx^4}}\,dx\\ &= -\frac{\sqrt{a+cx^4}}{2ax^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + c*x^4]/(a*x^2)
```

IntegrateAlgebraic [A] time = 0.06, size = 21, normalized size = 1.00

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + c*x^4]/(a*x^2)
```

fricas [A] time = 2.55, size = 17, normalized size = 0.81

$$-\frac{\sqrt{cx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

giac [A] time = 0.17, size = 31, normalized size = 1.48

$$\frac{\sqrt{c}}{\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c)/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\sqrt{c\,x^4+a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/2*(c*x^4+a)^(1/2)/a/x^2
```

maxima [A] time = 1.08, size = 17, normalized size = 0.81

$$-\frac{\sqrt{cx^4 + a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

mupad [B] time = 4.51, size = 17, normalized size = 0.81

$$-\frac{\sqrt{c\,x^4 + a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)^(1/2)),x)
[Out] -(a + c*x^4)^(1/2)/(2*a*x^2)
```

sympy [A] time = 0.84, size = 20, normalized size = 0.95

$$-\frac{\sqrt{c}\,\sqrt{\frac{a}{cx^4} + 1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+a)**(1/2),x)
[Out] -sqrt(c)*sqrt(a/(c*x**4) + 1)/(2*a)
```

3.781 $\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 321, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^4}{\sqrt{a+bx^2}}\,dx\\
&= \frac{x^3\sqrt{a+bx^2}}{4b}-\frac{(3a)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b}\\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}+\frac{\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2}\\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}+\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b^2}\\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.85

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\sqrt{b}\,x\sqrt{a+bx^2}\left(2bx^2-3a\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a + 2*b*x^2) + 3*a^2*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 63, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\left(2bx^3-3ax\right)}{8b^2}-\frac{3a^2\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a*x + 2*b*x^3))/(8*b^2) - (3*a^2*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/(8*b^(5/2))
```

fricas [A] time = 0.62, size = 124, normalized size = 1.70

$$\left[\frac{3\,a^{2}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,\left(2\,b^{2}x^{3}-3\,abx\right)\sqrt{bx^{2}+a}}{16\,b^{3}},-\frac{3\,a^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\left(2\,b^{2}x^{3}-3\,abx\right)\sqrt{bx^{2}+a}}{8\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.29, size = 54, normalized size = 0.74

$$\frac{1}{8}\,\sqrt{bx^{2}+a}\,x\left(\frac{2\,x^{2}}{b}-\frac{3\,a}{b^{2}}\right)-\frac{3\,a^{2}\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 59, normalized size = 0.81

$$\frac{\sqrt{b\,x^{2}+a}\;x^{3}}{4b}+\frac{3a^{2}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}-\frac{3\sqrt{b\,x^{2}+a}\;ax}{8b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^3*(b*x^2+a)^(1/2)/b-3/8*a*x*(b*x^2+a)^(1/2)/b^2+3/8*a^2/b^(5/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 0.97, size = 51, normalized size = 0.70

$$\frac{\sqrt{bx^{2}+a}\,x^{3}}{4\,b}-\frac{3\,\sqrt{bx^{2}+a}\,ax}{8\,b^{2}}+\frac{3\,a^{2}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*x^3/b - 3/8*sqrt(b*x^2 + a)*a*x/b^2 + 3/8*a^2*arcsinh(b
*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 4.61, size = 95, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x*
*2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*sqr
t(1 + b*x**2/a))
```

3.782 $\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}} dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5, 266, 43}

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right)dx, x, x^2\right)$$
$$= -\frac{a\sqrt{a+bx^2}}{b^2} + \frac{\left(a+bx^2\right)^{3/2}}{3b^2}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{\left(bx^2-2a\right)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 27, normalized size = 0.75

$$\frac{\left(bx^2-2a\right)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

fricas [A] time = 1.10, size = 23, normalized size = 0.64

$$\frac{\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

giac [A] time = 0.15, size = 30, normalized size = 0.83

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,b^2}-\frac{\sqrt{bx^2+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/3*(b*x^2 + a)^(3/2)/b^2 - sqrt(b*x^2 + a)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$-\frac{\sqrt{b\,x^2+a}\,\left(-b\,x^2+2a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2),x)
[Out] -1/3*(b*x^2+a)^(1/2)*(-b*x^2+2*a)/b^2
```

maxima [A] time = 1.02, size = 33, normalized size = 0.92

$$\frac{\sqrt{bx^2+a}\,x^2}{3\,b}-\frac{2\,\sqrt{bx^2+a}\,a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(b*x^2 + a)*x^2/b - 2/3*sqrt(b*x^2 + a)*a/b^2
```

mupad [B] time = 4.60, size = 24, normalized size = 0.67

$$-\frac{\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(1/2),x)
[Out] -((a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

sympy [A] time = 0.55, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2} + \frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne
(b, 0)), (x**4/(4*sqrt(a)), True))
```

3.783 $\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 321, 217, 206}

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$
$$= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$
$$= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{x\sqrt{a+bx^2}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 51, normalized size = 1.04

$$\frac{a\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{3/2}} + \frac{x\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) + (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3
/2))
```

fricas [A] time = 0.78, size = 93, normalized size = 1.90

$$\left[\frac{2\sqrt{bx^2+a}\,bx+a\sqrt{b}\,\log\left(-2\,bx^2+2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b^2},\frac{\sqrt{bx^2+a}\,bx+a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b^2]
```

giac [A] time = 0.18, size = 40, normalized size = 0.82

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}+\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.80

$$-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b\,x^2+a}\,x}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.02, size = 31, normalized size = 0.63

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}-\frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b - 1/2*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [B] time = 4.64, size = 56, normalized size = 1.14

$$\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{if } b = 0 \\ \frac{x\sqrt{b\,x^2+a}}{2\,b} - \frac{a\ln\left(2\sqrt{b}\,x+2\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, x^3/(3*a^(1/2)), b ~= 0, (x*(a + b*x^2)^(1/2))/(2*b) - (a
*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)))
```

sympy [A] time = 2.91, size = 42, normalized size = 0.86

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b} - \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)
)
```

3.784 $\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {5, 261}

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x}{\sqrt{a+bx^2}}\,dx\\ &= \frac{\sqrt{a+bx^2}}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] Sqrt[a + b*x^2]/b
```

IntegrateAlgebraic [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] Sqrt[a + b*x^2]/b
```

fricas [A] time = 0.90, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2 + a)/b
```

giac [A] time = 0.15, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] sqrt(b*x^2 + a)/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\sqrt{b\,x^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)/b
```

maxima [A] time = 1.05, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] sqrt(b*x^2 + a)/b
```

mupad [B] time = 4.33, size = 13, normalized size = 0.87

$$\frac{\sqrt{b\,x^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(1/2),x)
[Out] (a + b*x^2)^(1/2)/b
```

sympy [A] time = 0.40, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2),x)
[Out] Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

3.785 $\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.03, size = 28, normalized size = 1.12

$$-\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/Sqrt[b])
```

fricas [A] time = 1.58, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.18, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
[Out] ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.02, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 0.12, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

sympy [A] time = 1.20, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

3.786 $\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

IntegrateAlgebraic [A] time = 0.03, size = 25, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

fricas [A] time = 0.72, size = 60, normalized size = 2.40

$$\left[\frac{\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a))/a]
```

giac [A] time = 0.16, size = 22, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

maxima [A] time = 1.08, size = 17, normalized size = 0.68

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

mupad [B] time = 4.57, size = 19, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/2)),x)
[Out] -atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(1/2)
```

sympy [A] time = 1.20, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2),x)
[Out] -asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

3.787 $\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {5, 264}

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x^2\sqrt{a+bx^2}}\,dx\\ &= -\frac{\sqrt{a+bx^2}}{ax}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

IntegrateAlgebraic [A] time = 0.05, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

fricas [A] time = 0.97, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)/(a*x)
```

giac [A] time = 0.18, size = 30, normalized size = 1.58

$$\frac{2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$-\frac{\sqrt{b\,x^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] -(b*x^2+a)^(1/2)/a/x
```

maxima [A] time = 1.01, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -sqrt(b*x^2 + a)/(a*x)
```

mupad [B] time = 0.04, size = 17, normalized size = 0.89

$$-\frac{\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/2)),x)
[Out] -(a + b*x^2)^(1/2)/(a*x)
```

sympy [A] time = 0.75, size = 19, normalized size = 1.00

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2),x)
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

3.788 $\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5, 266, 51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2
))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x^3\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{2ax^2}-\frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a}\\
&= -\frac{\sqrt{a+bx^2}}{2ax^2}-\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a}\\
&= -\frac{\sqrt{a+bx^2}}{2ax^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.22

$$\frac{b\sqrt{a+bx^2}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\sqrt{\frac{bx^2}{a}+1}}-\frac{a}{2bx^2}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*(-1/2*a/(b*x^2) + ArcTanh[Sqrt[1 + (b*x^2)/a]]/(2*Sqrt[1
 + (b*x^2)/a])))/a^2
```

IntegrateAlgebraic [A] time = 0.07, size = 50, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -1/2*Sqrt[a + b*x^2]/(a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3
/2))
```

fricas [A] time = 2.01, size = 105, normalized size = 2.10

$$\left[\frac{\sqrt{a}bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}a}{4a^2x^2},-\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}a}{2a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

giac [A] time = 0.17, size = 51, normalized size = 1.02

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx^2+a}b}{ax^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] -1/2*(b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)*b
/(a*x^2))/b
```

maple [A] time = 0.01, size = 48, normalized size = 0.96

$$\frac{b\ln\left(\frac{2a+2\sqrt{bx^2+a}\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))
/x)
```

maxima [A] time = 1.03, size = 36, normalized size = 0.72

$$\frac{b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)/(a*x^2)
```

mupad [B] time = 4.54, size = 38, normalized size = 0.76

$$\frac{b\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^2)^(1/2)/(2*a*x
^2)
```

sympy [A] time = 3.47, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**
(3/2))
```

3.789 $\int \frac{1}{x^4 \sqrt{a+bx^2+(2+2c-2(1+c))x^4}} \, dx$

Optimal. Leaf size=44

$$\frac{2b\sqrt{a+bx^2}}{3a^2x} - \frac{\sqrt{a+bx^2}}{3ax^3}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5, 271, 264}

$$\frac{2b\sqrt{a+bx^2}}{3a^2x} - \frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2}}\,dx$$
$$= -\frac{\sqrt{a+bx^2}}{3ax^3} - \frac{(2b)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}$$
$$= -\frac{\sqrt{a+bx^2}}{3ax^3} + \frac{2b\sqrt{a+bx^2}}{3a^2x}$$

Mathematica [**A**] time = 0.01, size = 29, normalized size = 0.66

$$-\frac{(a-2bx^2)\sqrt{a+bx^2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -1/3*((a - 2*b*x^2)*Sqrt[a + b*x^2])/(a^2*x^3)
```

IntegrateAlgebraic [**A**] time = 0.07, size = 31, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\,(2bx^2-a)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] (Sqrt[a + b*x^2]*(-a + 2*b*x^2))/(3*a^2*x^3)
```

fricas [**A**] time = 0.90, size = 27, normalized size = 0.61

$$\frac{(2\,bx^2-a)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

giac [A] time = 0.28, size = 55, normalized size = 1.25

$$\frac{4\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3
```

maple [A] time = 0.00, size = 26, normalized size = 0.59

$$-\frac{\sqrt{b\,x^2+a}\,\left(-2b\,x^2+a\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3
```

maxima [A] time = 1.00, size = 36, normalized size = 0.82

$$\frac{2\sqrt{bx^2+a}\,b}{3\,a^2x}-\frac{\sqrt{bx^2+a}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^2 + a)*b/(a^2*x) - 1/3*sqrt(b*x^2 + a)/(a*x^3)
```

mupad [B] time = 4.55, size = 25, normalized size = 0.57

$$-\frac{\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

sympy [A] time = 1.05, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a**2)
```

3.790 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$\frac{x^5}{3\sqrt{cx^4}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^5/(3*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^4}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int x^2\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^5}{3\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] x^5/(3*Sqrt[c*x^4])
```

IntegrateAlgebraic [A] time = 0.02, size = 17, normalized size = 1.06

$$\frac{x\sqrt{cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] (x*Sqrt[c*x^4])/(3*c)
```

fricas [A] time = 1.79, size = 13, normalized size = 0.81

$$\frac{\sqrt{cx^4}\,x}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4)*x/c
```

giac [A] time = 0.15, size = 8, normalized size = 0.50

$$\frac{x^3}{3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="giac")
[Out] 1/3*x^3/sqrt(c)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^{5}}{3\sqrt{c\,x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4)^(1/2),x)
[Out] 1/3*x^5/(c*x^4)^(1/2)
```

maxima [A] time = 0.96, size = 12, normalized size = 0.75

$$\frac{x^{5}}{3\,\sqrt{cx^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="maxima")
[Out] 1/3*x^5/sqrt(c*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^4}{\sqrt{c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4)^(1/2),x)
[Out] int(x^4/(c*x^4)^(1/2), x)
```

sympy [A] time = 0.63, size = 15, normalized size = 0.94

$$\frac{x^{5}}{3\sqrt{c}\,\sqrt{x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4)**(1/2),x)
[Out] x**5/(3*sqrt(c)*sqrt(x**4))
```

3.791 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$\frac{x^4}{2\sqrt{cx^4}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^4/(2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^3}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2\int x\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^4}{2\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] x^4/(2*Sqrt[c*x^4])
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\sqrt{cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] Sqrt[c*x^4]/(2*c)
```

fricas [A] time = 0.83, size = 12, normalized size = 0.75

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(c*x^4)/c
```

giac [A] time = 0.15, size = 8, normalized size = 0.50

$$\frac{x^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/sqrt(c)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^4}{2\sqrt{c\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4)^(1/2),x)
```

```
[Out] 1/2*x^4/(c*x^4)^(1/2)
```

maxima [A] time = 1.04, size = 12, normalized size = 0.75

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^4)/c
```

mupad [B] time = 4.50, size = 10, normalized size = 0.62

$$\frac{\sqrt{x^4}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4)^(1/2),x)
```

```
[Out] (x^4)^(1/2)/(2*c^(1/2))
```

sympy [A] time = 0.53, size = 15, normalized size = 0.94

$$\frac{x^4}{2\sqrt{c}\,\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4)**(1/2),x)
```

```
[Out] x**4/(2*sqrt(c)*sqrt(x**4))
```

3.792 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$\frac{x^3}{\sqrt{cx^4}}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 8}

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^3/Sqrt[c*x^4]
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^2}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int 1\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^3}{\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] x^3/Sqrt[c*x^4]
```

IntegrateAlgebraic [A] time = 0.02, size = 16, normalized size = 1.23

$$\frac{\sqrt{cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] Sqrt[c*x^4]/(c*x)
```

fricas [A] time = 0.97, size = 14, normalized size = 1.08

$$\frac{\sqrt{cx^4}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^4)/(c*x)
```

giac [A] time = 0.17, size = 5, normalized size = 0.38

$$\frac{x}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] x/sqrt(c)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{x^{3}}{\sqrt{c\,x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4)^(1/2),x)
```

```
[Out] x^3/(c*x^4)^(1/2)
```

maxima [A] time = 0.98, size = 11, normalized size = 0.85

$$\frac{x^{3}}{\sqrt{cx^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] x^3/sqrt(c*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^{2}}{\sqrt{c\,x^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4)^(1/2),x)
```

```
[Out] int(x^2/(c*x^4)^(1/2), x)
```

sympy [A] time = 0.48, size = 14, normalized size = 1.08

$$\frac{x^{3}}{\sqrt{c}\,\sqrt{x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4)**(1/2),x)
```

```
[Out] x**3/(sqrt(c)*sqrt(x**4))
```

3.793 $\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=15

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1, 15, 29}

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx &= \int \frac{x}{\sqrt{cx^4}}\,dx\\ &= \frac{x^2\int \frac{1}{x}\,dx}{\sqrt{cx^4}}\\ &= \frac{x^2\log(x)}{\sqrt{cx^4}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^2 \log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 1.20

$$\frac{\sqrt{cx^4} \log(x)}{cx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] (Sqrt[c*x^4]*Log[x])/(c*x^2)
```

fricas [A] time = 1.06, size = 16, normalized size = 1.07

$$\frac{\sqrt{cx^4} \log(x)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^4)*log(x)/(c*x^2)
```

giac [A] time = 0.15, size = 7, normalized size = 0.47

$$\frac{\log(|x|)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="giac")
[Out] log(abs(x))/sqrt(c)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{x^2 \ln(x)}{\sqrt{c\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4)^(1/2),x)
```

```
[Out] x^2*ln(x)/(c*x^4)^(1/2)
```

maxima [A] time = 0.98, size = 13, normalized size = 0.87

$$\frac{x^2 \log(x)}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] x^2*log(x)/sqrt(c*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\sqrt{c\, x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4)^(1/2),x)
```

```
[Out] int(x/(c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(c*x**4), x)
```

3.794 $\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{cx^4}}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1, 15, 30}

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] -(x/Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x^2}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{x}{\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] -(x/Sqrt[c*x^4])
```

IntegrateAlgebraic [A] time = 0.02, size = 17, normalized size = 1.42

$$-\frac{\sqrt{cx^4}}{cx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] -(Sqrt[c*x^4]/(c*x^3))
```

fricas [A] time = 1.92, size = 15, normalized size = 1.25

$$-\frac{\sqrt{cx^4}}{cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4)/(c*x^3)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$-\frac{1}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="giac")
[Out] -1/(sqrt(c)*x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\frac{x}{\sqrt{c\,x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4)^(1/2),x)
[Out] -x/(c*x^4)^(1/2)
```

maxima [A] time = 0.98, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{cx^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="maxima")
[Out] -x/sqrt(c*x^4)
```

mupad [B] time = 4.30, size = 13, normalized size = 1.08

$$-\frac{\sqrt{x^{4}}}{\sqrt{c}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4)^(1/2),x)
[Out] -(x^4)^(1/2)/(c^(1/2)*x^3)
```

sympy [A] time = 0.47, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{c}\,\sqrt{x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4)**(1/2),x)
[Out] -x/(sqrt(c)*sqrt(x**4))
```

3.795 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2\sqrt{cx^4}}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x\sqrt{cx^4}}\,dx$$
$$= \frac{x^2\int \frac{1}{x^3}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{2\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/2*1/Sqrt[c*x^4]
```

IntegrateAlgebraic [A] time = 0.01, size = 19, normalized size = 1.46

$$-\frac{\sqrt{cx^4}}{2cx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/2*Sqrt[c*x^4]/(c*x^4)
```

fricas [A] time = 0.95, size = 15, normalized size = 1.15

$$-\frac{\sqrt{cx^4}}{2\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^4)/(c*x^4)
```

giac [A] time = 0.16, size = 8, normalized size = 0.62

$$-\frac{1}{2\,\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2/(sqrt(c)*x^2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{2\sqrt{c\,x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4)^(1/2),x)
```

```
[Out] -1/2/(c*x^4)^(1/2)
```

maxima [A] time = 1.03, size = 9, normalized size = 0.69

$$-\frac{1}{2\,\sqrt{cx^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2/sqrt(c*x^4)
```

mupad [B] time = 4.34, size = 10, normalized size = 0.77

$$-\frac{1}{2\,\sqrt{c}\,\sqrt{x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(2*c^(1/2)*(x^4)^(1/2))
```

sympy [A] time = 0.51, size = 15, normalized size = 1.15

$$-\frac{1}{2\sqrt{c}\,\sqrt{x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4)**(1/2),x)
```

```
[Out] -1/(2*sqrt(c)*sqrt(x**4))
```

3.796 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3x\sqrt{cx^4}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(3*x*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x^4}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{3x\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/3*1/(x*Sqrt[c*x^4])
```

IntegrateAlgebraic [A] time = 0.02, size = 19, normalized size = 1.19

$$-\frac{\sqrt{cx^4}}{3cx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/3*Sqrt[c*x^4]/(c*x^5)
```

fricas [A] time = 1.82, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{3\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(c*x^4)/(c*x^5)
```

giac [A] time = 0.16, size = 8, normalized size = 0.50

$$-\frac{1}{3\,\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="giac")
[Out] -1/3/(sqrt(c)*x^3)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{3\sqrt{c\,x^{4}}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4)^(1/2),x)
[Out] -1/3/x/(c*x^4)^(1/2)
```

maxima [A] time = 1.08, size = 12, normalized size = 0.75

$$-\frac{1}{3\,\sqrt{cx^{4}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="maxima")
[Out] -1/3/(sqrt(c*x^4)*x)
```

mupad [B] time = 4.31, size = 13, normalized size = 0.81

$$-\frac{1}{3\,\sqrt{c}\,x\,\sqrt{x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^4)^(1/2)),x)
[Out] -1/(3*c^(1/2)*x*(x^4)^(1/2))
```

sympy [A] time = 0.56, size = 17, normalized size = 1.06

$$-\frac{1}{3\sqrt{c}\,x\sqrt{x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4)**(1/2),x)
[Out] -1/(3*sqrt(c)*x*sqrt(x**4))
```

3.797 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(4*x^2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{cx^4}}\,dx$$
$$= \frac{x^2\int \frac{1}{x^5}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{4x^2\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.06

$$-\frac{cx^2}{4\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/4*(c*x^2)/(c*x^4)^(3/2)
```

IntegrateAlgebraic [A] time = 0.02, size = 19, normalized size = 1.19

$$-\frac{\sqrt{cx^4}}{4cx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/4*Sqrt[c*x^4]/(c*x^6)
```

fricas [A] time = 0.80, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{4\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(c*x^4)/(c*x^6)
```

giac [A] time = 0.16, size = 8, normalized size = 0.50

$$-\frac{1}{4\,\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4/(sqrt(c)*x^4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{4\sqrt{c\,x^4}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4)^(1/2),x)
```

```
[Out] -1/4/x^2/(c*x^4)^(1/2)
```

maxima [A] time = 1.05, size = 12, normalized size = 0.75

$$-\frac{1}{4\,\sqrt{cx^4}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4/(sqrt(c*x^4)*x^2)
```

mupad [B] time = 4.27, size = 13, normalized size = 0.81

$$-\frac{1}{4\,\sqrt{c}\,x^2\,\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(4*c^(1/2)*x^2*(x^4)^(1/2))
```

sympy [A] time = 0.66, size = 19, normalized size = 1.19

$$-\frac{1}{4\sqrt{c}\,x^2\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4)**(1/2),x)
```

```
[Out] -1/(4*sqrt(c)*x**2*sqrt(x**4))
```

3.798 $\int \frac{1}{x^4 \sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(5*x^3*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{cx^4}}\,dx$$
$$= \frac{x^2\int \frac{1}{x^6}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{5x^3\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{cx}{5\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/5*(c*x)/(c*x^4)^(3/2)
```

IntegrateAlgebraic [A] time = 0.02, size = 19, normalized size = 1.19

$$-\frac{\sqrt{cx^4}}{5cx^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/5*Sqrt[c*x^4]/(c*x^7)
```

fricas [A] time = 1.89, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{5\,cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -1/5*sqrt(c*x^4)/(c*x^7)
```

giac [A] time = 0.15, size = 8, normalized size = 0.50

$$-\frac{1}{5\sqrt{c}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="giac")
[Out] -1/5/(sqrt(c)*x^5)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{5\sqrt{c\,x^4}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4)^(1/2),x)
[Out] -1/5/x^3/(c*x^4)^(1/2)
```

maxima [A] time = 1.06, size = 12, normalized size = 0.75

$$-\frac{1}{5\,\sqrt{cx^4}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="maxima")
[Out] -1/5/(sqrt(c*x^4)*x^3)
```

mupad [B] time = 4.33, size = 13, normalized size = 0.81

$$-\frac{1}{5\,\sqrt{c}\;x^3\,\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c*x^4)^(1/2)),x)
[Out] -1/(5*c^(1/2)*x^3*(x^4)^(1/2))
```

sympy [A] time = 0.71, size = 19, normalized size = 1.19

$$-\frac{1}{5\sqrt{c}\,x^3\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4)**(1/2),x)
[Out] -1/(5*sqrt(c)*x**3*sqrt(x**4))
```

3.799 $\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^5}{5\sqrt{a}}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^5/(5*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^4}{\sqrt{a}}\,dx = \frac{\int x^4\,dx}{\sqrt{a}} = \frac{x^5}{5\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^5/(5*Sqrt[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] IntegrateAlgebraic[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4], x]
```

fricas [A] time = 1.84, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="fricas")
[Out] 1/5*x^5/sqrt(a)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*x^5/sqrt(a)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{x^{5}}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/a^(1/2),x)
```

```
[Out] 1/5*x^5/a^(1/2)
```

maxima [A] time = 1.01, size = 8, normalized size = 0.67

$$\frac{x^{5}}{5\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^5/sqrt(a)
```

mupad [B] time = 0.02, size = 8, normalized size = 0.67

$$\frac{x^{5}}{5\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/a^(1/2),x)
```

```
[Out] x^5/(5*a^(1/2))
```

sympy [A] time = 0.07, size = 8, normalized size = 0.67

$$\frac{x^{5}}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/a**(1/2),x)
```

```
[Out] x**5/(5*sqrt(a))
```

3.800 $\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^4}{4\sqrt{a}}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.125, Rules used = {2, 12, 30}

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^4/(4*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^3}{\sqrt{a}}\,dx$$
$$= \frac{\int x^3\,dx}{\sqrt{a}}$$
$$= \frac{x^4}{4\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^4/(4*Sqrt[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] IntegrateAlgebraic[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4], x]
```

fricas [A] time = 0.88, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="fricas")
[Out] 1/4*x^4/sqrt(a)
```

giac [A] time = 0.18, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*x^4/sqrt(a)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/a^(1/2),x)
```

```
[Out] 1/4*x^4/a^(1/2)
```

maxima [A] time = 1.04, size = 8, normalized size = 0.67

$$\frac{x^4}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/sqrt(a)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.67

$$\frac{x^4}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/a^(1/2),x)
```

```
[Out] x^4/(4*a^(1/2))
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/a**(1/2),x)
```

```
[Out] x**4/(4*sqrt(a))
```

3.801 $\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^3}{3\sqrt{a}}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^3/(3*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^2}{\sqrt{a}}\,dx$$
$$= \frac{\int x^2\,dx}{\sqrt{a}}$$
$$= \frac{x^3}{3\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^3/(3*Sqrt[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] IntegrateAlgebraic[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4], x]
```

fricas [A] time = 1.06, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="fricas")
[Out] 1/3*x^3/sqrt(a)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3/sqrt(a)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/a^(1/2),x)
```

```
[Out] 1/3*x^3/a^(1/2)
```

maxima [A] time = 0.95, size = 8, normalized size = 0.67

$$\frac{x^3}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/sqrt(a)
```

mupad [B] time = 0.01, size = 8, normalized size = 0.67

$$\frac{x^3}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/a^(1/2),x)
```

```
[Out] x^3/(3*a^(1/2))
```

sympy [A] time = 0.07, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/a**(1/2),x)
```

```
[Out] x**3/(3*sqrt(a))
```

3.802 $\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}} dx$

Optimal. Leaf size=12

$$\frac{x^2}{2\sqrt{a}}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2, 12, 30}

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^2/(2*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}} dx &= \int \frac{x}{\sqrt{a}} dx \\ &= \frac{\int x\,dx}{\sqrt{a}} \\ &= \frac{x^2}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^2/(2*Sqrt[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] IntegrateAlgebraic[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4], x]
```

fricas [A] time = 1.87, size = 8, normalized size = 0.67

$$\frac{x^2}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="fricas")
[Out] 1/2*x^2/sqrt(a)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$\frac{x^2}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="giac")
[Out] 1/2*x^2/sqrt(a)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/a^(1/2),x)
```

```
[Out] 1/2*x^2/a^(1/2)
```

maxima [A] time = 1.07, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/sqrt(a)
```

mupad [B] time = 0.02, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/a^(1/2),x)
```

```
[Out] x^2/(2*a^(1/2))
```

sympy [A] time = 0.08, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a**(1/2),x)
```

```
[Out] x**2/(2*sqrt(a))
```

3.803 $\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=7

$$\frac{x}{\sqrt{a}}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2, 8}

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x/Sqrt[a]

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}}\,dx\\ &= \frac{x}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x/Sqrt[a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] IntegrateAlgebraic[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4], x]
```

fricas [A] time = 2.58, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="fricas")
[Out] x/sqrt(a)
```

giac [A] time = 0.15, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="giac")
[Out] x/sqrt(a)
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a^(1/2),x)
[Out] x/a^(1/2)
```

maxima [A] time = 0.98, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(a)
```

mupad [B] time = 0.00, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a^(1/2),x)
```

```
[Out] x/a^(1/2)
```

sympy [A] time = 0.13, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a**(1/2),x)
```

```
[Out] x/sqrt(a)
```

3.804 $\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=8

$$\frac{\log(x)}{\sqrt{a}}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 29}

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] Log[x]/Sqrt[a]
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}\,x}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{\sqrt{a}}\\
&= \frac{\log(x)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] Log[x]/Sqrt[a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + (2 + 2c - 2(1 + c))x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] IntegrateAlgebraic[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]), x]
```

fricas [A] time = 1.14, size = 6, normalized size = 0.75

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="fricas")
[Out] log(x)/sqrt(a)
```

giac [A] time = 0.15, size = 7, normalized size = 0.88

$$\frac{\log(|x|)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="giac")
[Out] log(abs(x))/sqrt(a)
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\ln(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/a^(1/2),x)
```

```
[Out] ln(x)/a^(1/2)
```

maxima [A] time = 1.03, size = 6, normalized size = 0.75

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="maxima")
```

```
[Out] log(x)/sqrt(a)
```

mupad [B] time = 4.24, size = 6, normalized size = 0.75

$$\frac{\ln(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x),x)
```

```
[Out] log(x)/a^(1/2)
```

sympy [A] time = 0.08, size = 7, normalized size = 0.88

$$\frac{\log{(x)}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a**(1/2),x)
```

```
[Out] log(x)/sqrt(a)
```

3.805 $\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{\sqrt{a}\,x}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$-\frac{1}{\sqrt{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(1/(Sqrt[a]*x))
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}\,x^2}\,dx\\ &= \frac{\int \frac{1}{x^2}\,dx}{\sqrt{a}}\\ &= -\frac{1}{\sqrt{a}\,x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{\sqrt{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -(1/(Sqrt[a]*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] IntegrateAlgebraic[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]), x]
```

fricas [A] time = 2.66, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="fricas")
[Out] -1/(sqrt(a)*x)
```

giac [A] time = 0.18, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="giac")
[Out] -1/(sqrt(a)*x)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{1}{\sqrt{a}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/a^(1/2),x)
```

```
[Out] -1/x/a^(1/2)
```

maxima [A] time = 0.88, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(a)*x)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x^2),x)
```

```
[Out] -1/(a^(1/2)*x)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/a**(1/2),x)
```

```
[Out] -1/(sqrt(a)*x)
```

3.806 $\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/(2*Sqrt[a]*x^2)
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}\,x^3}\,dx\\ &= \frac{\int \frac{1}{x^3}\,dx}{\sqrt{a}}\\ &= -\frac{1}{2\sqrt{a}\,x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -1/2*1/(Sqrt[a]*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] IntegrateAlgebraic[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]), x]
```

fricas [A] time = 1.85, size = 8, normalized size = 0.67

$$-\frac{1}{2\,\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="fricas")
[Out] -1/2/(sqrt(a)*x^2)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$-\frac{1}{2\,\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="giac")
[Out] -1/2/(sqrt(a)*x^2)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$-\frac{1}{2\sqrt{a}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/a^(1/2),x)
[Out] -1/2/x^2/a^(1/2)
```

maxima [A] time = 1.04, size = 8, normalized size = 0.67

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="maxima")
[Out] -1/2/(sqrt(a)*x^2)
```

mupad [B] time = 4.40, size = 8, normalized size = 0.67

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x^3),x)
[Out] -1/(2*a^(1/2)*x^2)
```

sympy [A] time = 0.08, size = 12, normalized size = 1.00

$$-\frac{1}{2\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/a**(1/2),x)
[Out] -1/(2*sqrt(a)*x**2)
```

3.807 $\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/(3*Sqrt[a]*x^3)
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}\,x^4}\,dx\\ &= \frac{\int \frac{1}{x^4}\,dx}{\sqrt{a}}\\ &= -\frac{1}{3\sqrt{a}\,x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -1/3*1/(Sqrt[a]*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] IntegrateAlgebraic[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]), x]
```

fricas [A] time = 1.71, size = 8, normalized size = 0.67

$$-\frac{1}{3\,\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="fricas")
[Out] -1/3/(sqrt(a)*x^3)
```

giac [A] time = 0.18, size = 8, normalized size = 0.67

$$-\frac{1}{3\,\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="giac")
[Out] -1/3/(sqrt(a)*x^3)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$-\frac{1}{3\sqrt{a}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/a^(1/2),x)
[Out] -1/3/x^3/a^(1/2)
```

maxima [A] time = 1.00, size = 8, normalized size = 0.67

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="maxima")
[Out] -1/3/(sqrt(a)*x^3)
```

mupad [B] time = 4.33, size = 8, normalized size = 0.67

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x^4),x)
[Out] -1/(3*a^(1/2)*x^3)
```

sympy [A] time = 0.07, size = 12, normalized size = 1.00

$$-\frac{1}{3\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/a**(1/2),x)
[Out] -1/(3*sqrt(a)*x**3)
```

3.808 $\int x^{5/2} \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=31

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx &= \int \left(ax^{5/2}+bx^{9/2}+cx^{13/2}\right)\,dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.81

$$\frac{2x^{7/2}\left(165a+105bx^2+77cx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

IntegrateAlgebraic [A] time = 0.02, size = 29, normalized size = 0.94

$$\frac{2\left(165ax^{7/2}+105bx^{11/2}+77cx^{15/2}\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2 + c*x^4),x]
[Out] (2*(165*a*x^(7/2) + 105*b*x^(11/2) + 77*c*x^(15/2)))/1155
```

fricas [A] time = 1.38, size = 24, normalized size = 0.77

$$\frac{2}{1155}\left(77\,cx^{7}+105\,bx^{5}+165\,ax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 2/1155*(77*c*x^7 + 105*b*x^5 + 165*a*x^3)*sqrt(x)
```

giac [A] time = 0.16, size = 19, normalized size = 0.61

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.71

$$\frac{2\left(77c\,x^{4}+105b\,x^{2}+165a\right)x^{\frac{7}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a),x)
[Out] 2/1155*x^(7/2)*(77*c*x^4+105*b*x^2+165*a)
```

maxima [A] time = 0.98, size = 19, normalized size = 0.61

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

mupad [B] time = 4.29, size = 21, normalized size = 0.68

$$\frac{2\,x^{7/2}\,\left(77\,c\,x^4+105\,b\,x^2+165\,a\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2 + c*x^4),x)
[Out] (2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

sympy [A] time = 6.72, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{11}{2}}}{11}+\frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a),x)
[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

3.809 $\int x^{3/2} \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=31

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^{3/2} \left(a+bx^2+cx^4\right) \, dx &= \int \left(ax^{3/2}+bx^{7/2}+cx^{11/2}\right) \, dx \\ &= \frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.81

$$\frac{2}{585}x^{5/2}\left(117a+65bx^2+45cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

IntegrateAlgebraic [A] time = 0.02, size = 29, normalized size = 0.94

$$\frac{2}{585}\left(117ax^{5/2}+65bx^{9/2}+45cx^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2 + c*x^4),x]
[Out] (2*(117*a*x^(5/2) + 65*b*x^(9/2) + 45*c*x^(13/2)))/585
```

fricas [A] time = 1.64, size = 24, normalized size = 0.77

$$\frac{2}{585}\left(45\,cx^{6}+65\,bx^{4}+117\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 2/585*(45*c*x^6 + 65*b*x^4 + 117*a*x^2)*sqrt(x)
```

giac [A] time = 0.21, size = 19, normalized size = 0.61

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.71

$$\frac{2\left(45c\,x^{4}+65b\,x^{2}+117a\right)x^{\frac{5}{2}}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a),x)
[Out] 2/585*x^(5/2)*(45*c*x^4+65*b*x^2+117*a)
```

maxima [A] time = 1.04, size = 19, normalized size = 0.61

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.68

$$\frac{2\,x^{5/2}\,\left(45\,c\,x^4+65\,b\,x^2+117\,a\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

sympy [A] time = 2.67, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{9}{2}}}{9}+\frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

3.810 $\int \sqrt{x}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)dx &= \int \left(a\sqrt{x}+bx^{5/2}+cx^{9/2}\right)dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.81

$$\frac{2}{231}x^{3/2}\left(77a+33bx^2+21cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

IntegrateAlgebraic [A] time = 0.02, size = 29, normalized size = 0.94

$$\frac{2}{231}\left(77ax^{3/2}+33bx^{7/2}+21cx^{11/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2 + c*x^4),x]
[Out] (2*(77*a*x^(3/2) + 33*b*x^(7/2) + 21*c*x^(11/2)))/231
```

fricas [A] time = 2.21, size = 22, normalized size = 0.71

$$\frac{2}{231}\left(21\,cx^5+33\,bx^3+77\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 2/231*(21*c*x^5 + 33*b*x^3 + 77*a*x)*sqrt(x)
```

giac [A] time = 0.17, size = 19, normalized size = 0.61

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.71

$$\frac{2\left(21c\,x^4+33b\,x^2+77a\right)x^{\frac{3}{2}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a),x)
[Out] 2/231*x^(3/2)*(21*c*x^4+33*b*x^2+77*a)
```

maxima [A] time = 1.01, size = 19, normalized size = 0.61

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.68

$$\frac{2\,x^{3/2}\,\left(21\,c\,x^4+33\,b\,x^2+77\,a\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2 + c*x^4),x)
[Out] (2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

sympy [A] time = 2.10, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a),x)
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

3.811 $\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx&=\int \left(\frac{a}{\sqrt{x}}+bx^{3/2}+cx^{7/2}\right)dx\\&=2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{2}{45}\sqrt{x}\left(45a+9bx^2+5cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

IntegrateAlgebraic [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{2}{45}\left(45a\sqrt{x}+9bx^{5/2}+5cx^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/Sqrt[x],x]
[Out] (2*(45*a*Sqrt[x] + 9*b*x^(5/2) + 5*c*x^(9/2)))/45
```

fricas [A] time = 0.87, size = 21, normalized size = 0.72

$$\frac{2}{45}\left(5\,cx^4+9\,bx^2+45\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*c*x^4 + 9*b*x^2 + 45*a)*sqrt(x)
```

giac [A] time = 0.15, size = 19, normalized size = 0.66

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="giac")
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{2\left(5c\,x^4+9b\,x^2+45a\right)\sqrt{x}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(1/2),x)
[Out] 2/45*x^(1/2)*(5*c*x^4+9*b*x^2+45*a)
```

maxima [A] time = 1.03, size = 19, normalized size = 0.66

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.72

$$\frac{2\sqrt{x}\,\left(5\,c\,x^4+9\,b\,x^2+45\,a\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

sympy [A] time = 0.82, size = 27, normalized size = 0.93

$$2a\sqrt{x} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

3.812 $\int \frac{a+bx^2+cx^4}{x^{3/2}} dx$

Optimal. Leaf size=29

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+b\sqrt{x}+cx^{5/2}\right)dx\\ &= -\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{2\left(-21a+7bx^2+3cx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*(-21*a + 7*b*x^2 + 3*c*x^4))/(21*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 0.86

$$\frac{2\left(-21a+7bx^2+3cx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^(3/2),x]
[Out] (2*(-21*a + 7*b*x^2 + 3*c*x^4))/(21*Sqrt[x])
```

fricas [A] time = 1.00, size = 21, normalized size = 0.72

$$\frac{2\left(3\,cx^4+7\,bx^2-21\,a\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="fricas")
[Out] 2/21*(3*c*x^4 + 7*b*x^2 - 21*a)/sqrt(x)
```

giac [A] time = 0.15, size = 19, normalized size = 0.66

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="giac")
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-3c\,x^4-7b\,x^2+21a\right)}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(3/2),x)
[Out] -2/21*(-3*c*x^4-7*b*x^2+21*a)/x^(1/2)
```

maxima [A] time = 1.09, size = 19, normalized size = 0.66

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="maxima")
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.72

$$\frac{6\,c\,x^4+14\,b\,x^2-42\,a}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(3/2),x)
[Out] (14*b*x^2 - 42*a + 6*c*x^4)/(21*x^(1/2))
```

sympy [A] time = 1.04, size = 27, normalized size = 0.93

$$-\frac{2a}{\sqrt{x}}+\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(3/2),x)
[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

3.813 $\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}+cx^{3/2}\right)dx\\ &= -\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{2\left(-5a+15bx^2+3cx^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (2*(-5*a + 15*b*x^2 + 3*c*x^4))/(15*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 0.86

$$\frac{2\left(-5a+15bx^2+3cx^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^(5/2),x]
[Out] (2*(-5*a + 15*b*x^2 + 3*c*x^4))/(15*x^(3/2))
```

fricas [A] time = 1.06, size = 21, normalized size = 0.72

$$\frac{2\left(3\,cx^4+15\,bx^2-5\,a\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*c*x^4 + 15*b*x^2 - 5*a)/x^(3/2)
```

giac [A] time = 0.33, size = 19, normalized size = 0.66

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="giac")
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-3c\,x^4-15b\,x^2+5a\right)}{15x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(5/2),x)
[Out] -2/15*(-3*c*x^4-15*b*x^2+5*a)/x^(3/2)
```

maxima [A] time = 1.02, size = 19, normalized size = 0.66

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="maxima")
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.72

$$\frac{6\,c\,x^4+30\,b\,x^2-10\,a}{15\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(5/2),x)
[Out] (30*b*x^2 - 10*a + 6*c*x^4)/(15*x^(3/2))
```

sympy [A] time = 1.28, size = 27, normalized size = 0.93

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(5/2),x)
[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

3.814 $\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx &= \int \left(\frac{a}{x^{7/2}}+\frac{b}{x^{3/2}}+c\sqrt{x}\right)dx\\ &= -\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{2\left(-3a-15bx^2+5cx^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (2*(-3*a - 15*b*x^2 + 5*c*x^4))/(15*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 0.86

$$\frac{2(-3a-15bx^2+5cx^4)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/x^(7/2),x]
[Out] (2*(-3*a - 15*b*x^2 + 5*c*x^4))/(15*x^(5/2))
```

fricas [A] time = 1.75, size = 21, normalized size = 0.72

$$\frac{2\left(5\,cx^4-15\,bx^2-3\,a\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="fricas")
[Out] 2/15*(5*c*x^4 - 15*b*x^2 - 3*a)/x^(5/2)
```

giac [A] time = 0.21, size = 20, normalized size = 0.69

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="giac")
[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-5c\,x^4+15b\,x^2+3a\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(7/2),x)
[Out] -2/15*(-5*c*x^4+15*b*x^2+3*a)/x^(5/2)
```

maxima [A] time = 1.08, size = 20, normalized size = 0.69

$$\frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\left(5\,bx^2 + a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="maxima")
[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

mupad [B] time = 4.33, size = 21, normalized size = 0.72

$$-\frac{-10\,c\,x^4 + 30\,b\,x^2 + 6\,a}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(7/2),x)
[Out] -(6*a + 30*b*x^2 - 10*c*x^4)/(15*x^(5/2))
```

sympy [A] time = 1.87, size = 27, normalized size = 0.93

$$-\frac{2a}{5x^{\frac{5}{2}}} - \frac{2b}{\sqrt{x}} + \frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(7/2),x)
[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x) + 2*c*x**(3/2)/3
```

3.815 $\int x^{5/2} \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=64

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/2))/15 + (
4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2x^{5/2}+2abx^{9/2}+\left(b^2+2ac\right)x^{13/2}+2bcx^{17/2}+c^2x^{21/2}\right)dx\\ &= \frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}\left(b^2+2ac\right)x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}\end{aligned}$$

Mathematica [A] time = 3.70, size = 64, normalized size = 1.00

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/2))/15 + (
4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

IntegrateAlgebraic [A] time = 0.03, size = 62, normalized size = 0.97

$$\frac{2\left(72105a^2x^{7/2}+91770abx^{11/2}+67298acx^{15/2}+33649b^2x^{15/2}+53130bcx^{19/2}+21945c^2x^{23/2}\right)}{504735}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(72105*a^2*x^(7/2) + 91770*a*b*x^(11/2) + 33649*b^2*x^(15/2) + 67298*a*c
*x^(15/2) + 53130*b*c*x^(19/2) + 21945*c^2*x^(23/2)))/504735
```

fricas [A] time = 0.93, size = 49, normalized size = 0.77

$$\frac{2}{504735}\left(21945\,c^2x^{11}+53130\,bcx^9+33649\left(b^2+2\,ac\right)x^7+91770\,abx^5+72105\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*c^2*x^11 + 53130*b*c*x^9 + 33649*(b^2 + 2*a*c)*x^7 + 91770*
a*b*x^5 + 72105*a^2*x^3)*sqrt(x)
```

giac [A] time = 0.21, size = 46, normalized size = 0.72

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{15}\,acx^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2) + 4/15*a*c*x^(15/
2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.77

$$\frac{2\left(21945c^2x^8+53130bc\,x^6+67298ac\,x^4+33649b^2x^4+91770ab\,x^2+72105a^2\right)x^{\frac{7}{2}}}{504735}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/504735*x^(7/2)*(21945*c^2*x^8+53130*b*c*x^6+67298*a*c*x^4+33649*b^2*x^4+9
1770*a*b*x^2+72105*a^2)
```

maxima [A] time = 1.03, size = 44, normalized size = 0.69

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\left(b^2+2\,ac\right)x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*(b^2 + 2*a*c)*x^(15/2) + 4/11*
a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

mupad [B] time = 4.41, size = 45, normalized size = 0.70

$$x^{15/2}\left(\frac{2\,b^2}{15}+\frac{4\,a\,c}{15}\right)+\frac{2\,a^2\,x^{7/2}}{7}+\frac{2\,c^2\,x^{23/2}}{23}+\frac{4\,a\,b\,x^{11/2}}{11}+\frac{4\,b\,c\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(15/2)*((4*a*c)/15 + (2*b^2)/15) + (2*a^2*x^(7/2))/7 + (2*c^2*x^(23/2))/2
3 + (4*a*b*x^(11/2))/11 + (4*b*c*x^(19/2))/19
```

sympy [A] time = 22.35, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{4acx^{\frac{15}{2}}}{15}+\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 4*a*c*x**(15/2)/15 + 2*b**2*x**(15
/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

3.816 $\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=64

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*(b^2 + 2*a*c)*x^(13/2))/13 + (4*
b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx&=\int\left(a^2x^{3/2}+2abx^{7/2}+\left(b^2+2ac\right)x^{11/2}+2bcx^{15/2}+c^2x^{19/2}\right)dx\\&=\frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}\left(b^2+2ac\right)x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 1.03

$$2\left(\frac{1}{5}a^2x^{5/2}+\frac{1}{13}x^{13/2}\left(2ac+b^2\right)+\frac{2}{9}abx^{9/2}+\frac{2}{17}bcx^{17/2}+\frac{1}{21}c^2x^{21/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] 2*((a^2*x^(5/2))/5 + (2*a*b*x^(9/2))/9 + ((b^2 + 2*a*c)*x^(13/2))/13 + (2*b
*c*x^(17/2))/17 + (c^2*x^(21/2))/21)
```

IntegrateAlgebraic [A] time = 0.03, size = 62, normalized size = 0.97

$$\frac{2\left(13923a^2x^{5/2}+15470abx^{9/2}+10710acx^{13/2}+5355b^2x^{13/2}+8190bcx^{17/2}+3315c^2x^{21/2}\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(13923*a^2*x^(5/2) + 15470*a*b*x^(9/2) + 5355*b^2*x^(13/2) + 10710*a*c*x
^(13/2) + 8190*b*c*x^(17/2) + 3315*c^2*x^(21/2)))/69615
```

fricas [A] time = 0.81, size = 49, normalized size = 0.77

$$\frac{2}{69615}\left(3315\,c^2x^{10}+8190\,bcx^8+5355\left(b^2+2\,ac\right)x^6+15470\,abx^4+13923\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^2*x^10 + 8190*b*c*x^8 + 5355*(b^2 + 2*a*c)*x^6 + 15470*a*b*
x^4 + 13923*a^2*x^2)*sqrt(x)
```

giac [A] time = 0.17, size = 46, normalized size = 0.72

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{13}\,acx^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2) + 4/13*a*c*x^(13/
2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.77

$$\frac{2\left(3315c^2x^8+8190bc\,x^6+10710ac\,x^4+5355b^2x^4+15470ab\,x^2+13923a^2\right)x^{\frac{5}{2}}}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/69615*x^(5/2)*(3315*c^2*x^8+8190*b*c*x^6+10710*a*c*x^4+5355*b^2*x^4+15470
*a*b*x^2+13923*a^2)
```

maxima [A] time = 1.16, size = 44, normalized size = 0.69

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\left(b^2+2\,ac\right)x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*(b^2 + 2*a*c)*x^(13/2) + 4/9*a
*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

mupad [B] time = 0.03, size = 45, normalized size = 0.70

$$x^{13/2}\left(\frac{2\,b^2}{13}+\frac{4\,a\,c}{13}\right)+\frac{2\,a^2\,x^{5/2}}{5}+\frac{2\,c^2\,x^{21/2}}{21}+\frac{4\,a\,b\,x^{9/2}}{9}+\frac{4\,b\,c\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(13/2)*((4*a*c)/13 + (2*b^2)/13) + (2*a^2*x^(5/2))/5 + (2*c^2*x^(21/2))/2
1 + (4*a*b*x^(9/2))/9 + (4*b*c*x^(17/2))/17
```

sympy [A] time = 12.36, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{4acx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 4*a*c*x**(13/2)/13 + 2*b**2*x**(13/2
)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

3.817 $\int \sqrt{x} \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=64

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*(b^2 + 2*a*c)*x^(11/2))/11 + (4*
b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)^2 dx &= \int \left(a^2\sqrt{x}+2abx^{5/2}+\left(b^2+2ac\right)x^{9/2}+2bcx^{13/2}+c^2x^{17/2}\right)dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}\left(b^2+2ac\right)x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}\end{aligned}$$

Mathematica [A] time = 3.38, size = 50, normalized size = 0.78

$$\frac{2x^{3/2}\left(7315a^2+1995x^4\left(2ac+b^2\right)+6270abx^2+2926bcx^6+1155c^2x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*(7315*a^2 + 6270*a*b*x^2 + 1995*(b^2 + 2*a*c)*x^4 + 2926*b*c*x^6
 + 1155*c^2*x^8))/21945
```

IntegrateAlgebraic [A] time = 0.03, size = 62, normalized size = 0.97

$$\frac{2\left(7315a^2x^{3/2}+6270abx^{7/2}+3990acx^{11/2}+1995b^2x^{11/2}+2926bcx^{15/2}+1155c^2x^{19/2}\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(7315*a^2*x^(3/2) + 6270*a*b*x^(7/2) + 1995*b^2*x^(11/2) + 3990*a*c*x^(1
1/2) + 2926*b*c*x^(15/2) + 1155*c^2*x^(19/2)))/21945
```

fricas [A] time = 0.58, size = 47, normalized size = 0.73

$$\frac{2}{21945}\left(1155\,c^2x^9+2926\,bcx^7+1995\left(b^2+2\,ac\right)x^5+6270\,abx^3+7315\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*c^2*x^9 + 2926*b*c*x^7 + 1995*(b^2 + 2*a*c)*x^5 + 6270*a*b*x^
3 + 7315*a^2*x)*sqrt(x)
```

giac [A] time = 0.15, size = 46, normalized size = 0.72

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{11}\,acx^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2) + 4/11*a*c*x^(11/
2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.77

$$\frac{2\left(1155c^2x^8+2926bc\,x^6+3990ac\,x^4+1995b^2x^4+6270ab\,x^2+7315a^2\right)x^{\frac{3}{2}}}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/21945*x^(3/2)*(1155*c^2*x^8+2926*b*c*x^6+3990*a*c*x^4+1995*b^2*x^4+6270*a
*b*x^2+7315*a^2)
```

maxima [A] time = 1.11, size = 44, normalized size = 0.69

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\left(b^2+2\,ac\right)x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*(b^2 + 2*a*c)*x^(11/2) + 4/7*a
*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

mupad [B] time = 0.03, size = 45, normalized size = 0.70

$$x^{11/2}\left(\frac{2\,b^2}{11}+\frac{4\,a\,c}{11}\right)+\frac{2\,a^2\,x^{3/2}}{3}+\frac{2\,c^2\,x^{19/2}}{19}+\frac{4\,a\,b\,x^{7/2}}{7}+\frac{4\,b\,c\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(11/2)*((4*a*c)/11 + (2*b^2)/11) + (2*a^2*x^(3/2))/3 + (2*c^2*x^(19/2))/1
9 + (4*a*b*x^(7/2))/7 + (4*b*c*x^(15/2))/15
```

sympy [A] time = 3.45, size = 63, normalized size = 0.98

$$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{11}{2}}\left(2ac+b^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2
)/19 + 2*x**(11/2)*(2*a*c + b**2)/11
```

3.818 $\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*(b^2 + 2*a*c)*x^(9/2))/9 + (4*b*c*x^
(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2abx^{3/2}+\left(b^2+2ac\right)x^{7/2}+2bcx^{11/2}+c^2x^{15/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}\left(b^2+2ac\right)x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.02

$$2\left(a^2\sqrt{x}+\frac{1}{9}x^{9/2}\left(2ac+b^2\right)+\frac{2}{5}abx^{5/2}+\frac{2}{13}bcx^{13/2}+\frac{1}{17}c^2x^{17/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] 2*(a^2*Sqrt[x] + (2*a*b*x^(5/2))/5 + ((b^2 + 2*a*c)*x^(9/2))/9 + (2*b*c*x^(
13/2))/13 + (c^2*x^(17/2))/17)
```

IntegrateAlgebraic [A] time = 0.03, size = 62, normalized size = 1.00

$$\frac{2\left(9945a^2\sqrt{x}+3978abx^{5/2}+2210acx^{9/2}+1105b^2x^{9/2}+1530bcx^{13/2}+585c^2x^{17/2}\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] (2*(9945*a^2*Sqrt[x] + 3978*a*b*x^(5/2) + 1105*b^2*x^(9/2) + 2210*a*c*x^(9/
2) + 1530*b*c*x^(13/2) + 585*c^2*x^(17/2)))/9945
```

fricas [A] time = 0.97, size = 46, normalized size = 0.74

$$\frac{2}{9945}\left(585\,c^2x^8+1530\,bcx^6+1105\left(b^2+2\,ac\right)x^4+3978\,abx^2+9945\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*c^2*x^8 + 1530*b*c*x^6 + 1105*(b^2 + 2*a*c)*x^4 + 3978*a*b*x^2
+ 9945*a^2)*sqrt(x)
```

giac [A] time = 0.15, size = 46, normalized size = 0.74

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{9}\,acx^{\frac{9}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 4/9*a*c*x^(9/2) +
 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$\frac{2\left(585c^2x^8+1530bc\,x^6+2210ac\,x^4+1105b^2x^4+3978ab\,x^2+9945a^2\right)\sqrt{x}}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 2/9945*x^(1/2)*(585*c^2*x^8+1530*b*c*x^6+2210*a*c*x^4+1105*b^2*x^4+3978*a*b
*x^2+9945*a^2)
```

maxima [A] time = 1.14, size = 48, normalized size = 0.77

$$\frac{2}{17}c^2x^{\frac{17}{2}}+\frac{4}{13}bcx^{\frac{13}{2}}+\frac{2}{9}b^2x^{\frac{9}{2}}+2a^2\sqrt{x}+\frac{4}{45}\left(5cx^{\frac{9}{2}}+9bx^{\frac{5}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 2*a^2*sqrt(x) + 4
/45*(5*c*x^(9/2) + 9*b*x^(5/2))*a
```

mupad [B] time = 0.03, size = 45, normalized size = 0.73

$$x^{9/2}\left(\frac{2b^2}{9}+\frac{4ac}{9}\right)+2a^2\sqrt{x}+\frac{2c^2x^{17/2}}{17}+\frac{4abx^{5/2}}{5}+\frac{4bcx^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(1/2),x)
```

```
[Out] x^(9/2)*((4*a*c)/9 + (2*b^2)/9) + 2*a^2*x^(1/2) + (2*c^2*x^(17/2))/17 + (4*
a*b*x^(5/2))/5 + (4*b*c*x^(13/2))/13
```

sympy [A] time = 5.00, size = 68, normalized size = 1.10

$$2a^2\sqrt{x}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{4acx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{9}{2}}}{9}+\frac{4bcx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 4*a*c*x**(9/2)/9 + 2*b**2*x**(9/2)/9 +
4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

3.819 $\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}\left(2ac+b^2\right)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}\left(2ac+b^2\right)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*(b^2 + 2*a*c)*x^(7/2))/7 + (4*b*c
*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+2ab\sqrt{x}+\left(b^2+2ac\right)x^{5/2}+2bcx^{9/2}+c^2x^{13/2}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}\left(b^2+2ac\right)x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.87

$$\frac{2\left(-1155a^2+110a\left(7bx^2+3cx^4\right)+165b^2x^4+210bcx^6+77c^2x^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*(-1155*a^2 + 165*b^2*x^4 + 210*b*c*x^6 + 77*c^2*x^8 + 110*a*(7*b*x^2 + 3
*c*x^4)))/(1155*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.03, size = 52, normalized size = 0.84

$$\frac{2\left(-1155a^2+770abx^2+330acx^4+165b^2x^4+210bcx^6+77c^2x^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*(-1155*a^2 + 770*a*b*x^2 + 165*b^2*x^4 + 330*a*c*x^4 + 210*b*c*x^6 + 77*
c^2*x^8))/(1155*Sqrt[x])
```

fricas [A] time = 0.83, size = 46, normalized size = 0.74

$$\frac{2\left(77\,c^2x^8+210\,bcx^6+165\left(b^2+2\,ac\right)x^4+770\,abx^2-1155\,a^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*c^2*x^8 + 210*b*c*x^6 + 165*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2 - 11
55*a^2)/sqrt(x)
```

giac [A] time = 0.15, size = 46, normalized size = 0.74

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{7}\,acx^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2) + 4/7*a*c*x^(7/2) +
 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$-\frac{2\left(-77c^2x^8-210bc\,x^6-330ac\,x^4-165b^2x^4-770ab\,x^2+1155a^2\right)}{1155\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(3/2),x)
```

```
[Out] -2/1155*(-77*c^2*x^8-210*b*c*x^6-330*a*c*x^4-165*b^2*x^4-770*a*b*x^2+1155*a
^2)/x^(1/2)
```

maxima [A] time = 1.11, size = 44, normalized size = 0.71

$$\frac{2}{15}\,c^2 x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\left(b^2+2\,ac\right)x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*(b^2 + 2*a*c)*x^(7/2) + 4/3*a*b
*x^(3/2) - 2*a^2/sqrt(x)
```

mupad [B] time = 0.03, size = 45, normalized size = 0.73

$$x^{7/2}\left(\frac{2\,b^2}{7}+\frac{4\,a\,c}{7}\right)-\frac{2\,a^2}{\sqrt{x}}+\frac{2\,c^2\,x^{15/2}}{15}+\frac{4\,a\,b\,x^{3/2}}{3}+\frac{4\,b\,c\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(3/2),x)
```

```
[Out] x^(7/2)*((4*a*c)/7 + (2*b^2)/7) - (2*a^2)/x^(1/2) + (2*c^2*x^(15/2))/15 + (
4*a*b*x^(3/2))/3 + (4*b*c*x^(11/2))/11
```

sympy [A] time = 5.65, size = 68, normalized size = 1.10

$$-\frac{2a^2}{\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{4acx^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 4*a*c*x**(7/2)/7 + 2*b**2*x**(7/2)/7 +
 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

3.820 $\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}\left(2ac+b^2\right)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}\left(2ac+b^2\right)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*(b^2 + 2*a*c)*x^(5/2))/5 + (4*b*c
*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{\sqrt{x}}+\left(b^2+2ac\right)x^{3/2}+2bcx^{7/2}+c^2x^{11/2}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}\left(b^2+2ac\right)x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 53, normalized size = 0.85

$$\frac{-390a^2+468a\left(5bx^2+cx^4\right)+234b^2x^4+260bcx^6+90c^2x^8}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-390*a^2 + 234*b^2*x^4 + 260*b*c*x^6 + 90*c^2*x^8 + 468*a*(5*b*x^2 + c*x^4
))/(585*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 52, normalized size = 0.84

$$\frac{2\left(-195a^2+1170abx^2+234acx^4+117b^2x^4+130bcx^6+45c^2x^8\right)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (2*(-195*a^2 + 1170*a*b*x^2 + 117*b^2*x^4 + 234*a*c*x^4 + 130*b*c*x^6 + 45*
c^2*x^8))/(585*x^(3/2))
```

fricas [A] time = 0.96, size = 46, normalized size = 0.74

$$\frac{2\left(45\,c^2x^8+130\,bcx^6+117\left(b^2+2\,ac\right)x^4+1170\,abx^2-195\,a^2\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c^2*x^8 + 130*b*c*x^6 + 117*(b^2 + 2*a*c)*x^4 + 1170*a*b*x^2 - 19
5*a^2)/x^(3/2)
```

giac [A] time = 0.15, size = 46, normalized size = 0.74

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,b^2x^{\frac{5}{2}}+\frac{4}{5}\,acx^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2) + 4/5*a*c*x^(5/2) + 4
*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$-\frac{2\left(-45c^2x^8-130bc\,x^6-234ac\,x^4-117b^2x^4-1170ab\,x^2+195a^2\right)}{585x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(5/2),x)
```

```
[Out] -2/585*(-45*c^2*x^8-130*b*c*x^6-234*a*c*x^4-117*b^2*x^4-1170*a*b*x^2+195*a^
2)/x^(3/2)
```

maxima [A] time = 1.13, size = 44, normalized size = 0.71

$$\frac{2}{13}\,c^{2}x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\left(b^{2}+2\,ac\right)x^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^{2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*(b^2 + 2*a*c)*x^(5/2) + 4*a*b*sqr
t(x) - 2/3*a^2/x^(3/2)
```

mupad [B] time = 0.03, size = 45, normalized size = 0.73

$$x^{5/2}\left(\frac{2\,b^{2}}{5}+\frac{4\,a\,c}{5}\right)-\frac{2\,a^{2}}{3\,x^{3/2}}+\frac{2\,c^{2}\,x^{13/2}}{13}+4\,a\,b\,\sqrt{x}+\frac{4\,b\,c\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(5/2),x)
```

```
[Out] x^(5/2)*((4*a*c)/5 + (2*b^2)/5) - (2*a^2)/(3*x^(3/2)) + (2*c^2*x^(13/2))/13
 + 4*a*b*x^(1/2) + (4*b*c*x^(9/2))/9
```

sympy [A] time = 6.89, size = 68, normalized size = 1.10

$$-\frac{2a^{2}}{3x^{\frac{3}{2}}}+4ab\sqrt{x}+\frac{4acx^{\frac{5}{2}}}{5}+\frac{2b^{2}x^{\frac{5}{2}}}{5}+\frac{4bcx^{\frac{9}{2}}}{9}+\frac{2c^{2}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 4*a*c*x**(5/2)/5 + 2*b**2*x**(5/2)/5
 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

3.821 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{7/2}} \, dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}\left(2ac+b^2\right)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}\left(2ac+b^2\right)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*(b^2 + 2*a*c)*x^(3/2))/3 + (4*b
*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{7/2}} \, dx &= \int \left(\frac{a^2}{x^{7/2}}+\frac{2ab}{x^{3/2}}+\left(b^2+2ac\right)\sqrt{x}+2bcx^{5/2}+c^2x^{9/2}\right) dx\\ &= -\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}\left(b^2+2ac\right)x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.81

$$\frac{2\left(-231a^2+385x^4\left(2ac+b^2\right)-2310abx^2+330bcx^6+105c^2x^8\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*(-231*a^2 - 2310*a*b*x^2 + 385*(b^2 + 2*a*c)*x^4 + 330*b*c*x^6 + 105*c^2
*x^8))/(1155*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 52, normalized size = 0.84

$$\frac{2\left(-231a^2-2310abx^2+770acx^4+385b^2x^4+330bcx^6+105c^2x^8\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*(-231*a^2 - 2310*a*b*x^2 + 385*b^2*x^4 + 770*a*c*x^4 + 330*b*c*x^6 + 105
*c^2*x^8))/(1155*x^(5/2))
```

fricas [A] time = 1.50, size = 46, normalized size = 0.74

$$\frac{2\left(105\,c^2x^8+330\,bcx^6+385\left(b^2+2\,ac\right)x^4-2310\,abx^2-231\,a^2\right)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^2*x^8 + 330*b*c*x^6 + 385*(b^2 + 2*a*c)*x^4 - 2310*a*b*x^2 -
231*a^2)/x^(5/2)
```

giac [A] time = 0.16, size = 47, normalized size = 0.76

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\,b^2x^{\frac{3}{2}}+\frac{4}{3}\,acx^{\frac{3}{2}}-\frac{2\left(10\,abx^2+a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2) + 4/3*a*c*x^(3/2) - 2
/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$-\frac{2\left(-105c^2x^8-330bc\,x^6-770ac\,x^4-385b^2x^4+2310ab\,x^2+231a^2\right)}{1155x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(7/2),x)
```

```
[Out] -2/1155*(-105*c^2*x^8-330*b*c*x^6-770*a*c*x^4-385*b^2*x^4+2310*a*b*x^2+231*
a^2)/x^(5/2)
```

maxima [A] time = 1.12, size = 45, normalized size = 0.73

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\left(b^2+2\,ac\right)x^{\frac{3}{2}}-\frac{2\left(10\,abx^2+a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*(b^2 + 2*a*c)*x^(3/2) - 2/5*(10*a
*b*x^2 + a^2)/x^(5/2)
```

mupad [B] time = 0.05, size = 48, normalized size = 0.77

$$x^{3/2}\left(\frac{2\,b^2}{3}+\frac{4\,a\,c}{3}\right)-\frac{\frac{2\,a^2}{5}+4\,b\,a\,x^2}{x^{5/2}}+\frac{2\,c^2\,x^{11/2}}{11}+\frac{4\,b\,c\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(7/2),x)
```

```
[Out] x^(3/2)*((4*a*c)/3 + (2*b^2)/3) - ((2*a^2)/5 + 4*a*b*x^2)/x^(5/2) + (2*c^2*
x^(11/2))/11 + (4*b*c*x^(7/2))/7
```

sympy [A] time = 9.11, size = 68, normalized size = 1.10

$$-\frac{2a^2}{5x^{\frac{5}{2}}}-\frac{4ab}{\sqrt{x}}+\frac{4acx^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{3}{2}}}{3}+\frac{4bcx^{\frac{7}{2}}}{7}+\frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(7/2),x)
```

```
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 4*a*c*x**(3/2)/3 + 2*b**2*x**(3/2)/3
 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

3.822 $\int x^{5/2} \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=103

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{6}{11}a^2bx^{11/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(15/2))/5 +
(2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(23/2))/23 + (2*b*c^2*
x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=\int\left(a^3x^{5/2}+3a^2bx^{9/2}+3a\left(b^2+ac\right)x^{13/2}+b\left(b^2+6ac\right)x^{17/2}+3c\left(b^2+ac\right)x^{21/2}+\right.$$
$$=\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}a\left(b^2+ac\right)x^{15/2}+\frac{2}{19}b\left(b^2+6ac\right)x^{19/2}+\frac{6}{23}c\left(b^2+ac\right)x$$

Mathematica [A] time = 3.72, size = 103, normalized size = 1.00

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(15/2))/5 +
(2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(23/2))/23 + (2*b*c^2*
x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

IntegrateAlgebraic [A] time = 0.05, size = 111, normalized size = 1.08

$$\frac{2\left(6705765a^3x^{7/2}+12801915a^2bx^{11/2}+9388071a^2cx^{15/2}+9388071ab^2x^{15/2}+14823270abcx^{19/2}+6122655ac^2x^{23/2}+2470545b^3x^{19/2}+6122655b^2cx^{23/2}+5215595bc^2x^{27/2}+1514205c^3x^{31/2}\right)}{46940355}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(6705765*a^3*x^(7/2) + 12801915*a^2*b*x^(11/2) + 9388071*a*b^2*x^(15/2)
+ 9388071*a^2*c*x^(15/2) + 2470545*b^3*x^(19/2) + 14823270*a*b*c*x^(19/2) +
 6122655*b^2*c*x^(23/2) + 6122655*a*c^2*x^(23/2) + 5215595*b*c^2*x^(27/2) +
 1514205*c^3*x^(31/2)))/46940355
```

fricas [A] time = 0.93, size = 86, normalized size = 0.83

$$\frac{2}{46940355}\left(1514205\,c^3x^{15}+5215595\,bc^2x^{13}+6122655\left(b^2c+ac^2\right)x^{11}+2470545\left(b^3+6\,abc\right)x^9+12801915\,a^2bx^5+9388071\left(ab^2+a^2c\right)x^7+6705765\,a^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/46940355*(1514205*c^3*x^15 + 5215595*b*c^2*x^13 + 6122655*(b^2*c + a*c^2)
*x^11 + 2470545*(b^3 + 6*a*b*c)*x^9 + 12801915*a^2*b*x^5 + 9388071*(a*b^2 +
 a^2*c)*x^7 + 6705765*a^3*x^3)*sqrt(x)
```

giac [A] time = 0.16, size = 87, normalized size = 0.84

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{6}{23}\,ac^2x^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}+\frac{12}{19}\,abcx^{\frac{19}{2}}+\frac{2}{5}\,ab^2x^{\frac{15}{2}}+\frac{2}{5}\,a^2cx^{\frac{15}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 6/23*a*c^2*x
^(23/2) + 2/19*b^3*x^(19/2) + 12/19*a*b*c*x^(19/2) + 2/5*a*b^2*x^(15/2) + 2
/5*a^2*c*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7/2)
```

maple [A] time = 0.01, size = 90, normalized size = 0.87

$$\frac{2\left(1514205c^3x^{12}+5215595b\,c^2x^{10}+6122655a\,c^2x^8+6122655b^2c\,x^8+14823270abc\,x^6+2470545b^3x^6+9388071a^2c\,x^4+9388071a\,b^2x^4+12801915a^2b\,x^2+6705765a^3\right)x^{\frac{7}{2}}}{46940355}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/46940355*x^(7/2)*(1514205*c^3*x^12+5215595*b*c^2*x^10+6122655*a*c^2*x^8+6
122655*b^2*c*x^8+14823270*a*b*c*x^6+2470545*b^3*x^6+9388071*a^2*c*x^4+93880
71*a*b^2*x^4+12801915*a^2*b*x^2+6705765*a^3)
```

maxima [A] time = 1.00, size = 81, normalized size = 0.79

$$\frac{2}{31}\,c^{3}x^{\frac{31}{2}} + \frac{2}{9}\,bc^{2}x^{\frac{27}{2}} + \frac{6}{23}\left(b^{2}c + ac^{2}\right)x^{\frac{23}{2}} + \frac{2}{19}\left(b^{3} + 6\,abc\right)x^{\frac{19}{2}} + \frac{6}{11}\,a^{2}bx^{\frac{11}{2}} + \frac{2}{5}\left(ab^{2} + a^{2}c\right)x^{\frac{15}{2}} + \frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*(b^2*c + a*c^2)*x^(23/2) + 2/
19*(b^3 + 6*a*b*c)*x^(19/2) + 6/11*a^2*b*x^(11/2) + 2/5*(a*b^2 + a^2*c)*x^(
15/2) + 2/7*a^3*x^(7/2)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.74

$$x^{19/2}\left(\frac{2\,b^{3}}{19} + \frac{12\,a\,c\,b}{19}\right) + \frac{2\,a^{3}\,x^{7/2}}{7} + \frac{2\,c^{3}\,x^{31/2}}{31} + \frac{6\,a^{2}\,b\,x^{11/2}}{11} + \frac{2\,b\,c^{2}\,x^{27/2}}{9} + \frac{2\,a\,x^{15/2}\,\left(b^{2} + a\,c\right)}{5} + \frac{6\,c\,x^{23/2}\,\left(b^{2} + a\,c\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(19/2)*((2*b^3)/19 + (12*a*b*c)/19) + (2*a^3*x^(7/2))/7 + (2*c^3*x^(31/2)
)/31 + (6*a^2*b*x^(11/2))/11 + (2*b*c^2*x^(27/2))/9 + (2*a*x^(15/2)*(a*c +
b^2))/5 + (6*c*x^(23/2)*(a*c + b^2))/23
```

sympy [A] time = 60.63, size = 129, normalized size = 1.25

$$\frac{2a^{3}x^{\frac{7}{2}}}{7} + \frac{6a^{2}bx^{\frac{11}{2}}}{11} + \frac{2a^{2}cx^{\frac{15}{2}}}{5} + \frac{2ab^{2}x^{\frac{15}{2}}}{5} + \frac{12abcx^{\frac{19}{2}}}{19} + \frac{6ac^{2}x^{\frac{23}{2}}}{23} + \frac{2b^{3}x^{\frac{19}{2}}}{19} + \frac{6b^{2}cx^{\frac{23}{2}}}{23} + \frac{2bc^{2}x^{\frac{27}{2}}}{9} + \frac{2c^{3}x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a**2*c*x**(15/2)/5 + 2*a*b**2
*x**(15/2)/5 + 12*a*b*c*x**(19/2)/19 + 6*a*c**2*x**(23/2)/23 + 2*b**3*x**(1
9/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**3*x**(31/2)/3
1
```

3.823 $\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{5}a^3x^{5/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*(b^2 + a*c)*x^(13/2))/13 + (
2*b*(b^2 + 6*a*c)*x^(17/2))/17 + (2*c*(b^2 + a*c)*x^(21/2))/7 + (6*b*c^2*x^
(25/2))/25 + (2*c^3*x^(29/2))/29
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3x^{3/2}+3a^2bx^{7/2}+3a\left(b^2+ac\right)x^{11/2}+b\left(b^2+6ac\right)x^{15/2}+3c\left(b^2+ac\right)x^{19/2}+\right.\\ &= \frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}a\left(b^2+ac\right)x^{13/2}+\frac{2}{17}b\left(b^2+6ac\right)x^{17/2}+\frac{2}{7}c\left(b^2+ac\right)x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.10, size = 105, normalized size = 1.02

$$2\left(\frac{1}{5}a^3x^{5/2}+\frac{1}{3}a^2bx^{9/2}+\frac{1}{7}cx^{21/2}\left(ac+b^2\right)+\frac{1}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{3}{13}ax^{13/2}\left(ac+b^2\right)+\frac{3}{25}bc^2x^{25/2}+\frac{1}{29}c^3x^{29/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] 2*((a^3*x^(5/2))/5 + (a^2*b*x^(9/2))/3 + (3*a*(b^2 + a*c)*x^(13/2))/13 + (b
*(b^2 + 6*a*c)*x^(17/2))/17 + (c*(b^2 + a*c)*x^(21/2))/7 + (3*b*c^2*x^(25/2
))/25 + (c^3*x^(29/2))/29)
```

IntegrateAlgebraic [A] time = 0.05, size = 111, normalized size = 1.08

$$\frac{2\left(672945a^3x^{5/2}+1121575a^2bx^{9/2}+776475a^2cx^{13/2}+776475ab^2x^{13/2}+1187550abcx^{17/2}+480675ac^2x^{21/2}+197925b^3x^{17/2}+480675b^2cx^{21/2}+403767bc^2x^{25/2}+116025c^3x^{29/2}\right)}{3364725}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(672945*a^3*x^(5/2) + 1121575*a^2*b*x^(9/2) + 776475*a*b^2*x^(13/2) + 77
6475*a^2*c*x^(13/2) + 197925*b^3*x^(17/2) + 1187550*a*b*c*x^(17/2) + 480675
*b^2*c*x^(21/2) + 480675*a*c^2*x^(21/2) + 403767*b*c^2*x^(25/2) + 116025*c^
3*x^(29/2)))/3364725
```

fricas [A] time = 0.89, size = 86, normalized size = 0.83

$$\frac{2}{3364725}\left(116025\,c^3x^{14}+403767\,bc^2x^{12}+480675\left(b^2c+ac^2\right)x^{10}+197925\left(b^3+6\,abc\right)x^8+1121575\,a^2bx^4+776475\left(ab^2+a^2c\right)x^6+672945\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*c^3*x^14 + 403767*b*c^2*x^12 + 480675*(b^2*c + a*c^2)*x^1
0 + 197925*(b^3 + 6*a*b*c)*x^8 + 1121575*a^2*b*x^4 + 776475*(a*b^2 + a^2*c)
*x^6 + 672945*a^3*x^2)*sqrt(x)
```

giac [A] time = 0.17, size = 87, normalized size = 0.84

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{7}\,ac^2x^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{12}{17}\,abcx^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{6}{13}\,a^2cx^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/7*a*c^2*x^
(21/2) + 2/17*b^3*x^(17/2) + 12/17*a*b*c*x^(17/2) + 6/13*a*b^2*x^(13/2) + 6
/13*a^2*c*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/2)
```

maple [A] time = 0.01, size = 90, normalized size = 0.87

$$\frac{2\left(116025c^3x^{12}+403767b\,c^2x^{10}+480675a\,c^2x^8+480675b^2c\,x^8+1187550abc\,x^6+197925b^3x^6+776475a^2c\,x^4+776475a\,b^2x^4+1121575a^2b\,x^2+672945a^3\right)x^{\frac{5}{2}}}{3364725}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/3364725*x^(5/2)*(116025*c^3*x^12+403767*b*c^2*x^10+480675*a*c^2*x^8+48067
5*b^2*c*x^8+1187550*a*b*c*x^6+197925*b^3*x^6+776475*a^2*c*x^4+776475*a*b^2*
x^4+1121575*a^2*b*x^2+672945*a^3)
```

maxima [A] time = 1.06, size = 81, normalized size = 0.79

$$\frac{2}{29}\,c^3x^{\frac{29}{2}} + \frac{6}{25}\,bc^2x^{\frac{25}{2}} + \frac{2}{7}\left(b^2c + ac^2\right)x^{\frac{21}{2}} + \frac{2}{17}\left(b^3 + 6\,abc\right)x^{\frac{17}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{6}{13}\left(ab^2 + a^2c\right)x^{\frac{13}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*(b^2*c + a*c^2)*x^(21/2) + 2/
17*(b^3 + 6*a*b*c)*x^(17/2) + 2/3*a^2*b*x^(9/2) + 6/13*(a*b^2 + a^2*c)*x^(1
3/2) + 2/5*a^3*x^(5/2)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.74

$$x^{17/2}\left(\frac{2\,b^3}{17} + \frac{12\,a\,c\,b}{17}\right) + \frac{2\,a^3\,x^{5/2}}{5} + \frac{2\,c^3\,x^{29/2}}{29} + \frac{2\,a^2\,b\,x^{9/2}}{3} + \frac{6\,b\,c^2\,x^{25/2}}{25} + \frac{6\,a\,x^{13/2}\,\left(b^2 + a\,c\right)}{13} + \frac{2\,c\,x^{21/2}\,\left(b^2 + a\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(17/2)*((2*b^3)/17 + (12*a*b*c)/17) + (2*a^3*x^(5/2))/5 + (2*c^3*x^(29/2)
)/29 + (2*a^2*b*x^(9/2))/3 + (6*b*c^2*x^(25/2))/25 + (6*a*x^(13/2)*(a*c + b
^2))/13 + (2*c*x^(21/2)*(a*c + b^2))/7
```

sympy [A] time = 39.32, size = 129, normalized size = 1.25

$$\frac{2a^3x^{\frac{5}{2}}}{5} + \frac{2a^2bx^{\frac{9}{2}}}{3} + \frac{6a^2cx^{\frac{13}{2}}}{13} + \frac{6ab^2x^{\frac{13}{2}}}{13} + \frac{12abcx^{\frac{17}{2}}}{17} + \frac{2ac^2x^{\frac{21}{2}}}{7} + \frac{2b^3x^{\frac{17}{2}}}{17} + \frac{2b^2cx^{\frac{21}{2}}}{7} + \frac{6bc^2x^{\frac{25}{2}}}{25} + \frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a**2*c*x**(13/2)/13 + 6*a*b**2*
x**(13/2)/13 + 12*a*b*c*x**(17/2)/17 + 2*a*c**2*x**(21/2)/7 + 2*b**3*x**(17
/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**3*x**(29/2)/29
```

3.824 $\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/2))/11 + (
2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19/2))/19 + (6*b*c^2*x
^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3a\left(b^2+ac\right)x^{9/2}+b\left(b^2+6ac\right)x^{13/2}+3c\left(b^2+ac\right)x^{17/2}+\right.\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}a\left(b^2+ac\right)x^{11/2}+\frac{2}{15}b\left(b^2+6ac\right)x^{15/2}+\frac{6}{19}c\left(b^2+ac\right)x^{1}\end{aligned}$$

Mathematica [A] time = 3.38, size = 103, normalized size = 1.00

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/2))/11 + (
2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19/2))/19 + (6*b*c^2*x
^(23/2))/23 + (2*c^3*x^(27/2))/27
```

IntegrateAlgebraic [A] time = 0.05, size = 111, normalized size = 1.08

$$\frac{2\left(1514205a^3x^{3/2}+1946835a^2bx^{7/2}+1238895a^2cx^{11/2}+1238895ab^2x^{11/2}+1817046abcx^{15/2}+717255ac^2x^{19/2}+302841b^3x^{15/2}+717255b^2cx^{19/2}+592515bc^2x^{23/2}+168245c^3x^{27/2}\right)}{4542615}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(1514205*a^3*x^(3/2) + 1946835*a^2*b*x^(7/2) + 1238895*a*b^2*x^(11/2) +
1238895*a^2*c*x^(11/2) + 302841*b^3*x^(15/2) + 1817046*a*b*c*x^(15/2) + 717
255*b^2*c*x^(19/2) + 717255*a*c^2*x^(19/2) + 592515*b*c^2*x^(23/2) + 168245
*c^3*x^(27/2)))/4542615
```

fricas [A] time = 2.69, size = 84, normalized size = 0.82

$$\frac{2}{4542615}\left(168245\,c^3x^{13}+592515\,bc^2x^{11}+717255\left(b^2c+ac^2\right)x^9+302841\left(b^3+6\,abc\right)x^7+1946835\,a^2bx^3+1238895\left(ab^2+a^2c\right)x^5+1514205\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/4542615*(168245*c^3*x^13 + 592515*b*c^2*x^11 + 717255*(b^2*c + a*c^2)*x^9
 + 302841*(b^3 + 6*a*b*c)*x^7 + 1946835*a^2*b*x^3 + 1238895*(a*b^2 + a^2*c)
*x^5 + 1514205*a^3*x)*sqrt(x)
```

giac [A] time = 0.16, size = 87, normalized size = 0.84

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{6}{19}\,ac^2x^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{4}{5}\,abcx^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{11}\,a^2cx^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 6/19*a*c^2*
x^(19/2) + 2/15*b^3*x^(15/2) + 4/5*a*b*c*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6
/11*a^2*c*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/2)
```

maple [A] time = 0.01, size = 90, normalized size = 0.87

$$\frac{2\left(168245c^3x^{12}+592515b\,c^2x^{10}+717255a\,c^2x^8+717255b^2c\,x^8+1817046abc\,x^6+302841b^3x^6+1238895a^2c\,x^4+1238895a\,b^2x^4+1946835a^2b\,x^2+1514205a^3\right)x^{\frac{3}{2}}}{4542615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/4542615*x^(3/2)*(168245*c^3*x^12+592515*b*c^2*x^10+717255*a*c^2*x^8+71725
5*b^2*c*x^8+1817046*a*b*c*x^6+302841*b^3*x^6+1238895*a^2*c*x^4+1238895*a*b^
2*x^4+1946835*a^2*b*x^2+1514205*a^3)
```

maxima [A] time = 1.04, size = 81, normalized size = 0.79

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\left(b^2c+ac^2\right)x^{\frac{19}{2}}+\frac{2}{15}\left(b^3+6\,abc\right)x^{\frac{15}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{6}{11}\left(ab^2+a^2c\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*(b^2*c + a*c^2)*x^(19/2) + 2
/15*(b^3 + 6*a*b*c)*x^(15/2) + 6/7*a^2*b*x^(7/2) + 6/11*(a*b^2 + a^2*c)*x^(
11/2) + 2/3*a^3*x^(3/2)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.74

$$x^{15/2}\left(\frac{2\,b^3}{15}+\frac{4\,a\,c\,b}{5}\right)+\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,c^3\,x^{27/2}}{27}+\frac{6\,a^2\,b\,x^{7/2}}{7}+\frac{6\,b\,c^2\,x^{23/2}}{23}+\frac{6\,a\,x^{11/2}\,\left(b^2+a\,c\right)}{11}+\frac{6\,c\,x^{19/2}\,\left(b^2+a\,c\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(15/2)*((2*b^3)/15 + (4*a*b*c)/5) + (2*a^3*x^(3/2))/3 + (2*c^3*x^(27/2))/
27 + (6*a^2*b*x^(7/2))/7 + (6*b*c^2*x^(23/2))/23 + (6*a*x^(11/2)*(a*c + b^2
))/11 + (6*c*x^(19/2)*(a*c + b^2))/19
```

sympy [A] time = 6.05, size = 112, normalized size = 1.09

$$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}+\frac{2x^{\frac{19}{2}}\left(3ac^2+3b^2c\right)}{19}+\frac{2x^{\frac{15}{2}}\left(6abc+b^3\right)}{15}+\frac{2x^{\frac{11}{2}}\left(3a^2c+3ab^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*b*c**2*x**(23/2)/23 + 2*c**3*x*
*(27/2)/27 + 2*x**(19/2)*(3*a*c**2 + 3*b**2*c)/19 + 2*x**(15/2)*(6*a*b*c +
b**3)/15 + 2*x**(11/2)*(3*a**2*c + 3*a*b**2)/11
```

3.825 $\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=101

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{6}{5}a^2bx^{5/2}+2a^3\sqrt{x}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*(b^2 + a*c)*x^(9/2))/3 + (2*b*(b
^2 + 6*a*c)*x^(13/2))/13 + (6*c*(b^2 + a*c)*x^(17/2))/17 + (2*b*c^2*x^(21/2
))/7 + (2*c^3*x^(25/2))/25
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3a\left(b^2+ac\right)x^{7/2}+b\left(b^2+6ac\right)x^{11/2}+3c\left(b^2+ac\right)x^{15/2}+3bc^2x^{19}\right.\\ &= 2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}a\left(b^2+ac\right)x^{9/2}+\frac{2}{13}b\left(b^2+6ac\right)x^{13/2}+\frac{6}{17}c\left(b^2+ac\right)x^{17/2}+\frac{2}{7}b\end{aligned}$$

Mathematica [A] time = 0.07, size = 102, normalized size = 1.01

$$2\left(a^3\sqrt{x}+\frac{3}{5}a^2bx^{5/2}+\frac{3}{17}cx^{17/2}\left(ac+b^2\right)+\frac{1}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{1}{3}ax^{9/2}\left(ac+b^2\right)+\frac{1}{7}bc^2x^{21/2}+\frac{1}{25}c^3x^{25/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*(a^3*Sqrt[x] + (3*a^2*b*x^(5/2))/5 + (a*(b^2 + a*c)*x^(9/2))/3 + (b*(b^2
+ 6*a*c)*x^(13/2))/13 + (3*c*(b^2 + a*c)*x^(17/2))/17 + (b*c^2*x^(21/2))/7
+ (c^3*x^(25/2))/25)
```

IntegrateAlgebraic [A] time = 0.05, size = 111, normalized size = 1.10

$$\frac{2\left(116025a^3\sqrt{x}+69615a^2bx^{5/2}+38675a^2cx^{9/2}+38675ab^2x^{9/2}+53550abcx^{13/2}+20475ac^2x^{17/2}+8925b^3x^{13/2}+20475b^2cx^{17/2}+16575bc^2x^{21/2}+4641c^3x^{25/2}\right)}{116025}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*(116025*a^3*Sqrt[x] + 69615*a^2*b*x^(5/2) + 38675*a*b^2*x^(9/2) + 38675*
a^2*c*x^(9/2) + 8925*b^3*x^(13/2) + 53550*a*b*c*x^(13/2) + 20475*b^2*c*x^(1
7/2) + 20475*a*c^2*x^(17/2) + 16575*b*c^2*x^(21/2) + 4641*c^3*x^(25/2)))/11
6025
```

fricas [A] time = 1.61, size = 83, normalized size = 0.82

$$\frac{2}{116025}\left(4641\,c^3x^{12}+16575\,bc^2x^{10}+20475\left(b^2c+ac^2\right)x^8+8925\left(b^3+6\,abc\right)x^6+69615\,a^2bx^2+38675\left(ab^2+a^2c\right)x^4+116025\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*c^3*x^12 + 16575*b*c^2*x^10 + 20475*(b^2*c + a*c^2)*x^8 + 89
25*(b^3 + 6*a*b*c)*x^6 + 69615*a^2*b*x^2 + 38675*(a*b^2 + a^2*c)*x^4 + 1160
25*a^3)*sqrt(x)
```

giac [A] time = 0.17, size = 87, normalized size = 0.86

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{6}{17}\,ac^2x^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{12}{13}\,abcx^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{2}{3}\,a^2cx^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 6/17*a*c^2*x
^(17/2) + 2/13*b^3*x^(13/2) + 12/13*a*b*c*x^(13/2) + 2/3*a*b^2*x^(9/2) + 2/
3*a^2*c*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

maple [A] time = 0.01, size = 90, normalized size = 0.89

$$\frac{2\left(4641c^3x^{12}+16575b\,c^2x^{10}+20475a\,c^2x^8+20475b^2c\,x^8+53550abc\,x^6+8925b^3x^6+38675a^2c\,x^4+38675a\,b^2x^4+69615a^2b\,x^2+116025a^3\right)\sqrt{x}}{116025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 2/116025*x^(1/2)*(4641*c^3*x^12+16575*b*c^2*x^10+20475*a*c^2*x^8+20475*b^2*
c*x^8+53550*a*b*c*x^6+8925*b^3*x^6+38675*a^2*c*x^4+38675*a*b^2*x^4+69615*a^
2*b*x^2+116025*a^3)
```

maxima [A] time = 1.04, size = 88, normalized size = 0.87

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}+2\,a^3\sqrt{x}+\frac{2}{15}\left(5\,cx^{\frac{9}{2}}+9\,bx^{\frac{5}{2}}\right)a^2+\frac{2}{663}\left(117\,c^2x^{\frac{17}{2}}+306\,bcx^{\frac{13}{2}}+221\,b^2x^{\frac{9}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2) + 2*a^3*sqrt(x) + 2/15*(5*c*x^(9/2) + 9*b*x^(5/2))*a^2 + 2/663*(117*c
^2*x^(17/2) + 306*b*c*x^(13/2) + 221*b^2*x^(9/2))*a
```

mupad [B] time = 0.03, size = 76, normalized size = 0.75

$$x^{13/2}\left(\frac{2\,b^3}{13}+\frac{12\,a\,c\,b}{13}\right)+2\,a^3\,\sqrt{x}+\frac{2\,c^3\,x^{25/2}}{25}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{2\,b\,c^2\,x^{21/2}}{7}+\frac{2\,a\,x^{9/2}\,\left(b^2+a\,c\right)}{3}+\frac{6\,c\,x^{17/2}\,\left(b^2+a\,c\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(1/2),x)
```

```
[Out] x^(13/2)*((2*b^3)/13 + (12*a*b*c)/13) + 2*a^3*x^(1/2) + (2*c^3*x^(25/2))/25
 + (6*a^2*b*x^(5/2))/5 + (2*b*c^2*x^(21/2))/7 + (2*a*x^(9/2)*(a*c + b^2))/3
 + (6*c*x^(17/2)*(a*c + b^2))/17
```

sympy [A] time = 23.50, size = 128, normalized size = 1.27

$$2a^3\sqrt{x}+\frac{6a^2bx^{\frac{5}{2}}}{5}+\frac{2a^2cx^{\frac{9}{2}}}{3}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{12abcx^{\frac{13}{2}}}{13}+\frac{6ac^2x^{\frac{17}{2}}}{17}+\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{17}{2}}}{17}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a**2*c*x**(9/2)/3 + 2*a*b**2*x**(9
/2)/3 + 12*a*b*c*x**(13/2)/13 + 6*a*c**2*x**(17/2)/17 + 2*b**3*x**(13/2)/13
 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**3*x**(25/2)/25
```

3.826 $\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$2a^2bx^{3/2}-\frac{2a^3}{\sqrt{x}}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*(b^2 + a*c)*x^(7/2))/7 + (2*b*(b^
2 + 6*a*c)*x^(11/2))/11 + (2*c*(b^2 + a*c)*x^(15/2))/5 + (6*b*c^2*x^(19/2))
/19 + (2*c^3*x^(23/2))/23
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{3/2}}\,dx=\int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3a\left(b^2+ac\right)x^{5/2}+b\left(b^2+6ac\right)x^{9/2}+3c\left(b^2+ac\right)x^{13/2}+3bc^2x^1$$
$$=-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}a\left(b^2+ac\right)x^{7/2}+\frac{2}{11}b\left(b^2+6ac\right)x^{11/2}+\frac{2}{5}c\left(b^2+ac\right)x^{15/2}+\frac{6}{19}b$$

Mathematica [A] time = 0.08, size = 100, normalized size = 1.01

$$2\left(-\frac{a^3}{\sqrt{x}}+a^2bx^{3/2}+\frac{1}{5}cx^{15/2}\left(ac+b^2\right)+\frac{1}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{3}{7}ax^{7/2}\left(ac+b^2\right)+\frac{3}{19}bc^2x^{19/2}+\frac{1}{23}c^3x^{23/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] 2*(-(a^3/Sqrt[x]) + a^2*b*x^(3/2) + (3*a*(b^2 + a*c)*x^(7/2))/7 + (b*(b^2 +
 6*a*c)*x^(11/2))/11 + (c*(b^2 + a*c)*x^(15/2))/5 + (3*b*c^2*x^(19/2))/19 +
 (c^3*x^(23/2))/23)
```

IntegrateAlgebraic [A] time = 0.06, size = 93, normalized size = 0.94

$$\frac{2\left(-168245a^3+168245a^2bx^2+72105a^2cx^4+72105ab^2x^4+91770abcx^6+33649ac^2x^8+15295b^3x^6+33649b^2cx^8+26565bc^2x^{10}+7315c^3x^{12}\right)}{168245\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*(-168245*a^3 + 168245*a^2*b*x^2 + 72105*a*b^2*x^4 + 72105*a^2*c*x^4 + 15
295*b^3*x^6 + 91770*a*b*c*x^6 + 33649*b^2*c*x^8 + 33649*a*c^2*x^8 + 26565*b
*c^2*x^10 + 7315*c^3*x^12))/(168245*Sqrt[x])
```

fricas [A] time = 0.93, size = 83, normalized size = 0.84

$$\frac{2\left(7315\,c^3x^{12}+26565\,bc^2x^{10}+33649\left(b^2c+ac^2\right)x^8+15295\left(b^3+6\,abc\right)x^6+168245\,a^2bx^2+72105\left(ab^2+a^2c\right)x^4-168245\,a^3\right)}{168245\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*c^3*x^12 + 26565*b*c^2*x^10 + 33649*(b^2*c + a*c^2)*x^8 + 15
295*(b^3 + 6*a*b*c)*x^6 + 168245*a^2*b*x^2 + 72105*(a*b^2 + a^2*c)*x^4 - 16
8245*a^3)/sqrt(x)
```

giac [A] time = 0.16, size = 87, normalized size = 0.88

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\,b^2cx^{\frac{15}{2}}+\frac{2}{5}\,ac^2x^{\frac{15}{2}}+\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{12}{11}\,abcx^{\frac{11}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+\frac{6}{7}\,a^2cx^{\frac{7}{2}}+2\,a^2bx^{\frac{3}{2}}-\frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/5*a*c^2*x^
(15/2) + 2/11*b^3*x^(11/2) + 12/11*a*b*c*x^(11/2) + 6/7*a*b^2*x^(7/2) + 6/7
*a^2*c*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

maple [A] time = 0.01, size = 90, normalized size = 0.91

$$-\frac{2\left(-7315c^3x^{12}-26565b\,c^2x^{10}-33649a\,c^2x^8-33649b^2c\,x^8-91770abc\,x^6-15295b^3x^6-72105a^2c\,x^4-72105a\,b^2x^4-168245a^2b\,x^2+168245a^3\right)}{168245\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(3/2),x)
```

```
[Out] -2/168245*(-7315*c^3*x^12-26565*b*c^2*x^10-33649*a*c^2*x^8-33649*b^2*c*x^8-
91770*a*b*c*x^6-15295*b^3*x^6-72105*a^2*c*x^4-72105*a*b^2*x^4-168245*a^2*b*
x^2+168245*a^3)/x^(1/2)
```

maxima [A] time = 0.98, size = 81, normalized size = 0.82

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\left(b^2c+ac^2\right)x^{\frac{15}{2}}+\frac{2}{11}\left(b^3+6\,abc\right)x^{\frac{11}{2}}+2\,a^2bx^{\frac{3}{2}}+\frac{6}{7}\left(ab^2+a^2c\right)x^{\frac{7}{2}}-\frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*(b^2*c + a*c^2)*x^(15/2) + 2/
11*(b^3 + 6*a*b*c)*x^(11/2) + 2*a^2*b*x^(3/2) + 6/7*(a*b^2 + a^2*c)*x^(7/2)
 - 2*a^3/sqrt(x)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.77

$$x^{11/2}\left(\frac{2\,b^3}{11}+\frac{12\,a\,c\,b}{11}\right)-\frac{2\,a^3}{\sqrt{x}}+\frac{2\,c^3\,x^{23/2}}{23}+2\,a^2\,b\,x^{3/2}+\frac{6\,b\,c^2\,x^{19/2}}{19}+\frac{6\,a\,x^{7/2}\,\left(b^2+a\,c\right)}{7}+\frac{2\,c\,x^{15/2}\,\left(b^2+a\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(3/2),x)
```

```
[Out] x^(11/2)*((2*b^3)/11 + (12*a*b*c)/11) - (2*a^3)/x^(1/2) + (2*c^3*x^(23/2))/
23 + 2*a^2*b*x^(3/2) + (6*b*c^2*x^(19/2))/19 + (6*a*x^(7/2)*(a*c + b^2))/7
+ (2*c*x^(15/2)*(a*c + b^2))/5
```

sympy [A] time = 19.83, size = 126, normalized size = 1.27

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{\frac{3}{2}}+\frac{6a^2cx^{\frac{7}{2}}}{7}+\frac{6ab^2x^{\frac{7}{2}}}{7}+\frac{12abcx^{\frac{11}{2}}}{11}+\frac{2ac^2x^{\frac{15}{2}}}{5}+\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2cx^{\frac{15}{2}}}{5}+\frac{6bc^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a**2*c*x**(7/2)/7 + 6*a*b**2*x**(7/
2)/7 + 12*a*b*c*x**(11/2)/11 + 2*a*c**2*x**(15/2)/5 + 2*b**3*x**(11/2)/11 +
 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**3*x**(23/2)/23
```

3.827 $\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{13}cx^{13/2}\left(ac+b^2\right)+\frac{2}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{6}{5}ax^{5/2}\left(ac+b^2\right)+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$6a^2b\sqrt{x}-\frac{2a^3}{3x^{3/2}}+\frac{6}{13}cx^{13/2}\left(ac+b^2\right)+\frac{2}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{6}{5}ax^{5/2}\left(ac+b^2\right)+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*(b^2 + a*c)*x^(5/2))/5 + (2*b
*(b^2 + 6*a*c)*x^(9/2))/9 + (6*c*(b^2 + a*c)*x^(13/2))/13 + (6*b*c^2*x^(17/
2))/17 + (2*c^3*x^(21/2))/21
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{\sqrt{x}}+3a\left(b^2+ac\right)x^{3/2}+b\left(b^2+6ac\right)x^{7/2}+3c\left(b^2+ac\right)x^{11/2}+3bc^2x^{15/2}+\right.\\ &= -\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}a\left(b^2+ac\right)x^{5/2}+\frac{2}{9}b\left(b^2+6ac\right)x^{9/2}+\frac{6}{13}c\left(b^2+ac\right)x^{13/2}+\frac{6}{17}bc\end{aligned}$$

Mathematica [A] time = 0.07, size = 103, normalized size = 1.02

$$2\left(-\frac{a^3}{3x^{3/2}}+3a^2b\sqrt{x}+\frac{3}{13}cx^{13/2}\left(ac+b^2\right)+\frac{1}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{3}{5}ax^{5/2}\left(ac+b^2\right)+\frac{3}{17}bc^2x^{17/2}+\frac{1}{21}c^3x^{21/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] 2*(-1/3*a^3/x^(3/2) + 3*a^2*b*Sqrt[x] + (3*a*(b^2 + a*c)*x^(5/2))/5 + (b*(b
^2 + 6*a*c)*x^(9/2))/9 + (3*c*(b^2 + a*c)*x^(13/2))/13 + (3*b*c^2*x^(17/2))
/17 + (c^3*x^(21/2))/21)
```

IntegrateAlgebraic [A] time = 0.06, size = 93, normalized size = 0.92

$$\frac{2\left(-23205a^3+208845a^2bx^2+41769a^2cx^4+41769ab^2x^4+46410abcx^6+16065ac^2x^8+7735b^3x^6+16065b^2cx^8+12285bc^2x^{10}+3315c^3x^{12}\right)}{69615x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*(-23205*a^3 + 208845*a^2*b*x^2 + 41769*a*b^2*x^4 + 41769*a^2*c*x^4 + 773
5*b^3*x^6 + 46410*a*b*c*x^6 + 16065*b^2*c*x^8 + 16065*a*c^2*x^8 + 12285*b*c
^2*x^10 + 3315*c^3*x^12))/(69615*x^(3/2))
```

fricas [A] time = 1.14, size = 83, normalized size = 0.82

$$\frac{2\left(3315\,c^3x^{12}+12285\,bc^2x^{10}+16065\left(b^2c+ac^2\right)x^8+7735\left(b^3+6\,abc\right)x^6+208845\,a^2bx^2+41769\left(ab^2+a^2c\right)x^4-23205\,a^3\right)}{69615\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^3*x^12 + 12285*b*c^2*x^10 + 16065*(b^2*c + a*c^2)*x^8 + 773
5*(b^3 + 6*a*b*c)*x^6 + 208845*a^2*b*x^2 + 41769*(a*b^2 + a^2*c)*x^4 - 2320
5*a^3)/x^(3/2)
```

giac [A] time = 0.20, size = 87, normalized size = 0.86

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{6}{13}\,ac^2x^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{4}{3}\,abcx^{\frac{9}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+\frac{6}{5}\,a^2cx^{\frac{5}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 6/13*a*c^2*
x^(13/2) + 2/9*b^3*x^(9/2) + 4/3*a*b*c*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6/5*a^
2*c*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

maple [A] time = 0.01, size = 90, normalized size = 0.89

$$-\frac{2\left(-3315c^3x^{12}-12285b\,c^2x^{10}-16065a\,c^2x^8-16065b^2c\,x^8-46410abc\,x^6-7735b^3x^6-41769a^2c\,x^4-41769a\,b^2x^4-208845a^2b\,x^2+23205a^3\right)}{69615x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(5/2),x)
```

```
[Out] -2/69615*(-3315*c^3*x^12-12285*b*c^2*x^10-16065*a*c^2*x^8-16065*b^2*c*x^8-4
6410*a*b*c*x^6-7735*b^3*x^6-41769*a^2*c*x^4-41769*a*b^2*x^4-208845*a^2*b*x^
2+23205*a^3)/x^(3/2)
```

maxima [A] time = 1.06, size = 81, normalized size = 0.80

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\left(b^2c+ac^2\right)x^{\frac{13}{2}}+\frac{2}{9}\left(b^3+6\,abc\right)x^{\frac{9}{2}}+6\,a^2b\sqrt{x}+\frac{6}{5}\left(ab^2+a^2c\right)x^{\frac{5}{2}}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*(b^2*c + a*c^2)*x^(13/2) + 2
/9*(b^3 + 6*a*b*c)*x^(9/2) + 6*a^2*b*sqrt(x) + 6/5*(a*b^2 + a^2*c)*x^(5/2)
- 2/3*a^3/x^(3/2)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.75

$$x^{9/2}\left(\frac{2\,b^3}{9}+\frac{4\,a\,c\,b}{3}\right)-\frac{2\,a^3}{3\,x^{3/2}}+\frac{2\,c^3\,x^{21/2}}{21}+6\,a^2\,b\,\sqrt{x}+\frac{6\,b\,c^2\,x^{17/2}}{17}+\frac{6\,a\,x^{5/2}\,\left(b^2+a\,c\right)}{5}+\frac{6\,c\,x^{13/2}\,\left(b^2+a\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(5/2),x)
```

```
[Out] x^(9/2)*((2*b^3)/9 + (4*a*b*c)/3) - (2*a^3)/(3*x^(3/2)) + (2*c^3*x^(21/2))/
21 + 6*a^2*b*x^(1/2) + (6*b*c^2*x^(17/2))/17 + (6*a*x^(5/2)*(a*c + b^2))/5
+ (6*c*x^(13/2)*(a*c + b^2))/13
```

sympy [A] time = 25.44, size = 128, normalized size = 1.27

$$-\frac{2a^3}{3x^{\frac{3}{2}}}+6a^2b\sqrt{x}+\frac{6a^2cx^{\frac{5}{2}}}{5}+\frac{6ab^2x^{\frac{5}{2}}}{5}+\frac{4abcx^{\frac{9}{2}}}{3}+\frac{6ac^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a**2*c*x**(5/2)/5 + 6*a*b**2*x*
*(5/2)/5 + 4*a*b*c*x**(9/2)/3 + 6*a*c**2*x**(13/2)/13 + 2*b**3*x**(9/2)/9 +
 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3*x**(21/2)/21
```

3.828 $\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}} \, dx$

Optimal. Leaf size=99

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+\frac{6}{11}cx^{11/2}\left(ac+b^2\right)+\frac{2}{7}bx^{7/2}\left(6ac+b^2\right)+2ax^{3/2}\left(ac+b^2\right)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{5x^{5/2}}+\frac{6}{11}cx^{11/2}\left(ac+b^2\right)+\frac{2}{7}bx^{7/2}\left(6ac+b^2\right)+2ax^{3/2}\left(ac+b^2\right)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*(b^2 + a*c)*x^(3/2) + (2*b*(
b^2 + 6*a*c)*x^(7/2))/7 + (6*c*(b^2 + a*c)*x^(11/2))/11 + (2*b*c^2*x^(15/2)
)/5 + (2*c^3*x^(19/2))/19
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{7/2}}\,dx &= \int \left(\frac{a^3}{x^{7/2}}+\frac{3a^2b}{x^{3/2}}+3a\left(b^2+ac\right)\sqrt{x}+b\left(b^2+6ac\right)x^{5/2}+3c\left(b^2+ac\right)x^{9/2}+3bc^2x^{13/2}\right. \\ &= -\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2a\left(b^2+ac\right)x^{3/2}+\frac{2}{7}b\left(b^2+6ac\right)x^{7/2}+\frac{6}{11}c\left(b^2+ac\right)x^{11/2}+\frac{2}{5}bc^2x^{15}\end{aligned}$$

Mathematica [A] time = 0.08, size = 100, normalized size = 1.01

$$2\left(-\frac{a^3}{5x^{5/2}}-\frac{3a^2b}{\sqrt{x}}+\frac{3}{11}cx^{11/2}\left(ac+b^2\right)+\frac{1}{7}bx^{7/2}\left(6ac+b^2\right)+ax^{3/2}\left(ac+b^2\right)+\frac{1}{5}bc^2x^{15/2}+\frac{1}{19}c^3x^{19/2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(7/2),x]

[Out] 2*(-1/5*a^3/x^(5/2) - (3*a^2*b)/Sqrt[x] + a*(b^2 + a*c)*x^(3/2) + (b*(b^2 + 6*a*c)*x^(7/2))/7 + (3*c*(b^2 + a*c)*x^(11/2))/11 + (b*c^2*x^(15/2))/5 + (c^3*x^(19/2))/19)

IntegrateAlgebraic [A] time = 0.06, size = 93, normalized size = 0.94

$$\frac{2\left(-1463a^3-21945a^2bx^2+7315a^2cx^4+7315ab^2x^4+6270abcx^6+1995ac^2x^8+1045b^3x^6+1995b^2cx^8+1463bc^2x^{10}+385c^3x^{12}\right)}{7315x^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^3/x^(7/2),x]

[Out] (2*(-1463*a^3 - 21945*a^2*b*x^2 + 7315*a*b^2*x^4 + 7315*a^2*c*x^4 + 1045*b^3*x^6 + 6270*a*b*c*x^6 + 1995*b^2*c*x^8 + 1995*a*c^2*x^8 + 1463*b*c^2*x^10 + 385*c^3*x^12))/(7315*x^(5/2))

fricas [A] time = 1.92, size = 83, normalized size = 0.84

$$\frac{2\left(385\,c^3x^{12}+1463\,bc^2x^{10}+1995\left(b^2c+ac^2\right)x^8+1045\left(b^3+6\,abc\right)x^6-21945\,a^2bx^2+7315\left(ab^2+a^2c\right)x^4-1463\,a^3\right)}{7315\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="fricas")

[Out] 2/7315*(385*c^3*x^12 + 1463*b*c^2*x^10 + 1995*(b^2*c + a*c^2)*x^8 + 1045*(b^3 + 6*a*b*c)*x^6 - 21945*a^2*b*x^2 + 7315*(a*b^2 + a^2*c)*x^4 - 1463*a^3)/x^(5/2)

giac [A] time = 0.17, size = 88, normalized size = 0.89

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{6}{11}\,ac^2x^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{12}{7}\,abcx^{\frac{7}{2}}+2\,ab^2x^{\frac{3}{2}}+2\,a^2cx^{\frac{3}{2}}-\frac{2\left(15\,a^2bx^2+a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="giac")

[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 6/11*a*c^2*x^(11/2) + 2/7*b^3*x^(7/2) + 12/7*a*b*c*x^(7/2) + 2*a*b^2*x^(3/2) + 2*a^2*c*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)

maple [A] time = 0.01, size = 90, normalized size = 0.91

$$-\frac{2\left(-385c^{3}x^{12}-1463b\,c^{2}x^{10}-1995a\,c^{2}x^{8}-1995b^{2}c\,x^{8}-6270abc\,x^{6}-1045b^{3}x^{6}-7315a^{2}c\,x^{4}-7315a\,b^{2}x^{4}+21945a^{2}b\,x^{2}+1463a^{3}\right)}{7315x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(7/2),x)
```

```
[Out] -2/7315*(-385*c^3*x^12-1463*b*c^2*x^10-1995*a*c^2*x^8-1995*b^2*c*x^8-6270*a
*b*c*x^6-1045*b^3*x^6-7315*a^2*c*x^4-7315*a*b^2*x^4+21945*a^2*b*x^2+1463*a^
3)/x^(5/2)
```

maxima [A] time = 1.05, size = 82, normalized size = 0.83

$$\frac{2}{19}\,c^{3}x^{\frac{19}{2}}+\frac{2}{5}\,bc^{2}x^{\frac{15}{2}}+\frac{6}{11}\left(b^{2}c+ac^{2}\right)x^{\frac{11}{2}}+\frac{2}{7}\left(b^{3}+6\,abc\right)x^{\frac{7}{2}}+2\left(ab^{2}+a^{2}c\right)x^{\frac{3}{2}}-\frac{2\left(15\,a^{2}bx^{2}+a^{3}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*(b^2*c + a*c^2)*x^(11/2) + 2/
7*(b^3 + 6*a*b*c)*x^(7/2) + 2*(a*b^2 + a^2*c)*x^(3/2) - 2/5*(15*a^2*b*x^2 +
 a^3)/x^(5/2)
```

mupad [B] time = 0.04, size = 79, normalized size = 0.80

$$x^{7/2}\left(\frac{2\,b^{3}}{7}+\frac{12\,a\,c\,b}{7}\right)-\frac{\frac{2\,a^{3}}{5}+6\,b\,a^{2}\,x^{2}}{x^{5/2}}+\frac{2\,c^{3}\,x^{19/2}}{19}+\frac{2\,b\,c^{2}\,x^{15/2}}{5}+2\,a\,x^{3/2}\,\left(b^{2}+a\,c\right)+\frac{6\,c\,x^{11/2}\,\left(b^{2}+a\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(7/2),x)
```

```
[Out] x^(7/2)*((2*b^3)/7 + (12*a*b*c)/7) - ((2*a^3)/5 + 6*a^2*b*x^2)/x^(5/2) + (2
*c^3*x^(19/2))/19 + (2*b*c^2*x^(15/2))/5 + 2*a*x^(3/2)*(a*c + b^2) + (6*c*x
^(11/2)*(a*c + b^2))/11
```

sympy [A] time = 31.86, size = 124, normalized size = 1.25

$$-\frac{2a^{3}}{5x^{\frac{5}{2}}}-\frac{6a^{2}b}{\sqrt{x}}+2a^{2}cx^{\frac{3}{2}}+2ab^{2}x^{\frac{3}{2}}+\frac{12abcx^{\frac{7}{2}}}{7}+\frac{6ac^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{7}{2}}}{7}+\frac{6b^{2}cx^{\frac{11}{2}}}{11}+\frac{2bc^{2}x^{\frac{15}{2}}}{5}+\frac{2c^{3}x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(7/2),x)
```

```
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a**2*c*x**(3/2) + 2*a*b**2*x**(
3/2) + 12*a*b*c*x**(7/2)/7 + 6*a*c**2*x**(11/2)/11 + 2*b**3*x**(7/2)/7 + 6*
b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*x**(19/2)/19
```

3.829 $\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=389

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Rubi [A] time = 0.86, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1367, 1510, 298, 205, 208}

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-
b + Sqrt[b^2 - 4*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcT
anh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(
7/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c
])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3
/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2x^{3/2}}{3c} - \frac{2\,\text{Subst}\left(\int \frac{x^2(3a+3bx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{3c}$$
$$= \frac{2x^{3/2}}{3c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}}\right.}{c}$$
$$= \frac{2x^{3/2}}{3c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\,c^{3/2}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\right.}{}$$
$$= \frac{2x^{3/2}}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\left(b+\right.}{}$$

Mathematica [C] time = 0.05, size = 80, normalized size = 0.21

$$\frac{4x^{3/2} - 3\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4b\log\left(\sqrt{x}-\#1\right)+a\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*x^(3/2) - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b*Log
[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(6*c)
```

IntegrateAlgebraic [C] time = 0.13, size = 83, normalized size = 0.21

$$\frac{2x^{3/2}}{3c} - \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4b\log\left(\sqrt{x}-\#1\right)+a\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] +
b*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(2*c)
```

fricas [B] time = 11.73, size = 6649, normalized size = 17.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*
c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^
6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^
2*c^8 + 16*a^2*c^9)))*arctan(1/2*((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^
3*b^3*c^3 + 4*a^4*b*c^4 - (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3
*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*
b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*
c^16 - 64*a^3*c^17)))*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 -
62*a^13*b^6*c^3 + 46*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqr
t(1/2)*(a^7*b^17 - 17*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 9
24*a^11*b^9*c^4 - 1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 +
 12*a^15*b*c^8 - (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a
^10*b^8*c^10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 -
 64*a^14*c^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*
a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a
^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6
)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8
*a*b^2*c^8 + 16*a^2*c^9))) + (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*c^2 -
210*a^8*b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^6 - 4*
a^12*b*c^7 - (a^5*b^12*c^7 - 15*a^6*b^10*c^8 + 88*a^7*b^8*c^9 - 253*a^8*b^6
*c^10 + 362*a^9*b^4*c^11 - 224*a^10*b^2*c^12 + 32*a^11*c^13)*sqrt((b^12 - 1
0*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*
c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))
*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*
c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^
6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^
2*c^8 + 16*a^2*c^9)))/(a^7*b^12 - 10*a^8*b^10*c + 37*a^9*b^8*c^2 - 62*a^10*
b^6*c^3 + 46*a^11*b^4*c^4 - 12*a^12*b^2*c^5 + a^13*c^6)) - 12*c*sqrt(sqrt(1
/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*
b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^1
5 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))
*arctan(-1/2*((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*
c^4 + (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt((b^12 -
```

10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2
*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17))
)*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 - 62*a^13*b^6*c^3 + 46
*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqrt(1/2)*(a^7*b^17 - 1
7*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 924*a^11*b^9*c^4 - 10
78*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15*b*c^8 + (a^
7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a^10*b^8*c^10 + 997*a
^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 - 64*a^14*c^14)*sqrt(
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^
3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7
- 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*
a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b
^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*
c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b
^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c
^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c
^8 + 16*a^2*c^9))) + (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*c^2 - 210*a^8*
b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^6 - 4*a^12*b*c
^7 + (a^5*b^12*c^7 - 15*a^6*b^10*c^8 + 88*a^7*b^8*c^9 - 253*a^8*b^6*c^10 +
362*a^9*b^4*c^11 - 224*a^10*b^2*c^12 + 32*a^11*c^13)*sqrt((b^12 - 10*a*b^10
*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^
6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(x)
*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^
4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2
 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 -
12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 1
6*a^2*c^9))))/(a^7*b^12 - 10*a^8*b^10*c + 37*a^9*b^8*c^2 - 62*a^10*b^6*c^3
+ 46*a^11*b^4*c^4 - 12*a^12*b^2*c^5 + a^13*c^6)) - 3*c*sqrt(sqrt(1/2)*sqrt(
-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 +
 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 4
6*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^
2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/2*
sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^
4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17
*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^
5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a
^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b
^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a
*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b
^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^
2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^

```
10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*
c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 -
 a^8*c^3)*sqrt(x)) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a
*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b
^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c
 + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 +
 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9
- 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt
(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*
b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 -
8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4
*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^
9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) - 3*c*sqrt
(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*
b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)))*log(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*
b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7
+ (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b
^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4
*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^
5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*s
qrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4
- 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 6
4*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c
 + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt
((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a
^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c +
 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^
5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*s
qrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4
- 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 6
4*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^
```

```
14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 4
57*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 +
113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqr
t((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*
a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*
b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*
a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(
b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*
b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2
*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6
*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2
*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqr
t(x)) + 4*x^(3/2))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 19.14Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.06, size = 65, normalized size = 0.17

$$\frac{2x^{\frac{3}{2}}}{3c}-\frac{\left(\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6}b+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2}a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{2c\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7}c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/3/c*x^(3/2)-1/2/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R
=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,x^{\frac{3}{2}}}{3\,c}-\int\frac{bx^{\frac{5}{2}}+a\sqrt{x}}{c^{2}x^{4}+bcx^{2}+ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/c - integrate((b*x^(5/2) + a*sqrt(x))/(c^2*x^4 + b*c*x^2 + a*c)
, x)
```

mupad [B] time = 5.82, size = 12789, normalized size = 32.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^
3*c^5))/c^3 - (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*
a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6
*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6
*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^
2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^
3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*
a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11
 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (2
56*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/
4)*1i - (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*
b^3*c^5))/c^3 + (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(
4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b
^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b
^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c -
b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*
b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^
11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) -
(256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4
*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 2
80*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256
*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(
1/4)*1i)/((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^
5*b^3*c^5))/c^3 - (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112
```

*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(
-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a
*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3
*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*
c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)
+ (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-
(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 +
280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^
2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(2
56*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))
^(1/4) + (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5
*b^3*c^5))/c^3 + (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*
a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-
(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2
) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*
b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*
b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c -
b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) -
(256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 +
280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2
c^2(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(25
6*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^
(1/4) - (256*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7
- 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*2i + atan(((((
128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c
^3 - (256*x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2
)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96
*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 16
0*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a
^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^

7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (256*x^(1/2
)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)
^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*
c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 +
b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i - (
((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))
/c^3 + (256*x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
+ 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b
^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^
4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 +
96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 +
160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 +
a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*
c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (256*x^(1
/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^
2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^
3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*
a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11
+ b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i)/
((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5
))/c^3 - (256*x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c -
b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*
b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8
+ 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5
+ 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
+ a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^
8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (256*x^
(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c -
b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*
b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^
11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) +
(((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5)
)/c^3 + (256*x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
+ 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c -
b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b
^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 +
96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 +
160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^

(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
+ a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8
*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (256*x^(
1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b
^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4
*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^1
1 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (
256*(a^8*c - a^7*b^2))/c^3))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(
4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b
^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*2i + 2*atan(((((128*(51
2*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 - (x
^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*
c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
- 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^
9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c
^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3
*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(
1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16
*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (256*x^(1/2)*(
a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^
8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (((128*
(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 +
 (x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4
*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^
4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2)
- 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*
c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 -
 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (256*x^(1/2
)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)
^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*
c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 +

b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4))/((((1
28*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^
3 - (x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*
b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4
*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a
^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^
7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (256*x^(
1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b
^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4
*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^1
1 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i
+ (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^
5))/c^3 + (x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4
c(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 9
6*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 1
60*a^4*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^
5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c
^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 +
b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (2
56*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/
4)*1i + (256*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7
- 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + 2*atan(((((1
28*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^
3 - (x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*
b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4
*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/

2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a
^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^
7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (256*x^(
1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b
^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4
*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^1
1 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (
((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))
/c^3 + (x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a
^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*
a^4*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^
(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
+ a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8
*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (256*
x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*
c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4))
/((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^
5))/c^3 - (x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4
c(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 9
6*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 1
60*a^4*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^
5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c
^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 +
b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (2
56*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/
4)*1i + (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*
b^3*c^5))/c^3 + (x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5

```
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c
^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c
^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 - b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1
i + (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*
(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3
 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*
b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*
(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)
))^(1/4)*1i + (256*(a^8*c - a^7*b^2))/c^3))*(-(b^11 - b^6*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (2*x^(
3/2))/(3*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.830 $\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=385

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+$$

Rubi [A] time = 0.80, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\sqrt{x}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b - Sqrt[b
^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4
)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b +
Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*
c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\sqrt{x}}{c} - \frac{2\,\text{Subst}\left(\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}} \right.}{c}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\right.}{c\sqrt{-}}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b+\right.}{}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4b\log\left(\sqrt{x}-\#1\right)+a\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]-4\sqrt{x}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(-4*Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b
*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/c
```

IntegrateAlgebraic [C] time = 0.08, size = 83, normalized size = 0.22

$$\frac{2\sqrt{x}}{c} - \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4b\log\left(\sqrt{x}-\#1\right)+a\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b*Lo
g[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(2*c)
```

fricas [B] time = 4.48, size = 5319, normalized size = 13.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(-1/4*(sqrt(1/2)*(b^11 - 13
*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^
5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b
^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*
sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*x
 + 2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125
*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 + (b^11*c^5 - 15*a*b^9*c^6 + 85*a
^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^
4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) -
 2*sqrt(1/2)*(a*b^15 - 16*a^2*b^13*c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 +
 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5 + 224*a^7*b^3*c^6 - 32*a^8*b*c^7 + (a*b^
14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^
6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7*b^2*c^11 - 128*a^8*c^12)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt(1/
2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c
^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*
c^6 + 16*a^2*c^7)))/(a^5*b^8 - 6*a^6*b^6*c + 11*a^7*b^4*c^2 - 6*a^8*b^2*c^3
 + a^9*c^4)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
```

a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(sqrt(1/2)*(b
^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*
a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 3
52*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))*sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^
6*c^4)*x + 2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c
^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 - (b^11*c^5 - 15*a*b^9*c^
6 + 85*a^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt
(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^
2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b
^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))
/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) - 2*sqrt(1/2)*(a*b^15 - 16*a^2*b^13*
c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 + 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5
+ 224*a^7*b^3*c^6 - 32*a^8*b*c^7 - (a*b^14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*
b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7
*b^2*c^11 - 128*a^8*c^12)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 -
 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b
*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^5*b^8 - 6*a^
6*b^6*c + 11*a^7*b^4*c^2 - 6*a^8*b^2*c^3 + a^9*c^4)) - c*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(x) + (b^6 - 7*a*b^
4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*s
qrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10
- 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^
5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2
*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5

```
 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c + a
^3*c^2)*sqrt(x) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5
- 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c
^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a
*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))
*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(x) + (b^6 - 7*a*b^4*c + 13*a^2*
b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48
*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*
sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^
5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(
x) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6
 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(s
qrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16
*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)))) - 4*sqrt(x))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 14.51Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 64, normalized size = 0.17

$$\frac{\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4 b-a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{2c\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/c*x^(1/2)+1/2/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Root
Of(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 6.86, size = 10449, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (256
*x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4
 - 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (256*x^(1/2)*(
a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2
) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^
2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1
i - (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (256*x
^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^
```

2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b
^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 +
96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 -
 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*
a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 -
16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (256*x^(1/2)*(a^
4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)
/((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (256*x^(
1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
- 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2
c(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 1
28*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*
b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (256*x^(1/2)*(a^4*
b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9
 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((
512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (256*x^(1/2)*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*
b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^
4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^
2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
 - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^
6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (256*x^(1/2)*(a^4*b^4 +
 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^
4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)))*(-(b^9 +
 b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c -
 b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b
^4*c + 13*a^5*b^2*c^2))/c - (256*x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)

*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a
*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1
/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b
^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3
*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4
*c + 13*a^5*b^2*c^2))/c + (256*x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2
) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^
2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(
256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c
 - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4) + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9
 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b
^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c
 - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^
7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c
 + 13*a^5*b^2*c^2))/c - (256*x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(25
6*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c -
 b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))
/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(1/4) - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 -
 b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
 b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4) + (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13
*a^5*b^2*c^2))/c + (256*x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*
a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(
1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 +
b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5
*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)
^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*
a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*
(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))
^(1/4) + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3

- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i - 2*atan(((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (((

512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x^(1/2)*(-(b
^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3
*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^
3*c^5)*256i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a
^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*
c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (256*x^(1/2)*(a^4*b
^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 8
0*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)
^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(
-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*
a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - 2*atan(((((512*(a^3*b^6 - 4*a^6*c^3 - 7
*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^
(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256
*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/
4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 - b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c
))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((512*(a^3*b^6 - 4*a^6*c^3 - 7*
a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4
)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256
*a^3*b^2*c^8)))^(1/4)*1i - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c)
)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
- 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2
c(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a
^4*b^4*c + 13*a^5*b^2*c^2))/c - (x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -

```
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)
*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 - b^4*(
-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 -
 a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^
5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))
/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (((512*(a^3*b^6 - 4*a^6*c^3 - 7
*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^
(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256
*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/
4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 - b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(1/4)*1i - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c
))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(
256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^
(1/4) + (2*x^(1/2))/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.831 $\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan}{2^{3/4}c^{3/4}\sqrt{b^2}}$$

Rubi [A] time = 0.44, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1115, 1374, 298, 205, 208}

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c])) + ((-b + Sqrt
[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])
^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan
h[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/
4)*Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$
$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)+\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\Bigg(\int$$
$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\sqrt{c}}+\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\Bigg(\int \frac{}{\sqrt{-b+\sqrt{}}}}{\sqrt{2}\sqrt{}}$$
$$= -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Mathematica [C] time = 0.03, size = 48, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^3\log\left(\sqrt{x}-\#1\right)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1^3)/(b + 2*c*#1^4) & ]
/2
```

IntegrateAlgebraic [C] time = 0.09, size = 48, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^3\log\left(\sqrt{x}-\#1\right)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1^3)/(b + 2*c*#1^4) & ]
/2
```

fricas [B] time = 1.96, size = 4058, normalized size = 12.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*((b^4 -
 5*a*b^2*c + 4*a^2*c^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9
)))*sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x - 1/2*sqrt(1/2)*(a^3*b^7 - 6*a
^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 + (a^3*b^8*c^3 - 13*a^4*b^6*c^4 + 60
*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*
a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5))) + (a^2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^2 - 4*a^5*c^
3 + (a^2*b^7*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*c^5 - 16*a^5*b*c^6)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c
^9)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 4
8*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a^3*b
^4 - 2*a^4*b^2*c + a^5*c^2)) + 2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4
*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)))*arctan(-1/2*((b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^5*c^3 - 8*a*b^3*
c^4 + 16*a^2*b*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^
7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x
 - 1/2*sqrt(1/2)*(a^3*b^7 - 6*a^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 - (a^
3*b^8*c^3 - 13*a^4*b^6*c^4 + 60*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
```

```
- 64*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(sqrt(1/2)*sqrt(
-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))) + (a^2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^2
- 4*a^5*c^3 - (a^2*b^7*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*c^5 - 16*a^5*b*c^6)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^
4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
)))/(a^3*b^4 - 2*a^4*b^2*c + a^5*c^2)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*
a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2
 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^
6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 +
 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x
)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*
b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqr
t(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b
^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3
- 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3
*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^
2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c
^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
```

```
 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sq
rt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*
a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2
*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14
*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))
*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c
 - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

maple [C] time = 0.01, size = 45, normalized size = 0.14

$$\frac{\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^6 \ln\left(-\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right) + \sqrt{x}\right)}{4\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^7 c + 2\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 6.51, size = 8093, normalized size = 24.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan(((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 + b^2*(-(4*a*c -
b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c -
 b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5
- 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 + b^2*(-(4*a*c -
 b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5
 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*
b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b
^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c -
b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 -
 256*a^3*b^2*c^6)))^(1/4)*1i + (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) - (
-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^
6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*
(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*
a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b
^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a
^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-
(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*
b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6
*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/((x^(1/2)*(256*a^3*b^3
*c - 768*a^4*b*c^2) - (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3
+ 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*
c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(3
2768*a^5*c^5 - x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3
 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4
*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(
131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 -
 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - (
x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(
1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^
(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^
3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)
```

+ 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1
/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(
1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*
b^2*c^6)))^(1/4) + 256*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)
))^(1/4)*2i - atan(((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 - b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 - b^
2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a
c(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4
- 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 - b^2*
(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a
^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + (x^(1/2)*(256*a^3*b^3*c - 768*a^
4*b*c^2) - (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c
^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^
5 - x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b
^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*
c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*
b^2*c^4))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^
3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/((x^(1/2)*
(256*a^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 4
8*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(
32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6
)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/
(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^
6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a
^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)
))^(1/4) - (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 - b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 - b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*
c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a
^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 - b^2*(-(4*a*c

- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^
5 - 256*a^3*b^2*c^6)))^(1/4) + 256*a^4*b*c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5
)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^
5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*
a^3*b^2*c^6)))^(1/4)*2i - 2*atan(((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2)
+ (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 1
1*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a
*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/
2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*
a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 819
2*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^
4)*1i)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 -
 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) + (x^(1/2)*(256*a
^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(25
6*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3
/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(2
56*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(
1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i + 2048*a^3*
b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)
))^(1/4))/((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*(-(b^7 + b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*
c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a
^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7 + b^2*(-(
4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-
(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*
b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i - (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b
c^2) - (-(b^7 + b^2(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3
- 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 +
 x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3
 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6
 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 16384*a^4*
b^2*c^4)*1i)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*

```
b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8
*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + 256*a^
4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3
- 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - 2*atan(((x^(1/
2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2)
- 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2)
)/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*
c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2
*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i +
2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(
1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^
(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))^(1/4) + (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 - b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 - b^
2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a
*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4
- 65536*a^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7
- b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4
+ 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4))/((x^(1/2)*(256*a^3*b^3*c - 768
*a^4*b*c^2) + (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^
8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5
*c^5 - x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^
2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b
^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a
^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 1638
4*a^4*b^2*c^4)*1i)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 4
0*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7
 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i -
(x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^
3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)
^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5
)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)
*1i + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7 - b^2*(-(4*a*c - b^2
)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^
```

```
2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 2
56*a^3*b^2*c^6)))^(1/4)*1i + 256*a^4*b*c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^
3*b^2*c^6)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.832 $\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{}{\sqrt[4]{-}}\right.}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.40, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1115, 1374, 212, 208, 205}

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt
[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^
2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1
/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(
2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*
Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)+\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\Bigg(\int$$

$$= \frac{\sqrt{-b-\sqrt{b^2-4ac}}\ \text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}}+\frac{\sqrt{-b-\sqrt{b^2-4ac}}\ \text{Subst}\Bigg(}{\sqrt{}}$$

$$= \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\ \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\ \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-}}{}$$

Mathematica [C] time = 0.03, size = 46, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1\log\left(\sqrt{x}-\#1\right)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1)/(b + 2*c*#1^4) & ]/2
```

IntegrateAlgebraic [C] time = 0.08, size = 46, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1\log\left(\sqrt{x}-\#1\right)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1)/(b + 2*c*#1^4) & ]/2
```

fricas [B] time = 1.11, size = 2482, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)))*arctan(1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (b^7*c - 1
2*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b + (b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) + x)*sqrt(-(b + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*(b^4 - 8*a*b^2*
c + 16*a^2*c^2 - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqr
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x)*sqrt(-(b +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sq
rt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 2*sq
rt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)))*arctan(-1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5))*sqrt(sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) + x)*sqrt(-(b - (b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x)*sqrt(-(b - (b^
```

```
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt(-
(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 1/2*sqrt
(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b
 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(-(b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) +
sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b
^2*c^2 + 16*a^2*c^3)))*log(-(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/
2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

maple [C] time = 0.01, size = 45, normalized size = 0.14

$$\frac{\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^4 \ln\left(-\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right) + \sqrt{x}\right)}{4\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^7 c + 2\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 6.02, size = 8229, normalized size = 24.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2
 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c
^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6) - x^(1/2)*(65536*a^4*b*c^6 + 4
096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 1
6*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b
^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + (x^
(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2
*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 1
6*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 9830
4*a^3*b^4*c^5 - 393216*a^4*b^2*c^6) + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b
^5*c^4 - 32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b^5 + (-(4
*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 -
 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i)/((x^(1/2)*(51
2*a^3*c^4 - 256*a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
- 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
```

^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4
*c^5 - 393216*a^4*b^2*c^6) - x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 -
32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b
^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b^5 + (-(4*a*c - b^
2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6
*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - (x^(1/2)*(512*a^3*c^4 -
256*a^2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b
^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^
2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
c)/(32(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*
c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 39321
6*a^4*b^2*c^6) + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^
3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3
/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan(((x^(1/2)*(512*a^3*c^4 - 256
*a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c
^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4
)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a
^4*b^2*c^6)*1i + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^
3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3
/4)*1i - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + (x^(1/2)*(512*a^3*c^4 - 256*a^
2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/
(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)
))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(3
2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))
^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*
b^2*c^6)*1i - x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c
^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8
*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)
*1i - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4))/((x^(1/2)*(512*a^3*c^4 - 256*a^2*b
^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32
*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^

(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1
/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2
*c^6)*1i + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5)
)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c
+ 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i
 - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - (x^(1/2)*(512*a^3*c^4 - 256*a^2*b
^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32
*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^
(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1
/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2
*c^6)*1i - x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5)
)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c
+ 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i
 - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
 + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a
^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - atan(((x^(1/2)*(512*a^3*c^4 - 256*a
^2*b^2*c^3) - ((x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3
*c^5) + (-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1
/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2
c^6))(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/
4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4
*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + (x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^
3) - ((x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) - (
-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 2
56*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(5242
88*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6))*(-
(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 25
6*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) - 2048
*a^3*b*c^4 + 512*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
 256*a^3*b^2*c^4)))^(1/4))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 25
6*a^3*b^2*c^4)))^(1/4)*1i)/((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) - ((x^
(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) + (-(b^5 - (

-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^
5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^
7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6))*(-(b^5 - (-
(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5
 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^
4 - 512*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*
b^2*c^4)))^(1/4))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^
3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2
c^4)))^(1/4) - (x^(1/2)(512*a^3*c^4 - 256*a^2*b^2*c^3) - ((x^(1/2)*(65536
*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) - (-(b^5 - (-(4*a*c - b^
2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6
*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2
*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6))*(-(b^5 - (-(4*a*c - b^2
)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) - 2048*a^3*b*c^4 + 512*a^2*
b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*
c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*
2i + 2*atan(((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + ((x^(1/2)*(65536*a^
4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) - (-(b^5 - (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^
2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^
6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6)*1i)*(-(b^5 - (-(4*a*c - b^2
)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i + 2048*a^3*b*c^4 - 512*a
^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)^(1/4)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)^(1/4) + (x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + ((x^(1/2)*(65536*a^4*b
*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) + (-(b^5 - (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c
^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6)*1i)*(-(b^5 - (-(4*a*c - b^2)^5
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2
 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 2048*a^3*b*c^4 + 512*a^2*
b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4))/((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + ((x^(1/2)*(65536*a^4*b*c^

```
6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) - (-(b^5 - (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4
+ 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(
1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i + 2048*a^3*b*c^4 - 512*a^2*b^3
*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*1i - (x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + ((x^(1/2)*(65536*a^4*b*c^
6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) + (-(b^5 - (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4
+ 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(
1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 2048*a^3*b*c^4 + 512*a^2*b^3
*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*1i))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.833 $\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Rubi [A] time = 0.37, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1115, 1375, 298, 205, 208}

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((2^(1/4)*c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (2^(1/4)*c^
(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sq
rt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (2^(1/4)*c^(1/4)*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*
c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (2^(1/4)*c^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(\sqrt{2}\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{\left(\sqrt{2}\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\ldots}\right.}{\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt[4]{2}\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{2}\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{2}\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\ldots}{\sqrt[4]{-b-\ldots}}\right.}{\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2\ldots}}}$$

Mathematica [C] time = 0.03, size = 47, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^5) & ]/2
```

IntegrateAlgebraic [C] time = 0.08, size = 47, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^5) & ]/2
```

fricas [B] time = 0.90, size = 2769, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)))*arctan(((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(c^2*x - 1/2*sqrt
(1/2)*(b^3*c - 4*a*b*c^2 - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*
a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-
(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a*b^4*c - 8*a^2*b^2*c^2 +
16*a^3*c^3)*sqrt(x)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))/c) + 2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)))*arctan(((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt
(c^2*x - 1/2*sqrt(1/2)*(b^3*c - 4*a*b*c^2 + (a*b^6*c - 12*a^2*b^4*c^2 + 48*
a^3*b^2*c^3 - 64*a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a
*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b - (a*b
^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*
```

```
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/c) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b
^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*s
qrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a
*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*
a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqr
t(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sq
rt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*
c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c
^2)) + c*sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^
2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

maple [C] time = 0.01, size = 45, normalized size = 0.14

$$\frac{\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{4\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a),x)
[Out] 1/2*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 5.31, size = 6133, normalized size = 18.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2 + c*x^4),x)
[Out] 2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(
3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 - x^(1/2)*(-(b^5
 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^
5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^
4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i)*1i - 2
56*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b
^2*c^3)))^(1/4) - ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b
^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^
2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x^(1
/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^
8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*
(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*
1i)*1i + 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
 256*a^4*b^2*c^3)))^(1/4))/(((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
```

^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*
c^5 - x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3
)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3
*b^2*c^6)*1i)*1i - 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - 256*a*c^5 + ((-(b^5 - (-(4*a*c - b
^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*
b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a
^3*b*c^6 - 16384*a^2*b^3*c^5 + x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*
b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960
*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i)*1i + 256*a*b*c^5*x^(1/2))*(-(b^5 - (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^
4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i))*(-(b^5 -
(-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c
^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) - atan((((-(b
^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*
a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*
b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x^(1/2)*(-(b^5 - (-(4*a*c -
b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^
2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*
a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)) - 256*a*b*c^5*x^(1/2))
*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 +
256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i
- ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8
+ 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(
2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 - x^(1/2)*(-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4
- 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7
- 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)) + 256*a*b*c^5*x
^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1
/4)*1i)/(256*a*c^5 + ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*
a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4
*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x
^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a
*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/
4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^
6)) - 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 25
6*a^4*b^2*c^3)))^(1/4) + ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256
*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5

```
 - x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(3
2*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))
^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^
2*c^6)) + 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
- 256*a^4*b^2*c^3)))^(1/4)))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
256*a^4*b^2*c^3)))^(1/4)*2i - atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16
*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^
4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 + x^(1/2
)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8
+ 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(1
31072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6) -
16384*a^2*b^3*c^5) - 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
 + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a
^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96
*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 -
 x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(
1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*
c^6) - 16384*a^2*b^3*c^5) + 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i)/(256*a*c^5 + ((-(b^5 + (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4
- 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4
+ 32768*a^3*b*c^6 + x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^
2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^
5 - 131072*a^3*b^2*c^6) - 16384*a^2*b^3*c^5) - 256*a*b*c^5*x^(1/2))*(-(b^5
+ (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5
*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + ((-(b^5 +
 (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*
c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*
c^4 + 32768*a^3*b*c^6 - x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*
b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^
4*c^5 - 131072*a^3*b^2*c^6) - 16384*a^2*b^3*c^5) + 256*a*b*c^5*x^(1/2))*(-(
b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256
*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)))*(-(b^
5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a
^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*2i + 2*at
an((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b
^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)
*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1
```

```
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 9
6*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 +
 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i - 16384*a^2*b^3*c^5)*1i - 256*a
*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c
^3)))^(1/4) - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^
3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 + x^(1/2)*(-(b^5 + (-(4*a*c -
b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2
*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a
*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i - 16384*a^2*b^3*c^5)*
1i + 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256
*a^4*b^2*c^3)))^(1/4))/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*
a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - x^(1/2)*(-(b^5 + (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^
4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^
7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i - 16384*a^2
*b^3*c^5)*1i - 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4
*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - 256*a*c^5 + ((-(b^5 + (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*
c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b
*c^6 + x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^
3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^
3*b^2*c^6)*1i - 16384*a^2*b^3*c^5)*1i + 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*
a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 -
16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i))*(-(b^5 + (-(4
*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 -
 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.834 $\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=331

$$\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tanh^{-1}\cdots}{\sqrt{b^2-4ac}\left(\sqrt{\cdots}\right.}$$

Rubi [A] time = 0.42, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1115, 1347, 212, 208, 205}

$$\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/
4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[
b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (2^(3/4)*c^(3/4)*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}\,x \cdots}\right.}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-\cdots}}}$$

$$= \frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{2^{3/4}c^{3/4}\tanh \cdots}{\sqrt{b^2-4ac}\left(- \cdots\right.}$$

Mathematica [C] time = 0.03, size = 49, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ]/2
```

IntegrateAlgebraic [C] time = 0.05, size = 49, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ]/2
```

fricas [B] time = 3.46, size = 4045, normalized size = 12.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(-1/4*(sqrt(
1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (a^3*b^8 - 14*a^4*b
^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(
4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x + 2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 21*a
^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 - (a^3*b^9 - 13*a^4*b^7*c + 60*a^5*
b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + 2*sqrt(1/2)*(b
^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5 - (a^3
*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c
^5 - 128*a^8*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(x)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 -
8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b
```

```
^2*c^4 + a^2*c^5)) + 2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))
)*arctan(1/4*(sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 +
(a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)))*sqrt(4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x + 2*sqrt(1/2)*(b^8
 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 + (a^3*b^9 - 13*
a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
 - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + 2*sqrt(1/2)*(b^9
*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5 + (a^3*b
^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5
 - 128*a^8*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
- (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b^2
*c^4 + a^2*c^5)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))
)*log(-2*(b^2*c - a*c^2)*sqrt(x) + (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*
a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^
4*b^2*c + 16*a^5*c^2)))) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 -
12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) - (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)))) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
- (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
```

```
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) + (b^4 - 5*a*b^2*c + 4*a^2
*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 -
3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c
 + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(s
qrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x)), x)
```

maple [C] time = 0.01, size = 42, normalized size = 0.13

$$\frac{\ln\left(-\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right) + \sqrt{x}\right)}{4\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^7 c + 2\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a),x)
[Out] 1/2*sum(1/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{x}}{a} - \int \frac{cx^{\frac{7}{2}} + bx^{\frac{3}{2}}}{acx^4 + abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

[Out] 2*sqrt(x)/a - integrate((c*x^(7/2) + b*x^(3/2))/(a*c*x^4 + a*b*x^2 + a^2), x)

mupad [B] time = 6.26, size = 10401, normalized size = 31.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)*(a + b*x^2 + c*x^4)),x)

[Out] - atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) + 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i)/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 16384

0*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a
^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*
a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) + 512*c
^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b
^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) + ((-(b^7 +
b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c -
a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c +
96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b
^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^
5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6
*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4
*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096*b^7*c^4 -
45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4
*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b
^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c
^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*
a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32
*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))
)^(1/4)*2i - atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 +
256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(204
8*a*c^7 - 512*b^2*c^6 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c
^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b
^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)
*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^
6) + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^
2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b
^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) + 512*c^7*x
^(1/2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4
 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 - b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +
a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^
7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5
*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*
b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096*b^7*c^4 - 4
5056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*
a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4

*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^
4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 - b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^
2 - 256*a^6*b^2*c^3)))^(1/4)*1i)/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/
(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^
3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/
2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1
/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b
^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 39
3216*a^3*b^3*c^6) + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*
c^7 + 163840*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b
*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3
*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/
4)) + 512*c^7*x^(1/2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3
 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8
 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) +
 ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 1
1*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a
^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*
c^6 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 -
 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 -
 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096
*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7
 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
 + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 - b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +
a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^
6*b^2*c^3)))^(1/4)*2i - 2*atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)
))^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2
c^3)))^(1/4)(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 3932
16*a^3*b^3*c^6)*1i + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b
*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^
3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3

```
/4)*1i)*1i - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^
3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(
a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^
(1/4) - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4
 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 -
2048*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^
7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^
7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i - x^(
1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^
6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16
*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i)*1i + 512*c^7*x^(
1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 -
16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 + b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(
-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5
*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 + b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c -
a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7
 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i + x^(1/2)*(4096*b^7*c^4 - 450
56*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a
*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*
c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i)*1i - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*
a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4
*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^
4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^
2*c^3)))^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)
^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5
)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5
+ 393216*a^3*b^3*c^6)*1i - x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608
*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 4
8*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(
32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3
)))^(3/4)*1i)*1i + 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))
/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c
^3)))^(1/4)*1i))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*
```

a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256
*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) - 2*ata
n((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16
*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 - 2048*
a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4
 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4
 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i + x^(1/2)*
(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(
-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*
b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i)*1i - 512*c^7*x^(1/2))
(-(b^7 - b^2(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11
*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^
4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) - ((-(b^7 - b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a
c - b^2)^5)^(1/2))/(32(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*
c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 - b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(
-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5
*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98
304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i - x^(1/2)*(4096*b^7*c^4 - 45056*a*
b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b
^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c -
b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 -
 256*a^6*b^2*c^3)))^(3/4)*1i)*1i + 512*c^7*x^(1/2))*(-(b^7 - b^2*(-(4*a*c -
 b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2
 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a
^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*
(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))
^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c
^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216
*a^3*b^3*c^6)*1i + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c
^7 + 163840*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*
c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*
b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4
)*1i)*1i - 512*c^7*x^(1/2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^
3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1
/4)*1i + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^

```
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 -
 2048*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^
2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a
^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b
^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i - x^
(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c
^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
- 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 1
6*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i)*1i + 512*c^7*x^
(1/2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 -
 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i))*(-(b^7 - b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*
a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.835 $\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=371

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\right.}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-}}$$

Rubi [A] time = 0.57, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1368, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*
c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^
(1/4)) + (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{a\sqrt{x}} + \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-b-cx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)}{}$$
$$= -\frac{2}{a\sqrt{x}} + \frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\,a} - \frac{\left(\sqrt{c}\left(1-\right.\right.}{}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{}}{\sqrt[4]{-b+\sqrt{}}}\right.}{2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.05, size = 78, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4}{\sqrt{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(4/Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c*
Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/a
```

IntegrateAlgebraic [C] time = 0.12, size = 81, normalized size = 0.22

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{2a}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(a*Sqrt[x]) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c*L
og[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(2*a)
```

fricas [B] time = 4.52, size = 5384, normalized size = 14.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(1/2*((b^6 - 7*a*b^4*c +
13*a^2*b^2*c^2 - 4*a^3*c^3 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((b^8*c^8 - 6*a*b^6*c^9 +
11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/2*sqrt(1/2)*(b^13*c^5 -
13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 + 175*a^4*b^5*c^9 - 79*a^5
*b^3*c^10 + 12*a^6*b*c^11 + (a^5*b^12*c^5 - 16*a^6*b^10*c^6 + 100*a^7*b^8*c
^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b^2*c^10 + 64*a^11*c^11)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c
+ 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))) - (b^1
0*c^4 - 10*a*b^8*c^5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4
*a^5*c^9 + (a^5*b^9*c^4 - 11*a^6*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7
+ 16*a^9*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(x)
)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 -
 6*a^3*b^2*c^8 + a^4*c^9)) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-
1/2*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (a^5*b^5 - 8*a^6*b^3*c
 + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((
b^8*c^8 - 6*a*b^6*c^9 + 11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/
2*sqrt(1/2)*(b^13*c^5 - 13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 +
175*a^4*b^5*c^9 - 79*a^5*b^3*c^10 + 12*a^6*b*c^11 - (a^5*b^12*c^5 - 16*a^6*
b^10*c^6 + 100*a^7*b^8*c^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b
^2*c^10 + 64*a^11*c^11)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
```

```
c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*
c + 16*a^7*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a
^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))) - (b^10*c^4 - 10*a*b^8*c^
5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4*a^5*c^9 - (a^5*b^9
*c^4 - 11*a^6*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7 + 16*a^9*b*c^8)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c +
16*a^7*c^2))))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^
4*c^9)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*
c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*
a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5
 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^
2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*s
qrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c
 + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a
^2*c^6)*sqrt(x)) + a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2
+ (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 -
 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(-1/2*sqrt(1/2)*(
b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32
*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 +
352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8
*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^
4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
```

```
2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b
^2*c^5 + a^2*c^6)*sqrt(x)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12
*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sq
rt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3
*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b
^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a
^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^
4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) + a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*l
og(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 1
28*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 -
 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b
*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*
c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))
 + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) + 4*sqrt(x))/(a*x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 7.01Unable to convert to re
al 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 65, normalized size = 0.18

$$-\frac{\left(\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2 b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{2a\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8
*c+_Z^4*b+a))-2/a/x^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{a\sqrt{x}}-\int\frac{cx^{\frac{5}{2}}+b\sqrt{x}}{acx^4+abx^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/(a*sqrt(x)) - integrate((c*x^(5/2) + b*sqrt(x))/(a*c*x^4 + a*b*x^2 + a^2
), x)
```

mupad [B] time = 5.74, size = 10573, normalized size = 28.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] 2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
+ 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^
9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*
b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c
^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*
a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^
8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i +
 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4
 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 + b^4
```

*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)
))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800
*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^
6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i - 256*a^11*b*c^8*x^(1/
2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*
a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*
b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4
)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*
b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*
c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16
*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 32768
0*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^
4*c^6 - 114688*a^14*b^2*c^7)*1i + 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4
*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^
2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^
(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8
*b^2*c^3)))^(1/4)*1i + ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4
 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8
 + x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5
*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*
a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
 + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b
^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i
 + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^1
4*b^2*c^7)*1i - 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/
2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^
8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*
1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2
c(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96
*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - atan((((-(b^9 - b^4*(-(4*a*c - b^

2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)
))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(1310
72*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6
- 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^1
3*b^4*c^6 - 114688*a^14*b^2*c^7) + 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^
8*b^2*c^3)))^(1/4)*1i - ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8
- x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*
c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*
b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) +
2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*
b^2*c^7) - 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i)/(
((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12
0*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7
*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256
*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6
*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 225
28*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7) + 256*a^11*b*c^
8*x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*
c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9 - b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)
))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) +

80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(1310
72*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6
- 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^1
3*b^4*c^6 - 114688*a^14*b^2*c^7) - 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^
8*b^2*c^3)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a
*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 1
6*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i - atan((((-(b^9
+ b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^
3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2
 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 + b^4*(-(4*a*c
 - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2
c^3)))^(1/4)(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 2
04800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*
b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7) + 256*a^11*b*c^8*x^(1/2
))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a
^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9 + b^4*(-(4*a*c - b^2)^5)
^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c
 - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^
5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3
/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^
4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^
9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^
16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327
680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4
*c^6 - 114688*a^14*b^2*c^7) - 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c
 - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2
*c^3)))^(1/4)*1i)/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*
b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16
*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^
(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2

- 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7) + 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7) - 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i + 2*atan((((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i + 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^

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2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2
*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 2
04800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^
12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i - 256*a^11*b*c^8*
x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
+ 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))/(((-(b^9 - b^4*(-(4*a*c - b^2)
^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*
a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*
(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))
^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256
*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072
*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 -
327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^
13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i + 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2
) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9
*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15
*c^8 + x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^
2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
 + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*
b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a
^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^
7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 11468
8*a^14*b^2*c^7)*1i - 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5
*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*
a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
 + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - 2/(a*x^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.836 $\int \frac{1}{x^{5/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=371

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\ldots\right)}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\ldots$$

Rubi [A] time = 0.51, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1368, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*
a*c])^(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c]
)^(3/4)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x]
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4
))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{3ax^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{-3b-3cx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{3a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\cdots}{\cdots}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{a\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(c\left(1-\cdots\right.\right.}{\cdots}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{-b+\cdots}}\right)}{\sqrt[4]{2}\,a\left(-b+\sqrt{b^2-4ac}\right)^{3\cdots}}$$

Mathematica [C] time = 0.05, size = 82, normalized size = 0.22

$$-\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+\frac{4}{x^{3/2}}}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*(4/x^(3/2) + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] +
c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

IntegrateAlgebraic [C] time = 0.08, size = 85, normalized size = 0.23

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{2a}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*a*x^(3/2)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c
*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(2*a)
```

fricas [B] time = 9.25, size = 6671, normalized size = 17.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/6*(12*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*
a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^
6*b^2*c^6 - 16*a^7*c^7 + (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a
^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(4*(b^1
2*c^4 - 10*a*b^10*c^5 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 -
12*a^5*b^2*c^9 + a^6*c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^
14*c^2 - 546*a^3*b^12*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6
*b^6*c^6 - 592*a^7*b^4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 + (a^7*b^15 - 19*a
^8*b^13*c + 148*a^9*b^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*
a^12*b^5*c^5 + 928*a^13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c
^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7
 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a
^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4
*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^
2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 -
7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^
4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c
^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) + 2*sqrt(1/2)*(b^
20*c^2 - 21*a*b^18*c^3 + 188*a^2*b^16*c^4 - 935*a^3*b^14*c^5 + 2821*a^4*b^1
2*c^6 - 5292*a^5*b^10*c^7 + 6083*a^6*b^8*c^8 - 4071*a^7*b^6*c^9 + 1449*a^8*
b^4*c^10 - 248*a^9*b^2*c^11 + 16*a^10*c^12 + (a^7*b^17*c^2 - 22*a^8*b^15*c^
3 + 204*a^9*b^13*c^4 - 1032*a^10*b^11*c^5 + 3075*a^11*b^9*c^6 - 5417*a^12*b
^7*c^7 + 5324*a^13*b^5*c^8 - 2480*a^14*b^3*c^9 + 320*a^15*b*c^10)*sqrt((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5
*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c
^3)))*sqrt(x)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*
b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12
```

*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*
a^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b
*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a
^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8
*b^2*c + 16*a^9*c^2)))/(b^12*c^7 - 10*a*b^10*c^8 + 37*a^2*b^8*c^9 - 62*a^3*
b^6*c^10 + 46*a^4*b^4*c^11 - 12*a^5*b^2*c^12 + a^6*c^13)) - 12*a*x^2*sqrt(s
qrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 -
 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c
^2)))*arctan(1/4*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^
3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^
7 - (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^1
1*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*
a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^1
5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(4*(b^12*c^4 - 10*a*b^10*c^5
 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 - 12*a^5*b^2*c^9 + a^6*
c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^14*c^2 - 546*a^3*b^12
*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6*b^6*c^6 - 592*a^7*b^
4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 - (a^7*b^15 - 19*a^8*b^13*c + 148*a^9*b
^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*a^12*b^5*c^5 + 928*a^
13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^
15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2
*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2
*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3))
)/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^
5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*s
qrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4
- 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 6
4*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c
 + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt
((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a
^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) + 2*sqrt(1/2)*(b^20*c^2 -
21*a*b^18*c^3 + 188*a^2*b^16*c^4 - 935*a^3*b^14*c^5 + 2821*a^4*b^12*c^6 - 5
292*a^5*b^10*c^7 + 6083*a^6*b^8*c^8 - 4071*a^7*b^6*c^9 + 1449*a^8*b^4*c^10
- 248*a^9*b^2*c^11 + 16*a^10*c^12 - (a^7*b^17*c^2 - 22*a^8*b^15*c^3 + 204*a
^9*b^13*c^4 - 1032*a^10*b^11*c^5 + 3075*a^11*b^9*c^6 - 5417*a^12*b^7*c^7 +
5324*a^13*b^5*c^8 - 2480*a^14*b^3*c^9 + 320*a^15*b*c^10)*sqrt((b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqr
t(x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 +

(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c
 + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a
^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^
2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 -
 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c +
16*a^9*c^2)))/(b^12*c^7 - 10*a*b^10*c^8 + 37*a^2*b^8*c^9 - 62*a^3*b^6*c^10
+ 46*a^4*b^4*c^11 - 12*a^5*b^2*c^12 + a^6*c^13)) - 3*a*x^2*sqrt(sqrt(1/2)*s
qrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2
*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(
-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x) + (b^9 - 9*a*b
^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^
4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6
- 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^
7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*
a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^
4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b
^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^2*s
qrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*
b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*
a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x)
 - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*
b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c
 + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt
(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*
b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^
4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)
))) - 3*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a
^3*c^5)*sqrt(x) + (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^
4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5
*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c
^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3

```
- (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)))) + 3*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2
*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2
*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3))
)/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*
a^2*b^2*c^4 - a^3*c^5)*sqrt(x) - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3
*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10
*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*
c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2
 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b
^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 4*sqrt(x))/(a*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 13.91Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 64, normalized size = 0.17

$$\frac{\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4 c-b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{2a\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}-\frac{2}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z
^4*b+a))-2/3/a/x^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(3\,b\sqrt{x}+\frac{a}{x^{\frac{3}{2}}}\right)}{3\,a^2}+\int\frac{bcx^{\frac{7}{2}}+\left(b^2-ac\right)x^{\frac{3}{2}}}{a^2cx^4+a^2bx^2+a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b*sqrt(x) + a/x^(3/2))/a^2 + integrate((b*c*x^(7/2) + (b^2 - a*c)*x
^(3/2))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

mupad [B] time = 8.64, size = 16557, normalized size = 44.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 +
 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(
(x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 2498
56*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 -
 a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*
c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^1
7*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917
504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4
*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 9
6*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c
^7 + 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + (x^(1/2)*(512*a^10*c^1
0 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c
^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c
- b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a
*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c
 + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8
+ 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^1
4*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*
```

b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2

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*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*
c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
- 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^
2 - 256*a^10*b^2*c^3)))^(3/4) + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^
10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*
b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4
 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*2i + atan(((x^
(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^
(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
+ a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11
*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*
(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b
^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 11
2*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*
(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1
/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*
a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8
192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*
b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a
*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4
*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072
*a^10*b^3*c^8))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a
^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + (x^(1/2)*(512*a^10*c^10 - 256*a
^9*b^2*c^9) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a
^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)
^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-
(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^
11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7
) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 1
5*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b
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^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8

*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-
(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231
*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^
9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a
^10*b^2*c^3)))^(3/4) + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^
8))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 1
5*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b
^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(
4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8
*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*2i - 2*atan(((x^(1/2)*(
512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c
^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)
^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 -
16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680
*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6
 - 491520*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^1
3*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7
)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c
^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) -
 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c -
 b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2
 - 256*a^10*b^2*c^3)))^(3/4)*1i - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*
a^10*b^3*c^8)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2
)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96
*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + (x^(1/2)*(512*a^10*c^10 - 256*a^
9*b^2*c^9) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*
b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^1
1*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7)
 + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
- 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15

*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^
2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^1
4*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-
(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 +
280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^
2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a
^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))
^(3/4)*1i + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8)*1i)*(-(b
^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a
^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*
c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(
1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^1
0*b^2*c^3)))^(1/4))/((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 +
b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^
5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6
*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))
/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
c^3)))^(1/4)((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12
*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*
a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 28
0*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c
^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*
b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1
/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^1
5*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2)
- 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3
c^3(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^
5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4
- 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 4096*a^11*
b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8)*1i)*(-(b^11 + b^6*(-(4*a*c - b
^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4
*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 2
56*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i
- (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 + b^6*(-(4*a*c - b^2
)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3
*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256
*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(
1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a
^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2)
- 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3
c^3(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^
5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4

- 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^
8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*
a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2
)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96
*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 4096*a^11*b*c^9 + 512*a^9*b^5
*c^7 - 3072*a^10*b^3*c^8)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*
a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-
(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2
) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^
8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i))*(-(b^11 + b^6*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 +
280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2
c^2(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(1/4) - 2*atan(((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 - b^6*
(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3
 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*
b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*
(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)
))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*
c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 +
 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(
524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4
*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 11
2*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*
(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1
/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*
a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 4096*a^11*b*c^9
 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^
11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + (x^(1/
2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a
^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(32
7680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*
c^6 - 491520*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a

^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(
4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8
*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192
*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2
*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a
c - b^2)^5)^(1/2))/(32(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4
*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3
072*a^10*b^3*c^8)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c -
 b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*
b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c
 + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))/((x^(1/2)*(512*a^10*c^10 - 25
6*a^9*b^2*c^9) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 8
6*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)
^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*
a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096
*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*
c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*
c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
- 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^
2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496
*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 - b^
6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c
^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^
2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^
3)))^(3/4)*1i - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8)*1i)*
(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 2
31*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*
b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^
5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256
*a^10*b^2*c^3)))^(1/4)*1i - (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-
(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231
*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^
9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a
^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 532
48*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 - b^
6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c
^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^

```
2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^
3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491
520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^
11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 409
6*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8)*1i)*(-(b^11 - b^6*(-(4*
a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 28
0*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c
^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*
b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1
/4)*1i))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^
7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2
) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*
c^2 - 256*a^10*b^2*c^3)))^(1/4) - 2/(3*a*x^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.837 $\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=412

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\ldots}{\sqrt[4]{-\sqrt{b^2\ldots}}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Rubi [A] time = 0.98, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1368, 1504, 1510, 298, 205, 208}

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) + (c^(1/4)*(b - (b^2 - 2*a*c)/Sqrt[b
^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b -
(b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^
(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{-5b-5cx^4}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)}{5a}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-5\left(b^2-ac\right)-5bcx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{5a^2}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{\left(c\left(b+\right.\right.}{}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{\left(\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\,a^2} +$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a^2\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan}{2^{3/4}a^2\sqrt[4]{-b+}}$$

Mathematica [C] time = 0.07, size = 107, normalized size = 0.26

$$-\frac{-5\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)-ac\log\left(\sqrt{x}-\#1\right)+b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4a}{x^{5/2}}-\frac{20b}{\sqrt{x}}}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/10*((4*a)/x^(5/2) - (20*b)/Sqrt[x] - 5*RootSum[a + b*#1^4 + c*#1^8 & , (
b^2*Log[Sqrt[x] - #1] - a*c*Log[Sqrt[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)
/(b*#1 + 2*c*#1^5) & ])/a^2
```

IntegrateAlgebraic [C] time = 0.15, size = 109, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)-ac\log\left(\sqrt{x}-\#1\right)+b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{2a^2}-\frac{2\left(a-5bx^2\right)}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(a - 5*b*x^2))/(5*a^2*x^(5/2)) + RootSum[a + b*#1^4 + c*#1^8 & , (b^2*L
og[Sqrt[x] - #1] - a*c*Log[Sqrt[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#
1 + 2*c*#1^5) & ]/(2*a^2)
```

fricas [B] time = 32.95, size = 7995, normalized size = 19.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*(20*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 3
0*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((
b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 -
 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 -
12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c +
16*a^11*c^2)))*arctan(1/2*((b^11 - 11*a*b^9*c + 43*a^2*b^7*c^2 - 70*a^3*b^5
*c^3 + 41*a^4*b^3*c^4 - 4*a^5*b*c^5 - (a^9*b^6 - 10*a^10*b^4*c + 32*a^11*b^
2*c^2 - 32*a^12*c^3)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b
^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*
c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))
*sqrt((b^16*c^14 - 14*a*b^14*c^15 + 79*a^2*b^12*c^16 - 230*a^3*b^10*c^17 +
367*a^4*b^8*c^18 - 314*a^5*b^6*c^19 + 130*a^6*b^4*c^20 - 20*a^7*b^2*c^21 +
a^8*c^22)*x - 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^21*c^10 + 230*a^2*b^19*c^11
- 1311*a^3*b^17*c^12 + 4692*a^4*b^15*c^13 - 10947*a^5*b^13*c^14 + 16731*a^6
*b^11*c^15 - 16380*a^7*b^9*c^16 + 9711*a^8*b^7*c^17 - 3109*a^9*b^5*c^18 + 4
25*a^10*b^3*c^19 - 20*a^11*b*c^20 - (a^9*b^18*c^9 - 22*a^10*b^16*c^10 + 205
*a^11*b^14*c^11 - 1050*a^12*b^12*c^12 + 3206*a^13*b^10*c^13 - 5909*a^14*b^8
*c^14 + 6333*a^15*b^6*c^15 - 3580*a^16*b^4*c^16 + 880*a^17*b^2*c^17 - 64*a^
18*c^18)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 36
7*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^
8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(-(b^9
- 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*
a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*
a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7
*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c
^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))) - (b^19*c^7 - 18*a*b^17*c^8
+ 135*a^2*b^15*c^9 - 546*a^3*b^13*c^10 + 1287*a^4*b^11*c^11 - 1782*a^5*b^9*
c^12 + 1386*a^6*b^7*c^13 - 540*a^7*b^5*c^14 + 81*a^8*b^3*c^15 - 4*a^9*b*c^1
6 - (a^9*b^14*c^7 - 17*a^10*b^12*c^8 + 117*a^11*b^10*c^9 - 416*a^12*b^8*c^1
0 + 805*a^13*b^6*c^11 - 810*a^14*b^4*c^12 + 352*a^15*b^2*c^13 - 32*a^16*c^1
4)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*
```

b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^
18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(x))*sqrt(sqr
t(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c
^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a
^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^
6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b
^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))/(b^16*c^9
- 14*a*b^14*c^10 + 79*a^2*b^12*c^11 - 230*a^3*b^10*c^12 + 367*a^4*b^8*c^13
- 314*a^5*b^6*c^14 + 130*a^6*b^4*c^15 - 20*a^7*b^2*c^16 + a^8*c^17)) - 20*a
^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*
c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^
4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^
2)))*arctan(-1/2*((b^11 - 11*a*b^9*c + 43*a^2*b^7*c^2 - 70*a^3*b^5*c^3 + 41
*a^4*b^3*c^4 - 4*a^5*b*c^5 + (a^9*b^6 - 10*a^10*b^4*c + 32*a^11*b^2*c^2 - 3
2*a^12*c^3)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 +
 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8
*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt((b^
16*c^14 - 14*a*b^14*c^15 + 79*a^2*b^12*c^16 - 230*a^3*b^10*c^17 + 367*a^4*b
^8*c^18 - 314*a^5*b^6*c^19 + 130*a^6*b^4*c^20 - 20*a^7*b^2*c^21 + a^8*c^22)
*x - 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^21*c^10 + 230*a^2*b^19*c^11 - 1311*a^
3*b^17*c^12 + 4692*a^4*b^15*c^13 - 10947*a^5*b^13*c^14 + 16731*a^6*b^11*c^1
5 - 16380*a^7*b^9*c^16 + 9711*a^8*b^7*c^17 - 3109*a^9*b^5*c^18 + 425*a^10*b
^3*c^19 - 20*a^11*b*c^20 + (a^9*b^18*c^9 - 22*a^10*b^16*c^10 + 205*a^11*b^1
4*c^11 - 1050*a^12*b^12*c^12 + 3206*a^13*b^10*c^13 - 5909*a^14*b^8*c^14 + 6
333*a^15*b^6*c^15 - 3580*a^16*b^4*c^16 + 880*a^17*b^2*c^17 - 64*a^18*c^18)*
sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8
*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*
b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(-(b^9 - 9*a*b^7
*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*
c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*
c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7
+ a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^
9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c
 + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c
+ 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^
3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 +
a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*
b^4 - 8*a^10*b^2*c + 16*a^11*c^2))) - (b^19*c^7 - 18*a*b^17*c^8 + 135*a^2*b
^15*c^9 - 546*a^3*b^13*c^10 + 1287*a^4*b^11*c^11 - 1782*a^5*b^9*c^12 + 1386
*a^6*b^7*c^13 - 540*a^7*b^5*c^14 + 81*a^8*b^3*c^15 - 4*a^9*b*c^16 + (a^9*b^
14*c^7 - 17*a^10*b^12*c^8 + 117*a^11*b^10*c^9 - 416*a^12*b^8*c^10 + 805*a^1
3*b^6*c^11 - 810*a^14*b^4*c^12 + 352*a^15*b^2*c^13 - 32*a^16*c^14)*sqrt((b^
16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 3

```
14*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12
*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(
-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^
4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2
- 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 -
20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*
a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))))/(b^16*c^9 - 14*a*b^14
*c^10 + 79*a^2*b^12*c^11 - 230*a^3*b^10*c^12 + 367*a^4*b^8*c^13 - 314*a^5*b
^6*c^14 + 130*a^6*b^4*c^15 - 20*a^7*b^2*c^16 + a^8*c^17)) - 5*a^2*x^3*sqrt(
sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*
b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 7
9*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130
*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^2
0*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(1/2
*sqrt(1/2)*(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068
*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 26
4*a^8*b^2*c^8 - 16*a^9*c^9 - (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2
- 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6
)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b
^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^1
8*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqr
t(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*
b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^
2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6
- 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 6
4*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7
*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*
c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*
c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7
+ a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^
9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*
c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x)) + 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(
-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^
4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2
- 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 -
20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*
a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(-1/2*sqrt(1/2)*(b^
18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4
- 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8
- 16*a^9*c^9 - (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7
*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 -
 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a
^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^1
9*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*
b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b
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^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^
10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c
^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/
(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^
5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^
2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*
b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^
18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10
*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b
^2*c^10 + a^4*c^11)*sqrt(x)) - 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^
7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2
*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10
*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7
 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a
^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(1/2*sqrt(1/2)*(b^18 - 20*a*b^16*
c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a^5*b^8*
c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 +
(a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3 + 1204*a^1
3*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 - 14*a*b^14*c +
79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 13
0*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^
20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*
b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*
c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^
4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(
a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^
10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*
b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 -
 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a
^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^1
9*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^1
1*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c
^11)*sqrt(x)) + 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^
5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^
2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*
b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^
18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10
*b^2*c + 16*a^11*c^2)))*log(-1/2*sqrt(1/2)*(b^18 - 20*a*b^16*c + 168*a^2*b^
14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6
*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 + (a^9*b^13 - 19
*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 12
32*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^
2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6
- 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 6
4*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a
```

```
^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^1
6 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 31
4*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*
a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*
a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^
4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c +
 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 1
30*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a
^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8
*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x))
+ 4*(5*b*x^2 - a)*sqrt(x))/(a^2*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 14.71Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 82, normalized size = 0.20

$$\frac{\left(\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6} bc+\left(-ac+b^{2}\right)\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2}\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{2a^{2}\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7} c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3} b\right)}+\frac{2b}{a^{2}\sqrt{x}}-\frac{2}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a^2*sum((b*c*_R^6+(-a*c+b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R
=RootOf(_Z^8*c+_Z^4*b+a))-2/5/a/x^(5/2)+2*b/a^2/x^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\frac{5\,b}{\sqrt{x}}-\frac{a}{x^{\frac{5}{2}}}\right)}{5\,a^{2}}+\int\frac{bcx^{\frac{5}{2}}+\left(b^{2}-ac\right)\sqrt{x}}{a^{2}cx^{4}+a^{2}bx^{2}+a^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/5*(5*b/sqrt(x) - a/x^(5/2))/a^2 + integrate((b*c*x^(5/2) + (b^2 - a*c)*sq
rt(x))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

mupad [B] time = 6.48, size = 15149, normalized size = 36.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9
*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a
*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1
/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^
12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a
^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^
3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96
*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10
*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655
360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9
*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) + x
^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)
^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5
*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 1
5*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 1
6*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i + ((-(b^13 +
b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^
7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2
) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*
(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8
 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/
4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 14
4*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5
*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^
6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c +
 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b
^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 -
655360*a^27*b^2*c^8) - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^2
3*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*
b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a
```

^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i)/(256*a^20*c^12 - ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4) + ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8) - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c

^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5
+ a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*
a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*
b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*2i - (2/(5*a) - (2*b*x^2)/a^2)/x^(5/2)
+ atan((((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b
^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4
*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2
) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^
(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*
a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144
*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*
b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6
c(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c +
96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^
10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 6
55360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b
^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) +
x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^
2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b
^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c -
15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5
)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 -
16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i + ((-(b^13
- b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*
b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1
/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^
3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b
^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(
3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) +
144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a
^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^
2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*
b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c
+ 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23
*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7
- 655360*a^27*b^2*c^8) - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a
^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^2
1*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144
*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*
b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6
c(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c +
96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i)/(256*a^20*c^12 - ((-(b^13 -

b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b
^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/
2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3
*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^
8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3
/4)*(x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a
^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*
(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)
^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 -
256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^2
4*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8
) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^
23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21
*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*
a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b
^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-
(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 9
6*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4) + ((-(b^13 - b^8*(-(4*a*c - b^2)
^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5
*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 1
5*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^
(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 1
6*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c
^9 + x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a
^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*
(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)
^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 -
256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^2
4*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8
) - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*
a^24*b^5*c^7 - 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b
^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*
b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(
4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/
2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256
*a^12*b^2*c^3)))^(1/4)))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b
*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^
5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*
c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^1
1*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*2i - 2*atan((((-(b^13 + b^8*(-(4*a*c

- b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a
^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11
*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c
^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(
-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 3
90*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2
)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3
*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32
*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c
^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 29
9008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 131072*
a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6
- 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11
- 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^
6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 +
a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a
c - b^2)^5)^(1/2))/(32(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b
^4*c^2 - 256*a^12*b^2*c^3)))^(1/4) + ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2
) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 5
52*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^
4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) -
7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b
^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(
1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c
^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c
- b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) -
10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12
*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^
5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 2
048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*
b^5*c^7 - 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3
c^10))(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^
9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*
a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(
1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a
^12*b^2*c^3)))^(1/4))/(256*a^20*c^12 + ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1
/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 -
552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*
b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2)
- 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10
*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 + b^8

*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c
^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) -
 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(
4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 +
256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*
(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^
6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 131072*a^26*b*c^9 +
 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^2
4*b^5*c^7 + 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b
^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*
b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(
4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256
*a^12*b^2*c^3)))^(1/4)*1i - ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a
^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^
3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96
*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b
^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*
a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5
)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^
2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a
^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)
))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 29900
8*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 2048*a^21*
b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 -
 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(
-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 3
90*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2
)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3
*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32
*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c
^3)))^(1/4)*1i))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 1
15*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*
c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c -
b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^
2 - 256*a^12*b^2*c^3)))^(1/4) - 2*atan((((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(
1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4
- 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2
*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2)
 + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^1
0*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 - b^

8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*
c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2)
- 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-
(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 +
 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)
*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b
^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 131072*a^26*b*c^9
+ 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^
24*b^5*c^7 + 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*
b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2
*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-
(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1
/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5
)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 25
6*a^12*b^2*c^3)))^(1/4) + ((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6
*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*
c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*
a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(
-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a
^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^1
3 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^
3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^
(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*
c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9
*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))
^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*
a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 2048*a^21*b^
11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 3
93216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(
b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390
*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^
5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b
^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(
a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3
)))^(1/4))/(256*a^20*c^12 + ((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a
^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^
3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(
4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96
*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 - b^8*(-(4*a*c -
 b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^
4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*
c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2
)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^
4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^2

```
8*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696
320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 131072*a^26*b*c^9 + 2048*a^21*
b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 +
 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(
-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 3
90*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2
)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3
*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32
*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c
^3)))^(1/4)*1i - ((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 +
115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4
*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2
)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c -
 b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c
^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 - b^8*(
-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3
 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 1
7*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*
a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 25
6*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(1
31072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*
c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 2048*a^21*b^11*c^4 +
28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^2
5*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^
8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*
c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2)
- 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-
(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 +
 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)
*1i))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*
c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*
c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) +
 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/
2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^1
2*b^2*c^3)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.838 $\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=544

$$-\frac{bx^{3/2}}{2c\left(b^2-4ac\right)}+\frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\ldots$$

Rubi [A] time = 2.58, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1365, 1502, 1510, 298, 205, 208}

$$\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^{3/2}}{2c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x^(3/2))/(2*c*(b^2 - 4*a*c)) + (x^(7/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((3*b^3 - 20*a*b*c + (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*
a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b
^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)
^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b^3 - 20*a*b*c + (3*b^2 - 14*a
*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4
*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*
c])^(1/4)) + ((3*b^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTa
nh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^
(7/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^{14}}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{x^6\left(14a+3bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\mathrm{Subst}\left(\int \frac{x^2\left(9ab+3\left(3b^2-14ac\right)x^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right.}{6c\left(b^2-4ac\right)}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(3b^2-14ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{20abc}{\sqrt{b^2-4ac}}\right)\mathrm{Sub}}{4c\left(b^2-4\right.}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^2-14ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{20abc}{\sqrt{b^2-4ac}}\right)\mathrm{Subs}}{4\sqrt{2}\,c^{3/2}\left(b\right.}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(3b^3-20abc+\left(3b^2-14ac\right)\sqrt{b^2-4a}\right.}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b}}$$

Mathematica [C] time = 0.28, size = 144, normalized size = 0.26

$$\frac{\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{-14\#1^4ac\log\left(\sqrt{x}-\#1\right)+3\#1^4b^2\log\left(\sqrt{x}-\#1\right)+3ab\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right] - \frac{4x^{3/2}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{a+bx^2+cx^4}}{8c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*x^(3/2)*(b^2*x^2 + a*(b - 2*c*x^2)))/(a + b*x^2 + c*x^4) + RootSum[a +
 b*#1^4 + c*#1^8 & , (3*a*b*Log[Sqrt[x] - #1] + 3*b^2*Log[Sqrt[x] - #1]*#1^
4 - 14*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(8*c*(b^2 - 4*a*c
))
```

IntegrateAlgebraic [C] time = 0.65, size = 257, normalized size = 0.47

$$\frac{\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{-2\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4b^2c\log\left(\sqrt{x}-\#1\right)+13abc\log\left(\sqrt{x}-\#1\right)-4b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{8c^2\left(4ac-b^2\right)} - \frac{\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{b\log\left(\sqrt{x}-\#1\right)-\#1^4c\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{2c^2} + \frac{abx^{3/2}-2acx^{7/2}+b^2x^{7/2}}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*x^(3/2) + b^2*x^(7/2) - 2*a*c*x^(7/2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2
+ c*x^4)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] - c*Log[Sq
rt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(2*c^2) + RootSum[a + b*#1^4 + c*#1
^8 & , (-4*b^3*Log[Sqrt[x] - #1] + 13*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[S
qrt[x] - #1]*#1^4 - 2*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(
8*c^2*(-b^2 + 4*a*c))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 46.99Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 149, normalized size = 0.27

$$\frac{\left(\left(14ac-3b^{2}\right)\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6}-3\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2}ab\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{8\left(4ac-b^{2}\right)c\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7}c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3}b\right)}+\frac{\frac{ab\,x^{\frac{3}{2}}}{2\left(4ac-b^{2}\right)c}-\frac{\left(2ac-b^{2}\right)x^{\frac{7}{2}}}{2\left(4ac-b^{2}\right)c}}{c\,x^{4}+b\,x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/(4*a*c-b^2)/c*x^(7/2)+1/4/(4*a*c-b^2)*a*b/c*x^(3/2))/(c
*x^4+b*x^2+a)+1/8/c/(4*a*c-b^2)*sum(((14*a*c-3*b^2)*_R^6-3*_R^2*a*b)/(2*_R^
7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^{2}-2\,ac\right)x^{\frac{7}{2}}+abx^{\frac{3}{2}}}{2\left(\left(b^{2}c^{2}-4\,ac^{3}\right)x^{4}+ab^{2}c-4\,a^{2}c^{2}+\left(b^{3}c-4\,abc^{2}\right)x^{2}\right)}+\int\frac{\left(3\,b^{2}-14\,ac\right)x^{\frac{5}{2}}+3\,ab\sqrt{x}}{4\left(\left(b^{2}c^{2}-4\,ac^{3}\right)x^{4}+ab^{2}c-4\,a^{2}c^{2}+\left(b^{3}c-4\,abc^{2}\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^(7/2) + a*b*x^(3/2))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2
*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + integrate(1/4*((3*b^2 - 14*a*c)
*x^(5/2) + 3*a*b*sqrt(x))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
(b^3*c - 4*a*b*c^2)*x^2), x)
```

mupad [B] time = 7.01, size = 28774, normalized size = 52.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^(7/2)*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x^(3/2))/(2*c*(4*a*c
- b^2)))/(a + b*x^2 + c*x^4) - atan(((((46036680704*a^12*c^12 - 110592*a^3*
b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^1
2*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9
*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(1
6384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*
c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (x^(1/2)
*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 901
26*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a
^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a
^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c
^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c -
 b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^
22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 8
11008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 3244
0320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 503316
48*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302
528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683
200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576
279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81
*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^
2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^
13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^
```

7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i - (((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5

*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) +
4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2
c^3(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14
080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a
^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^
9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) + (x^
(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7
*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9
+ b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*
b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23
+ 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588
384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 8531747
84*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 20386938
88*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c +
10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a
^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^
10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 -
12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69
206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i)/((((46036680704*
a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*
c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^
8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728
*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b
^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a
^6*b^2*c^9)) - (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 7418
01984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^
4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7
*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^1
0*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746
*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^
15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^
19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 1
26720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 129761
28*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016
*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36
864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168
*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 88835358
72*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3
- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984
*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^1
5*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*
c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3

*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*
b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 +
b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720
*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^
7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10
*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) - (x^(1/2)*(9801*a^5*b^11 - 256
905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^
5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4
 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8
)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 -
90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 6470457
6*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 179962675
2*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^
4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b
^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a
*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11
- 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 3
2440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 503
31648*a^11*b^2*c^18)))^(1/4) + (((46036680704*a^12*c^12 - 110592*a^3*b^18*c
^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6
- 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c
^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a
^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 -
8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*((81*
b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2
*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^1
3*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7
*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(
4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^
15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8
 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*
a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a
^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^1
1*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^
4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^
7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040
*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8
*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(
-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19
*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5
 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8
+ 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c

- b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 10
56*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b
^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^
8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2
c^18)))^(3/4) + (x^(1/2)(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10
*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4)
)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3
*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^
15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*
a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^
6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^
9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/
2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^
3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9
- 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784
704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 576716
80*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)
- (107811*a^7*b^9 - 2531925*a^8*b^7*c + 128002112*a^11*b*c^4 + 22295196*a^9
*b^5*c^2 - 87242736*a^10*b^3*c^3)/(64*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^1
2*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*
b^4*c^8 - 28672*a^6*b^2*c^9))))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^2
3 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 105
88384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 85317
4784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 203869
3888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c
+ 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*
c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13
- 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 +
69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*2i - atan(((((4603
6680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a
^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 2140196044
8*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 1049
91817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 3
36*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 -
28672*a^6*b^2*c^9)) - (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/
2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10
588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 8531
74784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 20386
93888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*
c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*

c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677721
6*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18
*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13
 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 +
 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*
c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 -
185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 +
 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 +
b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^
4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) -
 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 105883
84*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 85317478
4*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 203869388
8*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c +
10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^
12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^1
0 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 1
2976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 692
06016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) - (x^(1/2)*(9801*a^5*
b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243
020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*
a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a
^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^1
1*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^
4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7
 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^1
0 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*
c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24
*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4
*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^
10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4
*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i - (((46036680704*a^12*c^12 - 110
592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632
*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 5940183
0400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)
/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*
a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) +
(x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c
^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 6
4704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 179
9626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9
604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15

93*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) + (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i)/((((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b

^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*
a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14
*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c
^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^
18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c
^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^
9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^
2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1
280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^
8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 -
1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 2795
71968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 24941
19936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)
^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) -
26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^
15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*
b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^1
2 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15
- 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18))
)^(3/4) - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5
+ 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(
4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^
6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b
^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^
17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11
*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*
c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4
023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*
c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(81
92*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 140
80*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^
6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9
*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4) + (((4
6036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 7778304
0*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 2140196
0448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 1
04991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4
- 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^
8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^
(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 +
10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 8
53174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 20
38693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^
21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4
*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677

7216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b
^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c
^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^1
6 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^
11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7
 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^1
0 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9
 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2
) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 105
88384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 85317
4784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 203869
3888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c
 + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*
c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13
 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 +
69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) + (x^(1/2)*(9801*a
^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13
243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*
a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15
*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c
^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*
c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b
^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1
/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b
^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*
a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7
*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*
b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4) - (107811*a^7*b^9 - 2531925*a^8*
b^7*c + 128002112*a^11*b*c^4 + 22295196*a^9*b^5*c^2 - 87242736*a^10*b^3*c^3
)/(64*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*
a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9))))*
(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 901
26*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a
^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a
^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c
^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c -
 b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^
22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 8
11008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 3244

0320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 503316
48*a^11*b^2*c^18)))^(1/4)*2i - 2*atan(((((46036680704*a^12*c^12 - 110592*a^
3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b
^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a
^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*
(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^
8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (x^(1/
2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 9
0126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576
*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752
*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4
c^4(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c
 - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^
6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*
b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 -
 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32
440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 5033
1648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 13
02528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 10616
83200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 115
76279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 +
240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))
*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 901
26*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a
^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a
^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c
^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c -
 b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^
22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 8
11008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 3244
0320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 503316
48*a^11*b^2*c^18)))^(3/4)*1i + (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c -
 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 3194564
8*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*
c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-
(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*
c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5
- 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 +
 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c
- b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1
/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c -
 b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 105
6*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^
14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8

*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*
c^18)))^(1/4) - (((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^
4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^
7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 1043123404
80*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*
c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^
7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*((81*b^8*(-(4*a*c -
b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 120
1623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 2795719
68*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 24941199
36*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15
)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 263
13*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^2
0*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 +
3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 5
7671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(
1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20
480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 388
6022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)
1i)/(16(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*
a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^
2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 12016
23*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968
*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936
*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^
(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313
*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*
c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3
784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 576
71680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/
4)*1i - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 +
2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(40
96*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
+ 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2)
- 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*
c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^
6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9
- 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023
*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3
*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*
a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b
^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^

6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4))/((((4603
6680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a
^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 2140196044
8*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 1049
91817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 3
36*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 -
28672*a^6*b^2*c^9)) - (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^2
3 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 105
88384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 85317
4784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 203869
3888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c
+ 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*
c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13
- 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 +
69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c
^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 1
85991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 +
8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9
+ b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*
b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23
+ 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588
384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 8531747
84*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 20386938
88*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c +
10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a
^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^
10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 -
12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69
206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i + (x^(1/2)*(9801*
a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 1
3243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*
a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15
*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c
^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*
c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b
^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1
/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b
^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*
a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7
*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*

b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i + (((46036680704*a^12*c^12 -
110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037
632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 5940
1830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^
11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 22
40*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9))
 + (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b
*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 +
 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1
799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 +
 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2
*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) -
1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^
7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^
16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*
c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^
17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16
*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^
8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c
^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c
^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 6
4704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 179
9626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9
604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
93*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7
- 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16
*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^
14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17
 - 50331648*a^11*b^2*c^18)))^(3/4)*1i - (x^(1/2)*(9801*a^5*b^11 - 256905*a^
6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3
+ 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240
*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((
81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*
a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*
b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*
b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*
(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^
2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*
c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 8110
08*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 3244032
0*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*

a^11*b^2*c^18)))^(1/4)*1i + (107811*a^7*b^9 - 2531925*a^8*b^7*c + 128002112
*a^11*b*c^4 + 22295196*a^9*b^5*c^2 - 87242736*a^10*b^3*c^3)/(64*(16384*a^7*
c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 896
0*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9))))*((81*b^8*(-(4*a*c
 - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 +
1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 2795
71968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 24941
19936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)
^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) -
26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^
15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*
b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^1
2 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15
- 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18))
)^(1/4) - 2*atan(((((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*
a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*
a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 10431234
0480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^1
4*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*
c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (x^(1/2)*(-(81*b^23 + 81*b^
8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 -
1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 2795
71968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 24941
19936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)
^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) -
26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^
15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*
b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^1
2 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15
- 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18))
)^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 +
 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 -
3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^
12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 12
80*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8
*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1
201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 27957
1968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 249411
9936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^
15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 2
6313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b
^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12
 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 -
 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))

^(3/4)*1i + (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^
5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16
*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c -
 b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*
b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^
11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^
5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) -
 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^
2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 1
4080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*
a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a
^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4) - ((
(46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783
040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401
960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 -
 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^
4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*
c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15
)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3
 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 -
 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 +
2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*
b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-
(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16
777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3
*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12
*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c
^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*
a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c
^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c
^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^
6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 384
0*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)
^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3
+ 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 -
853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2
038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b
^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(
4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(167
77216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*
b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*
c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^
16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i - (x^(1/2)

*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*
c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^1
2*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c
^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 74
1801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*
a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a
^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a
^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 107
46*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2
)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*
c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 +
126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 1297
6128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 692060
16*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4))/((((46036680704*a^12*c^
12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 7
88037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 -
59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b
^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5
+ 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*
c^9)) - (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*
a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15
*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c
^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*
c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b
^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1
/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b
^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*
a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7
*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*
b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^
3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b
^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9
*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 2
4*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144
*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a
^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*
c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^
7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c
^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^
4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^
24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a
^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*
b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b
^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i + (x^(1/2)*(9801*a^5*b^11 - 25

6905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i + (((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c -

```
29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648
*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c
^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 +
 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*
c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5
+ 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 -
 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c
- b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1
/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c -
 b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 105
6*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^
14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8
*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*
c^18)))^(1/4)*1i + (107811*a^7*b^9 - 2531925*a^8*b^7*c + 128002112*a^11*b*c
^4 + 22295196*a^9*b^5*c^2 - 87242736*a^10*b^3*c^3)/(64*(16384*a^7*c^10 - b^
14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6
*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9))))*(-(81*b^23 + 81*b^8*(-(4*a
*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*
a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^
6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^
9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/
2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^
3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9
 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784
704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 576716
80*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.839 $\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=520

$$-\frac{\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac \right.}{4\sqrt[4]{2}c^{5/4}\left(b^2-\right.}$$

Rubi [A] time = 1.37, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1115, 1365, 1502, 1422, 212, 208, 205}

$$-\frac{\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*Sqrt[x])/(2*c*(b^2 - 4*a*c)) + (x^(5/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^
(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*
c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*
a*c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b +
Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{12}}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^4\left(10a+bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{ab+\left(b^2-10ac\right)x^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2c\left(b^2-4ac\right)}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-10ac-\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\ldots}{\frac{b}{2}-\frac{1}{2}\ldots}\right.}{4c\left(b^2-4ac\right)}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2-10ac+\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt{-b-\ldots}}\right.}{4c\left(b^2-4ac\right)\sqrt{-b-\ldots}}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2-10ac+\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\ldots}{\sqrt[4]{-b-\sqrt{b}\ldots}}\right.}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(-b-\sqrt{b^2-4ac}\ldots\right.}$$

Mathematica [C] time = 0.27, size = 144, normalized size = 0.28

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{-10\#1^4ac\log\left(\sqrt{x}-\#1\right)+\#1^4b^2\log\left(\sqrt{x}-\#1\right)+ab\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right] - \frac{4\sqrt{x}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{a+bx^2+cx^4}}{8c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*Sqrt[x]*(b^2*x^2 + a*(b - 2*c*x^2)))/(a + b*x^2 + c*x^4) + RootSum[a +
 b*#1^4 + c*#1^8 & , (a*b*Log[Sqrt[x] - #1] + b^2*Log[Sqrt[x] - #1]*#1^4 -
10*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(8*c*(b^2 - 4*a*c))
```

IntegrateAlgebraic [C] time = 0.50, size = 262, normalized size = 0.50

$$\frac{\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-6\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+3\#1^4b^2c\log\left(\sqrt{x}-\#1\right)+15abc\log\left(\sqrt{x}-\#1\right)-4b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{8c^2\left(4ac-b^2\right)}-\frac{\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{b\log\left(\sqrt{x}-\#1\right)-\#1^4c\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{2c^2}+\frac{ab\sqrt{x}-2acx^{5/2}+b^2x^{5/2}}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*Sqrt[x] + b^2*x^(5/2) - 2*a*c*x^(5/2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2
+ c*x^4)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] - c*Log[Sq
rt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(2*c^2) + RootSum[a + b*#1^4 + c*
#1^8 & , (-4*b^3*Log[Sqrt[x] - #1] + 15*a*b*c*Log[Sqrt[x] - #1] + 3*b^2*c*L
og[Sqrt[x] - #1]*#1^4 - 6*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7)
 & ]/(8*c^2*(-b^2 + 4*a*c))
```

fricas [B] time = 75.66, size = 11906, normalized size = 22.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*
b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1
280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqr
t((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b
^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11
 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^
5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^1
8 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))*arctan
(-1/2*(sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c
^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6
- 497651072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 +
 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 - (b^25*c^5 - 70*a*b^23*c^
6 + 2192*a^2*b^21*c^7 - 40672*a^3*b^19*c^8 + 498432*a^4*b^17*c^9 - 4254720*
a^5*b^15*c^10 + 25976832*a^6*b^13*c^11 - 114475008*a^7*b^11*c^12 + 36195532
8*a^8*b^9*c^13 - 802029568*a^9*b^7*c^14 + 1183842304*a^10*b^5*c^15 - 104647
8848*a^11*b^3*c^16 + 419430400*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571
*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5
 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*
a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*
c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(
(81*a^2*b^16 - 8118*a^3*b^14*c + 358651*a^4*b^12*c^2 - 9129750*a^5*b^10*c^3
```

```
 + 146540625*a^6*b^8*c^4 - 1519250000*a^7*b^6*c^5 + 9937500000*a^8*b^4*c^6
- 37500000000*a^9*b^2*c^7 + 62500000000*a^10*c^8)*x + 1/2*sqrt(1/2)*(b^22 -
 112*a*b^20*c + 5735*a^2*b^18*c^2 - 176820*a^3*b^16*c^3 + 3634845*a^4*b^14*
c^4 - 52073994*a^5*b^12*c^5 + 527503968*a^6*b^10*c^6 - 3751826400*a^7*b^8*c
^7 + 18208800000*a^8*b^6*c^8 - 56920000000*a^9*b^4*c^9 + 102400000000*a^10*
b^2*c^10 - 80000000000*a^11*c^11 - (b^25*c^5 - 91*a*b^23*c^6 + 3641*a^2*b^2
1*c^7 - 84776*a^3*b^19*c^8 + 1280016*a^4*b^17*c^9 - 13215744*a^5*b^15*c^10
+ 95875584*a^6*b^13*c^11 - 493891584*a^7*b^11*c^12 + 1798938624*a^8*b^9*c^1
3 - 4533059584*a^9*b^7*c^14 + 7523860480*a^10*b^5*c^15 - 7405568000*a^11*b^
3*c^16 + 3276800000*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^
2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*
a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^
13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 5898
24*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(-(b^9 - 45*
a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5
- 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6
144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c
^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000
*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c
^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589
824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*
a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a
^5*b^2*c^10 + 4096*a^6*c^11)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470
625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*
b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 -
129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^
8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)
) - sqrt(1/2)*(9*a*b^30 - 1270*a^2*b^28*c + 82813*a^3*b^26*c^2 - 3305978*a^
4*b^24*c^3 + 90231255*a^5*b^22*c^4 - 1780615316*a^6*b^20*c^5 + 26199812170*
a^7*b^18*c^6 - 292147074792*a^8*b^16*c^7 + 2484388440192*a^9*b^14*c^8 - 160
82985454080*a^10*b^12*c^9 + 78485701504000*a^11*b^10*c^10 - 283191078400000
*a^12*b^8*c^11 + 730734080000000*a^13*b^6*c^12 - 1272576000000000*a^14*b^4*
c^13 + 1337600000000000*a^15*b^2*c^14 - 640000000000000*a^16*c^15 - (9*a*b^
33*c^5 - 1081*a^2*b^31*c^6 + 59923*a^3*b^29*c^7 - 2033390*a^4*b^27*c^8 + 47
234960*a^5*b^25*c^9 - 795781312*a^6*b^23*c^10 + 10050046208*a^7*b^21*c^11 -
 96993186304*a^8*b^19*c^12 + 722648002560*a^9*b^17*c^13 - 4169749463040*a^1
0*b^15*c^14 + 18574068219904*a^11*b^13*c^15 - 63226237812736*a^12*b^11*c^16
 + 161327426306048*a^13*b^9*c^17 - 298510607974400*a^14*b^7*c^18 + 37806407
6800000*a^15*b^5*c^19 - 293076992000000*a^16*b^3*c^20 + 104857600000000*a^1
7*b*c^21)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 +
 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 3
```

6*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^1
4 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 58982
4*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(x)*sqrt(-(b^9 - 45*a*b^7*c + 765*a
^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6
 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^1
0 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*
b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18
*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4
*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^1
7 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 24
0*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4
096*a^6*c^11)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470
625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*
b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 -
129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^
8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)
))/(6561*a^5*b^20 - 803358*a^6*b^18*c + 44473131*a^7*b^16*c^2 - 1466261550*
a^8*b^14*c^3 + 31889850625*a^9*b^12*c^4 - 478129875000*a^10*b^10*c^5 + 5004
993750000*a^11*b^8*c^6 - 36117500000000*a^12*b^6*c^7 + 171937500000000*a^13
*b^4*c^8 - 487500000000000*a^14*b^2*c^9 + 625000000000000*a^15*c^10)) - 4*(
(b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sq
rt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 +
 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b
^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 -
 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 -
2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^
2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^1
5 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 26214
4*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^
8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))*arctan(1/2*(sqr
t(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 13388
50*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 49765107
2*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 458240000
0*a^10*b^2*c^10 - 2560000000*a^11*c^11 + (b^25*c^5 - 70*a*b^23*c^6 + 2192*a
^2*b^21*c^7 - 40672*a^3*b^19*c^8 + 498432*a^4*b^17*c^9 - 4254720*a^5*b^15*c
^10 + 25976832*a^6*b^13*c^11 - 114475008*a^7*b^11*c^12 + 361955328*a^8*b^9*
c^13 - 802029568*a^9*b^7*c^14 + 1183842304*a^10*b^5*c^15 - 1046478848*a^11*
b^3*c^16 + 419430400*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c
^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000
*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c
^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589

824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt((81*a^2*b^
16 - 8118*a^3*b^14*c + 358651*a^4*b^12*c^2 - 9129750*a^5*b^10*c^3 + 1465406
25*a^6*b^8*c^4 - 1519250000*a^7*b^6*c^5 + 9937500000*a^8*b^4*c^6 - 37500000
000*a^9*b^2*c^7 + 62500000000*a^10*c^8)*x + 1/2*sqrt(1/2)*(b^22 - 112*a*b^2
0*c + 5735*a^2*b^18*c^2 - 176820*a^3*b^16*c^3 + 3634845*a^4*b^14*c^4 - 5207
3994*a^5*b^12*c^5 + 527503968*a^6*b^10*c^6 - 3751826400*a^7*b^8*c^7 + 18208
800000*a^8*b^6*c^8 - 56920000000*a^9*b^4*c^9 + 102400000000*a^10*b^2*c^10 -
 80000000000*a^11*c^11 + (b^25*c^5 - 91*a*b^23*c^6 + 3641*a^2*b^21*c^7 - 84
776*a^3*b^19*c^8 + 1280016*a^4*b^17*c^9 - 13215744*a^5*b^15*c^10 + 95875584
*a^6*b^13*c^11 - 493891584*a^7*b^11*c^12 + 1798938624*a^8*b^9*c^13 - 453305
9584*a^9*b^7*c^14 + 7523860480*a^10*b^5*c^15 - 7405568000*a^11*b^3*c^16 + 3
276800000*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*
a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(
b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256
*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4
*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(-(b^9 - 45*a*b^7*c +
765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^1
0*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^
2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950
*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/
(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 3225
6*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^
4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6
 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^1
0 + 4096*a^6*c^11)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c
^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a
^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096
*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3
+ 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 -
36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^
14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 5898
24*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 1800
0*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^
8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a
*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 26250
00*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^1
4*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 3
44064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9
*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3
840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)) - sqrt(1/2)*(9*a*b^30
 - 1270*a^2*b^28*c + 82813*a^3*b^26*c^2 - 3305978*a^4*b^24*c^3 + 90231255*a
^5*b^22*c^4 - 1780615316*a^6*b^20*c^5 + 26199812170*a^7*b^18*c^6 - 29214707
4792*a^8*b^16*c^7 + 2484388440192*a^9*b^14*c^8 - 16082985454080*a^10*b^12*c

^9 + 78485701504000*a^11*b^10*c^10 - 283191078400000*a^12*b^8*c^11 + 730734
080000000*a^13*b^6*c^12 - 1272576000000000*a^14*b^4*c^13 + 1337600000000000
*a^15*b^2*c^14 - 640000000000000*a^16*c^15 + (9*a*b^33*c^5 - 1081*a^2*b^31*
c^6 + 59923*a^3*b^29*c^7 - 2033390*a^4*b^27*c^8 + 47234960*a^5*b^25*c^9 - 7
95781312*a^6*b^23*c^10 + 10050046208*a^7*b^21*c^11 - 96993186304*a^8*b^19*c
^12 + 722648002560*a^9*b^17*c^13 - 4169749463040*a^10*b^15*c^14 + 185740682
19904*a^11*b^13*c^15 - 63226237812736*a^12*b^11*c^16 + 161327426306048*a^13
*b^9*c^17 - 298510607974400*a^14*b^7*c^18 + 378064076800000*a^15*b^5*c^19 -
293076992000000*a^16*b^3*c^20 + 104857600000000*a^17*b*c^21)*sqrt((b^12 -
78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2
625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2
*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15
 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144
*a^9*c^19)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c
^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a
^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096
*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3
+ 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 -
36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^
14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 5898
24*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 1800
0*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^
8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a
*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 26250
00*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^1
4*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 3
44064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9
*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3
840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))/(6561*a^5*b^20 - 803
358*a^6*b^18*c + 44473131*a^7*b^16*c^2 - 1466261550*a^8*b^14*c^3 + 31889850
625*a^9*b^12*c^4 - 478129875000*a^10*b^10*c^5 + 5004993750000*a^11*b^8*c^6
- 36117500000000*a^12*b^6*c^7 + 171937500000000*a^13*b^4*c^8 - 487500000000
000*a^14*b^2*c^9 + 625000000000000*a^15*c^10)) - ((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 -
45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*
c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9
- 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b
^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 625
0000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^
12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 -
589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 -
24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 61
44*a^5*b^2*c^10 + 4096*a^6*c^11)))*log((9*a*b^8 - 451*a^2*b^6*c + 8625*a^3*

b^4*c^2 - 75000*a^4*b^2*c^3 + 250000*a^5*c^4)*sqrt(x) + 1/2*(b^11 - 47*a*b^
9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*b*
c^5 - (b^14*c^5 - 44*a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b^8*c^8 + 294
40*a^4*b^6*c^9 - 82944*a^5*b^4*c^10 + 126976*a^6*b^2*c^11 - 81920*a^7*c^12)
*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a
^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*
c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 12902
4*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2
*c^18 - 262144*a^9*c^19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2
*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 +
240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10
+ 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^
6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c
^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b
^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17
+ 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*
a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 409
6*a^6*c^11)))) + ((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470
625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*
b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 -
129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^
8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
- 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)
))*log((9*a*b^8 - 451*a^2*b^6*c + 8625*a^3*b^4*c^2 - 75000*a^4*b^2*c^3 + 25
0000*a^5*c^4)*sqrt(x) - 1/2*(b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3
*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*b*c^5 - (b^14*c^5 - 44*a*b^12*c^6
+ 720*a^2*b^10*c^7 - 6080*a^3*b^8*c^8 + 29440*a^4*b^6*c^9 - 82944*a^5*b^4*c
^10 + 126976*a^6*b^2*c^11 - 81920*a^7*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571
*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5
+ 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*
a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*
c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(
sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18
000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*
c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78
*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 262
5000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b
^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 +
344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a
^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 +
3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))) - ((b^2*c^2 - 4*a

*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*s
qrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c
^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*
a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c +
2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^
2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 -
5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6
*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/
(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b
^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))*log((9*a*b^8 - 451*a^2*b^6*c
+ 8625*a^3*b^4*c^2 - 75000*a^4*b^2*c^3 + 250000*a^5*c^4)*sqrt(x) + 1/2*(b^1
1 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 4
0000*a^5*b*c^5 + (b^14*c^5 - 44*a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b^
8*c^8 + 29440*a^4*b^6*c^9 - 82944*a^5*b^4*c^10 + 126976*a^6*b^2*c^11 - 8192
0*a^7*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3
+ 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 -
36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c
^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589
824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*
c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a
*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^
5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 4
5950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c
^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 +
32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^
7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10
*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2
*c^10 + 4096*a^6*c^11)))) + ((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2
+ (b^3*c - 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2
*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 +
240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10
+ 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^
6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c
^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b
^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17
+ 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*
a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 409
6*a^6*c^11)))*log((9*a*b^8 - 451*a^2*b^6*c + 8625*a^3*b^4*c^2 - 75000*a^4*b
^2*c^3 + 250000*a^5*c^4)*sqrt(x) - 1/2*(b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2
- 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*b*c^5 + (b^14*c^5 - 44*
a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b^8*c^8 + 29440*a^4*b^6*c^9 - 8294
4*a^5*b^4*c^10 + 126976*a^6*b^2*c^11 - 81920*a^7*c^12)*sqrt((b^12 - 78*a*b^
10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*
a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c
^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 3440

```
64*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^
19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b
^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 12
80*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt
((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^
4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11
+ 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5
*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18
 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^
3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))) + 4*((
b^2 - 2*a*c)*x^2 + a*b)*sqrt(x))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.04Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 146, normalized size = 0.28

$$\frac{\left(\left(10ac-b^2\right)\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4-ab\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{8\left(4ac-b^2\right)c\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{\frac{ab\sqrt{x}}{2\left(4ac-b^2\right)c}-\frac{\left(2ac-b^2\right)x^{\frac{5}{2}}}{2\left(4ac-b^2\right)c}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/(4*a*c-b^2)/c*x^(5/2)+1/4/(4*a*c-b^2)*a*b/c*x^(1/2))/(c
*x^4+b*x^2+a)+1/8/c/(4*a*c-b^2)*sum(((10*a*c-b^2)*_R^4-a*b)/(2*_R^7*c+_R^3*
b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx^{\frac{9}{2}}+2\,ax^{\frac{5}{2}}}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}+\int-\frac{bx^{\frac{7}{2}}+10\,ax^{\frac{3}{2}}}{4\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(9/2) + 2*a*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) + integrate(-1/4*(b*x^(7/2) + 10*a*x^(3/2))/((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

mupad [B] time = 11.85, size = 31964, normalized size = 61.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] 2*atan(((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2
- 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^
3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c
^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 29
8844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 + b^6*(-(4*a*c - b^2)^
15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 +
 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 505036
80*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^
17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 12
6720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128
*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a
^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*
a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6
*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*
(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 3632
0*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*
b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*
c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a
^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12
*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c
^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21
+ b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2
- 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1506457
6*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^
9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*
b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(
```

8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 1
4080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^
6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9
*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i - (
x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2
 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 -
24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 614
4*a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*
c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 30013
44*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a
^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(
4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^
6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a
^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^
8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11
*b^2*c^16)))^(1/4) - (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549
*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 40
96*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6
*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b
^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 12
80*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 + b^6*(-(
4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a
^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9
*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9
 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(167
77216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*
b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^
11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14
 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*
c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 524
28800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(
-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^
17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +
15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13467
6480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 5
25*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*
c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 378
4704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671
680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4)
1i)(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*

a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2
*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10
+ 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 -
57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))
^(1/4)*1i + (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 6632
2*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 737
28000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^1
3*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7
+ 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b
^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3
9*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 -
48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c
^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12
+ 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 -
50331648*a^11*b^2*c^16)))^(1/4))/((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7
*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^1
0*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 +
53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 149
3172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2
*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b
^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*
c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 150
64576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13467648
0*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*
a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7
- 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 378470
4*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680
*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(1
67772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*
b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*
c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3
*b^2*c^4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10
+ 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^
5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^
5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69
*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1

056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^
14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8
*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*
c^16)))^(3/4)*1i)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b
*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001
344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*
a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-
(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c
^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*
a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a
^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^1
1*b^2*c^16)))^(1/4)*1i - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5
*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15
)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 4
04160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680
*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3
*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17
 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 1267
20*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a
^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^1
0*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i + (((9*a^3*b^9 - 397*a^4*b^
7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c +
256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)
*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 491520
0*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a
^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^
10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b
^2*c^6)) + ((-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10
+ 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^
5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^
5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69
*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1
056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^
14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8
*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*
c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^
6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 25
1658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728
000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*

c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 +
108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2
)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*
a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 4
8*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9
 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 +
32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50
331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) +
 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4
*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*
c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
 - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c
^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^
16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c
^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^1
5 - 50331648*a^11*b^2*c^16)))^(1/4)*1i + (x^(1/2)*(81*a^4*b^10 - 2000000*a^
9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a
^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 -
 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 + b^6*(-
(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*
a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^
9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^
9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*
(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16
777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3
*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c
^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^1
4 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i))*(-(b^21 +
 b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 -
 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576
*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9
*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14
080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6
*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*
b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) - atan(
((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800
*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 14
7456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160
*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*
c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 384
0*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/

2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160
*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*
b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4
*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^
24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^
4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^
10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4
*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^1
2*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 +
157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 -
16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*(-(4*a*c -
b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15
*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 -
50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a
^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^
8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12
976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 6920
6016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 + b^6*(-(4*a*
c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b
^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6
- 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2
500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677721
6*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18
*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 -
12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 6
9206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) + (x^(1/2)*(81*a^4*
b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^
4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 +
240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))
(-(b^21 + b^6(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*
b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5
+ 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134
676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c +
525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^2
0*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3
784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 576
71680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/
4)*1i - (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2
- 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^
3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c

^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 29
8844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 + b^6*(-(4*a*c - b^2)^
15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 +
 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 505036
80*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^
17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 12
6720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128
*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a
^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*
a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6
*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4
*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*(-(
4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a
^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9
*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9
 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(167
77216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*
b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^
11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14
 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 + b^6*
(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 3632
0*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*
b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*
c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a
^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12
*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c
^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) - (x^(1/2)*(
81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800
*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10
*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2
c^6)))(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 20
85*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^
11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^
8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b
^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*
a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c
^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^1

3 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16
)))^(1/4)*1i)/((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b
^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2
*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3
*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c
^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^1
0))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3
*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 + b^6*(-(4*a*c
- b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^1
5*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 -
50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 250
0*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*
a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c
^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 1
2976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 692
06016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 +
40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*
a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c +
256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 +
b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 -
36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*
a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*
b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(81
92*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 140
80*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*
b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b
^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21
+ b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2
- 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 150645
76*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a
^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2
*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/
(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 -
14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a
^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^
9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) + (x^
(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 -
547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24
*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*
a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^
10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344
*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8
*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) -

69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) + (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6

144*a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*
b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300
1344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160
*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*
c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008
*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*
a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^
11*b^2*c^16)))^(1/4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a
^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 10838
0160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)
^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b
^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 81
1008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440
320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 5033164
8*a^11*b^2*c^16)))^(1/4)*2i - atan(((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a
^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 -
16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a
^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6
+ 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1
493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a
^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-
(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^1
7*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1
5064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676
480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 52
5*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c
^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784
704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 576716
80*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*
(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^
5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^
2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*
b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 +
2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5
*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5
*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*
a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 10
56*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^1
4*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*

c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c
^16)))^(3/4))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^1
0 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*
a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*
b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) -
69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a
c - b^2)^15)^(1/2))/(8192(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 +
1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*
b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b
^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^
2*c^16)))^(1/4) + (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c
+ 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*
c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 384
0*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2)
+ 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a
^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^
7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a
*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/
2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24
*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*
b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10
*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c
^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i - (((9*a^3*b^9 - 397*a^4*b^7*c + 1
30000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4
*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(10066
32960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b
^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*
c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2
+ 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)
) - ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*
a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2
*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10
+ 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 -
57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))
^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 983
0400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240
*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 2
56*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b
*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001
344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*
a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2

) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i)/((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*

c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000
*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4
 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108
380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^1
5)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b
^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a
*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 -
811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 324
40320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331
648*a^11*b^2*c^16)))^(3/4))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728
000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*
c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 +
108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2
)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*
a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 4
8*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9
 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 +
32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50
331648*a^11*b^2*c^16)))^(1/4) + (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3
744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^
4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3
*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c -
b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*
c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 5
0503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*
a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^
12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8
 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 129
76128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206
016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) + (((9*a^3*b^9 - 397*a^
4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*
c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(
1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 49
15200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 9185525
76*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*
a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a
^5*b^2*c^6)) - ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c
^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300134
4*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^
8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
- 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6
 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^
5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8

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*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*
b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^1
0*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9
- 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 7372
8000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13
*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 +
 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^
2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39
*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 -
48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^
9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 +
 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 5
0331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 7
3728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b
^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^
7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c -
 b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5
 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16
*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^1
2 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15
- 50331648*a^11*b^2*c^16)))^(1/4) - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5
 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^
2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*
c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b
^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6
 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2
500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677721
6*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18
*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 -
 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 6
9206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)))*(-(b^21 - b^6*(-(
4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a
^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9
*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9
 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(167
77216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*
b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^
11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14
 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*2i - ((x^(5/2)*
(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x^(1/2))/(2*c*(4*a*c - b^2)))/(a
```

+ b*x^2 + c*x^4) + 2*atan(((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4
+ 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^1
1 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 537395
20*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224
*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^
3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 - b
^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 3
6320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a
^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b
^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2
c^2(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 1408
0*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b
^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^
6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(16777216
0*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7
 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1
i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^
4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*
a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2
*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10
 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 -
 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))
^(3/4)*1i)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 +
 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5
*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5
*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*
a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 10
56*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^1
4*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*
c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c
^16)))^(1/4)*1i - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c
+ 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*
c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 384
0*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2)
 + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a
^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^
7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a
*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/

2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24
*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*
b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10
*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c
^15 - 50331648*a^11*b^2*c^16)))^(1/4) - (((9*a^3*b^9 - 397*a^4*b^7*c + 1300
00*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^
5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(10066329
60*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11
*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9
 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) +
 ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2
*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5
 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13
4676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c
- 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^
20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 +
3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57
671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1
/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 983040
0*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^
8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 2
56*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b
*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001
344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*
a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-
(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c
^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*
a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a
^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^1
1*b^2*c^16)))^(3/4)*1i)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*
a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4
- 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1083
80160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^
4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*
b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 8
11008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 3244
0320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 503316
48*a^11*b^2*c^16)))^(1/4)*1i + (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 37
44*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4
))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*
b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c - b

```
^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c
^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50
503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a
^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c -
 b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^1
2*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8
+ 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 1297
6128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 692060
16*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4))/((((9*a^3*b^9 - 397*a^4
*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c
 + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1
/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 491
5200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 91855257
6*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a
*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^
5*b^2*c^6)) - ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^
10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344
*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8
*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) -
 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*
a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6
+ 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5
*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*
b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b
^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10
*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 -
 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73
728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^
13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7
 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5
- 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*
c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12
 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 -
 50331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2
) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*
a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b
^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^2
4*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4
*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^1
0*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*
```

c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i - (x^(1/2)*(81*a^4*b^10 - 2000000
*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 198000
0*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^
3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6
*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 363
20*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6
*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3
*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c
^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*
a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^1
2*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*
c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i + (((9*a
^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^
3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^
2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^
4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^
7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 409
6*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b
^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73
728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^
13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7
 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5
- 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*
c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12
 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 -
 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 -
 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 1572864
00*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2
)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3
 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 5050
3680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3
c^3(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*
c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 +
126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 129761
28*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016
*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21 - b^6*(-(4*a*c
 - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^
15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6
- 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 25
00*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a

```
*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*
c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 -
12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69
206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i + (x^(1/2)*(81*a^
4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*
b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2
+ 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)
))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^
2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^
5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 1
34676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c
 - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b
^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 +
 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 5
7671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(
1/4)*1i))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 +
2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*
b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*
c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a
*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c -
 b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 105
6*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14
*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c
^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^
16)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

3.840 $\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=471

$$\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(b\sqrt{b^2-4ac}+12ac+b\right.}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)}$$

Rubi [A] time = 0.92, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1365, 1510, 298, 205, 208}

$$\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2 + 12*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b - (b^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*
a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((b - (b
^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(6a-bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+12ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x\right.}{4\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2+12ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x\right.}{4\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b^2+\right.}{4}$$

Mathematica [C] time = 0.21, size = 124, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4b\log\left(\sqrt{x}-\#1\right)-6a\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{8\left(b^2-4ac\right)} - \frac{-2ax^{3/2}-bx^{7/2}}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(-2*a*x^(3/2) - b*x^(7/2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootS
um[a + b*#1^4 + c*#1^8 & , (-6*a*Log[Sqrt[x] - #1] + b*Log[Sqrt[x] - #1]*#1
^4)/(b*#1 + 2*c*#1^5) & ]/(8*(b^2 - 4*a*c))
```

IntegrateAlgebraic [C] time = 0.46, size = 196, normalized size = 0.42

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)+10ac\log\left(\sqrt{x}-\#1\right)-4b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{8c\left(4ac-b^2\right)} + \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{2c} + \frac{2ax^{3/2}+bx^{7/2}}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]

[Out] (2*a*x^(3/2) + b*x^(7/2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^5) &]/(2*c) - RootSum[a + b*#1^4 + c*#1^8 & , (-4*b^2*Log[Sqrt[x] - #1] + 10*a*c*Log[Sqrt[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) &]/(8*c*(-b^2 + 4*a*c))

fricas [B] time = 79.68, size = 11032, normalized size = 23.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*arctan(-1/2*((b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 - (b^14*c^3 - 12*a*b^12*c^4 - 48*a^2*b^10*c^5 + 1600*a^3*b^8*c^6 - 11520*a^4*b^6*c^7 + 39936*a^5*b^4*c^8 - 69632*a^6*b^2*c^9 + 49152*a^7*c^10)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt((117649*a^4*b^20 + 9983358*a^5*b^18*c + 404714961*a^6*b^16*c^2 + 9897860448*a^7*b^14*c^3 + 158656107456*a^8*b^12*c^4 + 1707655509504*a^9*b^10*c^5 + 12338818573824*a^10*b^8*c^6 + 58812305154048*a^11*b^6*c^7 + 177024646692864*a^12*b^4*c^8 + 304679870005248*a^13*b^2*c^9 + 228509902503936*a^14*c^10)*x - 1/2*sqrt(1/2)*(2401*a^3*b^25 + 294294*a^4*b^23*c + 13335105*a^5*b^21*c^2 + 323354360*a^6*b^19*c^3 + 4269253584*a^7*b^17*c^4 + 24537890304*a^8*b^15*c^5 - 79436754432*a^9*b^13*c^6 - 1621756588032*a^10*b^11*c^7 - 3506876964864*a^11*b^9*c^8 + 27305557622784*a^12*b^7*c^9 + 100201644490752*a^13*b^5*c^10 - 142936235311104*a^14*b^3*c^11 - 677066377789440*a^15*b*c^12 - (2401*a^3*b^30*c^3 - 49049*a^4*b^28*c^4 - 1432760*a^5*b^26*c^5 - 6473264*a^6*b^24*c^6 + 373184512*a^7*b^22*c^7 - 319185152*a^8*b^20*c^8 - 27408852992*a^9*b^18*c^9 + 93871525888*a^10*b^16*c^10 + 774145638400*a^11*b^14*c^11 - 4486009651200*a^12*b^12*c^12 - 5590781263872*a^13*b^10*c^13 + 81717925773312*a^14*b^8*c^14 - 108093958520832*a^15*b^6*c^15 - 454721122861056*a^16*b^4*c^16 + 1497904875307008*a^17*b^2*c^17 - 1283918464548864*a^18*c^18)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496* a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15))) *sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))) - (343*a^2*b^19 + 21070*a^3*b^17*c + 600271*a^4*b^15*c^2 + 8903196*a^5*b^13*c^3 + 62719920*a^6*b^11*c^4 - 15909696*a^7*b^9*c^5 - 2396812032*a^8*b^7*c^6 - 6953610240*a^9*b^5*c^7 + 19591041024*a^10*b^3*c^8 + 78364164096*a^11*b*c^9 - (343*a^2*b^24*c^3 + 10437*a^3*b^22*c^4 + 90132*a^4*b^20*c^5 - 1028432*a^5*b^18*c^6 - 14041152*a^6*b^16*c^7 + 70390272*a^7*b^14*c^8 + 646137856*a^8*b^12*c^9 - 3121520640*a^9*b^10*c^10 - 11091935232*a^10*b^8*c^11 + 68335239168*a^11*b^6*c^12 + 24652283904*a^12*b^4*c^13 - 557256278016*a^13*b^2*c^14 + 743008370688*a^14*c^15)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))/(2401*a^3*b^16 + 179046*a^4*b^14*c + 6354369*a^5*b^12*c^2 + 131902344*a^6*b^10*c^3 + 1713103344*a^7*b^8*c^4 + 13740938496*a^8*b^6*c^5 + 65167421184*a^9*b^4*c^6 + 166523848704*a^10*b^2*c^7 + 176319369216*a^11*c^8)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*arctan(1/2*((b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 + (b^14*c^3 - 12*a*b^12*c^4 - 48*a^2*b^10*c^5 + 1600*a^3*b^8*c^6 - 11520*a^4*b^6*c^7 + 39936*a^5*b^4*c^8 - 69632*a^6*b^2*c^9 + 49152*a^7*c^10)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3

*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5
376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b
^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sq
rt((117649*a^4*b^20 + 9983358*a^5*b^18*c + 404714961*a^6*b^16*c^2 + 9897860
448*a^7*b^14*c^3 + 158656107456*a^8*b^12*c^4 + 1707655509504*a^9*b^10*c^5 +
 12338818573824*a^10*b^8*c^6 + 58812305154048*a^11*b^6*c^7 + 17702464669286
4*a^12*b^4*c^8 + 304679870005248*a^13*b^2*c^9 + 228509902503936*a^14*c^10)*
x - 1/2*sqrt(1/2)*(2401*a^3*b^25 + 294294*a^4*b^23*c + 13335105*a^5*b^21*c^
2 + 323354360*a^6*b^19*c^3 + 4269253584*a^7*b^17*c^4 + 24537890304*a^8*b^15
*c^5 - 79436754432*a^9*b^13*c^6 - 1621756588032*a^10*b^11*c^7 - 35068769648
64*a^11*b^9*c^8 + 27305557622784*a^12*b^7*c^9 + 100201644490752*a^13*b^5*c^
10 - 142936235311104*a^14*b^3*c^11 - 677066377789440*a^15*b*c^12 + (2401*a^
3*b^30*c^3 - 49049*a^4*b^28*c^4 - 1432760*a^5*b^26*c^5 - 6473264*a^6*b^24*c
^6 + 373184512*a^7*b^22*c^7 - 319185152*a^8*b^20*c^8 - 27408852992*a^9*b^18
*c^9 + 93871525888*a^10*b^16*c^10 + 774145638400*a^11*b^14*c^11 - 448600965
1200*a^12*b^12*c^12 - 5590781263872*a^13*b^10*c^13 + 81717925773312*a^14*b^
8*c^14 - 108093958520832*a^15*b^6*c^15 - 454721122861056*a^16*b^4*c^16 + 14
97904875307008*a^17*b^2*c^17 - 1283918464548864*a^18*c^18)*sqrt((b^8 + 54*a
*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 -
 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10
 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824
*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^
2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3
*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 +
54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c
^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*
c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 58
9824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*
b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6
*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3
*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3
840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c +
 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^
16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 12902
4*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2
*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 12
80*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))) - (3
43*a^2*b^19 + 21070*a^3*b^17*c + 600271*a^4*b^15*c^2 + 8903196*a^5*b^13*c^3
 + 62719920*a^6*b^11*c^4 - 15909696*a^7*b^9*c^5 - 2396812032*a^8*b^7*c^6 -
6953610240*a^9*b^5*c^7 + 19591041024*a^10*b^3*c^8 + 78364164096*a^11*b*c^9
+ (343*a^2*b^24*c^3 + 10437*a^3*b^22*c^4 + 90132*a^4*b^20*c^5 - 1028432*a^5
*b^18*c^6 - 14041152*a^6*b^16*c^7 + 70390272*a^7*b^14*c^8 + 646137856*a^8*b
^12*c^9 - 3121520640*a^9*b^10*c^10 - 11091935232*a^10*b^8*c^11 + 6833523916
8*a^11*b^6*c^12 + 24652283904*a^12*b^4*c^13 - 557256278016*a^13*b^2*c^14 +
743008370688*a^14*c^15)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a

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^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 -
 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6
*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*
sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3
*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3
840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c +
 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^
16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 12902
4*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2
*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 12
80*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))))/(24
01*a^3*b^16 + 179046*a^4*b^14*c + 6354369*a^5*b^12*c^2 + 131902344*a^6*b^10
*c^3 + 1713103344*a^7*b^8*c^4 + 13740938496*a^8*b^6*c^5 + 65167421184*a^9*b
^4*c^6 + 166523848704*a^10*b^2*c^7 + 176319369216*a^11*c^8)) + ((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b
^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*
c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*
c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b
^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 537
6*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6
*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^1
2*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c
^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*log(1/2*sqrt(1/2)*(b^18 + 25*a*b^16
*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5
*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c
^8 + 71663616*a^9*c^9 - (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 1171
2*a^3*b^17*c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*
b^11*c^9 - 50823168*a^7*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b
^5*c^12 + 250609664*a^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a
*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 -
 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10
 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824
*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c +
 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8
*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^
9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a
^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 +
32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^
7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10
*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2
*c^8 + 4096*a^6*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3
*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3
840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c +
 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^
16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 12902
```

4*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2
*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 12
80*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (34
3*a^2*b^10 + 14553*a^3*b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 1
0077696*a^6*b^2*c^4 + 15116544*a^7*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x^4 +
a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^
5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^
2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a
^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104
976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c
^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 5898
24*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a
*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^
5*b^2*c^8 + 4096*a^6*c^9)))*log(-1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^
2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 -
10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 716636
16*a^9*c^9 - (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*
c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 -
50823168*a^7*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 2
50609664*a^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 13
77*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*
c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a
^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^
14 - 262144*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^
3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280
*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^
8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^
18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b
^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13
+ 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*
a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096
*a^6*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b
^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4
*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b
^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 57
6*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c
^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262
144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10
+ 14553*a^3*b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6
*b^2*c^4 + 15116544*a^7*c^5)*sqrt(x)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a
^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 -
1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqr
t((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4

)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*
a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*
c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 +
240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 +
4096*a^6*c^9)))*log(1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 -
5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6
*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 +
(b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 + 195072
*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7
*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a^1
0*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c
^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^
2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11
+ 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*
a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024
*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
+ 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6
*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*
a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 1
29024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8
*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5
- 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*
sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24
*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 174
96*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c
^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064
*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15
)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a
^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10 + 14553*a^3*
b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 1
5116544*a^7*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 +
3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^
6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*
a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6
- 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^1
0 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 58982
4*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8
*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^
9)))*log(-1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b
^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 +
44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 + (b^23*c^3
- 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 + 195072*a^4*b^15*c
^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7*b^9*c^10 +

```
 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a^10*b^3*c^13
- 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*
a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8
- 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^
6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))
*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3
- (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a
^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7
+ 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b
^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 -
 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*
b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sqrt(-(b^7
+ 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4
 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8
 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*
c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a
^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^
12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c
^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7
- 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10 + 14553*a^3*b^8*c + 281
232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 15116544*a^7
*c^5)*sqrt(x)) - 4*(b*x^3 + 2*a*x)*sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.21Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 120, normalized size = 0.25

$$-\frac{\left(\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6} b-6\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2} a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{8\left(4ac-b^{2}\right)\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7} c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3} b\right)}+\frac{-\frac{b\,x^{\frac{7}{2}}}{2\left(4ac-b^{2}\right)}-\frac{a\,x^{\frac{3}{2}}}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(7/2)-1/2*a/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)-1/
8/(4*a*c-b^2)*sum((_R^6*b-6*_R^2*a)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Ro
otOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx^{\frac{7}{2}} + 2\,ax^{\frac{3}{2}}}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \int -\frac{bx^{\frac{5}{2}} - 6\,a\sqrt{x}}{4\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(7/2) + 2*a*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) - integrate(-1/4*(b*x^(5/2) - 6*a*sqrt(x))/((b^2*c - 4*a
*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

mupad [B] time = 6.44, size = 23808, normalized size = 50.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((a*x^(3/2))/(4*a*c - b^2) + (b*x^(7/2))/(2*(4*a*c - b^2)))/(a + b*x^2 +
c*x^4) - atan(((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*
b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b
^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 1638
4*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*
a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4*a*c -
 b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^
13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10
665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*
c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12
976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 6920
6016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11
- 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 3329
22880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 26508
00128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*
b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a
*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^
3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6
- 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677
```

7216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i - (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324* a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i)/((279936*a^8*c^5 + 343*a^4*b^8*c + 7350*a^5*b^6*c^2 + 58968*a^6*b^4*c^3 + 209952*a^7*b^2*c^4)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 3237

47840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 81705
04192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3
*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*
b^12*c)) + (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b
*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^
5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c
^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a
c - b^2)^15)^(1/2))/(8192(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 +
 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*
b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8
*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*
c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^1
2*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*
c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a
^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b
^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a
^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 58521
6*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b
^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c
^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*
a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8
*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*
b^2*c^14)))^(3/4) - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*
c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240
*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*
a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 9
6*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^
5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324
*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^
2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8
 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 -
57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^
(1/4)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^
2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 +
3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2
c^2(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^
20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3
784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 5767
1680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4
)*2i - 2*atan(((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*
b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b
^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 1638

4*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*
a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4*a*c -
 b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^
13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10
665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*
c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12
976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 6920
6016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11
 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 3329
22880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 26508
00128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a
^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(
4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752
*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c
^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1
6777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^
3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c
^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12
 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i - (x^(1/2)*(
49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712
*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^
3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b
^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13
*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 1066
5984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a
^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^
6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 1297
6128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 692060
16*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4) - (((5435817984*a^10*b*c
^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 32
3747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 817
0504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a
^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*
a*b^12*c)) + (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9
*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*
a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3
*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4
 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^
5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b
^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^

2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b
^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^
6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4
096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*
a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386
304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 -
585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*
a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b
^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 81
1008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 3244032
0*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*
a^11*b^2*c^14)))^(3/4)*1i + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*
a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c
^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c
^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b
*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^
5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c
^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a
c - b^2)^15)^(1/2))/(8192(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 +
1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*
b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8
*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*
c^14)))^(1/4))/((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4
*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*
b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 163
84*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504
*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4*a*c
- b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b
^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 1
0665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677721
6*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18
*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 1
2976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 692
06016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11
 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332
922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650
800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-
(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 275
2*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*
c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c -
b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a

^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i - (279936*a^8*c^5 + 343*a^4*b^8*c + 7350*a^5*b^6*c^2 + 58968*a^6*b^4*c^3 + 209952*a^7*b^2*c^4)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 1

2386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4) - atan(((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^

6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i - (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i)/((279936*a^8*c^5 + 343*a^4*b^8*c + 7350*a^5*b^6*c^2 + 58968*a^6*b^4*c^3 + 209952*a^7*b^2*c^4)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((5435817984*a^10*b*c^10 - 409

6*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a
^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a
^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^
3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)
) - (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 -
 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*
c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 3
24*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*
a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c
^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11
- 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14))
)^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5
- 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 +
2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6
 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5
 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*
c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5
*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^
8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*
c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 +
1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b
^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*
c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c
^14)))^(3/4) + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 +
 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*
b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^1
0*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^
2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 -
3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2
c^2(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^
20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3
784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 5767
1680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4
) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 -
32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 51
21245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 +
 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5
 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^
2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 5
5296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7
*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3
*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15
 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 1267

20*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*2i - 2*atan(((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^

11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^1
4)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c
^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8
 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a
^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b
^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*
a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 5852
16*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*
b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*
c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008
*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^
8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11
*b^2*c^14)))^(3/4)*1i - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*
b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 +
 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^
9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b
^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8
+ 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 10
56*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^1
4*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^
11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^1
4)))^(1/4) - (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^
13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7
*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*
a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^
5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(b^19 + b^4*(-(4
*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^1
3*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 106
65984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^
(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*
a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c
^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 129
76128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206
016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 -
 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 33292
2880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 265080
0128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 +
b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752
*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c
^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1

6777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4))/((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b

^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i - (279936*a^8*c^5 + 343*a^4*b^8*c + 7350*a^5*b^6*c^2 + 58968*a^6*b^4*c^3 + 209952*a^7*b^2*c^4)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15

```
*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 335052
8*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5
 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704
*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a
^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.841 $\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=483

$$\frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\mathrm{t}}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b\right.}}$$

Rubi [A] time = 1.03, antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1365, 1422, 212, 208, 205}

$$\frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2 + 4
*a*c + 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2
- 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((3*b^2 + 4*a*c + 3*b*Sqrt
[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(
1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4
)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2
)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{2a-3bx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right.}{4\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}}\right.}{4\left(b^2-4ac\right)^{3/2}\sqrt{-b+\sqrt{b^2-4ac}}}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^2+4ac+3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\,\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{(3}{}$$

Mathematica [C] time = 0.21, size = 127, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{3\#1^4b\log\left(\sqrt{x}-\#1\right)-2a\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{8\left(b^2-4ac\right)} - \frac{-2a\sqrt{x}-bx^{5/2}}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(-2*a*Sqrt[x] - b*x^(5/2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootS
um[a + b*#1^4 + c*#1^8 & , (-2*a*Log[Sqrt[x] - #1] + 3*b*Log[Sqrt[x] - #1]*
#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(8*(b^2 - 4*a*c))
```

IntegrateAlgebraic [C] time = 0.38, size = 201, normalized size = 0.42

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{3\#1^4bc\log\left(\sqrt{x}-\#1\right)+14ac\log\left(\sqrt{x}-\#1\right)-4b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{8c\left(4ac-b^2\right)} + \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{2c} + \frac{2a\sqrt{x}+bx^{5/2}}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a*Sqrt[x] + b*x^(5/2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum[a
 + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ]/(2*c) - Ro
otSum[a + b*#1^4 + c*#1^8 & , (-4*b^2*Log[Sqrt[x] - #1] + 14*a*c*Log[Sqrt[x
] - #1] + 3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(8*c*(-b^2
+ 4*a*c))
```

fricas [B] time = 12.62, size = 9245, normalized size = 19.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^
10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b
^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2
 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^
6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a
^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
- 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*
arctan(-1/2*(sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 -
1990720*a^3*b^9*c^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a
^6*b^3*c^6 - 180224*a^7*b*c^7 - (27*b^22*c - 820*a*b^20*c^2 + 10064*a^2*b^1
8*c^3 - 57024*a^3*b^16*c^4 + 44544*a^4*b^14*c^5 + 1505280*a^5*b^12*c^6 - 10
838016*a^6*b^10*c^7 + 38436864*a^7*b^8*c^8 - 79233024*a^8*b^6*c^9 + 9201254
4*a^9*b^4*c^10 - 49283072*a^10*b^2*c^11 + 4194304*a^11*c^12)*sqrt((6561*b^4
 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 -
 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b
^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt
((1476225*b^8 + 641520*a*b^6*c + 30816*a^2*b^4*c^2 - 8448*a^3*b^2*c^3 + 256
*a^4*c^4)*x + sqrt(1/2)*(111537*b^12 - 1375704*a*b^10*c + 5803760*a^2*b^8*c
^2 - 8961280*a^3*b^6*c^3 + 2522880*a^4*b^4*c^4 - 186368*a^5*b^2*c^5 + 4096*
a^6*c^6 + 8*(81*b^19*c - 2596*a*b^17*c^2 + 36416*a^2*b^15*c^3 - 292096*a^3*
b^13*c^4 + 1465856*a^4*b^11*c^5 - 4716544*a^5*b^9*c^6 + 9519104*a^6*b^7*c^7
 - 11075584*a^7*b^5*c^8 + 5832704*a^8*b^3*c^9 - 262144*a^9*b*c^10)*sqrt((65
61*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14
*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064
*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11))
)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4
096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^
16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024
*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^1
```

0 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)) + sqrt(1/2)*(2657205*b^19 - 57028212*a*b^17*c + 502044480*a^2*b^15*c^2 - 2306152704*a^3*b^13*c^3 + 5758457344*a^4*b^11*c^4 - 7169792000*a^5*b^9*c^5 + 2897625088*a^6*b^7*c^6 + 946012160*a^7*b^5*c^7 - 111345664*a^8*b^3*c^8 + 2883584*a^9*b*c^9 - (32805*b^26*c - 989172*a*b^24*c^2 + 12010848*a^2*b^22*c^3 - 66614144*a^3*b^20*c^4 + 38905600*a^4*b^18*c^5 + 1841587200*a^5*b^16*c^6 - 12771508224*a^6*b^14*c^7 + 43815469056*a^7*b^12*c^8 - 85947383808*a^8*b^10*c^9 + 90262732800*a^9*b^8*c^10 - 34319892480*a^10*b^6*c^11 - 9386852352*a^11*b^4*c^12 + 1895825408*a^12*b^2*c^13 - 67108864*a^13*c^14)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(x)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))/(332150625*a*b^12 + 321489000*a^2*b^10*c + 107535600*a^3*b^8*c^2 + 12061440*a^4*b^6*c^3 - 463104*a^5*b^4*c^4 - 104448*a^6*b^2*c^5 + 4096*a^7*c^6)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*arctan(1/2*(sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5177600*a^4*b

^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7*b*c^7 + (27
*b^22*c - 820*a*b^20*c^2 + 10064*a^2*b^18*c^3 - 57024*a^3*b^16*c^4 + 44544*
a^4*b^14*c^5 + 1505280*a^5*b^12*c^6 - 10838016*a^6*b^10*c^7 + 38436864*a^7*
b^8*c^8 - 79233024*a^8*b^6*c^9 + 92012544*a^9*b^4*c^10 - 49283072*a^10*b^2*
c^11 + 4194304*a^11*c^12)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*
c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10
*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 58982
4*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt((1476225*b^8 + 641520*a*b^6*c + 30
816*a^2*b^4*c^2 - 8448*a^3*b^2*c^3 + 256*a^4*c^4)*x + sqrt(1/2)*(111537*b^1
2 - 1375704*a*b^10*c + 5803760*a^2*b^8*c^2 - 8961280*a^3*b^6*c^3 + 2522880*
a^4*b^4*c^4 - 186368*a^5*b^2*c^5 + 4096*a^6*c^6 - 8*(81*b^19*c - 2596*a*b^1
7*c^2 + 36416*a^2*b^15*c^3 - 292096*a^3*b^13*c^4 + 1465856*a^4*b^11*c^5 - 4
716544*a^5*b^9*c^6 + 9519104*a^6*b^7*c^7 - 11075584*a^7*b^5*c^8 + 5832704*a
^8*b^3*c^9 - 262144*a^9*b*c^10)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/
(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^
4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 +
589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240
*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 +
3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a
*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^
3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 -
589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24
*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*
a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c -
240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 64
8*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376
*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^
8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c -
24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 61
44*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^
2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4
*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c +
16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^
5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a
^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c
^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c
^6 + 4096*a^6*c^7)) + sqrt(1/2)*(2657205*b^19 - 57028212*a*b^17*c + 5020444
80*a^2*b^15*c^2 - 2306152704*a^3*b^13*c^3 + 5758457344*a^4*b^11*c^4 - 71697
92000*a^5*b^9*c^5 + 2897625088*a^6*b^7*c^6 + 946012160*a^7*b^5*c^7 - 111345
664*a^8*b^3*c^8 + 2883584*a^9*b*c^9 + (32805*b^26*c - 989172*a*b^24*c^2 + 1
2010848*a^2*b^22*c^3 - 66614144*a^3*b^20*c^4 + 38905600*a^4*b^18*c^5 + 1841
587200*a^5*b^16*c^6 - 12771508224*a^6*b^14*c^7 + 43815469056*a^7*b^12*c^8 -
85947383808*a^8*b^10*c^9 + 90262732800*a^9*b^8*c^10 - 34319892480*a^10*b^6
*c^11 - 9386852352*a^11*b^4*c^12 + 1895825408*a^12*b^2*c^13 - 67108864*a^13

```
*c^14)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3
 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b
^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 26
2144*a^9*c^11)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a
^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3
840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b
^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*
b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 5
89824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a
*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^
5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (
b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*
c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^
2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 3
2256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^
4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 +
240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 +
4096*a^6*c^7)))/(332150625*a*b^12 + 321489000*a^2*b^10*c + 107535600*a^3*b^
8*c^2 + 12061440*a^4*b^6*c^3 - 463104*a^5*b^4*c^4 - 104448*a^6*b^2*c^5 + 40
96*a^7*c^6)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x
^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c -
24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 614
4*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b
^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*
b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 5
89824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b
^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*
c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) + (81*b^6 - 652*a
*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(b^13*c - 24*a*b^11*c^2 + 240*a^
2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a
^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*
c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^
5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 -
 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*
c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a
^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c
+ 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*
c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824
*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10
*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2
*c^6 + 4096*a^6*c^7)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 +
 (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^
4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*
a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 +
```

32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*
b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2
+ 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6
+ 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) - (81*
b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(b^13*c - 24*a*b^11*c
^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c
^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 -
36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6
- 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8
*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c -
240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c
^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 6
48*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 537
6*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c
^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c
- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6 + 4096*a^6*c^7)))) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*
a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 +
3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*
b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3
*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 -
589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*
a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a
^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqr
t(x) + (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(b^13*c -
24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 614
4*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/
(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^
4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 +
589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 76
0*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 128
0*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6
561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^1
4*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 34406
4*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)
))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) - ((b^2*c - 4*a*c^2)*x^4 + a*
b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b
^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3
*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b
^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4
- 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6
*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b
^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c

```
^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a
^2*c^2)*sqrt(x) - (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4
*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^
5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 1
6*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5
 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^
7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(
81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^
8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c
^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 +
576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*
c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 26214
4*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) - 4*(b*x^2 + 2*a)*
sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.2Unable to convert to re
al 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 118, normalized size = 0.24

$$\frac{\left(-3\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4 b+2a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{8\left(4ac-b^2\right)\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+\frac{-\frac{b\,x^{\frac{5}{2}}}{2\left(4ac-b^2\right)}-\frac{a\sqrt{x}}{4ac-b^2}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(5/2)-1/2*a/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1/
8/(4*a*c-b^2)*sum((-3*_R^4*b+2*a)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Root
Of(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,cx^{\frac{9}{2}}+bx^{\frac{5}{2}}}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}-\int -\frac{2\,cx^{\frac{7}{2}}+5\,bx^{\frac{3}{2}}}{4\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^(9/2) + b*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) - integrate(-1/4*(2*c*x^(7/2) + 5*b*x^(3/2))/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

mupad [B] time = 10.88, size = 26432, normalized size = 54.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan((((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3
*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b
^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 409
6*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^
5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2)
- 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9
*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 11
84*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^1
2*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^
4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10
*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^1
1 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c
^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 786
43200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*
b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^
2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 7
19360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7
*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c +
 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^
4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297
6128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016
*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) - (405*a^2*b^6*c^3 - 32*a^
5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^
4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)
^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719
360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b
^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 1
6777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4
+ 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 129761
28*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a
^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 20
```

25*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16
*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c
^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^
15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 7193
60*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^
3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16
777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 +
126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297612
8*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^
10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + ((((x^(1/2)*(603979776*a
^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*
c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^
9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((
-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b
^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5
259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c -
b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*
a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c
^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 -
57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^
(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 491
5200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*
a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16
*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^
8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^
5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^2
2*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 8110
08*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a
^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11
*b^2*c^12)))^(3/4) + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a
^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a
*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8
+ 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*
b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*
c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008
*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8
*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b
^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c
^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^
2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b
^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 +
960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b

^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*
(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c
^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*
a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*
b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^
2*c^12)))^(1/4)*1i)/(((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^
4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 -
581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/
(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^
4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 - 81*b^2*(-(4*a*c -
b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 -
719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a
^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c
+ 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*
c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12
976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 692060
16*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 2
0480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^
5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^
4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^
2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a
^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^
6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/
(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14
080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*
b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*
c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) - (405*a^2*
b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*
c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*
(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3
*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6
+ 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 1408
0*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^
12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^
10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(1
28*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^
5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 +
3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(
-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*
b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 +
4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(81
92*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080
*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^1
2*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^1

0 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - ((((x^(1/2)*
(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 281149
44*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 119537664
0*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^
2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b
^10*c)) + ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8
 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5
*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22
*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 81100
8*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^
8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*
b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^
11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8
- 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*
b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983
040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4
+ 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*
b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^1
3 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^1
6*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8
+ 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5
0331648*a^11*b^2*c^12)))^(3/4) + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^
4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^
2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 98304
0*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 +
2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^
15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13
- 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*
c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 +
32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 503
31648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 2
70*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*
c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*
c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040
*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2
727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^1
5*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 -
 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c
^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 3
2440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5033
1648*a^11*b^2*c^12)))^(1/4)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2)
 - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^
9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1
184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^

12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a
^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^1
0*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^
11 - 50331648*a^11*b^2*c^12)))^(1/4)*2i - ((a*x^(1/2))/(4*a*c - b^2) + (b*x
^(5/2))/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + atan((((((x^(1/2)*(6039797
76*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b
^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^
5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^
2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
- ((-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a
^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5
 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a
c - b^2)^15)^(1/2))/(8192(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1
056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^
14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^
9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12
)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 +
 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829
120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
- 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*
b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 272793
6*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c -
 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a
*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 -
811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 324403
20*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*
a^11*b^2*c^12)))^(3/4) - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 +
96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*
c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*
a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4
*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b
^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 81
1008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320
*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^
11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b
^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 24
0*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24
*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c
^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a
^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*
a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^
22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811
008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*
a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^1

1*b^2*c^12)))^(1/4)*1i + ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^1
5*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^
7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^1
0))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^
4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 + 81*b^2*(-(4*a*
c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c
^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 45875
20*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^
24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b
^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7
- 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69
206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10
 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 2621440
0*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 25
6*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 8
1*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 844
80*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^
5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/
2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3
- 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*
a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*
b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) + (405*
a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*
a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*
b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480
*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*
c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2)
)/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 -
14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^
6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^
6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2
)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 86
4*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c
^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b
^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*
a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c
^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))
/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 1
4080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6
*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6
*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i)/(((((x
^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 -
 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 11
95376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 +
 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -

24*a*b^10*c)) - ((-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^
8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727
936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c
 - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48
*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5
 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 3244
0320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5033164
8*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520
*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b
^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 2
56*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2
) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b
^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 -
1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a
^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*
a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^
10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c
^11 - 50331648*a^11*b^2*c^12)))^(3/4) - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918
*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2)
 - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9
*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 11
84*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^1
2*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^
4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10
*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^1
1 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*
c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 40
96*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a
^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) -
 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*
c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 118
4*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12
*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4
*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*
c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11
 - 50331648*a^11*b^2*c^12)))^(1/4) - ((((x^(1/2)*(603979776*a^9*b*c^11 - 10
2400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 16646144
0*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064
*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c
^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 + 81
b^2(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 8448
0*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5
*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 -

14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*2i + 2*atan((((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^

3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 458752
0*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^2
4*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^
18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 -
 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 692
06016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10
 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400
*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 -
 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 8
4480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*
b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^
3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 378470
4*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^
9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i +
(405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 +
 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17
 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 +
84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6
*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c
^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 37847
04*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a
^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i +
 (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*
c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a
^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^1
7 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2
 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a
^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20
*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 378
4704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680
*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) +
((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*
c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8
 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*
c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*
c^5 - 24*a*b^10*c)) + ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9830
40*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 +
 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b
^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13
 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16
*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 +

32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50
331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 4
91520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*
a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4
*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^
15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 7193
60*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^
3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16
777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 +
126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297612
8*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^
10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i - (405*a^2*b^6*c^3 - 32*a^
5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^
4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)
^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719
360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b
^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 1
6777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4
+ 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 129761
28*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a
^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + (x^(1/2)*(128*a^6*c^7 +
2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/
(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^
4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^
2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 7
19360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7
*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c +
16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^
4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297
6128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016
*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4))/(((((x^(1/2)*(603979776*a
^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*
c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^
9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((
-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b
^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5
259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c -
b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*
a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c
^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 -
57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^
(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 491
5200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*
a^7*b^3*c^9)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -

16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b
*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936
*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c +
4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*
b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 8
11008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 3244032
0*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a
^11*b^2*c^12)))^(3/4)*1i + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4
+ 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
- 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*
b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 272793
6*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c +
4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a
*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 -
811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 324403
20*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*
a^11*b^2*c^12)))^(1/4)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*
a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6
+ 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5
- 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^
8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727
936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c
+ 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48
*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5
- 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 3244
0320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5033164
8*a^11*b^2*c^12)))^(1/4)*1i - ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^
2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b
^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^
3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 - 81*b^2*(-
(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b
^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 +
4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*
a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12
*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10
+ 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b
*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26
214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(
b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*
b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c
^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 525926
4*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b

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^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 +
3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671
680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)
*1i - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*
(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81
*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*
c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 52592
64*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*
b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 +
 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 5767
1680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4
)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^
4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(
81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^1
3*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 525
9264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^
2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6
 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57
671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1
/4)*1i))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 +
 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b
^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*
(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c
^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*
a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*
b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^
2*c^12)))^(1/4) + 2*atan((((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^
15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c
^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^
10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a
^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 + 81*b^2*(-(4*a
*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*
c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587
520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b
^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*
b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7
 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 6
9206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^1
0 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 262144
00*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8
+ 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17
 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 +
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84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) + ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976

128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*
a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i - (405*a^2*b^6*c^3 - 32*
a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*
b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^
2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 7
19360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7
*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c +
16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^
4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297
6128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016
*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + (x^(1/2)*(128*a^6*c^7
+ 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6)
)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*
b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c -
b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 -
719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a
^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c
+ 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*
c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12
976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 692060
16*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4))/(((((x^(1/2)*(603979776
*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^1
1*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*
c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2
- 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) -
((-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2
*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 -
5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 105
6*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14
*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9
- 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12))
)^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4
915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 12582912
0*a^7*b^3*c^9)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
- 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8
*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 27279
36*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c
- 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*
a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 -
811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440
320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648
*a^11*b^2*c^12)))^(3/4)*1i + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^
4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^
3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^

8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727
936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c
 - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48
*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5
- 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 3244
0320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5033164
8*a^11*b^2*c^12)))^(1/4)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 27
0*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c
^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c
^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*
a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 27
27936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15
*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 -
48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^
5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32
440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331
648*a^11*b^2*c^12)))^(1/4)*1i - ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*
a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5
*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*
b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 +
3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 + 81*b^2*
(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3
*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6
+ 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 1408
0*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^
12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^
10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8
*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 -
26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2
*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(8
1*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13
*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259
264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^
2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2
*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6
+ 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 576
71680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/
4)*1i - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(
2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(
81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^1
3*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 525
9264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^
2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6

```
 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57
671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1
/4)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*
a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2
- 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(
-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b
^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5
259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c -
 b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*
a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c
^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 -
57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^
(1/4)*1i))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8
 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5
*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22
*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 81100
8*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^
8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*
b^2*c^12)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.842 $\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=450

$$-\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\cdots}{\sqrt[4]{\sqrt{b^2-4\cdots}}}\right.}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}\,-\cdots}}$$

Rubi [A] time = 0.71, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1364, 1510, 298, 205, 208}

$$-\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (c^(1/4)*(
4*b + Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4)) + (c^(1/4)*(4*b + Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(
3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4
)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4+cx^8\right)^2}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b-2cx^4\right)}{a+bx^4+cx^8}\,dx,x,\sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(4b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx,x,\sqrt{\right.}{2\left(b^2-4ac\right)^{3/2}}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(\sqrt{c}\left(4b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\right.}{2\sqrt{2}\left(b^2-4ac\right)^{3/2}}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt[4]{c}\left(4b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(4b\right.}{2\,2^{3/4}}$$

Mathematica [C] time = 0.21, size = 109, normalized size = 0.24

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{2\#1^4c\log\left(\sqrt{x}-\#1\right)-3b\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4x^{3/2}\left(b+2cx^2\right)}{a+bx^2+cx^4}}{8\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/8*((4*x^(3/2)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 +
c*#1^8 & , (-3*b*Log[Sqrt[x] - #1] + 2*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*
c*#1^5) & ])/(b^2 - 4*a*c)
```

IntegrateAlgebraic [C] time = 0.32, size = 121, normalized size = 0.27

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{3b\log\left(\sqrt{x}-\#1\right)-2\#1^4c\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{8\left(b^2-4ac\right)} - \frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(x^(3/2)*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum[
a + b*#1^4 + c*#1^8 & , (3*b*Log[Sqrt[x] - #1] - 2*c*Log[Sqrt[x] - #1]*#1^4
)/(b*#1 + 2*c*#1^5) & ]/(8*(b^2 - 4*a*c))
```

fricas [B] time = 31.67, size = 9757, normalized size = 21.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*
b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b
^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18
 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^
4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a
^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
- 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*
arctan(-((81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(a*b^13
- 24*a^2*b^11*c + 240*a^3*b^9*c^2 - 1280*a^4*b^7*c^3 + 3840*a^5*b^5*c^4 - 6
144*a^6*b^3*c^5 + 4096*a^7*b*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2
)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*
a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7
 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt((74733890625*b^16*c^2 + 11
2193100000*a*b^14*c^3 + 68088600000*a^2*b^12*c^4 + 20761920000*a^3*b^10*c^5
 + 3063744000*a^4*b^8*c^6 + 113909760*a^5*b^6*c^7 - 19021824*a^6*b^4*c^8 -
1179648*a^7*b^2*c^9 + 65536*a^8*c^10)*x - 1/2*sqrt(1/2)*(2989355625*b^21*c
- 23678649000*a*b^19*c^2 + 7135160400*a^2*b^17*c^3 + 277460328960*a^3*b^15*
c^4 - 338956033536*a^4*b^13*c^5 - 492326940672*a^5*b^11*c^6 - 183476674560*
a^6*b^9*c^7 - 21980119040*a^7*b^7*c^8 + 750059520*a^8*b^5*c^9 + 190316544*a
^9*b^3*c^10 - 7340032*a^10*b*c^11 + (36905625*a*b^28*c - 1159839000*a^2*b^2
6*c^2 + 15854324400*a^3*b^24*c^3 - 122710429440*a^4*b^22*c^4 + 584418357504
*a^5*b^20*c^5 - 1728949905408*a^6*b^18*c^6 + 2983008514048*a^7*b^16*c^7 - 2
317983285248*a^8*b^14*c^8 - 462348419072*a^9*b^12*c^9 + 1339972648960*a^10*
b^10*c^10 + 254402363392*a^11*b^8*c^11 - 161849802752*a^12*b^6*c^12 - 51220
840448*a^13*b^4*c^13 - 2550136832*a^14*b^2*c^14 + 268435456*a^15*c^15)*sqrt
((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*
b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 34
4064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c
^9)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c
 + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5
```

```
 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*
a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 12
9024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^
2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280
*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))) + (221
43375*b^14*c - 161619300*a*b^12*c^2 + 233100720*a^2*b^10*c^3 + 224213184*a^
3*b^8*c^4 + 48450816*a^4*b^6*c^5 + 185344*a^5*b^4*c^6 - 487424*a^6*b^2*c^7
+ 16384*a^7*c^8 - 4*(273375*a*b^21*c - 6355800*a^2*b^19*c^2 + 60732720*a^3*
b^17*c^3 - 301810176*a^4*b^15*c^4 + 798453248*a^5*b^13*c^5 - 951914496*a^6*
b^11*c^6 + 38461440*a^7*b^9*c^7 + 557711360*a^8*b^7*c^8 + 179503104*a^9*b^5
*c^9 + 11010048*a^10*b^3*c^10 - 1048576*a^11*b*c^11)*sqrt((6561*b^4 - 648*a
*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^
5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 -
 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(x))*sqr
t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*
b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b
^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18
 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^
4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a
^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
- 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))/
(332150625*b^12*c + 321489000*a*b^10*c^2 + 107535600*a^2*b^8*c^3 + 12061440
*a^3*b^6*c^4 - 463104*a^4*b^4*c^5 - 104448*a^5*b^2*c^6 + 4096*a^6*c^7)) - 4
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(
1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c
+ 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5
+ 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a
^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129
024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2
*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*arctan(
((81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(a*b^13 - 24*a^2
*b^11*c + 240*a^3*b^9*c^2 - 1280*a^4*b^7*c^3 + 3840*a^5*b^5*c^4 - 6144*a^6*
b^3*c^5 + 4096*a^7*b*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b
^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10
*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 58982
4*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt((74733890625*b^16*c^2 + 1121931000
00*a*b^14*c^3 + 68088600000*a^2*b^12*c^4 + 20761920000*a^3*b^10*c^5 + 30637
44000*a^4*b^8*c^6 + 113909760*a^5*b^6*c^7 - 19021824*a^6*b^4*c^8 - 1179648*
a^7*b^2*c^9 + 65536*a^8*c^10)*x - 1/2*sqrt(1/2)*(2989355625*b^21*c - 236786
49000*a*b^19*c^2 + 7135160400*a^2*b^17*c^3 + 277460328960*a^3*b^15*c^4 - 33
8956033536*a^4*b^13*c^5 - 492326940672*a^5*b^11*c^6 - 183476674560*a^6*b^9*
c^7 - 21980119040*a^7*b^7*c^8 + 750059520*a^8*b^5*c^9 + 190316544*a^9*b^3*c
^10 - 7340032*a^10*b*c^11 - (36905625*a*b^28*c - 1159839000*a^2*b^26*c^2 +
15854324400*a^3*b^24*c^3 - 122710429440*a^4*b^22*c^4 + 584418357504*a^5*b^2
```

0*c^5 - 1728949905408*a^6*b^18*c^6 + 2983008514048*a^7*b^16*c^7 - 231798328
5248*a^8*b^14*c^8 - 462348419072*a^9*b^12*c^9 + 1339972648960*a^10*b^10*c^1
0 + 254402363392*a^11*b^8*c^11 - 161849802752*a^12*b^6*c^12 - 51220840448*a
^13*b^4*c^13 - 2550136832*a^14*b^2*c^14 + 268435456*a^15*c^15)*sqrt((6561*b
^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2
 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8
*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sq
rt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a
^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*
a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16
*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7
*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 -
262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6
*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(sqrt(1/2)
*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 24
0*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 40
96*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b
^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*
a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8
 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))) + (22143375
*b^14*c - 161619300*a*b^12*c^2 + 233100720*a^2*b^10*c^3 + 224213184*a^3*b^8
*c^4 + 48450816*a^4*b^6*c^5 + 185344*a^5*b^4*c^6 - 487424*a^6*b^2*c^7 + 163
84*a^7*c^8 + 4*(273375*a*b^21*c - 6355800*a^2*b^19*c^2 + 60732720*a^3*b^17*
c^3 - 301810176*a^4*b^15*c^4 + 798453248*a^5*b^13*c^5 - 951914496*a^6*b^11*
c^6 + 38461440*a^7*b^9*c^7 + 557711360*a^8*b^7*c^8 + 179503104*a^9*b^5*c^9
+ 11010048*a^10*b^3*c^10 - 1048576*a^11*b*c^11)*sqrt((6561*b^4 - 648*a*b^2*
c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^1
2*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 5898
24*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(x)*sqrt(sqrt
(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c
 + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5
 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*
a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 12
9024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^
2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280
*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))))/(3321
50625*b^12*c + 321489000*a*b^10*c^2 + 107535600*a^2*b^8*c^3 + 12061440*a^3*
b^6*c^4 - 463104*a^4*b^4*c^5 - 104448*a^5*b^2*c^6 + 4096*a^6*c^7)) - ((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sq
rt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a
^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*
a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16
*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7
*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 -

262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6
*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(1/2*sqrt(1
/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3
 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224
*a^7*b*c^7 - (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b
^16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6
 + 38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 49
283072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16
*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3
+ 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9
*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(8
1*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8
*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^
6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 5
76*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c
^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144
*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*
a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((656
1*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*
c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*
a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))
/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^
4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^
2 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + ((b^2*c
 - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt
(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3
*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^
7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c
 + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b
^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 26
2144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c
^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(-1/2*sqrt(1/
2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3
+ 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*
a^7*b*c^7 - (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^
16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6
+ 38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 492
83072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*
a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 +
 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*
b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81
*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*
c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6

)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^2 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(1/2*sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7*b*c^7 + (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6 + 38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 49283072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^2 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + ((b^2*c - 4

```
*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(
81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^
8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c
^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c +
576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*
c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 26214
4*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3
+ 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(-1/2*sqrt(1/2)*
(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5
177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7
*b*c^7 + (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^16*
c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6 + 3
8436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 492830
72*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2
*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32
256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4
*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b^
5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*s
qrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a
^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 +
 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^1
1*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 384
0*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a*b^
3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^
4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2
- 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*
b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*
b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^
4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^2 +
47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + 4*(2*c*x^3 +
 b*x)*sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x
^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.37Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 121, normalized size = 0.27

$$\frac{\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6 c-3\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2 b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{8\left(4ac-b^2\right)\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+\frac{\frac{2c\,x^{\frac{7}{2}}}{8ac-2b^2}+\frac{2b\,x^{\frac{3}{2}}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(7/2)+1/4*b/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)+1/8
/(4*a*c-b^2)*sum((2*_R^6*c-3*_R^2*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=R
ootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,cx^{\frac{7}{2}}+bx^{\frac{3}{2}}}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}+\int-\frac{2\,cx^{\frac{5}{2}}-3\,b\sqrt{x}}{4\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^(7/2) + b*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) + integrate(-1/4*(2*c*x^(5/2) - 3*b*sqrt(x))/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

mupad [B] time = 6.06, size = 21913, normalized size = 48.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((b*x^(3/2))/(2*(4*a*c - b^2)) + (c*x^(7/2))/(4*a*c - b^2))/(a + b*x^2 + c*
x^4) - atan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c
^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8
+ 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11
)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a
^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/
2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a
^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5
 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a
*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1
056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^
14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^
8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11
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)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 94
69952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807
296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^
12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^
(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a
^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^
7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 167772
16*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126
720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^
8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b
^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) + (x^(1/2)*(576*a^4*b*c^8 - 5625*
a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6
+ 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5
- 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8
*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 27279
36*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c
- 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*
a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 -
811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440
320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648
*a^12*b^2*c^11)))^(1/4)*1i - (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 24
33024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693
440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296
256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*
b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b
^12*c)) + (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a
^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 272
7936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*
c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 4
8*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4
- 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 324
40320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 503316
48*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a
^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*
b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^
2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4
*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^1
1*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 45
87520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^
4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c
^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 +
69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) - (x^(1/2)*(576*a

^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i)/(((16875*a*b^7*c^5 + 320*a^4*b*c^8 + 13500*a^2*b^5*c^6 + 3600*a^3*b^3*c^7)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^1

1*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) + (((110592*a*b^16*c^4 - 13421
7728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4
*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*
b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^
10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^
6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15
)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360
*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*
c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 1677
7216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 1
26720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*
a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11
*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b
^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^
7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10
 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 128
0*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*
b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c
^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 525926
4*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b
^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 +
3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671
680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)
 - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b
^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4
*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^1
1*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 45
87520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^
4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c
^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 +
 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)))*(-(81*b^17 + 81
b^2(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 8448
0*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5
*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 -
 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a
^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*
b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*2i - 2*a
tan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 212
00896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812
352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(
b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c

^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81
*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*
c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 52592
64*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*
b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 +
 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 5767
1680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4
)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^
3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*
b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 +
4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144
*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2)
 - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^
9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1
184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^
13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a
^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^1
0*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^
10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*
b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 +
240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b
*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936
*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c +
4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^
2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 8
11008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 3244032
0*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a
^12*b^2*c^11)))^(1/4) - (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024
*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a
^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a
^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c
^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c
)) + (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*
c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*
a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4
*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2
*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 81
1008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320
*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^
12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^
14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c
^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^1
1)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840

*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a
*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*
c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587
520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a
*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*
b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6
 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 6
9206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(576*
a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^1
2 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(
1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^
4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7
 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 1677721
6*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 1267
20*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8
*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^
4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4))/((((110592*a*b^16*c^4 - 134217728
*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^1
0*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*
c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c
^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^
2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1
/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4
*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7
 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216
*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 12672
0*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*
b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4
*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*
c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 +
180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 30
1989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280
*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b
^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^
2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264
*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^
20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3
784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 576716
80*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*
1i - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3
*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 +
3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-
(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b

^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 +
4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*
a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12
*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9
 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i - (16875*a*b
^7*c^5 + 320*a^4*b*c^8 + 13500*a^2*b^5*c^6 + 3600*a^3*b^3*c^7)/(64*(b^14 -
16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21
504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((110592*a*b^16*c^4
- 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687
168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 10317987
84*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336
*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 2
8672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c -
b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 -
 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a
^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24
 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*
c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12
976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 692060
16*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36
864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*
b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^
4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8
*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c
)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*
a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^
5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*
a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c +
1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b
^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c
^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^1
1)))^(3/4)*1i + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6
 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3
*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17
- 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 +
84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6
*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c
^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 37847
04*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a
^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i))
*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2
*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 -
 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c

- b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) - 2*atan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) - (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184

*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*
c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*
b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c
^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10
- 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 -
909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879
360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 30198988
8*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b
^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 +
81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84
480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b
^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2
- 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704
*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^1
0*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i + (
x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c
^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a
^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c
- b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^
3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 458752
0*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b
^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^
18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 -
12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 692
06016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4))/((((110592*a*b^16*c^
4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 876
87168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 103179
8784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 3
36*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 +
28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c
- b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3
- 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520
*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^
24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^1
8*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 -
12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 6920
6016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 +
36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^
4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*
b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b
^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10
c)))(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 96
0*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*
c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(

4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i - (16875*a*b^7*c^5 + 320*a^4*b*c^8 + 13500*a^2*b^5*c^6 + 3600*a^3*b^3*c^7)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5

- 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) - atan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4

*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i - (((110592*a*b^16*c^4 - 1342177
28*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b
^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^
4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10
*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*
b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^
(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a
^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^
7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 167772
16*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126
720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^
8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b
^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^1
6*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7
 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 -
301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^
17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2
 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*
a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^2
0*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 37
84704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 5767168
0*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) -
 (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3
c^7))/(16(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840
*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a
*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*
c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587
520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a
*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*
b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6
 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 6
9206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i)/((16875*a*b^7*c
^5 + 320*a^4*b*c^8 + 13500*a^2*b^5*c^6 + 3600*a^3*b^3*c^7)/(64*(b^14 - 1638
4*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*
a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((110592*a*b^16*c^4 - 13
4217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*
a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a
^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2
*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672
*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)
^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719
360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b
^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 1

6777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) + (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^2

```
2*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008
*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9
*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b
^2*c^11)))^(3/4) - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*
c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^
17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2
 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*
a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^2
0*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 37
84704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 5767168
0*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)))
*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2
*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 -
 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 105
6*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14
*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8
- 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11))
)^(1/4)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.843 $\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=442

$$\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tanh^{-1}\left(\cdots\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4a\cdots\right.}$$

Rubi [A] time = 0.70, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1364, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (c^(3/4)*(
3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 -
 4*a*c])^(3/4)) + (c^(3/4)*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(
-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*Arc
Tanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*
(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{b-6cx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x\right.}{2\left(b^2-4ac\right)\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{c^{3/4}\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(3-\right.}{2\sqrt[4]{2}\left(b^2\right.}$$

Mathematica [C] time = 0.23, size = 111, normalized size = 0.25

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{6\#1^4c\log\left(\sqrt{x}-\#1\right)-b\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+\frac{4\sqrt{x}\left(b+2cx^2\right)}{a+bx^2+cx^4}}{8\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/8*((4*Sqrt[x]*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 +
c*#1^8 & , (-(b*Log[Sqrt[x] - #1]) + 6*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 +
2*c*#1^7) & ])/(b^2 - 4*a*c)
```

IntegrateAlgebraic [C] time = 0.27, size = 122, normalized size = 0.28

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{b\log\left(\sqrt{x}-\#1\right)-6\#1^4c\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{8\left(b^2-4ac\right)} - \frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(Sqrt[x]*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum[
a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] - 6*c*Log[Sqrt[x] - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(8*(b^2 - 4*a*c))
```

fricas [B] time = 29.11, size = 10570, normalized size = 23.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a
^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4
*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b
^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 57
6*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*
c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262
144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*
c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*arctan(1/2*(sqr
t(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^
4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*
c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 - (a^3*b^23 - 20*a^4*b^21*c +
 432*a^5*b^19*c^2 - 11712*a^6*b^17*c^3 + 195072*a^7*b^15*c^4 - 1935360*a^8*
b^13*c^5 + 12214272*a^9*b^11*c^6 - 50823168*a^10*b^9*c^7 + 139788288*a^11*b
^7*c^8 - 245628928*a^12*b^5*c^9 + 250609664*a^13*b^3*c^10 - 113246208*a^14*
b*c^11)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104
976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c
^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 5898
24*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt((49*b^12*c^
2 + 3150*a*b^10*c^3 + 95985*a^2*b^8*c^4 + 1621296*a^3*b^6*c^5 + 15746400*a^
4*b^4*c^6 + 75582720*a^5*b^2*c^7 + 136048896*a^6*c^8)*x + 1/2*sqrt(1/2)*(b^
18 + 52*a*b^16*c + 1269*a^2*b^14*c^2 + 14294*a^3*b^12*c^3 + 48608*a^4*b^10*
c^4 - 679392*a^5*b^8*c^5 - 4209408*a^6*b^6*c^6 - 4105728*a^7*b^4*c^7 + 2149
90848*a^8*b^2*c^8 - 483729408*a^9*c^9 - (a^3*b^23 + 7*a^4*b^21*c - 152*a^5*
b^19*c^2 - 2960*a^6*b^17*c^3 + 44032*a^7*b^15*c^4 + 60928*a^8*b^13*c^5 - 44
44160*a^9*b^11*c^6 + 36855808*a^10*b^9*c^7 - 153681920*a^11*b^7*c^8 + 36352
8192*a^12*b^5*c^9 - 467140608*a^13*b^3*c^10 + 254803968*a^14*b*c^11)*sqrt((
b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(
a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^1
0*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^
7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*
a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2
- 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sq
```

rt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^
4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256
*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^
4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
+ 4096*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^
3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a
^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377
*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*
c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^1
1*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8
 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)) - sqrt(1/2
)*(7*b^24*c + 400*a*b^22*c^2 + 7843*a^2*b^20*c^3 + 22574*a^3*b^18*c^4 - 139
5688*a^4*b^16*c^5 - 11961472*a^5*b^14*c^6 + 98703360*a^6*b^12*c^7 + 1408361
472*a^7*b^10*c^8 - 12100202496*a^8*b^8*c^9 + 1218281472*a^9*b^6*c^10 + 2412
19731456*a^10*b^4*c^11 - 812665405440*a^11*b^2*c^12 + 835884417024*a^12*c^1
3 - (7*a^3*b^29*c + 85*a^4*b^27*c^2 + 1764*a^5*b^25*c^3 - 37920*a^6*b^23*c^
4 - 103296*a^7*b^21*c^5 - 2564352*a^8*b^19*c^6 + 145468416*a^9*b^17*c^7 - 1
602797568*a^10*b^15*c^8 + 6543507456*a^11*b^13*c^9 + 7533166592*a^12*b^11*c
^10 - 193399619584*a^13*b^9*c^11 + 890247315456*a^14*b^7*c^12 - 20785209016
32*a^15*b^5*c^13 + 2556193406976*a^16*b^3*c^14 - 1320903770112*a^17*b*c^15)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(x)*sqrt(-(b^7 + 21
*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4
096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3
+ 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b
^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 -
 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5 + 4096*a^9*c^6)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 1
68*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c
^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10
*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c
^5 + 4096*a^9*c^6)))/(2401*b^16*c^3 + 179046*a*b^14*c^4 + 6354369*a^2*b^12*
c^5 + 131902344*a^3*b^10*c^6 + 1713103344*a^4*b^8*c^7 + 13740938496*a^5*b^6
*c^8 + 65167421184*a^6*b^4*c^9 + 166523848704*a^7*b^2*c^10 + 176319369216*a
^8*c^11)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^

2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^
3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a
^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377
*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*
c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^1
1*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8
 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*arctan(-1
/2*(sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 -
2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a
^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 + (a^3*b^23 - 20*a^4*b
^21*c + 432*a^5*b^19*c^2 - 11712*a^6*b^17*c^3 + 195072*a^7*b^15*c^4 - 19353
60*a^8*b^13*c^5 + 12214272*a^9*b^11*c^6 - 50823168*a^10*b^9*c^7 + 139788288
*a^11*b^7*c^8 - 245628928*a^12*b^5*c^9 + 250609664*a^13*b^3*c^10 - 11324620
8*a^14*b*c^11)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^
3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9
*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6
 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt((49*
b^12*c^2 + 3150*a*b^10*c^3 + 95985*a^2*b^8*c^4 + 1621296*a^3*b^6*c^5 + 1574
6400*a^4*b^4*c^6 + 75582720*a^5*b^2*c^7 + 136048896*a^6*c^8)*x + 1/2*sqrt(1
/2)*(b^18 + 52*a*b^16*c + 1269*a^2*b^14*c^2 + 14294*a^3*b^12*c^3 + 48608*a^
4*b^10*c^4 - 679392*a^5*b^8*c^5 - 4209408*a^6*b^6*c^6 - 4105728*a^7*b^4*c^7
 + 214990848*a^8*b^2*c^8 - 483729408*a^9*c^9 + (a^3*b^23 + 7*a^4*b^21*c - 1
52*a^5*b^19*c^2 - 2960*a^6*b^17*c^3 + 44032*a^7*b^15*c^4 + 60928*a^8*b^13*c
^5 - 4444160*a^9*b^11*c^6 + 36855808*a^10*b^9*c^7 - 153681920*a^11*b^7*c^8
+ 363528192*a^12*b^5*c^9 - 467140608*a^13*b^3*c^10 + 254803968*a^14*b*c^11)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(-(b^7 + 21*a*b^5*c
 + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b
^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*
c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976
*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3
+ 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*
a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*
b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b
^2*c^5 + 4096*a^9*c^6)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b
^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b
^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a
^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10
*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7
 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240
*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 409

6*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (
a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^
4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*
b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 5
76*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8
*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 26
2144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6
*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)) - sqrt(1/2)*(7*
b^24*c + 400*a*b^22*c^2 + 7843*a^2*b^20*c^3 + 22574*a^3*b^18*c^4 - 1395688*
a^4*b^16*c^5 - 11961472*a^5*b^14*c^6 + 98703360*a^6*b^12*c^7 + 1408361472*a
^7*b^10*c^8 - 12100202496*a^8*b^8*c^9 + 1218281472*a^9*b^6*c^10 + 241219731
456*a^10*b^4*c^11 - 812665405440*a^11*b^2*c^12 + 835884417024*a^12*c^13 + (
7*a^3*b^29*c + 85*a^4*b^27*c^2 + 1764*a^5*b^25*c^3 - 37920*a^6*b^23*c^4 - 1
03296*a^7*b^21*c^5 - 2564352*a^8*b^19*c^6 + 145468416*a^9*b^17*c^7 - 160279
7568*a^10*b^15*c^8 + 6543507456*a^11*b^13*c^9 + 7533166592*a^12*b^11*c^10 -
 193399619584*a^13*b^9*c^11 + 890247315456*a^14*b^7*c^12 - 2078520901632*a^
15*b^5*c^13 + 2556193406976*a^16*b^3*c^14 - 1320903770112*a^17*b*c^15)*sqrt
((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)
/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a
^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*
c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(
-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*
b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b
^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^
3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 -
5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12
*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(
a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^
4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^
2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 -
1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt
((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)
/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a
^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*
c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c +
240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 +
4096*a^9*c^6)))/(2401*b^16*c^3 + 179046*a*b^14*c^4 + 6354369*a^2*b^12*c^5 +
 131902344*a^3*b^10*c^6 + 1713103344*a^4*b^8*c^7 + 13740938496*a^5*b^6*c^8
+ 65167421184*a^6*b^4*c^9 + 166523848704*a^7*b^2*c^10 + 176319369216*a^8*c^
11)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt
(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^
3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^
4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576
*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c

^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 2621
44*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 2
25*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) + 1/2*(b^9 + 19*a*
b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 - (a^3*b^14 - 1
2*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 399
36*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c
 + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^
7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129
024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*
b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a
^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 -
 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqr
t((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4
)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*
a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4
*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c +
 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 +
 4096*a^9*c^6)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^
3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 +
3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c
+ 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7
*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 1290
24*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b
^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1
280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log
((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) - 1/2
*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 -
(a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*
b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8
 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6
*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b
^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 +
 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*
b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*
a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096
*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 1
04976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12
*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 58
9824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24
*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*
a^8*b^2*c^5 + 4096*a^9*c^6)))) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c +
 (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*
c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a

^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) + 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 + (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) - 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 + (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3

```
+ 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) + 4*(2*c*x^2 + b)*
sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.03Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 118, normalized size = 0.27

$$\frac{\left(6\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4 c-b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{8\left(4ac-b^2\right)\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+\frac{\frac{2c\,x^{\frac{5}{2}}}{8ac-2b^2}+\frac{2b\sqrt{x}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(5/2)+1/4*b/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1/8
/(4*a*c-b^2)*sum((6*_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_
Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bcx^{\frac{9}{2}}+\left(b^2-2\,ac\right)x^{\frac{5}{2}}}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}+\int -\frac{bcx^{\frac{7}{2}}+\left(b^2+6\,ac\right)x^{\frac{3}{2}}}{4\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(9/2) + (b^2 - 2*a*c)*x^(5/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-1/4*(b*c*x^(7/2) + (b
^2 + 6*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2), x)
```

mupad [B] time = 10.63, size = 28713, normalized size = 64.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(a + b*x^2 + c*x^4)^2,x)

[Out] atan(((((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 692060

16*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - ((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i)/(((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 -

12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) + ((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3

350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^2
0*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 37
84704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671
680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)
*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^
10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 -
134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*
b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 10066
3296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b
^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^
11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^
4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/
2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 5529
6*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^
5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*
b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a
^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*
a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b
^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4
*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 116
64*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16
*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 9
6*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^
5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324
*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^
5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5
+ 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 -
57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^
(1/4) + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a
^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 +
3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)
^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^
3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 1066598
4*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2
) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16
777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 +
126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 1297612
8*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*
a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)))*((b^4*(-(4*a*c - b^2)^15)
^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 +
55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^
7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +

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3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 167772
16*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126
720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^
10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13
*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*2i + 2*atan((((((((b^4*(-(4*a*c
 - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*
b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 -
10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^
24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^1
8*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 -
12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69
206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11
+ 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b
^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^
10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6
*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 +
73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*
a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 40
96*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a
^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 123863
04*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 5
85216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a
^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*
b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811
008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320
*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a
^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(
2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b
^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2
- 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6
*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14
080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*
b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b
^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i + (x^(
1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))
/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^
4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) -
 b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^
4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^
7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17
*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*
```

c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*
b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*
c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^1
0 - 50331648*a^14*b^2*c^11)))^(1/4) - ((((((b^4*(-(4*a*c - b^2)^15)^(1/2) -
b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^
4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^
7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17
*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*
c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*
b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*
c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^1
0 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 -
73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a
^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 2
56*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(20
48*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6
+ 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485
760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^
2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b
^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a
^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 +
3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b
^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 +
3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 576
71680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/
4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^
4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^
15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3
+ 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984
*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 167
77216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 +
126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128
*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a
^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(1225*b^6*c^7
- 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^
6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^
2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^
9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216
*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^
3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(
4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*
c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a

^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4))/(((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680

*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i
 + ((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2
*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3
350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^2
0*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 37
84704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671
680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)
*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^
10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 -
 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a
^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 10
0663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^
4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3
c^11))/(8(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840
*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^
(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 5
5296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7
*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3
*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 1677721
6*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 1267
20*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^1
0*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*
b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6
 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^
3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c
^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*
b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8
 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1
056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^
14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*
c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^
11)))^(1/4)*1i - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8
+ 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*
b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a
*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^
3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6
- 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*
b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b
^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6
- 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 +
69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i))*((b^4*(-(4*a*

c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3
*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 -
 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b
^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^
18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 -
 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 6
9206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) + ((b*x^(1/2))/(2*(
4*a*c - b^2)) + (c*x^(5/2))/(4*a*c - b^2))/(a + b*x^2 + c*x^4) + atan((((((
(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*
c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528
*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
)/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2
 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*
a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^
12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(1006
63296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6
 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 13421
7728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^
3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a
^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8
 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(
8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*
c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(
1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4
*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7
 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*
c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c
^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b
^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c
^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10
 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^
2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^
6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^
2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 -
3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2
c^2(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^
20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3
784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 5767
1680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4
) - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b
^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 384
0*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c -

b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - (((((-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 +

585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328
*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^
4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 8
11008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 324403
20*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648
*a^14*b^2*c^11)))^(1/4)*1i)/((((((-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) +
12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c
^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 1789
1328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*
a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 4
8*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4
 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32
440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 5033
1648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a
^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c
^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4
 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4
 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a
^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5
*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-
(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^
2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a
^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/
(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 -
14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^
9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12
*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232
*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^
4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(
1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4
*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7
 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*
c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c
^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b
^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c
^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10
 - 50331648*a^14*b^2*c^11)))^(1/4) - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^1
0 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^
10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a
^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 -
 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^

2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b
^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 +
3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 576
71680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/
4) + (((((-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*
a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5
- 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^
15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*
b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 +
3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57
671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1
/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3
*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^
9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a
^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 10
0663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^
4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3
c^11))/(8(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840
*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b
^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 -
55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^
7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 167772
16*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126
720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^
10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13
*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 +
11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
- 16*a*b^6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^
9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b
^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8
+ 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 10
56*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^1
4*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c
^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^1
1)))^(1/4) + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14
256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*
c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-
(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b
^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 1
0665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^2

4 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18
*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 1
2976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 692
06016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)))*(-(b^19 + b^4*(-(4*
a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13
*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 1066
5984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 +
16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^
3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 1297
6128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 692060
16*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*2i + 2*atan(((((((-(b^19
+ b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2
752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^
7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192
*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080
*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^1
2*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*
c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a
^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 6553
60*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^
7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b
*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 2
6738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b
^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4
- 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2)
+ 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^1
1*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 1
7891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c +
27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12
- 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*
c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 +
32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 5
0331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2
*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6
c)))(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2
*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3
350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^2
0*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 37
84704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671
680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)

1i + (x^(1/2)(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2
*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3
840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c
- b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3
- 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984
*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
- 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 167
77216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 +
126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128
*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a
^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) - (((((-(b^19 + b^4*(-(4*a*c
- b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^
3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 1066598
4*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2
) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16
777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 +
126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 1297612
8*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*
a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*
a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7
- 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)
/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) +
(x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a
^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7
*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*
c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*
c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9
*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*
a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3
*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c
+ 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^
8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*
b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^
2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8
+ 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 +
b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 275
2*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*
c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a
^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*
c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^
9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - (x^(1/2)*
(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(

b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4))/((((((-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2 *c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i + (((((-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96

```
*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5
 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5
*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5
+ 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 5
7671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(
1/4)*1i))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96
*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5
 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5
*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5
+ 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 5
7671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(
1/4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.844 $\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=489

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\cdots$$

Rubi [A] time = 1.00, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1366, 1510, 298, 205, 208}

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(1/4)*(b - (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)
*a*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 20*
a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
 - (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b +
Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-b^2+10ac-bcx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(\sqrt{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x\right.}{4\sqrt{2}a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(b+\frac{b^2-2}{\sqrt{b^2-}}\right.}{4\,2^{3/4}a\left(b^2-\right.}$$

Mathematica [C] time = 0.24, size = 149, normalized size = 0.30

$$-\frac{\left(a+bx^2+cx^4\right)\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)-10ac\log\left(\sqrt{x}-\#1\right)+b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+4x^{3/2}\left(-2ac+b^2+bcx^2\right)}{8a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/8*(4*x^(3/2)*(b^2 - 2*a*c + b*c*x^2) + (a + b*x^2 + c*x^4)*RootSum[a + b
*#1^4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqrt[x] - #1] + b*c*
Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(a*(-b^2 + 4*a*c)*(a + b*x^2
 + c*x^4))
```

IntegrateAlgebraic [C] time = 0.35, size = 156, normalized size = 0.32

$$\frac{x^{3/2}\left(2ac-b^2-bcx^2\right)}{2a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} - \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)-10ac\log\left(\sqrt{x}-\#1\right)+b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{8a\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(-b^2 + 2*a*c - b*c*x^2))/(2*a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4))
 - RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqrt
[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(8*a*(-b^2 +
4*a*c))
```

fricas [B] time = 152.99, size = 12411, normalized size = 25.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*
b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1
280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqr
t((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b
^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c
 + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^
15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^
8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a
^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*arctan
(1/2*((b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b
^3*c^4 - 40000*a^5*b*c^5 - (a^5*b^14 - 44*a^6*b^12*c + 720*a^7*b^10*c^2 - 6
080*a^8*b^8*c^3 + 29440*a^9*b^6*c^4 - 82944*a^10*b^4*c^5 + 126976*a^11*b^2*
c^6 - 81920*a^12*c^7)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a
^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a
^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*
a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4
*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt((531441*b^24*c^8 - 768
81798*a*b^22*c^9 + 5113978011*a^2*b^20*c^10 - 206852401350*a^3*b^18*c^11 +
5667080000625*a^4*b^16*c^12 - 110792866500000*a^5*b^14*c^13 + 1584936775000
000*a^6*b^12*c^14 - 16715805000000000*a^7*b^10*c^15 + 128988375000000000*a^
8*b^8*c^16 - 710150000000000000*a^9*b^6*c^17 + 2647500000000000000*a^10*b^4
*c^18 - 6000000000000000000*a^11*b^2*c^19 + 6250000000000000000*a^12*c^20)*
x - 1/2*sqrt(1/2)*(6561*b^31*c^5 - 1032993*a*b^29*c^6 + 75634965*a^2*b^27*c
^7 - 3414264975*a^3*b^25*c^8 + 106186248955*a^4*b^23*c^9 - 2407919378459*a^
5*b^21*c^10 + 41083864936232*a^6*b^19*c^11 - 536376931701360*a^7*b^17*c^12
+ 5394460343808000*a^8*b^15*c^13 - 41720627697600000*a^9*b^13*c^14 + 245614
092480000000*a^10*b^11*c^15 - 1078472304000000000*a^11*b^9*c^16 + 341052480
0000000000*a^12*b^7*c^17 - 7314160000000000000*a^13*b^5*c^18 + 948800000000
0000000*a^14*b^3*c^19 - 5600000000000000000*a^15*b*c^20 - (6561*a^5*b^34*c^
5 - 895212*a^6*b^32*c^6 + 56697732*a^7*b^30*c^7 - 2212069617*a^8*b^28*c^8 +
 59497163992*a^9*b^26*c^9 - 1169816993840*a^10*b^24*c^10 + 17397456159488*a
```

^11*b^22*c^11 - 199763116583168*a^12*b^20*c^12 + 1791922585643008*a^13*b^18*c^13 - 12624164431147008*a^14*b^16*c^14 + 69835076189159424*a^15*b^14*c^15 - 301610411758387200*a^16*b^12*c^16 + 1004700278784000000*a^17*b^10*c^17 - 2527971917824000000*a^18*b^8*c^18 + 4641908326400000000*a^19*b^6*c^19 - 5864652800000000000*a^20*b^4*c^20 + 4554752000000000000*a^21*b^2*c^21 - 1638400000000000000*a^22*c^22)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6))) - (729*b^23*c^4 - 86994*a*b^21*c^5 + 4700619*a^2*b^19*c^6 - 151648714*a^3*b^17*c^7 + 3240737969*a^4*b^15*c^8 - 48070563100*a^5*b^13*c^9 + 503690450000*a^6*b^11*c^10 - 3715387000000*a^7*b^9*c^11 + 18824300000000*a^8*b^7*c^12 - 62050000000000*a^9*b^5*c^13 + 119000000000000*a^10*b^3*c^14 - 100000000000000*a^11*b*c^15 - (729*a^5*b^26*c^4 - 84807*a^6*b^24*c^5 + 4445469*a^7*b^22*c^6 - 138927340*a^8*b^20*c^7 + 2884712240*a^9*b^18*c^8 - 41968650816*a^10*b^16*c^9 + 439511597568*a^11*b^14*c^10 - 3350499342336*a^12*b^12*c^11 + 18578963128320*a^13*b^10*c^12 - 74005426176000*a^14*b^8*c^13 + 205936435200000*a^15*b^6*c^14 - 379514880000000*a^16*b^4*c^15 + 415744000000000*a^17*b^2*c^16 - 204800000000000*a^18*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))/(6561*b^20*c^5 - 803358*a*b^18*c^6 + 44473131*a^2*b^16*c^7 - 1466261550*a^3*b^14*c^8 + 31889850625*a^4*b^12*c^9 - 478129875000*a^5*b^10*c^10 + 5004993750000*a^6*b^8*c^11 - 36117500000000*a^7*b^6*c^12 + 171937500000000*a^8*b^4*c^13 - 487500000000000*a^9*b^2*c^14 + 625000000000000*a^10*c^15)) - 4*

```
((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*s
qrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3
+ 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*
b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12
- 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 -
 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a
^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c
^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 2621
44*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c
^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*arctan(-1/2*((
b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4
- 40000*a^5*b*c^5 + (a^5*b^14 - 44*a^6*b^12*c + 720*a^7*b^10*c^2 - 6080*a^8
*b^8*c^3 + 29440*a^9*b^6*c^4 - 82944*a^10*b^4*c^5 + 126976*a^11*b^2*c^6 - 8
1920*a^12*c^7)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*
c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^1
8 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^
10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 +
589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt((531441*b^24*c^8 - 76881798*a
*b^22*c^9 + 5113978011*a^2*b^20*c^10 - 206852401350*a^3*b^18*c^11 + 5667080
000625*a^4*b^16*c^12 - 110792866500000*a^5*b^14*c^13 + 1584936775000000*a^6
*b^12*c^14 - 16715805000000000*a^7*b^10*c^15 + 128988375000000000*a^8*b^8*c
^16 - 710150000000000000*a^9*b^6*c^17 + 2647500000000000000*a^10*b^4*c^18 -
 6000000000000000000*a^11*b^2*c^19 + 6250000000000000000*a^12*c^20)*x - 1/2
*sqrt(1/2)*(6561*b^31*c^5 - 1032993*a*b^29*c^6 + 75634965*a^2*b^27*c^7 - 34
14264975*a^3*b^25*c^8 + 106186248955*a^4*b^23*c^9 - 2407919378459*a^5*b^21*
c^10 + 41083864936232*a^6*b^19*c^11 - 536376931701360*a^7*b^17*c^12 + 53944
60343808000*a^8*b^15*c^13 - 41720627697600000*a^9*b^13*c^14 + 2456140924800
00000*a^10*b^11*c^15 - 1078472304000000000*a^11*b^9*c^16 + 3410524800000000
000*a^12*b^7*c^17 - 7314160000000000000*a^13*b^5*c^18 + 9488000000000000000
*a^14*b^3*c^19 - 5600000000000000000*a^15*b*c^20 + (6561*a^5*b^34*c^5 - 895
212*a^6*b^32*c^6 + 56697732*a^7*b^30*c^7 - 2212069617*a^8*b^28*c^8 + 594971
63992*a^9*b^26*c^9 - 1169816993840*a^10*b^24*c^10 + 17397456159488*a^11*b^2
2*c^11 - 199763116583168*a^12*b^20*c^12 + 1791922585643008*a^13*b^18*c^13 -
 12624164431147008*a^14*b^16*c^14 + 69835076189159424*a^15*b^14*c^15 - 3016
10411758387200*a^16*b^12*c^16 + 1004700278784000000*a^17*b^10*c^17 - 252797
1917824000000*a^18*b^8*c^18 + 4641908326400000000*a^19*b^6*c^19 - 586465280
0000000000*a^20*b^4*c^20 + 4554752000000000000*a^21*b^2*c^21 - 163840000000
0000000*a^22*c^22)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*
b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10
*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^1
4*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^
7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(-(b^9 - 45*a*b^7*c + 765*
a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*
c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c
^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3
```

*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^1
0*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^
14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c
^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 2
40*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 +
4096*a^11*c^6)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b
^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^1
1*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 47
0625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a
^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 -
129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a
^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6
))) - (729*b^23*c^4 - 86994*a*b^21*c^5 + 4700619*a^2*b^19*c^6 - 151648714*a
^3*b^17*c^7 + 3240737969*a^4*b^15*c^8 - 48070563100*a^5*b^13*c^9 + 50369045
0000*a^6*b^11*c^10 - 3715387000000*a^7*b^9*c^11 + 18824300000000*a^8*b^7*c^
12 - 62050000000000*a^9*b^5*c^13 + 119000000000000*a^10*b^3*c^14 - 10000000
0000000*a^11*b*c^15 + (729*a^5*b^26*c^4 - 84807*a^6*b^24*c^5 + 4445469*a^7*
b^22*c^6 - 138927340*a^8*b^20*c^7 + 2884712240*a^9*b^18*c^8 - 41968650816*a
^10*b^16*c^9 + 439511597568*a^11*b^14*c^10 - 3350499342336*a^12*b^12*c^11 +
18578963128320*a^13*b^10*c^12 - 74005426176000*a^14*b^8*c^13 + 20593643520
0000*a^15*b^6*c^14 - 379514880000000*a^16*b^4*c^15 + 415744000000000*a^17*b
^2*c^16 - 204800000000000*a^18*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^
8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250
000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^
12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 -
589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(x)*sqrt
(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 1
8000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6
*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 7
8*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 26
25000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12
*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5
+ 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*
a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6))))/(6561*b^20*c^5 -
803358*a*b^18*c^6 + 44473131*a^2*b^16*c^7 - 1466261550*a^3*b^14*c^8 + 3188
9850625*a^4*b^12*c^9 - 478129875000*a^5*b^10*c^10 + 5004993750000*a^6*b^8*c
^11 - 36117500000000*a^7*b^6*c^12 + 171937500000000*a^8*b^4*c^13 - 48750000
0000000*a^9*b^2*c^14 + 625000000000000*a^10*c^15)) - ((a*b^2*c - 4*a^2*c^2)
*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b
^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a
^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4
*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a

^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(1/2*sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 497651072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 - (a^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^2 - 40672*a^8*b^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832*a^11*b^13*c^6 - 114475008*a^12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 802029568*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a^16*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)) + (729*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a^3*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 2500000000*a^6*c^10)*sqrt(x)) + ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8

- 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(-1/2
sqrt(1/2)(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1
338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 4976
51072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 45824
00000*a^10*b^2*c^10 - 2560000000*a^11*c^11 - (a^5*b^25 - 70*a^6*b^23*c + 21
92*a^7*b^21*c^2 - 40672*a^8*b^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b
^15*c^5 + 25976832*a^11*b^13*c^6 - 114475008*a^12*b^11*c^7 + 361955328*a^13
*b^9*c^8 - 802029568*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a
^16*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b
^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 625
0000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b
^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 -
589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1
/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^
4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 +
3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^1
0*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a
^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c
^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 34406
4*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^
9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*
a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c
+ 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6
*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10
*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 459
50*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6
)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32
256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17
*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10
*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*
c^5 + 4096*a^11*c^6)) + (729*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*
c^6 - 26110000*a^3*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9
+ 2500000000*a^6*c^10)*sqrt(x)) - ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4
*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c +
765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b
^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b
^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950
*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/
(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 3225
6*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b
^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c
+ 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^
5 + 4096*a^11*c^6)))*log(1/2*sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*
c^2 - 87230*a^3*b^16*c^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 1

00253344*a^6*b^10*c^6 - 497651072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 36
27264000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 + (a
^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^2 - 40672*a^8*b^19*c^3 + 498432*a
^9*b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832*a^11*b^13*c^6 - 114475008*a^
12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 802029568*a^14*b^7*c^9 + 1183842304*
a^15*b^5*c^10 - 1046478848*a^16*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^1
2 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4
 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576
*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8
*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 26
2144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b
^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^1
1*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 47
0625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a
^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 -
 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a
^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6
)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^
4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 +
3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^1
0*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a
^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c
^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 34406
4*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^
9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*
a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)) + (729*b^12*c^4 - 52731*a
*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a^3*b^6*c^7 + 241500000*a^4*b^4*
c^8 - 1200000000*a^5*b^2*c^9 + 2500000000*a^6*c^10)*sqrt(x)) + ((a*b^2*c -
4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2
)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*
b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 38
40*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*
c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5
*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2
 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*
a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)
))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^
9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(-1/2*sqrt(1/2)*(b^22 -
 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1338850*a^4*b^14*c^
4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 497651072*a^7*b^8*c^7
+ 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^1
0 - 2560000000*a^11*c^11 + (a^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^2 -
40672*a^8*b^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832

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*a^11*b^13*c^6 - 114475008*a^12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 8020295
68*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a^16*b^3*c^11 + 419
430400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3
*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^1
0*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^
14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c
^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 4
5*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^1
2 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 -
 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8
*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 62500
00*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^1
2*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 5
89824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 2
4*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144
*a^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2
- 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*
b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^
11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 4
70625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*
a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4
- 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*
a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c
^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^
6)) + (729*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a^3
*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 2500000000*a^6*
c^10)*sqrt(x)) - 4*(b*c*x^3 + (b^2 - 2*a*c)*x)*sqrt(x))/((a*b^2*c - 4*a^2*c
^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.23Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 146, normalized size = 0.30

$$-\frac{\left(\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6} b c+\left(-10 a c+b^{2}\right) \mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2}\right) \ln \left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{8\left(4 a c-b^{2}\right) a\left(2 \mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7} c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3} b\right)}+\frac{-\frac{b c x^{\frac{7}{2}}}{2\left(4 a c-b^{2}\right) a}+\frac{\left(2 a c-b^{2}\right) x^{\frac{3}{2}}}{2\left(4 a c-b^{2}\right) a}}{c x^{4}+b x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/a/(4*a*c-b^2)*c*x^(7/2)+1/4*(2*a*c-b^2)/(4*a*c-b^2)/a*x^(3/2))/(c
*x^4+b*x^2+a)-1/8/a/(4*a*c-b^2)*sum((_R^6*b*c+(-10*a*c+b^2)*_R^2)/(2*_R^7*c
+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bcx^{\frac{7}{2}} + \left(b^2 - 2\,ac\right)x^{\frac{3}{2}}}{2\left(\left(ab^2c - 4\,a^2c^2\right)x^4 + a^2b^2 - 4\,a^3c + \left(ab^3 - 4\,a^2bc\right)x^2\right)} - \int -\frac{bcx^{\frac{5}{2}} + \left(b^2 - 10\,ac\right)\sqrt{x}}{4\left(\left(ab^2c - 4\,a^2c^2\right)x^4 + a^2b^2 - 4\,a^3c + \left(ab^3 - 4\,a^2bc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(7/2) + (b^2 - 2*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - integrate(-1/4*(b*c*x^(5/2) + (b
^2 - 10*a*c)*sqrt(x))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b
^3 - 4*a^2*b*c)*x^2), x)
```

mupad [B] time = 6.56, size = 26373, normalized size = 53.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 28528
64*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18574213
12*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386
073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^
4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 2867
2*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 737280
00*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c
^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1
08380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a
*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48
*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4
- 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 3
2440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 503
31648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 1966
08*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*
a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762524467
2*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 2
```

4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c
^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300134
4*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^
8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
- 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c
 + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^
10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^1
3*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*
b^2*c^11)))^(3/4) + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^
11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 24
0*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(
-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^
17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +
15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13467
6480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 5
25*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*
c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 378
4704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671
680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)
*1i - (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 28528
64*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18574213
12*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386
073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^
4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 2867
2*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 737280
00*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c
^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1
08380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a
*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48
*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4
- 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 3
2440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 503
31648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 1966
08*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*
a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762524467
2*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c
^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300134
4*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^
8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)

```
- 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c
 + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^
10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^1
3*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*
b^2*c^11)))^(3/4) - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^
11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 24
0*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(
-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^
17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +
15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13467
6480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 5
25*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*
c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 378
4704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671
680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)
*1i)/((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 28528
64*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18574213
12*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386
073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^
4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 2867
2*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 737280
00*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c
^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1
08380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a
*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48
*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4
- 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 3
2440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 503
31648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 1966
08*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*
a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762524467
2*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c
^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300134
4*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^
8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
- 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c
 + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^
10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^1
3*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*
```

b^2*c^11)))^(3/4) + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^
11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 24
0*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(
-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^
17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +
15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13467
6480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 5
25*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*
c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 378
4704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671
680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)
 - (5000000*a^3*c^12 - 3645*b^6*c^9 + 121500*a*b^4*c^10 - 1350000*a^2*b^2*c
^11)/(32*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 224
0*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6))
 + (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a
^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a
^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 193860730
88*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^
10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^
8*b^2*c^6)) + (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a
^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 10838
0160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)
^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6
*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 81
1008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440
320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 5033164
8*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a
^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*
b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^
8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^
3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7
*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10
 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^
5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^
5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69
*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1
056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b
^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^
8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*
c^11)))^(3/4) - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 -
 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^

4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*2i + atan(((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811

008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 324403
20*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648
*a^16*b^2*c^11)))^(3/4) + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^
3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*
c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^
5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*
a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7
*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5
+ 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 -
57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))
^(1/4)*1i - (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 +
2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18
57421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 +
19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c +
336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5
+ 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) +
73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*
b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c
^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^1
2 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^1
6*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c
^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10
- 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4
+ 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 3204
71040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762
5244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c
^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4
- 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^
10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380
160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^
(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*
b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811
008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 324403
20*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648
*a^16*b^2*c^11)))^(3/4) - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^
3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*
c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^

5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*
a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7
*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5
+ 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 -
57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))
^(1/4)*1i)/((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 +
2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18
57421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 +
19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c +
336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5
+ 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) +
73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*
b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c
^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^1
2 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^1
6*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c
^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10
- 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4
+ 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 3204
71040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762
5244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c
^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4
- 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^
10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380
160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^
(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*
b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811
008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 324403
20*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648
*a^16*b^2*c^11)))^(3/4) + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^
3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*
c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^
5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*
a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)

^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7
*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5
 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 -
 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))
^(1/4) - (5000000*a^3*c^12 - 3645*b^6*c^9 + 121500*a*b^4*c^10 - 1350000*a^2
*b^2*c^11)/(32*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2
 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*
c^6)) + (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 285
2864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 185742
1312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 193
86073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*
a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28
672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 7372
8000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13
*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 +
 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^
2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39
*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 -
48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^
4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 +
 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 5
0331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 19
6608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 32047104
0*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244
672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 -
 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 61
44*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b
*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001
344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*
a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-
(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22
*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*
a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a
^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^1
6*b^2*c^11)))^(3/4) - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*
c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c +
240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))
(-(b^21 - b^6(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*
b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5
+ 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134
676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c -
 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^2
0*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3

784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 576
71680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/
4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*
a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7
*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5
+ 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 -
57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))
^(1/4)*2i + 2*atan(((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*
b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*
c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^
5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b
^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*
b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^
(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404
160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a
^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(
-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*
a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720
*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^1
2*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*
b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b
^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^
7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^
10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 +
4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a
^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) +
73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*
b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c
^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
- 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^1
2 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^1
6*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c
^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10
- 50331648*a^16*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c
^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6
- 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6
144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*
b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300
1344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160

*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^2
2*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008
*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*
a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^
16*b^2*c^11)))^(1/4) - (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a
^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^
11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7
*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^
3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a
^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^1
5)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 +
404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 5050368
0*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^
3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 167772
16*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126
720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*
a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^
15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*
a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12
*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6
*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^1
2 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 384
0*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2)
 + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a
^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^
7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a
*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/
2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*
c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*
b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^1
0*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c
^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^
5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c
^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4
- 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^
10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380
160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^
(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*
b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811
008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 324403

20*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648
*a^16*b^2*c^11)))^(1/4))/((5000000*a^3*c^12 - 3645*b^6*c^9 + 121500*a*b^4*c
^10 - 1350000*a^2*b^2*c^11)/(32*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c +
336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5
+ 28672*a^8*b^2*c^6)) + (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400
*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*
b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a
^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*
a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504
*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)
^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3
+ 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503
680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*
c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^
2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 1677
7216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 1
26720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 1297612
8*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*
a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 409
6*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^
12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b
^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b
^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3
840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/
2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160
*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*
b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4
*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^1
7*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^
9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b
^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4
*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(81*b^7*c^8 + 3060*a*
b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8
*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^
4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*
a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4
- 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1083
80160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^
4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^
6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 8
11008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 3244
0320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 503316
48*a^16*b^2*c^11)))^(1/4)*1i + (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905

190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 33570816
0*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 1504287
1296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7
 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 -
 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 + b^6*(-(4*a*c
- b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^1
5*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 -
 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 250
0*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24
+ 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c
^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 1
2976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 692
06016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13
 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*
a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672
*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*
(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c
^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^1
5)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 +
404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 5050368
0*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^
3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 167772
16*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126
720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*
a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^
15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(81*b^7*c^8 + 3
060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 40
96*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*
b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 737
28000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^1
3*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7
+ 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b
^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3
9*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 -
 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c
^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7
+ 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 -
50331648*a^16*b^2*c^11)))^(1/4)*1i))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2
) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*
a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b
^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17

*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4) + 2*atan(((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4) - (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^

3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4))/((5000000*a^3*c^12 - 3645*b^6*c^9 + 121500*a*b^4*c^10 - 1350000*a^2*b^2*c^11)/(32*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a

^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c
^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*
(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b
^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +
15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 1346
76480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c -
525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^
(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20
*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 37
84704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 5767
1680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4
)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a
^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a
^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792
*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b
^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21
- b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2
- 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1506457
6*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^
9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*
b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 1
4080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^
11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^1
4*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i - (
x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^
10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*
a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c
 - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^
15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6
- 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 25
00*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24
 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*
c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 -
12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69
206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*1i + (((2048*b^19*c^
4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 392
47872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670
516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/
(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5
*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^
(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085
*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11
*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8

- 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*1i))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4) + ((x^(3/2)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^(7/2))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.845 $\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=503

$$\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\cdots$$

Rubi [A] time = 1.28, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1345, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(3/4)*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 -
 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b - Sq
rt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 - 4*a*c])
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1
/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1345

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(
x*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^
2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + n*
(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^
(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*
c, 0] && ILtQ[p, -1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{b^2-2ac-4\left(b^2-4ac\right)-3bcx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4}}\right.}{4a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{}{\sqrt{-b-\sqrt{b^2}}}\right.}{4a\left(b^2-4ac\right)^{3/2}\sqrt{-b-\sqrt{b^2-4}}}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{c^{3/4}\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4a}}}\right.}{4\sqrt[4]{2}\,a\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] time = 0.25, size = 153, normalized size = 0.30

$$-\frac{\left(a+bx^2+cx^4\right)\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{3\#1^4bc\log\left(\sqrt{x}-\#1\right)-14ac\log\left(\sqrt{x}-\#1\right)+3b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+4\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{8a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/8*(4*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2) + (a + b*x^2 + c*x^4)*RootSum[a + b
*#1^4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sqrt[x] - #1] + 3*
b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*(-b^2 + 4*a*c)*(a +
 b*x^2 + c*x^4))
```

IntegrateAlgebraic [C] time = 0.30, size = 160, normalized size = 0.32

$$\frac{\sqrt{x}\left(2ac-b^2-bcx^2\right)}{2a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} - \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{3\#1^4bc\log\left(\sqrt{x}-\#1\right)-14ac\log\left(\sqrt{x}-\#1\right)+3b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{8a\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (Sqrt[x]*(-b^2 + 2*a*c - b*c*x^2))/(2*a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4))
 - RootSum[a + b*#1^4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sq
rt[x] - #1] + 3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(8*a*(-
b^2 + 4*a*c))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.36Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 144, normalized size = 0.29

$$\frac{\left(-3\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{4} bc+14ac-3b^{2}\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{8\left(4ac-b^{2}\right)a\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7} c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3} b\right)}+\frac{-\frac{bc\,x^{\frac{5}{2}}}{2\left(4ac-b^{2}\right)a}+\frac{\left(2ac-b^{2}\right)\sqrt{x}}{2\left(4ac-b^{2}\right)a}}{c\,x^{4}+b\,x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/a/(4*a*c-b^2)*c*x^(5/2)+1/4*(2*a*c-b^2)/(4*a*c-b^2)/a*x^(1/2))/(c
*x^4+b*x^2+a)+1/8/a/(4*a*c-b^2)*sum((-3*_R^4*b*c+14*a*c-3*b^2)/(2*_R^7*c+_R
^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,b^{2}c-14\,ac^{2}\right)x^{\frac{9}{2}}+\left(3\,b^{3}-13\,abc\right)x^{\frac{5}{2}}+4\left(ab^{2}-4\,a^{2}c\right)\sqrt{x}}{2\left(a^{3}b^{2}-4\,a^{4}c+\left(a^{2}b^{2}c-4\,a^{3}c^{2}\right)x^{4}+\left(a^{2}b^{3}-4\,a^{3}bc\right)x^{2}\right)}-\int\frac{\left(3\,b^{2}c-14\,ac^{2}\right)x^{\frac{7}{2}}+\left(3\,b^{3}-17\,abc\right)x^{\frac{3}{2}}}{4\left(a^{3}b^{2}-4\,a^{4}c+\left(a^{2}b^{2}c-4\,a^{3}c^{2}\right)x^{4}+\left(a^{2}b^{3}-4\,a^{3}bc\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*b^2*c - 14*a*c^2)*x^(9/2) + (3*b^3 - 13*a*b*c)*x^(5/2) + 4*(a*b^2 -
 4*a^2*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*
b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((3*b^2*c - 14*a*c^2)*x^(7/2) + (3*b^
3 - 17*a*b*c)*x^(3/2))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (
a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

mupad [B] time = 7.29, size = 35171, normalized size = 69.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^(1/2)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^(5/2))/(2*a*(4*a*c -
b^2)))/(a + b*x^2 + c*x^4) + atan(((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) -
 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^
3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6
+ 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 -
 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a
*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(
-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a
^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^
10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^
12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6
*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*
a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c
^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 -
 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^
6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a
^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*
b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a
^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a
^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^
3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/
2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^1
7*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*
c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c
^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 40
23*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 1408
0*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^1
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3*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16
*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (5378
24*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a
^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 25
6*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a
^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*
c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^
7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c
^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^
4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^1
9*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a
^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14
*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b
^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + (x^(1/2)*(15059072*a^4*c^13 + 9
801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^
12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280
*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b
^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201
623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 27957196
8*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 249411993
6*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)
^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 2631
3*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^
(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20
*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 +
3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57
671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1
/4)*1i - ((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*
b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4
+ 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 -
1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10
+ 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) -
 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^
12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*
b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^1
0*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c
^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^
15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7
- 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10))
/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^
3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*
b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*
b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*

a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i)/(((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b

^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*
a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^1
4*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*
c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^
11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c
^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 +
342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4
 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(1268357529
6*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^
15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*
b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576
*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^
8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*
(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^1
9*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^
5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8
 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*
c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^
(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1
056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12
*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*
b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^
2*c^11)))^(3/4) - (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358
*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*
c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2)
- 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c
^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6
 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9
- 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*
a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a
^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b
^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^
6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + (x^(1/2)
*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^
11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c +
 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5))
)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90
126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*
a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*
a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*
c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c
- b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6

c(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8
*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 -
811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 324
40320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331
648*a^18*b^2*c^11)))^(1/4) + ((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^
23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10
588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 8531
74784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 20386
93888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*
c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*
c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^2
4 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^1
8*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6
 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 +
 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b
*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 3
2440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 47815
0656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*
c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^1
9*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c
^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7
*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^1
2 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 38
40*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 8
1*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3
 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 +
853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2
038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b
^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(
4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7
*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10
*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12
*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c
^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (537824*a^4
*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2
c^10)/(2(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*
b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*
c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 +
64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 17
99626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 +
9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*
(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1
593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12
 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^
16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*

c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^1
0 - 50331648*a^18*b^2*c^11)))^(1/4) - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^
8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(
16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b
^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15
)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^
3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*
b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*
b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2)
 + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*
b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))
/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 -
 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 378470
4*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680
*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)))*
((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 9012
6*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^
5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^
8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^
4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b
^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 81
1008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440
320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 5033164
8*a^18*b^2*c^11)))^(1/4)*2i + atan(((((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)
^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*
c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^
6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9
 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023
*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3
*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*
a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*
b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b
^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(28521267
2*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11
*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9
 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*
a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864
*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^
5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800
*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*
(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*
c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c -

b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*
b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^
11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^
5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) -
4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^
2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 1
4080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*
a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a
^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (5
37824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 51038
4*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 -
256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 7418019
84*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b
^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^
9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b
^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^
2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)
^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216
*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 1267
20*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*
a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^
17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + (x^(1/2)*(15059072*a^4*c^13
+ 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^
2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 -
1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b
^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 -
1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279
571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494
119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2
)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) -
26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9
*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c
^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8
- 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)
))^(1/4)*1i - (((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984
*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^1
5*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*
c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3
*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*
b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a
^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720
*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^

14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17
*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*
a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^
9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*
c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*
b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 141312
0*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 145942118
4*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 285757
93152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*
c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4
 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 7418
01984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^
4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7
*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^1
0*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746
*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^
15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777
216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 1
26720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 129761
28*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016
*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (537824*a^4*c^11 + 891*b
^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^
4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-
(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126
*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5
*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8
*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4
*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b
^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*
(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^
22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811
008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 324403
20*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648
*a^18*b^2*c^11)))^(1/4) - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 2275
02*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12
 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 384
0*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)
^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3
+ 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 -
853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2
038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b
^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(
4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7
*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10
*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12

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*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c
^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i)/((((((-(8
1*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a
^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b
^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b
^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(
-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-
(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22
*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 81100
8*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320
*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a
^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*
a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*
b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 2
56*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(
12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891
968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 60238
06976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 2
8705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c +
240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))
*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90
126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*
a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*
a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*
c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c
- b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6
*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8
*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 -
811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 324
40320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331
648*a^18*b^2*c^11)))^(3/4) - (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c
^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 -
16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4
*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 120162
3*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*
a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*
a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(
1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*
a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c
^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 37
84704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 5767
1680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4
) + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 209210
```

4*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*
a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a
^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^1
1*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^
4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7
+ 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^1
0 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*
c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*
c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^1
1*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b
^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4
*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + (((((-(81*b^23 + 81*b^8*(-(4*a*c
- b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3
*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b
^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b
^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2)
- 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b
^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/
(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 -
14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704
*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*
a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(28
5212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^
6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b
^5*c^9 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c
 + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 -
 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816
064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436
428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12)
)/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^
7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4
*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 120162
3*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*
a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*
a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(
1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*
a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c
^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 37
84704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 5767
1680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4
) - (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 -
 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4
*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 7

41801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384
*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*
a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*
a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10
746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^
2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16
777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3
+ 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 129
76128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206
016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) - (x^(1/2)*(15059072*a^
4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*
a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*
c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23
+ 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19
*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5
+ 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8
- 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c
- b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 10
56*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*
b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b
^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2
c^11)))^(1/4)))(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*
a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15
*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c
^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*
c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b
^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1
/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^
19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*
a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^1
4*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*
b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*2i + 2*atan(((((((((81*b^8*(-(4*a
*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2
+ 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 27
9571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 249
4119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^
2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2)
- 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^
9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*
c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^
8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11
)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5

- 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 34
2884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^
4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(126835752
96*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b
^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7
*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 2870581657
6*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b
^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8
*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^
19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c
^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^
8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a
*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)
^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*
c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c +
1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^1
2*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15
*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b
^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 15
5358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*
b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1
/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^
17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11
*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*
c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4
023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*
c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(81
92*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 140
80*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^
13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^1
6*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i - (
x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2
*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b
^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^
2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^
11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64
704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799
626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 96
04*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-
(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 159
3*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 -
48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16
*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^
7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10
- 50331648*a^18*b^2*c^11)))^(1/4) - ((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2)

- 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c
^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6
 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9
- 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*
a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a
^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b
^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^
6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672
*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*
c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9
- 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 9
6*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 368
64*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*
a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 164364288
00*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(1
6*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^
6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)
^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3
*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b
^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b
^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2)
+ 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b
^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/
(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 -
14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704
*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*
a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i
+ (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 5
10384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c
^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 7418
01984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^
4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7
*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^1
0*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746
*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^
15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777
216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 1
26720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 129761
28*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016
*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(15059072*a^
4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*
a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*
c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-

(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*
c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5
- 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 +
2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c
- b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1
/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c -
b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 105
6*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b
^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^
8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*
c^11)))^(1/4))/(((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 74180198
4*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^
15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9
*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^
3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2
*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^
(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*
a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 12672
0*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a
^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^1
7*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288
*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b
^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3
*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*
a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 14
13120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 14594
21184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28
575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a
^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4
*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 7
41801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384
*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*
a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*
a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10
746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^
2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16
777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3
+ 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 129
76128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206
016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 +
891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/
(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3
)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 -
90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 6470457
6*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 179962675

2*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^
4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b
^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a
^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4
- 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 3
2440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 503
31648*a^18*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^
9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(
a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c
^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1
/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^
17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11
*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*
c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4
023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*
c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(81
92*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 140
80*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^
13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^1
6*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (
(((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 9
0126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576
*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752
*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4
c^4(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c
 - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^
6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^
8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 -
 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32
440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 5033
1648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 3
64544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 13631488
0*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^
4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (
x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^
5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^
8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5
*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*
b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b
^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c
^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 6
4704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 179
9626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9
604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(

-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
93*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12
- 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^1
6*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c
^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10
- 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19
548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256
*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4
*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^
2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 -
279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2
494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c -
b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2
) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*
a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^1
4*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*
c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^
11)))^(1/4)*1i + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*
c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a
^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4
*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 7
41801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384
*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*
a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*
a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10
746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^
2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16
777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3
+ 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 129
76128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206
016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i))*((81*b^8*(-(4*a*c
- b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1
201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 27957
1968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 249411
9936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^
15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 2
6313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b
^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5
+ 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 -
57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))
^(1/4) + 2*atan(((((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801
984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*
b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b

^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6

*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9
*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2
) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3
*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2)
)/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2
- 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 37847
04*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 5767168
0*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) -
(((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11
+ 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704
576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626
752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*
a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a
*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48
*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^
4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 +
32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 5
0331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4
+ 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 13631
4880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*
(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))
+ (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17
*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11
*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*
b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a
^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^
9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11
*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4
- 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 +
1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10
+ 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c
^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2)
- 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c
^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11
*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^
10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*
c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7
- 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 +
256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^2
3 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^
19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c
^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^
8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a
*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)

^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4))/((((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^2

2*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 8110
08*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 3244032
0*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*
a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^
8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 1
6*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*
a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623
*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a
^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a
^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1
/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a
^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/
2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^
2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 378
4704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671
680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)
1i - (x^(1/2)(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092
104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 2
4*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144
*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a
^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*
c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^
7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c
^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^
4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^1
9*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a
^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14
*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b
^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (((((-(81*b^23 + 81*b^8*(-(4
*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 120162
3*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*
a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*
a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(
1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*
a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c
^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 37
84704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 5767
1680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4
)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 46202
88*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*
a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a
^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*
b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6

+ 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^2

```
1*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b
^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b
^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c
^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9
 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.846 $\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=573

$$-\frac{5b^2-18ac}{2a^2\sqrt{x}\left(b^2-4ac\right)}+\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2\ldots}\right.}{4\,2^{3/4}a^2\left(\ldots\right.}$$

Rubi [A] time = 2.45, antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1366, 1504, 1510, 298, 205, 208}

$$-\frac{5b^2-18ac}{2a^2\sqrt{x}\left(b^2-4ac\right)}+\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{-2ac+b^2+bcx^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(5*b^2 - 18*a*c)/(2*a^2*(b^2 - 4*a*c)*Sqrt[x]) + (b^2 - 2*a*c + b*c*x^2)/(
2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) + (c^(1/4)*(5*b^3 - 28*a*b*c
 - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt
[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^3 - 28*a*b*c + (5*b^2 - 18*a*c)*Sqrt[
b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/
4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (
c^(1/4)*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2
 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^3 - 28*a*b*
c + (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sq
rt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-5b^2+18ac-5bcx^4}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(-b\left(5b^2-23ac\right)-\right.}{a+bx^4\ldots}\right.}{2a^2\left(b^2\ldots\right.}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(5b^2-18ac+\frac{5b^3}{\sqrt{b^2-4ac}}\right.\right.}{\ldots}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} + \frac{\left(\sqrt{c}\left(5b^2-18ac+\frac{5b\ldots}{\sqrt{b^2-\ldots}}\right.\right.}{\ldots}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\sqrt[4]{c}\left(5b^2-18ac-\frac{5b^3}{\sqrt{b^2-4a\ldots}}\right.}{4\,2^{3/4}a^2\left(b^2-\ldots\right.}$$

Mathematica [C] time = 0.34, size = 190, normalized size = 0.33

$$-\frac{\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-18\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+5\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-23abc\log\left(\sqrt{x}-\#1\right)+5b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{b^2-4ac}+\frac{4x^{3/2}\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{16}{\sqrt{x}}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/8*(16/Sqrt[x] + (4*x^(3/2)*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^3*
Log[Sqrt[x] - #1] - 23*a*b*c*Log[Sqrt[x] - #1] + 5*b^2*c*Log[Sqrt[x] - #1]*
#1^4 - 18*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(b^2 - 4*a*c)
)/a^2
```

IntegrateAlgebraic [C] time = 0.54, size = 280, normalized size = 0.49

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-2\#1^4ac^2\log(\sqrt{x}-\#1)+\#1^4b^2c\log(\sqrt{x}-\#1)-7abc\log(\sqrt{x}-\#1)+b^3\log(\sqrt{x}-\#1)}{2\#1^5c+\#1b}\&\right]}{8a^2\left(4ac-b^2\right)}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(\sqrt{x}-\#1)+b\log(\sqrt{x}-\#1)}{2\#1^5c+\#1b}\&\right]}{2a^2}+\frac{-16a^2c+4ab^2-19abcx^2-18ac^2x^4+5b^3x^2+5b^2cx^4}{2a^2\sqrt{x}\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (4*a*b^2 - 16*a^2*c + 5*b^3*x^2 - 19*a*b*c*x^2 + 5*b^2*c*x^4 - 18*a*c^2*x^4
)/(2*a^2*(-b^2 + 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) - RootSum[a + b*#1^4 +
 c*#1^8 & , (b*Log[Sqrt[x] - #1] + c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1
^5) & ]/(2*a^2) + RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] -
7*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 - 2*a*c^2*Log[Sqrt
[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(8*a^2*(-b^2 + 4*a*c))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 50.43Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.03, size = 245, normalized size = 0.43

$$-\frac{c^2x^{\frac{7}{2}}}{\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)a}+\frac{b^2cx^{\frac{7}{2}}}{2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)a^2}-\frac{3bcx^{\frac{3}{2}}}{2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)a}+\frac{b^3x^{\frac{3}{2}}}{2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)a^2}-\frac{\left(\left(18ac-5b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6c+\left(23ac-5b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2b\right)\ln\left(-\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{8\left(4ac-b^2\right)a^2\left(2\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}-\frac{2}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^(7/2)+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a
*c-b^2)*x^(7/2)*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^(3/2)*c+1/2/a^2/(
```

```
c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)*x^(3/2)-1/8/a^2/(4*a*c-b^2)*sum((c*(18*a*c-5
*b^2)*_R^6+b*(23*a*c-5*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Root
Of(_Z^8*c+_Z^4*b+a))-2/a^2/x^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(5\,b^2c-18\,ac^2\right)x^{\frac{7}{2}}+\left(5\,b^3-19\,abc\right)x^{\frac{3}{2}}+\frac{4\left(ab^2-4\,a^2c\right)}{\sqrt{x}}}{2\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)}-\int\frac{\left(5\,b^2c-18\,ac^2\right)x^{\frac{5}{2}}+\left(5\,b^3-23\,abc\right)\sqrt{x}}{4\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((5*b^2*c - 18*a*c^2)*x^(7/2) + (5*b^3 - 19*a*b*c)*x^(3/2) + 4*(a*b^2
- 4*a^2*c)/sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2
*b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((5*b^2*c - 18*a*c^2)*x^(5/2) + (5*b
^3 - 23*a*b*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 +
(a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

mupad [B] time = 11.42, size = 31145, normalized size = 54.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan(((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 260
4992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^1
7*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175
670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143
360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*
b^3*c^20) + (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a
^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17
*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^
11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^
10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)
^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181
990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b
^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 +
16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*
c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 -
 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 6
9206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34
*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040
*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8
 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 - 649291
```

7279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088
955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436
224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 13483557107143802
88*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a
^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625
b^10(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*
c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5
 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*
c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^1
1*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 684
75*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^
2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*
b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 -
811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 324
40320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331
648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^3
4*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 19860742471
68*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24
*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 -
 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543
721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 21466
20531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721
914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 6124895493
22387456*a^39*b^2*c^20)))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2)
+ 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 714
83001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 599
6689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 +
21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*
a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^
4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14
080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a
^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^
18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i +
(x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 260499200
0*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13
 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703
423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^
28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^
20) + (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*
c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 -
 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7
 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5

*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2)
 - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^
3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15
)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 167772
16*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 +
126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976
128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 6920601
6*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(25649407252758528*a^38*c
^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*
b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153
599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 649291727949
0048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^12 + 104398826088955904
*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14 + 641705669216436224*a^
32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 + 1348355710714380288*a^3
4*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695801744382230528*a^36*b^
4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*
(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 -
8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 189
8983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 -
21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*
c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2
*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)
^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*
a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c
 + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008
*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*
a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^
20*b^2*c^11)))^(1/4)*(9119789245425254 4*a^40*c^21 - 52428800*a^23*b^34*c^4
 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^2
6*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 -
 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694
329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556
635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 21466205313
72195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474
102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 6124895493223874
56*a^39*b^2*c^20)))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105
423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*
a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 599668992
0*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 214830
12096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c -
b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/
2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4
*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9
*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^
12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^

12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6
*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i)/((x^(1/
2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21
*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 655
7747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488
*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7
*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) +
(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 +
 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 43447
8624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 135
24825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10
- 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 296
25*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*
c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2
) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^2
1*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720
*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^
16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19
*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34*c^4 - 25649
407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040*a^23*b^30*c
^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8 - 153599583
715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 - 6492917279490048*a
^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088955904*a^30*
b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436224*a^32*b^1
2*c^15 + 1077743462209552384*a^33*b^10*c^16 - 1348355710714380288*a^34*b^8*
c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a^36*b^4*c^19
 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a
*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 826499
0*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 189898336
0*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 211223
10144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 +
 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c
^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c -
b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 105
6*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*
b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b
^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2
c^11)))^(1/4)(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418
357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28
*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 19246
94567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453
871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811
840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 21466205313721958
40*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784

*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^3
9*b^2*c^20)))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360
*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^
17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*
b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*
a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^1
5)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 1
81990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c -
b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24
+ 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^1
8*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6
- 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 +
69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) - (x^(1/2)*(60233
2119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^1
1 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 655774764236
8*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^1
1*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1
742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) + (-(625*b^
25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a
^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*
b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*
a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 1257504
7680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23
*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875
*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 -
48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^1
6*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c
^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10
- 50331648*a^20*b^2*c^11)))^(3/4)*(25649407252758528*a^38*c^21 - 32768000*
a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*b^30*c^6 + 1213
399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153599583715328*a^
26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 6492917279490048*a^28*b^20*
c^11 - 29298398985191424*a^29*b^18*c^12 + 104398826088955904*a^30*b^16*c^13
- 293000581579014144*a^31*b^14*c^14 + 641705669216436224*a^32*b^12*c^15 -
1077743462209552384*a^33*b^10*c^16 + 1348355710714380288*a^34*b^8*c^17 - 11
98053158392168448*a^35*b^6*c^18 + 695801744382230528*a^36*b^4*c^19 - 223957
324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)
^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^1
9*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^1
3*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9
*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^
5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a
*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801
*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^

(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(9119789245425254 4*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) - 89161004482560*a^29*b*c^21 + 175760000*a^20*b^19*c^12 - 6846528000*a^21*b^17*c^13 + 118362316800*a^22*b^15*c^14 - 1191953858560*a^23*b^13*c^15 + 7705795952640*a^24*b^11*c^16 - 33166059110400*a^25*b^9*c^17 + 95038786764800*a^26*b^7*c^18 - 174846482841600*a^27*b^5*c^19 + 187403222384640*a^28*b^3*c^20))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*2i - (2/a - (x^2*(5*b^3 - 19*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(18*a*c - 5*b^2))/(2*a^2*(4*a*c - b^2)))/(a*x^(1/2) + b*x^(5/2) + c*x^(9/2)) + atan(((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c

^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 152
0311317037056*a^30*b^3*c^20) + (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(
1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3
+ 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6
- 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c
^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*
(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b
^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b
^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))
/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2
 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784
704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 576716
80*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*
(32768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^3
2*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 158983630
35648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27
*b^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*
c^12 - 104398826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^1
4 - 641705669216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16
- 1348355710714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 6
95801744382230528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2
)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12
 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434
478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 1
3524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^1
0 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 2
9625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^
4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1
/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a
^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 1267
20*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*
a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^
19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21
- 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^3
0*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340
683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 112301336669716
48*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a
^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^3
4*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36
*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^
4*c^19 - 612489549322387456*a^39*b^2*c^20)))*(-(625*b^25 + 625*b^10*(-(4*a*
c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990
*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360
*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 2112231
0144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 -

26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) + (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(25649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14 + 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 + 1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(9119789245425254 4*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6

+ 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059
200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^3
0*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^
16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12
*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c
^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19
 - 612489549322387456*a^39*b^2*c^20)))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^
2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b
^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b
^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a
^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*
a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4
*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 1718
01*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*
b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^
5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8
 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11))
)^(1/4)*1i)/((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^1
0 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a
^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^1
5 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 119782
1248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 152031131703705
6*a^30*b^3*c^20) + (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 31054
23360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a
^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920
*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 2148301
2096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b
^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2
) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*
a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*
b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^1
2*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^1
2*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*
c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^
21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 6439
8295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b
^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 -
 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 10439
8826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 64170566
9216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 1348355710
714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 6958017443822
30528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^2
5 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^
2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b

^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) - (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) + (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 692

06016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(25649407252758528*a^
38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a
^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 +
153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 64929172
79490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^12 + 10439882608895
5904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14 + 64170566921643622
4*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 + 1348355710714380288
*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695801744382230528*a^3
6*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 + 625*b
^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^
2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 +
1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^
8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*
b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475
*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)
^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^
22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 81
1008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440
320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 5033164
8*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*
c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168
*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c
^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 5
1694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 54372
1556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620
531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 265772191
4474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322
387456*a^39*b^2*c^20)))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) +
3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483
001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 59966
89920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21
483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*
c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)
^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*
(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 1408
0*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^1
5*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18
*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) - 89161
004482560*a^29*b*c^21 + 175760000*a^20*b^19*c^12 - 6846528000*a^21*b^17*c^1
3 + 118362316800*a^22*b^15*c^14 - 1191953858560*a^23*b^13*c^15 + 7705795952
640*a^24*b^11*c^16 - 33166059110400*a^25*b^9*c^17 + 95038786764800*a^26*b^7
*c^18 - 174846482841600*a^27*b^5*c^19 + 187403222384640*a^28*b^3*c^20))*(-(
625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 63

8475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 43447862
4*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 135248
25600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 1
2575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*
a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3
*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) -
10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c
^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^
13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*
b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^
4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*2i + 2*atan(((x^(1/2)*(60233211917
1072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57
034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24
*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16
 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819
580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 - 6
25*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^2
1*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c
^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^
9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a
^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 6
8475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^1
0*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4
- 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 3
2440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 503
31648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34*c^4 - 25649407252758528*a^
38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564
288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8 - 153599583715328*a^26*b^2
4*c^9 + 1132021560639488*a^27*b^22*c^10 - 6492917279490048*a^28*b^20*c^11 +
 29298398985191424*a^29*b^18*c^12 - 104398826088955904*a^30*b^16*c^13 + 293
000581579014144*a^31*b^14*c^14 - 641705669216436224*a^32*b^12*c^15 + 107774
3462209552384*a^33*b^10*c^16 - 1348355710714380288*a^34*b^8*c^17 + 11980531
58392168448*a^35*b^6*c^18 - 695801744382230528*a^36*b^4*c^19 + 223957324438
437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(
1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3
+ 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6
- 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c
^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*
(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b
^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b
^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))
/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2
 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784

704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 576716
80*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*
(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^3
2*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715
265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^2
9*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^1
8*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c
^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^
16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18
+ 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)
1i)(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c
^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 -
434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7
+ 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*
c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2)
- 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3
*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)
^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 1677721
6*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 1
26720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 129761
28*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016
*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) + (x^(1/2)*(60233211917107
2*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034
444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^
15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 +
548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580
444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 - 625*
b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c
^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5
+ 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c
^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11
*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 6847
5*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2
)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*
(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b
^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 8
11008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 3244
0320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 503316
48*a^20*b^2*c^11)))^(3/4)*(2564940725275852 8*a^38*c^21 - 32768000*a^21*b^34
*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288
*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c
^9 - 1132021560639488*a^27*b^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29
298398985191424*a^29*b^18*c^12 + 104398826088955904*a^30*b^16*c^13 - 293000
581579014144*a^31*b^14*c^14 + 641705669216436224*a^32*b^12*c^15 - 107774346
2209552384*a^33*b^10*c^16 + 1348355710714380288*a^34*b^8*c^17 - 11980531583

92168448*a^35*b^6*c^18 + 6958017443822305 28*a^36*b^4*c^19 - 223957324438437
888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2
) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 7
1483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5
996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9
+ 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(
4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*
c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 -
14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704
*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*
a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91
197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c
^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265
024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b
^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c
^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14
 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16
+ 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1
675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i
)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12
 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434
478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 1
3524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^1
0 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 2
9625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^
4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1
/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a
^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 1267
20*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*
a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^
19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4))/((x^(1/2)*(602332119171072*a
^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444
800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*
c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548
447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444
672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 - 625*b^1
0*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2
- 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1
898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8
- 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^
3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a
^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^1
5)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(

4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 1348355710714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444

800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(25649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14 + 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 + 1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478

624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 1352
4825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 -
12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 2962
5*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2)
+ 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21
*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*
a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^1
6*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*
b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i - 89161004482560*a^29*b*c^21
+ 175760000*a^20*b^19*c^12 - 6846528000*a^21*b^17*c^13 + 118362316800*a^22*
b^15*c^14 - 1191953858560*a^23*b^13*c^15 + 7705795952640*a^24*b^11*c^16 - 3
3166059110400*a^25*b^9*c^17 + 95038786764800*a^26*b^7*c^18 - 17484648284160
0*a^27*b^5*c^19 + 187403222384640*a^28*b^3*c^20))*(-(625*b^25 - 625*b^10*(-
(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 82
64990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 18989
83360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21
122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^
11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b
^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*
c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c +
1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a
^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^
17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20
*b^2*c^11)))^(1/4) + 2*atan(((x^(1/2)*(602332119171072*a^31*b*c^21 - 540800
00*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12
+ 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 401692297789
44*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b
^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 -
1520311317037056*a^30*b^3*c^20) - (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^1
5)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*
c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*
c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b
^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*
c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c
- b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a
^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20
*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 +
3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57
671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3
/4)*(32768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22
*b^32*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898
363035648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*

a^27*b^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 1348355710714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(9119789245425254 4*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) + (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*

c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5
*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*
b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2)
)/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^
2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 378
4704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671
680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)
*(25649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^
32*c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363
035648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^2
7*b^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18
*c^12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^
14 + 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16
 + 1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 +
695801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/
2)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^1
2 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 43
4478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 +
13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^
10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) -
29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b
^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(
1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*
a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126
720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128
*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a
^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21
 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^
30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 25734
0683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971
648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*
a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^
34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^3
6*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b
^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 + 625*b^10*
(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 -
8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 189
8983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 -
21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*
c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2
*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)
^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*
a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c
 + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008

*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*
a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^
20*b^2*c^11)))^(1/4))/((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^2
0*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 7491
18545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^2
5*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^1
7 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 15203
11317037056*a^30*b^3*c^20) - (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/
2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 +
71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 -
5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9
 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-
(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2
c^4(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 -
 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 378470
4*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680
*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(3
2768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*
c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035
648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b
^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^
12 - 104398826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14
- 641705669216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 -
1348355710714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695
801744382230528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*
(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 +
 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 43447
8624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 135
24825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10
- 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 296
25*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*
c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2
) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^2
1*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720
*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^
16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19
*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 -
52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*
c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 25734068
3059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648
*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^3
2*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*
b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b

^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*
c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 + 625*b^10*(-(
4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 826
4990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 189898
3360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 211
22310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^1
1 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^
6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1
/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c +
1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^
14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^1
7*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*
b^2*c^11)))^(1/4)*1i - (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^2
0*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 7491
18545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^2
5*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^1
7 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 15203
11317037056*a^30*b^3*c^20) - (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/
2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 +
71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 -
5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9
 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-
(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2
c^4(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 -
 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 378470
4*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680
*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(2
5649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*
c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035
648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b
^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^
12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14
+ 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 +
1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695
801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*
(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 +
 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 43447
8624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 135
24825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10
- 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 296
25*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*
c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2

```
) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^2
1*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720
*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^
16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19
*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 -
52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*
c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 25734068
3059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648
*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^3
2*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*
b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b
^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*
c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 + 625*b^10*(-(
4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 826
4990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 189898
3360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 211
22310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^1
1 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^
6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1
/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c +
1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^
14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^1
7*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*
b^2*c^11)))^(1/4)*1i - 89161004482560*a^29*b*c^21 + 175760000*a^20*b^19*c^1
2 - 6846528000*a^21*b^17*c^13 + 118362316800*a^22*b^15*c^14 - 1191953858560
*a^23*b^13*c^15 + 7705795952640*a^24*b^11*c^16 - 33166059110400*a^25*b^9*c^
17 + 95038786764800*a^26*b^7*c^18 - 174846482841600*a^27*b^5*c^19 + 1874032
22384640*a^28*b^3*c^20))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) +
 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 7148
3001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996
689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 2
1483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a
*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15
)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4
*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192
*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 140
80*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^
15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^1
8*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.847 $\int \frac{x^{15/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=621

$$-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right.}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Rubi [A] time = 1.77, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1115, 1365, 1498, 1502, 1422, 212, 208, 205}

$$-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x^{5/2}\left(x^2\left(12ac+b^2\right)+8ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt{x}\left(12ac+b^2\right)}{16c\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*(b^2 + 12*a*c)*Sqrt[x])/(16*c*(b^2 - 4*a*c)^2) + (x^(9/2)*(2*a + b*x^2)
)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(5/2)*(8*a*b + (b^2 + 12*a
*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (3*(b^3 - 28*a*b*c + (
b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^
2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c
- 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
- 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c + (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqr
t[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1
/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1498

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(f^(n - 1)*(f*x)^(m - n + 1)*(a +
 b*x^n + c*x^(2*n))^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^n))/(n*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{16}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^8\left(18a-3bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^4\left(-120ab-3\right.}{a+bx^4}\right.}{16\left(b^2\right.}$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

Mathematica [C] time = 0.44, size = 254, normalized size = 0.41

$$\frac{3c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-28\#1^4abc\log\left(\sqrt{x}-\#1\right)+\#1^4b^3\log\left(\sqrt{x}-\#1\right)+12a^2c\log\left(\sqrt{x}-\#1\right)+ab^2\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+16\sqrt{x}\left(b^2-4ac\right)\left(-2a^2c+ab\left(b-3cx^2\right)+b^3x^2\right)+4\sqrt{x}\left(-68a^2c^2+21ab^2c-28abc^2x^2-4b^4+b^3cx^2\right)\left(a+bx^2+cx^4\right)}{64c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*Sqrt[x]*(-4*b^4 + 21*a*b^2*c - 68*a^2*c^2 + b^3*c*x^2 - 28*a*b*c^2*x^2)*
(a + b*x^2 + c*x^4) + 16*(b^2 - 4*a*c)*Sqrt[x]*(-2*a^2*c + b^3*x^2 + a*b*(b
 - 3*c*x^2)) + 3*c*(a + b*x^2 + c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (a
*b^2*Log[Sqrt[x] - #1] + 12*a^2*c*Log[Sqrt[x] - #1] + b^3*Log[Sqrt[x] - #1]
*#1^4 - 28*a*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*c^2*(
b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

IntegrateAlgebraic [C] time = 1.19, size = 527, normalized size = 0.85

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{6\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+3\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+28a^2c^2\log\left(\sqrt{x}-\#1\right)-22ab^2c\log\left(\sqrt{x}-\#1\right)+3b^4\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{8ac^3\left(4ac-b^2\right)}-\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-36\#1^4a^2bc^3\log\left(\sqrt{x}-\#1\right)-17\#1^4ab^3c^2\log\left(\sqrt{x}-\#1\right)+8\#1^4b^5c\log\left(\sqrt{x}-\#1\right)-140a^3c^3\log\left(\sqrt{x}-\#1\right)+223a^2b^2c^2\log\left(\sqrt{x}-\#1\right)-80ab^4c\log\left(\sqrt{x}-\#1\right)+8b^6\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{64ac^3\left(4ac-b^2\right)^2}+\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{2c^2}-\frac{\sqrt{x}\left(36a^3c+3a^2b^2+48a^2bcx^2+68a^2c^2x^4+6ab^3x^2+7ab^2cx^4+28abc^2x^6+3b^4x^4-b^3cx^6\right)}{16c\left(4ac-b^2\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/16*(Sqrt[x]*(3*a^2*b^2 + 36*a^3*c + 6*a*b^3*x^2 + 48*a^2*b*c*x^2 + 3*b^4
*x^4 + 7*a*b^2*c*x^4 + 68*a^2*c^2*x^4 - b^3*c*x^6 + 28*a*b*c^2*x^6))/(c*(-b
^2 + 4*a*c)^2*(a + b*x^2 + c*x^4)^2) + RootSum[a + b*#1^4 + c*#1^8 & , Log[
Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ]/(2*c^2) - RootSum[a + b*#1^4 + c*#1^8
 & , (3*b^4*Log[Sqrt[x] - #1] - 22*a*b^2*c*Log[Sqrt[x] - #1] + 28*a^2*c^2*L
og[Sqrt[x] - #1] + 3*b^3*c*Log[Sqrt[x] - #1]*#1^4 + 6*a*b*c^2*Log[Sqrt[x] -
 #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(8*a*c^3*(-b^2 + 4*a*c)) - (3*RootSum[a
+ b*#1^4 + c*#1^8 & , (8*b^6*Log[Sqrt[x] - #1] - 80*a*b^4*c*Log[Sqrt[x] - #
1] + 223*a^2*b^2*c^2*Log[Sqrt[x] - #1] - 140*a^3*c^3*Log[Sqrt[x] - #1] + 8*
b^5*c*Log[Sqrt[x] - #1]*#1^4 - 17*a*b^3*c^2*Log[Sqrt[x] - #1]*#1^4 - 36*a^2
*b*c^3*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*a*c^3*(-b^2 + 4
*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 192.56Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 275, normalized size = 0.44

$$\frac{3\left(\left(-28ac+b^{2}\right)\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{4}b+12a^{2}c+a\,b^{2}\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{64\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7}c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3}b\right)}+\frac{-\frac{\left(28ac-b^{2}\right)b\,x^{\frac{13}{2}}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3\left(8ac+b^{2}\right)ab\,x^{\frac{5}{2}}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c}-\frac{\left(68a^{2}c^{2}+7a\,b^{2}c+3b^{4}\right)x^{\frac{9}{2}}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c}-\frac{3\left(12ac+b^{2}\right)a^{2}\sqrt{x}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-3/32*a^2*(12*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-3/16*a/c*b*(
8*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-1/32*(68*a^2*c^2+7*a*b^2*c+3*
b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)-1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/c/(16*a^2*c^2-8*a*b^2*c+b^4)
*sum((b*(-28*a*c+b^2)*_R^4+12*a^2*c+a*b^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)
),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(b^{2}c+12\,ac^{2}\right)x^{\frac{17}{2}}+\left(7\,b^{3}+44\,abc\right)x^{\frac{13}{2}}+24\,a^{2}bx^{\frac{5}{2}}+\left(35\,ab^{2}+4\,a^{2}c\right)x^{\frac{9}{2}}}{16\left(\left(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}\right)x^{8}+2\left(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3}\right)x^{6}+a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}+\left(b^{6}-6\,ab^{4}c+32\,a^{3}c^{3}\right)x^{4}+2\left(ab^{5}-8\,a^{2}b^{3}c+16\,a^{3}bc^{2}\right)x^{2}\right)}-\int\frac{3\left(\left(b^{2}+12\,ac\right)x^{\frac{7}{2}}+40\,abx^{\frac{3}{2}}\right)}{32\left(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+\left(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3}\right)x^{4}+\left(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2}\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c + 12*a*c^2)*x^(17/2) + (7*b^3 + 44*a*b*c)*x^(13/2) + 24*a^2*
b*x^(5/2) + (35*a*b^2 + 4*a^2*c)*x^(9/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2
*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*((b^2 + 12*a*c)*x^(7/2) + 40*a*b*x
^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

mupad [B] time = 9.35, size = 50970, normalized size = 82.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(15/2)/(a + b*x^2 + c*x^4)^3,x)

[Out] atan(((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8 *b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354 024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^ 7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a ^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6 *c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^ 5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^ 32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7* b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 19373070 7456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16 *c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 1664 7293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a ^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b* c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14 0233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^1 3*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253 1438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^ 3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b ^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1 /2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20 *c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22* c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 21134258 99520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b ^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 1 9585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880* a^19*b^2*c^24)))^(1/4) - (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^ 6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a* b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 8 11008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 3244032 0*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a ^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280* a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c ^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19 *c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a ^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560* a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 400 9*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2) ^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(10995116277

76*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^3
4*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*
c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^
9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2
113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560
*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c
^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558
138880*a^19*b^2*c^24)))^(1/4)*1i - (((3*(3159*a^3*b^14 - 20155392*a^10*c^7
- 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 19451
79360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536
*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^
12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589
824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 + b^8*(-(4*a*c - b
^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*
b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b
^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528
*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 -
3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a
^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^2
5)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 5464
8*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(
1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040
*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^
30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a
^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 70
4475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^
13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^2
0 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 130567005
79840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640
*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 48318382
0800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^
9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 13194139
53331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*
a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*
b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 34
4064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(
16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^2
3*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 20918638
2151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9
*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*
c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14
+ 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13
- 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*
c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 +
32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 503

31648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104
225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4
*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a
^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 131511
74656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9
*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765
570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*
c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c
 - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099
511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*
a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^
6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 4402970
6240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c
^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 1040455
8274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^1
6*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 -
5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)
^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^
3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6
 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^
15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358
219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5
*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2
) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b
^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^
36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10
 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24
*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299
840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14
*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 208
09116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a
^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) + (9*x^(1/2)*(123201*a^
4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2
 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6
*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c
 + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*
c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12
976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 692060
16*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b
^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*
b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b
^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528
*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 -
3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a

^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^2
5)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 5464
8*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(
1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040
*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^
30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a
^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 70
4475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^
13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^2
0 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 130567005
79840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i)/((((3*(3159*
a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 -
299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 1
64042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 5
76*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c
^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(
-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 105
09*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^
5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4337679974
4*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9
86354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^
13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1
296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(
4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*
a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^
40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a
^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680
*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193
730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12
*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 -
16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869
760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c
^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 268435
45600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8
+ 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 5772436045
82400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11
*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^
10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*
c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824
*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15
*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 187475322
47040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^
15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11
- 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 1522
42778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*

(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) - (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^

5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4337679974
4*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9
86354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^
13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1
296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(
4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*
a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^
40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a
^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680
*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193
730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12
*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 -
 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869
760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c
^24)))^(1/4) + (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c
+ 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 +
2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^
9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b
^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 58
9824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 4711
04225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a
^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992
*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 1315
1174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b
^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 42137
65570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^3
1*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(10
99511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 7296
0*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*
a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029
706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18
*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404
558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a
^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23
- 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 67108
8640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 -
5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*
a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^
12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(655
36*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*
b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 5
89824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^
4 - 3124372214882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 100019050905
6*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8

+ 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 2878301
5391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 1309738251006
77120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064
*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 10
56*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^1
4*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9
 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)
))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 +
 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14023372
8*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 433767
99744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10
 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253143859
2*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15
 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) -
107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25
+ b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 12403
20*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 127008
7680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 +
 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*
a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^
19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 1958505
0869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b
^2*c^24)))^(3/4))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 47110422528
0*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25
*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^
19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656
*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12
 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 421376557056
0*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4
009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^
2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(109951162
7776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b
^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^2
8*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*
a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 +
 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 104045582745
60*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8
*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 54975
58138880*a^19*b^2*c^24)))^(1/4) + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a
^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^
10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^
10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^1
3 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^1
6*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8

+ 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5
0331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 4711
04225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a
^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992
*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 1315
1174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b
^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 42137
65570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^3
1*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(10
99511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 7296
0*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*
a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029
706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18
*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404
558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a
^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23
- 5497558138880*a^19*b^2*c^24)))^(1/4)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^
25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27
*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*
c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9
*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840
358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*
b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(
1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^
3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2
*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c
^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b
^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475
299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b
^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 +
20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 1305670057984
0*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*2i - ((3*x^(1/2)*(12
*a^3*c + a^2*b^2))/(16*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^(5/2)*(a*b^
3 + 8*a^2*b*c))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^(13/2)*(28*a*c
- b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(9/2)*(3*b^4 + 68*a^2*c^2
+ 7*a*b^2*c))/(16*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a
^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((3*(3159*a^3*b^14 - 2015539
2*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*
c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c
^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 -
5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^
6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 - b^8*(
-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 -

394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243
68896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1
08493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b
^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212
262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a
*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c -
 b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38
*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 1587
6096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8
255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^2
0*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 52022
79137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^
15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22
+ 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(7036
87441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6
 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*
a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11
 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 175921
8604441600*a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2
 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b
^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (
9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755
584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7
 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 792555469
0916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 7387068871213056
0*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^1
3*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216
*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 12672
0*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*
b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4
*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/
2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 +
9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9
732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c
^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 38403582197
76*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^1
4 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) -
157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c
^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(335
54432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c
^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 1
58760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^1
3 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*
a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^1

8 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 2080911
6549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*
b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 - b^8*(-(4*a*c
 - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*
a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a
^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 10849307
8528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^1
1 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 82122626826
24*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^
2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) +
54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 +
3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^
5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 82555699
20*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15
 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 520227913728
0*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10
*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056
700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) - (9*x^(1/2)
*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^
6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210
048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194
304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 1408
0*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^
12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^
10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(
-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 -
394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243
68896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1
08493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b
^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212
262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a
*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c -
 b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38
*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 1587
6096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8
255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^2
0*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 52022
79137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^
15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22
+ 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i -
(((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*
b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*
b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b

^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 12902
4*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^
9)) + ((3*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b
*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 1
40233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 +
 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^
13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75625
31438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b
^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*
b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(
1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^2
0*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8
+ 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 -
 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22
*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425
899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*
b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 -
19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880
*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c
^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a
^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 +
 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929
996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*c - 26
2144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 3225
6*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c
^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414
882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6
+ 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 154495127597
8752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10
*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*
c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15)
)/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3
 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704
*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9
*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33
- b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29
*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5
+ 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*
c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448
*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13
 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4
*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2
)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(
4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80
*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9

- 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880

*a^19*b^2*c^24)))^(1/4)*1i)/((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 3674
97*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*
a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18
*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4
 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^
7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25
)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c
^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^
6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b
^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035
8219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^
5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/
2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*
b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))
/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b
^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^1
0 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^2
4*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 70447529
9840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^1
4*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20
809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*
a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*
b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a
^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 17
3173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 13194139533312
00*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b
^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c
^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a
^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(167772
16*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5
- 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 20918638215168
0*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*
c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 +
 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 1038
64266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a
*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 -
811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 324403
20*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*
a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280
*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*
c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^1
9*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*
a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12
+ 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560
*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 40

09*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2
)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627
776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^
34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28
*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a
^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 +
2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 1040455827456
0*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*
c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 549755
8138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2)
 - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 92
19696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 973
2052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9
 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776
*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14
+ 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 15
7*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3
*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554
432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7
 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158
760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13
- 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^
11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18
+ 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 208091165
49120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^
4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) - (9*x^(1/2)*(123201*a^4*b^16
 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 1310
5503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 +
 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 167
77216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 +
126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128
*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^1
0*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25
)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c
^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^
6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b
^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035
8219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^
5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/
2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*
b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))
/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b
^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^1
0 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^2
4*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 70447529

9840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^1
4*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20
809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*
a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) + (((3*(3159*a^3*b^14
- 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340
*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*
a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^
14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 3440
64*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^3
3 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^
29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^
5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^1
7*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9863540244
48*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^
13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c
^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b
^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(
-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 -
80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c
^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26
*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456
*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^1
7 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293
239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*
b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1
/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4
*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 360777
25286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9
*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13
 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a
*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129
024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*
c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 +
 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6
*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 -
7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 7387068
8712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 1522427780283
76064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c +
 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^
4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297
6128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016
*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2
)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^
27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^2
1*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a

^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 38
40358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^1
4*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)
^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*
a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/
2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a
^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30
*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8
*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 7044
75299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13
*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20
+ 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579
840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 - b^8
*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2
- 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 142
4368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 -
 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11
*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 82
12262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4
*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)
^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c
 - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^
38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15
876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 +
 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b
^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 520
2279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*
a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^2
2 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) + (
9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653
342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 -
 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^
7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c
^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 37847
04*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a
^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^3
3 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^
29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^
5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^1
7*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9863540244
48*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^
13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c
^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b
^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(
-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 -

80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c
^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26
*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456
*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^1
7 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293
239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*
b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1
/4)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^1
6 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14023
3728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 433
76799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c
^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253143
8592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c
^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*
c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2)
 + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^
25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 12
40320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 127
0087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^1
4 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 21134258995
20*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12
*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 1958
5050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^1
9*b^2*c^24)))^(1/4)*2i + 2*atan(((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 -
367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179
360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(
b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12
*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 58982
4*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 + b^8*(-(4*a*c - b^2)^
25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27
*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*
c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9
*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840
358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*
b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(
1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^
3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2
*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c
^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b
^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475
299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b
^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 +
20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 1305670057984
0*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^1

3*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800
*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 -
173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 131941395333
1200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12
*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b
^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344
064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(1
6777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23
*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382
151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*
b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c
^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 +
 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 -
 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c
^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 3
2440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5033
1648*a^11*b^2*c^12))))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 4711042
25280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*
b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^
7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 1315117
4656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*
c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 42137655
70560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c
 + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c
- b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(10995
11627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a
^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6
*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706
240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^
16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558
274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16
*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5
497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^2
5)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*
c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c
^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*
b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 38403
58219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b
^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1
/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3
*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2))
)/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*
b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^
10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^
24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 7044752

99840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) - (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12 *b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 3124372241488281 6*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^4

0*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^
4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*
a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 1937
30707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*
b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 -
16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 195850508697
60*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^
24)))^(1/4)*1i + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 1461985
2*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 10230607
1520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261
608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2
 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^
5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^
8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*
c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c
^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140
233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4
3376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13
*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531
438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3
*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^
4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/
2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*
c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 +
1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1
270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c
^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 211342589
9520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^
12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19
585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a
^19*b^2*c^24)))^(1/4))/((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^
4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b
^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c -
262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32
256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4
*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2)
 - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 92
19696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 973
2052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9
 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776
*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14
+ 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 15
7*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3
*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554
432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7

 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158
760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13
- 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^
11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18
+ 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 208091165
49120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^
4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15
+ 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17
*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081
374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10
*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14
)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 -
 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b
^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a
^3*b^25*c^4 - 3124372241488 2816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 10
00190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^
7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10
 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130
973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 10386426
6406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^2
2*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 8110
08*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a
^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11
*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^1
6*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4
- 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^
7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10
*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75
62531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^1
5*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a
^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25
)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*
a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c
^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^1
1 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b
^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113
425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^
14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21
 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138
880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2)
- 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 921
9696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732
052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9
+ 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*
a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 +

4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157
*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*
(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7
- 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 1587
60960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 -
44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^1
1*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 +
10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 2080911654
9120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4
*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (9*x^(1/2)*(123201*a^4*b^
16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13
105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5
+ 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 1
6777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4
+ 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 129761
28*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a
^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^
25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27
*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*
c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9
*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840
358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*
b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(
1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^
3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2
*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c
^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b
^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475
299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b
^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 +
20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 1305670057984
0*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i + (((3*(3159*a^3*
b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 2995
49340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 16404
2496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a
^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 +
344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(
b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2
*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23
*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*
b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9863540
24448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7
*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^

4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23* c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c -

b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(109951
1627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^
3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*
b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 440297062
40*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^1
6 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 104045582
74560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*
b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 54
97558138880*a^19*b^2*c^24)))^(1/4) + 2*atan(((((3*(3159*a^3*b^14 - 20155392
*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c
^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^
6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5
376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6
*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 - b^8*(-(4
*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394
248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243688
96*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1084
93078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11
*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262
682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c
- b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2
) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^
2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^
6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 1587609
6*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255
569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c
^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 52022791
37280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*
b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 1
3056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(7036874
41776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 +
483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7
*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 -
1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 175921860
4441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2
 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b
^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (
9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755
584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7
 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 792555469
0916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 7387068871213056
0*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^1
3*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216
*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 12672
0*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*

b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448* a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) - (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*

b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c
^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20
*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 378
4704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680
*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b
^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*
b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*
c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b
^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 98635402
4448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*
c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4
c^4(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c -
b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c
*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5
- 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32
*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^
26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 1937307074
56*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c
^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 166472
93239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^1
7*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^
(3/4)*1i)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b
*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 1
40233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 +
43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^
13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75625
31438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b
^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*
b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(
1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^2
0*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8
+ 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 -
1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22
*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425
899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*
b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 -
19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880
*a^19*b^2*c^24)))^(1/4)*1i + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c
^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^
3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^
4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 4
8*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5
- 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 324
40320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 503316
48*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225

280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4))/((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b

^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186
382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a
^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^
9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^1
4 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^1
3 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^1
6*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8
+ 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5
0331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 4711
04225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a
^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992
*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 1315
1174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b
^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 42137
65570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^3
1*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a
*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(10
99511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 7296
0*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*
a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029
706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18
*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404
558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a
^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23
- 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 - b^8*(-(4*a*c - b^2
)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^
27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^2
1*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a
^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 38
40358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^1
4*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)
^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*
a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/
2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a
^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30
*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8
*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 7044
75299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13
*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20
+ 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579
840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (9*x^(1/2)*(1
23201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b
^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048
*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304
*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a

^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*
c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10
+ 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4
*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394
248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243688
96*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1084
93078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11
*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262
682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c
- b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2
) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^
2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^
6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 1587609
6*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255
569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c
^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 52022791
37280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*
b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 1
3056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i + (((
3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^1
0*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4
*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16
*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a
^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9))
 - (((-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16
 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233
728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4337
6799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^
10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438
592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^
15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c
^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^2
5 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 124
0320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270
087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14
 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 211342589952
0*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*
c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585
050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19
*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 -
26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^
15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 5772
43604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 197912092999680
0*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 2621

44*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 1

```
4619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 10
2306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6
+ 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^
22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811
008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*
a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^1
1*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^
16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4
 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c
^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^1
0*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7
562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^
15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*
a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^2
5)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776
*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*
c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^
11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*
b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 211
3425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a
^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^2
1 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 549755813
8880*a^19*b^2*c^24)))^(1/4)*1i))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2
) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9
219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 97
32052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^
9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035821977
6*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14
 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 1
57*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^
3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355
4432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^
7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 15
8760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13
 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a
^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18
 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116
549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b
^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.848 $\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=569

$$\frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^{3/2}\left(x^2\left(28ac+5b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tan^{-1}\left(\right.}{32\, 2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}\,-}}$$

Rubi [A] time = 1.91, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1365, 1498, 1510, 298, 205, 208}

$$\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\, 2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\, 2^{3/4}c^{3/4}\left(b^2-4ac\right)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\, 2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\, 2^{3/4}c^{3/4}\left(b^2-4ac\right)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^{3/2}\left(x^2\left(28ac+5b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(7/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^(3/2)*
(24*a*b + (5*b^2 + 28*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2
- 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((5*b^2 + 28*a*c - (5*b^3
+ 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt
[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 -
 4*a*c])^(1/4)) - ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((5*b^2
+ 28*a*c - (5*b^3 + 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)
^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1498

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(f^(n - 1)*(f*x)^(m - n + 1)*(a +
 b*x^n + c*x^(2*n))^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^n))/(n*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{14}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^6\left(14a-5bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(-72ab+\left(5b^2\right.\right.}{a+bx^4+}\right.}{16\left(b^2-\right.}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(5b^3+172abc+\sqrt{b^2}\right.}{}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(5b^3+172abc+\sqrt{b^2}\right.}{}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(5b^3+172abc+\sqrt{b^2}\right.}{32\,2^{3/4}c^{3/4}\left(\right.}$$

Mathematica [C] time = 0.41, size = 216, normalized size = 0.38

$$\frac{c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{28\#1^4ac\log\left(\sqrt{x}-\#1\right)+5\#1^4b^2\log\left(\sqrt{x}-\#1\right)-72ab\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]-16x^{3/2}\left(b^2-4ac\right)\left(a\left(b-2cx^2\right)+b^2x^2\right)+4x^{3/2}\left(8abc+28ac^2x^2+4b^3+5b^2cx^2\right)\left(a+bx^2+cx^4\right)}{64c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*x^(3/2)*(4*b^3 + 8*a*b*c + 5*b^2*c*x^2 + 28*a*c^2*x^2)*(a + b*x^2 + c*x^
4) - 16*(b^2 - 4*a*c)*x^(3/2)*(b^2*x^2 + a*(b - 2*c*x^2)) + c*(a + b*x^2 +
c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (-72*a*b*Log[Sqrt[x] - #1] + 5*b^2
*Log[Sqrt[x] - #1]*#1^4 + 28*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5)
& ])/(64*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

IntegrateAlgebraic [C] time = 1.12, size = 397, normalized size = 0.70

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-36\#1^4a^2c^3\log\left(\sqrt{x}-\#1\right)-11\#1^4ab^2c^2\log\left(\sqrt{x}-\#1\right)+8\#1^4b^4c\log\left(\sqrt{x}-\#1\right)+344a^2bc^2\log\left(\sqrt{x}-\#1\right)-136ab^3c\log\left(\sqrt{x}-\#1\right)+8b^5\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{64ac^2\left(4ac-b^2\right)^2}+\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{2\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-13abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{8ac^2\left(4ac-b^2\right)}+\frac{x^{3/2}\left(24a^2b-4a^2cx^2+37ab^2x^2+36abcx^4+28ac^2x^6+9b^3x^4+5b^2cx^6\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(24*a^2*b + 37*a*b^2*x^2 - 4*a^2*c*x^2 + 9*b^3*x^4 + 36*a*b*c*x^4
+ 5*b^2*c*x^6 + 28*a*c^2*x^6))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) +
 RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 13*a*b*c*Log[Sqrt
[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 2*a*c^2*Log[Sqrt[x] - #1]*#1^4)/
(b*#1 + 2*c*#1^5) & ]/(8*a*c^2*(-b^2 + 4*a*c)) + RootSum[a + b*#1^4 + c*#1^
8 & , (8*b^5*Log[Sqrt[x] - #1] - 136*a*b^3*c*Log[Sqrt[x] - #1] + 344*a^2*b*
c^2*Log[Sqrt[x] - #1] + 8*b^4*c*Log[Sqrt[x] - #1]*#1^4 - 11*a*b^2*c^2*Log[S
qrt[x] - #1]*#1^4 - 36*a^2*c^3*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) &
]/(64*a*c^2*(-b^2 + 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.18Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 242, normalized size = 0.43

$$\frac{\left(\left(28ac+5b^{2}\right)\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6}-72\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2}ab\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{64\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7}c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3}b\right)}+\frac{\frac{2\left(28ac+5b^{2}\right)c\,x^{\frac{15}{2}}}{512a^{2}c^{2}-256ab^{2}c+32b^{4}}+\frac{9\left(4ac+b^{2}\right)b\,x^{\frac{11}{2}}}{16\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)}+\frac{3a^{2}b\,x^{\frac{3}{2}}}{2\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)}-\frac{\left(4ac-37b^{2}\right)a\,x^{\frac{7}{2}}}{16\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*b*x^(3/2)-1/32*a*(4*a*c-37*b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+9/32*b*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^(11/2)+1/32*c*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^4
```

```
+b*x^2+a)^2+1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((28*a*c+5*b^2)*_R^6-72*_R^
2*a*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(5\,b^{2}c+28\,ac^{2}\right)x^{\frac{15}{2}}+9\left(b^{3}+4\,abc\right)x^{\frac{11}{2}}+24\,a^{2}bx^{\frac{3}{2}}+\left(37\,ab^{2}-4\,a^{2}c\right)x^{\frac{7}{2}}}{16\left(\left(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}\right)x^{8}+2\left(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3}\right)x^{6}+a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}+\left(b^{6}-6\,ab^{4}c+32\,a^{3}c^{3}\right)x^{4}+2\left(ab^{5}-8\,a^{2}b^{3}c+16\,a^{3}bc^{2}\right)x^{2}\right)}+\int\frac{\left(5\,b^{2}+28\,ac\right)x^{\frac{5}{2}}-72\,ab\sqrt{x}}{32\left(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+\left(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3}\right)x^{4}+\left(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2}\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c + 28*a*c^2)*x^(15/2) + 9*(b^3 + 4*a*b*c)*x^(11/2) + 24*a^2*b
*x^(3/2) + (37*a*b^2 - 4*a^2*c)*x^(7/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*
c + 16*a^3*b*c^2)*x^2) + integrate(1/32*((5*b^2 + 28*a*c)*x^(5/2) - 72*a*b*
sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

mupad [B] time = 8.01, size = 39697, normalized size = 69.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((9*x^(11/2)*(b^3 + 4*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(7/2
)*(37*a*b^2 - 4*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^(15/2)*(
28*a*c + 5*b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a^2*b*x^(3/2))/(2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*
b*x^2 + 2*b*c*x^6) - atan(((((3861836680470200032*a^16*c^16 + 2097152000*a^3
*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 515
4027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 3505726682667
41760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032
*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*
a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^
14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 2684354
56*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4
 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 1
96804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 -
 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c
^13 - 56*a*b^26*c)) - (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1
/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c
^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*
b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 144629
70429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*
```

b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4) - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21

*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 66645041479
68*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^1
0 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 26745984
4112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*
a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627
776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^
34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*
c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9
*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 21
13425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*
a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^
19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 54975581
38880*a^19*b^2*c^22)))^(1/4)*1i - (((386183668047020032*a^16*c^16 + 2097152
000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^
5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572
668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 462823696696
0300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753
743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 1182321565924274
9952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 +
268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b
^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*
c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8
*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^1
3*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)
^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3
*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 2651888332
80*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 +
14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 7045524226048
0*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*
c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1
/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 5437
5*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b
^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*
a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*
a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 1937
30707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*
b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 -
16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 195850508697
60*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^
22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 40925
66962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b
^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 +
112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 5077
43474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582

645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4) + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*1i)/((((38618366804702003 2*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 -

350572668266741760*a^8*b^16*c^8 + 15602952356222273024*a^9*b^14*c^9 - 46282
36966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 792402
6369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 118232156
59242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(
b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 25625
6*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^
7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a
^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524
096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c
 - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186
416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265
188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^1
5*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 704552
42260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^
13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)
^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
 + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(109951627776*a^20*c
^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1
240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270
087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12
 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 211342589952
0*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*
c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585
050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19
*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4
 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 83799106912256
0*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14
*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11
 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 43
6356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(419430
4*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126
720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^
7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^
4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4
*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 2
7186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 -
 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8
*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70
455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 26745984411238
4*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c -
b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(109951627776*a^
20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6
 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 -
1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*

c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 21134258 99520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b ^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 1 9585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880* a^19*b^2*c^22)))^(3/4) - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c ^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9 *c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a ^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 1408 0*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^ 12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^ 9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625 *b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89 000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 254924 09600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520 *a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c ^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416 *a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(335544 32*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 15876 0960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 4 4029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11* b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1 0404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 208091165491 20*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c ^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) + (((386183668047020032*a^16*c^1 6 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784* a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c ^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4 628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 79 24026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823 215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(2684354 56*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 2 56256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 5622988 8*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 10496245 76*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 93 9524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4 *a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 2 7186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8 *b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70 455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 26745984411238 4*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(

1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4) + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b

^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 3
8416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c
^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33
554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*
c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 1
58760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11
 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a
^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16
 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116
549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b
^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - (285333125*a^4*b^15*c + 48
189030400*a^11*b*c^8 + 22337507500*a^5*b^13*c^2 + 657473586000*a^6*b^11*c^3
 + 8657411576000*a^7*b^9*c^4 + 43867083462400*a^8*b^7*c^5 + 13299491251200*
a^9*b^5*c^6 + 1381697515520*a^10*b^3*c^7)/(134217728*(b^28 + 268435456*a^14
*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050
048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 19680460
8*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 152672
6656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 5
6*a*b^26*c))))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 1519210463
2320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*
a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 16888
16578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^1
3*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 20666
9464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*
a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c +
1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^
2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4
 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096
*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569
920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13
 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 52022791372
80*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^1
0*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 1305
6700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*2i - 2*atan
(((((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128
*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^
6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560
295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139
182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 194235326
1163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 84191980283
92431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b
^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 +
 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 5
24812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11
 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^

(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^

4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32
440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 503316
48*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625
*b^31 + 1519210463232O*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25
*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^
6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 1446
2970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^1
1*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13
- 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) -
23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b
^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c
^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b
^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b
^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707
456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*
c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647
293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^
17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))
^(1/4) - (((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 761531
2560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*
b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8
+ 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 +
8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 19
42353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419
198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 145
6*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^1
8*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*
c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b
^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)
) + (x^(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 1519210463232
0*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4
*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 16888165
78560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c
^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 20666946
4207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^1
4*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 191
1000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^
25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 +
3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^
5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920
*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 -
704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*
a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c
^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 1305670
0579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(2758454771764

4288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75
824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880
*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^1
2*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^
12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 -
170573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 +
1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*
b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*
c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^1
1 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 1519210
4632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 13422976
00*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 16
88816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*
b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 20
6669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 1500091147878
40*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c
+ 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c -
b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*
c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876
096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255
569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c
^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 52022791
37280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*
b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 1
3056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i - (x^
(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2
+ 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*
c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24
+ 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*
b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c
^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 -
50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2
) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a
^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 26518883
3280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8
- 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260
480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^
5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^
(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54
375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 +
b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 124032
0*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 127008768
0*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 19
3730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^1
2*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17

- 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 1958505086 9760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2* c^22)))^(1/4))/((((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 51540273271930 88*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b ^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12* c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^ 12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^1 4 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048* a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^ 8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656 *a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a* b^26*c)) - (x^(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 151921 04632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297 600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1 688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9 *b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 2 06669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787 840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29* c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(109951162777 6*a^20*c^23 + b^40*c^3 - 80*a*b^38 *c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 1587 6096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 825 5569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20* c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279 137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15 *b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(275845 47717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c ^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 613334214 7706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688* a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11 *b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4* c^14 - 170573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12 *c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8110 08*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a ^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11* b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1 342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c ^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 144629704294 40*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^ 11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009 114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a

*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(
4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*
a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7
- 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10
+ 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10
*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5
202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 1664729323929
6*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c
^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1
i + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^
13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a
^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304
*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 1267
20*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7
*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4
*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^2
5)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 271
86416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 2
65188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b
^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 7045
5242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*
a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^
2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20
*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 +
1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 12
70087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^
12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899
520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^1
2*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 195
85050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^
19*b^2*c^22)))^(1/4)*1i + (((386183668047020032*a^16*c^16 + 2097152000*a^3*
b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154
027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 35057266826674
1760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*
a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a
^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^1
4*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 26843545
6*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4
- 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 19
6804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 -
1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^
13 - 56*a*b^26*c)) + (x^(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^3
1 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3
+ 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^

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19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970
429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^
9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 15
0009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 231
25*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c
*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 -
 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*
c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*
c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*
a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15
 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 166472932
39296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b
^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/
4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*
a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 +
 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150
323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 5077434745906
79040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464
*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 167
77216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*
c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033
1648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 6
25*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^
25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*
a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14
462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a
^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^1
3 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2)
 - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a
*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40
*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4
*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7
*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 1937307
07456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^1
6*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 166
47293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*
a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)
))^(3/4)*1i - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098
125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 3874
69862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7)
)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*
c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12
976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 6920601
6*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*
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c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27
*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^
21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 666450414
7968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c
^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459
844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(
4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^
2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(10995116
27776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*
b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^2
8*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a
^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 +
2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1040455827456
0*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*
c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 549755
8138880*a^19*b^2*c^22)))^(1/4)*1i + (285333125*a^4*b^15*c + 48189030400*a^1
1*b*c^8 + 22337507500*a^5*b^13*c^2 + 657473586000*a^6*b^11*c^3 + 8657411576
000*a^7*b^9*c^4 + 43867083462400*a^8*b^7*c^5 + 13299491251200*a^9*b^5*c^6 +
1381697515520*a^10*b^3*c^7)/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*
a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*
c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^
8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6
*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)))
)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^
15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 -
25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b
^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 416332
6443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^1
2*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14
- 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^
2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/
(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^
36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8
+ 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c
^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 70447529984
0*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c
^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809
116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^1
8*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - 2*atan(((((386183668047
020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 2
95658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 5282129021763
5840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a
^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11
*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b
^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2

c^15)/(268435456(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a
^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^1
6*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10
*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^1
2*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(625*b^31
 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a
^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600
*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 66
64504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10
*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 -
 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*
c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a
*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1
099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 729
60*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*
a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 440297
06240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*
c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 104045
58274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^
16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 -
 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 998915
44064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c
^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 3118847
1955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 28653712824493
6704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120
*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14
*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14
080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*
b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*
c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c))*(-(6
25*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 -
89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 2549
2409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c
^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 41633264435
20*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7
*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 384
16*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(3355
4432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^
5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158
760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 -
 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^1
1*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 +
 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 2080911654
9120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4

*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - (((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^1

1 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*
a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^1
6 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 2080911
6549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*
b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^1
6 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*
a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8
 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 2865
37128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 59936577
8533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 1705738358866
57536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^2
0*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 37
84704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 5767168
0*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22
c)))(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15
*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*
c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*
a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4
163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 20666946420736
0*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*
c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a
^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1
/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a
^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30
*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b
^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475
299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b
^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 +
20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 1305670057984
0*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i - (x^(1/2)*(37056
25*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000
*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953
639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a
^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8
11008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3244032
0*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^
11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2
) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3
 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^
19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970
429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^
9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 15
0009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 231
25*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c
*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 -

80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*
c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*
c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*
a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15
- 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 166472932
39296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b
^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/
4))/((((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560
128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20
*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1
560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604
139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 194235
3261163970560*a^13*b^6*c^13 + 1182321565924274995 2*a^14*b^4*c^14 - 84191980
28392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^
2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^
5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8
- 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c
^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) -
(x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a
^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^
23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 16888165785
60*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9
- 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 20666946420
7360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b
^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 191100
0*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)
^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 304
0*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b
^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^
8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704
475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^1
3*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18
+ 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 1305670057
9840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(2758454771764428
8*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824
426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^
7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c
^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12
- 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170
573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 10
56*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^1
4*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8
- 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 -
48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 151921046
32320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600

*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*1i + (((38618366804702003 2*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*

a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 15267266
56*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*
a*b^26*c)) + (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 151
92104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342
297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6
+ 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*
a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11
+ 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114
787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^
29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a
c - b^2)^25)^(1/2))/(33554432(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b
^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 1
5876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 +
8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^
20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202
279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a
^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20
 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(275
84547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^2
2*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 613334
2147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 1123431503238266
88*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a
^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b
^4*c^14 - 170573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a
^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8
11008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3244032
0*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^
11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2
) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3
 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^
19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970
429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^
9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 15
0009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 231
25*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c
*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 -
 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*
c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*
c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*
a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15
 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 166472932
39296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b
^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/
4)*1i - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^
4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 3874698624

00*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(419
4304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 +
126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128
*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10
*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(
-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2
+ 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^
5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*
a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 -
70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 26745984411
2384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c
- b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25
)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776
*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*
c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9
- 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^
22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 21134
25899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^1
4*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19
- 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 54975581388
80*a^19*b^2*c^22)))^(1/4)*1i + (285333125*a^4*b^15*c + 48189030400*a^11*b*c
^8 + 22337507500*a^5*b^13*c^2 + 657473586000*a^6*b^11*c^3 + 8657411576000*a
^7*b^9*c^4 + 43867083462400*a^8*b^7*c^5 + 13299491251200*a^9*b^5*c^6 + 1381
697515520*a^10*b^3*c^7)/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b
^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 +
12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 5
24812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11
+ 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c))))*(-(
625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 -
89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 254
92409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*
c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443
520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^
7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38
416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^
2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(335
54432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c
^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 15
8760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11
- 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^
11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16
+ 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 208091165
49120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^
4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - atan(((((386183668047020032
*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 2956585

69334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a ^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^1 4*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10* c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^1 3 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15) /(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^2 2*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4* c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*((625*b^6*(-(4*a* c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27 *c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^ 21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 666450414 7968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c ^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459 844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^ 2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(10995116 27776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3* b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^2 8*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a ^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1040455827456 0*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8* c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 549755 8138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a ^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 83 7991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587 072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^ 10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b ^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^ 15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^ 18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 6920 6016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4 *a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b ^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5 *b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 666450 4147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^1 1*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267 459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3* (-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(10995 11627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a ^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*

b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 4402970624
0*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14
 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1040455827
4560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b
^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 549
7558138880*a^19*b^2*c^22)))^(3/4) - (x^(1/2)*(3705625*a^3*b^15*c - 64022568
96*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940
400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117
420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20
*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 378
4704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680
*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*
c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b
*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^
4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^
7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 416
3326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*
a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^
14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2
*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2
))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2
*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c
^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^2
4*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 70447529
9840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^1
4*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20
809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*
a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*1i - (((38618366804702
0032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295
658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 528212902176358
40*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9
*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b
^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6
*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c
^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3
*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*
c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c
^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*
b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*((625*b^6*(-(
4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*
b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^
5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 66645
04147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^
11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 26
7459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3

*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4) + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 +

4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207
360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^
3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000
*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^
(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040
*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^
30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8
*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 7044
75299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13
*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18
+ 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579
840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*1i)/((((3861836680
47020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 +
295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217
635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024
*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^
11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13
*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b
^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296
*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b
^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^
10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a
^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*((625*b^6
*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*
a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 2549240960
0*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6
664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^1
0*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12
+ 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3
c^3(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*
a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(
1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72
960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960
*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029
706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18
*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404
558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a
^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21
- 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891
544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*
c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 311884
71955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 2865371282449
36704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 59936577853325312
0*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^1
4*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 1408

0*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4) - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) + (((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a

^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^1
3*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*
b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 2329
6*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*
b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b
^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*
a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*((625*b^
6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000
*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 254924096
00*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 +
6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^
10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12
+ 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^
3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4
*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*
(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 7
2960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 15876096
0*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 4402
9706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^1
8*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1040
4558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*
a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21
- 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 9989
1544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20
*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188
471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244
936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 5993657785332531
20*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^
14*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 140
80*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b
^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c
^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625
b^6(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89
000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 254924
09600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7
+ 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520
*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c
^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416
*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(335544
32*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5
- 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 15876
0960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 4
4029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*
b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1

0404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 208091165491
20*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c
^21 - 5497558138880*a^19*b^2*c^22)))^(3/4) + (x^(1/2)*(3705625*a^3*b^15*c -
6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 +
95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*
c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056
*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*
c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 -
57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 4
8*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 151921046323
20*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^
4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816
578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*
c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 2066694
64207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^
14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 19
11000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)
^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 +
3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a
^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 825556992
0*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 -
704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280
*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*
c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 130567
00579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - (285333125*
a^4*b^15*c + 48189030400*a^11*b*c^8 + 22337507500*a^5*b^13*c^2 + 6574735860
00*a^6*b^11*c^3 + 8657411576000*a^7*b^9*c^4 + 43867083462400*a^8*b^7*c^5 +
13299491251200*a^9*b^5*c^6 + 1381697515520*a^10*b^3*c^7)/(134217728*(b^28 +
268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*
b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14
*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^
8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^
13*b^2*c^13 - 56*a*b^26*c))))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^3
1 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3
+ 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^
19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970
429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^
9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 15
0009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 231
25*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c
*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 -
80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*
c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*
c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*
a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15

```
 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 166472932
39296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b
^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/
4)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.849 $\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=569

$$\frac{\sqrt{x}\left(x^2\left(20ac+7b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tan^{-1}\left(\right.}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right.}$$

Rubi [A] time = 1.96, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1365, 1498, 1422, 212, 208, 205}

$$\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(x^2\left(20ac+7b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(5/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[x]*
(24*a*b + (7*b^2 + 20*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) -
 (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*c))*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2
 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7*b^2 + 20*a*c - (7*b
^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
 - 4*a*c])^(3/4)) - (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*
c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*
2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7
*b^2 + 20*a*c - (7*b^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*
a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1498

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(f^(n - 1)*(f*x)^(m - n + 1)*(a +
 b*x^n + c*x^(2*n))^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^n))/(n*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{12}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^4\left(10a-7bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{-24ab+3\left(7b^2+2\right.}{a+bx^4+cx}\right.}{16\left(b^2-4\right.}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3\left(7b^3+36abc+\sqrt{b}\right.\right.}{}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\left(7b^3+36abc+\sqrt{b}\right.\right.}{}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{3\left(7b^3+36abc+\sqrt{b^2}\right.}{32\sqrt[4]{2}\sqrt[4]{c}\left(b\right.}$$

Mathematica [C] time = 0.39, size = 219, normalized size = 0.38

$$\frac{3c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{20\#1^4ac\log\left(\sqrt{x}-\#1\right)+7\#1^4b^2\log\left(\sqrt{x}-\#1\right)-8ab\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]-16\sqrt{x}\left(b^2-4ac\right)\left(a\left(b-2cx^2\right)+b^2x^2\right)+4\sqrt{x}\left(8abc+20ac^2x^2+4b^3+7b^2cx^2\right)\left(a+bx^2+cx^4\right)}{64c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*Sqrt[x]*(4*b^3 + 8*a*b*c + 7*b^2*c*x^2 + 20*a*c^2*x^2)*(a + b*x^2 + c*x^
4) - 16*(b^2 - 4*a*c)*Sqrt[x]*(b^2*x^2 + a*(b - 2*c*x^2)) + 3*c*(a + b*x^2
+ c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (-8*a*b*Log[Sqrt[x] - #1] + 7*b^
2*Log[Sqrt[x] - #1]*#1^4 + 20*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^
7) & ])/(64*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

IntegrateAlgebraic [C] time = 0.93, size = 413, normalized size = 0.73

$$\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-44\#1^4a^2c^3\log\left(\sqrt{x}-\#1\right)-9\#1^4ab^2c^2\log\left(\sqrt{x}-\#1\right)+8\#1^4b^4c\log\left(\sqrt{x}-\#1\right)+152a^2bc^2\log\left(\sqrt{x}-\#1\right)-72ab^3c\log\left(\sqrt{x}-\#1\right)+8b^5\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{64ac^2\left(4ac-b^2\right)^2}+\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{2\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-5abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{8ac^2\left(4ac-b^2\right)}+\frac{24a^2b\sqrt{x}-12a^2cx^{5/2}+39ab^2x^{5/2}+28abcx^{9/2}+20ac^2x^{13/2}+11b^3x^{9/2}+7b^2cx^{13/2}}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (24*a^2*b*Sqrt[x] + 39*a*b^2*x^(5/2) - 12*a^2*c*x^(5/2) + 11*b^3*x^(9/2) +
28*a*b*c*x^(9/2) + 7*b^2*c*x^(13/2) + 20*a*c^2*x^(13/2))/(16*(b^2 - 4*a*c)^
2*(a + b*x^2 + c*x^4)^2) + (3*RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt
[x] - #1] - 5*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 2*a*
c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(8*a*c^2*(-b^2 + 4*a*c
)) + (3*RootSum[a + b*#1^4 + c*#1^8 & , (8*b^5*Log[Sqrt[x] - #1] - 72*a*b^3
*c*Log[Sqrt[x] - #1] + 152*a^2*b*c^2*Log[Sqrt[x] - #1] + 8*b^4*c*Log[Sqrt[x
] - #1]*#1^4 - 9*a*b^2*c^2*Log[Sqrt[x] - #1]*#1^4 - 44*a^2*c^3*Log[Sqrt[x]
- #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*a*c^2*(-b^2 + 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.55Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 241, normalized size = 0.42

$$\frac{3\left(\left(20ac+7b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4-8ab\right)\ln\left(-\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8ab^2c+b^4\right)\left(2\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{\frac{2\left(20ac+7b^2\right)cx^{\frac{13}{2}}}{512a^2c^2-256ab^2c+32b^4}+\frac{2\left(28ac+11b^2\right)bx^{\frac{9}{2}}}{512a^2c^2-256ab^2c+32b^4}+\frac{3a^2b\sqrt{x}}{2\left(16a^2c^2-8ab^2c+b^4\right)}-\frac{3\left(4ac-13b^2\right)ax^{\frac{5}{2}}}{16\left(16a^2c^2-8ab^2c+b^4\right)}}{\left(cx^4+bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*b*x^(1/2)-3/32*(4*a*c-13*b^2)/(16*a^2
*c^2-8*a*b^2*c+b^4)*a*x^(5/2)+1/32*b*(28*a*c+11*b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^(9/2)+1/32*c*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*
x^4+b*x^2+a)^2+3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((20*a*c+7*b^2)*_R^4-8*a
*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{24\,bc^2x^{\frac{17}{2}} + \left(41\,b^2c - 20\,ac^2\right)x^{\frac{13}{2}} + \left(13\,b^3 + 20\,abc\right)x^{\frac{9}{2}} + 3\left(3\,ab^2 + 4\,a^2c\right)x^{\frac{5}{2}}}{16\left(\left(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4\right)x^8 + 2\left(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3\right)x^6 + a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + \left(b^6 - 6\,ab^4c + 32\,a^3c^3\right)x^4 + 2\left(ab^5 - 8\,a^2b^3c + 16\,a^3bc^2\right)x^2\right)} + \int \frac{3\left(8\,bcx^{\frac{7}{2}} + 5\left(3\,b^2 + 4\,ac\right)x^{\frac{3}{2}}\right)}{32\left(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^4 + \left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(17/2) + (41*b^2*c - 20*a*c^2)*x^(13/2) + (13*b^3 + 20*a*
b*c)*x^(9/2) + 3*(3*a*b^2 + 4*a^2*c)*x^(5/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3
*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2) + integrate(3/32*(8*b*c*x^(7/2) + 5*(3*b^2 + 4*
a*c)*x^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

mupad [B] time = 8.52, size = 45495, normalized size = 79.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(9/2)*(11*b^3 + 28*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^(
5/2)*(13*a*b^2 - 4*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^(13/2
)*(20*a*c + 7*b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a^2*b*x^(1/2))
/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2
*a*b*x^2 + 2*b*c*x^6) - atan((((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2
) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*
a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 398985216
0*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977
994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^1
1 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 30
40*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*
b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^
8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 7044
75299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13
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*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^

40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i - ((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 4

6901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 20615843020800
0*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^
13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^
2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 +
344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)
) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 14
7907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b
^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 223
3932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 159978941841
4080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^1
2*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^
12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 81
1008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320
*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^1
1*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401
*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23
*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^
17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a
^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 2307
70606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b
^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6
 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c
^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840
*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^
14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091
16549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18
*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8
+ 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879
403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9
 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b
^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*
a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 70464307
2000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040
*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799
680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66
059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^
5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^
40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*
a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6
*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240
*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12
+ 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274

560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i)/(((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)

) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 14
7907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b
^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 223
3932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 159978941841
4080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^1
2*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^
12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 81
1008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320
*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^1
1*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401
*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23
*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^
17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a
^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 2307
70606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b
^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6
 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c
^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840
*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^
14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091
16549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18
*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8
 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879
403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9
 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b
^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*
a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 70464307
2000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040
*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799
680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66
059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^
5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^
40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*
a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6
*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240
*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12
+ 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274
560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^
8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497
558138880*a^19*b^2*c^20)))^(1/4) - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 1036
8000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404
429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 223

94880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^1
4*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^2
1*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*
b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 6605923942
4*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 +
887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 940
0*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1
099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34
*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^
7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^2
2*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 211342
5899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14
*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 -
19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 549755813888
0*a^19*b^2*c^20)))^(1/4) + ((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) -
2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3
*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a
^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994
240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 -
230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-
(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^
(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*
a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^3
0*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b
^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 7044752
99840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^
14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 2
0809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840
*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13
*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4
*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 4690
1042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a
^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13
- 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b
^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344
064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) +
(9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 14790
7936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19
*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 223393
2749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 159978941841408
0*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b

^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*
c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 81100
8*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^
8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b
^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^
29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^
3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*
c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*
b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 2307706
06080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c -
 b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36
*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 +
158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9
- 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^
11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14
+ 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091165
49120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^
4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2
917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403
392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 +
576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*
c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b
^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 70464307200
0*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^
4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680
*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059
239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c
^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
+ 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*
c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3
*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^
28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^
9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2
113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560
*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c
^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558
138880*a^19*b^2*c^20)))^(1/4) + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 1036800
0000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429
344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 223948
80000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2
 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704
*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9
*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))
((81(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b

*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c
^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^1
5*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a
^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 88
7850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a
*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099
511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^
4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 -
 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c
^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 211342589
9520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^
12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19
585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a
^19*b^2*c^20)))^(1/4)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^2
9 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3
 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c
^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b
^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 23077060
6080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c -
b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(
33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*
c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 1
58760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 -
 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^1
1*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 +
 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 2080911654
9120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4
*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*2i - atan((((((3*(-(81*(2401*b
^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 13236
00*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 14372843
52*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327
073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10
+ 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^
13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 940
0*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^2
0*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a
^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a
^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 1937307
07456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^1
6*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 166
47293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*
a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)
))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a
^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 1095
2166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8

*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*

c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 130567
00579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) - (9*x^(1/2)*
(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 41
19250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 -
 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 167772
16*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4
 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 324
40320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033164
8*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2
)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^
3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*
a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 1997799
4240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11
+ 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)
^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040
*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^
30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*
b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475
299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b
^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 +
20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 1305670057984
0*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i - ((((3*(-(81*(24
01*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1
323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437
284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 1
3327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c
^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 88785027072
0*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c +
 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776
*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 12403
20*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 12700876
80*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193
730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12
*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 -
 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869
760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c
^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 97307852
80*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 +
10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040
*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^1
2 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*
(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b
^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 58
9824*a^8*b^2*c^8 - 36*a*b^16*c)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 42

22124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*
b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777
217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 37273444181606
40*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b
^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15
))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18
*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 1
2976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 692060
16*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29
 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*
a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*
a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073
280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 1
43696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*
b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a
*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c
^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*
b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*
b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 1937307074
56*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c
^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 166472
93239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^1
7*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^
(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c
^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7
))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 3
2256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^
4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(
4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 +
28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 -
3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^
8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^1
1*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 1000
0*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c
 - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38
*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 1587
6096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255
569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^
11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 520227913
7280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b
^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13
056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) + (9*x^(1
/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4
+ 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c
^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16

777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i)/(((((3*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3* c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 282

43200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 398
9852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 -
19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b
^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a
^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c -
b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^
2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 1587609
6*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569
920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11
- 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 520227913728
0*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10
*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056
700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) + ((((3*(-(8
1*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^1
4 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 +
1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^
7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*
b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850
270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^2
7*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 10995116
27776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 +
1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 127
0087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10
+ 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520
*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c
^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 195850
50869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*
b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 973
0785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c
^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 1298798110
31040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^
6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(6
5536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*
a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7
+ 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4
- 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400
*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8
- 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 372734441
8160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a
^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3
c^15))/(4194304(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3
*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^
6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 6
9206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c))*(-(81*(2401
*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 132

3600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 143728
4352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 133
27073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^1
0 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*
a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9
400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a
^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320
*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680
*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 19373
0707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b
^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 1
6647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 1958505086976
0*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^2
0)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*
b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^
3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^
3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a
^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2401*b^29 + 2401*b^
4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c
^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c
^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^
13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 14369685504
0*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 +
10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(
4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a
*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 -
15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 +
8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^
20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202
279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a
^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18
+ 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) + (9
x^(1/2)(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12
*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*
b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24
+ 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4
*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*
c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10
- 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*
a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28
243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 39
89852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8
- 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*
b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*
a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c -

b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c
^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 158760
96*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 825556
9920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11
- 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 52022791372
80*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^1
0*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 1305
6700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)))*(-(81*(24
01*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1
323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437
284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 1
3327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c
^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 88785027072
0*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c +
9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776
*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 12403
20*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 12700876
80*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193
730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12
*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 -
16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869
760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c
^20)))^(1/4)*2i - 2*atan(((((((((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 24
01*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^
23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*
b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240
*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 23
0770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*
a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/
2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2
*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c
^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24
*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 7044752998
40*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*
c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 2080
9116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^
18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^
15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^
18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 4690104
2872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*
b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 6
15726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b
^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344
064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) -
(9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 14790

7936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^

20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3
784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 576716
80*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^2
2*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*
a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*
b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a
^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 6605923
9424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^1
2 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) +
9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c
+ 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b
^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28
*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*
b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 211
3425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a
^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^1
7 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 549755813
8880*a^19*b^2*c^20)))^(1/4) - ((((((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2)
- 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^
3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*
a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1997799
4240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11
- 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)
^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040
*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^
30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*
b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475
299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b
^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 +
20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 1305670057984
0*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^1
3*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^
4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 469
01042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*
a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13
 - 615726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a
^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 +
 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c
)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 1
47907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*
b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 22
33932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 15997894184
14080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^
12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a

^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8
11008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3244032
0*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^
11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 240
1*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^2
3*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b
^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*
a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230
770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a
*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2
)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*
b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^
6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*
c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 70447529984
0*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c
^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809
116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^1
8*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*
c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 +
879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9
*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a
^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 -
36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 7046
43072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 27141
5040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 279
3799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9
+ 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^1
2*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)
^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432
*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72
960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960
*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 4402970
6240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c
^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 1040455
8274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^1
6*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 -
5497558138880*a^19*b^2*c^20)))^(1/4)*1i - (9*x^(1/2)*(43758225*a^2*b^14*c^3
- 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5
+ 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^
8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^
2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5
+ 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57
671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a
*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072
000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*

a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 27937996 80*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 660 59239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5 *c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^4 0*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a ^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6* b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240* a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 104045582745 60*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8 *c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 54975 58138880*a^19*b^2*c^20)))^(1/4))/(((((((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1 /2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 2824320 0*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852 160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 199 77994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c ^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c ^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2) ^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^ 5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920* a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 70 4475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^ 13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^1 6 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 130567005 79840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320 *a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 16777216000 0*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208 000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4* c^13 - 615726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 5 76*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c ^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^ 16*c)) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720* a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789 418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 1600888930041856 0*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 167772 16*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 324 40320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033164 8*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3 *b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*

(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 729
60*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*
a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706
240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^
12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558
274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16
*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5
497558138880*a^19*b^2*c^20)))^(1/4)*1i + ((((((81*(2401*b^4*(-(4*a*c - b^2)
^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 -
28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 +
3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^
8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^1
1*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 1000
0*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c
 - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38
*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 1587
6096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255
569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^
11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 520227913
7280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b
^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13
056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(35184372
0888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 1677
72160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^1
4*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 20615
8430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^
11*b^4*c^13 - 615726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*
c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^
5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 -
36*a*b^16*c)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14
*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322
398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^
15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 +
1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 160088893
00418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 +
 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b
^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^
7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 -
50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^
(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243
200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 39898
52160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1
9977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7
*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2
c^2(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^

2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 1

0404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091165491
20*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c
^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i))*((81*(2401*b^4*(-(4*a*c - b
^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2
 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5
 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13
*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*
a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 1
0000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*
a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b
^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 1
5876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8
255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20
*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 520227
9137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^1
5*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 +
 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) - 2*at
an(((((((-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 70464307200
0*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^
4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680
*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059
239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c
^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
- 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*
c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3
*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^
28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^
9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2
113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560
*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c
^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558
138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^
22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 169114337280
0*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9
+ 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 8246337208
3200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b
^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^
12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589
824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (9*x^(1/2)*(37748736
00*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5
+ 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 17632988233728
0*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c
^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 712
4835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858
822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*

c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^
14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^
21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7
*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 660592394
24*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12
- 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 94
00*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c +
1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^3
4*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c
^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^
22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 21134
25899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^1
4*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17
- 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 54975581388
80*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c
^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 +
1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2
- 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*
b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))*(-(81*(2
401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 -
1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 143
7284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 +
13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*
c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 8878502707
20*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c
+ 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 109951162777
6*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240
320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087
680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 19
3730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^1
2*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15
- 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 1958505086
9760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*
c^20)))^(1/4)*1i + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10
+ 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c
^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2
c^9))/(4194304(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*
b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6
- 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69
206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*
b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323
600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284

352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 1332
7073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10
 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a
^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 94
00*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^
20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*
a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*
a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730
707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^
16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16
647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760
*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20
)))^(1/4) - (((((-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704
643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 2714
15040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 27
93799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9
 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^
12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25
)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355443
2*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 7
2960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 15876096
0*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 440297
06240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*
c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 104045
58274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^
16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 -
 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 25165824
0*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691
143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b
^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82
463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 61572651155456
0*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 537
6*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c
^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (9*x^(1/2)*(
3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b
^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329
882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^
8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^
12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 137
92273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a
^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^
5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 5
7671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*
a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 7046430
72000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 27141504

0*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 279379
9680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 6
6059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b
^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1
/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b
^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960
*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^
6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 4402970624
0*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12
 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 1040455827
4560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b
^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 549
7558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^
2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b
^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b
^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344
064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))*
(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b
*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c
^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^1
5*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a
^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 88
7850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a
*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099
511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^
4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 -
 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c
^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 211342589
9520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^
12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19
585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a
^19*b^2*c^20)))^(1/4)*1i - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*
a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a
^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000
*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14
080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*
b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*
c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c))*(-(8
1*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^1
4 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 +
 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^
7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*
b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850
270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^2
7*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 10995116

27776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 +
1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 127
0087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10
+ 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520
*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c
^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 195850
50869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*
b^2*c^20)))^(1/4))/((((((-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/
2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^
3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*
c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*
b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 2307706
06080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c -
 b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36
*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 +
158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9
- 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^
11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14
+ 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091165
49120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^
4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 +
251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^
6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 469010428723
20*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c
^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726
511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c
^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a
^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (9*x
^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 1479079362
56*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7
+ 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 22339327497
33888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^1
0*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^
14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12
+ 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5
*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8
*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^
11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) +
 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 -
271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6
+ 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11
*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 23077060608
0*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2
)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(335

54432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3
 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 1587
60960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44
029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b
^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10
404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 2080911654912
0*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^
19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 29
17215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 8794033
92*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 5
76*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c
^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^
16*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 70464307200
0*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^
4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680
*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059
239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c
^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
 - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*
c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3
*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^
28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^
9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2
113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560
*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c
^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558
138880*a^19*b^2*c^20)))^(1/4)*1i + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 1036
8000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404
429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 223
94880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^
14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^
21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7
*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 660592394
24*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12
 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 94
00*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c +
1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^3
4*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c
^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^
22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 21134
25899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^1

4*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i + (((((-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^1

4*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 39

```
89852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8
- 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*
b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*
a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c -
 b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c
^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 158760
96*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 825556
9920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11
 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 52022791372
80*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^1
0*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 1305
6700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.850 $\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=533

$$-\frac{3x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{-\sqrt{\cdots}}}\right.}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b\cdots}}$$

Rubi [A] time = 1.45, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1115, 1365, 1500, 1510, 298, 205, 208}

$$-\frac{3x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(-4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(-4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*x^(3/2
)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (
3*c^(1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^
2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(
1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b
- Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^2 -
4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1500

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2
 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(6a-9bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a\left(7b^2+20\cdots\right.\right.}{a+bx\cdots}\cdots\right)}{16a\left(b^2\cdots\right.}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(11b^2+20ac-4\cdots\right.\right.}{\cdots}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\sqrt{c}\left(11b^2+20ac-\cdots\right.\right.}{\cdots}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{3\sqrt[4]{c}\left(11b^2+20ac+\cdots\right.}{16\,2^{3/4}\left(b^2-\cdots\right.}$$

Mathematica [C] time = 0.38, size = 176, normalized size = 0.33

$$\frac{-3\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{8\#1^4bc\log\left(\sqrt{x}-\#1\right)-20ac\log\left(\sqrt{x}-\#1\right)-7b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right] - \frac{12x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{a+bx^2+cx^4} + \frac{16x^{3/2}\left(b^2-4ac\right)\left(2a+bx^2\right)}{\left(a+bx^2+cx^4\right)^2}}{64\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((16*(b^2 - 4*a*c)*x^(3/2)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 - (12*x^(3/
2)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(a + b*x^2 + c*x^4) - 3*RootSum[a + b*#1^4
+ c*#1^8 & , (-7*b^2*Log[Sqrt[x] - #1] - 20*a*c*Log[Sqrt[x] - #1] + 8*b*c*L
og[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(64*(b^2 - 4*a*c)^2)
```

IntegrateAlgebraic [C] time = 0.97, size = 344, normalized size = 0.65

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{-8\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+8\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+260a^2c^2\log\left(\sqrt{x}-\#1\right)-133ab^2c\log\left(\sqrt{x}-\#1\right)+8b^4\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{64ac\left(4ac-b^2\right)^2} - \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)-10ac\log\left(\sqrt{x}-\#1\right)+b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{8ac\left(4ac-b^2\right)} - \frac{x^{3/2}\left(20a^2c+7ab^2+28abcx^2-12ac^2x^4+11b^3x^2+39b^2cx^4+24bc^2x^6\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/16*(x^(3/2)*(7*a*b^2 + 20*a^2*c + 11*b^3*x^2 + 28*a*b*c*x^2 + 39*b^2*c*x
^4 - 12*a*c^2*x^4 + 24*b*c^2*x^6))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
- RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqrt[
x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(8*a*c*(-b^2 +
 4*a*c)) - RootSum[a + b*#1^4 + c*#1^8 & , (8*b^4*Log[Sqrt[x] - #1] - 133*a
*b^2*c*Log[Sqrt[x] - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1] + 8*b^3*c*Log[Sqrt
[x] - #1]*#1^4 - 8*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(6
4*a*c*(-b^2 + 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.51Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 244, normalized size = 0.46

$$-\frac{3\left(8\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6} bc+\left(-20ac-7b^{2}\right)\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2}\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{64\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7} c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3} b\right)}+\frac{-\frac{3b\,c^{2}x^{\frac{15}{2}}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{3\left(4ac-13b^{2}\right)c\,x^{\frac{11}{2}}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(28ac+11b^{2}\right)b\,x^{\frac{7}{2}}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(20ac+7b^{2}\right)a\,x^{\frac{3}{2}}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-1/32*a*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32*b*(28*a*c
+11*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/32*(4*a*c-13*b^2)*c/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^(11/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(
```

```
c*x^4+b*x^2+a)^2-3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((8*_R^6*b*c+(-20*a*c-7
*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{24\,bc^{2}x^{\frac{15}{2}} + 3\left(13\,b^{2}c - 4\,ac^{2}\right)x^{\frac{11}{2}} + \left(11\,b^{3} + 28\,abc\right)x^{\frac{7}{2}} + \left(7\,ab^{2} + 20\,a^{2}c\right)x^{\frac{3}{2}}}{16\left(\left(b^{4}c^{2} - 8\,ab^{2}c^{3} + 16\,a^{2}c^{4}\right)x^{8} + 2\left(b^{5}c - 8\,ab^{3}c^{2} + 16\,a^{2}bc^{3}\right)x^{6} + a^{2}b^{4} - 8\,a^{3}b^{2}c + 16\,a^{4}c^{2} + \left(b^{6} - 6\,ab^{4}c + 32\,a^{3}c^{3}\right)x^{4} + 2\left(ab^{5} - 8\,a^{2}b^{3}c + 16\,a^{3}bc^{2}\right)x^{2}\right)} - \int \frac{3\left(8\,bcx^{\frac{5}{2}} - \left(7\,b^{2} + 20\,ac\right)\sqrt{x}\right)}{32\left(ab^{4} - 8\,a^{2}b^{2}c + 16\,a^{3}c^{2} + \left(b^{4}c - 8\,ab^{2}c^{2} + 16\,a^{2}c^{3}\right)x^{4} + \left(b^{5} - 8\,ab^{3}c + 16\,a^{2}bc^{2}\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(15/2) + 3*(13*b^2*c - 4*a*c^2)*x^(11/2) + (11*b^3 + 28*a
*b*c)*x^(7/2) + (7*a*b^2 + 20*a^2*c)*x^(3/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3
*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*(8*b*c*x^(5/2) - (7*b^2 + 20*a
*c)*sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

mupad [B] time = 7.66, size = 37678, normalized size = 70.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - atan(((((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161
128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^2
1*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 196180
3621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 1581647476555776
0*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*
b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c
^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^
3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16
*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*
c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12
*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (9*x^(1/2)*((81*(2401
*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 132
3600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 143728
4352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 133
27073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^1
0 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*
a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9
400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a
^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320
*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680
```

*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 19373
0707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b
^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 1
6647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 1958505086976
0*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^1
9)))^(1/4)*(822083584*a*b^26*c^4 - 140737488355328000*a^14*c^17 - 2795084185
6*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 1
0329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a
^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^1
2 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 4707
6689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b
^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*
a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^
10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^
10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)
^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 -
28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 +
3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^
8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^1
1*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 1000
0*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c
 - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^
38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 1587
6096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255
569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^
10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 520227913
7280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b
^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13
056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) - (9*x^(1
/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6
 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^
9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^
2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5
 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57
671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a
*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072
000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*
a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 27937996
80*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 660
59239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5
*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2
) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b
^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a
^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*
b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*

a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 104045582745 60*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8 *c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 54975 58138880*a^20*b^2*c^19)))^(1/4)*1i - (((27*(5754585088*a*b^27*c^4 + 3096224 74381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^2 3*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 55781317 2535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8 *b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c ^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5 *b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b ^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^ 11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^2 6*c)) + (9*x^(1/2)*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 7 04643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 27 1415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c ^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080* a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^ 25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554 432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760 960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 4402 9706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^1 8*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 1040 4558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120* a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835 532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2 968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6 *b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 285697101 36131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 4025092166962380 8*a^13*b^2*c^16))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a ^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b ^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c ^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15* c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^1 0*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 8878 50270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b

^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 109951
1627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3
+ 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1
270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^
9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 21134258995
20*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12
*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 1958
5050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^2
0*b^2*c^19)))^(3/4) + (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c
^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^
7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 +
16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b
^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^
7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 -
50331648*a^11*b^2*c^11 - 48*a*b^22*c))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^
(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243
200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 39898
52160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1
9977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7
*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2
c^2(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^
2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c
+ 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*
a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 825556992
0*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 -
704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*
a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c
^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 1305670
0579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*1i)/((27*(1036
80000000*a^8*c^12 + 1406514375*a*b^14*c^5 + 22129159500*a^2*b^12*c^6 + 1402
97799600*a^3*b^10*c^7 + 460920922560*a^4*b^8*c^8 + 844743271680*a^5*b^6*c^9
+ 869387904000*a^6*b^4*c^10 + 469670400000*a^7*b^2*c^11))/(134217728*(b^28
+ 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^
4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^
14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*
b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*
a^13*b^2*c^13 - 56*a*b^26*c)) + (((27*(5754585088*a*b^27*c^4 + 309622474381
721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6
- 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 5578131725352
96*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13
*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 -
32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 51706
9532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456
*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18
*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c

^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (9*x^(1/2)*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3

2440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331
648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2)
 - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a
^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160
*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 199779
94240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11
 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*
(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25
)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 304
0*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b
^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9
*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 70447
5299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*
b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 +
 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 130567005798
40*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4) + (((27*(5754585088
*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 +
1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^
5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10
+ 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 3929654
5576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 30075601261533
5936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 +
268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b
^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*
c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8
*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^1
3*b^2*c^13 - 56*a*b^26*c)) + (9*x^(1/2)*((81*(2401*b^4*(-(4*a*c - b^2)^25)^
(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243
200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 39898
52160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1
9977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7
*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2
c^2(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^
2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c
+ 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*
a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 825556992
0*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 -
704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*
a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c
^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 1305670
0579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b
^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 39943195
8528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^
8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 6580577092
23936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10

*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^
15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b^24 + 16777216*a^12*c^12
+ 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5
*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8
*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^
11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 -
704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 2
71415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 -
2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*
c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080
*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355
4432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2
- 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 15876
0960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 440
29706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^
18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 104
04558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120
*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^1
8 - 5497558138880*a^20*b^2*c^19)))^(3/4) + (9*x^(1/2)*(200930625*a*b^13*c^5
- 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7
+ 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^
10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^
18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 -
12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 6920
6016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4
*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600
*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352
*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 1332707
3280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 -
143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13
*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*
a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*
c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5
*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8
*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707
456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*
c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647
293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^
18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))
^(1/4)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 70464307200
0*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^
4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680
*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059
239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c

^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
+ 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^4
0 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4
*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^
28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^
10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2
113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560
*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c
^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558
138880*a^20*b^2*c^19)))^(1/4)*2i - atan(((((27*(5754585088*a*b^27*c^4 + 309
622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3
*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 5578
13172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176
*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b
^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^1
5 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^
14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048
*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a
^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 152672665
6*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a
*b^26*c)) - (9*x^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2
) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3
 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c
^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b
^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 23077060
6080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c -
b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(
33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*
c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 1
58760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 -
 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^1
2*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 +
 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 2080911654
9120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4
*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 140737
48835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^
6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 626206231756
8*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c
^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 2856
9710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669
623808*a^13*b^2*c^16))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^

14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*1i - (((27*(5754585088*a*b^27*c^4 + 30962247438172160 0*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (9*x^(1/2)*(-(81*(2401*b^29 + 2

401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*
b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*
b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*
a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 14369
6855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*
c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2
c(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20
- 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32
*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26
*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a
^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12
- 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 1664729323
9296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^
6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4
)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^2
4*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 1032939634
6880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c
^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 86531
56510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 4707668985485
7216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b^24 + 167
77216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*
c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033
1648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c -
b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200
*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 39898521
60*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 1997
7994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^
11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^
2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3
040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6
*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a
^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704
475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^1
4*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15
 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 1305670057
9840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) + (9*x^(1/2)*(200
930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 75234
54960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453
568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c
^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 37847
04*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a
^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)
))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^1

4*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*1i)/((27*(103680000000*a^8*c^12 + 1406514375*a*b^14*c^5 + 22129159500*a^2*b^12*c^6 + 140297799600*a^3*b^10*c^7 + 460920922560*a^4*b^8*c^8 + 844743271680*a^5*b^6*c^9 + 869387904000*a^6*b^4*c^10 + 469670400000*a^7*b^2*c^11))/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (9*x^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*

c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*
(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*
c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 103293963468
80*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^1
0 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156
510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 470766898548572
16*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b^24 + 16777
216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^
4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32
440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 503316
48*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^
2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a
^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160
*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 199779
94240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11
 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*
(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25
)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 304
0*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b
^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9
*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 70447
5299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*
b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 +
 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 130567005798
40*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) - (9*x^(1/2)*(20093
0625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454
960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 545356
8000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2
 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704
*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9
*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))
(-(81(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*
b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*
c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^
15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*
a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 8
87850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*
a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 109
9511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c
^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6
 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22
*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 21134258
99520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b
^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 1
9585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*

a^20*b^2*c^19)))^(1/4) + (((27*(5754585088*a*b^27*c^4 + 309622474381721600*
a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983
582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*
b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 +
 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 327566
50414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217
475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^
24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 +
12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 52
4812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11
+ 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (9*x
^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000
*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4
*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*
a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 660592
39424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^
12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) -
 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40
 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*
b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^2
8*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^1
0*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 21
13425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*
a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^
16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 54975581
38880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14
*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 296889614336
0*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 -
 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 3463461627
49440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^
11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*
c^16))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*
b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6
 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69
206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*
b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323
600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284
352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 1332
7073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10
 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a
^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 94
00*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^
21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*
a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*
a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730

707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^
16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16
647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760
*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19
)))^(3/4) + (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093
250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 23
54261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*
a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 -
811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 324403
20*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a
^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^2
5)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b
^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6
*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 1997799424
0*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 2
30770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4
*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1
/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^
3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*
c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^2
4*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299
840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14
*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 208
09116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a
^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4))*(-(81*(2401*b^29 + 24
01*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b
^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b
^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a
^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696
855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c
^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*
c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 -
80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*
c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*
c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^
11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 -
5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239
296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6
*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)
*2i - 2*atan(((((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17
- 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a
^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 -
1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 1581647476
5557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592
*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13

*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23
296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^
6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9
*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 152672665
6*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*((81*(
2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 +
 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 14
37284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 -
 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9
*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270
720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c
 + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 10995116277
76*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 124
0320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 127008
7680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 1
93730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^
13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14
 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 195850508
69760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2
c^19)))^(1/4)(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 279508
41856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7
 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 2028598953246
72*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10
*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 -
47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(419
4304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 +
126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128
*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10
*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c
 - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25
*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19
*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*
b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855
040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13
 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(
-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80
*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4
 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7
 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*
b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 52
02279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296
*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^
17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i
 + (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^
2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*

a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4) - (((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^

6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 626206231756
8*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c
^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 2856
9710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669
623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^
20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3
784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 576716
80*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^2
2*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*
a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*
b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a
^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 6605923
9424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^1
2 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) +
9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40
+ 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b
^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28
*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10
*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 211
3425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a
^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^1
6 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 549755813
8880*a^20*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400
000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 1032858
0864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194
304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 1
26720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*
a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*
b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c
- b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*
c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*
c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b
^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 1436968550
40*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13
+ 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-
(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*
a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4
- 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7
+ 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b
^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 520
2279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*
a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^1
7 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4))/((
((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464
*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 5

6328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 196180362185932
8*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^1
1*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14
+ 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(26
8435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^
3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56
229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 104
9624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12
 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*((81*(2401*b^4*(-(4*a
*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^
25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^
19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^
8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 1436968
55040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^
13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c
*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 -
80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c
^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c
^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^1
1*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 -
5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 166472932392
96*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*
c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*
(822083584*a*b^26*c^4 - 140737488355328000*a^14*c^17 - 27950841856*a^2*b^24*
c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 103293963468
80*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^1
0 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156
510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 470766898548572
16*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16
777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16
*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 503
31648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/
2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200
*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 39898521
60*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1997
7994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^
11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^
2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3
040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6
*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a
^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704
475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^1
4*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15

+ 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 1305670057
9840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(
200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 75
23454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5
453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^2
0*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 37
84704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 5767168
0*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22
c)))((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a
^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b
^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^
7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239
424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12
 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9
400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 +
 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^
34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*
c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*
b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113
425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^
15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16
 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138
880*a^20*b^2*c^19)))^(1/4)*1i - (27*(103680000000*a^8*c^12 + 1406514375*a*b
^14*c^5 + 22129159500*a^2*b^12*c^6 + 140297799600*a^3*b^10*c^7 + 4609209225
60*a^4*b^8*c^8 + 844743271680*a^5*b^6*c^9 + 869387904000*a^6*b^4*c^10 + 469
670400000*a^7*b^2*c^11))/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*a^2*
b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5
+ 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 -
524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^1
1 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + ((
(27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*
a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56
328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328
*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11
*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 +
 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268
435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3
 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 562
29888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049
624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12
- 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*((81*(2401*b^4*(-(4*a*
c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^2
5*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^1
9*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8
*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 14369685

5040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12

+ 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 94
00*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 +
1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^3
4*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c
^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b
^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 21134
25899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^1
5*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16
- 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 54975581388
80*a^20*b^2*c^19)))^(1/4)*1i))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2
401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b
^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6
*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1997799424
0*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 2
30770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4
*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1
/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^
3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*
c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^2
4*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299
840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14
*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 208
09116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a
^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4) - 2*atan(((((27*(575458
5088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^
5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 5632849608704
0*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c
^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 392
96545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 3007560126
15335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^2
8 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a
^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b
^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10
*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096
*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*
a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28
243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 39
89852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8
- 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*
b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*
a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c -
b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38
*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 158760
96*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 825556
9920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10

 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 52022791372
80*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^1
0*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 1305
6700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*
a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 39943
1958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18
*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 6580577
09223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a
^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4
*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16777216*a^12
*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8110
08*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a
^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*
b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(
1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*
c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^1
7*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^
9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 23077
0606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c
 - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2))
)/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^
36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5
+ 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^
8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*
a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^1
3 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 2080911
6549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*
b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(200930625*
a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a
^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*
a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14
080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*
b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*
c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(8
1*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^1
4 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 +
 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^
7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*
b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850
270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^2
7*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 10995116
27776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 +
1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 127
0087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9
+ 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520

*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c
^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 195850
50869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*
b^2*c^19)))^(1/4) - (((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*
b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 398358216
7040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*
c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 1581
6474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414
702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 51706953221747507
2*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^
2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300
288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 5248122
88*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 152
6726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*
(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b
*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c
^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^1
5*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a
^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 88
7850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a
*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099
511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^
3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 -
 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*
c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 211342589
9520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^
12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19
585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a
^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 -
 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b
^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859
895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a
^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*
c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9
i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18
*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 1
2976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 692060
16*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29
 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*
a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*
a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073
280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 1
43696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*
b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a
*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c

^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*
b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*
b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 1937307074
56*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c
^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 166472
93239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^1
8*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^
(3/4)*1i - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 20932
50600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 235
4261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a
^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8
11008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3244032
0*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^
11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25
)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^
23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*
b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240
*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 23
0770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*
a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/
2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3
*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c
^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24
*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 7044752998
40*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*
c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 2080
9116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^
19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4))/((((27*(5754585088*a*b^
27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 16261
81992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^1
9*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715
782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 392965455767
14240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*
a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 26843
5456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c
^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 +
196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10
- 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2
*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2
)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^
3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*
a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 1997799
4240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11
+ 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)

^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040
*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^
30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*
b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475
299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b
^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 +
20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 1305670057984
0*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^
4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a
^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 62
62062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*
a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c
^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 4
0250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1
056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^
14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^
8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11
- 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 70
4643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271
415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2
793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^
9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a
^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^2
5)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 -
72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 1587609
60*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029
706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18
*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404
558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a
^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18
- 5497558138880*a^20*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(200930625*a*b^13*c^
5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^
7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c
^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b
^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6
- 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 692
06016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b
^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 13236
00*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 14372843
52*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327
073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10
+ 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^
13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 940
0*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^2

1*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a
^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a
^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 1937307
07456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^1
6*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 166
47293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*
a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)
))^(1/4)*1i - (27*(103680000000*a^8*c^12 + 1406514375*a*b^14*c^5 + 22129159
500*a^2*b^12*c^6 + 140297799600*a^3*b^10*c^7 + 460920922560*a^4*b^8*c^8 + 8
44743271680*a^5*b^6*c^9 + 869387904000*a^6*b^4*c^10 + 469670400000*a^7*b^2*
c^11))/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a
^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^1
6*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10
*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^1
2*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (((27*(5754585088*a*
b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 162
6181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b
^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 7
15782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 3929654557
6714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 30075601261533593
6*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268
435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20
*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7
 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^
10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b
^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b
^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*
a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 398985216
0*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977
994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^1
1 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 30
40*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*
b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^
9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 7044
75299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14
*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15
+ 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579
840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*
c^4 - 140737488355328000*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528
*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 +
6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 65805770922393
6*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8
*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 +

40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*1i))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25

```
)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355443
2*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 7
2960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 15876096
0*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 440297
06240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*
c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 104045
58274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^
17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 -
 5497558138880*a^20*b^2*c^19)))^(1/4) - ((x^(7/2)*(11*b^3 + 28*a*b*c))/(16*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(3/2)*(7*a*b^2 + 20*a^2*c))/(16*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) - (3*x^(11/2)*(4*a*c^2 - 13*b^2*c))/(16*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^(15/2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.851 $\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=533

$$\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Rubi [A] time = 1.36, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1365, 1430, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\sqrt{x}\left(-4ac+13b^2+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (Sqrt[x]*
(13*b^2 - 4*a*c + 24*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
c^(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(
-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2
- 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*
2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-11bx^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{a\left(5b^2+28ac\right)-}{a+bx^4+c}\right.}{16a\left(b^2-\right.}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(41b^2+28ac-36\right.\right.}{}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(41b^2+28ac-36\right.\right.}{}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{c^{3/4}\left(41b^2+28ac+3\right.}{16\sqrt[4]{2}\left(b^2-\right.}$$

Mathematica [C] time = 0.44, size = 177, normalized size = 0.33

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{72\#1^4bc\log\left(\sqrt{x}-\#1\right)-28ac\log\left(\sqrt{x}-\#1\right)-5b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+\frac{4\sqrt{x}\left(28a^2c+a\left(5b^2+36bcx^2-4c^2x^4\right)+bx^2\left(9b^2+37bcx^2+24c^2x^4\right)\right)}{\left(a+bx^2+cx^4\right)^2}}{64\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/64*((4*Sqrt[x]*(28*a^2*c + a*(5*b^2 + 36*b*c*x^2 - 4*c^2*x^4) + b*x^2*(9
*b^2 + 37*b*c*x^2 + 24*c^2*x^4)))/(a + b*x^2 + c*x^4)^2 + RootSum[a + b*#1^
4 + c*#1^8 & , (-5*b^2*Log[Sqrt[x] - #1] - 28*a*c*Log[Sqrt[x] - #1] + 72*b*
c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(b^2 - 4*a*c)^2
```

IntegrateAlgebraic [C] time = 0.80, size = 350, normalized size = 0.66

$$-\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-8\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+8\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+140a^2c^2\log\left(\sqrt{x}-\#1\right)-71ab^2c\log\left(\sqrt{x}-\#1\right)+8b^4\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{64ac\left(4ac-b^2\right)^2}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{3\#1^4bc\log\left(\sqrt{x}-\#1\right)-14ac\log\left(\sqrt{x}-\#1\right)+3b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{8ac\left(4ac-b^2\right)}-\frac{\sqrt{x}\left(28a^2c+5ab^2+36abcx^2-4ac^2x^4+9b^3x^2+37b^2cx^4+24bc^2x^6\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/16*(Sqrt[x]*(5*a*b^2 + 28*a^2*c + 9*b^3*x^2 + 36*a*b*c*x^2 + 37*b^2*c*x^
4 - 4*a*c^2*x^4 + 24*b*c^2*x^6))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) -
RootSum[a + b*#1^4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sqrt[
x] - #1] + 3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(8*a*c*(-b
^2 + 4*a*c)) - (3*RootSum[a + b*#1^4 + c*#1^8 & , (8*b^4*Log[Sqrt[x] - #1]
- 71*a*b^2*c*Log[Sqrt[x] - #1] + 140*a^2*c^2*Log[Sqrt[x] - #1] + 8*b^3*c*Lo
g[Sqrt[x] - #1]*#1^4 - 8*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7
) & ])/(64*a*c*(-b^2 + 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.21Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 237, normalized size = 0.44

$$\frac{\left(-72\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4bc+28ac+5b^2\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8a\,b^2c+b^4\right)\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{-\frac{3b\,c^2x^{\frac{13}{2}}}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{2\left(4ac-37b^2\right)c\,x^{\frac{9}{2}}}{512a^2c^2-256a\,b^2c+32b^4}-\frac{9\left(4ac+b^2\right)b\,x^{\frac{5}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(28ac+5b^2\right)a\sqrt{x}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-1/32*a*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-9/32*b*(4*a*c+
b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*c*(4*a*c-37*b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(9/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x^4
+b*x^2+a)^2+1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((-72*_R^4*b*c+28*a*c+5*b^2)
/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(5\,b^{2}c^{2}+28\,ac^{3}\right)x^{\frac{17}{2}}+2\left(5\,b^{3}c+16\,abc^{2}\right)x^{\frac{13}{2}}+\left(5\,b^{4}+ab^{2}c+60\,a^{2}c^{2}\right)x^{\frac{9}{2}}+\left(ab^{3}+20\,a^{2}bc\right)x^{\frac{5}{2}}}{16\left(\left(ab^{4}c^{2}-8\,a^{2}b^{2}c^{3}+16\,a^{3}c^{4}\right)x^{8}+a^{3}b^{4}-8\,a^{4}b^{2}c+16\,a^{5}c^{2}+2\left(ab^{5}c-8\,a^{2}b^{3}c^{2}+16\,a^{3}bc^{3}\right)x^{6}+\left(ab^{6}-6\,a^{2}b^{4}c+32\,a^{4}c^{3}\right)x^{4}+2\left(a^{2}b^{5}-8\,a^{3}b^{3}c+16\,a^{4}bc^{2}\right)x^{2}\right)}-\int\frac{\left(5\,b^{2}c+28\,ac^{2}\right)x^{\frac{7}{2}}+5\left(b^{3}+20\,abc\right)x^{\frac{3}{2}}}{32\left(a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}+\left(ab^{4}c-8\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3}\right)x^{4}+\left(ab^{5}-8\,a^{2}b^{3}c+16\,a^{3}bc^{2}\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c^2 + 28*a*c^3)*x^(17/2) + 2*(5*b^3*c + 16*a*b*c^2)*x^(13/2) +
 (5*b^4 + a*b^2*c + 60*a^2*c^2)*x^(9/2) + (a*b^3 + 20*a^2*b*c)*x^(5/2))/((a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*
c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) - integ
rate(1/32*((5*b^2*c + 28*a*c^2)*x^(7/2) + 5*(b^3 + 20*a*b*c)*x^(3/2))/(a^2*
b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^4
 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

mupad [B] time = 8.38, size = 47803, normalized size = 89.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856
800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(
b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^
10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589
824*a^8*b^2*c^8 - 36*a*b^16*c)) + ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 6
25*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^
25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*
a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14
462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a
^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^1
3 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2)
 - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a
*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*
c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7
*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^1
0*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 1937307
07456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^1
```

6*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 166
47293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*
a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)
))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a
^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186
227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^1
1*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 47
27899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(65536*(b^1
8 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*
c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824
*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b
^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 129842
2300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^1
7*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 +
63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 3426518
03680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429
742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*
c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4
 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7
*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163
326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a
^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^1
4 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*
b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2)
)/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*
b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^
5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^2
4*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299
840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14
*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 208
09116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a
^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*((625*b^6*(-(4*a*c - b
^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2
- 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^
5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*
a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 +
 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 26745984411
2384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c
 - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25
)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 10
99511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*

c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) - (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i - (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 211342

5899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17
*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 -
19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 549755813888
0*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^1
5 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^
17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 3607772
52864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a
^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^
14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 +
32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*
b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4
- 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b
^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 1972356
35650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*
a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^
9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^1
5 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12
+ 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5
*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8
*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^
11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 151921
04632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297
600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1
688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9
*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 2
06669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787
840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*
c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c
- b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^
38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 1587
6096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 825
5569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*
c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279
137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18
*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 +
13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*((625*
b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 890
00*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 2549240
9600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7
+ 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*
a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^
12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*
a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-
(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(3355443
2*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 -

72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760
960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44
029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b
^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10
404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 2080911654912
0*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^
18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (x^(1/2)*(481890304*a^6*c^13 +
441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 12
3896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^1
2))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^1
8*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 -
12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206
016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*
a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^
27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*
b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504
147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11
*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 2674
59844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(
-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c -
b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^
40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^
6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b
^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240
*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11
+ 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274
560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^
8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497
558138880*a^22*b^2*c^19)))^(1/4)*1i)/((((171894580*a*b^8*c^7 - 48125*b^10*c
^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 +
167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 -
5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^
6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + ((((625*b
^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 8900
0*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409
600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 +
6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a
^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^1
2 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a
^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(
4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432
*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 -
72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 1587609
60*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 440
29706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^

18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 104
04558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120
*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^1
8 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 17592186044
41600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563
714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^
13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 30
51144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 431008558088
1920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 537
6*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c
^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(20
9715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 9
4623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^
19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15
146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733
873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655
360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 167
77216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*
c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033
1648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 6
25*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^
25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*
a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14
462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a
^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^1
3 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2)
 - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a
*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*
c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7
*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^1
0*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 1937307
07456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^1
6*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 166
47293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*
a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)
))^(3/4))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a
^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^
23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 16888165785
60*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9
+ 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 20666946420
7360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b
^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 191100
0*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)
^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 304

0*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b
^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^
11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704
475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^1
6*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15
 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 1305670057
9840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) - (x^(1/2)*(48189
0304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*
a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 742012
7232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2
 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704
*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9
*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))
*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^1
5 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 -
25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^
17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326
443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12
*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 -
 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2
c^2(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(
33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^3
6*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 +
 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c
^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840
*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^
13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 208091
16549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21
*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (((171894580*a*b^8*c^7 -
 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a
^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^
2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 +
344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)
) + ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*
b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c
^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a
^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 41
63326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360
*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c
^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^
2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/
2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^
5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*
c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b
^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 7044752

99840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4

*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 1
5876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 +
8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^
20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202
279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a
^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17
 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (x
^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 +
151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*
c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056
*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*
c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 -
 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 4
8*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 151921046323
20*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^
4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816
578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*
c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 2066694
64207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^
14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 19
11000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)
^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c +
 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a
^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 825556992
0*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 -
 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280
*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*
c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 130567
00579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)))*((625*b^6*(
-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^
2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*
a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 666
4504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*
b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 +
267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c
^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^
3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 7296
0*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a
^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 4402970
6240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c
^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 1040455
8274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^1
9*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 -
5497558138880*a^22*b^2*c^19)))^(1/4)*2i - ((9*x^(5/2)*(b^3 + 4*a*b*c))/(16*

(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(1/2)*(5*a*b^2 + 28*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^(9/2)*(4*a*c^2 - 37*b^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^(13/2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520 *a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) - (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730

707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i - (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13 *c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^

12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2
113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560
*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c
^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558
138880*a^22*b^2*c^19)))^(1/4)*1i)/((((171894580*a*b^8*c^7 - 48125*b^10*c^6
- 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167
976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 537
6*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c
^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (((-(625*b^3
1 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*
a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 2549240960
0*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6
664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^1
0*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12
- 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3
c^3(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*
a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(
a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72
960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960
*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029
706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18
*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404
558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a
^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18
- 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441
600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 56371
4457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13
*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051
144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 43100855808819
20*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*
a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6
- 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(2097
15200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 946
23498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19
*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 1514
6459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 18014673387
3889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 41930975436865536
0*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777
216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^
4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32
440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 503316
48*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)
^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^2
5*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a
^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 144

62970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) - (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*

c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 70447529984
0*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c
^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809
116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^2
1*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (((171894580*a*b^8*c^7
- 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*
a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a
^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 +
344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c
)) + (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^1
5*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23
*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560
*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 -
4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 2066694642073
60*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3
*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*
a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(
1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*
a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^3
0*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11
*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 70447
5299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*
b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 +
20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 130567005798
40*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^
4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3
*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 685476
78044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^
8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13
- 4310085580881920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2
*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 3
44064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))
+ (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*
a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 18038
86264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6
*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c
^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 -
419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097
152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 1
26720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*
a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*
b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-
(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 +
27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5
- 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a

^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255

569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c
^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 52022791
37280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*
b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 1
3056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)))*(-(625
*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89
000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 254924
09600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7
 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520
*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c
^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416
*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(335544
32*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2
 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 15876
0960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 4
4029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*
b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 1
0404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 208091165491
20*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c
^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*2i + 2*atan(((((171894580*a*b^8*
c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738
432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 5
76*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c
^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^
16*c)) - ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*
a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b
^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578
560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9
 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 2066694642
07360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*
b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 19110
00*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25
)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 30
40*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*
b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a
^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 70
4475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^
16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^1
5 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 130567005
79840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23
*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*
a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 685
47678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600
*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c

^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1i)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i - (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400

*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 122959
17312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^
12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 81
1008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320
*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^1
1*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31
+ 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 +
1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19
*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 1446297042
9440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*
c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 1500
09114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125
*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(
-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 8
0*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^
4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c
^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^
13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 -
5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239
296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6
*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)
- (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800
*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^1
8 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*
c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824
*a^8*b^2*c^8 - 36*a*b^16*c)) - ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*
b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*
c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6
*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462
970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11
*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 -
150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) -
23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^
4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^2
0 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^
32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b
^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 1937307074
56*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c
^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 166472
93239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^2
0*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^
(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*
b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227
200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c
^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 47278

99999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^1
8 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*
c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824
*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b
^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 129842
2300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^1
7*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 +
63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 3426518
03680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429
742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*
c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4
 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7
*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163
326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a
^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^1
4 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*
b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2)
)/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*
b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^
5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^2
4*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299
840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14
*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 208
09116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a
^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1i)*((625*b^6*(-(4*a*c
- b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c
^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21
*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 66645041479
68*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^1
0 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 26745984
4112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*
a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 +
 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^
34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*
c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^1
2*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 21
13425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*
a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^
16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 54975581
38880*a^22*b^2*c^19)))^(1/4)*1i + (x^(1/2)*(481890304*a^6*c^13 + 441265825*

b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360
*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(20971
52*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 12
6720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a
^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b
^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)
^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 2
7186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 +
 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8
*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70
455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 26745984411238
4*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c -
b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 10995
11627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3
 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 -
1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22
*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 21134258
99520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b
^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 1
9585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*
a^22*b^2*c^19)))^(1/4))/((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368
*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4
*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12
*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 58982
4*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - ((((625*b^6*(-(4*a*c -
 b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^
2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*
c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 666450414796
8*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10
 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844
112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^
25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 +
1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^3
4*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c
^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12
*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 211
3425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a
^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^1
6 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 549755813
8880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*
c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4
*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 3607
77252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 305114476707840

0*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3
*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12
*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 58982
4*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(209715200*b^
27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240
*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 1
97235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 151464598673
81760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*
a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b
^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12
*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8110
08*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a
^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*
b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 +
15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1
342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c
^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 144629704294
40*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^
11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009
114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a
*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(
4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*
a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4
- 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7
+ 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13
*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5
202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 1664729323929
6*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c
^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1
i)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c
^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4
- 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*
b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 41633
26443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^
12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14
- 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b
^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))
/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b
^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5
+ 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24
*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 7044752998
40*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*
c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 2080
9116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^
21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i - (x^(1/2)*(481890304

*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*
b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232
*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 1
4080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6
*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6
*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c))))*((6
25*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 +
89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 2549
2409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c
^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 41633264435
20*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7
*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 384
16*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(3355
4432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^
2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158
760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 -
44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^1
4*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 +
10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 2080911654
9120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4
*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i + (((171894580*a*b^8*c^7 -
48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^
3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2
*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 3
44064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))
- ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b
*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^
4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^
7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 416
3326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*
a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^
14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2
*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2
))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5
*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c
^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^
24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 70447529
9840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^1
4*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20
809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*
a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 +
1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^
19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 685476780
44160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b

^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1i)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)

1i + (x^(1/2)(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^1
0*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*
a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^1
2 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a
^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b
^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*
c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 1519
2104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 13422
97600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 -
1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a
^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 -
206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 1500091147
87840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^2
9*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*
c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*
b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15
876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8
255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^2
0*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 52022
79137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^
18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17
+ 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i))*
((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15
+ 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 2
5492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^1
7*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 41633264
43520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*
b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 -
38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*
c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(3
3554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36
*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 +
158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^
8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*
a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^1
3 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 2080911
6549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*
b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + 2*atan(((((171894580*a*b^
8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 35127
38432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 +
576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8
*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*
b^16*c)) - (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 151921046323
20*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^
4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816

578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*
c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 2066694
64207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^
14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 19
11000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)
^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c +
3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a
^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 825556992
0*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 -
704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280
*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*
c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 130567
00579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b
^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 67108864
00*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 +
68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289
600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^
5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9
+ 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^
8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a
*b^16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 916201049
19318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^
7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 23306210535
01440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a
^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^
7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^1
6))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^1
8*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 -
12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206
016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 6
25*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b
^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5
*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 666450
4147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^1
1*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267
459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*
(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c -
b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b
^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a
^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*
b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 4402970624
0*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11
+ 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 1040455827
4560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b
^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 549

7558138880*a^22*b^2*c^19)))^(3/4)*1i)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)
^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3
*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 2651888332
80*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 +
14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 7045524226048
0*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*
c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1
/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 5437
5*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^
23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*
a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*
a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 1937
30707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*
b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 -
16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 195850508697
60*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^
19)))^(1/4)*1i - (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 167182
55400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 1
2295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 167772
16*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4
- 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 324
40320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033164
8*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^
(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25
*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^
6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 1446
2970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^1
1*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13
+ 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) +
23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b
^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^
20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b
^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*
b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707
456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*
c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647
293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^
20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))
^(1/4) - (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520
856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(6553
6*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4
*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 +
589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2
)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^
3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833

280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 704552422604 80*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5 *c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 543 75*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a ^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320 *a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680 *a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193 730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15 *b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869 760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c ^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 16777216 00*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 837 5186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7 *b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(655 36*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^ 4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145 600*a*b^25*c^5 - 9162010491931852 8*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560* a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13* c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 29695 6100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^ 2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57 671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a *b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320 *a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4* b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 168881657 8560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^ 9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464 207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14 *b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911 000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^2 5)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3 040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8 *b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920* a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 7 04475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a ^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^ 15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700

579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1i)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i + (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4))/((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 2549240960

0*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 1958505086976

0*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^1
9)))^(3/4)*1i)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 1519210463
2320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*
a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 16888
16578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^1
3*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 20666
9464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*
a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c +
1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^
2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c
 + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096
*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569
920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10
 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 52022791372
80*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^1
0*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 1305
6700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i - (x^(1/
2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 1518
43979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11
 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2
*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5
 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 576
71680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*
b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*
a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b
^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578
560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9
 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 2066694642
07360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*
b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 19110
00*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25
)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 30
40*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*
b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a
^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 70
4475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^
16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^1
5 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 130567005
79840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i + (((17189458
0*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 +
 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9
*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a
^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 -
 36*a*b^16*c)) - (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 151921
04632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297

600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120* a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1i)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i + (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 1500091147878

```
40*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c
 + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c -
 b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^3
8*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876
096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255
569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c
^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 52022791
37280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*
b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 1
3056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.852 $\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=594

$$\frac{3x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt[4]{c}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tan^{-}}{32\,2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4a}}}$$

Rubi [A] time = 2.31, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1364, 1500, 1510, 298, 205, 208}

$$\frac{3x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt[4]{c}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(\sqrt{b^2-4ac}\left(12ac+b^2\right)-68abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(\sqrt{b^2-4ac}\left(12ac+b^2\right)-68abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(3/
2)*(b*(b^2 + 4*a*c) + c*(b^2 + 12*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) + (3*c^(1/4)*(b^2 + 12*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c
)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) +
 (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 12*a*c))*ArcTan[(2^(
1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 -
 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^2 + 12*a*c -
b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2
*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 -
4*a*c]*(b^2 + 12*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4
*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1500

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2
 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^6}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{x^2\left(3b-18cx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{x^2(\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(b^2+12\cdots\right.\right.}{\cdots}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\sqrt{c}\left(b^2+\cdots\right.\right.}{\cdots}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt[4]{c}\left(b^2+1\cdots\right.}{32\cdots}$$

Mathematica [C] time = 0.41, size = 222, normalized size = 0.37

$$\frac{3\left(a+bx^2+cx^4\right)^2 \mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{12\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-28abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]-16ax^{3/2}\left(b^2-4ac\right)\left(b+2cx^2\right)+12x^{3/2}\left(4abc+12ac^2x^2+b^3+b^2cx^2\right)\left(a+bx^2+cx^4\right)}{64a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-16*a*(b^2 - 4*a*c)*x^(3/2)*(b + 2*c*x^2) + 12*x^(3/2)*(b^3 + 4*a*b*c + b^
2*c*x^2 + 12*a*c^2*x^2)*(a + b*x^2 + c*x^4) + 3*(a + b*x^2 + c*x^4)^2*RootS
um[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 28*a*b*c*Log[Sqrt[x] -
#1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 12*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
 + 2*c*#1^5) & ])/(64*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

IntegrateAlgebraic [C] time = 0.56, size = 245, normalized size = 0.41

$$\frac{3\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{12\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-28abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{64a\left(4ac-b^2\right)^2}+\frac{x^{3/2}\left(28a^2bc+68a^2c^2x^2-ab^3+7ab^2cx^2+48abc^2x^4+36ac^3x^6+3b^4x^2+6b^3cx^4+3b^2c^2x^6\right)}{16a\left(4ac-b^2\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(-(a*b^3) + 28*a^2*b*c + 3*b^4*x^2 + 7*a*b^2*c*x^2 + 68*a^2*c^2*x^
2 + 6*b^3*c*x^4 + 48*a*b*c^2*x^4 + 3*b^2*c^2*x^6 + 36*a*c^3*x^6))/(16*a*(-b
^2 + 4*a*c)^2*(a + b*x^2 + c*x^4)^2) + (3*RootSum[a + b*#1^4 + c*#1^8 & , (
b^3*Log[Sqrt[x] - #1] - 28*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1
]*#1^4 + 12*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(64*a*(-b^
2 + 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.33Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 277, normalized size = 0.47

$$\frac{3\left(\left(12ac+b^2\right)\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6 c+\left(-28ac+b^2\right)\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2 b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8ab^2c+b^4\right)a\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+\frac{\frac{3\left(12ac+b^2\right)c^2x^{\frac{15}{2}}}{16\left(16a^2c^2-8ab^2c+b^4\right)a}+\frac{3\left(8ac+b^2\right)bc\,x^{\frac{11}{2}}}{8\left(16a^2c^2-8ab^2c+b^4\right)a}+\frac{\left(68a^2c^2+7ab^2c+3b^4\right)x^{\frac{7}{2}}}{16\left(16a^2c^2-8ab^2c+b^4\right)a}+\frac{2\left(28ac-b^2\right)b\,x^{\frac{3}{2}}}{512a^2c^2-256ab^2c+32b^4}}{\left(cx^4+bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+1/32*(68*a^2*c^2+
7*a*b^2*c+3*b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/16/a*c*b*(8*a*c+b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/2)+3/32*c^2*(12*a*c+b^2)/a/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(15/2))/(c*x^4+b*x^2+a)^2+3/64/a/(16*a^2*c^2-8*a*b^2*c+b^4)
*sum((c*(12*a*c+b^2)*_R^6+b*(-28*a*c+b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^
(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(b^{2}c^{2}+12\,ac^{3}\right)x^{\frac{15}{2}}+6\left(b^{3}c+8\,abc^{2}\right)x^{\frac{11}{2}}+\left(3\,b^{4}+7\,ab^{2}c+68\,a^{2}c^{2}\right)x^{\frac{7}{2}}-\left(ab^{3}-28\,a^{2}bc\right)x^{\frac{3}{2}}}{16\left(\left(ab^{4}c^{2}-8\,a^{2}b^{2}c^{3}+16\,a^{3}c^{4}\right)x^{8}+a^{3}b^{4}-8\,a^{4}b^{2}c+16\,a^{5}c^{2}+2\left(ab^{5}c-8\,a^{2}b^{3}c^{2}+16\,a^{3}bc^{3}\right)x^{6}+\left(ab^{6}-6\,a^{2}b^{4}c+32\,a^{4}c^{3}\right)x^{4}+2\left(a^{2}b^{5}-8\,a^{3}b^{3}c+16\,a^{4}bc^{2}\right)x^{2}\right)}+\int\frac{3\left(\left(b^{2}c+12\,ac^{2}\right)x^{\frac{5}{2}}+\left(b^{3}-28\,abc\right)\sqrt{x}\right)}{32\left(a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}+\left(ab^{4}c-8\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3}\right)x^{4}+\left(ab^{5}-8\,a^{2}b^{3}c+16\,a^{3}bc^{2}\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c^2 + 12*a*c^3)*x^(15/2) + 6*(b^3*c + 8*a*b*c^2)*x^(11/2) + (3
*b^4 + 7*a*b^2*c + 68*a^2*c^2)*x^(7/2) - (a*b^3 - 28*a^2*b*c)*x^(3/2))/((a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c
 + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + integr
ate(3/32*((b^2*c + 12*a*c^2)*x^(5/2) + (b^3 - 28*a*b*c)*sqrt(x))/(a^2*b^4 -
 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^4 + (a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

mupad [B] time = 8.02, size = 42197, normalized size = 71.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*x^(11/2)*(b^3*c + 8*a*b*c^2))/(8*(a*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)) -
(x^(3/2)*(b^3 - 28*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(7/2)*(
3*b^4 + 68*a^2*c^2 + 7*a*b^2*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3
*c^2*x^(15/2)*(12*a*c + b^2))/(16*(a*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)))/(x^4
*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan(((((27*(3799
912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461
120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 -
 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 356438262153
2160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^
9*b^12*c^13 + 65529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8
*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17
- 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^
14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^
20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c
^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^
8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^
15*b^2*c^13)) - (9*x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 47
1104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696
*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 97320529
92*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13
151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12
*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 421
```

3765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b
^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4
*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(
a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72
960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 15876096
0*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 440
29706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^
18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 104
04558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120
*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^1
8 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 1677
7216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491
964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*
b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11
+ 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 8079
8711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131
445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24
 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*
c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12
976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206
016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c -
b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3
*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*
b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 10849307852
8*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 -
 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*
a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^
25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 546
48*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^
(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 304
0*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*
b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*
a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 7
04475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a
^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^
15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700
579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) + (9*x^(1/2)*(2
982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640
*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 449
37566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^
22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 81100
8*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^
10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13
*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^1
6*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4

- 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^
7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10
*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75
62531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^1
5*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a
^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25
)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 109
9511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c
^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^
6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*
b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113
425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^
19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16
 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138
880*a^24*b^2*c^19)))^(1/4)*1i - (((27*(3799912185593856*a^15*c^19 + 2097152
*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*
a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 6
45335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105
196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*
a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b
^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^1
8))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^
24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 +
12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 5
24812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^1
1 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (9*x^(1/2)*(-(81
(b^33 + b^8(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a
^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^
23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^
8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 98635
4024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b
^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*
a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*
c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^
6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^
20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b
^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^1
2*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 1937307
07456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^1
6*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 166
47293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*
a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)
))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2
*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 2044082
3103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*

b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17

+ 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i)/(
(((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5
 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4
*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3
564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 356948203
62027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 337676513564426
24*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^1
3*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435
456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 25
6256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888
*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624
576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 9
39524096*a^15*b^2*c^13)) - (9*x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)
^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^
3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6
 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^
15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358
219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5
*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2
) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b
^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^
36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5
 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^2
4*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299
840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14
*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 208
09116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a
^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*
c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24
*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273
051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8
*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*
c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 +
67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(419430
4*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 1408
0*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^
12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*
c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*
(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 -
 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424
368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 -
108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*
b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 821
2262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*

a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4) - (27*(2114129160*a*b^11*c^10 - 24024195*b^13*c^9 + 1209323520*a^6*b*c^15 - 61748341200*a^2*b^9*c^11 + 590751532800*a^3*b^7*c^12 + 227993875200*a^4*b^5*c^13 + 28822210560*a^5*b^3*c^14))/(16777216*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51

215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034
782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^
3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2
050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 19680
4608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1
526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^1
3)) + (9*x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280
*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*
c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^1
9*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*
a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12
+ 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560
*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 40
09*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2
)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 +
 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^
34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^2
8*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a
^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 +
2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 1040455827456
0*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*
c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 549755
8138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^2
8*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*
a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 +
 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541
706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747
136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^1
3*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 1677721
6*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 1267
20*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9
*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b
^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(
1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3
+ 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 -
 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15
*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035821
9776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c
^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2)
- 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2
c^3(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(3
3554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36
*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 +
 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*
c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 70447529984

0*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c
^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809
116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^2
3*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) - (9*x^(1/2)*(2982998016*
a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c
^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a
^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 105
6*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14
*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8
 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)
))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 +
 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14023372
8*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 433767
99744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10
 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253143859
2*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15
 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) -
107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776
*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 12403
20*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 127008
7680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 +
 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*
a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^
14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 1958505
0869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b
^2*c^19)))^(1/4)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 4711042252
80*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^2
5*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b
^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 1315117465
6*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^1
2 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 42137655705
60*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c +
4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b
^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40
 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*
b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b
^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240
*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11
+ 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274
560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^
8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497
558138880*a^24*b^2*c^19)))^(1/4)*2i - atan(((((27*(3799912185593856*a^15*c^
19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 4
02594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*

b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (9*x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 729

60*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960
*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 4402
9706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^1
8*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 1040
4558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*
a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18
 - 5497558138880*a^24*b^2*c^19)))^(3/4) + (9*x^(1/2)*(2982998016*a^6*b*c^14
 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 1667
26460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^1
3))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20
*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 378
4704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 5767168
0*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(
b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2
*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23
*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*
b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9863540
24448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7
*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^
4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c
 - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*
c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20
 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^3
2*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*
b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707
456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*
c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647
293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^
22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))
^(1/4)*1i - (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 26633830
4*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 70745
49334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b
^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12
 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 337
67651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 1145427233
35192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b
^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b
^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^
6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c
^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*
b^4*c^12 - 939524096*a^15*b^2*c^13)) + (9*x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*
c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248
*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*
a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1084930
78528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^

11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682
624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b
^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) +
 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c +
 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a
^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569
920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10
 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 52022791372
80*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^1
0*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 1305
6700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580
791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724
800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8
 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 57270811
91178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 4717344687810150
4*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12
*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^
17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^2
0*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 37
84704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 576716
80*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*
(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^
2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^2
3*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8
*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354
024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^
7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a
^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c
 - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6
c(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^2
0 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^
32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12
*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 19373070
7456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16
*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 1664
7293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a
^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19))
)^(3/4) - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201
*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 14758194
8160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 1677721
6*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 1267
20*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9
*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b
^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(

1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3
+ 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 -
9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15
*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035821
9776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c
^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2)
- 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2
c^3(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(3
3554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36
*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 +
158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*
c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 70447529984
0*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c
^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809
116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^2
3*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i)/((((27*(3799912185593
856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b
^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933
056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*
b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^
13 + 65529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 5
1215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 7061503
4782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a
^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 -
2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 1968
04608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 -
1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^
13)) - (9*x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 47110422528
0*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25
*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^
19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656
*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12
+ 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 421376557056
0*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4
009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^
2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40
+ 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b
^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^
28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*
a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 +
2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 104045582745
60*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8
*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 54975
58138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^
28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448

*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 1938054 1706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 8079871147874 7136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^ 13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 167772 16*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126 720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^ 9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12* b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^ (1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^1 5*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 38403582 19776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5* c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^ 2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^3 6*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24 *c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 7044752998 40*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14* c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 2080 9116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^ 23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) + (9*x^(1/2)*(2982998016 *a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9* c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208* a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 10 56*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^1 4*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^ 8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 1402337 28*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376 799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^1 0 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75625314385 92*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^1 5 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^ 2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 109951162777 6*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240 320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 12700 87680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520 *a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c

^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 195850
50869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*
b^2*c^19)))^(1/4) - (27*(2114129160*a*b^11*c^10 - 24024195*b^13*c^9 + 12093
23520*a^6*b*c^15 - 61748341200*a^2*b^9*c^11 + 590751532800*a^3*b^7*c^12 + 2
27993875200*a^4*b^5*c^13 + 28822210560*a^5*b^3*c^14))/(16777216*(a^2*b^28 +
268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c
^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 5
6229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 +
1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c
^12 - 939524096*a^15*b^2*c^13)) + (((27*(3799912185593856*a^15*c^19 + 20971
52*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 40259446374
4*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 +
645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 137283991
05196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 5652960363570790
4*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12
*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c
^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*
b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5
+ 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 -
524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c
^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (9*x^(1/2)*(-(
81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509
*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*
b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*
a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986
354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13
*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 129
6*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*
a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*
b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*
c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9
*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a
^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 19373
0707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b
^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 1
6647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 1958505086976
0*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^1
9)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a
^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440
823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^
7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c
^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 -
93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 2758454
7717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3
*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 81

1008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320
*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a
^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*
a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c
^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19
*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a
^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 +
 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*
a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 400
9*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)
^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 +
1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^3
4*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28
*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^
14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2
113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560
*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c
^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558
138880*a^24*b^2*c^19)))^(3/4) - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 1731384
72*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a
^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(41943
04*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 140
80*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b
^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6
*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8
*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2
 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 142
4368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 -
 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11
*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 82
12262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4
*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)
^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c
 - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*
b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15
876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 +
 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b
^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 520
2279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*
a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^1
7 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)))*(
-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 105
09*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^
5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4337679974
4*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9

86354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^
13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1
296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(
4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*
a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^2
5*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a
^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680
*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193
730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17
*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 -
16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869
760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c
^19)))^(1/4)*2i - 2*atan(((((27*(3799912185593856*a^15*c^19 + 2097152*b^30*
c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^
24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 6453354
79222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032
*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b
^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^1
6 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(3
3554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2
- 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 123002
88*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 5248122
88*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 15
26726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (x^(1/2)*(-(81*(b^33 +
b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*
c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 +
1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c
^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*
a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13
- 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*
(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)
^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4
*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*
a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4
- 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c
^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^
15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 -
5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239
296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6
*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)
*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^
5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*
a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^1
0 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 4717
3446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 9341450789

5848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288
*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c
 + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^
7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b
^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2
c^11)))(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*
c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14
0233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 +
43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^1
3*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253
1438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^
3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b
^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1
/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511
627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 +
 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 -
1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22
*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 21134258
99520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b
^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 1
9585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*
a^24*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a
*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b
^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(
a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a
^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*
c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9
 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(
4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 39
4248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368
896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108
493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^1
1*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 821226
2682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c
 - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/
2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b
^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38
*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 158760
96*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 825
5569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*
c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279
137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20
*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 +
13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4) - (((27
*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14

019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22
*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 356438
2621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027
008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^
11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4
*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a
^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*
a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*
b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a
^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524
096*a^15*b^2*c^13)) + (x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2)
- 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 921
9696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732
052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9
+ 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*
a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 +
 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157
*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*
(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2
- 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 1587
60960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 -
 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^1
6*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 +
 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 2080911654
9120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4
*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 +
16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 -
1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*
a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c
^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 +
80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 6790583
8131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a
^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^
5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c
^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9
+ 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4
*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394
248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243688
96*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1084
93078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11
*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262
682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c
- b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2
) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^

2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4))/((27*(2114129160*a*b^11*c^10 - 24024195*b^13*c^9 + 1209323520*a^6*b*c^15 - 61748341200*a^2*b^9*c^11 + 590751532800*a^3*b^7*c^12 + 227993875200*a^4*b^5*c^13 + 28822210560*a^5*b^3*c^14))/(16777216*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3

*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 20
50048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804
608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 15
26726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13
)) - (x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^
16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4
 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c
^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^1
0*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7
562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^
15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*
a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^2
5)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 10
99511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*
c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c
^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14
*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 211
3425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a
^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^1
6 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 549755813
8880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c
^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4
*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 12
25740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706
993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136
*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b
^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 1677721
6*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 1267
20*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9
*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b
^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(
1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3
+ 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 -
 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15
*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035821
9776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c
^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2)
- 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2
c^3(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(3
3554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36
*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 +
 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*
c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 70447529984
0*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c
^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809

116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^2
3*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(29829980
16*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^
9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 4493756620
8*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c +
1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b
^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*
c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^
11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^1
6 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14023
3728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 433
76799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c
^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253143
8592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c
^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*
c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2)
 - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627
776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 12
40320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 127
0087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^
9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 21134258995
20*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12
*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 1958
5050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^2
4*b^2*c^19)))^(1/4)*1i + (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c
^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^2
4*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 64533547
9222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*
a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^
10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16
 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33
554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2
- 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 1230028
8*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 52481228
8*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 152
6726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (x^(1/2)*(-(81*(b^33 +
b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c
^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 +
1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^
8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a
^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 -
 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(
-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^
25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*
a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a

^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262

682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4) - 2*atan(((((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4* a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a

^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 10849307
8528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^1
1 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 82122626826
24*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^
2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) +
54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c +
3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^
10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 82555699
20*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10
- 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 520227913728
0*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10
*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056
700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4) - (((27*(379
9912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 1401946
1120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8
- 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 35643826215
32160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a
^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^
8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17
- 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c
^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b
^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*
c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b
^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a
^15*b^2*c^13)) + (x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471
104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*
a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 973205299
2*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 131
51174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*
b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213
765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^
31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*
a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a
^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 729
60*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960
*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 4402
9706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^1
8*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 1040
4558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*
a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18
- 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777
216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 14919
64264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b
^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 +

19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19))))^(3/4)*1i + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 54975

58138880*a^24*b^2*c^19)))^(1/4))/((27*(2114129160*a*b^11*c^10 - 24024195*b^13*c^9 + 1209323520*a^6*b*c^15 - 61748341200*a^2*b^9*c^11 + 590751532800*a^3*b^7*c^12 + 227993875200*a^4*b^5*c^13 + 28822210560*a^5*b^3*c^14))/(16777216*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^

10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2)
- 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 921
9696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732
052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9
+ 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*
a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 +
4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157
*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*
(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2
- 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 1587
60960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 -
44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^1
6*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 +
10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 2080911654
9120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4
*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(2982998016*a^
6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^1
0 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5
*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*
a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c
^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 -
57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))
(-(81(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 1
0509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*
a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799
744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 +
986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*
a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 -
1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 10
7*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a
^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320
*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 12700876
80*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 1
93730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^
17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14
- 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 195850508
69760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2
*c^19)))^(1/4)*1i + (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 -
266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7
+ 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 6453354792222
72*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b
^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^
14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 11

4542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(3355443
2*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 232
96*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8
*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^1
1*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 15267266
56*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (x^(1/2)*(-(81*(b^33 - b^8*(
-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 -
394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243
68896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1
08493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b
^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212
262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a
*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c -
b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^
38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 1587
6096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8
255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^2
0*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 52022
79137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^
20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17
+ 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066
549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67
947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^
20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 57
27081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 4717344687
8101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 9341450789584896
0*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*
b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 105
6*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14
*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8
- 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)
))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 +
10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14023372
8*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 433767
99744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10
+ 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253143859
2*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15
- 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) +
107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776
*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 12403
20*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 127008
7680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 +
193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*

a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.853 $\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=594

$$-\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(cx^2\left(44ac+b^2\right)+b\left(20ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3c^{3/4}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\text{ta}}{32\sqrt[4]{2}\,a\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4\right.}}$$

Rubi [A] time = 2.37, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1364, 1430, 1422, 212, 208, 205}

$$-\frac{3c^{3/4}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\left(44ac+b^2\right)-68abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\left(44ac+b^2\right)-68abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(cx^2\left(44ac+b^2\right)+b\left(20ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[x]
*(b*(b^2 + 20*a*c) + c*(b^2 + 44*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) - (3*c^(3/4)*(b^2 + 44*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)
/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c
])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) -
(3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 44*a*c))*ArcTan[(2^(1
/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 -
4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^2 + 44*a*c - b
^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4
*a*c]*(b^2 + 44*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^3}\,dx &= 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{b-22cx^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)} \\
&= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \text{Subst}\Big(\int - \\
&= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \Big(3c\Big(b^2+4 \\
&= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \Big(3c\Big(b^2+4 \\
&= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - 3c^{3/4}\Big(b^2+
\end{aligned}
$$

Mathematica [C] time = 0.41, size = 224, normalized size = 0.38

$$
\frac{3\left(a+bx^2+cx^4\right)^2 \text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{44\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-12abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]-16a\sqrt{x}\left(b^2-4ac\right)\left(b+2cx^2\right)+4\sqrt{x}\left(20abc+44ac^2x^2+b^3+b^2cx^2\right)\left(a+bx^2+cx^4\right)}{64a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-16*a*(b^2 - 4*a*c)*Sqrt[x]*(b + 2*c*x^2) + 4*Sqrt[x]*(b^3 + 20*a*b*c + b^
2*c*x^2 + 44*a*c^2*x^2)*(a + b*x^2 + c*x^4) + 3*(a + b*x^2 + c*x^4)^2*RootS
um[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 12*a*b*c*Log[Sqrt[x] -
#1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 44*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
^3 + 2*c*#1^7) & ])/(64*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

IntegrateAlgebraic [C] time = 0.49, size = 245, normalized size = 0.41

$$\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{44\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-12abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{64a\left(4ac-b^2\right)^2}+\frac{\sqrt{x}\left(36a^2bc+76a^2c^2x^2-3ab^3+13ab^2cx^2+64abc^2x^4+44ac^3x^6+b^4x^2+2b^3cx^4+b^2c^2x^6\right)}{16a\left(4ac-b^2\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (Sqrt[x]*(-3*a*b^3 + 36*a^2*b*c + b^4*x^2 + 13*a*b^2*c*x^2 + 76*a^2*c^2*x^2
 + 2*b^3*c*x^4 + 64*a*b*c^2*x^4 + b^2*c^2*x^6 + 44*a*c^3*x^6))/(16*a*(-b^2
+ 4*a*c)^2*(a + b*x^2 + c*x^4)^2) + (3*RootSum[a + b*#1^4 + c*#1^8 & , (b^3
*Log[Sqrt[x] - #1] - 12*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#
1^4 + 44*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*a*(-b^2
 + 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.63Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 270, normalized size = 0.45

$$\frac{3\left(\left(44ac+b^2\right)\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4c-12abc+b^3\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8ab^2c+b^4\right)a\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{\frac{\left(44ac+b^2\right)c^2x^{\frac{13}{2}}}{16\left(16a^2c^2-8ab^2c+b^4\right)a}+\frac{\left(32ac+b^2\right)bc\,x^{\frac{9}{2}}}{8\left(16a^2c^2-8ab^2c+b^4\right)a}+\frac{\left(76a^2c^2+13ab^2c+b^4\right)x^{\frac{5}{2}}}{16\left(16a^2c^2-8ab^2c+b^4\right)a}+\frac{3\left(12ac-b^2\right)b\sqrt{x}}{16\left(16a^2c^2-8ab^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/32*b*(12*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)+1/32*(76*a^2*c^2+
13*a*b^2*c+b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/16/a*c*b*(32*a*c+b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)+1/32*c^2*(44*a*c+b^2)/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/a/(16*a^2*c^2-8*a*b^2*c+b^4)*
sum((c*(44*a*c+b^2)*_R^4-12*a*b*c+b^3)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R
=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(b^3c^2-12\,abc^3\right)x^{\frac{17}{2}}+\left(6\,b^4c-71\,ab^2c^2+44\,a^2c^3\right)x^{\frac{13}{2}}+\left(3\,b^5-28\,ab^3c-8\,a^2bc^2\right)x^{\frac{9}{2}}+\left(7\,ab^4-59\,a^2b^2c+76\,a^3c^2\right)x^{\frac{5}{2}}}{16\left(\left(a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4\right)x^8+a^4b^4-8\,a^5b^2c+16\,a^6c^2+2\left(a^2b^5c-8\,a^3b^3c^2+16\,a^4bc^3\right)x^6+\left(a^2b^6-6\,a^3b^4c+32\,a^5c^3\right)x^4+2\left(a^3b^5-8\,a^4b^3c+16\,a^5bc^2\right)x^2\right)}+\int-\frac{3\left(\left(b^3c-12\,abc^2\right)x^{\frac{7}{2}}+\left(b^4-13\,ab^2c-44\,a^2c^2\right)x^{\frac{3}{2}}\right)}{32\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2+\left(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3\right)x^4+\left(a^2b^5-8\,a^3b^3c+16\,a^4bc^2\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^3*c^2 - 12*a*b*c^3)*x^(17/2) + (6*b^4*c - 71*a*b^2*c^2 + 44*a^2*
c^3)*x^(13/2) + (3*b^5 - 28*a*b^3*c - 8*a^2*b*c^2)*x^(9/2) + (7*a*b^4 - 59*
a^2*b^2*c + 76*a^3*c^2)*x^(5/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 +
 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + integrate(-3/32*((b^3*c - 12*a*b*c^2)*
x^(7/2) + (b^4 - 13*a*b^2*c - 44*a^2*c^2)*x^(3/2))/(a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

mupad [B] time = 9.35, size = 54027, normalized size = 90.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 36
79344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227
840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576
*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5
 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*
(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 +
 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a
^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^
8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626
713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b
^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 299
19144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a
*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4
*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*
```

b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*
c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^
11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680
*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193
730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19
*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 -
16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869
760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c
^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448
*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045
478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*
b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 -
764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 170644204
6308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c
+ 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^
8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) -
(9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627
904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5
968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a
^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^
12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39
951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 225179981
36852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b
^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 8110
08*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320
*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a
^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 125050657177
60*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*
c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^
7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^
15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291
284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^1
5*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^
25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*
a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^2
5)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 109
9511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*
c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*
c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^1
6*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 21
13425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*
a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^
16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 54975581
38880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2)
+ 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 132

9320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 1811904
00*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490
242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^
11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 3197
4471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-
(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)
^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(
4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(
a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72
960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760
960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 4
4029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*
b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 1
0404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 208091165491
20*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c
^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (9*x^(1/2)*(245025*b^14*c^9 -
1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 2
2497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*
c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 -
48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c
^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 +
32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 5
0331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 125
05065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*
a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^
7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 8349024256
0*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^
12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 319744712
37632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*
c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2
) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c
- b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b
^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a
^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a
^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 440297
06240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*
c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 104045
58274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^
23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 -
5497558138880*a^26*b^2*c^19)))^(1/4)*1i - (((3*(230850*a*b^11*c^8 - 4455*b
^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^
10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 -
262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32
256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b
^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)

^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^
3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 -
181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9
- 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*
a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14
+ 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5
c^5(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b
^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*
c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(335
54432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c
^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 +
158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*
c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 70447529984
0*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c
^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809
116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^2
5*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^1
6 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c
^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 1038523092172
8*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c
^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 15876
94790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15))/(65536*(a^4*b^
18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3
+ 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a
^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817216*a^16*b*c
^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^2
5*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 4045000199
3728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^1
5*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 +
21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 428897
49576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4
*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*
b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^
6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9
+ 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(
4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 9
1335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800
*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 109128704
00*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11
- 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 201149592371
20*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c
^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*
c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2)
- 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c +

3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096
*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 82555
69920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^
10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 520227913
7280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b
^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13
056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b
^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^
2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*
c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c
^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a
^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13
- 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837
120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c
+ 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b
^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-
(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 8
0*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*
c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^
26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 19373070745
6*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^
12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 1664729
3239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24
*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(
1/4) - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^
12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016
*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(419
4304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 1
4080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^1
0*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13
*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 +
b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^3
1*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 +
70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 +
10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b
^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 201
14959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a
^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510
*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^2
5)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*
c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8
*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 -
15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^
7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^1
7*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 -

5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 166472932392
96*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*
c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*
1i)/((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 367
9344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 97602278
40*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*
a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5
+ 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*(
-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 +
3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^
5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8
*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 5026267
13600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^
9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 2991
9144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*
b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*
a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b
^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c
^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^1
1*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*
a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 1937
30707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*
b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 -
16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 195850508697
60*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^
19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*
a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 20454
78174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b
^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 -
764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046
308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c +
 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8
*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) -
(9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 11576279
04*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 59
68393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^
8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^1
2 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 399
51854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 2251799813
6852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^
22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 81100
8*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*
a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^
15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 1250506571776
0*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c

^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 3197447123

7632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c
 - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2)
 + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c
 - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^
40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^
10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^
13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 4402970
6240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c
^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 1040455
8274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^2
3*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 -
5497558138880*a^26*b^2*c^19)))^(1/4) + (((3*(230850*a*b^11*c^8 - 4455*b^13*
c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 -
 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 2621
44*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*
a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c
^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/
2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 +
1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 1811
90400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83
490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12
*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 3
1974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5
*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^
25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*
(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355443
2*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 -
 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158
760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8
- 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^
18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13
+ 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 208091165
49120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^
4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 -
16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 +
 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^
9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11
+ 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 158769479
0508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15))/(65536*(a^4*b^18 -
 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 3
2256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*
b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817216*a^16*b*c^18
- 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^
6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728
*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^

11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 211
86489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 4288974957
6286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^2
4 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18
*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 -
12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69
206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11))))*(-(81*(b^35 + b^10*(-(4*a*
c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335
*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6
*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a
^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 23
79389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a
^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16
- 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*
(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33
880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^
(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 304
0*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^1
2*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 825556992
0*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 -
704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280
*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*
c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 130567
00579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35
+ b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^
31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5
+ 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 +
10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*
b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20
114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*
a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 51
0*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^
25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a
c - b^2)^25)^(1/2)))/(33554432(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^
8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4
- 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c
^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^
17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 -
5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239
296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6
*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)
- (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c
^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4
*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304
*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080

*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*2i + atan(((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^

7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 -

44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840

*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i - (((3*(230850*a*b
^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 83
09952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65
536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a
^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6
- 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 - b^10*(-(
4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 9
1335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800
*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 109128704
00*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11
- 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 201149592371
20*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c
^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*
c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c +
3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096
*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 82555
69920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^
10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 520227913
7280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b
^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13
056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(77405618
5954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 200655
50336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8
+ 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 3009913081
0368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*
b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)
)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5
376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^
6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(309622474
3817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 341
75188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*
c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 3508816
48214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*
a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b
^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17
))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*
c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784
704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 576716
80*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*
(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*
a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^2
5*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19
*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600

*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^1
3 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 299191448
37120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*
c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c -
 b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*
(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 -
 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^3
2*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*
b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707
456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*
c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647
293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^
24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))
^(3/4))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*
b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12
356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 6687
23200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10
+ 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 829128441856
0*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^
15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2
) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*
c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2)
 + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 10995116277
76*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 12
40320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 12
70087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c
^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899
520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^1
2*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 195
85050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^
26*b^2*c^19)))^(1/4) - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16
 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^1
2 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6
*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*
a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c
^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8
 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)
))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^1
7 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 1235681
6*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200
*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502
626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^1
3*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 -
29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 9
5*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(

-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45
*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^
27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320
*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087
680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 +
193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a
^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^1
4 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050
869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^
2*c^19)))^(1/4)*1i)/((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*
a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5
*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a
^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129
024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b
^2*c^8)) + ((3*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 1250506571776
0*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c
^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7
 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^1
5*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 82912
84418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15
*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^2
5)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a
^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25
)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099
511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c
^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c
^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16
*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 211
3425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a
^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^1
6 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 549755813
8880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*
c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*
b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026
843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*
a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c
^14 - 1706442046308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9 -
 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4
- 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a
^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^2
9*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a
^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 17
1227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 5236424127283
20*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^1
2*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c

^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*
c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*
b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10
*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^
10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2)
 + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 13
29320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190
400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 8349
0242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b
^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 319
74471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(
-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25
)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-
(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*
(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 7
2960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 15876
0960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 -
44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18
*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 +
10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549
120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*
c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 - b^10*(-(4*a*c -
b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3
*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^2
3*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b
^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 237938
9337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*
b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 23
4256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4
*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*
a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2
)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^
9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^
30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^
15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704
475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^2
0*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15
 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 1305670057
9840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (9*x^(1/2)*(245
025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^
2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614
170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777
216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 12
6720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*
a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^

14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15))/(6

5536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*
a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^
6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817
216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 3417518
8992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8
+ 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 35088164821
4016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11
*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c
^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(
4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2
- 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*
a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a
^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^3
5 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*
b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^
5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8
 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^1
1*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 -
20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 2991914483712
0*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c -
510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2
)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4
*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*
a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^
4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26
*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*
a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12
 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 166472932
39296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b
^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/
4))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^
17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 123568
16*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 66872320
0*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 50
2626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^
13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 -
 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) -
95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*
(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 4
5*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a
^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 124032
0*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 127008
7680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 +
 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*
a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^

14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 1958505
0869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b
^2*c^19)))^(1/4) - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 1
3142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 +
112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2
c^15))/(4194304(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*
b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 +
3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 5
7671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(
-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 +
3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^
5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8
*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 5026267
13600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^
9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 2991
9144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*
b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*
a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b
^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c
^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^1
1*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*
a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 1937
30707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*
b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 -
16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 195850508697
60*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^
19)))^(1/4)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760
*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^
4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7
- 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15
*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 829128
4418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*
b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25
)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^
3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)
^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 10995
11627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^
3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^
6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*
b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113
425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^
21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16
- 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138
880*a^26*b^2*c^19)))^(1/4)*2i + 2*atan(((((3*(230850*a*b^11*c^8 - 4455*b^13
*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10

- 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262
144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256
*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*
c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2
) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1
329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 18119
0400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 834
90242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*
b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31
974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*
(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^2
5)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(
-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432
*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 -
72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 1587
60960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 -
44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^1
8*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 +
10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 2080911654
9120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4
*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 1
6777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 +
256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9
*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 +
156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790
508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18
- 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 +
32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^1
1*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^1
8 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*
c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 404500019937
28*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*
c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 2
1186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749
576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b
^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^
18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6
- 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 +
69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*
a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 913
35*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a
^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400
*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 -
2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120
*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^1

6 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^
2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) -
33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25
)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3
040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a
^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569
920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10
 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 52022791372
80*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^1
0*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 1305
6700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(
b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a
^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25
*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*
c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*
a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13
 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 2991914483
7120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c
 + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c -
b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(
-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 -
80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32
*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b
^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 1937307074
56*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c
^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 166472
93239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^2
4*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^
(1/4)*1i + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*
a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 11200511
0016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/
(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2
 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704
*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*
a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^
35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2
*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c
^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^
8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^
11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 -
 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 299191448371
20*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c +
 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^
2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(
4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80

*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c
^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^2
6*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456
*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^1
2 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293
239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*
b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1
/4) - (((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 36
79344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227
840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576
*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5
 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-
(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3
910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5
*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*
b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 50262671
3600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9
*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919
144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b
^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^
8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^
20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11
*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a
^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 19373
0707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b
^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 1
6647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 1958505086976
0*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^1
9)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a
^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 204547
8174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^
12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 7
64160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 17064420463
08352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c
 + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b
^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8))
+ (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 115762
7904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 -
5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*
a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c
^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 3
9951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998
136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*
b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811

008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 3244032
0*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*
a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717
760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27
*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c
^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b
^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 829
1284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^
15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)
^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015
*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^
25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 10
99511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34
*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28
*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^
16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2
113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560
*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c
^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558
138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1
/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 +
1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181
190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 8
3490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^1
2*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 +
31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^
5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4
*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2
- 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 15
8760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8
- 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a
^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13
+ 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116
549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b
^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i - (9*x^(1/2)*(245025*b^14
*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c
^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a
^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16
*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8
*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^1
0*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c
^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2
) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1

329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 18119
0400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 834
90242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*
b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31
974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*
(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^2
5)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(
-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432
*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 -
72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 1587
60960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 -
44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^1
8*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 +
10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 2080911654
9120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4
*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4))/((((3*(230850*a*b^11*c^8 - 44
55*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^
7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^1
8 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3
+ 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^
11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 + b^10*(-(4*a*c - b^2)^2
5)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*
c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6
- 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^
9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 237938933760
0*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^
14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a
^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c -
b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^
2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(3
3554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36
*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5
+ 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^2
4*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299
840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14
*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 208
09116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a
^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c
^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20
*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921
728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10
*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 158
7694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a
^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^1
2*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589

824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c

^19)))^(1/4)*1i + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13
142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 1
12005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*
c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b
^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 +
3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57
671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-
(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3
910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5
*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*
b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 50262671
3600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9
*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919
144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b
^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^
8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^
20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11
*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a
^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 19373
0707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b
^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 1
6647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 1958505086976
0*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^1
9)))^(1/4)*1i + (((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b
*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11
 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^
16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a
^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^
8)) - (((-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*
b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12
356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 6687
23200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10
+ 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 829128441856
0*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^
15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2
) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*
c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2)
 - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 10995116277
76*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 12
40320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 12
70087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c
^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899
520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^1
2*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 195

85050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a

^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9
*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^3
0*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^1
5*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 7044
75299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20
*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15
+ 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579
840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i - (9*x^(1/2)*(2
45025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*
a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 6176
14170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 167
77216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 +
126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 1297612
8*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*
a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2
)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^
29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c
^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17
*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 237938933
7600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7
*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 23425
6*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4
*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))
/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b
^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*
c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*
b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475
299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b
^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 +
20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 1305670057984
0*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i))*(-(81*(b^35 + b
^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*
c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 7
0316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10
912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^1
3*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114
959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^1
6*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a
^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)
^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c
- b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b
^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 1
5876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7

+ 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + 2*atan(((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 5033164

8*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 125050657
17760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^
27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21
*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10
*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8
291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*
a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^
2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 20
15*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2
)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 +
1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^
34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^
28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*
a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 +
2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 104045582745
60*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8
*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 54975
58138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^
(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3
+ 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 1
81190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 -
83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a
^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14
+ 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*
c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^
2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c
^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355
4432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^
2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 +
158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c
^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840
*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^
13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 208091
16549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25
*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(245025*b^
14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10
*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624
*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^
16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a
^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b
^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4
*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1
/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 +
1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181
190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 8

3490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^1
2*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 +
31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^
5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)
^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4
*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2
- 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 15
8760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8
 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a
^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13
 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116
549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b
^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) - (((3*(230850*a*b^11*c^8 -
4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*
b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b
^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^
3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*
a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 - b^10*(-(4*a*c - b^2)
^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^2
9*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^
6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*
c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337
600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*
c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256
*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c
 - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*
b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^
36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c
^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b
^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 7044752
99840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^
14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 2
0809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840
*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16
*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^
20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 103852309
21728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^
10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1
587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*
(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b
^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 5
89824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817216*a
^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*

a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 404
50001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*
a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11
*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 +
 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(41943
04*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 140
80*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*
b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b
^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b
^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*
c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 7
0316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10
912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^1
3*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114
959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^1
6*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a
^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)
^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c
- b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b
^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 1
5876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7
+ 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*
b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 52
02279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296
*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^
17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i
)*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17
 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816
*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*
a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 5026
26713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13
*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 2
9919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95
*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-
(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*
a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^2
7*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*
a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 12700876
80*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 1
93730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^
19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14
 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 195850508
69760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2
*c^19)))^(1/4)*1i - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 -
13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 +

112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4))/((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 88

9192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9* b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i + (((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25

)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3
040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a
^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569
920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10
 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 52022791372
80*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^1
0*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 1305
6700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(7740561859
54304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550
336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 +
 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 300991308103
68*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^
6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3
i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 -
5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b
^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(30962247
43817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34
175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21
*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881
648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488
*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*
b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^1
7))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20
*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 378
4704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671
680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81
(b^35 - b^10(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910
*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^
25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^1
9*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 50262671360
0*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^
13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144
837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33
*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c
- b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c
*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20
- 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^
32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14
*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 19373070
7456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16
*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 1664
7293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a
^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19))
)^(3/4)*1i)*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a
^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4

- 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 319744

```
71237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4
*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(
1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*
a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^
7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 7296
0*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 15876096
0*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 440
29706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^
18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 104
04558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120
*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^1
8 - 5497558138880*a^26*b^2*c^19)))^(1/4) + ((x^(9/2)*(b^3*c + 32*a*b*c^2))/
(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*x^(1/2)*(b^3 - 12*a*b*c))/(16*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(5/2)*(b^4 + 76*a^2*c^2 + 13*a*b^2*c))/(1
6*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^(13/2)*(44*a*c + b^2))/(16*a*(
b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*
x^2 + 2*b*c*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.854 $\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=658

$$\frac{x^{3/2}\left(52a^2c^2+bcx^2\left(5b^2-44ac\right)-45ab^2c+5b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Rubi [A] time = 5.48, antiderivative size = 658, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1366, 1500, 1510, 298, 205, 208}

$$\frac{x^{3/2}\left(52a^2c^2+bcx^2\left(5b^2-44ac\right)-45ab^2c+5b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + b*c*(5*b^2 - 44*a*c)*x^2))/(
16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c
+ 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^
(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*
a^2*c^2 + b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*
(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^
2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 + b
*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1500

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2
 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(-5b^2+26ac-9bcx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} +$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} +$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

Mathematica [C] time = 0.49, size = 254, normalized size = 0.39

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{-44\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+5\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+260a^2c^2\log\left(\sqrt{x}-\#1\right)-49ab^2c\log\left(\sqrt{x}-\#1\right)+5b^4\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4x^{3/2}\left(52a^2c^2-45ab^2c-44abc^2x^2+5b^4+5b^3cx^2\right)}{a+bx^2+cx^4}-\frac{16ax^{3/2}\left(4ac-b^2\right)\left(-2ac+b^2+bcx^2\right)}{\left(a+bx^2+cx^4\right)^2}}{64a^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-16*a*(-b^2 + 4*a*c)*x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4)
^2 + (4*x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + 5*b^3*c*x^2 - 44*a*b*c^2
*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^4*Log[Sqr
t[x] - #1] - 49*a*b^2*c*Log[Sqrt[x] - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1] +
 5*b^3*c*Log[Sqrt[x] - #1]*#1^4 - 44*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
+ 2*c*#1^5) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

IntegrateAlgebraic [C] time = 0.68, size = 294, normalized size = 0.45

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-44\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+5\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+260a^2c^2\log\left(\sqrt{x}-\#1\right)-49ab^2c\log\left(\sqrt{x}-\#1\right)+5b^4\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{64a^2\left(4ac-b^2\right)^2}+\frac{x^{3/2}\left(84a^3c^2-69a^2b^2c-8a^2bc^2x^2+52a^2c^3x^4+9ab^4-36ab^3cx^2-89ab^2c^2x^4-44abc^3x^6+5b^5x^2+10b^4cx^4+5b^3c^2x^6\right)}{16a^2\left(4ac-b^2\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(9*a*b^4 - 69*a^2*b^2*c + 84*a^3*c^2 + 5*b^5*x^2 - 36*a*b^3*c*x^2
- 8*a^2*b*c^2*x^2 + 10*b^4*c*x^4 - 89*a*b^2*c^2*x^4 + 52*a^2*c^3*x^4 + 5*b^
3*c^2*x^6 - 44*a*b*c^3*x^6))/(16*a^2*(-b^2 + 4*a*c)^2*(a + b*x^2 + c*x^4)^2
) + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^4*Log[Sqrt[x] - #1] - 49*a*b^2*c*L
og[Sqrt[x] - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1] + 5*b^3*c*Log[Sqrt[x] - #1
]*#1^4 - 44*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(64*a^2*(
-b^2 + 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.53Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.05, size = 321, normalized size = 0.49

$$-\frac{\left(\left(44ac-5b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6bc+\left(-260a^2c^2+49a\,b^2c-5b^4\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2\right)\ln\left(-\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8a\,b^2c+b^4\right)a^2\left(2\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{-\frac{\left(44ac-5b^2\right)b\,c^2x^{\frac{15}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}+\frac{\left(52a^2c^2-89a\,b^2c+10b^4\right)c\,x^{\frac{11}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}-\frac{\left(8a^2c^2+36a\,b^2c-5b^4\right)b\,x^{\frac{7}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}+\frac{3\left(28a^2c^2-23a\,b^2c+3b^4\right)x^{\frac{3}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a}}{\left(c\,x^4+b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/32*(28*a^2*c^2-23*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(3/2)-
1/32*b*(8*a^2*c^2+36*a*b^2*c-5*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+
1/32/a^2*c*(52*a^2*c^2-89*a*b^2*c+10*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/
2)-1/32*c^2*b*(44*a*c-5*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^
4+b*x^2+a)^2-1/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(44*a*c-5*b^2)*_R
^6+(-260*a^2*c^2+49*a*b^2*c-5*b^4)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),
_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(5 b^3c^2-44 abc^3\right)x^{\frac{15}{2}}+\left(10 b^4c-89 ab^2c^2+52 a^2c^3\right)x^{\frac{11}{2}}+\left(5 b^5-36 ab^3c-8 a^2bc^2\right)x^{\frac{7}{2}}+3\left(3 ab^4-23 a^2b^2c+28 a^3c^2\right)x^{\frac{3}{2}}}{16\left(\left(a^2b^4c^2-8 a^3b^2c^3+16 a^4c^4\right)x^8+a^4b^4-8 a^5b^2c+16 a^6c^2+2\left(a^2b^5c-8 a^3b^3c^2+16 a^4bc^3\right)x^6+\left(a^2b^6-6 a^3b^4c+32 a^5c^3\right)x^4+2\left(a^3b^5-8 a^4b^3c+16 a^5bc^2\right)x^2\right)}-\int -\frac{\left(5 b^3c-44 abc^2\right)x^{\frac{5}{2}}+\left(5 b^4-49 ab^2c+260 a^2c^2\right)\sqrt{x}}{32\left(a^3b^4-8 a^4b^2c+16 a^5c^2+\left(a^2b^4c-8 a^3b^2c^2+16 a^4c^3\right)x^4+\left(a^2b^5-8 a^3b^3c+16 a^4bc^2\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^3*c^2 - 44*a*b*c^3)*x^(15/2) + (10*b^4*c - 89*a*b^2*c^2 + 52*a^2
*c^3)*x^(11/2) + (5*b^5 - 36*a*b^3*c - 8*a^2*b*c^2)*x^(7/2) + 3*(3*a*b^4 -
23*a^2*b^2*c + 28*a^3*c^2)*x^(3/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*
c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^
2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b
^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - integrate(-1/32*((5*b^3*c - 44*a*b*
c^2)*x^(5/2) + (5*b^4 - 49*a*b^2*c + 260*a^2*c^2)*sqrt(x))/(a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

mupad [B] time = 8.75, size = 46948, normalized size = 71.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(11/2)*(10*b^4*c + 52*a^2*c^3 - 89*a*b^2*c^2))/(16*(a^2*b^4 + 16*a^4*c^
2 - 8*a^3*b^2*c)) - (x^(7/2)*(8*a^2*b*c^2 - 5*b^5 + 36*a*b^3*c))/(16*a*(a*b
^4 + 16*a^3*c^2 - 8*a^2*b^2*c)) + (3*x^(3/2)*(3*b^4 + 28*a^2*c^2 - 23*a*b^2
*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^(15/2)*(44*a*c - 5*b^
2))/(16*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c
^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((2097152000*a*b^33*c^4 + 46617885
6428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b
^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23
398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 163109930
0505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 452077026065
68226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 4878820944
58626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946
621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418
770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^1
```

4 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^

(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4) + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i - (((2097152000*a*b^33*c^4 + 466178856428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c

^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^
7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8
*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^1
9*b^2*c^13)) + (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) +
11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^
3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b
^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 826293
38933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a
^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^
11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15
 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 2856
10000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c
^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2
) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*
(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(3355
4432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*
c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5
 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24
*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 7044752998
40*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*
c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 2080
9116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^
27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^1
8*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a
^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 +
2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 988625794
21544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677
275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 92018897786712
88320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 1384160234849068
6464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*b^24
 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*
c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 -
 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 6
9206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-
(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*
c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^2
7*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 1331706844
8000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17
*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 -
 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851
227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 355771891266
35520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a
*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^
3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2
) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a

c - b^2)^25)^(1/2))/(33554432(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^1
0*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4
 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*
c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a
^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12
- 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 1664729323
9296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^
6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4
) - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*
a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 197541
4457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*
c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b
^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5
+ 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 -
57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11))))*
(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b
*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*
c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*
a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 +
419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 58070005
41921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282
176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^1
6*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c -
 b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4
*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2
) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511
627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^
3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^
6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*
b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113
425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^
23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16
 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138
880*a^28*b^2*c^19)))^(1/4)*1i)/((((2097152000*a*b^33*c^4 + 4661788564281884
67200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6
- 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 2339859098
6584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 16310993005051904
00*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816
*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375
680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 177194662141347
9153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 141877011651
0434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a
^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 -
 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 +

196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^1
0 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^
2*c^13)) - (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1127
9020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 +
1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*
c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 8262933893
3248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*
b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c
^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5
2725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 28561000
0*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) -
28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4
*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432
*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2
- 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 15
8760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8
- 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a
^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13
+ 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116
549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b
^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^
19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*b
^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2159
815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 9886257942154
4448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 16354394336772751
36*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 920188977867128832
0*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464
*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*b^24 + 1
6777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3
+ 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 129
76128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206
016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a
*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2
- 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^
5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000
*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^1
0 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 154
22593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 488512278
86223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 3557718912663552
0*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^3
5*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-
(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) +
121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c -
b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^

38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 1
5876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7
+ 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*
b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 52
02279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296
*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^
17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4) +
(x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*
b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457
856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16
))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*
c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 37
84704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 5767
1680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(6
25*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^1
8 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4
- 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*
b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 4197
01532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 580700054192
1280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*
a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^
5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2
)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2)
+ 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(
4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) +
21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 10995116277
76*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 +
1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 -
1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22
*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 21134258
99520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b
^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 1
9585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*
a^28*b^2*c^19)))^(1/4) + (((2097152000*a*b^33*c^4 + 4661788564281884672OO*a
^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 12030
0087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064
*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*
b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b
^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^1
2*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664
*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197
504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26
*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 205004
8*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 1968046
08*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 152

6726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)
) + (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326
912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954
201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1
756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^
9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^
11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3
1764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360
025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c
^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c
 - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 2854520
1*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c -
b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b
^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960
*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960
*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 4402
9706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^1
8*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 1040
4558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*
a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18
 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^19 - 41
9430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6
 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 21598155728
48640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421544448*a^
10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677275136*a^12
*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288320*a^14*
b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464*a^16*b
^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*b^24 + 16777216
*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 12672
0*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a
^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^1
6*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^
2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758
230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188
531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^
21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 173
7502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259399
1966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 488512278862233б
0*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*
b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 3
80775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c
- b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 1215786
00*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^2
5)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c +
3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096

*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 82555
69920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^
10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 520227913
7280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b
^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13
056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4) - (x^(1/2
)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17
+ 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4
*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(419
4304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 1
4080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a
^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^
15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37
- 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 216
8275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285
749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^
7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 41970153273
3440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^
12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^
9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16
- 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(
1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075
730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c -
b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a
*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29
*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320
*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087
680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 +
193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a
^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^1
4 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050
869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^
2*c^19)))^(1/4) + (80318101760000000*a^7*c^19 - 6746163125*b^14*c^12 + 5724
89781500*a*b^12*c^13 - 15194313373200*a^2*b^10*c^14 + 226647361174720*a^3*b
^8*c^15 - 2095830057168640*a^4*b^6*c^16 + 12493373163648000*a^5*b^4*c^17 -
44688231411200000*a^6*b^2*c^18)/(134217728*(a^6*b^28 + 268435456*a^20*c^14
- 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20
*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*
c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b
^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a
^19*b^2*c^13))))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1127902
0326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 110
9954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6
- 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 8262933893324
8*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^1

5*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259399196672O*a^13*b^11*c^13
 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5272
5360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a
^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4
*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 285
45201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*
c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a
^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 7
2960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 15876
0960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 -
44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20
*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 +
10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549
120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*
c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*2i + atan(((((2097152000*a*b^33
*c^4 + 4661788564281884672OO*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340
020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^
5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c
^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12
 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^
14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9
*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*
b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268
435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 +
 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56
229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 +
1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c
^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c
- b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 5
7758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 +
 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^
8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 -
 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 154225
93991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 488512278862
23360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a
^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c
 + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*
a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121
578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^
2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*
c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 1587
6096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8
255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^2
0*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 52022
79137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^

24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4) + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^

13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52
725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000
*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-
(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 2
8545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*
a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*
(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 -
72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158
760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8
- 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^
20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13
+ 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 208091165
49120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^
4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i - (((2097152000*a*b^33*c^4
+ 4661788564281884672000*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 53400200
80640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^
25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10
- 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 4
5207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 -
487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^1
6 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*
c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 2684354
56*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256
256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 562298
88*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049
624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12
- 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^
2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758
230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188
531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^
21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 173
7502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259399
1966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 488512278862233б
0*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*
b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 3
80775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c
- b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 1215786
00*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^2
5)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c +
3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096
*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 82555
69920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^
10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 520227913
7280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b
^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13

056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(23784635
53205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 -
814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*
a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c
^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12
+ 1635439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 +
9201889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 1
3841602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(41
94304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 -
14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*
a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a
^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^3
7 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 21
68275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 1628
5749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c
^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 4197015327
33440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a
^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b
^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16
- 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^
(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 407
5730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c
- b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*
a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^2
9*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 124032
0*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 127008
7680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 +
193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*
a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^
14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 1958505
0869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b
^2*c^19)))^(3/4) - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 +
99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^
9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483
712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22
*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008
*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*
a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^
17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 112790203
26912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 11099
54201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 -
1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*
a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*
c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 +
31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 527253

60025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i)/((((2097152000*a*b^33*c^4 + 4661788564281884672 00*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(237846355320

5043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814
718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*
b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10
+ 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 16
35439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 920
1889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841
602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(419430
4*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 1408
0*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12
*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*
b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11))))*(-(625*b^37 +
625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 216827
5*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749
400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 +
13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 41970153273344
0*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*
b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c
^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 3
5577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2
) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730
*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^
2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^
10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^
20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^
13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680
*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193
730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21
*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 -
16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869
760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c
^19)))^(3/4) + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 9980
3558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^
13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 109904837120
00*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c +
1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^1
1*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14
*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b
^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1127902032691
2000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 110995420
1*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 175
6313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*
b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11
+ 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 317
64369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5272536002
5927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6

*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4) + (((2097152000*a*b^33*c^4 + 466178856428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 8147183861

76*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^
8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862
579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 163543943
3677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778
671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 138416023484
90686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*
b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b
^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c
^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9
 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^1
2*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b
^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5
*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 133170
68448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*
b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^
12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4
8851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189
126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 526
25*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^
6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^
(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-
(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80
*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32
*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b
^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 1937307074
56*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c
^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 166472
93239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^2
6*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^
(3/4) - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400
000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 19
75414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*
b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a
^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*
c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^
8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11
)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^
18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b
^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913
600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^
9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807
000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3176436974
3282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680
*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a
*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^2

5)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4
c^4(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^
(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 109
9511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^3
4*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^2
8*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a
^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 +
2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 1040455827456
0*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*
c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 549755
8138880*a^28*b^2*c^19)))^(1/4) + (80318101760000000*a^7*c^19 - 6746163125*b
^14*c^12 + 572489781500*a*b^12*c^13 - 15194313373200*a^2*b^10*c^14 + 226647
361174720*a^3*b^8*c^15 - 2095830057168640*a^4*b^6*c^16 + 12493373163648000*
a^5*b^4*c^17 - 44688231411200000*a^6*b^2*c^18)/(134217728*(a^6*b^28 + 26843
5456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 2
56256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 5622
9888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 10
49624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^1
2 - 939524096*a^19*b^2*c^13))))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^
(1/2) + 1127902032691200*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3
*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 1885317804
00*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8
- 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295
326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720
*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*
b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^1
7 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a
^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^
25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*
b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2
))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^
11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b
^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a
^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 70
4475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^
22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^1
5 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 130567005
79840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*2i + 2*atan((((
2097152000*a*b^33*c^4 + 466178856428188467200*a^17*b*c^20 - 151833804800*a^
2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 19
33149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263
505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588
032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 16802707228761207
6032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 10826732229231
22114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 20140680186

80264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(26843545
6*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 232
96*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a
^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288
*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526
726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 - 6
25*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275
*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 162857494
00*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 +
13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440
*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b
^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^
14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35
577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2)
 - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*
a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2
)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^1
0*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^2
0 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^1
3*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*
a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 1937
30707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*
b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 -
16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 195850508697
60*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^
19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675
773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c
^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 1661536
0157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 4569839705385
86112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 448054836609417
2160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155
776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 850251462149878579
2*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*
c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*
a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a
^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^1
7*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1127902032
6912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 110995
4201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 -
1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a
^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c
^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 +
31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5272536
0025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*
c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*

c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 285452
01*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c -
 b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*
b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 7296
0*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 15876096
0*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 440
29706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^
18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 104
04558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120
*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^1
8 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i - (x^(1/2)*(30525625*b^15*c^10
- 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11
*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591
360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777
216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 12
6720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 1297612
8*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*
a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c -
 b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57
758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 +
188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8
*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 -
1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259
3991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4885122788622
3360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^
17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c
- 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 1215
78600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2
)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c
 + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876
096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 82
55569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20
*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 520227
9137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^2
4*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 +
 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4) - (((2
097152000*a*b^33*c^4 + 4661788564281884672OO*a^17*b*c^20 - 151833804800*a^2
*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 193
3149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 2198782522635
05920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 96250148040285880
32*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076
032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 108267322292312
2114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 201406801868
0264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456

*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 2329
6*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^
12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*
a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 15267
26656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 - 62
5*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*
a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 1628574940
0*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 1
3317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*
a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^
13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^1
4 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 355
77189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2)
 - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a
^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)
^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10
c(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20
 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13
*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a
^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 19373
0707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b
^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 1
6647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 1958505086976
0*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^1
9)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 266757
73440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^
7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360
157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 45698397053858
6112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 4480548366094172
160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 136750395310221557
76*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792
*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c
 + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a
^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^
14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17
*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326
912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954
201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1
756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^
9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^
11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3
1764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360
025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c
^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c
 - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 2854520

1*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4))/((((2097152000*a*b^33*c^4 + 466178856428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296

*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^1
2*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a
^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 152672
6656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 - 625
b^12(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a
^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400
*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13
317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a
^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^1
3*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14
 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 3557
7189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) -
 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^
3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^
25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*
c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20
 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*
b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^
16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730
707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^
16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16
647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760
*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19
)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 2667577
3440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7
 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 166153601
57450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586
112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 44805483660941721
60*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 1367503953102215577
6*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792*
a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c
 + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^
11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^1
4*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*
b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 112790203269
12000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 11099542
01*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 17
56313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9
*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^1
1 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31
764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 527253600
25927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^
6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c
 - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201
*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b

^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^
40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*
a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*
a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029
706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18
*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404
558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a
^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18
- 5497558138880*a^28*b^2*c^19)))^(3/4)*1i - (x^(1/2)*(30525625*b^15*c^10 -
1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c
^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 475398059136
0*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 1677721
6*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 1267
20*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*
a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^
16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b
^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 5775
8230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 18
8531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b
^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 17
37502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 154225939
91966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 488512278862233
60*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17
*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c -
380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c
 - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578
600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^
25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c +
 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 1587609
6*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255
569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c
^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 52022791
37280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*
b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 1
3056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i + (((
2097152000*a*b^33*c^4 + 4661788564281884672000*a^17*b*c^20 - 151833804800*a^
2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 19
33149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263
505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588
032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 16802707228761207
6032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 10826732229231
22114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 20140680186
80264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(26843545
6*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 232
96*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a

^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288 *a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526 726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 - 6 25*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275 *a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 162857494 00*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440 *a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b ^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^ 14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35 577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730* a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^1 0*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^2 0 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^1 3*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680* a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 1937 30707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21* b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 195850508697 60*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^ 19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675 773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c ^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 1661536 0157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 4569839705385 86112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 448054836609417 2160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155 776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 850251462149878579 2*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22* c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008* a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a ^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^1 7*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1127902032 6912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 110995 4201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a ^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c ^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5272536 0025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6* c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a* c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 285452 01*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*

b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 7296
0*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 15876096
0*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 440
29706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^
18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 104
04558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120
*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^1
8 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i + (x^(1/2)*(30525625*b^15*c^10
- 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11
*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591
360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777
216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 12
6720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 1297612
8*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*
a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c -
 b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57
758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 +
188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8
*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 -
1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259
3991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4885122788622
3360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^
17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c
- 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 1215
78600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2
)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c
 + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876
096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 82
55569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20
*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 520227
9137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^2
4*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 +
 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i - (
80318101760000000*a^7*c^19 - 6746163125*b^14*c^12 + 572489781500*a*b^12*c^1
3 - 15194313373200*a^2*b^10*c^14 + 226647361174720*a^3*b^8*c^15 - 209583005
7168640*a^4*b^6*c^16 + 12493373163648000*a^5*b^4*c^17 - 44688231411200000*a
^6*b^2*c^18)/(134217728*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1
456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11
*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^1
4*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 152672665
6*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13))))*(-
(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c
^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^
4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^

7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 41
9701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541
921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3176436974328217
6*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*
b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b
^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(
-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2)
+ 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 109951162
7776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3
+ 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6
- 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^
22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 211342
5899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23
*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 -
 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 549755813888
0*a^28*b^2*c^19)))^(1/4) + 2*atan(((((2097152000*a*b^33*c^4 + 4661788564281
88467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c
^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 2339859
0986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 16310993005051
90400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226
816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626
375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 177194662141
3479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 141877011
6510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 5
6*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^
4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7
 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*
c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19
*b^2*c^13)) - (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1
1279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3
 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^
25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 8262933
8933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^
11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^1
1*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15
+ 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 28561
0000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^
2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2)
 + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(
-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554
432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c
^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 +
 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*
c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 70447529984

0*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c
^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809
116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^2
7*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18
*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^
5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2
159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 9886257942
1544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 16354394336772
75136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 920188977867128
8320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686
464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^
24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^1
8*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6
 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 +
 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*
(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^3
3*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b
^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068
448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^
17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12
 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 488
51227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 3557718912
6635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625
*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*
c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1
/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4
*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a
^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c
^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^2
6*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456
*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^1
2 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293
239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*
b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3
/4)*1i - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 9980355840
0000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1
975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6
*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*
a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14
*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c
^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^1
1)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a
^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*
b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 175631391
3600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c

^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 580
7000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 317643697
43282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5272536002592768
0*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*
a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^
25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^
4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)
^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 10
99511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^
34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^
28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*
a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 +
2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 104045582745
60*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8
*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 54975
58138880*a^28*b^2*c^19)))^(1/4) - (((2097152000*a*b^33*c^4 + 46617885642818
8467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^
6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590
986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 163109930050519
0400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 452077026065682268
16*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 4878820944586263
75680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413
479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116
510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56
*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4
- 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7
+ 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c
^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*
b^2*c^13)) + (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11
279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3
+ 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^2
5*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338
933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^1
1*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11
*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 +
52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610
000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-
(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(335544
32*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^
2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 +
158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c
^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840
*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^

13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 208091
16549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27
*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*
c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5
*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 21
59815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421
544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 163543943367727
5136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288
320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 138416023484906864
64*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^2
4 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18
*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6
- 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 +
69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(
-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33
*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^
27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 133170684
48000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^1
7*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12
- 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4885
1227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126
635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*
a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c
^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/
2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*
a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^
10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^
4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26
*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*
a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12
- 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 166472932
39296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b
^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/
4)*1i + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400
000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 19
75414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*
b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a
^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*
c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^
8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11
)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^
18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b
^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913
600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^
9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807

000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3176436974
3282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680
*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a
*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^2
5)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4
c^4(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^
(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 109
9511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^3
4*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^2
8*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a
^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 +
2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 1040455827456
0*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*
c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 549755
8138880*a^28*b^2*c^19)))^(1/4))/((((2097152000*a*b^33*c^4 + 466178856428188
467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6
 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 233985909
86584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190
400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 4520770260656822681
6*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 48788209445862637
5680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 17719466214134
79153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 14187701165
10434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*
a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4
- 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 +
 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^
10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b
^2*c^13)) - (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 112
79020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 +
 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25
*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 826293389
33248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11
*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*
c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 +
52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 2856100
00*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*
(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) +
 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(
4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(3355443
2*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2
 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 1
58760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^
8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*
a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^1
3 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 2080911

6549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*
b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c
^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*
b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 215
9815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 988625794215
44448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 163543943367727
5136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 92018897786712883
20*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 1384160234849068646
4*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24
 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*
c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 -
 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 6
9206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-
(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*
c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^2
7*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 1331706844
8000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17
*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 -
 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851
227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 355771891266
35520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a
*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^
3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2
) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a
c - b^2)^25)^(1/2))/(33554432(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^1
0*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4
 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*
c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a
^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12
 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 1664729323
9296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^
6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4
)*1i - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 998035584000
00*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 197
5414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b
^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^
8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c
^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8
 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)
))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^1
8*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^
29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 17563139136
00*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9
 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 58070
00541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743

282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i + (((2097152000*a*b^33*c^4 + 466178856428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^2

7*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5272536002592768

```
0*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*
a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^
25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^
4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)
^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 10
99511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^
34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^
28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*
a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 +
 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 104045582745
60*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8
*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 54975
58138880*a^28*b^2*c^19)))^(1/4)*1i - (80318101760000000*a^7*c^19 - 67461631
25*b^14*c^12 + 572489781500*a*b^12*c^13 - 15194313373200*a^2*b^10*c^14 + 22
6647361174720*a^3*b^8*c^15 - 2095830057168640*a^4*b^6*c^16 + 12493373163648
000*a^5*b^4*c^17 - 44688231411200000*a^6*b^2*c^18)/(134217728*(a^6*b^28 + 2
68435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3
 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 -
56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9
+ 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4
*c^12 - 939524096*a^19*b^2*c^13))))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^
25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230
*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531
780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*
c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 173750
2295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259399196
6720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a
^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3
*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 3807
75*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b
^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*
a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^
(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 304
0*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^
14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 82555699
20*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10
- 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 520227913728
0*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10
*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056
700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.855 $\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=658

$$\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\text{t}}{32\sqrt[4]{2}\,a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/}}$$

Rubi [A] time = 5.79, antiderivative size = 658, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1115, 1345, 1430, 1422, 212, 208, 205}

$$\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a^2\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a^2\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + b*c*(7*b^2 - 52*a*c)*x^2))/(
16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*
c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c
)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c +
280*a^2*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5
/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*
a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a^2
*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-
b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1345

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(
x*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^
2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + n*
(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^
(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*
c, 0] && ILtQ[p, -1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{b^2-2ac-8\left(b^2-4ac\right)-11bcx^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right.}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right.}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right.}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right.}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [C] time = 0.46, size = 258, normalized size = 0.39

$$\frac{3\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{-52\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+7\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+140a^2c^2\log\left(\sqrt{x}-\#1\right)-59ab^2c\log\left(\sqrt{x}-\#1\right)+7b^4\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+\frac{4\sqrt{x}\left(60a^2c^2-55ab^2c-52abc^2x^2+7b^4+7b^3cx^2\right)}{a+bx^2+cx^4}-\frac{16a\sqrt{x}\left(4ac-b^2\right)\left(-2ac+b^2+bcx^2\right)}{\left(a+bx^2+cx^4\right)^2}}{64a^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-16*a*(-b^2 + 4*a*c)*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4)
^2 + (4*Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + 7*b^3*c*x^2 - 52*a*b*c^2
*x^2))/(a + b*x^2 + c*x^4) + 3*RootSum[a + b*#1^4 + c*#1^8 & , (7*b^4*Log[S
qrt[x] - #1] - 59*a*b^2*c*Log[Sqrt[x] - #1] + 140*a^2*c^2*Log[Sqrt[x] - #1]
 + 7*b^3*c*Log[Sqrt[x] - #1]*#1^4 - 52*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#
1^3 + 2*c*#1^7) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

IntegrateAlgebraic [C] time = 0.59, size = 296, normalized size = 0.45

$$\frac{3\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-52\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+7\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+140a^2c^2\log\left(\sqrt{x}-\#1\right)-59ab^2c\log\left(\sqrt{x}-\#1\right)+7b^4\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{64a^2\left(4ac-b^2\right)^2}+\frac{\sqrt{x}\left(92a^3c^2-79a^2b^2c-8a^2bc^2x^2+60a^2c^3x^4+11ab^4-44ab^3cx^2-107ab^2c^2x^4-52abc^3x^6+7b^5x^2+14b^4cx^4+7b^3c^2x^6\right)}{16a^2\left(4ac-b^2\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (Sqrt[x]*(11*a*b^4 - 79*a^2*b^2*c + 92*a^3*c^2 + 7*b^5*x^2 - 44*a*b^3*c*x^2
 - 8*a^2*b*c^2*x^2 + 14*b^4*c*x^4 - 107*a*b^2*c^2*x^4 + 60*a^2*c^3*x^4 + 7*
b^3*c^2*x^6 - 52*a*b*c^3*x^6))/(16*a^2*(-b^2 + 4*a*c)^2*(a + b*x^2 + c*x^4)
^2) + (3*RootSum[a + b*#1^4 + c*#1^8 & , (7*b^4*Log[Sqrt[x] - #1] - 59*a*b^
2*c*Log[Sqrt[x] - #1] + 140*a^2*c^2*Log[Sqrt[x] - #1] + 7*b^3*c*Log[Sqrt[x]
 - #1]*#1^4 - 52*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(
64*a^2*(-b^2 + 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.11Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 316, normalized size = 0.48

$$\frac{3\left(\left(-52ac+7b^2\right)\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4bc+140a^2c^2-59a\,b^2c+7b^4\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8a\,b^2c+b^4\right)a^2\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{-\frac{\left(52ac-7b^2\right)b\,c^2x^{\frac{13}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}+\frac{\left(60a^2c^2-107a\,b^2c+14b^4\right)c\,x^{\frac{9}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}-\frac{\left(8a^2c^2+44a\,b^2c-7b^4\right)b\,x^{\frac{5}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}+\frac{\left(92a^2c^2-79a\,b^2c+11b^4\right)\sqrt{x}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a}}{\left(c\,x^4+b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(92*a^2*c^2-79*a*b^2*c+11*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(1/2)
-1/32*b*(8*a^2*c^2+44*a*b^2*c-7*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)
+1/32/a^2*c*(60*a^2*c^2-107*a*b^2*c+14*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9
/2)-1/32*c^2*b*(52*a*c-7*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x
^4+b*x^2+a)^2+3/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(-52*a*c+7*b^2)*
_R^4+140*a^2*c^2-59*a*b^2*c+7*b^4)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Roo
tOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(7\,b^4c^2-59\,ab^2c^3+140\,a^2c^4\right)x^{\frac{17}{2}}+\left(42\,b^5c-347\,ab^3c^2+788\,a^2bc^3\right)x^{\frac{13}{2}}+\left(21\,b^6-121\,ab^4c-41\,a^2b^2c^2+900\,a^3c^3\right)x^{\frac{9}{2}}+\left(49\,ab^5-398\,a^2b^3c+832\,a^3bc^2\right)x^{\frac{5}{2}}+32\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2\right)\sqrt{x}}{16\left(a^5b^4-8\,a^6b^2c+16\,a^7c^2+\left(a^3b^4c^2-8\,a^4b^2c^3+16\,a^5c^4\right)x^8+2\left(a^3b^5c-8\,a^4b^3c^2+16\,a^5bc^3\right)x^6+\left(a^3b^6-6\,a^4b^4c+32\,a^6c^3\right)x^4+2\left(a^4b^5-8\,a^5b^3c+16\,a^6bc^2\right)x^2\right)}-\int\frac{3\left(\left(7\,b^4c-59\,ab^2c^2+140\,a^2c^3\right)x^{\frac{7}{2}}+\left(7\,b^5-66\,ab^3c+192\,a^2bc^2\right)x^{\frac{3}{2}}\right)}{32\left(a^4b^4-8\,a^5b^2c+16\,a^6c^2+\left(a^3b^4c-8\,a^4b^2c^2+16\,a^5c^3\right)x^4+\left(a^3b^5-8\,a^4b^3c+16\,a^5bc^2\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(7*b^4*c^2 - 59*a*b^2*c^3 + 140*a^2*c^4)*x^(17/2) + (42*b^5*c - 347
*a*b^3*c^2 + 788*a^2*b*c^3)*x^(13/2) + (21*b^6 - 121*a*b^4*c - 41*a^2*b^2*c
^2 + 900*a^3*c^3)*x^(9/2) + (49*a*b^5 - 398*a^2*b^3*c + 832*a^3*b*c^2)*x^(5
/2) + 32*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*sqrt(x))/(a^5*b^4 - 8*a^6*b^2
*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8 + 2*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*
c^3)*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2) - integrate(3/32*(
(7*b^4*c - 59*a*b^2*c^2 + 140*a^2*c^3)*x^(7/2) + (7*b^5 - 66*a*b^3*c + 192*
a^2*b*c^2)*x^(3/2))/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + (a^3*b^4*c - 8*a^
4*b^2*c^2 + 16*a^5*c^3)*x^4 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2),
x)
```

mupad [B] time = 9.85, size = 60099, normalized size = 91.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] ((x^(9/2)*(14*b^4*c + 60*a^2*c^3 - 107*a*b^2*c^2))/(16*a^2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) + (x^(1/2)*(11*b^4 + 92*a^2*c^2 - 79*a*b^2*c))/(16*a*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) - (x^(5/2)*(8*a^2*b*c^2 - 7*b^5 + 44*a*b^3*c))/(1
6*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^(13/2)*(52*a*c - 7*b^2))/(
16*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8
+ 2*a*b*x^2 + 2*b*c*x^6) - atan((((((9*x^(1/2)*(1546704997025054720*a^19*b*
c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^
6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3
041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892
427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 154540674867
0558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965
799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 965089734210617
```

3440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24
 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^1
8*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6
 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 +
 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-(81*(2401*b^39 -
2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 744506
0*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 4030266
3491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7
+ 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373
440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^1
2*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^1
1*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16
- 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010
000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c
^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2
) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(
-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2)
+ 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511
627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^
3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^
6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*
b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113
425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^
25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16
 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138
880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^
25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 182804545536
0*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*
c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 52
94148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463
624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 -
262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 +
32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^
15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c -
 b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180
851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 +
406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8
*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 -
 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 117565
81147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701
511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*
a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c
 - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^2
5)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6
c^4(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(

1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(
4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 8
0*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^3
2*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*
b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707
456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*
c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647
293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^
28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))
^(3/4) + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*
c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260
*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8
763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^1
6*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024
*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2
c^8)))(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566
784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 311254
4495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 -
3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*
a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*
c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 +
 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836
636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 112249500440
98560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a
*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c
^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/
2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6
*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33
554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^
36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c
^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b
^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 7044752
99840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^
14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2
0809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840
*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784
832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 11724
33971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*
b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 15
56843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*
a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^
4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 +
 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 5
0331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(
1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*

b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 40693634220 0*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551 027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200 *a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15 *b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^ 17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25) ^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a *c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 1080 25400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38 *c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 158 76096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^ 20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202 279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a ^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7* b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 2 1176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 47572 0885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206 695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 101174948 92562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 56720022556 96429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b ^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 8 11008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 3244 0320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 503316 48*a^19*b^2*c^11)) + (3*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1 /2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b ^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200 *a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489235510 27200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200* a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15* b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^1 7 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^ (1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7 556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a* c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 10802 5400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2) ^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38* c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 1587

6096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8
255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^2
0*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 52022
79137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^
26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17
+ 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377
699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5
 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040
*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*
c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9
906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 960533358
0251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c
+ 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^1
3*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8
)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165667840
00*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495
*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276
813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*
b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11
 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216
83350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366360
93972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004409856
0*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^3
7*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(
-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) +
 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(
4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c
^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 +
 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*
c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447529984
0*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c
^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809
116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^2
9*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) - (3*(4356374400000*a^8*c
^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10
- 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^
5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(655
36*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a
^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*
c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*
b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2
*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*
a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 213
41140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a

^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i)/(((((9*x^(1/2)*(1546704997025054

720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 14736
77099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b
^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10
+ 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1
545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 73
15227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650
897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(419430
4*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 140
80*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^
14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^1
7*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-(81*(2
401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^
19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^
4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^
7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 4
92398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 505264416
1945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216833504234700
80*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^1
6*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c
^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660
*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^
2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^
5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)
^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^4
0 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a
^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a
^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 440297
06240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*
c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 104045
58274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^
27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 -
 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 11744
0512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1
828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096
*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^1
1*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 +
12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(
a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*
b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6
- 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14
*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^3
5*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*
b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 2134114
0889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*
b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^

12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 -
30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 2435987
4477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c
^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a
*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052
295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c -
b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a
*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^
31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 12403
20*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 12700
87680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9
+ 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520
*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c
^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 195850
50869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*
b^2*c^19)))^(3/4) + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712
223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 6
81741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b
^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 -
36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c
^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 5898
24*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) -
2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c
^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*
b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 11333
0748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200
*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*
b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c
^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1
1224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2)
- 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 755611
5*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b
^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*
a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^
(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3
040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*
a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 825556
9920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^1
0 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137
280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^
10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 130
56700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/
2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c
^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373
024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^

4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20
*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a
^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15
*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b
^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c -
b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180
851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 +
406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8
*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 -
1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 117565
81147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701
511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*
a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c
- b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^2
5)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6
c^4(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(
1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(
4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 8
0*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^3
2*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*
b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707
456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*
c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647
293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^
28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))
^(1/4) - ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^3
1*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976
960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21
*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11
- 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 -
3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 1
011749489256221900 8*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 567
2002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 -
48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16
*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^
7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10
- 50331648*a^19*b^2*c^11)) + (3*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2
)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 1808519
65*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 4069
36342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^2
3*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 174
8923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 1175658114
7443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 309290257015111
68*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17
*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b

^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4
*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2)
 - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*
c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^
12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^
4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26
*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*
a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12
 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 166472932
39296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b
^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/
4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*
b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 171369
19511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^
13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*
c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9
605333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9
*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 12
9024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16
*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 240541
6566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 31
12544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^
6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025
600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b
^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^
13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 3
2836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950
044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 1937
95*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b
^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)
^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2
c^6(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))
/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^1
3*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^
30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^
19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704
475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^2
4*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15
 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 1305670057
9840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) - (3*(43563744000
00*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^
12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 26948575
97280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^1
5))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2

- 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a
^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39
 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 744
5060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 4030
2663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c
^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189
373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*
a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*
b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^
16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24
010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^1
0*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(
1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^
5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/
2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099
511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34
*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28
*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^
20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2
113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560
*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c
^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558
138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 175
5191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 -
 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 3336032513018
88*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c
^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b
^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5
 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 -
 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))
(-(81(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*
a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^
4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813
600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^2
1*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 +
5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216833
50423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366360939
72480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a
^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c
 - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4
*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87
808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a
*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*
(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2
- 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 15

8760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*2i - atan((((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 -

24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 2401000
0*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(
4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) -
73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951162
7776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3
+ 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6
- 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^
22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211342
5899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25
*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 -
19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755813888
0*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25
*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*
a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^
9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294
148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 1252563646362
4192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 26
2144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32
256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15
*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b
^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 18085
1965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 40
6936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b
^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1
748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581
147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 3092902570151
1168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^
17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c -
b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)
^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c
^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/
2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*
a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*
a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*
c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^
26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 19373070745
6*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^
12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 1664729
3239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28
*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(
3/4) + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^
9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a
^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 876

3424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c

+ 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876
096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 82
55569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20
*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 520227
9137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^2
6*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 +
13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (
(((9*x^(1/2)*(154670499702505472O*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50
851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^
25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 211
76692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 4757208
85626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 386720669
5260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892
562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696
429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^2
2*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811
008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 324403
20*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648
*a^19*b^2*c^11)) + (3*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2
) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^3
3*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a
^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 11
3330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027
200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^
13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^
9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17
+ 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1
/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 755
6115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c
- b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 1080254
00*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c
+ 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 158760
96*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 825
5569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*
c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279
137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26
*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 +
13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(337769
9720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 +
132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a
^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^
10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 990
6599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 96053335802
51136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c +

576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*
b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8))
)*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000
*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a
^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681
3600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^
21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 +
 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683
350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093
972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*
a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*
c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(
4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 8
7808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*
a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432
*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2
 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1
58760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^
8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*
a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^1
3 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911
6549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*
b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) - (3*(4356374400000*a^8*c^1
6 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 -
110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*
b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536
*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^1
1*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^
6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^
14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b
^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^
5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341
140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^1
0*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*
c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14
 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359
874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7
c^7(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4
*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 340
52295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c
 - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745
*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*
a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 124
0320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 127
0087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^

9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 21134258995
20*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12
*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 1958
5050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^3
0*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^
16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 119117324
72304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6
*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(419
4304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 -
14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704
*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*
a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(24
01*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^1
9 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4
 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7
*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49
2398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161
945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216833504234700
80*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16
*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^
18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*
a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2
)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5
*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^
25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40
 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^
14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^
17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 4402970
6240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c
^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 1040455
8274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^2
7*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 -
5497558138880*a^30*b^2*c^19)))^(1/4)*1i)/(((((9*x^(1/2)*(154670499702505472
0*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677
099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^2
3*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 +
113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 154
5406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315
227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 965089
7342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*
(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080
*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14
*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*
b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-(81*(240
1*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19

+ 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a

^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^1
4 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050
869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^
2*c^19)))^(3/4) + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 68571222
3*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681
741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4
*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 3
6*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4
 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824
*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2
405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3
 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^
27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 1133307
48025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a
^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^
13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^1
5 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 112
24950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) -
 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*
a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2
)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^
6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1
/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 304
0*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^
16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 82555699
20*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10
 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 520227913728
0*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10
*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056
700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)
*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^1
2 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 7762637302
4736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*
c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c
^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^1
2*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b
^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4
*c^10 - 50331648*a^19*b^2*c^11))))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b
^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 18085
1965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 40
6936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b
^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1
748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581
147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 3092902570151
1168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^

17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) + (3*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)

*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^
23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919
511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13
*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^
12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 960
5333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b
^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 1290
24*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b
^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165
66784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112
544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6
- 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 11333074802560
0*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^1
7*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13
 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328
36636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004
4098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795
*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8
c^3(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(
1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c
^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(
33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*
b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30
*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19
*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447
5299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*
b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 +
 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 130567005798
40*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) - (3*(4356374400000
*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12
*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597
280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15)
)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 -
5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^1
4*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 +
 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 74450
60*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403026
63491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7
 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818937
3440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^
12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^
11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16
 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 2401
0000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*
c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/

2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*
(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2)
 - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951
1627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c
^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c
^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20
*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211
3425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a
^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^1
6 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755813
8880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 17551
91025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 1
1911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888
*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^1
8))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^2
0*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 +
 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 5
7671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(
-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^
19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*
b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681360
0400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*
c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 50
52644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350
423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972
480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^1
8*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c +
 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a
*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 8780
8681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c
 - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a
^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 -
72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1587
60960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 -
 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^2
2*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 +
 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911654
9120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4
*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)))*(-(81*(2401*b^39 + 2401*b^14
*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^3
5*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*
b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 2134114
0889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*
b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^
12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 -

30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 2435987
4477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c
^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a
*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052
295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c -
 b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a
*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^
31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 12403
20*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 12700
87680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9
+ 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520
*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c
^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 195850
50869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*
b^2*c^19)))^(1/4)*2i - 2*atan((((((9*x^(1/2)*(154670499702505472O*a^19*b*c^
19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*
b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 304
1476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 11381289242
7485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 15454067486705
58208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 731522788096579
9936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 96508973421061734
40*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 +
 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*
c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 -
 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 6
9206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - ((-(81*(2401*b^39 - 2401
b^14(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^
2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491
*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21
341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*
a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^
15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^
14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24
359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*
a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) -
34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*
a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73
745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 10995116277
76*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 +
1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 -
1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22
*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 21134258
99520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b
^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 1

9585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*
a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c
^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^
10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9
+ 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 529414
8487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 125256364636241
92*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 2
62144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 3
2256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^1
5*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c -
b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 1808
51965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 4
06936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*
b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 -
1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 1175658
1147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 309290257015
11168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a
^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c
- b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25
)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*
c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1
/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4
*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80
*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32
*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b
^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 1937307074
56*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c
^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 166472
93239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^2
8*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^
(3/4)*1i - (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^1
4*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 6817412352
60*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 -
8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b
^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 1290
24*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b
^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165
66784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112
544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6
- 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 11333074802560
0*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^1
7*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13
+ 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328
36636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004
4098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795

*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8
c^3(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(
1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c
^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(
33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*
b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30
*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19
*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447
5299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*
b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 +
20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 130567005798
40*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(12
19784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 +
1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736
*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17
- 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12
- 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^
16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*
c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^1
0 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^
25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965
*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936
342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*
c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489
23551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 117565811474
43200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168
*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b
^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2
)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(
-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) -
108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c
- b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12
*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4
- 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c
^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^
21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 -
5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239
296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6
*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)
+ ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4
+ 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^
7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 -
21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475
720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 38672

06695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 1011749
4892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 567200225
5696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9
*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 -
811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32
440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 5033
1648*a^19*b^2*c^11)) + ((-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1
/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b
^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200
*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 -
113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489235510
27200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*
a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*
b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^1
7 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^
(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7
556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*
c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 10802
5400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)
^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*
c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 1587
6096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8
255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^2
0*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 52022
79137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^
26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17
+ 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377
699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5
+ 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040
*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*
c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9
906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 960533358
0251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16
*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*
a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*
c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165667
84000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544
495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3
276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a
^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c
^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 +
21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366
36093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004409
8560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*
b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^

3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2
) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*
(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(335
54432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^3
6*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^
5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^
24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447529
9840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^1
4*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20
809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*
a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4)*1i + (3*(4356374400000
*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12
*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597
280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15)
)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 -
5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^1
4*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 -
 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 74450
60*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403026
63491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7
 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818937
3440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^
12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^
11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16
 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 2401
0000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*
c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/
2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*
(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2)
 + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951
1627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c
^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c
^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20
*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211
3425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a
^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^1
6 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755813
8880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 17
55191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13
- 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301
888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*
c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*
b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^
5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8
- 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11))

)*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000
*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a
^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681
3600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^
21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 +
5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683
350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093
972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*
a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*
c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(
4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 8
7808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*
a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432
*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2
- 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1
58760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^
8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*
a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^1
3 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911
6549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*
b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4))/(((((9*x^(1/2)*(1546704997
025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 -
1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736
*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19
*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^
12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^1
4 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16
+ 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(
4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2
- 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784
704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 576716
80*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - ((-(8
1*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*
b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^3
1*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681360040
0*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9
+ 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 50526
44161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423
470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480
*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b
^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 99
6660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c
- b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 8780868
1*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c -
b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11

*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 729
60*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1587609
60*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44
029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b
^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10
404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911654912
0*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^
18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 1
17440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6
 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 11457254758
8096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14
*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^1
3 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(
65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 537
6*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b
^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 24
01*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*
a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403026634
91*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 +
21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818937344
0*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*
b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*
c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 -
24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 2401000
0*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2)
- 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(
4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) +
73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951162
7776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3
+ 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6
- 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^
22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211342
5899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25
*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 -
 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755813888
0*a^30*b^2*c^19)))^(3/4)*1i - (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^
8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^1
0*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 658229519
8080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*
a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a
^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*
c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^2
5)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*
a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 4069363

42200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 1958

5050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i - ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) + ((-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c

^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4)*1i + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 240541656678400*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 117243

3971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b
^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 155
6843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a
^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4
 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 +
32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50
331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1
/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b
^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200
*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 -
113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489235510
27200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*
a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*
b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^1
7 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^
(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7
556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*
c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 10802
5400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)
^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*
c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 1587
6096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8
255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^2
0*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 52022
79137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^
26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17
+ 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i))*
(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a
^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4
*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 32768136
00400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21
*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5
052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 2168335
0423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 3283663609397
2480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^
18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c
- 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*
a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 878
08681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*
c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(
a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 -
 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158
760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8
- 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^
22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13

+ 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 208091165 49120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^ 4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - 2*atan((((((9*x^(1/2)*(1546 704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29 *c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626 612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^1 0*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b ^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^ 11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7 *c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^ 18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^ 20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - ((-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000 *a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a ^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681 3600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^ 21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683 350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093 972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560* a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37* c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 8 7808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4* a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432 *(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1 58760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^ 8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840* a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^1 3 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911 6549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29* b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^ 16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^ 21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 11457 2547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 201458017999257 6*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b ^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15) *3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064* a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^3 9 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 74 45060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403

02663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*
c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818
9373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600
*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14
*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c
^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 2
4010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^
10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^
(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c
^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1
/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109
9511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^3
4*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^2
8*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a
^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 +
2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 1040455827456
0*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*
c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755
8138880*a^30*b^2*c^19)))^(3/4)*1i - (3*(4356374400000*a^8*c^16 + 18475695*b
^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a
^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 658
2295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 2
62144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 3
2256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^1
5*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c -
b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 1808
51965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 4
06936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*
b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 -
1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 1175658
1147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 309290257015
11168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a
^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c
- b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25
)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*
c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1
/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4
*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80
*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32
*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b
^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 1937307074
56*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c
^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 166472
93239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^2
8*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^

(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) + ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) + ((-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15* c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4)*1i + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c

^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*
a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 87
63424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16
*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*
a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*
c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165667
84000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544
495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3
276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a
^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c
^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 +
21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366
36093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004409
8560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*
b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^
3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2
) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*
(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(335
54432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^3
6*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^
5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^
24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447529
9840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^1
4*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20
809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*
a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(12197
84832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 117
2433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^
4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 -
1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 4
8*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*
c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7
 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 -
 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)
^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^
3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342
200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8
 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489235
51027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 117565811474432
00*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^
15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*
c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^2
5)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2)
- 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4
*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 10

8025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b
^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^
38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 1
5876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7
+ 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*
b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 52
02279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296
*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^
17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4))/(
((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 5
0851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b
^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21
176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720
885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 38672066
95260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 1011749489
2562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 567200225569
6429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^
22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 81
1008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440
320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 5033164
8*a^19*b^2*c^11)) - ((-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2)
 - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33
*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^
6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113
330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489235510272
00*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^1
3*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9
*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 +
 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/
2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556
115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c -
 b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 10802540
0*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25
)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c +
 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 1587609
6*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255
569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c
^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 52022791
37280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*
b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 1
3056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699
720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 +
132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^
11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^1
0 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906

599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 960533358025
1136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c
+ 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^1
3*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8
)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165667840
00*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495
*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276
813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*
b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11
 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216
83350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366360
93972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004409856
0*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^3
7*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(
-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) -
 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(
4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c
^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 +
 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*
c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447529984
0*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c
^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809
116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^2
9*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4)*1i - (3*(4356374400000*a^
8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^
10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280
*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(
65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 537
6*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b
^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 24
01*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*
a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403026634
91*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 +
21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818937344
0*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*
b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*
c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 -
24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 2401000
0*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(
4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) -
73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951162
7776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3

+ 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 240541656678400*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i - ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 576716

80*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) + ((-(8
1*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*
b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^3
1*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681360040
0*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9
 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 50526
44161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423
470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480
*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b
^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 99
6660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c
- b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 8780868
1*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c -
b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11
*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 729
60*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1587609
60*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44
029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b
^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10
404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911654912
0*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^
18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 1
17440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6
 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 11457254758
8096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14
*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^1
3 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(
65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 537
6*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b
^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 24
01*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*
a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403026634
91*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 +
21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818937344
0*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*
b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*
c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 -
24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 2401000
0*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(
4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) -
73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951162
7776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3
+ 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6

- 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4)*1i + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*

```
c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14
- 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359
874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7
*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4
*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 340
52295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c
 - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745
*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*
a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 124
0320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 127
0087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^
9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 21134258995
20*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12
*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 1958
5050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^3
0*b^2*c^19)))^(1/4)*1i))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(
1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*
b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 40693634220
0*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 -
 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551
027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200
*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15
*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^
17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)
^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) -
7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a
*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 1080
25400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2
)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38
*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 158
76096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 +
8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^
20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202
279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a
^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17
 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.856 $\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=156

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b(dx)^{m+3}}{d^3(m+3)}+\frac{3c\left(ac+b^2\right)(dx)^{m+9}}{d^9(m+9)}+\frac{b\left(6ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{3a\left(ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)}+\frac{c^3}{d^1}$$

Rubi [A] time = 0.10, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{3a^2b(dx)^{m+3}}{d^3(m+3)}+\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a\left(ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{b\left(6ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{3c\left(ac+b^2\right)(dx)^{m+9}}{d^9(m+9)}+\frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)}+\frac{c^3(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
3*a*(b^2 + a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (b*(b^2 + 6*a*c)*(d*x)^(7 +
m))/(d^7*(7 + m)) + (3*c*(b^2 + a*c)*(d*x)^(9 + m))/(d^9*(9 + m)) + (3*b*c^
2*(d*x)^(11 + m))/(d^11*(11 + m)) + (c^3*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx = \int \left(a^3(dx)^m+\frac{3a^2b(dx)^{2+m}}{d^2}+\frac{3a\left(b^2+ac\right)(dx)^{4+m}}{d^4}+\frac{b\left(b^2+6ac\right)(dx)^{6+m}}{d^6}+\frac{3c}{}\right.$$

$$=\frac{a^3(dx)^{1+m}}{d(1+m)}+\frac{3a^2b(dx)^{3+m}}{d^3(3+m)}+\frac{3a\left(b^2+ac\right)(dx)^{5+m}}{d^5(5+m)}+\frac{b\left(b^2+6ac\right)(dx)^{7+m}}{d^7(7+m)}+\frac{3c\,(}{}$$

Mathematica [A] time = 0.13, size = 111, normalized size = 0.71

$$x(dx)^m\left(\frac{a^3}{m+1}+\frac{3a^2bx^2}{m+3}+\frac{3cx^8\left(ac+b^2\right)}{m+9}+\frac{bx^6\left(6ac+b^2\right)}{m+7}+\frac{3ax^4\left(ac+b^2\right)}{m+5}+\frac{3bc^2x^{10}}{m+11}+\frac{c^3x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^2)/(3 + m) + (3*a*(b^2 + a*c)*x^4)/(5 +
 m) + (b*(b^2 + 6*a*c)*x^6)/(7 + m) + (3*c*(b^2 + a*c)*x^8)/(9 + m) + (3*b*
c^2*x^10)/(11 + m) + (c^3*x^12)/(13 + m))
```

IntegrateAlgebraic [F] time = 0.68, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + bx^2 + cx^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a + b*x^2 + c*x^4)^3, x]
```

fricas [B] time = 0.76, size = 594, normalized size = 3.81

$$\frac{\left(\left(c^3m^6 + 36c^3m^5 + 505c^3m^4 + 3480c^3m^3 + 12139c^3m^2 + 19524c^3m + 10395c^3\right)x^{13} + 3\left(bc^2m^6 + 38bc^2m^5 + 555bc^2m^4 + 3940bc^2m^3 + 14039bc^2m^2 + 22902bc^2m + 12285bc^2\right)x^{11} + 3\left(\left(b^2c + ac^2\right)m^6 + 40\left(b^2c + ac^2\right)m^5 + 613\left(b^2c + ac^2\right)m^4 + 4528\left(b^2c + ac^2\right)m^3 + 15015b^2c + 15015ac^2 + 16627\left(b^2c + ac^2\right)m^2 + 27688\left(b^2c + ac^2\right)m\right)x^9 + \left(\left(b^3 + 6abc\right)m^6 + 42\left(b^3 + 6abc\right)m^5 + 679\left(b^3 + 6abc\right)m^4 + 5292\left(b^3 + 6abc\right)m^3 + 19305b^3 + 115830abc + 20335\left(b^3 + 6abc\right)m^2 + 34986\left(b^3 + 6abc\right)m\right)x^7 + 3\left(\left(ab^2 + a^2c\right)m^6 + 44\left(ab^2 + a^2c\right)m^5 + 753\left(ab^2 + a^2c\right)m^4 + 6280\left(ab^2 + a^2c\right)m^3 + 27027ab^2 + 27027a^2c + 25979\left(ab^2 + a^2c\right)m^2 + 47436\left(ab^2 + a^2c\right)m\right)x^5 + 3\left(a^2bm^6 + 46a^2bm^5 + 835a^2bm^4 + 7540a^2bm^3 + 34759a^2bm^2 + 73054a^2bm + 45045a^2b\right)x^3 + \left(a^3m^6 + 48a^3m^5 + 925a^3m^4 + 9120a^3m^3 + 48259a^3m^2 + 129072a^3m + 135135a^3\right)x\right)(dx)^m}{m^7 + 49m^6 + 973m^5 + 10045m^4 + 57379m^3 + 177331m^2 + 264207m + 135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 36*c^3*m^5 + 505*c^3*m^4 + 3480*c^3*m^3 + 12139*c^3*m^2 + 19524
*c^3*m + 10395*c^3)*x^13 + 3*(b*c^2*m^6 + 38*b*c^2*m^5 + 555*b*c^2*m^4 + 39
40*b*c^2*m^3 + 14039*b*c^2*m^2 + 22902*b*c^2*m + 12285*b*c^2)*x^11 + 3*((b^
2*c + a*c^2)*m^6 + 40*(b^2*c + a*c^2)*m^5 + 613*(b^2*c + a*c^2)*m^4 + 4528*
(b^2*c + a*c^2)*m^3 + 15015*b^2*c + 15015*a*c^2 + 16627*(b^2*c + a*c^2)*m^2
 + 27688*(b^2*c + a*c^2)*m)*x^9 + ((b^3 + 6*a*b*c)*m^6 + 42*(b^3 + 6*a*b*c)
*m^5 + 679*(b^3 + 6*a*b*c)*m^4 + 5292*(b^3 + 6*a*b*c)*m^3 + 19305*b^3 + 115
830*a*b*c + 20335*(b^3 + 6*a*b*c)*m^2 + 34986*(b^3 + 6*a*b*c)*m)*x^7 + 3*((
a*b^2 + a^2*c)*m^6 + 44*(a*b^2 + a^2*c)*m^5 + 753*(a*b^2 + a^2*c)*m^4 + 628
0*(a*b^2 + a^2*c)*m^3 + 27027*a*b^2 + 27027*a^2*c + 25979*(a*b^2 + a^2*c)*m
^2 + 47436*(a*b^2 + a^2*c)*m)*x^5 + 3*(a^2*b*m^6 + 46*a^2*b*m^5 + 835*a^2*b
*m^4 + 7540*a^2*b*m^3 + 34759*a^2*b*m^2 + 73054*a^2*b*m + 45045*a^2*b)*x^3
+ (a^3*m^6 + 48*a^3*m^5 + 925*a^3*m^4 + 9120*a^3*m^3 + 48259*a^3*m^2 + 1290
72*a^3*m + 135135*a^3)*x)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 573
79*m^3 + 177331*m^2 + 264207*m + 135135)
```

giac [B] time = 0.23, size = 1132, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^3*m^6*x^13 + 36*(d*x)^m*c^3*m^5*x^13 + 3*(d*x)^m*b*c^2*m^6*x^11
+ 505*(d*x)^m*c^3*m^4*x^13 + 114*(d*x)^m*b*c^2*m^5*x^11 + 3480*(d*x)^m*c^3*
m^3*x^13 + 3*(d*x)^m*b^2*c*m^6*x^9 + 3*(d*x)^m*a*c^2*m^6*x^9 + 1665*(d*x)^m
*b*c^2*m^4*x^11 + 12139*(d*x)^m*c^3*m^2*x^13 + 120*(d*x)^m*b^2*c*m^5*x^9 +
120*(d*x)^m*a*c^2*m^5*x^9 + 11820*(d*x)^m*b*c^2*m^3*x^11 + 19524*(d*x)^m*c^
3*m*x^13 + (d*x)^m*b^3*m^6*x^7 + 6*(d*x)^m*a*b*c*m^6*x^7 + 1839*(d*x)^m*b^2
*c*m^4*x^9 + 1839*(d*x)^m*a*c^2*m^4*x^9 + 42117*(d*x)^m*b*c^2*m^2*x^11 + 10
395*(d*x)^m*c^3*x^13 + 42*(d*x)^m*b^3*m^5*x^7 + 252*(d*x)^m*a*b*c*m^5*x^7 +
 13584*(d*x)^m*b^2*c*m^3*x^9 + 13584*(d*x)^m*a*c^2*m^3*x^9 + 68706*(d*x)^m*
b*c^2*m*x^11 + 3*(d*x)^m*a*b^2*m^6*x^5 + 3*(d*x)^m*a^2*c*m^6*x^5 + 679*(d*x
)^m*b^3*m^4*x^7 + 4074*(d*x)^m*a*b*c*m^4*x^7 + 49881*(d*x)^m*b^2*c*m^2*x^9
+ 49881*(d*x)^m*a*c^2*m^2*x^9 + 36855*(d*x)^m*b*c^2*x^11 + 132*(d*x)^m*a*b^
2*m^5*x^5 + 132*(d*x)^m*a^2*c*m^5*x^5 + 5292*(d*x)^m*b^3*m^3*x^7 + 31752*(d
*x)^m*a*b*c*m^3*x^7 + 83064*(d*x)^m*b^2*c*m*x^9 + 83064*(d*x)^m*a*c^2*m*x^9
 + 3*(d*x)^m*a^2*b*m^6*x^3 + 2259*(d*x)^m*a*b^2*m^4*x^5 + 2259*(d*x)^m*a^2*
c*m^4*x^5 + 20335*(d*x)^m*b^3*m^2*x^7 + 122010*(d*x)^m*a*b*c*m^2*x^7 + 4504
5*(d*x)^m*b^2*c*x^9 + 45045*(d*x)^m*a*c^2*x^9 + 138*(d*x)^m*a^2*b*m^5*x^3 +
 18840*(d*x)^m*a*b^2*m^3*x^5 + 18840*(d*x)^m*a^2*c*m^3*x^5 + 34986*(d*x)^m*
b^3*m*x^7 + 209916*(d*x)^m*a*b*c*m*x^7 + (d*x)^m*a^3*m^6*x + 2505*(d*x)^m*a
^2*b*m^4*x^3 + 77937*(d*x)^m*a*b^2*m^2*x^5 + 77937*(d*x)^m*a^2*c*m^2*x^5 +
19305*(d*x)^m*b^3*x^7 + 115830*(d*x)^m*a*b*c*x^7 + 48*(d*x)^m*a^3*m^5*x + 2
2620*(d*x)^m*a^2*b*m^3*x^3 + 142308*(d*x)^m*a*b^2*m*x^5 + 142308*(d*x)^m*a^
2*c*m*x^5 + 925*(d*x)^m*a^3*m^4*x + 104277*(d*x)^m*a^2*b*m^2*x^3 + 81081*(d
*x)^m*a*b^2*x^5 + 81081*(d*x)^m*a^2*c*x^5 + 9120*(d*x)^m*a^3*m^3*x + 219162
*(d*x)^m*a^2*b*m*x^3 + 48259*(d*x)^m*a^3*m^2*x + 135135*(d*x)^m*a^2*b*x^3 +
 129072*(d*x)^m*a^3*m*x + 135135*(d*x)^m*a^3*x)/(m^7 + 49*m^6 + 973*m^5 + 1
0045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

maple [B] time = 0.01, size = 782, normalized size = 5.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] x*(c^3*m^6*x^12+36*c^3*m^5*x^12+3*b*c^2*m^6*x^10+505*c^3*m^4*x^12+114*b*c^2
*m^5*x^10+3480*c^3*m^3*x^12+3*a*c^2*m^6*x^8+3*b^2*c*m^6*x^8+1665*b*c^2*m^4*
x^10+12139*c^3*m^2*x^12+120*a*c^2*m^5*x^8+120*b^2*c*m^5*x^8+11820*b*c^2*m^3
*x^10+19524*c^3*m*x^12+6*a*b*c*m^6*x^6+1839*a*c^2*m^4*x^8+b^3*m^6*x^6+1839*
b^2*c*m^4*x^8+42117*b*c^2*m^2*x^10+10395*c^3*x^12+252*a*b*c*m^5*x^6+13584*a
*c^2*m^3*x^8+42*b^3*m^5*x^6+13584*b^2*c*m^3*x^8+68706*b*c^2*m*x^10+3*a^2*c*
m^6*x^4+3*a*b^2*m^6*x^4+4074*a*b*c*m^4*x^6+49881*a*c^2*m^2*x^8+679*b^3*m^4*
x^6+49881*b^2*c*m^2*x^8+36855*b*c^2*x^10+132*a^2*c*m^5*x^4+132*a*b^2*m^5*x^
```

```
4+31752*a*b*c*m^3*x^6+83064*a*c^2*m*x^8+5292*b^3*m^3*x^6+83064*b^2*c*m*x^8+
3*a^2*b*m^6*x^2+2259*a^2*c*m^4*x^4+2259*a*b^2*m^4*x^4+122010*a*b*c*m^2*x^6+
45045*a*c^2*x^8+20335*b^3*m^2*x^6+45045*b^2*c*x^8+138*a^2*b*m^5*x^2+18840*a
^2*c*m^3*x^4+18840*a*b^2*m^3*x^4+209916*a*b*c*m*x^6+34986*b^3*m*x^6+a^3*m^6
+2505*a^2*b*m^4*x^2+77937*a^2*c*m^2*x^4+77937*a*b^2*m^2*x^4+115830*a*b*c*x^
6+19305*b^3*x^6+48*a^3*m^5+22620*a^2*b*m^3*x^2+142308*a^2*c*m*x^4+142308*a*
b^2*m*x^4+925*a^3*m^4+104277*a^2*b*m^2*x^2+81081*a^2*c*x^4+81081*a*b^2*x^4+
9120*a^3*m^3+219162*a^2*b*m*x^2+48259*a^3*m^2+135135*a^2*b*x^2+129072*a^3*m
+135135*a^3)*(d*x)^m/(m+13)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.31, size = 195, normalized size = 1.25

$$\frac{c^3 d^m x^{13} x^m}{m+13} + \frac{3\, b c^2 d^m x^{11} x^m}{m+11} + \frac{3\, b^2 c d^m x^9 x^m}{m+9} + \frac{3\, a c^2 d^m x^9 x^m}{m+9} + \frac{b^3 d^m x^7 x^m}{m+7} + \frac{6\, abcd^m x^7 x^m}{m+7} + \frac{3\, ab^2 d^m x^5 x^m}{m+5} + \frac{3\, a^2 c d^m x^5 x^m}{m+5} + \frac{3\, a^2 b d^m x^3 x^m}{m+3} + \frac{(dx)^{m+1} a^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] c^3*d^m*x^13*x^m/(m + 13) + 3*b*c^2*d^m*x^11*x^m/(m + 11) + 3*b^2*c*d^m*x^9
*x^m/(m + 9) + 3*a*c^2*d^m*x^9*x^m/(m + 9) + b^3*d^m*x^7*x^m/(m + 7) + 6*a*
b*c*d^m*x^7*x^m/(m + 7) + 3*a*b^2*d^m*x^5*x^m/(m + 5) + 3*a^2*c*d^m*x^5*x^m
/(m + 5) + 3*a^2*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^3/(d*(m + 1))
```

mupad [B] time = 4.83, size = 546, normalized size = 3.50

$$\frac{a^3 x (dx)^m \left(m^6+48 m^5+925 m^4+9120 m^3+48259 m^2+129072 m+135135\right)}{m^7+49 m^6+973 m^5+10045 m^4+57379 m^3+177331 m^2+264207 m+135135} + \frac{c^3 x^{13} (dx)^m \left(m^6+36 m^5+505 m^4+3480 m^3+12139 m^2+19524 m+10395\right)}{m^7+49 m^6+973 m^5+10045 m^4+57379 m^3+177331 m^2+264207 m+135135} + \frac{3 a^2 b x^3 (dx)^m \left(m^6+46 m^5+835 m^4+7540 m^3+34759 m^2+73054 m+45045\right)}{m^7+49 m^6+973 m^5+10045 m^4+57379 m^3+177331 m^2+264207 m+135135} + \frac{3 b c^2 x^{11} (dx)^m \left(m^6+38 m^5+555 m^4+3940 m^3+14039 m^2+22902 m+12285\right)}{m^7+49 m^6+973 m^5+10045 m^4+57379 m^3+177331 m^2+264207 m+135135} + \frac{3 a x^5 (dx)^m \left(b^2+ac\right) \left(m^6+44 m^5+753 m^4+6280 m^3+25979 m^2+47436 m+27027\right)}{m^7+49 m^6+973 m^5+10045 m^4+57379 m^3+177331 m^2+264207 m+135135} + \frac{b x^7 (dx)^m \left(b^2+6ac\right) \left(m^6+42 m^5+679 m^4+5292 m^3+20335 m^2+34986 m+19305\right)}{m^7+49 m^6+973 m^5+10045 m^4+57379 m^3+177331 m^2+264207 m+135135} + \frac{3 c x^9 (dx)^m \left(b^2+ac\right) \left(m^6+40 m^5+613 m^4+4528 m^3+16627 m^2+27688 m+15015\right)}{m^7+49 m^6+973 m^5+10045 m^4+57379 m^3+177331 m^2+264207 m+135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (a^3*x*(d*x)^m*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 +
135135))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6
+ m^7 + 135135) + (c^3*x^13*(d*x)^m*(19524*m + 12139*m^2 + 3480*m^3 + 505*m
^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
+ 973*m^5 + 49*m^6 + m^7 + 135135) + (3*a^2*b*x^3*(d*x)^m*(73054*m + 34759*
m^2 + 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45045))/(264207*m + 177331*m^2 +
57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (3*b*c^2*x^11*(d
*x)^m*(22902*m + 14039*m^2 + 3940*m^3 + 555*m^4 + 38*m^5 + m^6 + 12285))/(2
64207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135
135) + (3*a*x^5*(d*x)^m*(a*c + b^2)*(47436*m + 25979*m^2 + 6280*m^3 + 753*m
^4 + 44*m^5 + m^6 + 27027))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
+ 973*m^5 + 49*m^6 + m^7 + 135135) + (b*x^7*(d*x)^m*(6*a*c + b^2)*(34986*m
+ 20335*m^2 + 5292*m^3 + 679*m^4 + 42*m^5 + m^6 + 19305))/(264207*m + 17733
1*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (3*c*x^9
*(d*x)^m*(a*c + b^2)*(27688*m + 16627*m^2 + 4528*m^3 + 613*m^4 + 40*m^5 + m
```

```
^6 + 15015))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*
m^6 + m^7 + 135135)
```

sympy [A] time = 7.27, size = 4451, normalized size = 28.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**3,x)

[Out] Piecewise(((-a**3/(12*x**12) - 3*a**2*b/(10*x**10) - 3*a**2*c/(8*x**8) - 3*
a*b**2/(8*x**8) - a*b*c/x**6 - 3*a*c**2/(4*x**4) - b**3/(6*x**6) - 3*b**2*c
/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*log(x))/d**13, Eq(m, -13)), ((-a**3/(1
0*x**10) - 3*a**2*b/(8*x**8) - a**2*c/(2*x**6) - a*b**2/(2*x**6) - 3*a*b*c/
(2*x**4) - 3*a*c**2/(2*x**2) - b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2
*log(x) + c**3*x**2/2)/d**11, Eq(m, -11)), ((-a**3/(8*x**8) - a**2*b/(2*x**
6) - 3*a**2*c/(4*x**4) - 3*a*b**2/(4*x**4) - 3*a*b*c/x**2 + 3*a*c**2*log(x)
 - b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4)/d**9, E
q(m, -9)), ((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a**2*c/(2*x**2) - 3*a*b
**2/(2*x**2) + 6*a*b*c*log(x) + 3*a*c**2*x**2/2 + b**3*log(x) + 3*b**2*c*x*
*2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/d**7, Eq(m, -7)), ((-a**3/(4*x**4) -
3*a**2*b/(2*x**2) + 3*a**2*c*log(x) + 3*a*b**2*log(x) + 3*a*b*c*x**2 + 3*a*
c**2*x**4/4 + b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8)/
d**5, Eq(m, -5)), ((-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a**2*c*x**2/2 + 3*
a*b**2*x**2/2 + 3*a*b*c*x**4/2 + a*c**2*x**6/2 + b**3*x**4/4 + b**2*c*x**6/
2 + 3*b*c**2*x**8/8 + c**3*x**10/10)/d**3, Eq(m, -3)), ((a**3*log(x) + 3*a*
*2*b*x**2/2 + 3*a**2*c*x**4/4 + 3*a*b**2*x**4/4 + a*b*c*x**6 + 3*a*c**2*x**
8/8 + b**3*x**6/6 + 3*b**2*c*x**8/8 + 3*b*c**2*x**10/10 + c**3*x**12/12)/d,
 Eq(m, -1)), (a**3*d**m*m**6*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**3*d**m*m**5*x*x**m
/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26420
7*m + 135135) + 925*a**3*d**m*m**4*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 1004
5*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9120*a**3*d**m*m**
3*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
 + 264207*m + 135135) + 48259*a**3*d**m*m**2*x*x**m/(m**7 + 49*m**6 + 973*m
**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 129072*a
**3*d**m*m*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17
7331*m**2 + 264207*m + 135135) + 135135*a**3*d**m*x*x**m/(m**7 + 49*m**6 +
973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a
**2*b*d**m*m**6*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m
**3 + 177331*m**2 + 264207*m + 135135) + 138*a**2*b*d**m*m**5*x**3*x**m/(m*
*7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
+ 135135) + 2505*a**2*b*d**m*m**4*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10
045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 22620*a**2*b*d**
m*m**3*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
```

331*m**2 + 264207*m + 135135) + 104277*a**2*b*d**m*m**2*x**3*x**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1351
35) + 219162*a**2*b*d**m*m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**2*b*d**m*x**3
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 3*a**2*c*d**m*m**6*x**5*x**m/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 132*a**2*c
*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 2259*a**2*c*d**m*m**4*x**5*x**m/(m**7 +
49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 18840*a**2*c*d**m*m**3*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045
*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 77937*a**2*c*d**m*m
2*x5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331
*m**2 + 264207*m + 135135) + 142308*a**2*c*d**m*m*x**5*x**m/(m**7 + 49*m**6
+ 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) +
81081*a**2*c*d**m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379
*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a*b**2*d**m*m**6*x**5*x**m/(m*
*7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
+ 135135) + 132*a*b**2*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 100
45*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 2259*a*b**2*d**m*
m**4*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 18840*a*b**2*d**m*m**3*x**5*x**m/(m**7 + 49*m
6 + 973*m5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 77937*a*b**2*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 142308*a*b**2*d**m*m*x**
5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 81081*a*b**2*d**m*x**5*x**m/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 6*a*b*c*d*
*m*m**6*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17
7331*m**2 + 264207*m + 135135) + 252*a*b*c*d**m*m**5*x**7*x**m/(m**7 + 49*m
6 + 973*m5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 4074*a*b*c*d**m*m**4*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 +
57379*m**3 + 177331*m**2 + 264207*m + 135135) + 31752*a*b*c*d**m*m**3*x**7
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 122010*a*b*c*d**m*m**2*x**7*x**m/(m**7 + 49*m**6 + 973
*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 209916
*a*b*c*d**m*m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**
3 + 177331*m**2 + 264207*m + 135135) + 115830*a*b*c*d**m*x**7*x**m/(m**7 +
49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135
135) + 3*a*c**2*d**m*m**6*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 120*a*c**2*d**m*m**5*x**
9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 1839*a*c**2*d**m*m**4*x**9*x**m/(m**7 + 49*m**6 + 973
*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 13584*
a*c**2*d**m*m**3*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*

m**3 + 177331*m**2 + 264207*m + 135135) + 49881*a*c**2*d**m*m**2*x**9*x**m/
(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207
*m + 135135) + 83064*a*c**2*d**m*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 1
0045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 45045*a*c**2*d*
*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*
m**2 + 264207*m + 135135) + b**3*d**m*m**6*x**7*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 42*b**3
*d**m*m**5*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
 177331*m**2 + 264207*m + 135135) + 679*b**3*d**m*m**4*x**7*x**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 5292*b**3*d**m*m**3*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 20335*b**3*d**m*m**2*x**7
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 34986*b**3*d**m*m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5
 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 19305*b**3*
d**m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 3*b**2*c*d**m*m**6*x**9*x**m/(m**7 + 49*m**6
+ 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 1
20*b**2*c*d**m*m**5*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573
79*m**3 + 177331*m**2 + 264207*m + 135135) + 1839*b**2*c*d**m*m**4*x**9*x**
m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 2642
07*m + 135135) + 13584*b**2*c*d**m*m**3*x**9*x**m/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 49881*b**2
*c*d**m*m**2*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3
 + 177331*m**2 + 264207*m + 135135) + 83064*b**2*c*d**m*m*x**9*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 45045*b**2*c*d**m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*b*c**2*d**m*m**6*x**11
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 114*b*c**2*d**m*m**5*x**11*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 1665*b*
c**2*d**m*m**4*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m
3 + 177331*m2 + 264207*m + 135135) + 11820*b*c**2*d**m*m**3*x**11*x**m/
(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207
*m + 135135) + 42117*b*c**2*d**m*m**2*x**11*x**m/(m**7 + 49*m**6 + 973*m**5
 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 68706*b*c**
2*d**m*m*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 36855*b*c**2*d**m*x**11*x**m/(m**7 + 49*
m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + c**3*d**m*m**6*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573
79*m**3 + 177331*m**2 + 264207*m + 135135) + 36*c**3*d**m*m**5*x**13*x**m/(
m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*
m + 135135) + 505*c**3*d**m*m**4*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10
045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3480*c**3*d**m*m
3*x13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733

```
1*m**2 + 264207*m + 135135) + 12139*c**3*d**m*m**2*x**13*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 19524*c**3*d**m*m*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57
379*m**3 + 177331*m**2 + 264207*m + 135135) + 10395*c**3*d**m*x**13*x**m/(m
**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
 + 135135), True))
```

3.857 $\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + ((b
^2 + 2*a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (2*b*c*(d*x)^(7 + m))/(d^7*(7 +
m)) + (c^2*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{2+m}}{d^2}+\frac{\left(b^2+2ac\right)(dx)^{4+m}}{d^4}+\frac{2bc(dx)^{6+m}}{d^6}+\frac{c^2(dx)^{8+m}}{d^8}\right) dx\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{\left(b^2+2ac\right)(dx)^{5+m}}{d^5(5+m)}+\frac{2bc(dx)^{7+m}}{d^7(7+m)}+\frac{c^2(dx)^{9+m}}{d^9(9+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.69

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{x^4\left(2ac+b^2\right)}{m+5}+\frac{2abx^2}{m+3}+\frac{2bcx^6}{m+7}+\frac{c^2x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + ((b^2 + 2*a*c)*x^4)/(5 + m)
+ (2*b*c*x^6)/(7 + m) + (c^2*x^8)/(9 + m))
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + bx^2 + cx^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.75, size = 241, normalized size = 2.39

$$\frac{\left(\left(c^2m^4 + 16\,c^2m^3 + 86\,c^2m^2 + 176\,c^2m + 105\,c^2\right)x^9 + 2\left(bcm^4 + 18\,bcm^3 + 104\,bcm^2 + 222\,bcm + 135\,bc\right)x^7 + \left(\left(b^2 + 2\,ac\right)m^4 + 20\left(b^2 + 2\,ac\right)m^3 + 130\left(b^2 + 2\,ac\right)m^2 + 189\,b^2 + 378\,ac + 300\left(b^2 + 2\,ac\right)m\right)x^5 + 2\left(abm^4 + 22\,abm^3 + 164\,abm^2 + 458\,abm + 315\,ab\right)x^3 + \left(a^2m^4 + 24\,a^2m^3 + 206\,a^2m^2 + 744\,a^2m + 945\,a^2\right)x\right)(dx)^m}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^9 + 2*(b*c*m^4
 + 18*b*c*m^3 + 104*b*c*m^2 + 222*b*c*m + 135*b*c)*x^7 + ((b^2 + 2*a*c)*m^4
 + 20*(b^2 + 2*a*c)*m^3 + 130*(b^2 + 2*a*c)*m^2 + 189*b^2 + 378*a*c + 300*(
b^2 + 2*a*c)*m)*x^5 + 2*(a*b*m^4 + 22*a*b*m^3 + 164*a*b*m^2 + 458*a*b*m + 3
15*a*b)*x^3 + (a^2*m^4 + 24*a^2*m^3 + 206*a^2*m^2 + 744*a^2*m + 945*a^2)*x)
*(d*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B] time = 0.18, size = 449, normalized size = 4.45

$$\frac{(dx)^m c^2m^4x^9 + 16\,(dx)^m c^2m^3x^9 + 2\,(dx)^m bcm^4x^7 + 86\,(dx)^m c^2m^2x^9 + 36\,(dx)^m bcm^3x^7 + 176\,(dx)^m c^2mx^9 + (dx)^m b^2m^4x^5 + 2\,(dx)^m acm^4x^5 + 208\,(dx)^m bcm^2x^7 + 105\,(dx)^m c^2x^9 + 20\,(dx)^m b^2m^3x^5 + 40\,(dx)^m acm^3x^5 + 444\,(dx)^m bcmx^7 + 2\,(dx)^m abm^4x^3 + 130\,(dx)^m b^2m^2x^5 + 260\,(dx)^m acm^2x^5 + 270\,(dx)^m bcx^7 + 44\,(dx)^m abm^3x^3 + 300\,(dx)^m b^2mx^5 + 600\,(dx)^m acmx^5 + (dx)^m a^2m^4x + 328\,(dx)^m abm^2x^3 + 189\,(dx)^m b^2x^5 + 378\,(dx)^m acx^5 + 24\,(dx)^m a^2m^3x + 916\,(dx)^m abmx^3 + 206\,(dx)^m a^2m^2x + 630\,(dx)^m abx^3 + 744\,(dx)^m a^2mx + 945\,(dx)^m a^2x}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^4*x^9 + 16*(d*x)^m*c^2*m^3*x^9 + 2*(d*x)^m*b*c*m^4*x^7 + 86*
(d*x)^m*c^2*m^2*x^9 + 36*(d*x)^m*b*c*m^3*x^7 + 176*(d*x)^m*c^2*m*x^9 + (d*x
)^m*b^2*m^4*x^5 + 2*(d*x)^m*a*c*m^4*x^5 + 208*(d*x)^m*b*c*m^2*x^7 + 105*(d*
x)^m*c^2*x^9 + 20*(d*x)^m*b^2*m^3*x^5 + 40*(d*x)^m*a*c*m^3*x^5 + 444*(d*x)^
m*b*c*m*x^7 + 2*(d*x)^m*a*b*m^4*x^3 + 130*(d*x)^m*b^2*m^2*x^5 + 260*(d*x)^m
*a*c*m^2*x^5 + 270*(d*x)^m*b*c*x^7 + 44*(d*x)^m*a*b*m^3*x^3 + 300*(d*x)^m*b
^2*m*x^5 + 600*(d*x)^m*a*c*m*x^5 + (d*x)^m*a^2*m^4*x + 328*(d*x)^m*a*b*m^2*
```

```
x^3 + 189*(d*x)^m*b^2*x^5 + 378*(d*x)^m*a*c*x^5 + 24*(d*x)^m*a^2*m^3*x + 91
6*(d*x)^m*a*b*m*x^3 + 206*(d*x)^m*a^2*m^2*x + 630*(d*x)^m*a*b*x^3 + 744*(d*
x)^m*a^2*m*x + 945*(d*x)^m*a^2*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*
m + 945)
```

maple [B] time = 0.01, size = 301, normalized size = 2.98

$$\frac{\left(c^2m^4x^8+16c^2m^3x^8+2bc\,m^4x^6+86c^2m^2x^8+36bc\,m^3x^6+176c^2m\,x^8+2ac\,m^4x^4+b^2m^4x^4+208bc\,m^2x^6+105c^2x^8+40ac\,m^3x^4+20b^2m^3x^4+444bcm\,x^6+2ab\,m^4x^2+260ac\,m^2x^4+130b^2m^2x^4+270bc\,x^6+44ab\,m^3x^2+600acm\,x^4+300b^2m\,x^4+a^2m^4+328ab\,m^2x^2+378ac\,x^4+189b^2x^4+24a^2m^3+916abm\,x^2+206a^2m^2+630ab\,x^2+744a^2m+945a^2\right)x\left(dx\right)^m}{\left(m+9\right)\left(m+7\right)\left(m+5\right)\left(m+3\right)\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(c^2*m^4*x^8+16*c^2*m^3*x^8+2*b*c*m^4*x^6+86*c^2*m^2*x^8+36*b*c*m^3*x^6+1
76*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+208*b*c*m^2*x^6+105*c^2*x^8+40*a*c*m
^3*x^4+20*b^2*m^3*x^4+444*b*c*m*x^6+2*a*b*m^4*x^2+260*a*c*m^2*x^4+130*b^2*m
^2*x^4+270*b*c*x^6+44*a*b*m^3*x^2+600*a*c*m*x^4+300*b^2*m*x^4+a^2*m^4+328*a
*b*m^2*x^2+378*a*c*x^4+189*b^2*x^4+24*a^2*m^3+916*a*b*m*x^2+206*a^2*m^2+630
*a*b*x^2+744*a^2*m+945*a^2)*(d*x)^m/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.11, size = 110, normalized size = 1.09

$$\frac{c^2d^mx^9x^m}{m+9}+\frac{2\,bcd^mx^7x^m}{m+7}+\frac{b^2d^mx^5x^m}{m+5}+\frac{2\,acd^mx^5x^m}{m+5}+\frac{2\,abd^mx^3x^m}{m+3}+\frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x^9*x^m/(m + 9) + 2*b*c*d^m*x^7*x^m/(m + 7) + b^2*d^m*x^5*x^m/(m +
5) + 2*a*c*d^m*x^5*x^m/(m + 5) + 2*a*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*
a^2/(d*(m + 1))
```

mupad [B] time = 4.58, size = 260, normalized size = 2.57

$$(d\,x)^m\,\left(\frac{c^2\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{x^5\,\left(b^2+2\,a\,c\right)\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{a^2\,x\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{2\,a\,b\,x^3\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{2\,b\,c\,x^7\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (d*x)^m*((c^2*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2
+ 230*m^3 + 25*m^4 + m^5 + 945) + (x^5*(2*a*c + b^2)*(300*m + 130*m^2 + 20*
m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (a^2*
x*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 230*m^3 + 25*
m^4 + m^5 + 945) + (2*a*b*x^3*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689
```

```
*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (2*b*c*x^7*(222*m + 104*m^2
+ 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))
```

sympy [A] time = 2.87, size = 1486, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) -
b*c/x**2 + c**2*log(x))/d**9, Eq(m, -9)), ((-a**2/(6*x**6) - a*b/(2*x**4) -
 a*c/x**2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/d**7, Eq(m, -7)), (
(-a**2/(4*x**4) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x
**4/4)/d**5, Eq(m, -5)), ((-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*
x**2/2 + b*c*x**4/2 + c**2*x**6/6)/d**3, Eq(m, -3)), ((a**2*log(x) + a*b*x*
*2 + a*c*x**4/2 + b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8)/d, Eq(m, -1)), (a
**2*d**m*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 24*a**2*d**m*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 206*a**2*d**m*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 744*a**2*d**m*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 945*a**2*d**m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 2*a*b*d**m*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 44*a*b*d**m*m**3*x**3*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 328*a*b*d**m*m**2*x**3*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a*b*d**m*m*x**3*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a*b*d**m*x**3*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*a*c*d**m*m**4*x**5
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 40*a*c*d**m*m
**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 260*a
*c*d**m*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 600*a*c*d**m*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 378*a*c*d**m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 16
89*m + 945) + b**2*d**m*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 20*b**2*d**m*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 130*b**2*d**m*m**2*x**5*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 300*b**2*d**m*m*x**5*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 189*b**2*d**m*x**5*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*b*c*d**m*m**4*x**7*x**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 36*b*c*d**m*m**3*
x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 208*b*c*d
**m*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
444*b*c*d**m*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 270*b*c*d**m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + c**2*d**m*m**4*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
```

```
1689*m + 945) + 16*c**2*d**m*m**3*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + 86*c**2*d**m*m**2*x**9*x**m/(m**5 + 25*m**4 + 230*
m**3 + 950*m**2 + 1689*m + 945) + 176*c**2*d**m*m*x**9*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*c**2*d**m*x**9*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

3.858 $\int (dx)^m \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(3 + m))/(d^3*(3 + m)) + (c*(d*x)^
(5 + m))/(d^5*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int (dx)^m \left(a+bx^2+cx^4\right) \, dx &= \int \left(a(dx)^m+\frac{b(dx)^{2+m}}{d^2}+\frac{c(dx)^{4+m}}{d^4}\right) \, dx \\ &= \frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{3+m}}{d^3(3+m)}+\frac{c(dx)^{5+m}}{d^5(5+m)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.67

$$x(dx)^m \left(\frac{a}{m+1}+\frac{bx^2}{m+3}+\frac{cx^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + (b*x^2)/(3 + m) + (c*x^4)/(5 + m))
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+bx^2+cx^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 0.79, size = 71, normalized size = 1.37

$$\frac{\left(\left(cm^2+4\,cm+3\,c\right)x^5+\left(bm^2+6\,bm+5\,b\right)x^3+\left(am^2+8\,am+15\,a\right)x\right)(dx)^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 4*c*m + 3*c)*x^5 + (b*m^2 + 6*b*m + 5*b)*x^3 + (a*m^2 + 8*a*m + 1
5*a)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

giac [B] time = 0.16, size = 119, normalized size = 2.29

$$\frac{(dx)^m\,cm^2x^5+4\,(dx)^m\,cmx^5+(dx)^m\,bm^2x^3+3\,(dx)^m\,cx^5+6\,(dx)^m\,bmx^3+(dx)^m\,am^2x+5\,(dx)^m\,bx^3+8\,(dx)^m\,amx+15\,(dx)^m\,ax}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m^2*x^5 + 4*(d*x)^m*c*m*x^5 + (d*x)^m*b*m^2*x^3 + 3*(d*x)^m*c*x^
5 + 6*(d*x)^m*b*m*x^3 + (d*x)^m*a*m^2*x + 5*(d*x)^m*b*x^3 + 8*(d*x)^m*a*m*x
 + 15*(d*x)^m*a*x)/(m^3 + 9*m^2 + 23*m + 15)
```

maple [A] time = 0.00, size = 78, normalized size = 1.50

$$\frac{\left(c\,m^2x^4+4cm\,x^4+b\,m^2x^2+3c\,x^4+6bm\,x^2+a\,m^2+5b\,x^2+8am+15a\right)x\,(dx)^m}{(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(c*m^2*x^4+4*c*m*x^4+b*m^2*x^2+3*c*x^4+6*b*m*x^2+a*m^2+5*b*x^2+8*a*m+15*a
)*(d*x)^m/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.09, size = 50, normalized size = 0.96

$$\frac{cd^m x^5 x^m}{m+5} + \frac{bd^m x^3 x^m}{m+3} + \frac{(dx)^{m+1}\, a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] c*d^m*x^5*x^m/(m + 5) + b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [B] time = 4.40, size = 89, normalized size = 1.71

$$(d\,x)^m\left(\frac{b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}+\frac{c\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{a\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4),x)
```

```
[Out] (d*x)^m*((b*x^3*(6*m + m^2 + 5))/(23*m + 9*m^2 + m^3 + 15) + (c*x^5*(4*m +
m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (a*x*(8*m + m^2 + 15))/(23*m + 9*m^2
+ m^3 + 15))
```

sympy [A] time = 0.99, size = 314, normalized size = 6.04

$$\begin{cases} \frac{-\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x)}{d^5} & \text{for}\ m=-5 \\ \frac{-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}}{d^3} & \text{for}\ m=-3 \\ \frac{a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}}{d} & \text{for}\ m=-1 \\ \frac{ad^m m^2 x x^m}{m^3+9m^2+23m+15}+\frac{8ad^m m x x^m}{m^3+9m^2+23m+15}+\frac{15ad^m x x^m}{m^3+9m^2+23m+15}+\frac{bd^m m^2 x^3 x^m}{m^3+9m^2+23m+15}+\frac{6bd^m m x^3 x^m}{m^3+9m^2+23m+15}+\frac{5bd^m x^3 x^m}{m^3+9m^2+23m+15}+\frac{cd^m m^2 x^5 x^m}{m^3+9m^2+23m+15}+\frac{4cd^m m x^5 x^m}{m^3+9m^2+23m+15}+\frac{3cd^m x^5 x^m}{m^3+9m^2+23m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise((((-a/(4*x**4) - b/(2*x**2) + c*log(x))/d**5, Eq(m, -5)), ((-a/(2*
x**2) + b*log(x) + c*x**2/2)/d**3, Eq(m, -3)), ((a*log(x) + b*x**2/2 + c*x*
*4/4)/d, Eq(m, -1)), (a*d**m*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a*
d**m*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a*d**m*x*x**m/(m**3 + 9*m**2
 + 23*m + 15) + b*d**m*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 6*b*d**
m*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*b*d**m*x**3*x**m/(m**3 + 9*m*
*2 + 23*m + 15) + c*d**m*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*c*d
**m*m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*c*d**m*x**5*x**m/(m**3 + 9*
m**2 + 23*m + 15), True))
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
```

```
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```